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Rotation and deformation of strangeon stars in Lennard-Jones model

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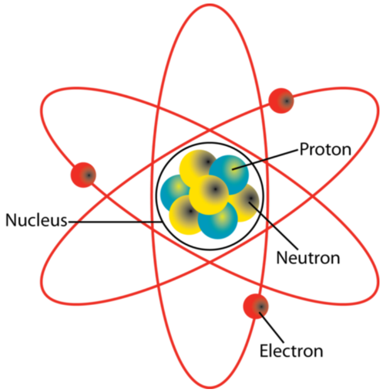
Outline

- Strangeon stars in Lennard Jones model
- Rotation and tidal deformation
- Observations and possible constraints
- Summary

Two perspectives of neutron-star interiors

Compression of normal baryonic matter by gravity in supernova explosions

Normal baryonic matter



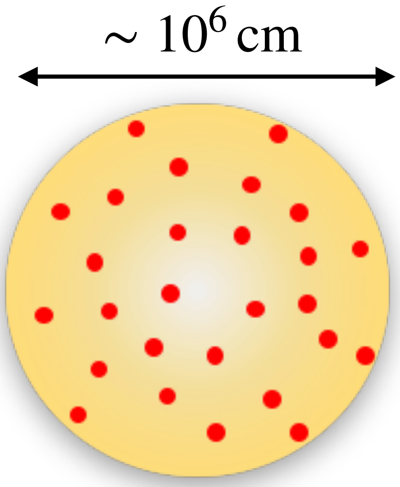
$p_F \propto n_e^{1/3}$

Compression

$$l_{ep} \sim \frac{1}{\alpha_{em}} \frac{\hbar c}{m_e c^2}$$

$$\rho_{EM} \simeq \frac{m_p}{l_{ep}^3} \sim 10\ \mathrm{g/cm^3}$$

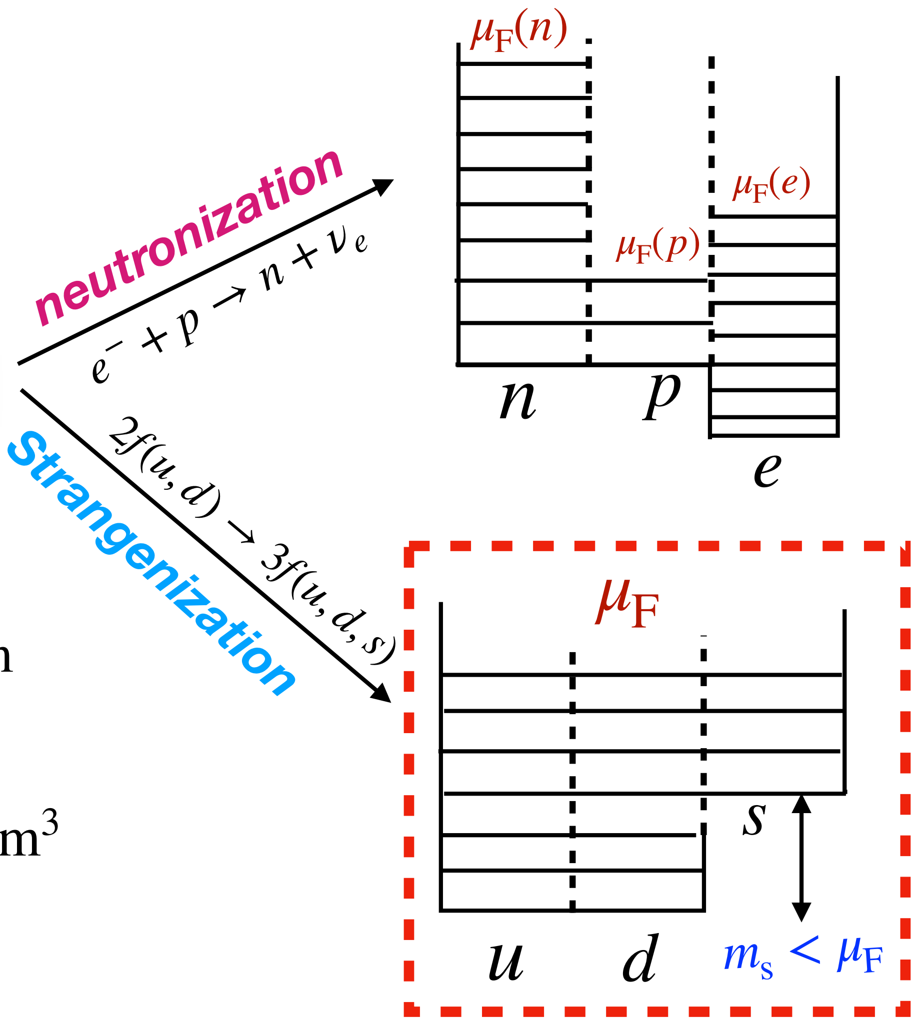
Energetic electrons ($E_e \sim 100\,\mathrm{MeV}$) in a gigantic nucleus



$$l_q \sim \frac{1}{\alpha_s} \frac{\hbar c}{m_q c^2} \simeq \frac{1}{\alpha_s}\mathrm{fm}$$

$$\rho_{SM} \simeq \frac{m_q}{l_q^3} \sim 10^{15}\ \mathrm{g/cm^3}$$

Two ways to kill energetic electrons



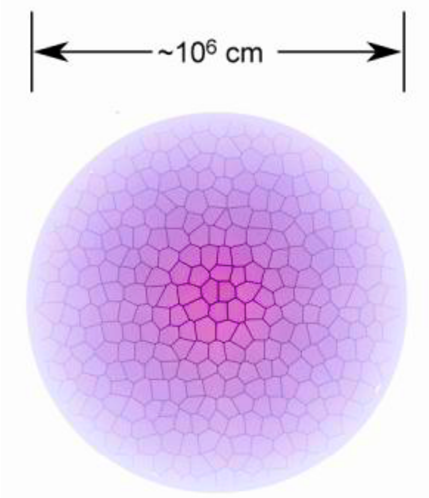
[Landau 1932, Witten 1984, R.-X. Xu 2003]

Localized or non-localized ?

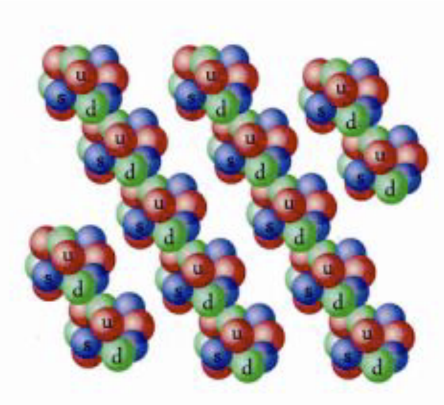
$$\Delta m_{\rm uds} \sim 0.1\,\mathrm{GeV}$$
$$E_{\rm scale} \sim 0.5\,\mathrm{GeV}$$
$$\Lambda_\chi > 1\mathrm{GeV}$$

$$\longrightarrow \quad \Delta m_{\rm uds} \ll E_{\rm scale} < \Lambda_\chi$$

Conjecture: quarks would be clustered or localized. Strangeon star, not strange quark star.



