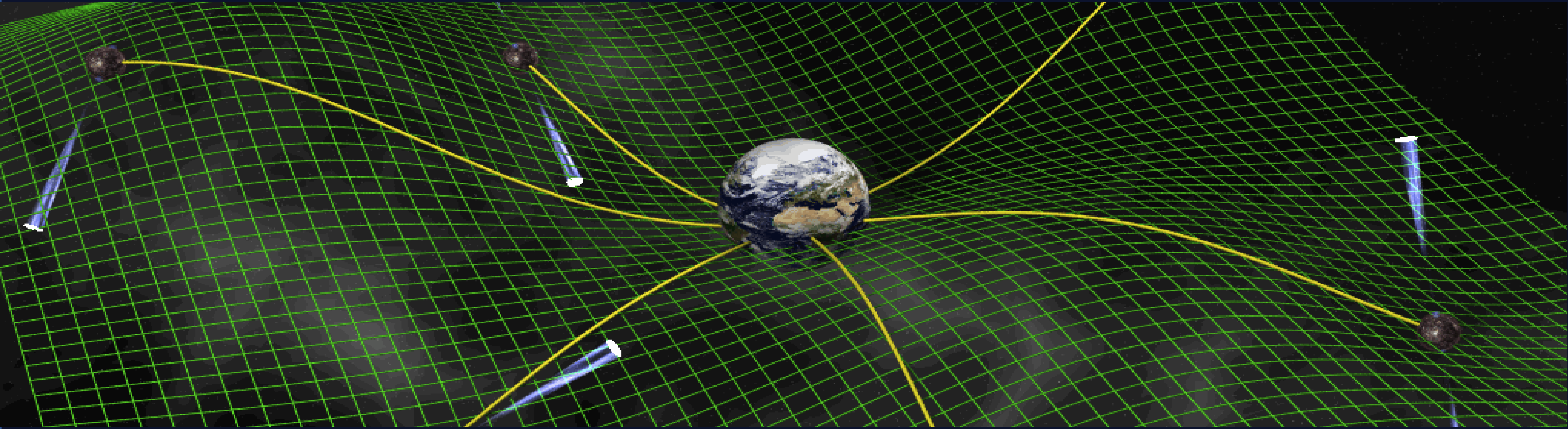
Search for a gravitational-wave background by the European Pulsar Timing Array: Examination of a common red signal

FAST/Future Pulsar Science 10 (FPS10) – July 13 2021,
Jinan/China



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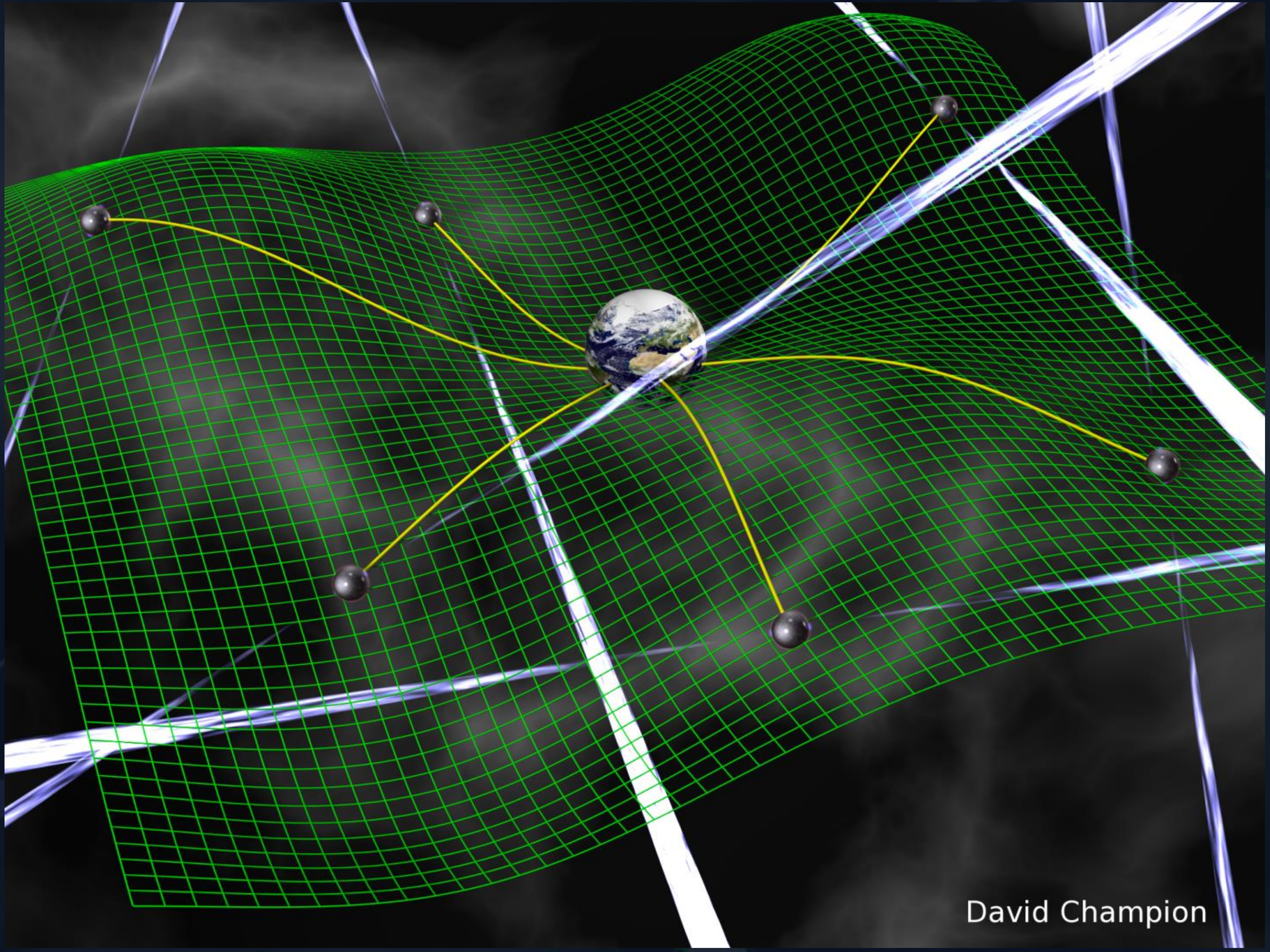
Today’s talk:

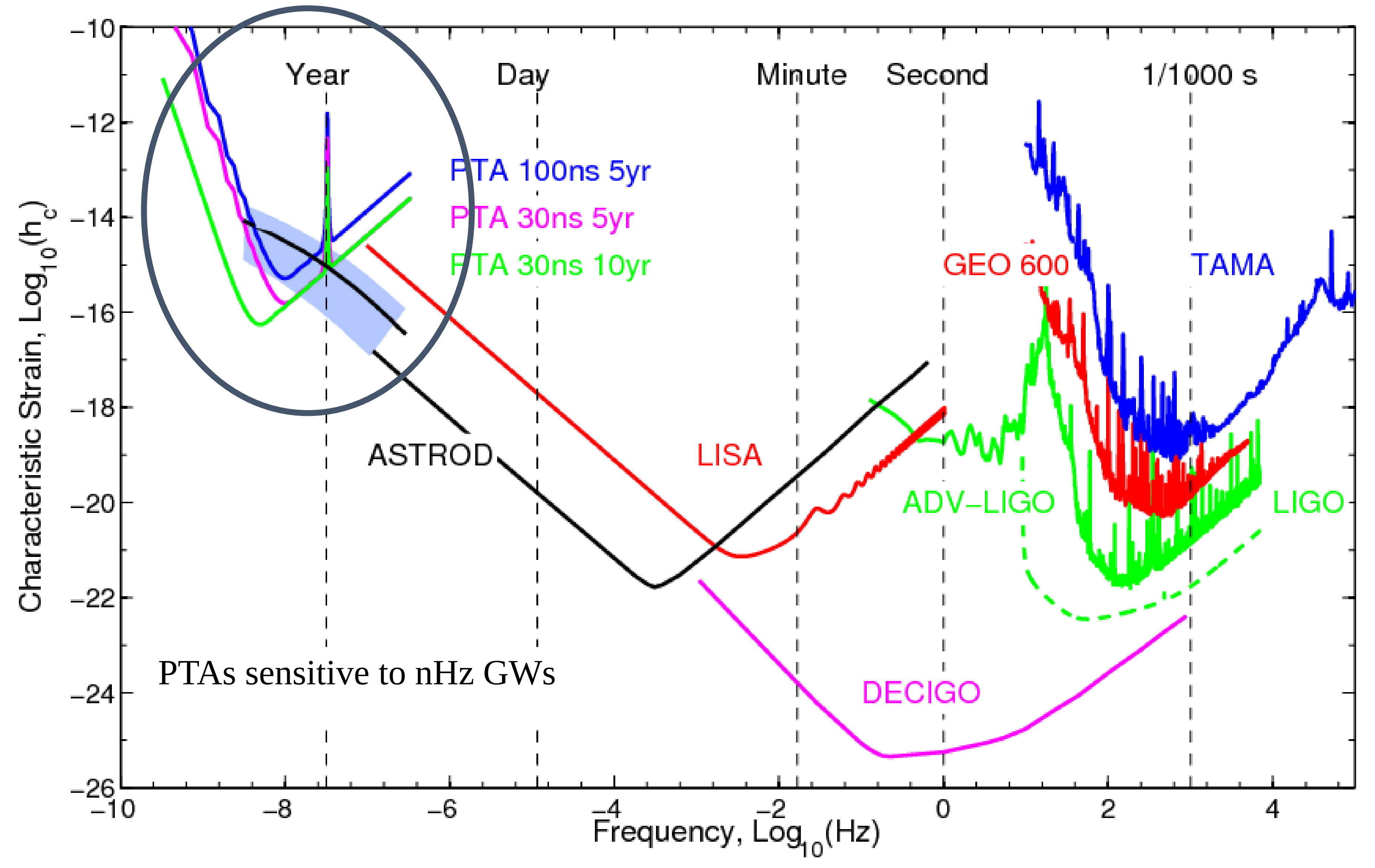
- Analysis of a common red signal (CRS) in European Pulsar Timing Data (EPTA)
- Results from Chen, Caballero + EPTA (in prep.)
- Credits to everyone in the EPTA
- Results based on Bayesian analysis.
  Some results verified by frequentist analyses (not discussed)

Pulsar Timing Arrays as GW detectors

PTA = Array of pulsars, at different sky locations



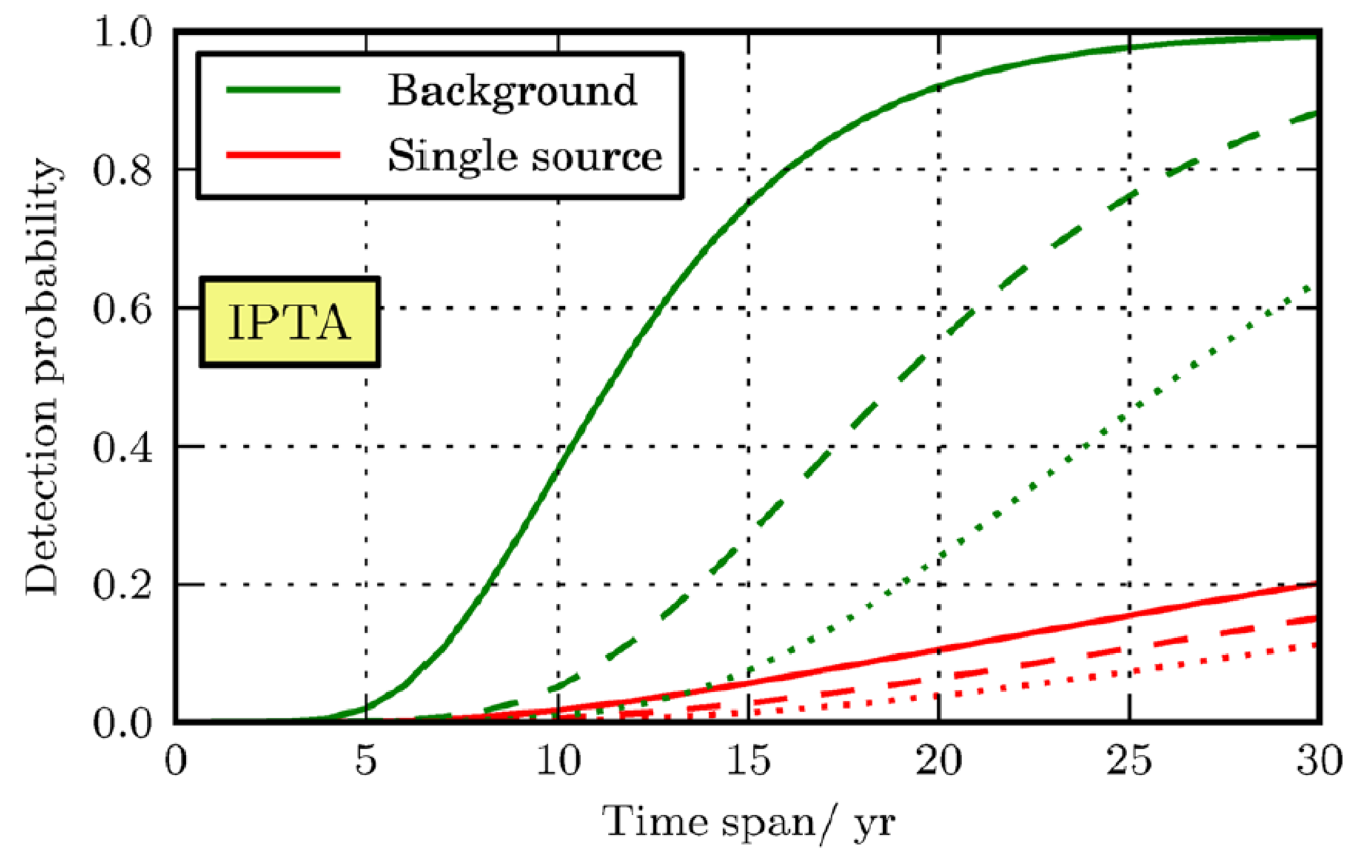
Pulsar Timing Arrays: GW sensitivity



PTAs sensitive to nHz GWs

Astrophysical sources:
Supermassive black-hole binaries (SMBHBs)

Rosado, Sesana & Gair (2015)



The GWB search parameters

(eg Maggiore et al. 2000; Phinney 2001)

Spectral index

Dimensionless strain

$$h_c(f) = A\left(\frac{f}{f_r}\right)^{\alpha} \Rightarrow S(f) = \frac{A^2}{12\pi^2}\left(\frac{f}{f_r}\right)^{-\gamma}$$

$$\gamma \equiv 3 - 2\alpha$$

Strain amplitude($f_r = \mathrm{yr}^{-1}$)

TOA Power Spectral density

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Strain amplitude(f_r = yr^{-1})

TOA Power Spectral density

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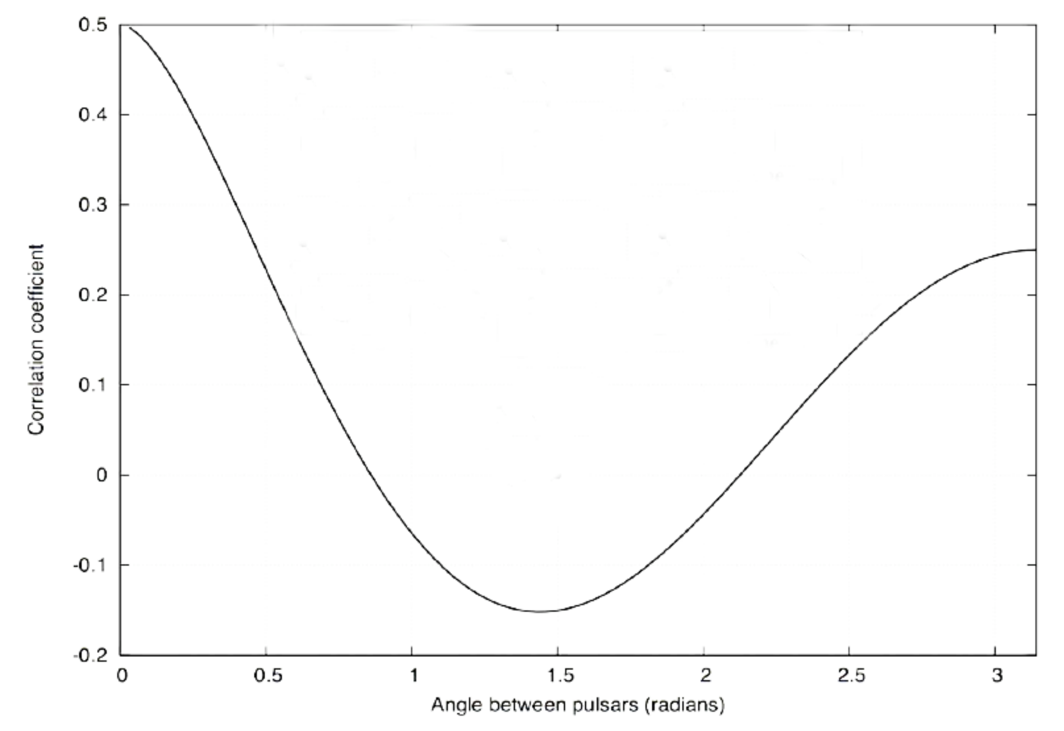
Overlap Reduction Function (ORF)
GR: Hellings-Downs Curve
(Helligs & Downs 1983)



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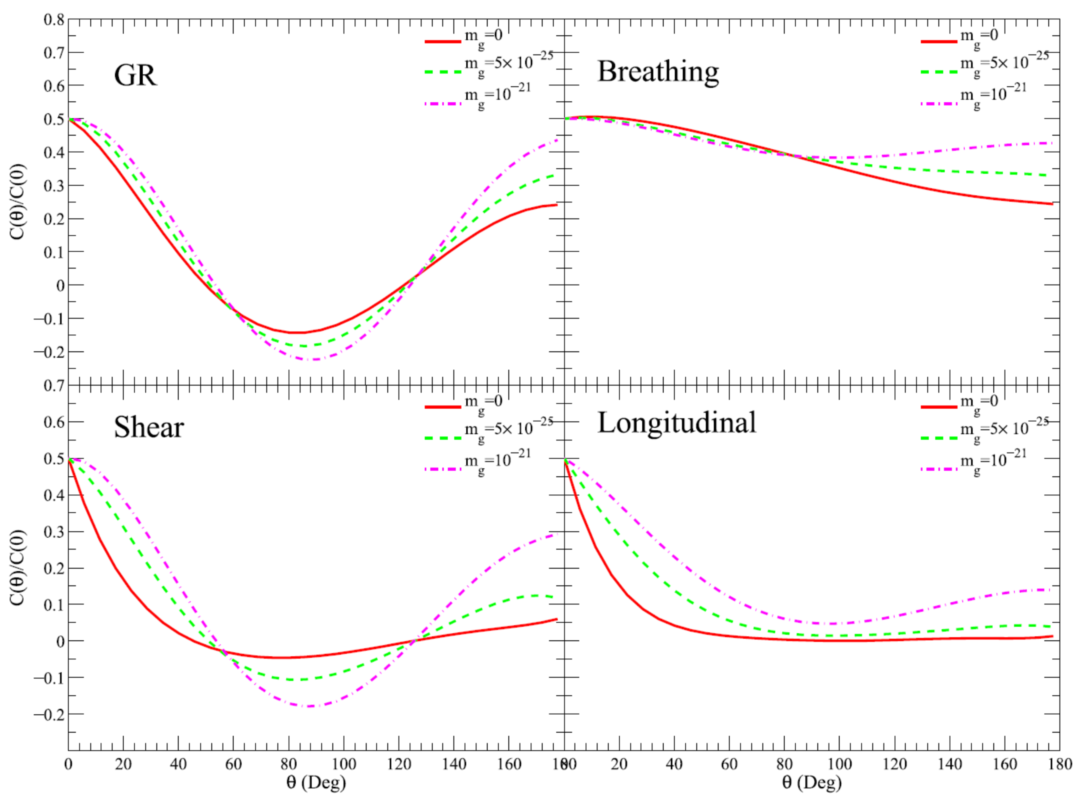
Dimensionless strain

