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Precession of magnetars

dynamical evolutions and modulations on polarized electromagnetic waves

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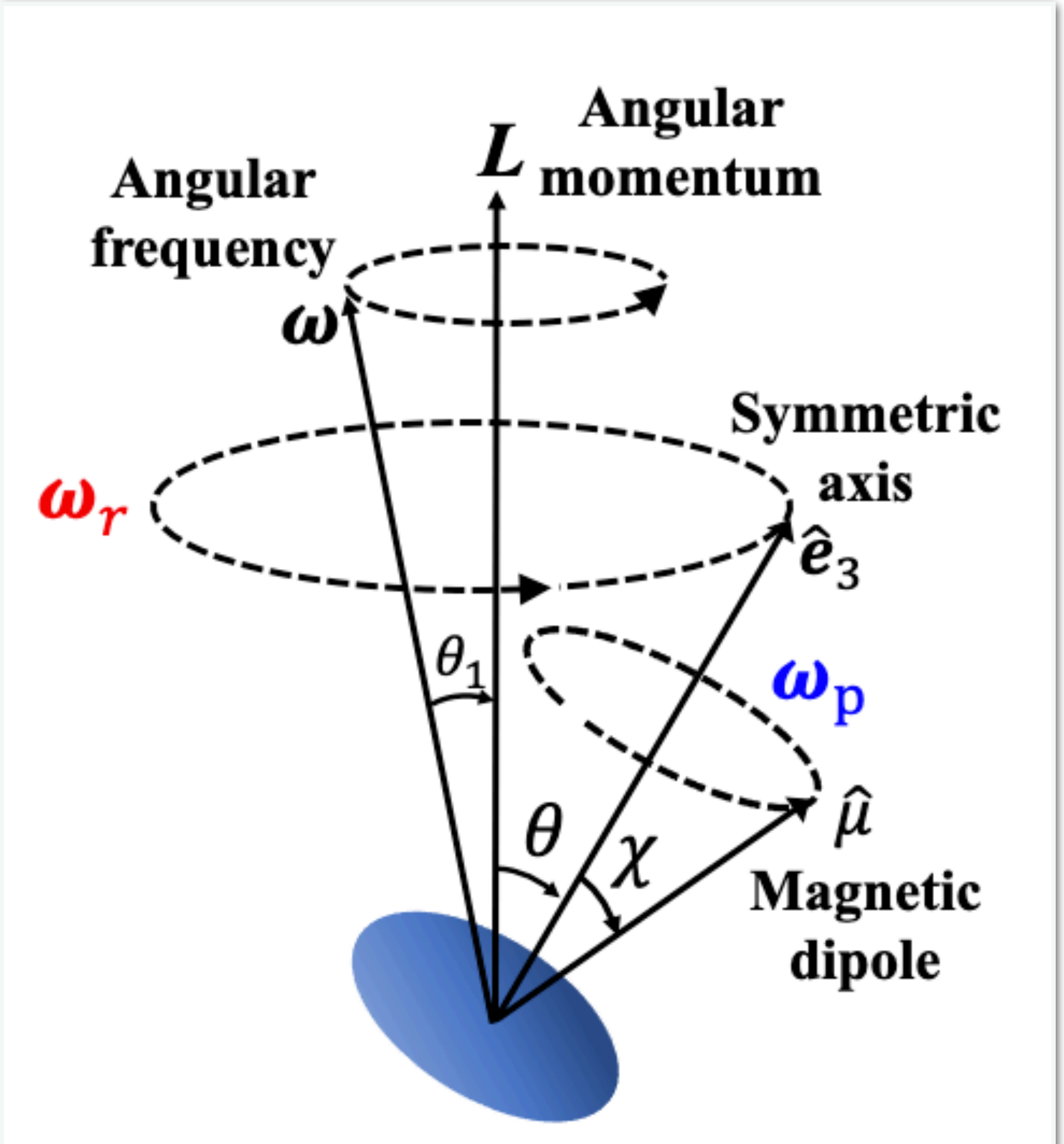
Outline

- NS free precession and magnetars as condidates
- Precession dynamics of deformed magnetars
- Timing residuals and modulations on polarized radio/X-ray emissions
- Summary

What is free precession?