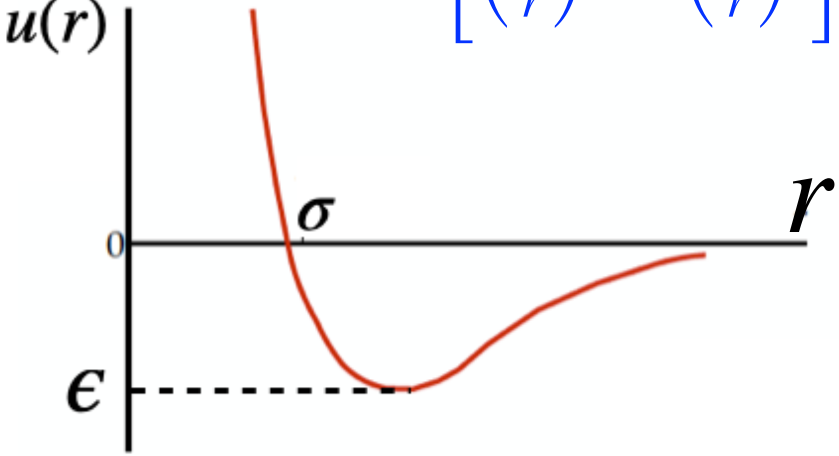
Credit: R.-X. Xu



Strangeon star = Quark-cluster Star

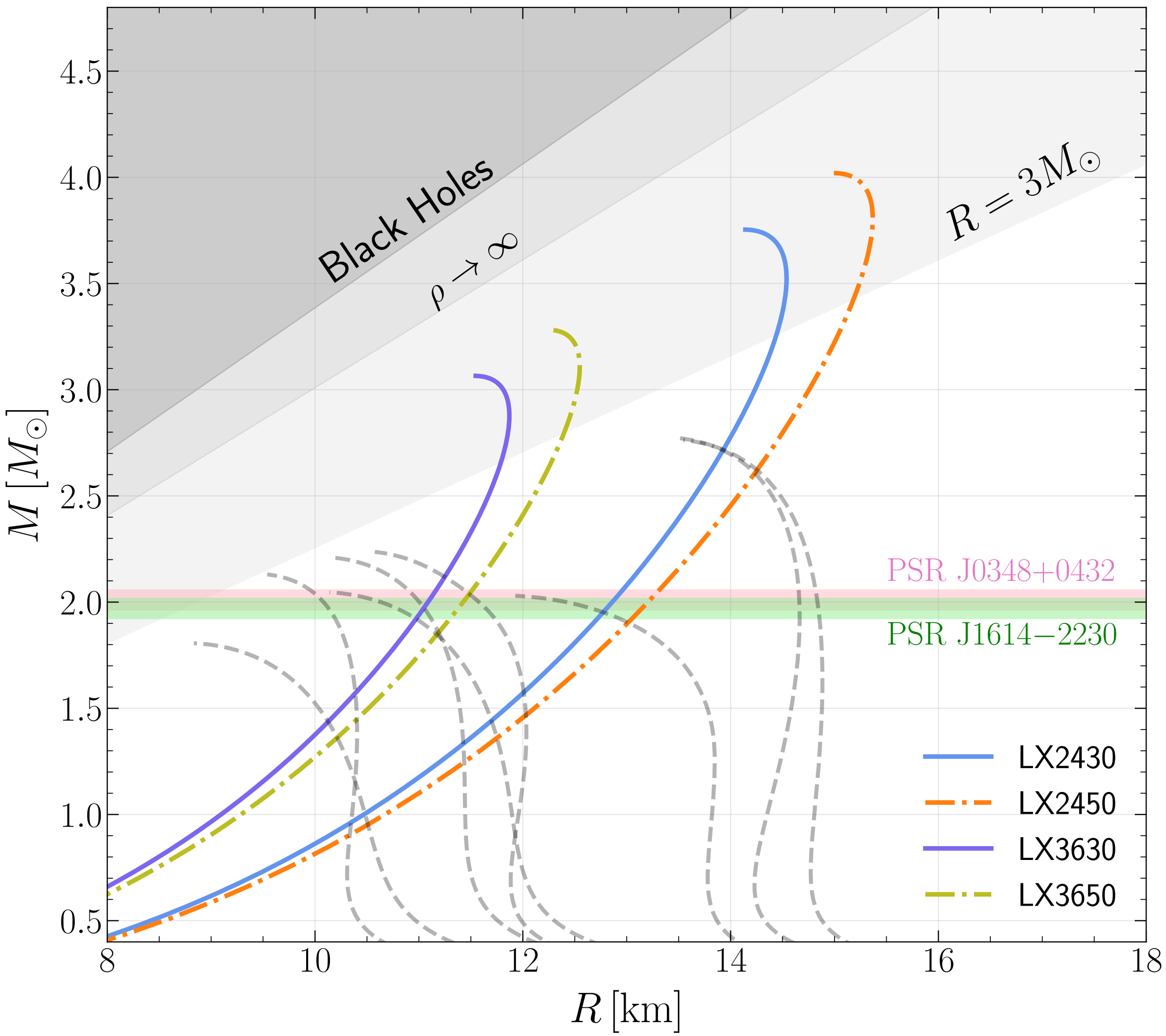
Lennard-Jones model

$$u(r) = 4\epsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right]$$



[R.-X. Xu, 2003, 2013, 2018; Xiaoyu Lai & R.-X. Xu, 2009]

Mass-radius relation



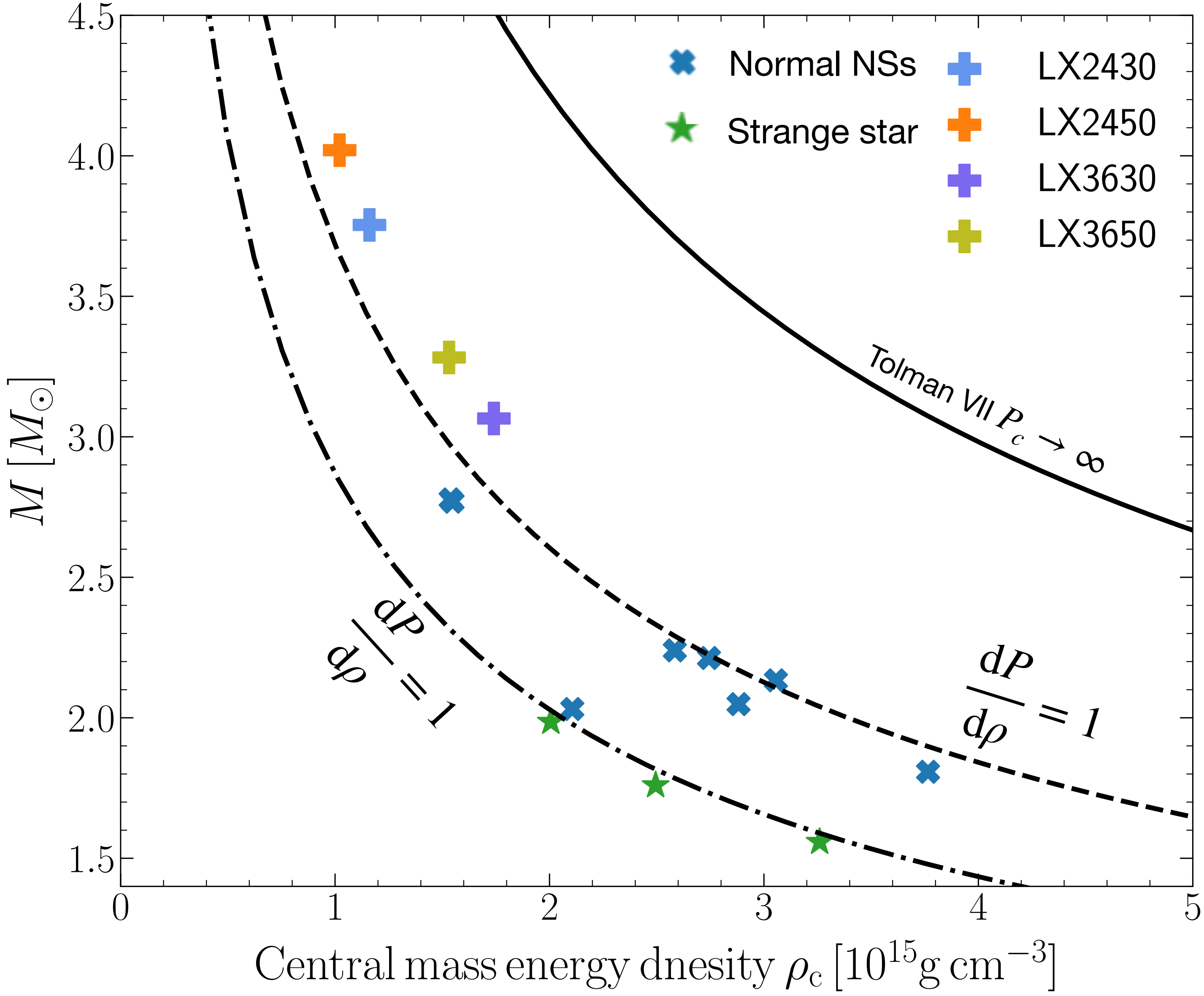
LX2450: $n_s = 0.24\,\mathrm{fm}^{-3}$, $\epsilon = 50\,\mathrm{MeV}$