Strain amplitude(f_r = yr^{-1})

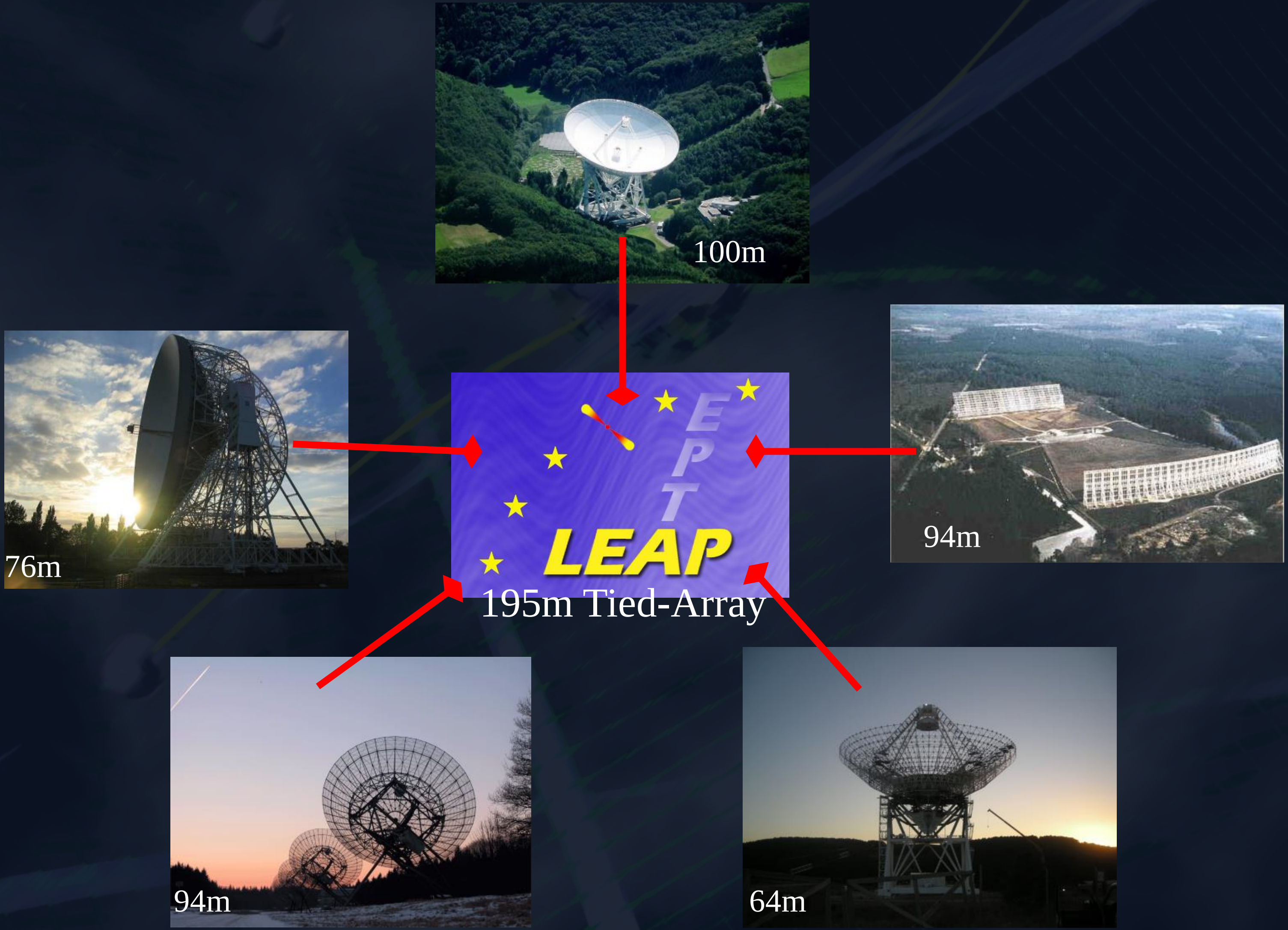
TOA Power Spectral density

$$\gamma \equiv 3 - 2\alpha$$

ORF depends on gravity theory!
(Fig.: Lee 2013)



The European Pulsar Timing Array (EPTA)



24-yr of high-precision EPTA data!

GWB search with the EPTA - 2014/15 analysis

- 2nd GWB strain upper limit (6 MSPs): Lentati et al. 2015: $A<3\times10^{-15}$ for $\gamma=13/3$ but already having signs of some "common red signal" (CRS)

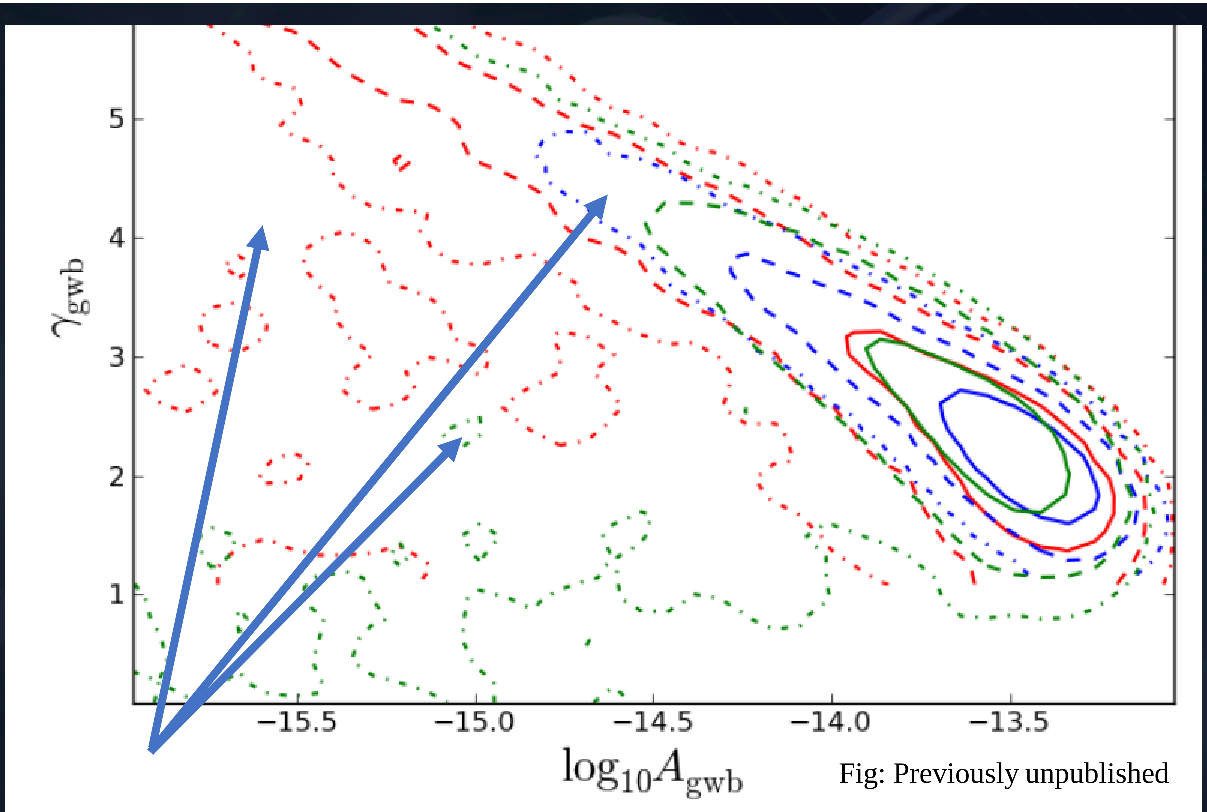
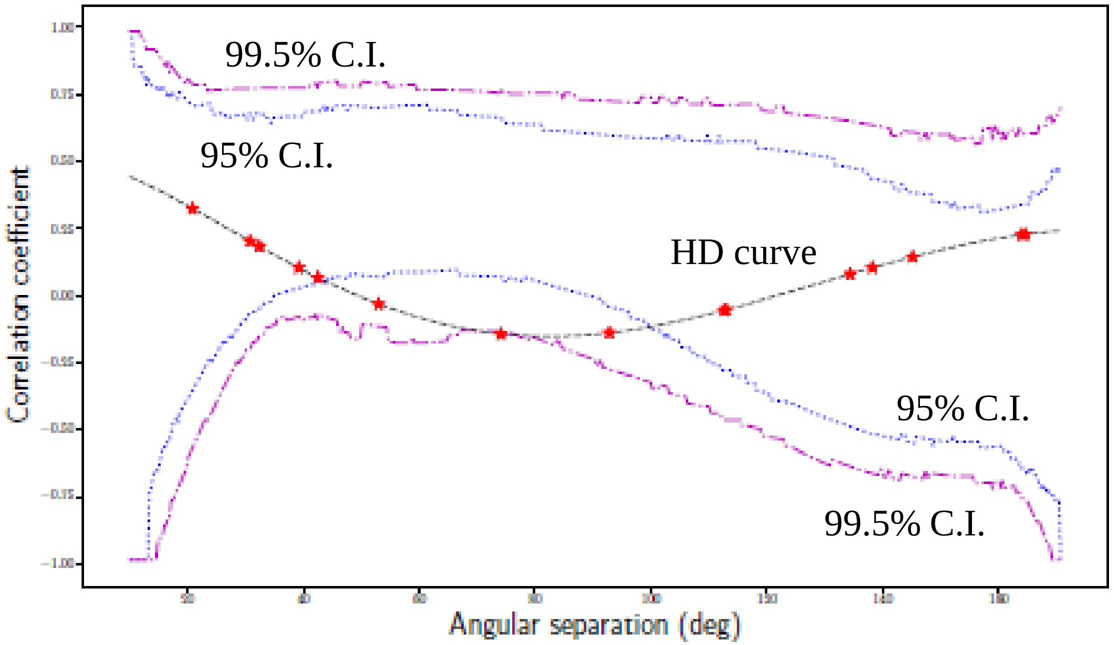


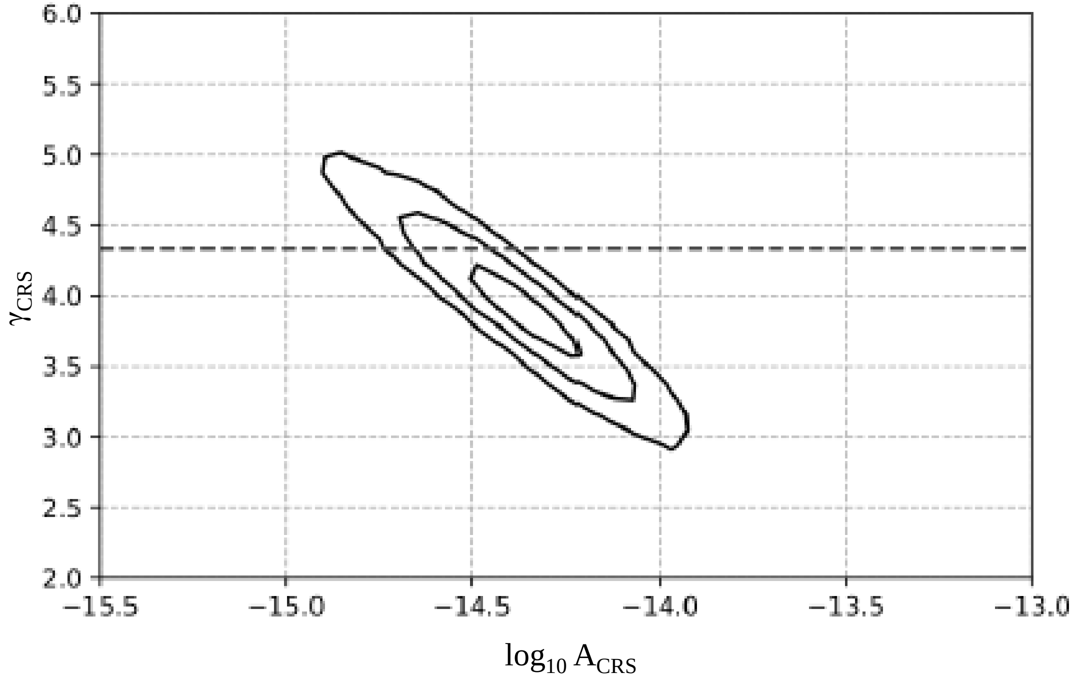
Fig: Previously unpublished

- EPTA decision = reanalyze after adding more+better data
- Better = more precision, wider bandwidth, better polarization calibration

EPTA DR2 search for a “CRS”

Chen, Caballero+EPTA (in prep.)





- Hellings-Downs curve still acceptable, but flat ORF also fits
- We compare evidence for different (physically motivated) models

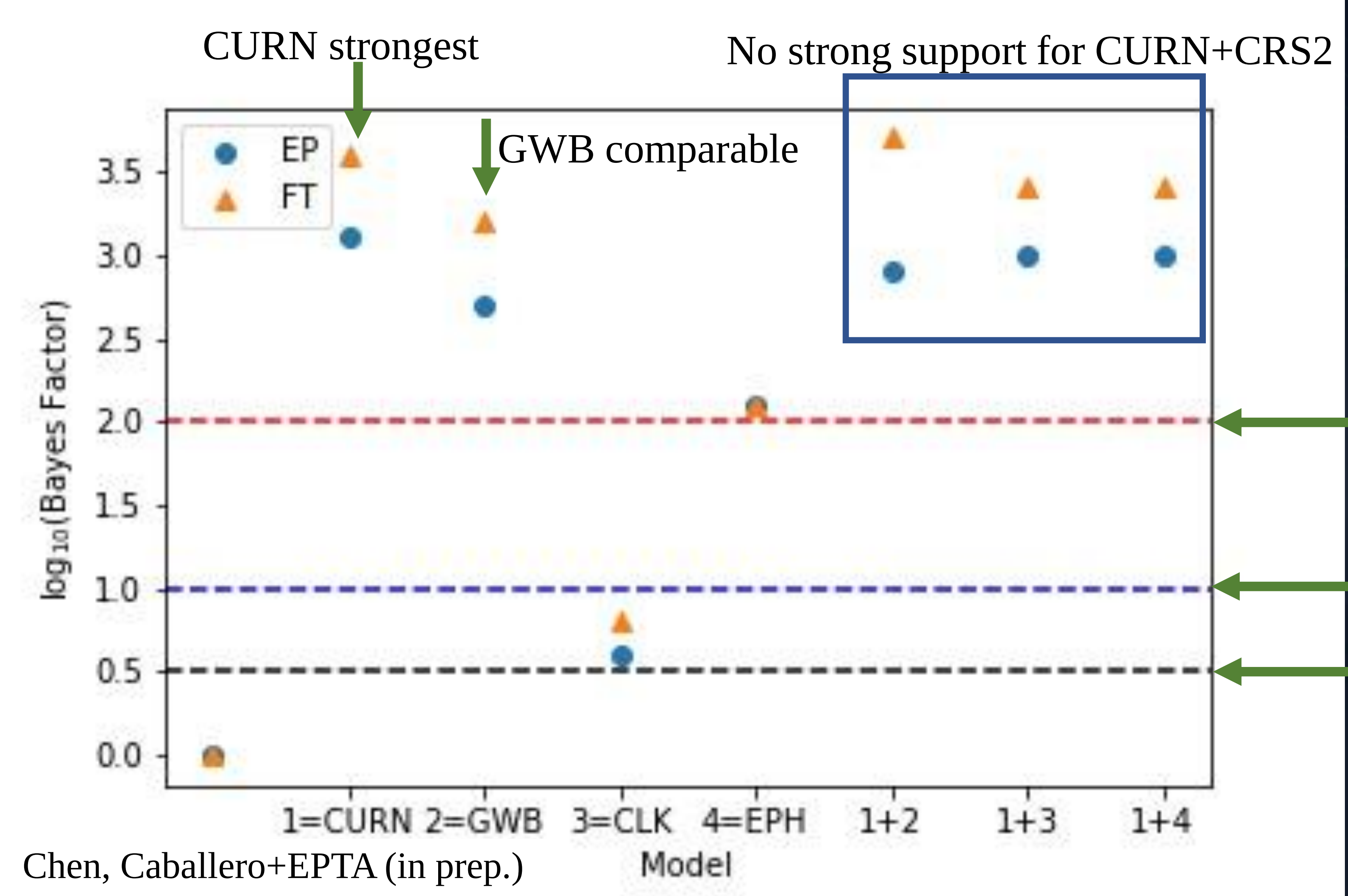
What else can the CRS be?

1) **Clock-error noise (CLK)** ← Inaccuracy in terrestrial time standard.
   Effect: Exact same TOA shifts in all pulsars ➔ common signal with *monopolar* ORF
2) **Solar-system ephemeris noise (EPH)** ← Inaccuracy in planetary parameters (mass, orbital elements) in the used SSE.
   Effect: Oscillation in time of calculated SSB position ➔ common signal with *dipolar* ORF
3) **Common Uncorrelated Red Noise (CURN)** ← Individual red noise in MSPs with similar spectral properties.
   Effect: An apparent CRS with correlation coefficients consistent with zero
   Demonstrated clearly using simulations; PPTA: Goncharov et al. (submitted)

↓

Used Bayes Factor analysis to investigated data support of CRS models

What can the CRS be?



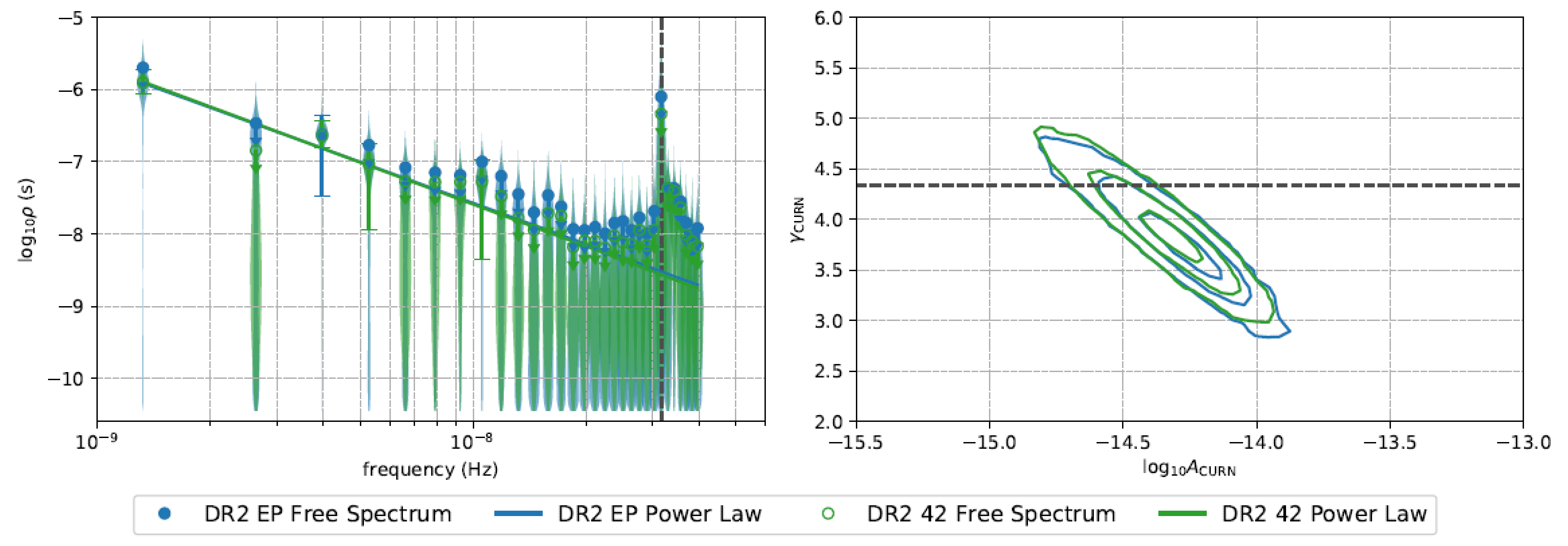
Chen, Caballero+EPTA (in prep.)