Global properties:

- Self-bounded solid stars, surface density $\neq 0$, $M \sim R^3$ at low mass
- non-relativistic quark cluster, very stiff and can support $M_{max} > 3M_{\odot}$
- Photon ring > Radius for massive ones (GW echoes?)
- Violate causality? No

[Bludman & Ruderman, 1968; Caporaso & Brecher, 1979; G. Ellis et al., 2007; Jiguang Lu et al., 2018]

Mass-radius relation



Stiffer equation of state

↓

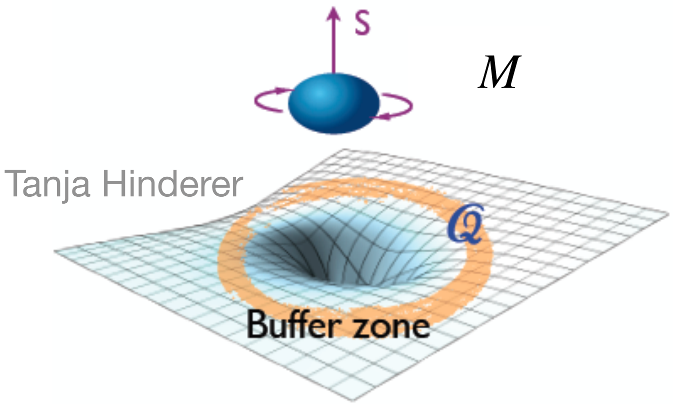
Lower central density for maximal mass

↓

Lower energy scale possibly do not support deconfined quarks

[Lattimer, 2012]

Slow rotation of strangeon stars



- Global properties we care

M $\quad Q \quad$ $S = I\Omega$

- Hartle-Thorne formalism ($\Omega^2 \ll GM/R^3$):

  - First order
  - Second order
  - Third order

$$ds^2 = -e^{\nu(r)}[1+2h(r,\theta)]dt^2 + e^{\lambda(r)}\left[1+\frac{2m^*(r,\theta)}{r-2m(r)}\right]dr^2$$

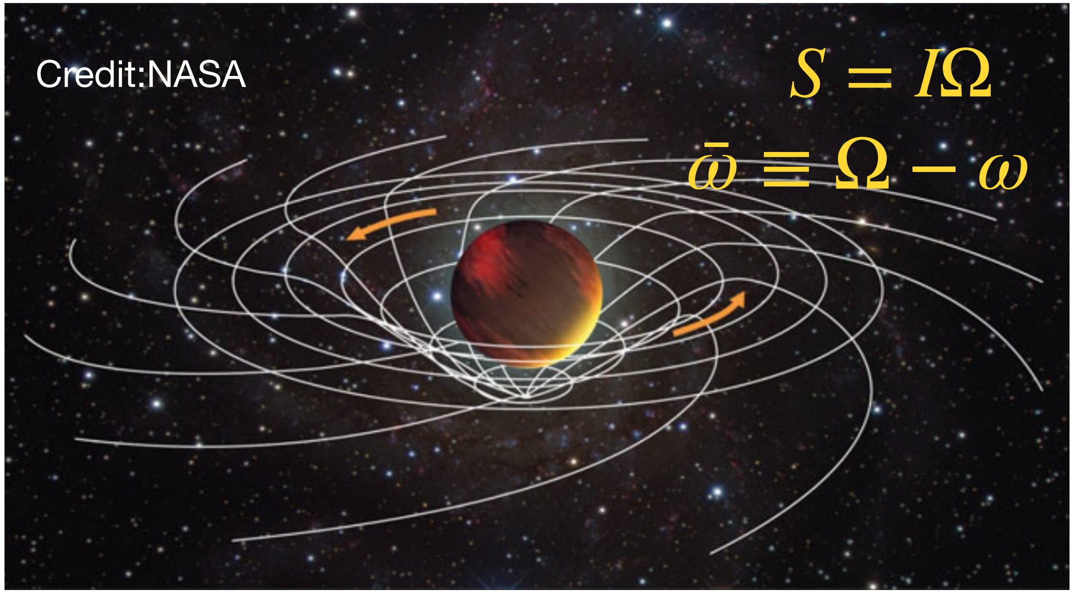
$$+r^2\left[1+2k_2(r,\theta)\right]\left\{d\theta^2+\sin^2\theta[d\phi-\omega(r)-w_1(r,\theta)-w_3(r,\theta)dt]^2\right\}$$

$$T_\mu^\nu = -(\rho+\delta\rho+P+\delta P)u_\mu u^\nu + (P+\delta P)\delta_\mu^\nu$$

[Hartle, ApJ, 1967; Hartle and Thorne, ApJ, 1968; Glendenning et al., 1992; Benhar et al., PRD, 2013]

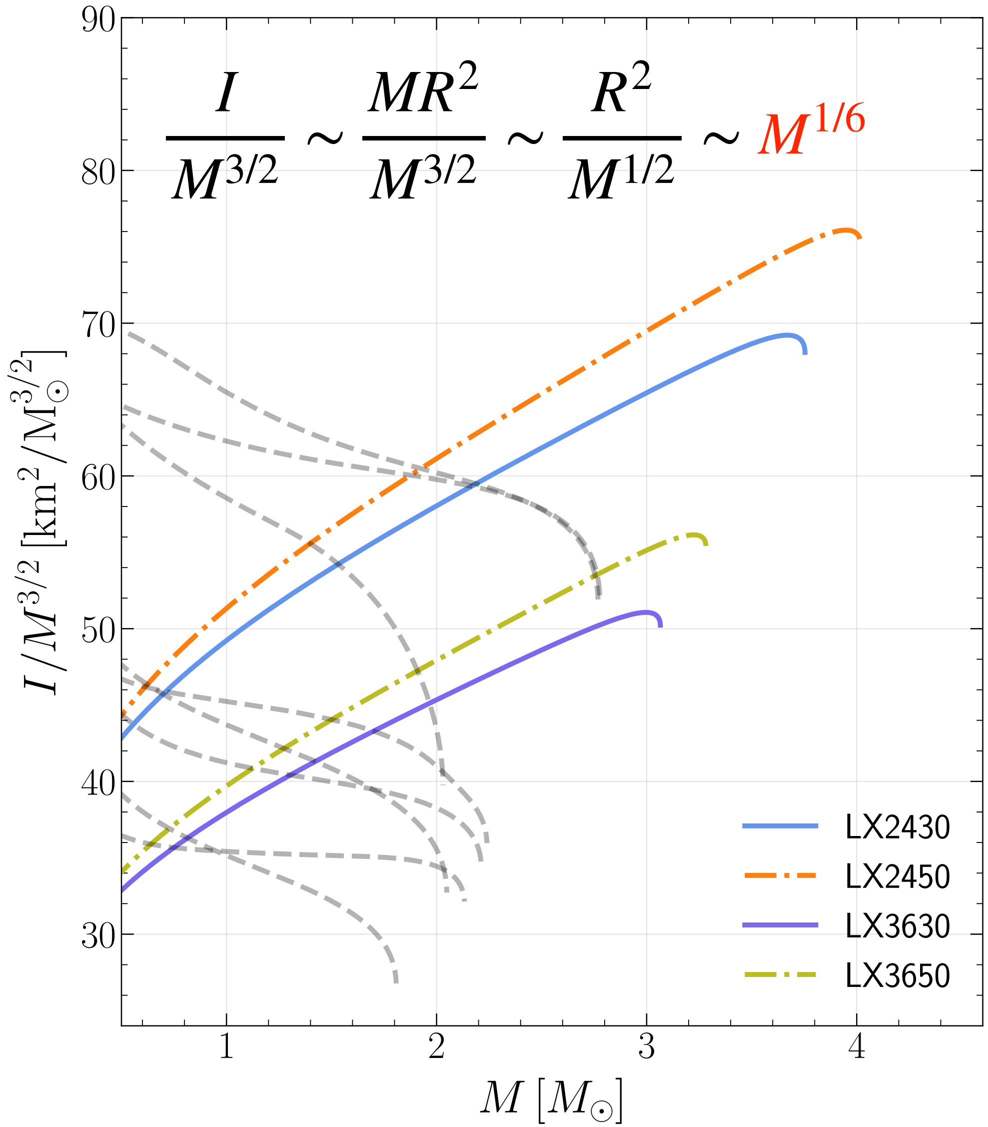
Slow rotation $O(\Omega)$

- The deviation of spherical spacetime ~ O(Ω), moment of inertia (I_0) and frame dragging