‘decisive’

‘strong’

‘substantial’

CURN – Spectrum model

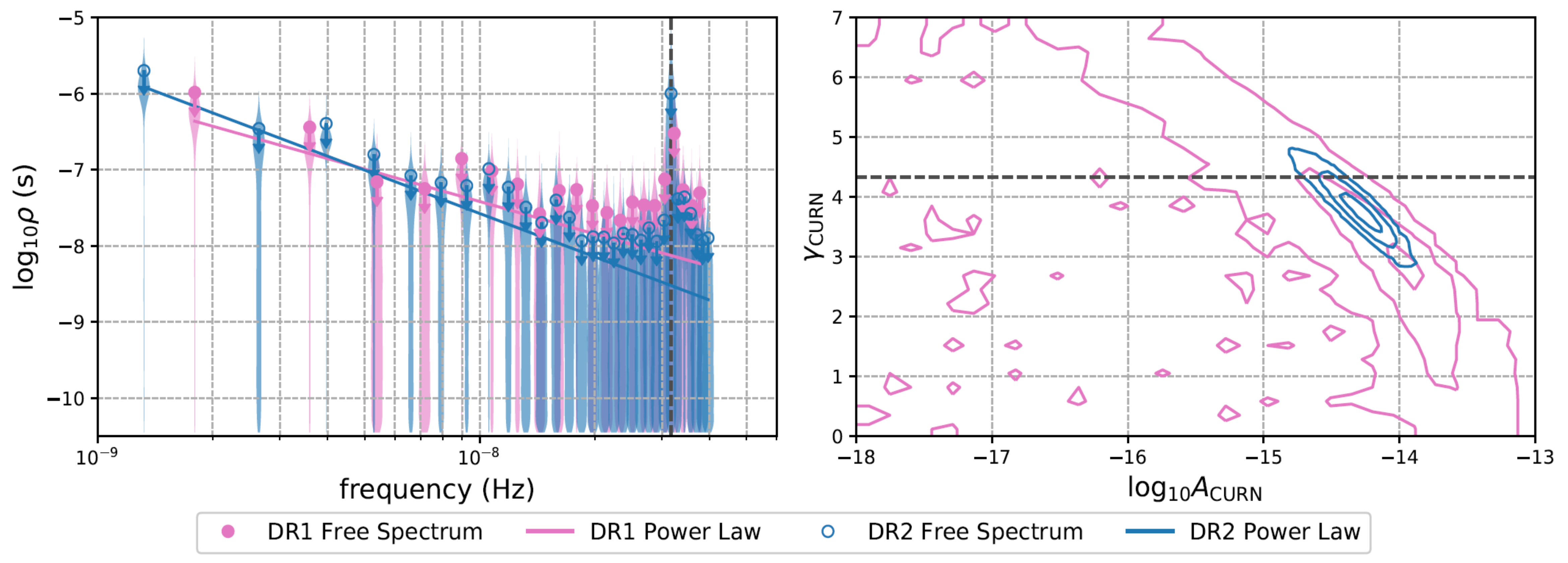


Chen, Caballero+EPTA (in prep.)

$$A = 5.13^{+4.20}_{-2.73} \times 10^{-15} \quad , \quad \gamma = 3.78^{+0.69}_{-0.59}$$

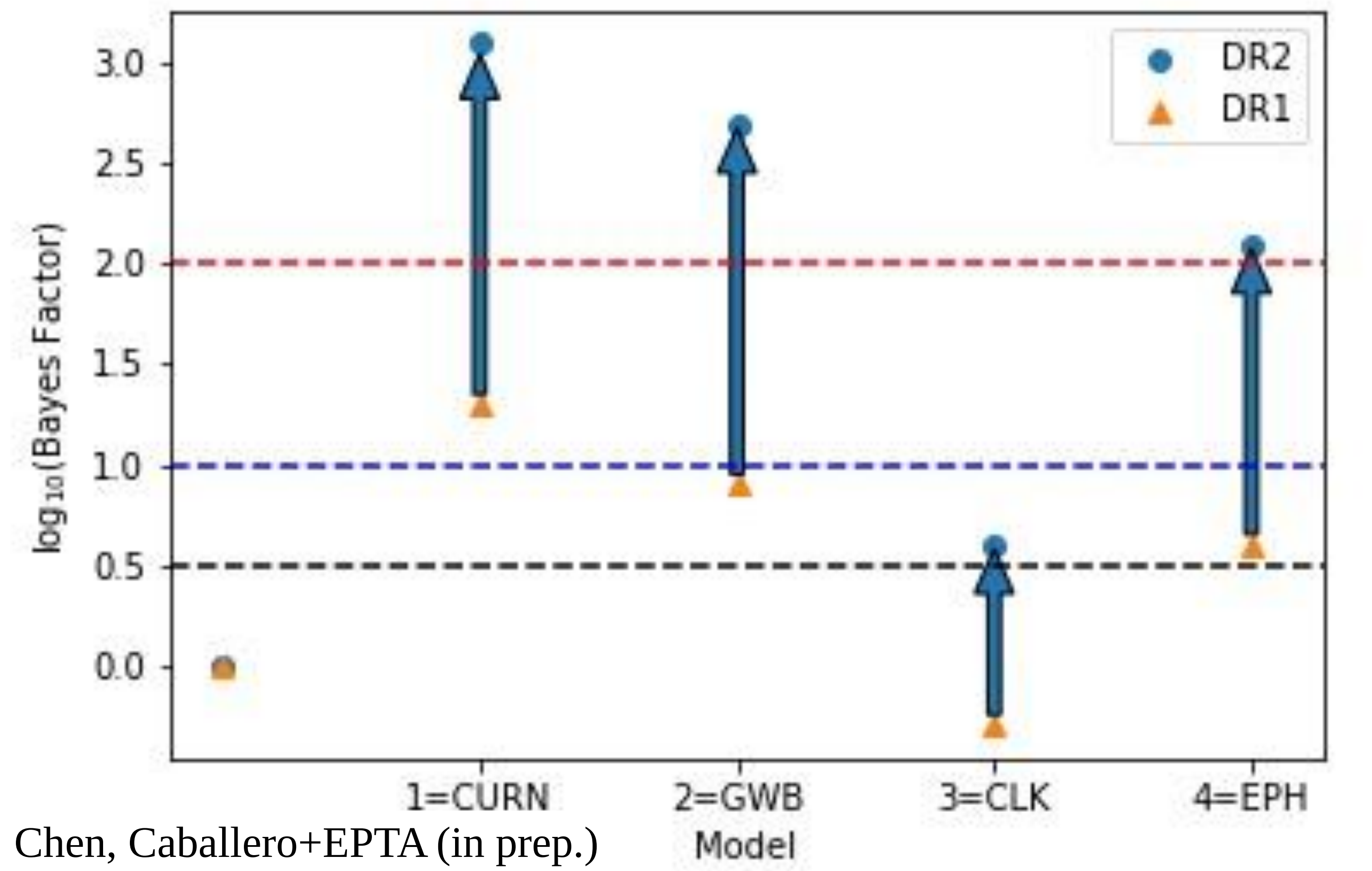
$A = 2.95^{+0.89}_{-0.72} \times 10^{-15}$ for fixed $\gamma = 13/3$

CURN – DR2/DR1 spectrum consistency



Chen, Caballero+EPTA (in prep.)

CURN – DR2/DR1 model selection consistency



Chen, Caballero+EPTA (in prep.)

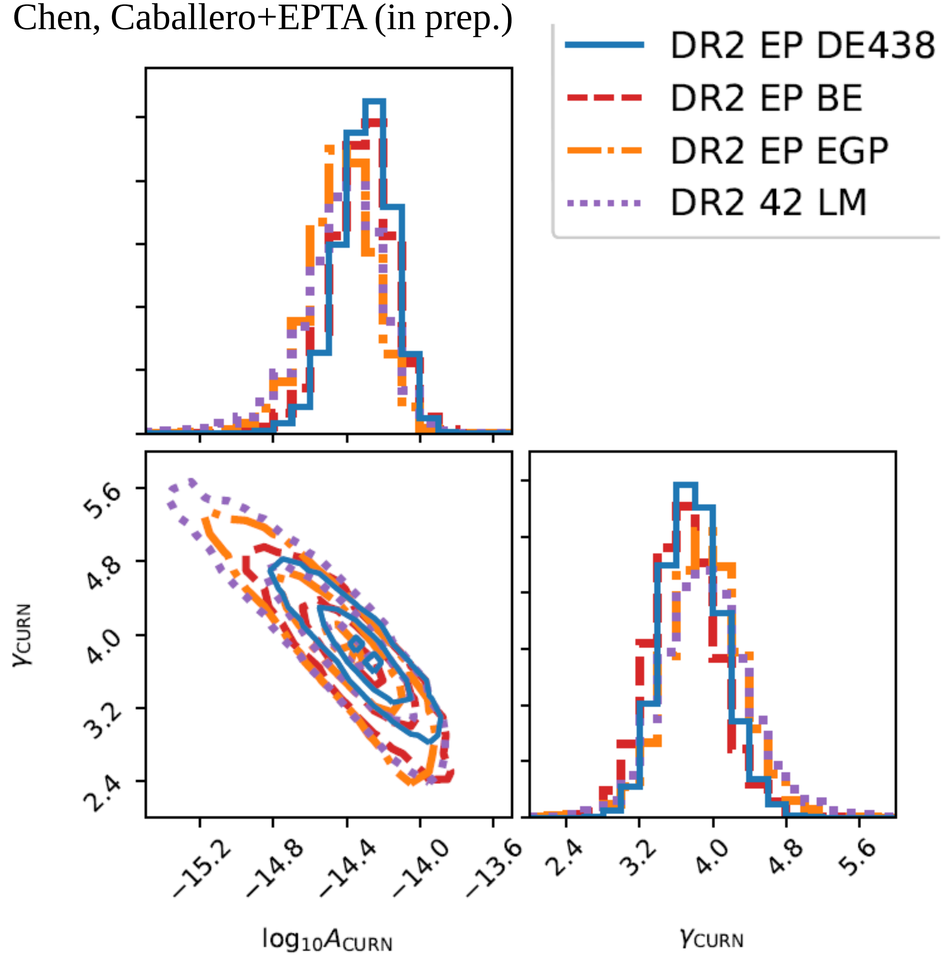
CURN – SSE effects (parameter estimation)

Used 3 algorithms in different ways to gauge multiple insights.

Algorithm	Planet Mass	Planet Orbit	Fit
BayesEphem	JUP,SAT,URAN,NEP	JUP,SAT	MC Sample
LINIMOSS	JUP,SAT	JUP,SAT	Analytical Marg.
EphemGP	-	JUP,SAT	MC Sample

- General agreement
- Marginal evidence (log10BF=0.4) for need of SSE parameter fitting
- Broader priors = broader spectral parameter distributions
- Effect seems mostly by orbital elements

Chen, Caballero+EPTA (in prep.)



CURN – Spectrum in context

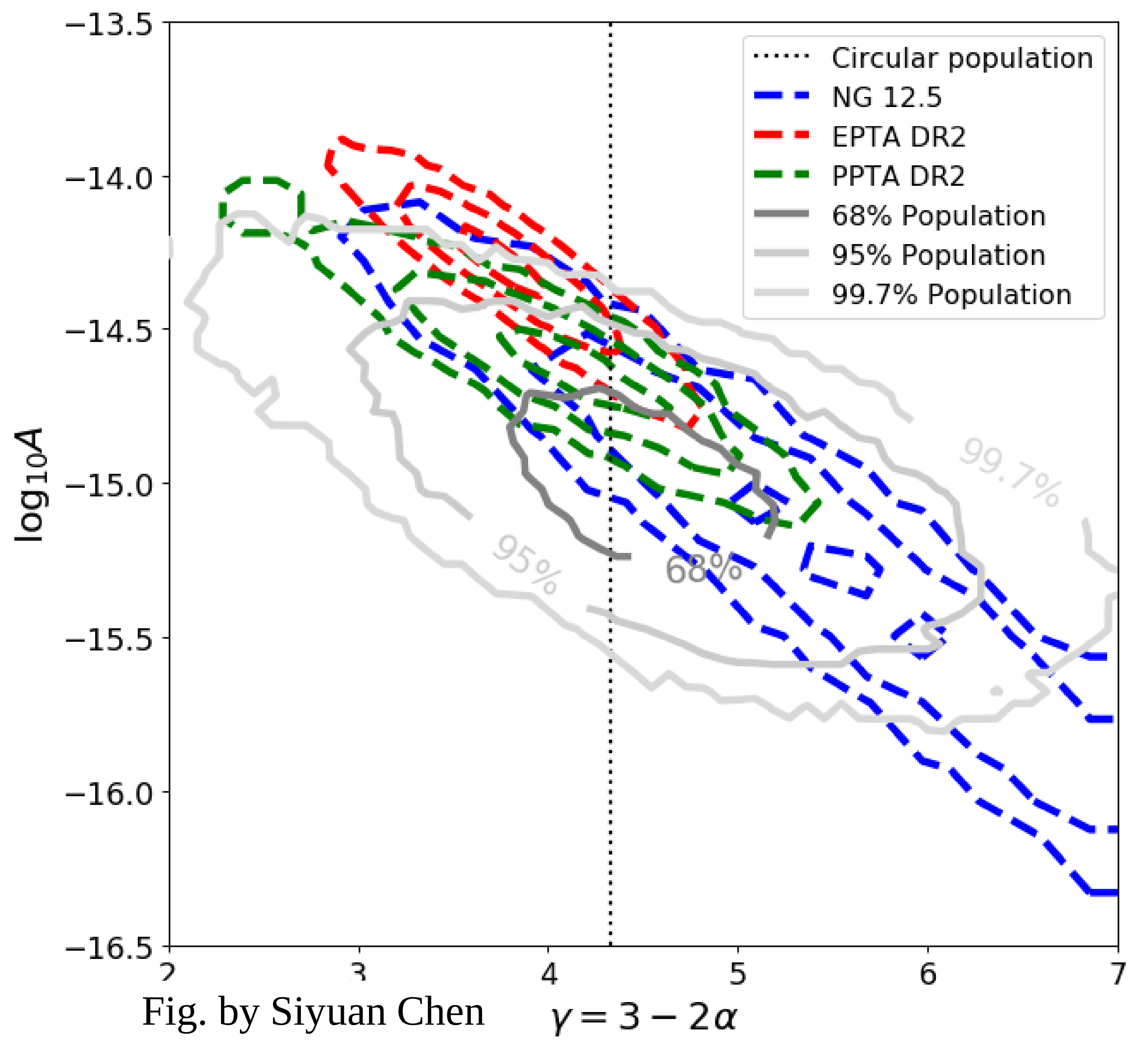


Fig. by Siyuan Chen

EPTA DR2: *Chen, Caballero et al (in prep.)*
PPTA DR2: *Goncharov et al. (submitted)*
NANOGrav 12.5: *Arzoumanian et al. (2020)*

SMBHB GWB theoretical contours:
As in *Middleton et al. (2021)*
Based on cosmological simulations from
Rosado, Sesana & Gair (2015).
{{234000 simulated Universes;
only GW-driven, circular SMBHB inspirals;
ΛCDM Universe}}

CURN – Spectrum in context

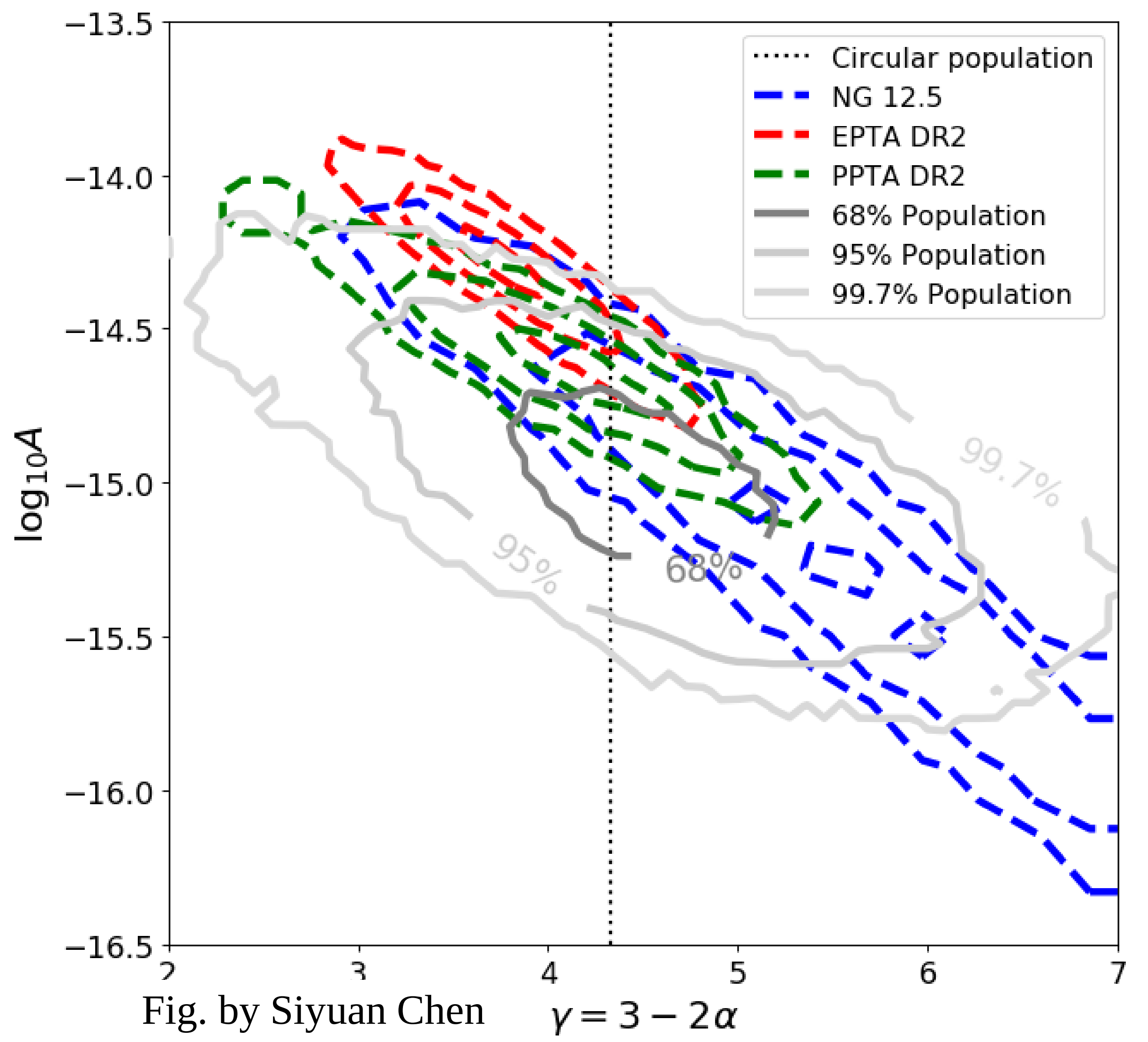


Fig. by Siyuan Chen

- General Agreement among PTAs
- Careful with 2-parameter spectrum model; depends on number of fitted frequency bins and data timespan
- Under the **ASSUMPTION** that the CRS is a stochastic GWB: Observational-Theoretical agreement remains (careful with model prediction)
- PTAs can start informing SMBHB population models {{eg *Middleton et al. (2021)*}}