$$I \sim \frac{2}{5}MR^2 \sim 1.1\times 10^{45}\left(\frac{M}{1.4M_\odot}\right)\left(\frac{R}{10\text{km}}\right)^2 \text{g cm}^2$$

$$J^\mu = T^\mu_\nu \xi^\nu_{(\phi)} \quad I_0 = \frac{S}{\Omega} = \frac{1}{\Omega}\int_{t=\text{const}} \sqrt{-g}\,T^t_\phi d^3x$$

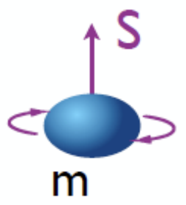


Slow rotation $O(\Omega^2)$

- Centrifugal force ~ $O(\Omega^2)$, induces mass quadrupole moment and distortion of the star

$$e \sim \frac{\Omega^2 R^3}{GM} \sim 2.1 \times 10^{-3} \left(\frac{f}{100\ \mathrm{Hz}}\right)^2 \left(\frac{R}{10\mathrm{km}}\right)^3 \left(\frac{1.4 M_\odot}{M}\right)$$

$$\sigma = (-g_{tt})^{1/2} \qquad e = \left(\frac{3Q}{Mr^*}\right)^{1/2} + O\left(\frac{1}{r^{*2}}\right)$$

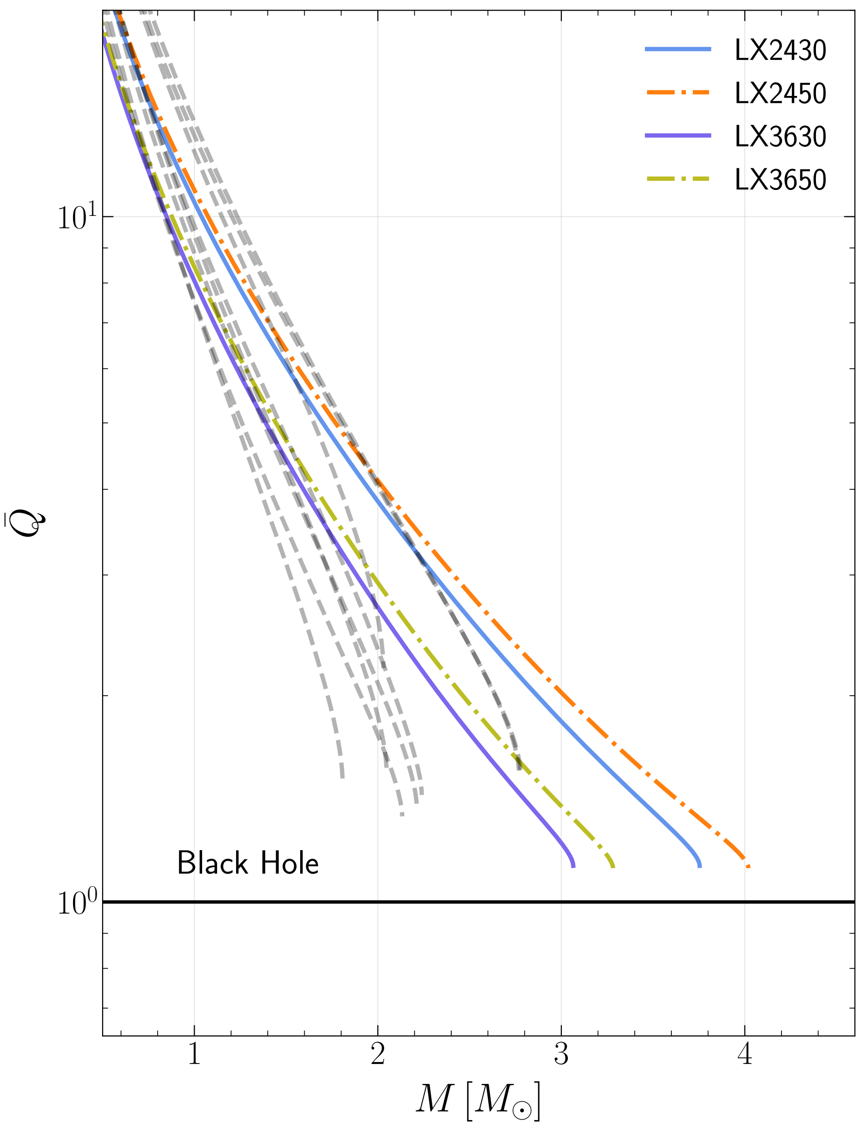


Tanja Hinderer

$$Q = -\bar{Q}\,\chi^2 m^3$$

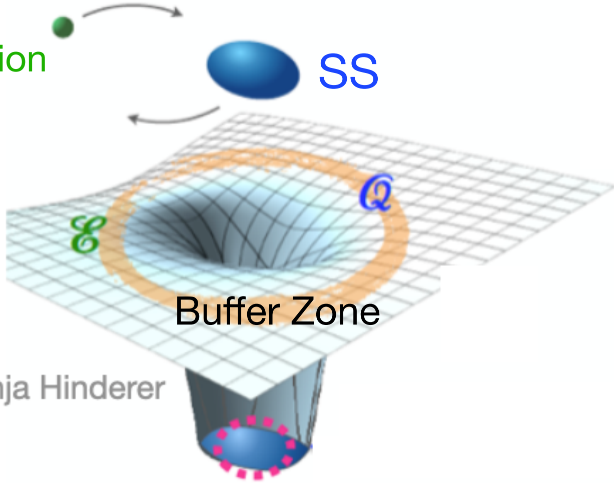
Matter dependent
=1 for BH, $\sim 1 - 15$
for NSs, SQM, SS

dimensionless spin parameter
$\lesssim 1$ for BH
$\lesssim 0.4$ for millisecond pulsars



Tidal deformation of strangeon stars

$$U_{\rm eff} = -\frac{1+g_{tt}}{2} = -\frac{GM}{r} - \frac{\left(3n^i n^j - \delta^{ij}\right) Q_{ij}}{2r^3} + O\left(r^{-4}\right) + \frac{1}{2} n^i n^j \mathcal{E}_{ij} r^2 + O\left(r^3\right)$$



$\sim \frac{1}{r}$, mass monopole of SS, M

$\sim \frac{1}{r^3}$, tidal induced mass quadrupole Q_{ij}

$\sim r^2$, tidal field produced by companion $\mathcal{E}_{ij}$

$Q_{ij} = -\lambda \mathcal{E}_{ij}$, λ **is tidal deformability**, which measures the **ability of deformation** in external tidal field.