Immediate next steps (EPTA)

- Finalize EPTA DR2 phase 2
- Identified 25+ priority MSPs based on estimated contribution to ORF measurement improvement
- More advance noise models, incl. possible data systematics (standard noise models used)
- Prepare codes for demanding 25+ pulsar analysis
- INVESTIGATING: Dependence of results on
  - Statistical methods
  - Pulsar noise modelling (GW detector noise characterization)
  - Details of CRS spectrum modelling
  - Estimation of detection significance
  - Data quality/systematics

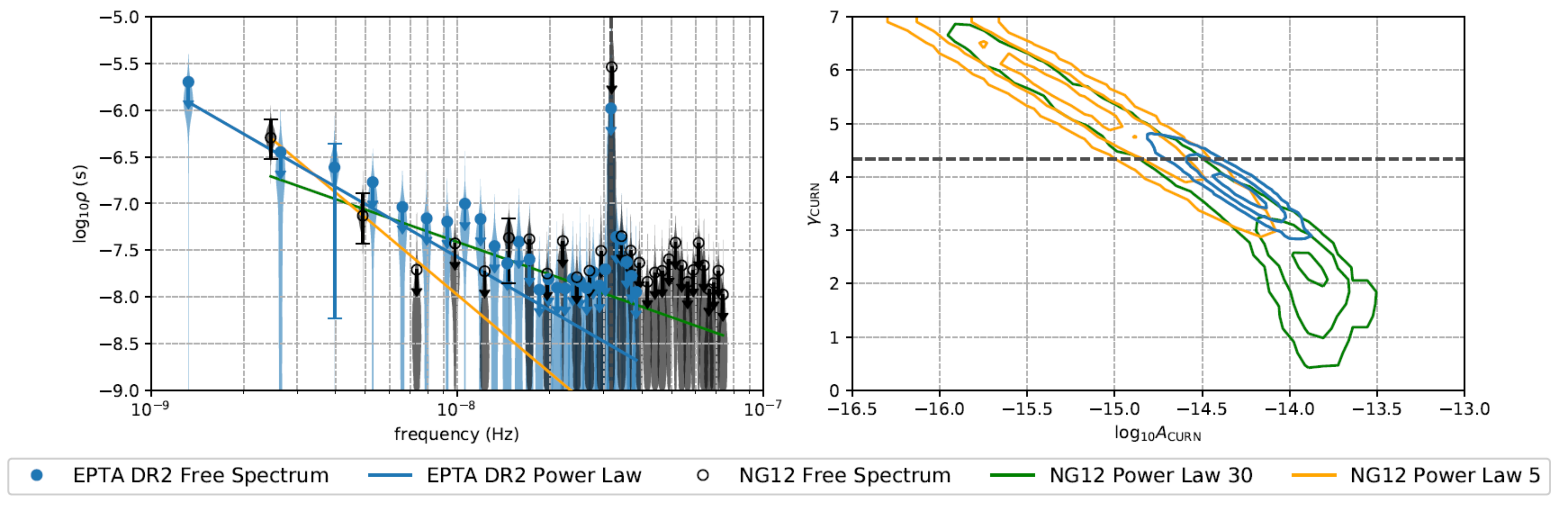
Summary

- EPTA DR2(6-pulsar) data show a common red signal; **NO** significant Hellings-Downs curve measurement.
- EPTA 25+ pulsar analysis in preparation
- EPTA, PPTA, NANOGrav, all have independent evidence for a Common Red Signal.
  Differences in data → some parameter variations; overall statistically consistent results
  Spectral properties still compatible with SMBHB GWB model predictions.
  ORF still compatible with Helling-Downs curve
- Details of analysis methods, effects on results are being examined

Thanks for your attention!

Extra

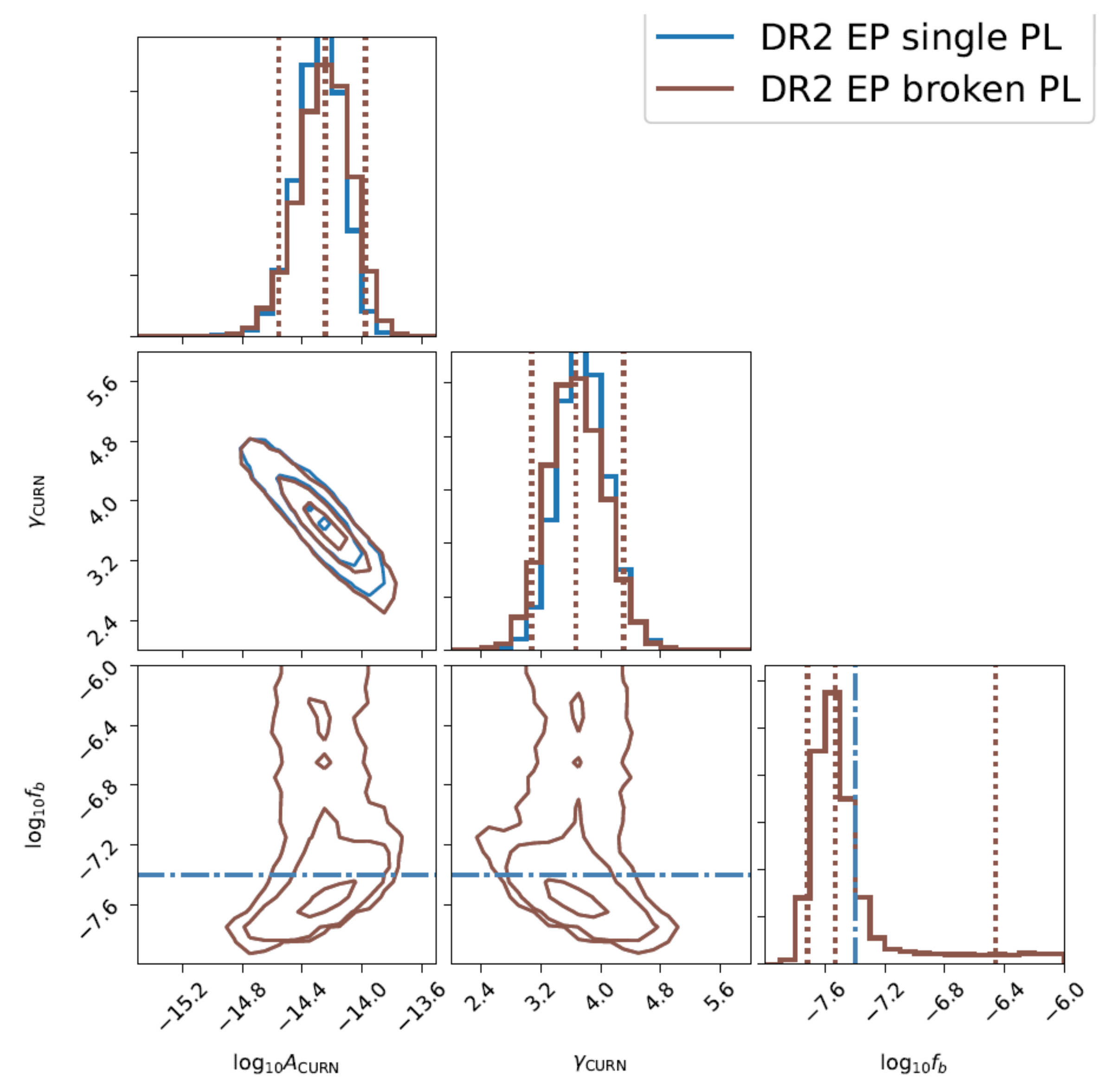
CURN – Effects of number of fitted freq. bins



NANOGrav results from
Arzoumanian+2020

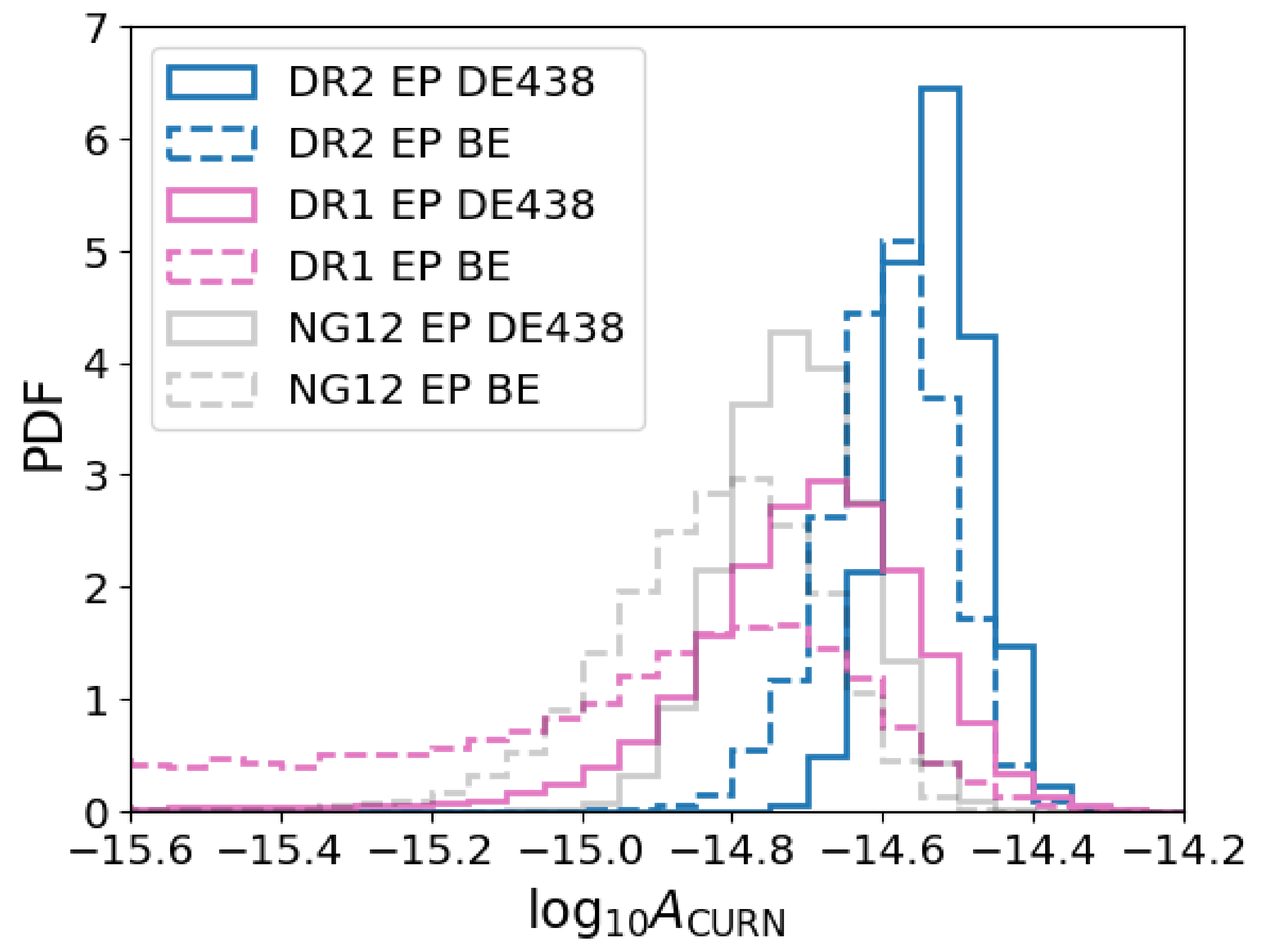
Number of freq. bins fitted can matter!

Broken PL – bend frequency

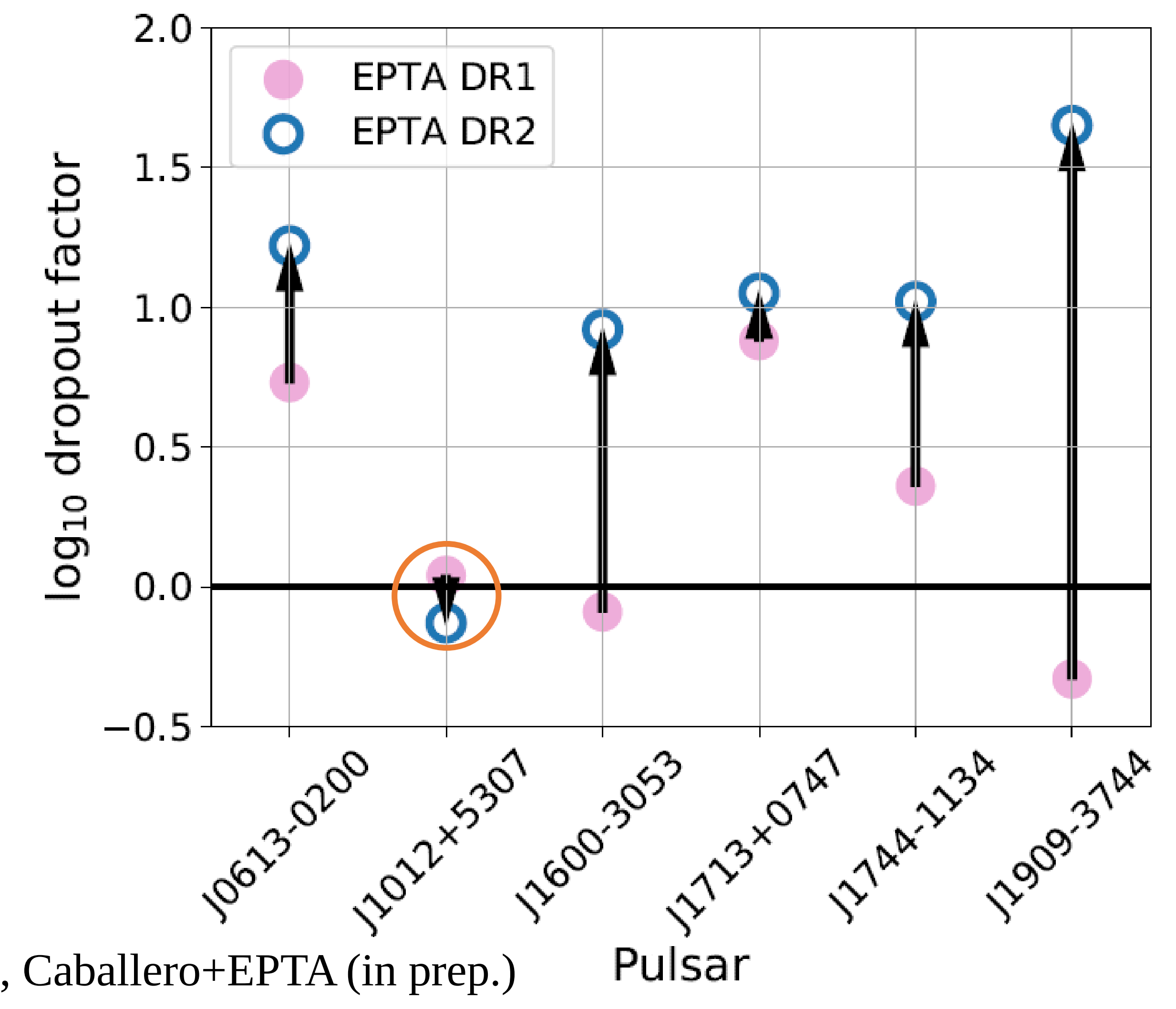


Parameters stabilize after ~10 bins

γ=13/3 – DR1-DR2-NG12: Timespan effect?

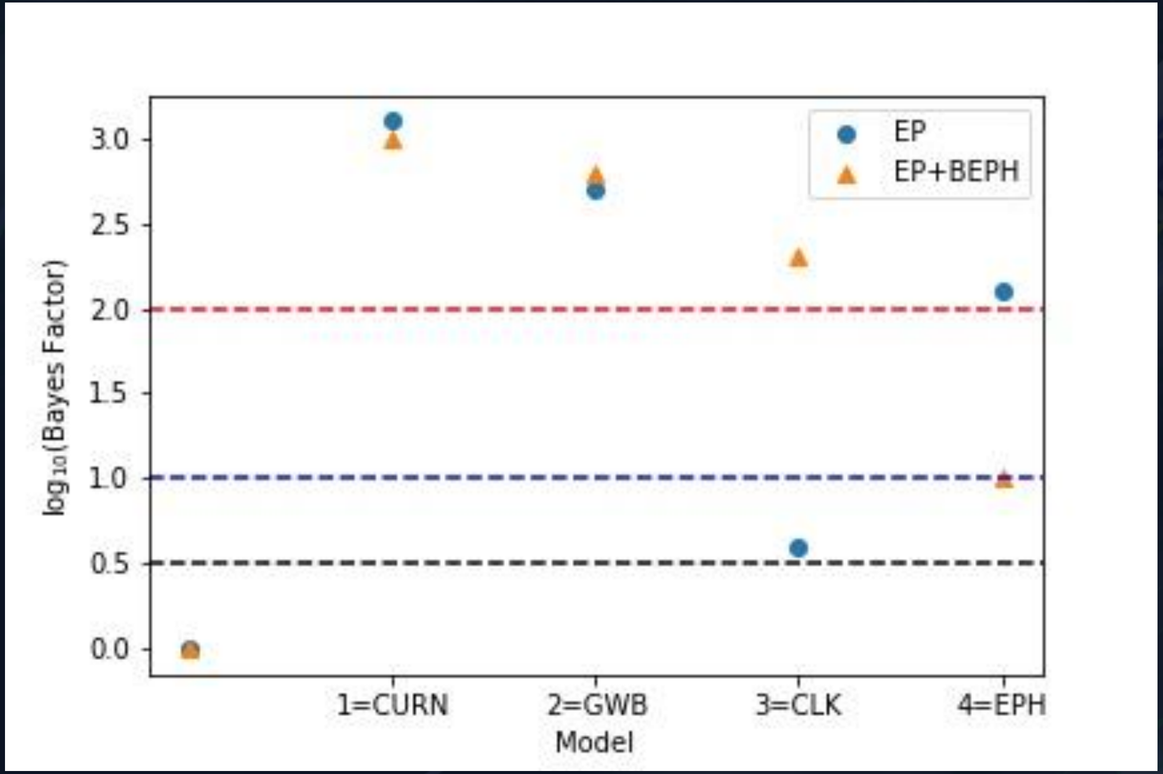


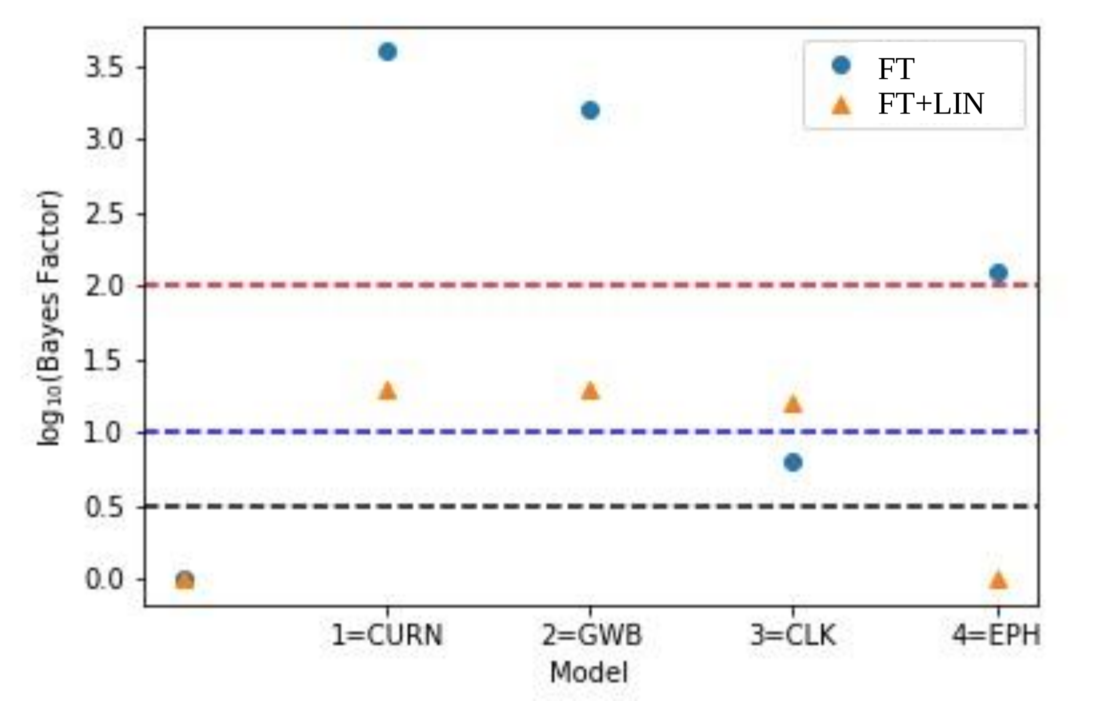
CURN –DR2/DR1 pulsar contribution consistency



Chen, Caballero+EPTA (in prep.)