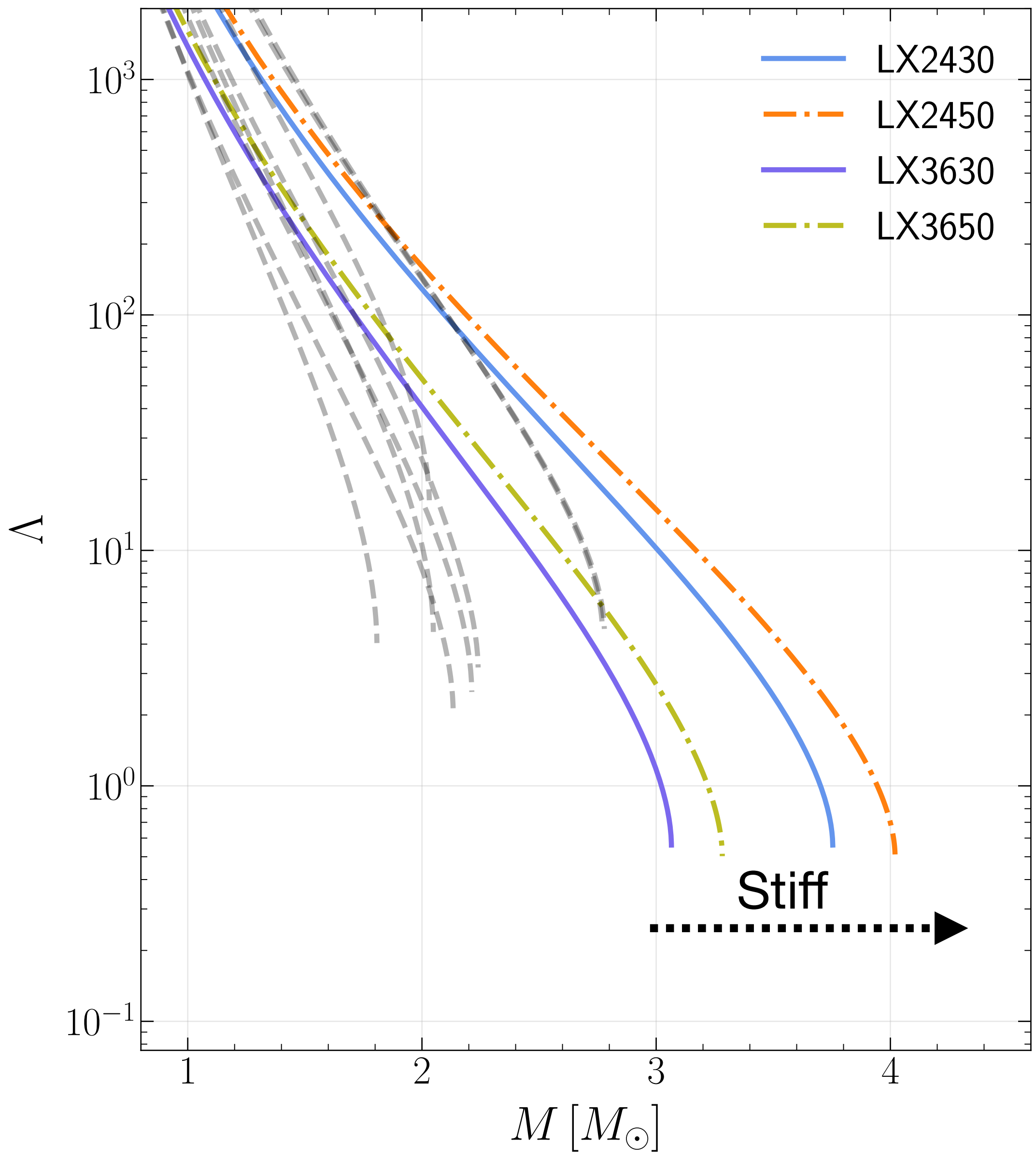
$l = 2$, even parity perturbation

$$ds^2 = -e^{\nu(r)}\left[1+H_0(r)Y_{20}(\theta)\right]dt^2 + e^{\lambda(r)}\left[1+H_2(r)Y_{20}(\theta)\right]dr^2 + r^2\left[1-K(r)Y_{20}(\theta)\right]\left(d\theta^2+\sin^2\theta d\varphi^2\right)$$

$$T_\mu^\nu = -(\rho+\delta\rho+P+\delta P)u_\mu u^\nu + (P+\delta P)\delta_\mu^\nu$$

[Hinderer & Flanangan, 2008; Damour & Nagar, 2008]

Tidal deformation of strangeon stars



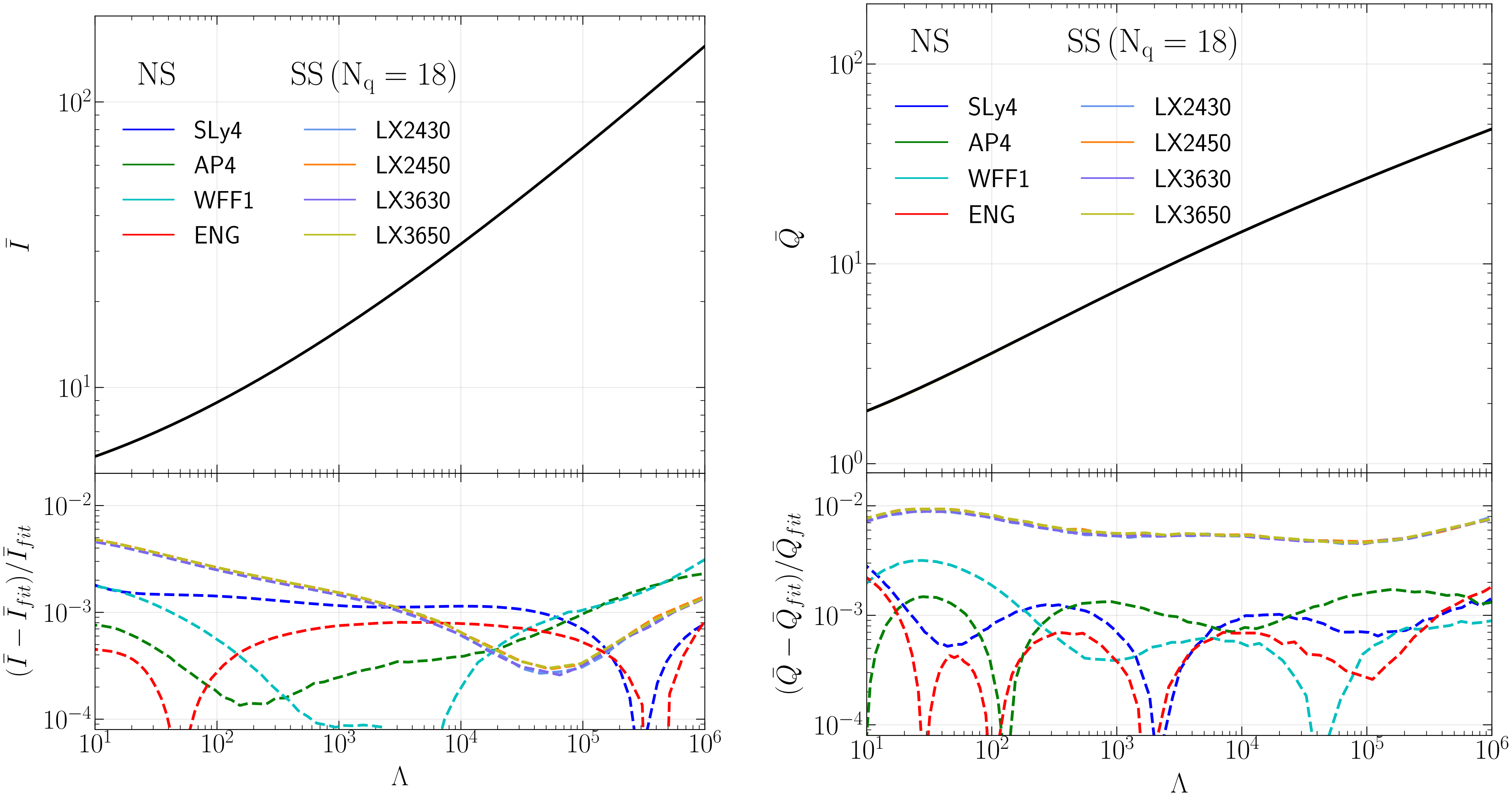
[Hinderer & Flanangan, 2008; Damour & Nagar, 2008; Enping Zhou et al., 2018]

Tidal properties:

- Surface density $\neq 0$, corrections to master equation compared to neutron star
- M increase, Λ decrease, hard to probe for large mass stars
- Stiffer EoS, larger radius for a given mass, larger Λ, easier to be deformed

$\bar{I} - \Lambda - \bar{Q}$ relation

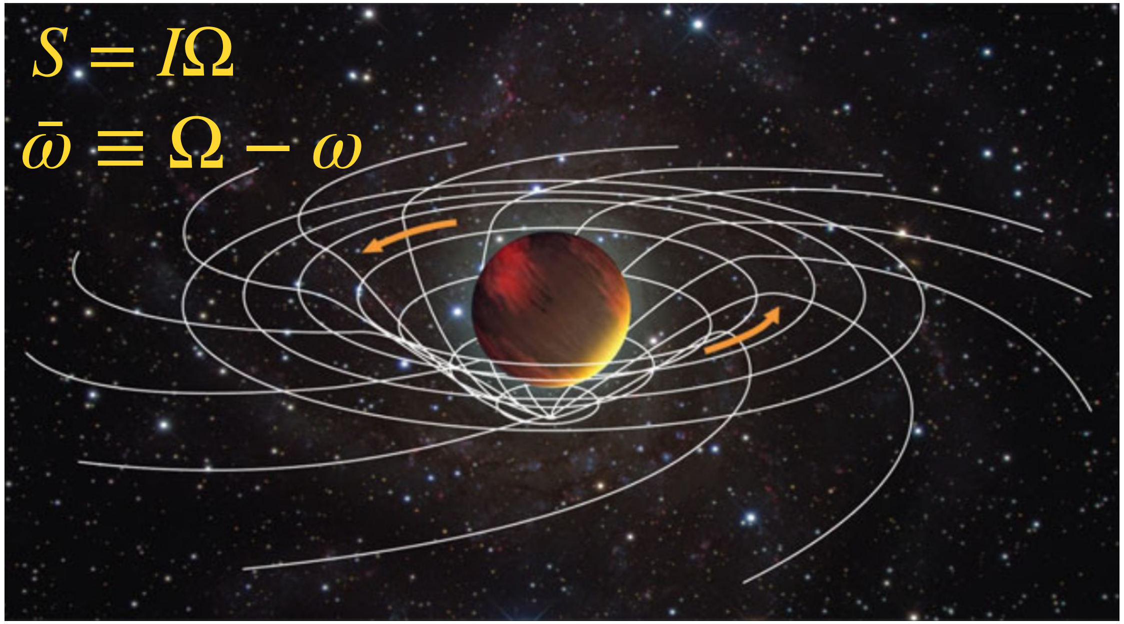
$$\bar{I} \equiv \frac{I}{M^3} \quad , \quad \Lambda \equiv \frac{\lambda}{M^5} \quad , \quad \bar{Q} \equiv -\frac{Q_{\rm rot}}{M^3 \chi^2}$$



[Yagi & Yunes, 2013]

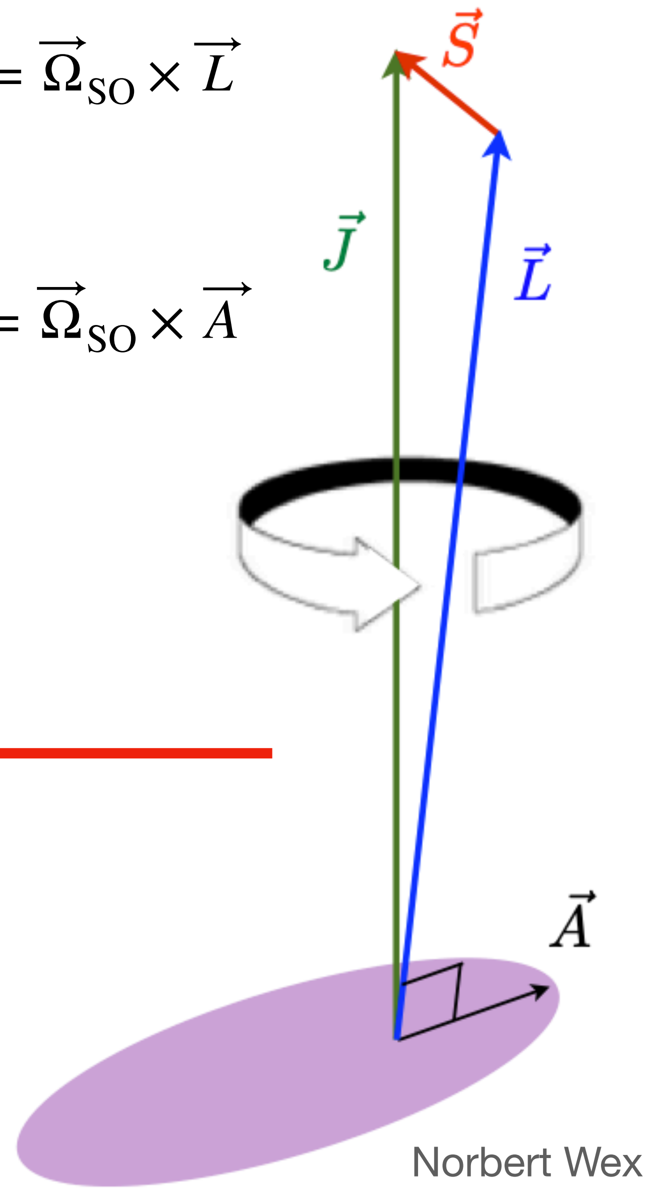
Still hold this universal relation up to 1% !

Constrain I: frame dragging



$$\left\langle \frac{d\vec{L}}{dt} \right\rangle = \vec{\Omega}_{\rm SO} \times \vec{L}$$

$$\left\langle \frac{d\vec{A}}{dt} \right\rangle = \vec{\Omega}_{\rm SO} \times \vec{A}$$