CURN – DR2/DR1 model selection consistency





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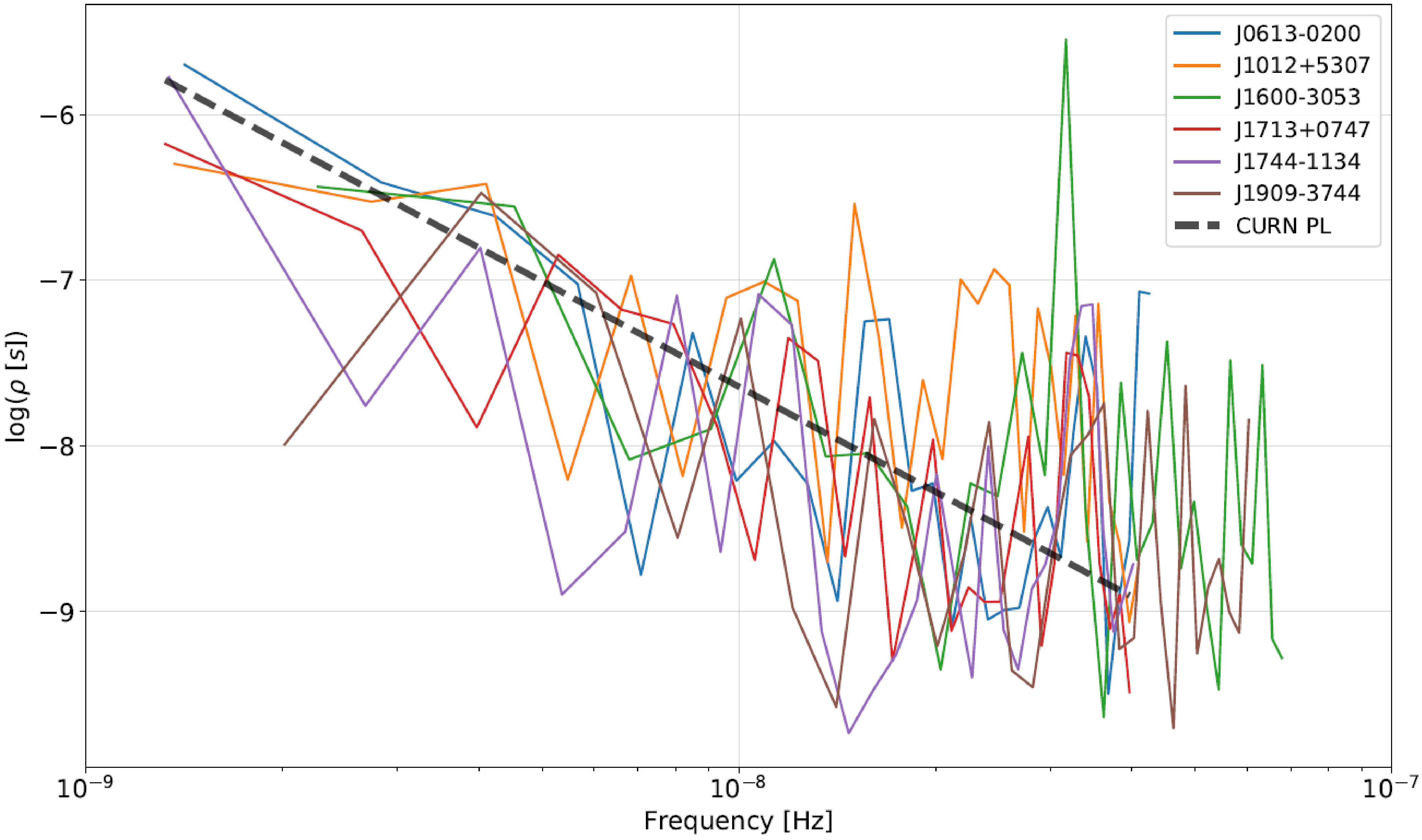
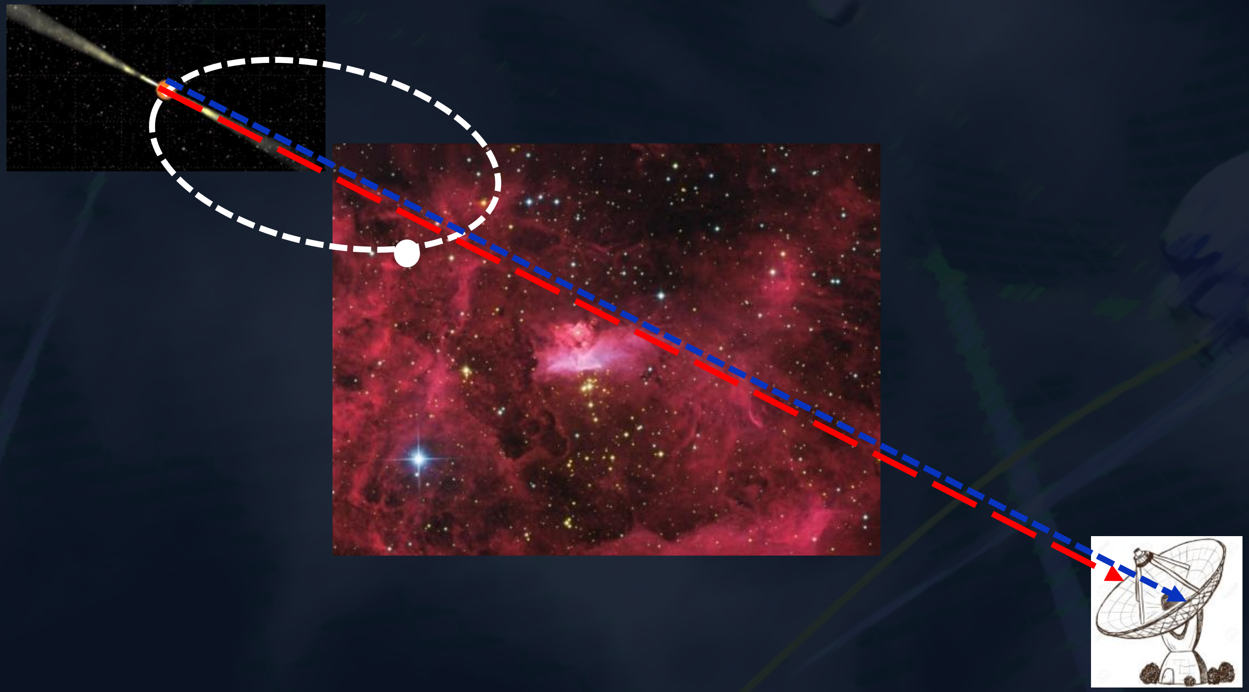

J0613-0200
J1012+5307
J1600-3053
J1713+0747
J1744-1134
J1909-3744
CURN PL
log(ρ [s])
−6
−7
−8
−9
10^{-9}
10^{-8}
10^{-7}
Frequency [Hz]

Pulsar Timing & Timing Models



TIMING MODEL

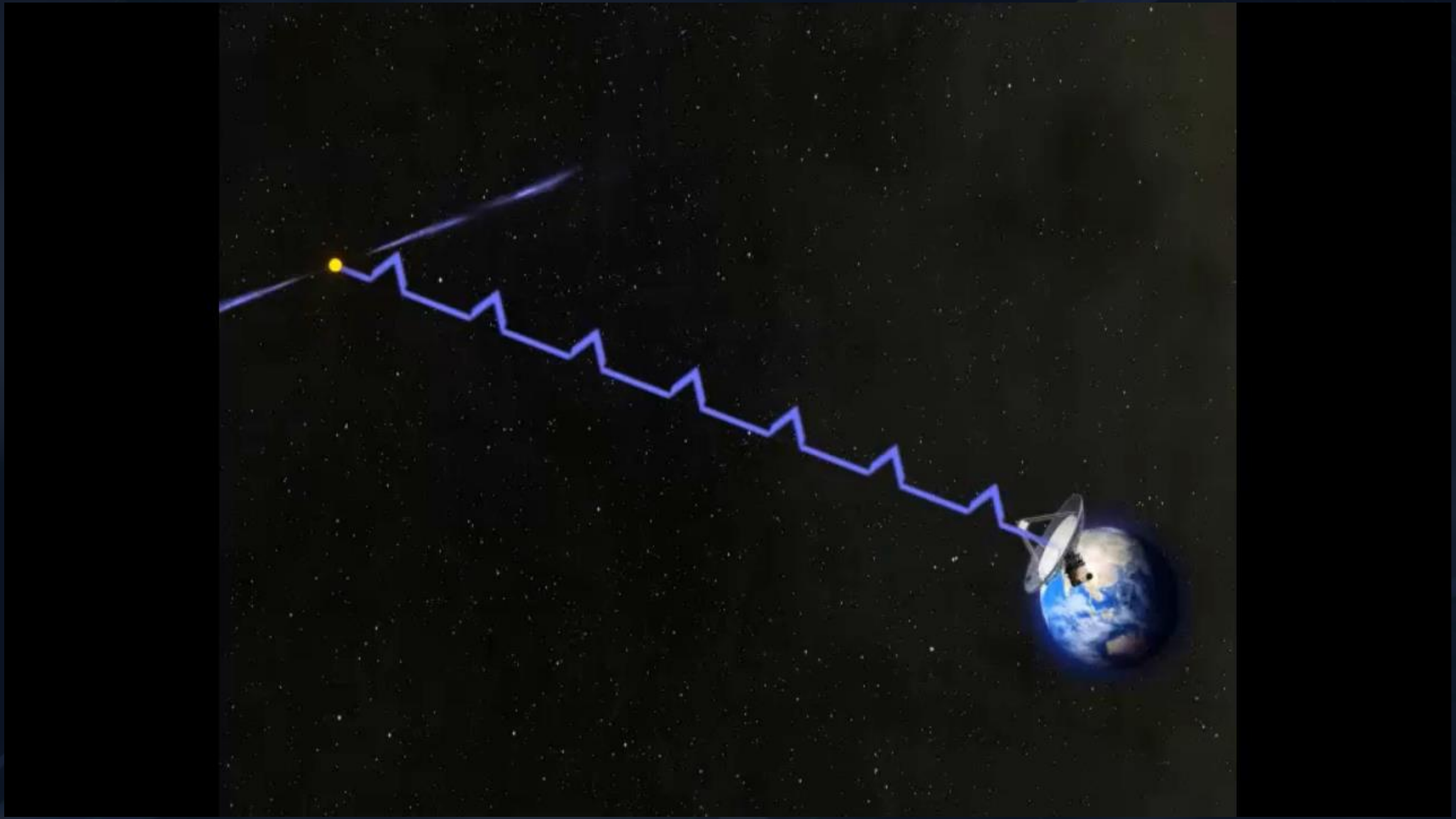
- Model: Correctly predict future pulse times-of-arrival (TOAs)
- Pulsar rotation, orbit (sometime relativistic), astrometry
- IISM signal propagation (Dispersion Measure - DM)
- Reference time-standards (from BIPM)
- Solar-system ephemeris (SSE): transfer TOAs to (quasi-)inertial reference of Solar-system barycenter
- Stochastic Noise (spin/red noise, DM noise, jitter noise, etc)

$$t_{\mathrm{residuals}} = t_{\mathrm{OBS}} - t_{\mathrm{PRED}}$$

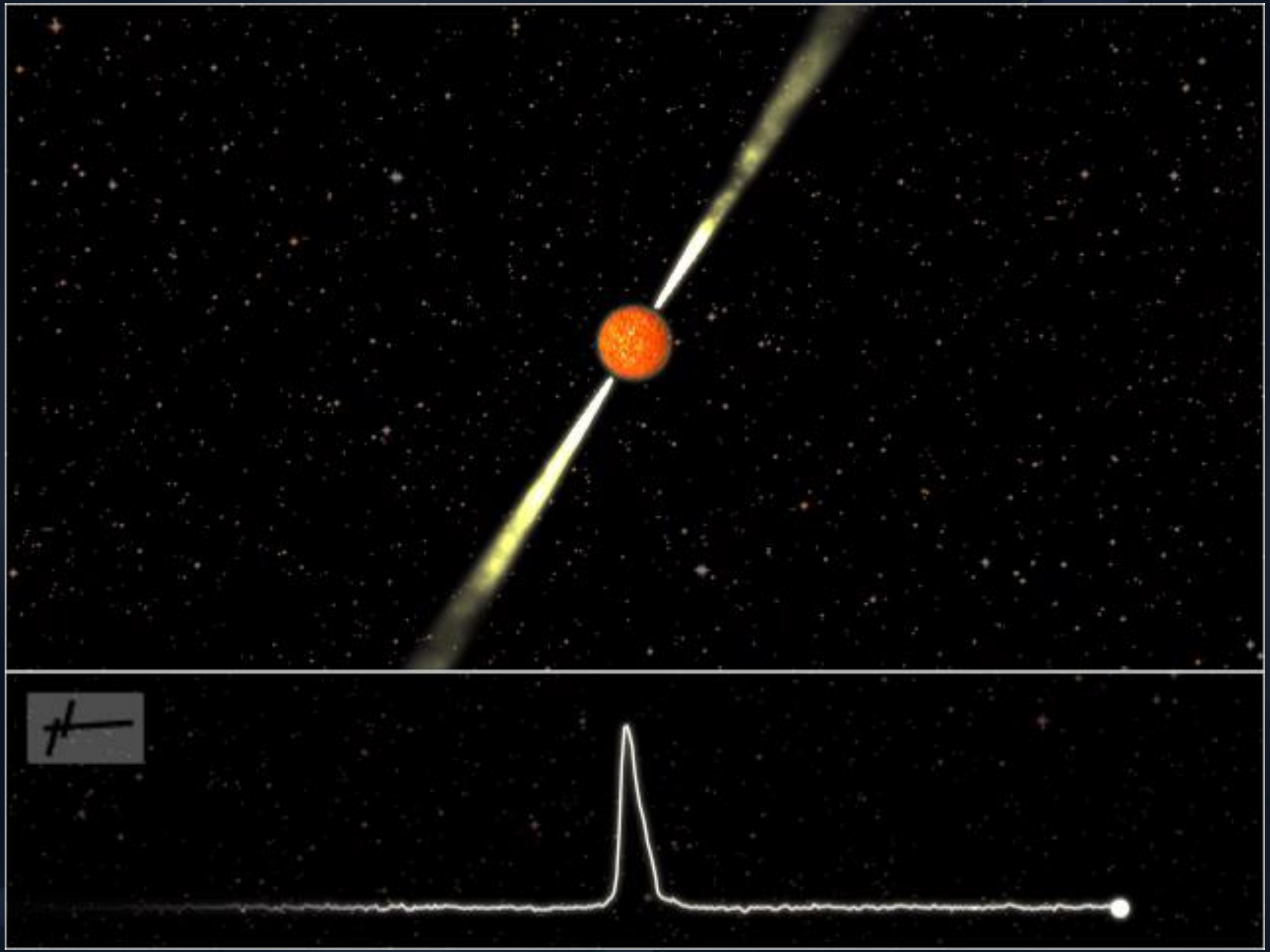
Pulsar Timing & Timing Models

- Millisecond pulsars (MSPs): Fastest periods, most stable rotations
- MSPs’ TOA measurement precision (50-400) ns
- MSP Period measurement precision $\leq 10^{-17}$sec
- MSPs extremely accurate cosmic clocks
- Can probe small changes in light propagation times due to spacetime deformations by GWs

Pulsar Timing Arrays as GW detectors



Pulsar Timing & Timing Models



Animation Credits: "Joeri van Leeuwen". License: CC-BY-AS

Pulsar Timing Arrays: GW sources

- Spatially resolved, inspiralling supermassive black-hole binaries/SMBHBs (deterministic waveforms)
- Permanent memory term of SMBHB mergers
- **Stochastic GW backgrounds/GWBs**

  - ***incoherent superposition of GWs from (>100) unresolved SMBHBs***

  - Relic GWB from inflationary era (quantum fluctuations of gravitational field boosted by inflation)

  - Cosmic strings (topological defects) oscillations (quantum field theory, string/superstring theory)

GWB model: input data

1) Galaxy merger rate
2) The relation between SBHs and their hosts
3) The efficiency of SBH coalescence following galaxy mergers
4) When and how accretion is triggered during a merger event

Outcome, for each SMBHB:

1) Proper chirp mass = $[m_1 m_2]^{3/5}/[m_1+m_2]^{1/5}$
2) Observe GW frequency (red-shifted)
3) redshift

Bayes Theorem and Evidence

Evidence in favour of some model? Bayesian model comparison via Bayes Factors

$$p(\vec{\mu}|\vec{d},\mathcal{H}) = \frac{L(\vec{\mu},\mathcal{H}|d)\pi(\vec{\mu}|\mathcal{H})}{\boxed{p(\vec{d}|\mathcal{H})}}$$

$$p(\vec{d}|\mathcal{H})=\int \mathcal{L}(\vec{\mu})\pi(\vec{\mu})d^N\mu$$

Evidence/Marginal Likelihood (Z)

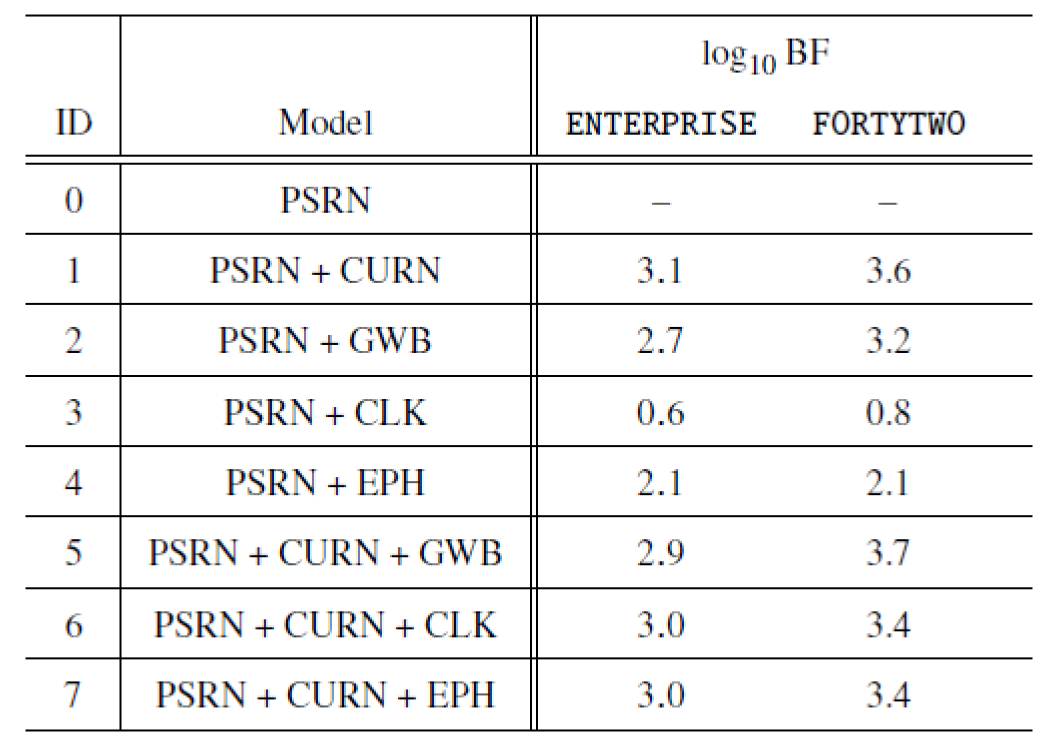
$$R_{10}=(Z_1/Z_2)(\pi_1/\pi_2)=BF_{10}(\pi_1/\pi_2)$$

$$\boxed{R_{10}=BF_{10}}\ \text{,if } \pi_1=\pi_2$$

$\log_{10}(BF)$	Evidence against 0
0-0.5	bare mention
0.5-1	substantial
1-2	strong
>2	decisive

Kass & Raftery (1995)

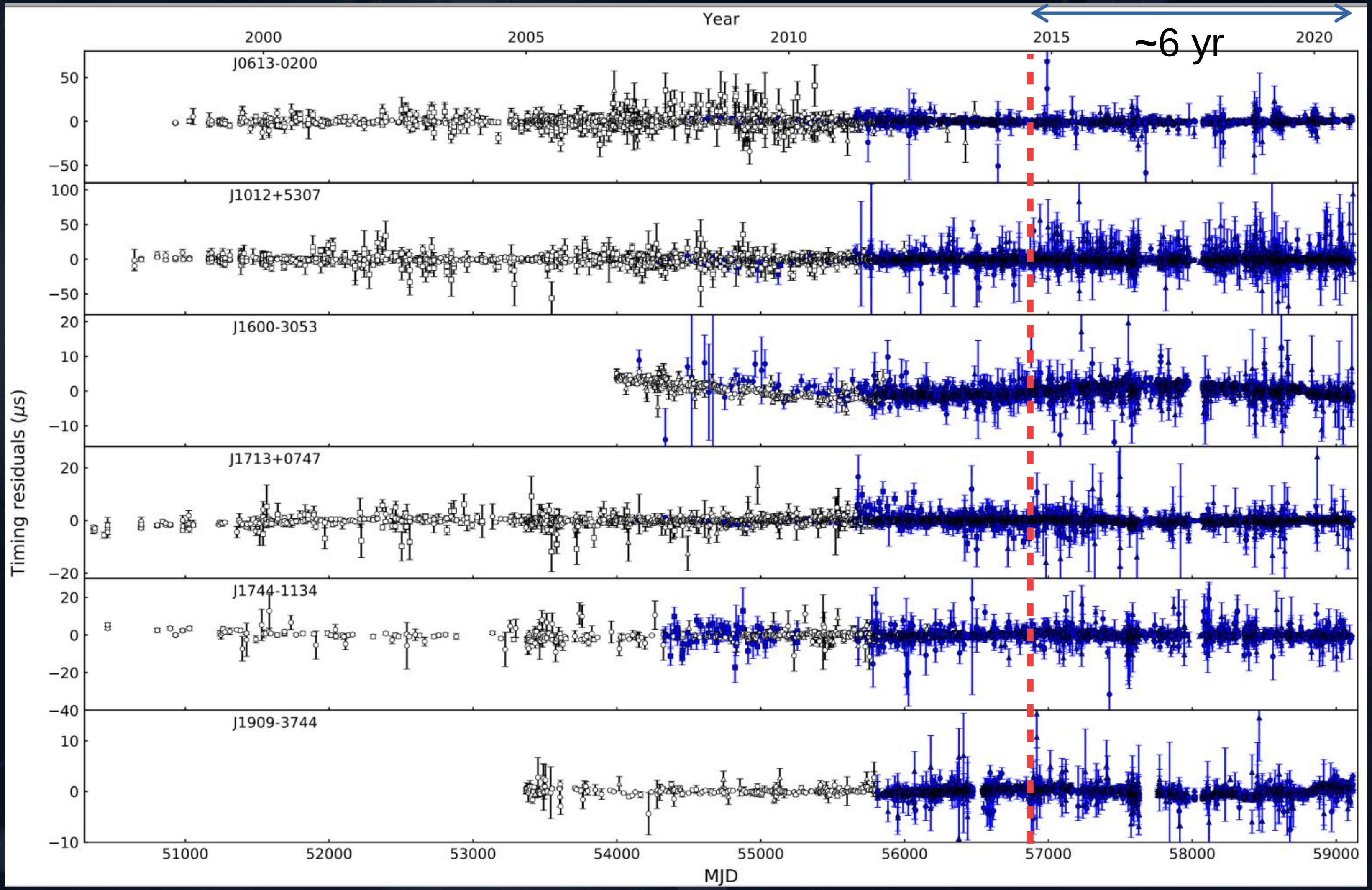
Bayes Factors for ORF-CRS model selection



ID	Model	$\log_{10}$ BF	
		`ENTERPRISE`	`FORTYTWO`
0	PSRN	–	–
1	PSRN + CURN	3.1	3.6
2	PSRN + GWB	2.7	3.2
3	PSRN + CLK	0.6	0.8
4	PSRN + EPH	2.1	2.1
5	PSRN + CURN + GWB	2.9	3.7
6	PSRN + CURN + CLK	3.0	3.4
7	PSRN + CURN + EPH	3.0	3.4

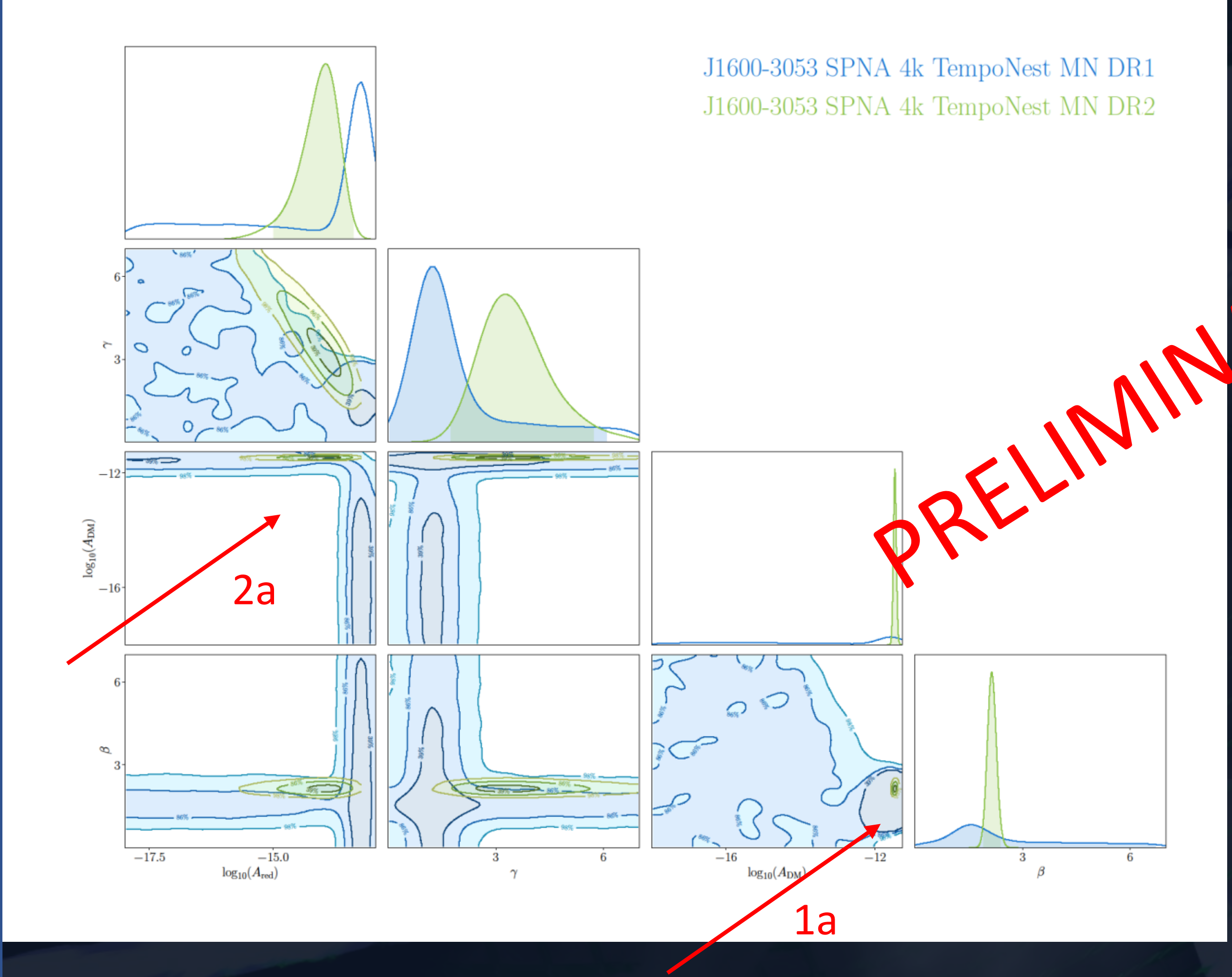
- Hypermodel BF & Nested sampling evidence methods
- CURN, GWB, EPH, clearly preferred over base PSRN model, with log_10(BF) > 2
- CURN marginal advantage over GWB, with log_10(BF) < 0.5
- Adding a 2^{nd} CRS to CURN does not improve the evidence (=> models 5,6,7 not further examined later)
- =>Further analysis assuming the CURN model

GWB search with EPTA DR2

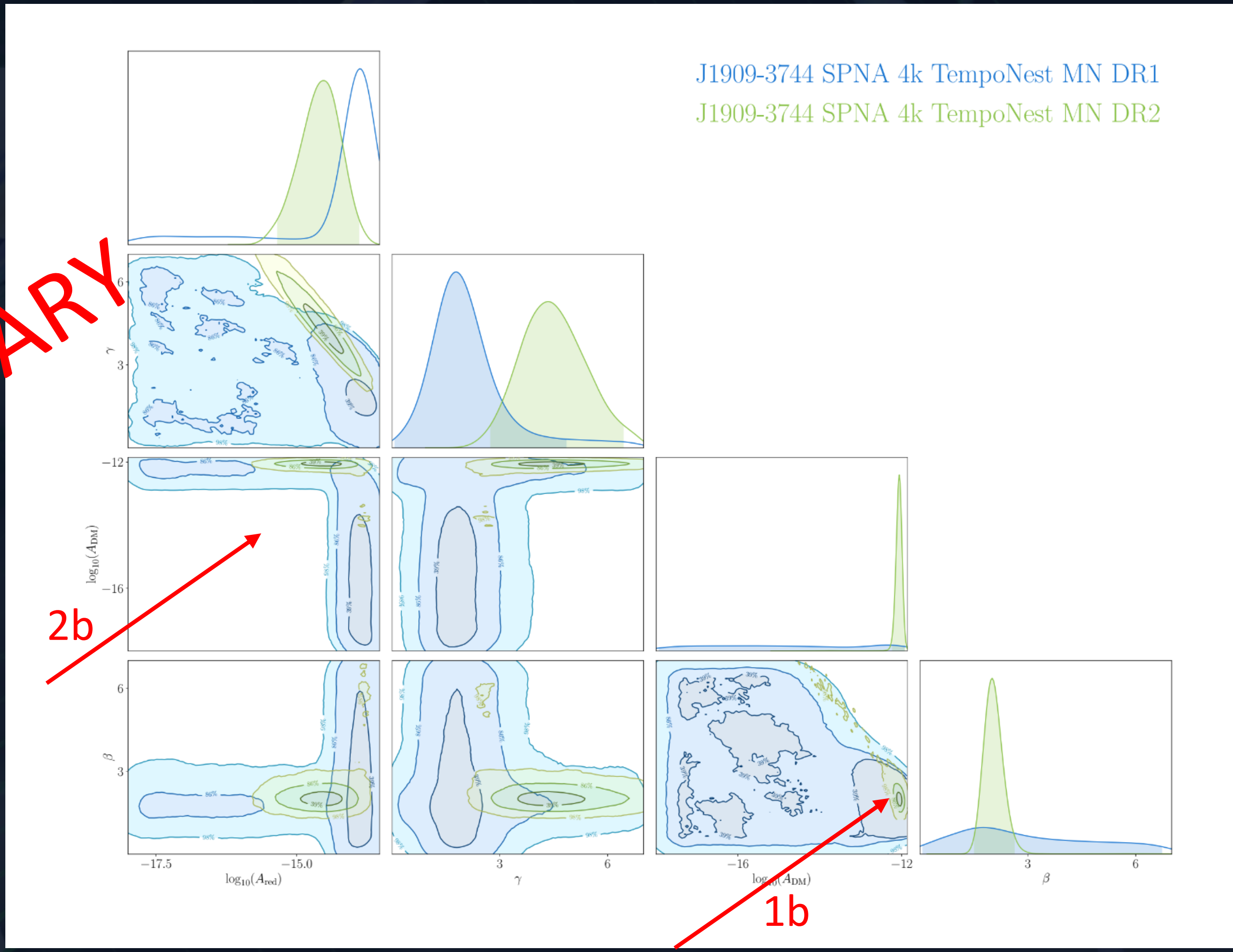


Chen, Caballero+EPTA (in prep.)

EPTA DR2: Pulsar noise measurement

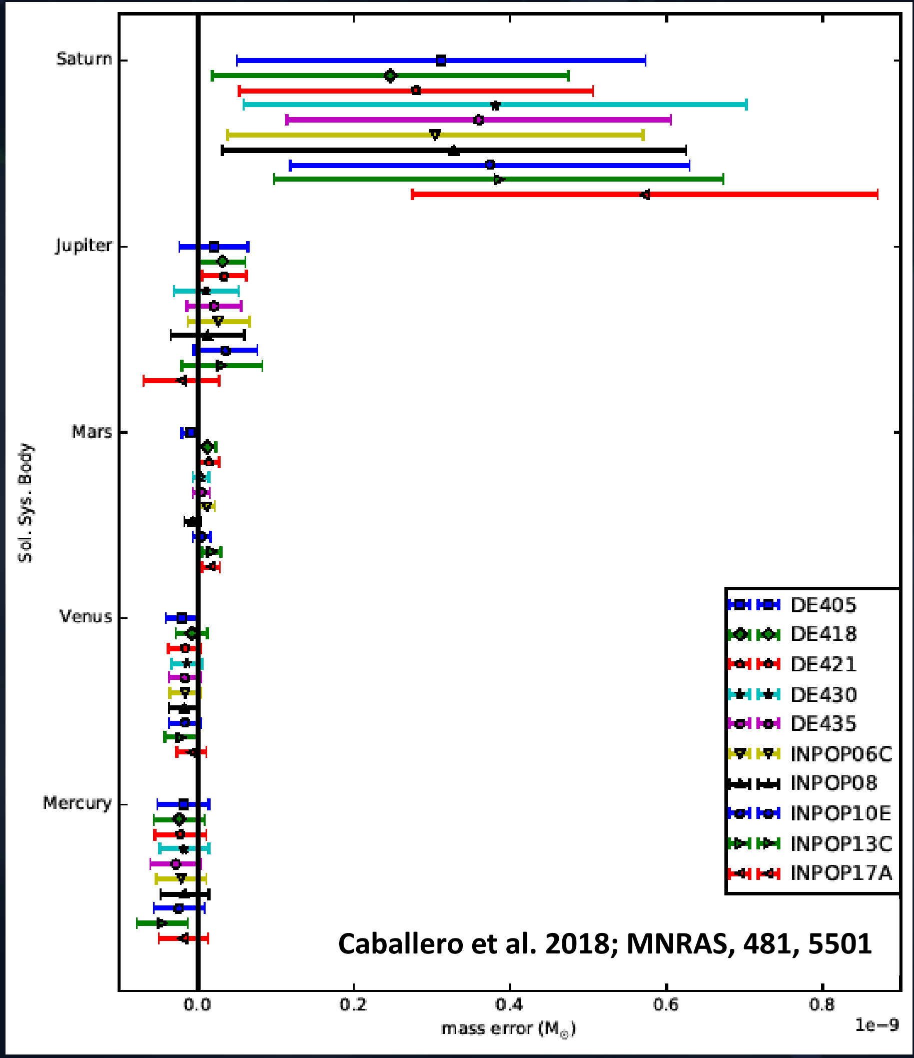


PRELIMINARY



CRS decorrelation is even better than GWB detection!