Relativistic binary, pulsar rotates rapidly

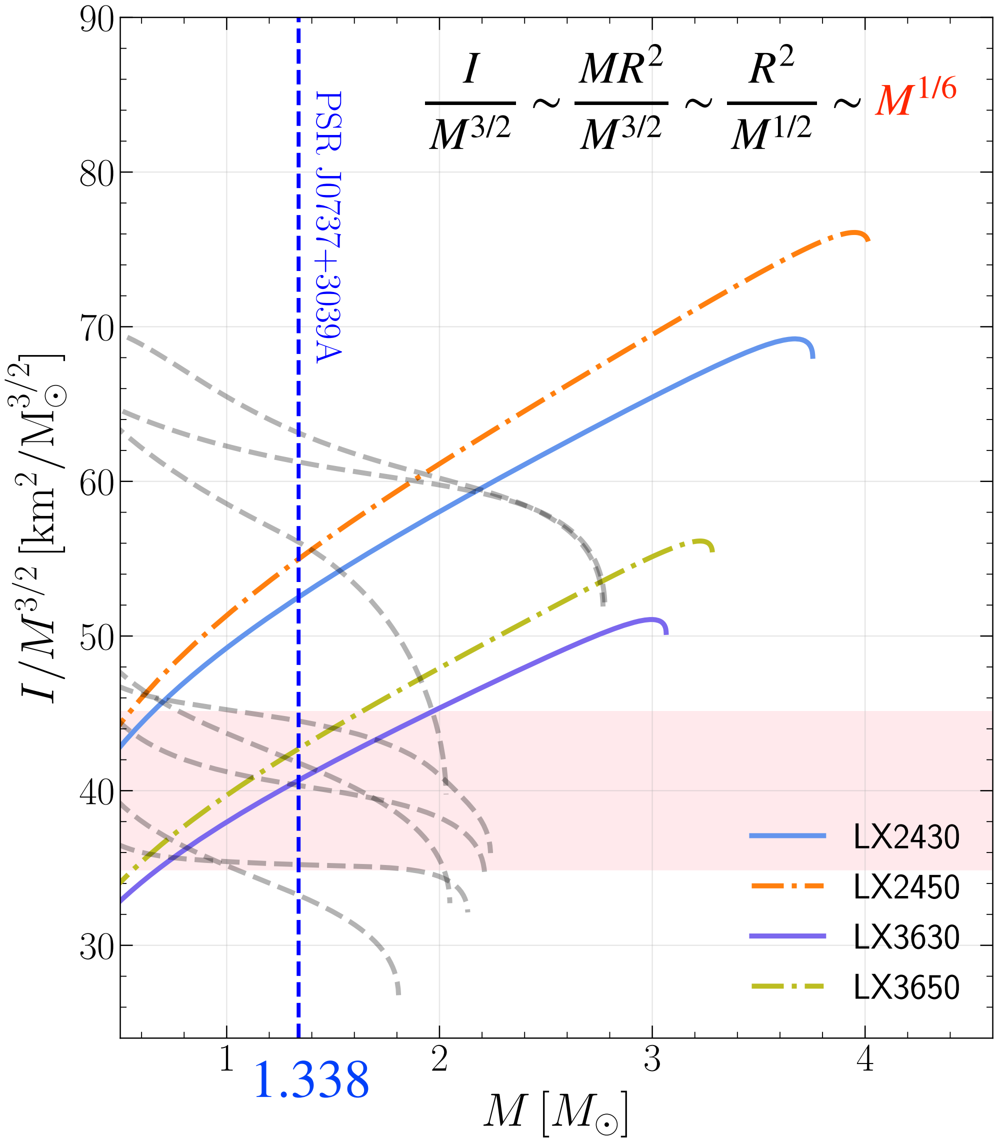
A good candidate: PSR J0737–3039A

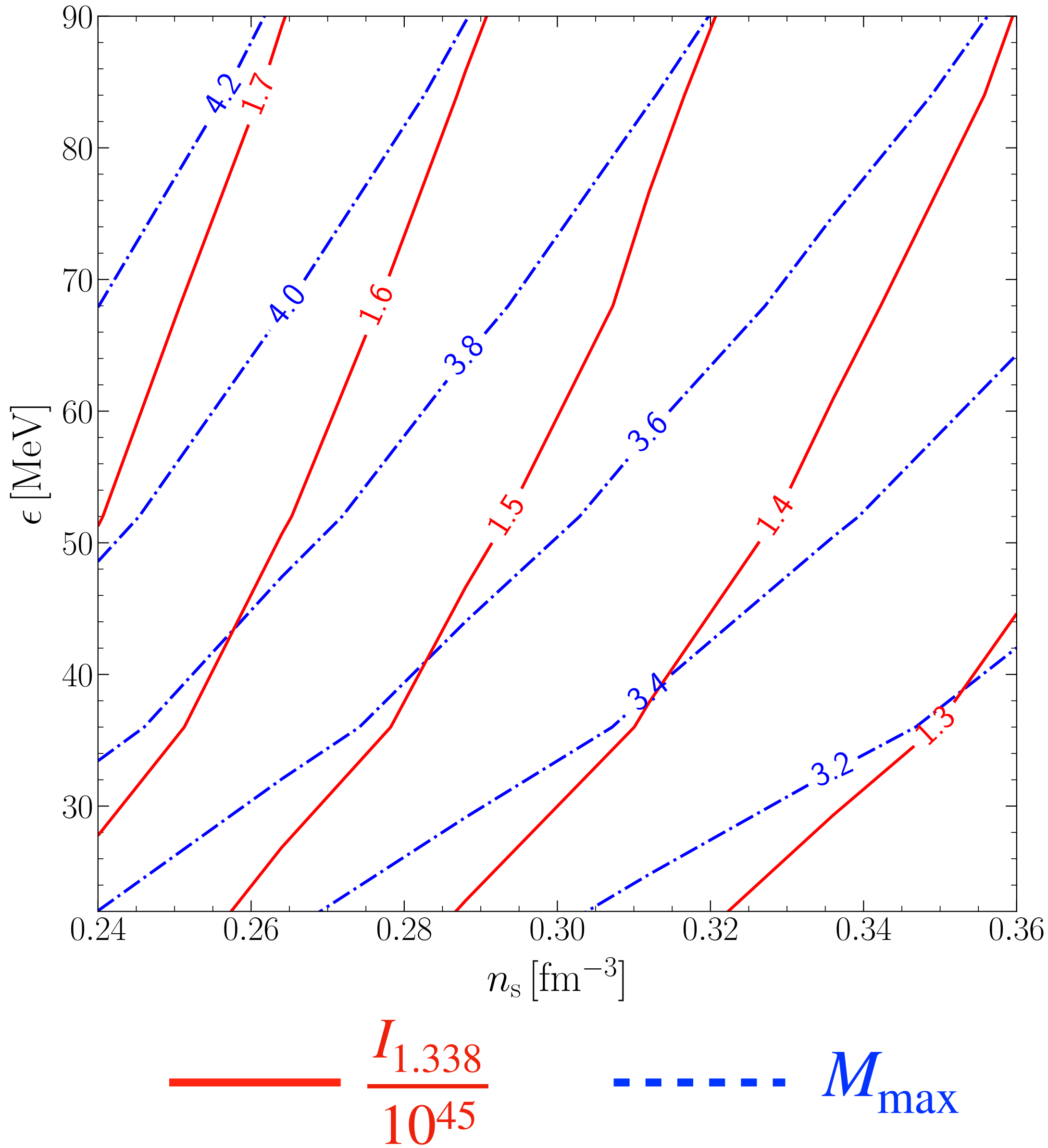
$$\dot{\omega}^{\rm intr} = \dot{\omega}^{\rm 1PN} + \dot{\omega}^{\rm 2PN} + \dot{\omega}^{\rm LT,A}$$

$$= \frac{3\beta_{\rm O}^2 n_{\rm b}}{1 - e_{\rm T}^2} \left[1 + f_{\rm O}\beta_{\rm O}^2 - g_{\rm S_A}^{\parallel} \beta_{\rm O} \beta_{\rm S_A} \right]$$