- PTAs can probe clock and SSE errors
- This can be noise to GW search but:
- PTAs can create independent pulsar-based timescale
  Guinot & Petit 1991; Hobbs et al. 2012,2019 etc.
- PTAs can provide additional data to SSE by finding deviations in the measured vs SSE predicted location of the SSB



Caballero et al. 2018; MNRAS, 481, 5501

GWB search with the EPTA/priors

- Upper limit analysis Vs. Search in Bayesian inference depends on the chosen “prior” distribution of A

- *A* = scale invariant => following priors
  1) Uniform A distribution ⬄ Upper limit analysis
  2) Uniform logA distribution (~non-informative) ⬄ Search analysis
- In high S/N regime, posterior probability distribution (inferred parameters) → insensitive to the prior type (information in data dominates information in assumptions)

Bayesian GWB search

Model parameters

Model

$$p(\vec{\mu}|\vec{d},\mathcal{H}) = \frac{L(\vec{\mu},\mathcal{H}|d)\,\pi(\vec{\mu}|\mathcal{H})}{p(\vec{d}|\mathcal{H})}$$

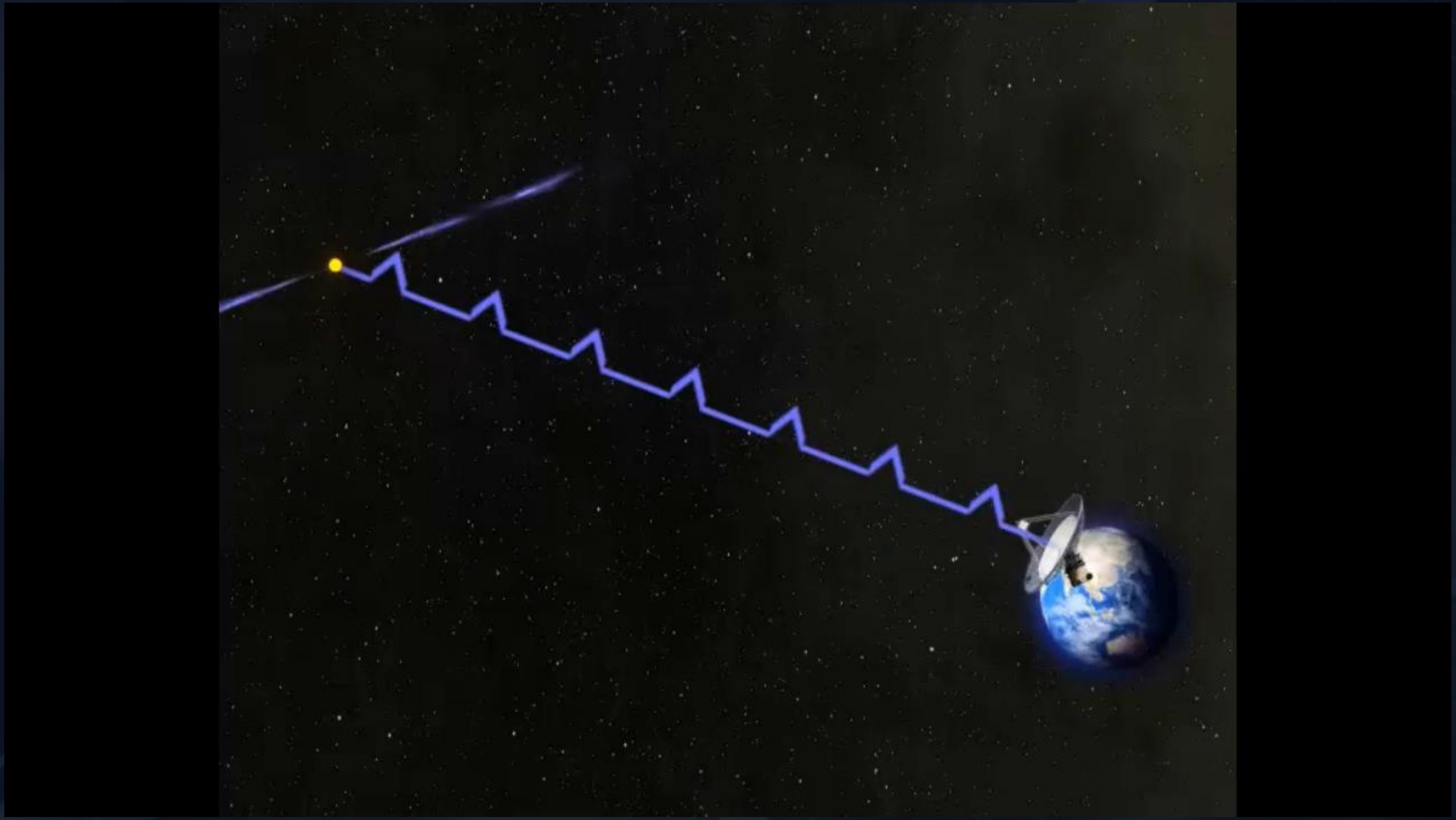
data

TOA

$$L_{tim} = L(\epsilon,\xi|d) = \frac{1}{\sqrt{(2\pi)^n|\mathbf{C}|}} \times e^{-\frac{1}{2}(\mathbf{t}-\boldsymbol{\tau}(\boldsymbol{\epsilon}))^{\mathbf{T}}\mathbf{C}^{-1}(\mathbf{t}-\boldsymbol{\tau}(\boldsymbol{\epsilon}))}$$

Timing signal

Pulsar Timing Arrays as GW detectors



Single-pulsar noise analysis

- Same models as in DR1 (Caballero+2016, Desvignes+2016)
  1 (EFAC,EQUAD) pair per observing system (telescope+backend)

$$\hat{\sigma}^2 = (\sigma \cdot \mathrm{EFAC})^2 + \mathrm{EQUAD}^2$$

  1 single power-law (PL) time-stationary stochastic red noise per pulsar (30 freq. bins)
  1 single-PL time-stationary stochastic DM noise (100 freq. bins)

$$S \propto A^2 f^{-\gamma}$$

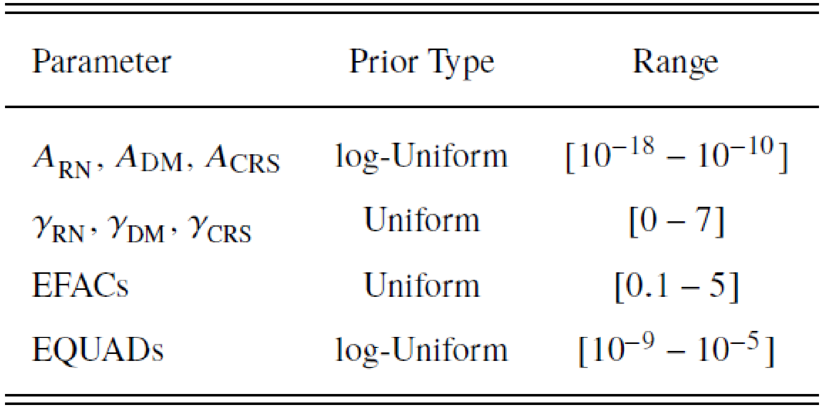
- Analytical marginalization of timing model (van Haasteren+2009)

CRS spectrum+Overlap Reduction Function (ORF) search

- Single power-law spectrum models for CRS (30 fr.bins – not same as PSR 30 bins)

$$S \propto A^2 f^{-\gamma}$$

- Keep simultaneous fit of pulsar Red+DM noise PLs (own freq. bins)
- White noise fixed at SPNA values (checks suggest this to be ok)
- Analytically marginalize over timing parameters.
- Chebyshev (4 coeff.) polynomial fit for ORF (tested method from DR1)



Parameter	Prior Type	Range
$A_{\mathrm{RN}}, A_{\mathrm{DM}}, A_{\mathrm{CRS}}$	log-Uniform	$[10^{-18} - 10^{-10}]$
$\gamma_{\mathrm{RN}}, \gamma_{\mathrm{DM}}, \gamma_{\mathrm{CRS}}$	Uniform	$[0 - 7]$
EFACs	Uniform	$[0.1 - 5]$
EQUADs	log-Uniform	$[10^{-9} - 10^{-5}]$

Inconclusive ORF-CRS model selection

- Follow the same models as DR1 (see Lentati+2015), now widely used

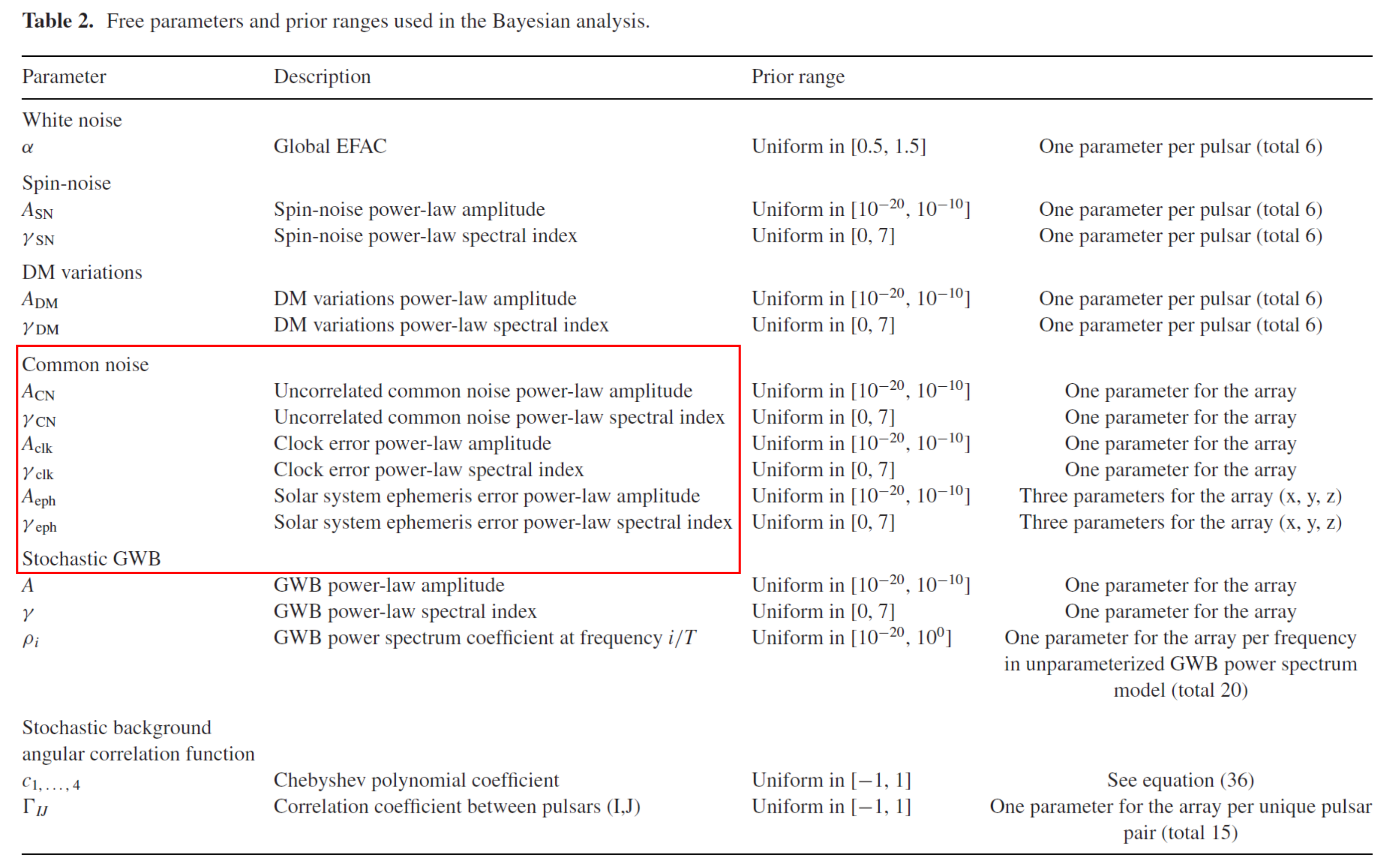


Table 2. Free parameters and prior ranges used in the Bayesian analysis.

Parameter	Description	Prior range	
White noise			
α	Global EFAC	Uniform in [0.5, 1.5]	One parameter per pulsar (total 6)
Spin-noise			
$A_{\rm SN}$	Spin-noise power-law amplitude	Uniform in $[10^{-20}, 10^{-10}]$	One parameter per pulsar (total 6)
$\gamma_{\rm SN}$	Spin-noise power-law spectral index	Uniform in [0, 7]	One parameter per pulsar (total 6)
DM variations			
$A_{\rm DM}$	DM variations power-law amplitude	Uniform in $[10^{-20}, 10^{-10}]$	One parameter per pulsar (total 6)
$\gamma_{\rm DM}$	DM variations power-law spectral index	Uniform in [0, 7]	One parameter per pulsar (total 6)
Common noise			
$A_{\rm CN}$	Uncorrelated common noise power-law amplitude	Uniform in $[10^{-20}, 10^{-10}]$	One parameter for the array
$\gamma_{\rm CN}$	Uncorrelated common noise power-law spectral index	Uniform in [0, 7]	One parameter for the array
$A_{\rm clk}$	Clock error power-law amplitude	Uniform in $[10^{-20}, 10^{-10}]$	One parameter for the array
$\gamma_{\rm clk}$	Clock error power-law spectral index	Uniform in [0, 7]	One parameter for the array
$A_{\rm eph}$	Solar system ephemeris error power-law amplitude	Uniform in $[10^{-20}, 10^{-10}]$	Three parameters for the array (x, y, z)
$\gamma_{\rm eph}$	Solar system ephemeris error power-law spectral index	Uniform in [0, 7]	Three parameters for the array (x, y, z)
Stochastic GWB			
A	GWB power-law amplitude	Uniform in $[10^{-20}, 10^{-10}]$	One parameter for the array
γ	GWB power-law spectral index	Uniform in [0, 7]	One parameter for the array
ρ_i	GWB power spectrum coefficient at frequency i/T	Uniform in $[10^{-20}, 10^{0}]$	One parameter for the array per frequency in unparameterized GWB power spectrum model (total 20)
Stochastic background angular correlation function			
$c_{1,\dots,4}$	Chebyshev polynomial coefficient	Uniform in $[-1, 1]$	See equation (36)
Γ_{IJ}	Correlation coefficient between pulsars (I,J)	Uniform in $[-1, 1]$	One parameter for the array per unique pulsar pair (total 15)

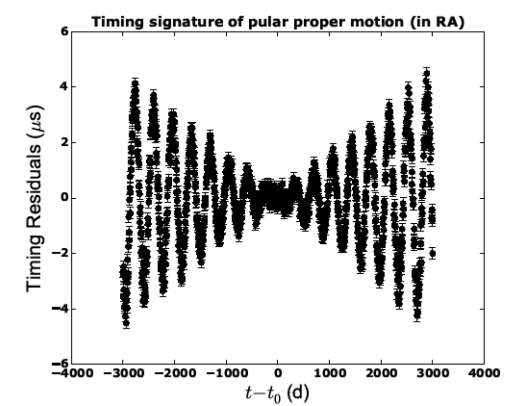
Pulsar Timing & Timing Models

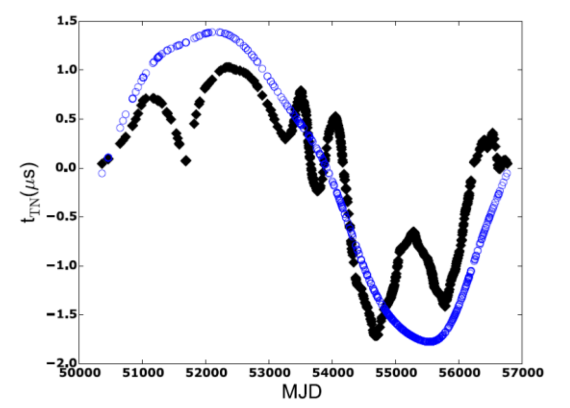
- Millisecond pulsars (MSPs): Fastest periods, most stable rotations
- MSPs’ TOA measurement precision (50-400) ns
- MSP Period measurement precision $\leq 10^{-17}$sec
- MSPs extremely accurate cosmic clocks, multiple applications

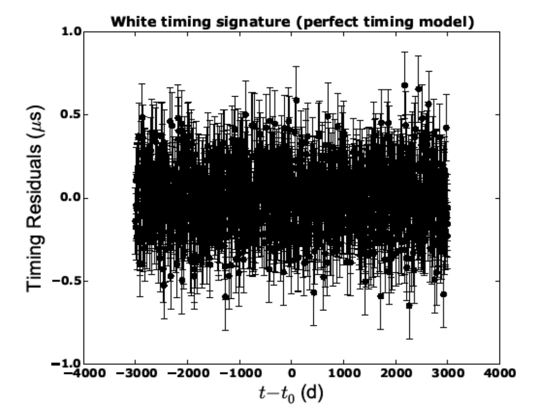
- Pulsars/NS interior models, mass measurements
- Testing gravity theories via binary pulsar orbital dynamics (***Nobel prize 1993***, Hulse&Taylor)
- ISM studies
- NS Astrometry
- Binary stellar evolution
- **Gravitational-wave detectors at nHz-mHz frequencies, via Pulsar Timing Arrays (PTAs)**

Pulsar Timing & Timing Models

$$t_{\rm residuals} = t_{\rm OBS} - t_{\rm PRED}$$







- Millisecond pulsars (MSPs): Fastest periods, most stable rotations
- MSPs’ TOA measurement precision (50-400) ns
- Pulsar Period measurement precision:
- Pulsar/MSPs extremely accurate cosmic clocks, multiple applications

Pulsar Timing Arrays

- PTA = Array of pulsars, at different sky locations (Foster & Backer, 1990)
- To probe “common, correlated signals”, ie from physical process not specific to a pulsar
- “Favourite” target: space-time fluctuations by GWs (but more signals of interest exist)
- Most rotationally stable pulsars required: Millisecond pulsars (MSPs)
- Signal must be present in TRs of all PTA pulsar
- Signal must be spatially correlated in agreement with theory, e.g. General Relativity (GR)
- Signal can be deterministic (coherent waveforms)
  or stochastic (signal spectrum + angular correlations curve)

Single-pulsar analysis (Aurélien’s talk)

- Same models as in DR1 (Caballero+2016, Desvignes+2016)
  1 (EFAC,EQUAD) pair per observing system (telescope+backend)

$$\hat{\sigma}^2 = (\sigma \cdot \mathrm{EFAC})^2 + \mathrm{EQUAD}^2$$

  1 single power-law (PL) time-stationary stochastic red noise per pulsar (30 freq. bins)
  1 single-PL time-stationary stochastic DM noise (100 freq. bins)

$$S \propto A^2 f^{-\gamma}$$

  DM noise follows $\Delta T_{\mathrm{dm}} \propto \nu^{-2}$ (cold plasma). DM+DM1+DM2 included in timing model

- Analytical marginalization of timing model (van Haasteren+2009)

$$L_{\mathrm{PSR}} \propto \frac{1}{\sqrt{(-\frac{1}{2} \times |\mathbf{C}||\mathbf{D}^{\mathsf{T}}\mathbf{C}^{-1}\mathbf{D}|}} e^{-\frac{1}{2}(\delta \boldsymbol{t})^{\mathsf{T}}\mathbf{C}'^{-1}(\delta \boldsymbol{t})}$$

$$\mathbf{C} = \mathbf{C}_{\mathrm{W}} + \mathbf{C}_{\mathrm{R}} + \mathbf{C}_{\mathrm{DM}}$$

where,

$$\mathbf{C}' = \mathbf{C}^{-1} - \mathbf{C}^{-1}\mathbf{D}(\mathbf{D}^{\mathsf{T}}\mathbf{C}^{-1}\mathbf{D})^{-1}\mathbf{D}^{\mathsf{T}}\mathbf{C}^{-1}$$

Bayesian GWB search

- Use stationary, power-law spectrum models for pulsar intrinsic stochastic noise

  $$S \propto A^2 f^{-\gamma}$$

  DM noise follows $\Delta\tau \propto \nu^{-2}$ (cold plasma)

- Search for GWB model simultaneously with re-fitting pulsar noise parameters (possibly correlated)
  while we analytically marginalize over timing parameters (nuance parameters).

European Pulsar Timing Array (EPTA)

- Max Planck Institute for Radio Astronomy
- Jodrell Bank Centre for Astrophysics
- Nançay Radio Observatory/Paris Observatory
- ASTRON
- Cagliari Observatory
- + multiple Universities and Institutes (data analysis, algorithm development, theory, GE sources, signal predictions)
- Experience in Radio Pulsar Observations since pulsar discovery
- High-precision radio pulsar timing data since 1996

Recipe for GWB search

Search for a stochastic signal common to all pulsars,
with expected power-law spectrum (lower frequencies)
and which has a spatial correlation described by Hellings-Downs curve

1) Build radio telescope with pulsar observing backend
2) Observe pulsars
3) Create pulsar timing/noise model
4) Search for common red signal and define spectral parameters
5) Measure the ORF

Bayesian GWB search

- Van Haasteren et al. (2011), first GWB strain upper limit
- Based on van Haasteren et al. (2009) methodology
- Most used approach currently (with modifications/improvements)

Bayesian GWB search

$$p\Big(\overset{\substack{\text{Model}\\ \text{parameters}}}{\overset{\uparrow}{\vec{\mu}}}\Big|\underset{\underset{\text{data}}{\downarrow}}{\vec{d}},\ \overset{\text{Model}}{\overset{\uparrow}{\mathcal{H}}}\Big) = \frac{L(\vec{\mu},\mathcal{H}|d)\,\pi(\vec{\mu}|\mathcal{H})}{p(\vec{d}|\mathcal{H})}$$

Bayesian GWB search

$$p(\overset{\text{Model parameters}}{\vec{\mu}}|\underset{\text{data}}{\vec{d}}, \overset{\text{Model}}{\mathcal{H}}) = \frac{\overbrace{\boxed{L(\vec{\mu}, \mathcal{H}|d)\pi(\vec{\mu}|\mathcal{H})}}^{\text{Parameter estimation}}}{\underbrace{\boxed{p(\vec{d}|\mathcal{H})}}_{\text{‘Evidence’ : Model Selection}}}$$

Bayesian GWB search

- Use MC sampler to sample the (unnormalized) parameter space and calculate the posterior probability distribution (PDF) of parameters
- Prior probability distributions (priors) *required*

  * log-uniform for amplitudes (~ uninformative for scale invariant properties – see e.g. Gregory 2005)

  * uniform (flat) otherwise (eg spectral index)

  * If Amplitude unconstrained, use uniform priors to produce upper limits

EPTA DR2: EPH modelling

✓ 3 different algorithms

BayesEphem

Model of planetary mass/orbital effects
(Vallisnery et al. 2020)

EphemGP

Orbital parameter effects from INPOP19a

LINIMOSS

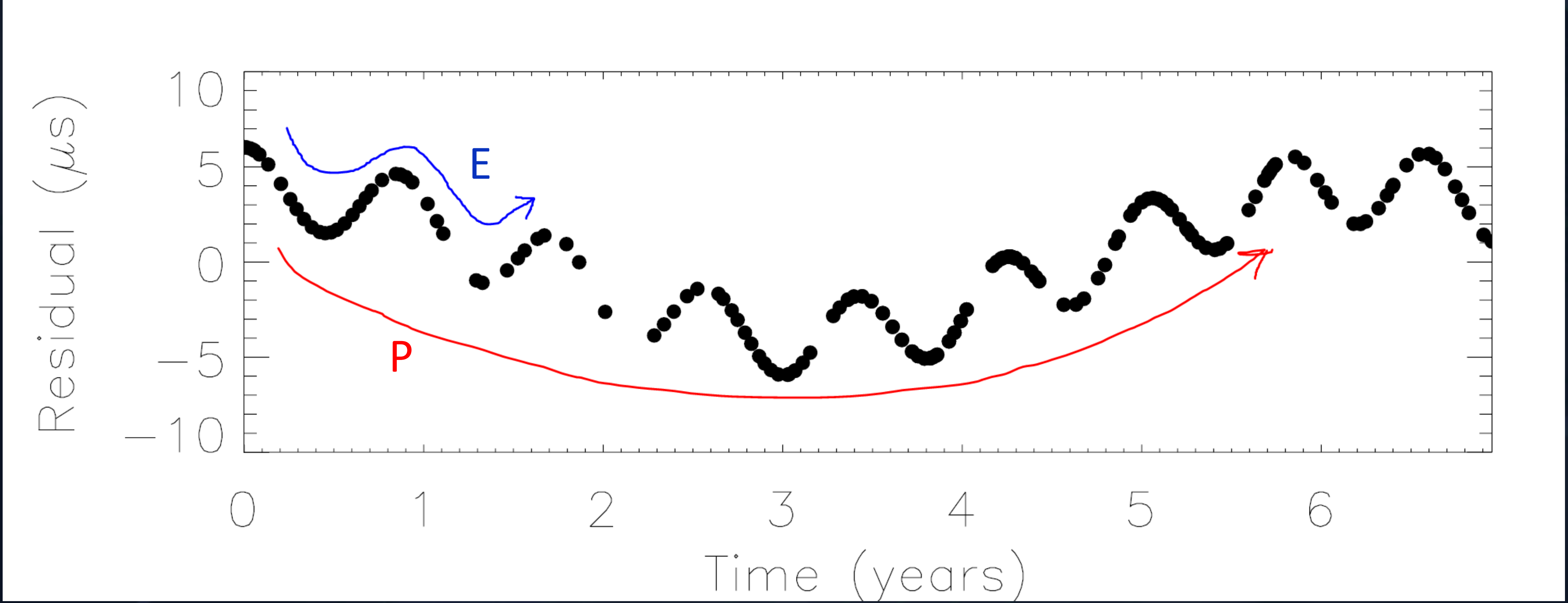
Direct access to modified PMOE ephemeris.
Fit mass/orbital parameters
(Guo et al. 2019)

- ✓ Consistent results
- ✓ Small SSE effects on parameter estimation
- ✓ Marginal evidence for the benefit of fitting the SSE
- ✓ Fitting the SSE does decrease evidence for dipolar CRS
- ✓ More pulsars needed

PRELIMINARY

Pulsar Timing Arrays as GW detectors

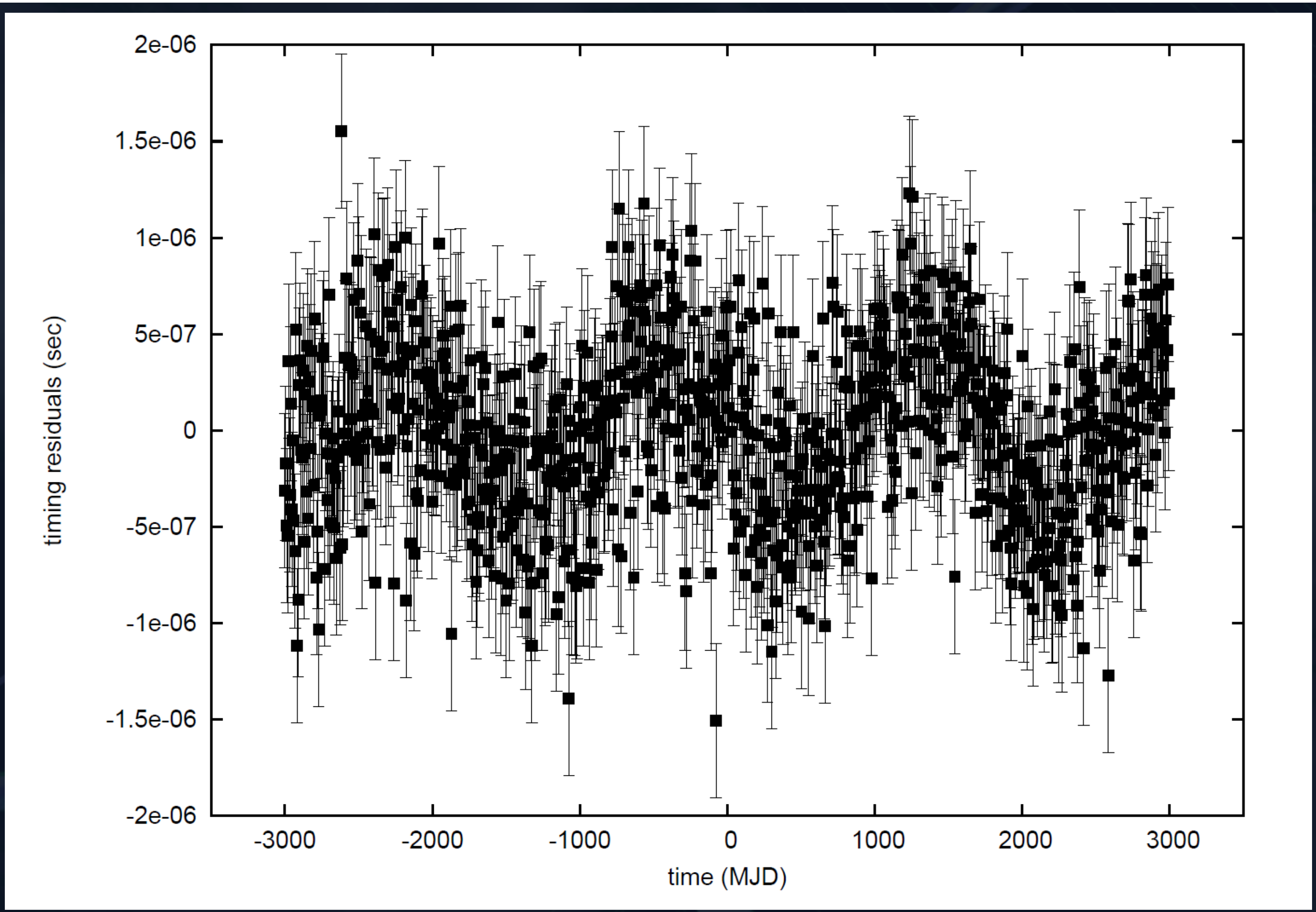
- A SMBHB signal: Earth term (blue) + Pulsar term (red)



- Only Earth term is coherent across all PTA pulsars
- Unless pulsar distances known, pulsar terms act as noise

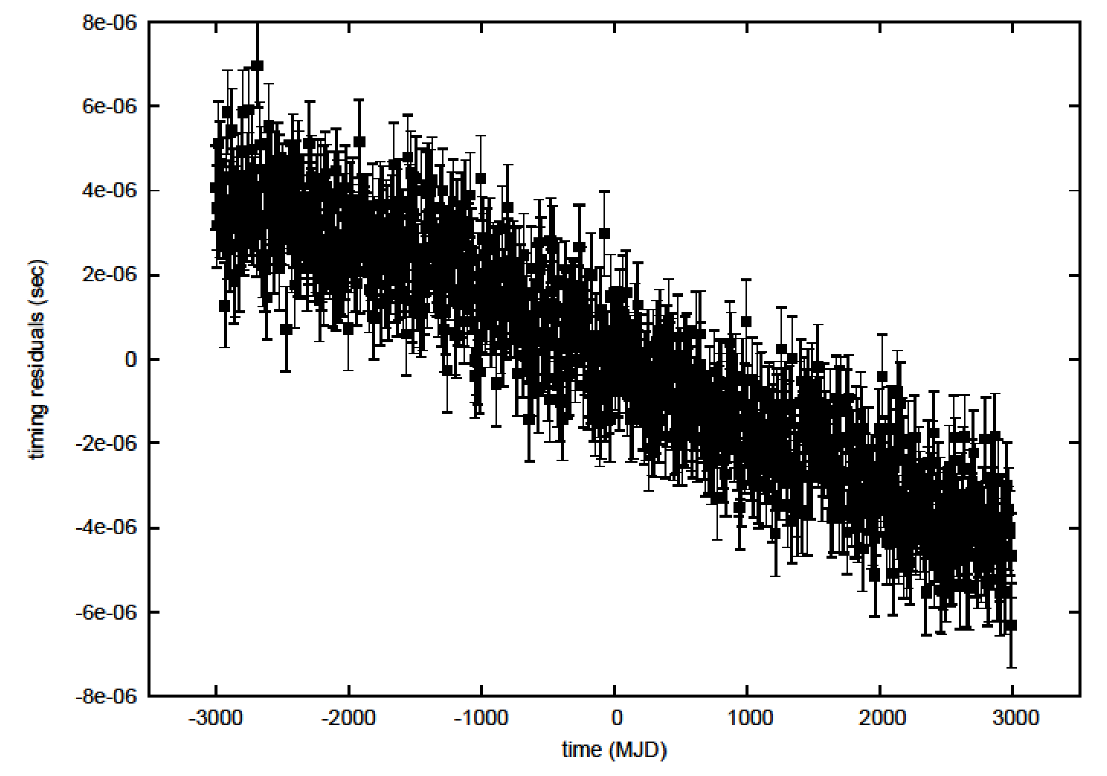
Pulsar Timing Arrays as GW detectors

- A SMBHB signal: How it would really look

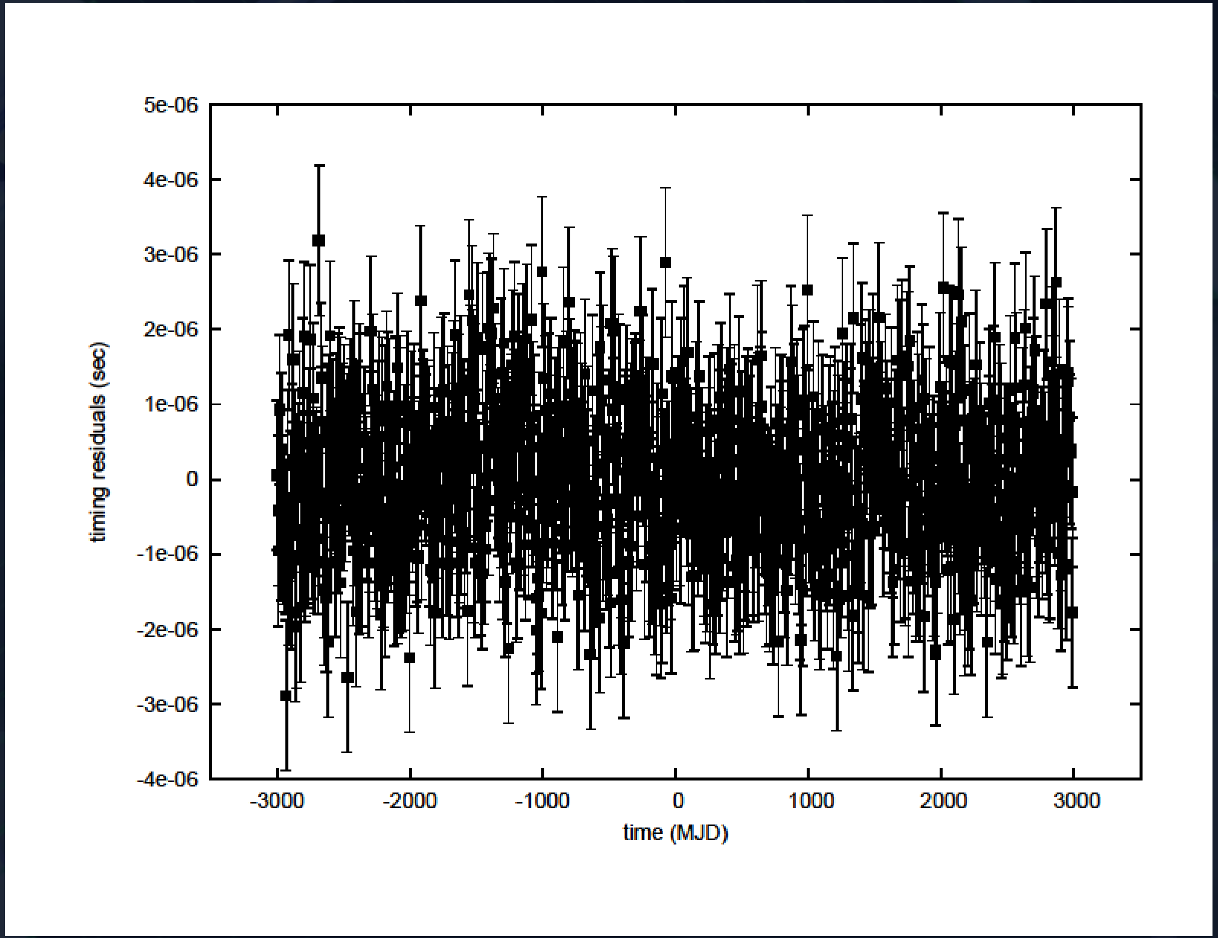


Pulsar Timing Arrays as GW detectors

- A GWB signal (A=1e-15 , σ_{TOA}= 1μs, T~16 yr)

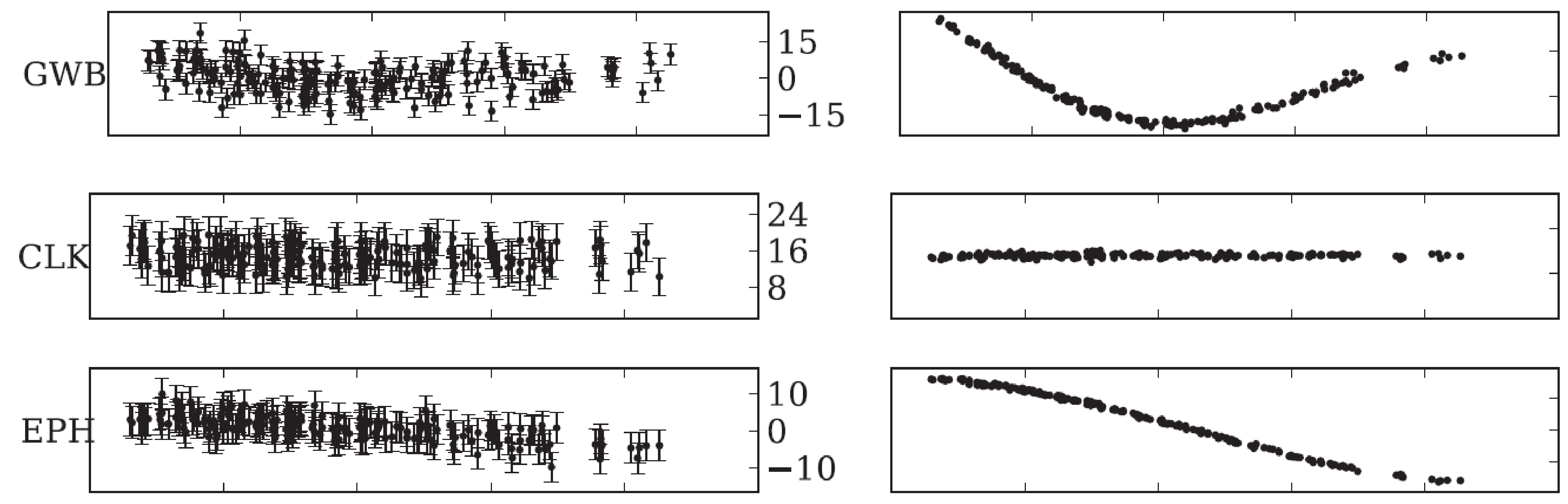


"pre-fit"



"post-fit"

Pulsar Timing: Correlated Noise in TRs



Tiburzi et al. (2016)

- Note: We can actually also treat them as scientific signals of interest:
- Independent Pulsar-based timescales (Petit & Tavella 1996; Hobbs et al. 2012; Hobbs et al. 2019)
- Independent measurement of planetary parameters (Champion et al. 2010; Caballero et al. 2018; Guo et al. 2019)

(I)PTA timescale prospects (I)

σ_z statistic (Matsakis, et al. 1997), measure of fractional clock frequency offsets over given time lags. Similar to Allan variance but appropriate for pulsars