$$\beta_{\rm S_A} = \frac{c I_{\rm A} \Omega_{\rm A}}{G m_{\rm A}^2}$$

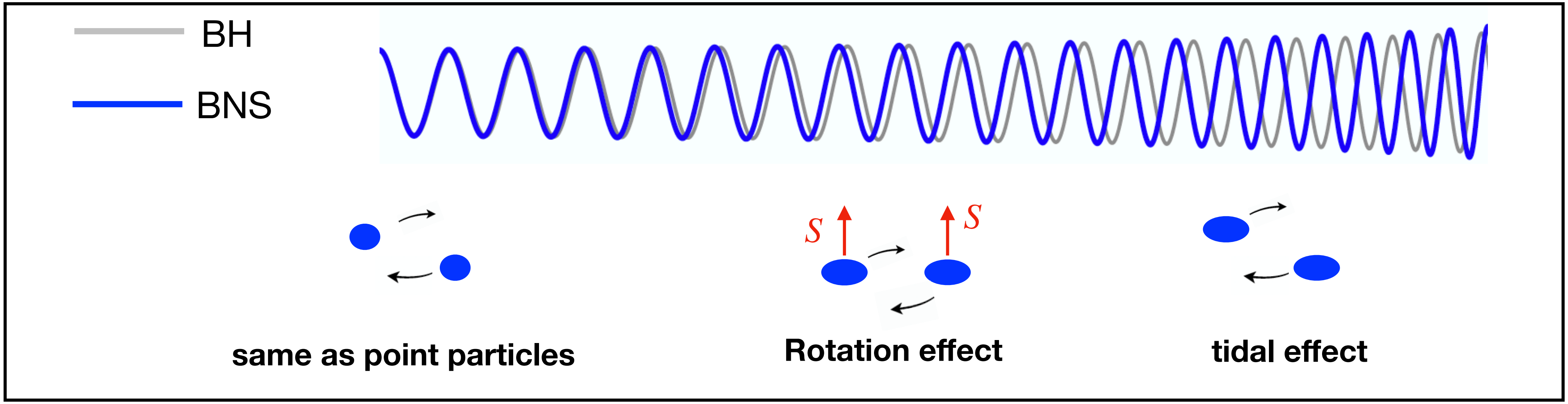
Constrain I: frame dragging

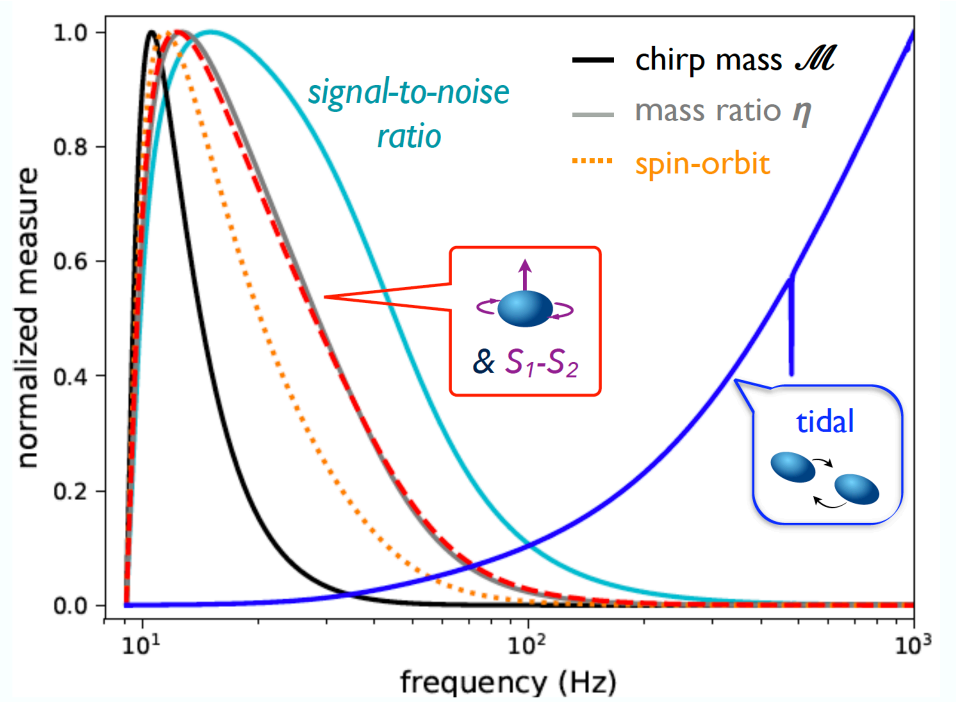




[Huanchen Hu et al., 2020, Xiaoli Miao et al., 2021]

Constrain Λ and $Q_{\rm rot}$: GWs from multipoles





- Spin induced deformation, Leading order enters at 2PN

$$\Delta\phi^{\rm spin-Q} \sim \bar{Q}\chi^2 \left(\frac{v^2}{c^2}\right)^2$$

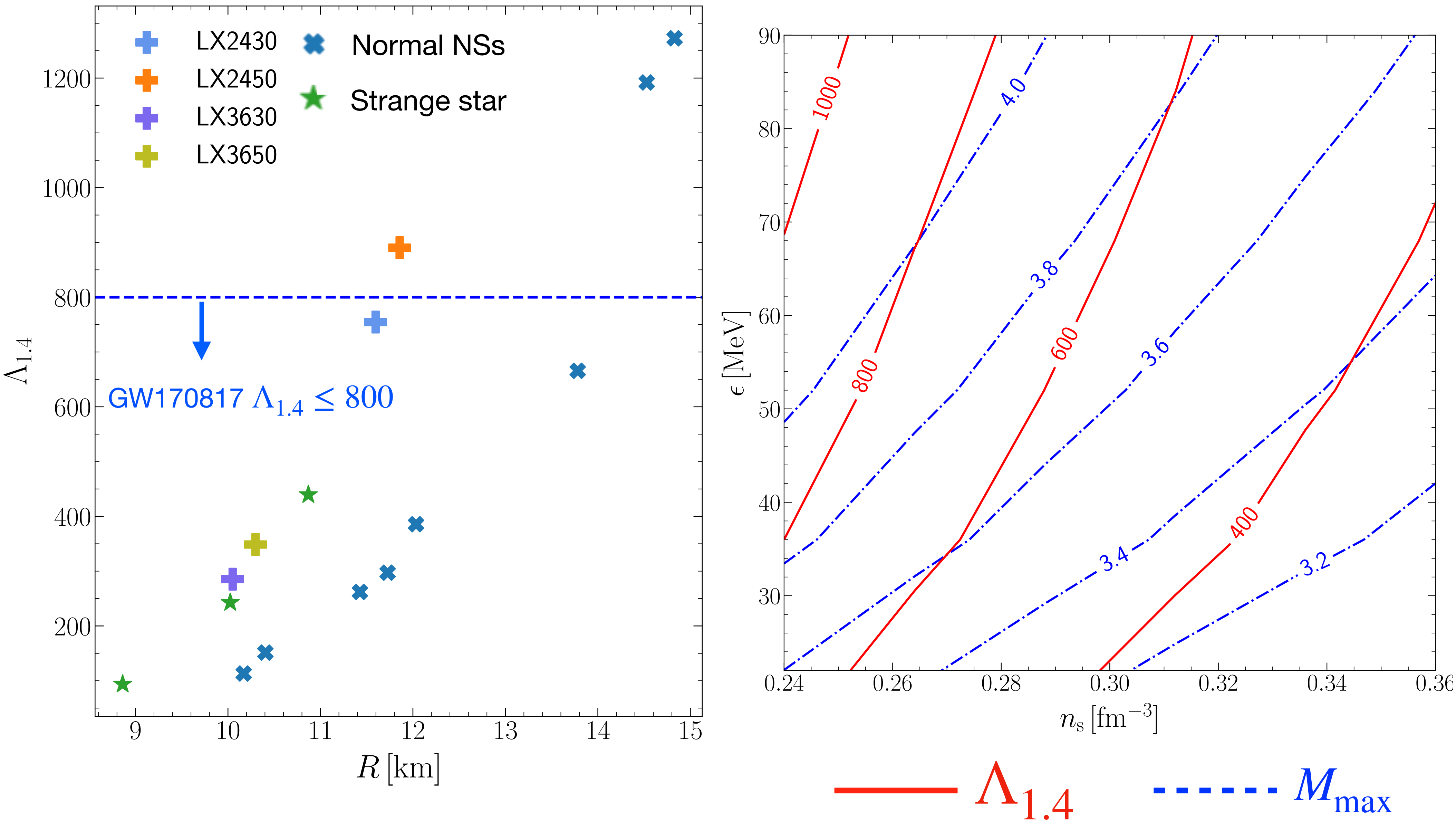
- Tidal induced deformation, Leading order enters at 5PN

$$\Delta\phi^{\rm tidal}_{\rm GW} \sim \Lambda \left(\frac{v^2}{c^2}\right)^5$$

Ian Harry & Tanja Hinderer, 2018

[Vines & Marsat et al., 2017; Nagar 2017; Tanja & Flanangan 2008, Steinhoff et al. 2017]

Constraints of tidal deformability



[Abbott et al., 2017, 2018; Xiaoyu Lai, Enping Zhou and R.-X. Xu, 2018]

Summary

- Study the static, rotating, and tidally-deformed strangeon stars
- MOI, quadrupole, tidal measurements constrain EOS
- More work needed on solid physics: shear (may violate universal relation), anisotropic matter, and microscopic physics on EOS) *—a long way to go*

Thanks!