Free precession of a biaxial star

Jones & Andersson, MNRAS, 2001, 2002
Link & Epstein, ApJ, 2001

- A **deformed** NS will precess when the angular momentum and the deformation axis are **not aligned**

ellipticity $\epsilon = \dfrac{\Delta I_{\rm d}}{I_0}$ wobble angle: θ

- The ellipticity for NSs is quite small

$\epsilon \ll 10^{-4}$ from current calculations

$\theta_1 \sim \epsilon\theta$, $\boldsymbol{\omega}$ and $\boldsymbol{L}$ are nearly aligned

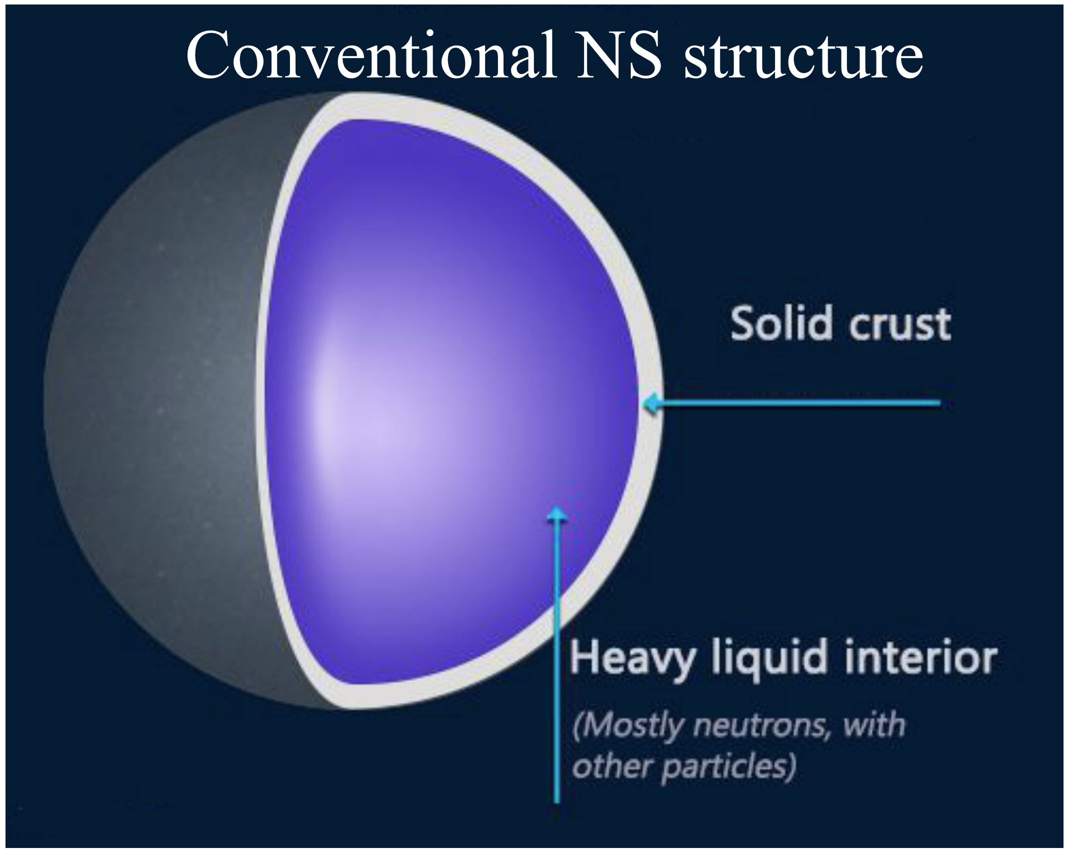
- **Two superimposed motion:**

$$\boldsymbol{\omega} = \omega_{\rm r}\hat{\boldsymbol{L}} - \omega_{\rm p}\hat{\boldsymbol{e}}_3 \qquad \omega_{\rm p} = \epsilon\cos\theta\,\omega_{\rm r}$$

$$\text{Precession period } P_{\rm f} = \frac{P}{\epsilon\cos\theta}$$

Precession of NSs: why we study it

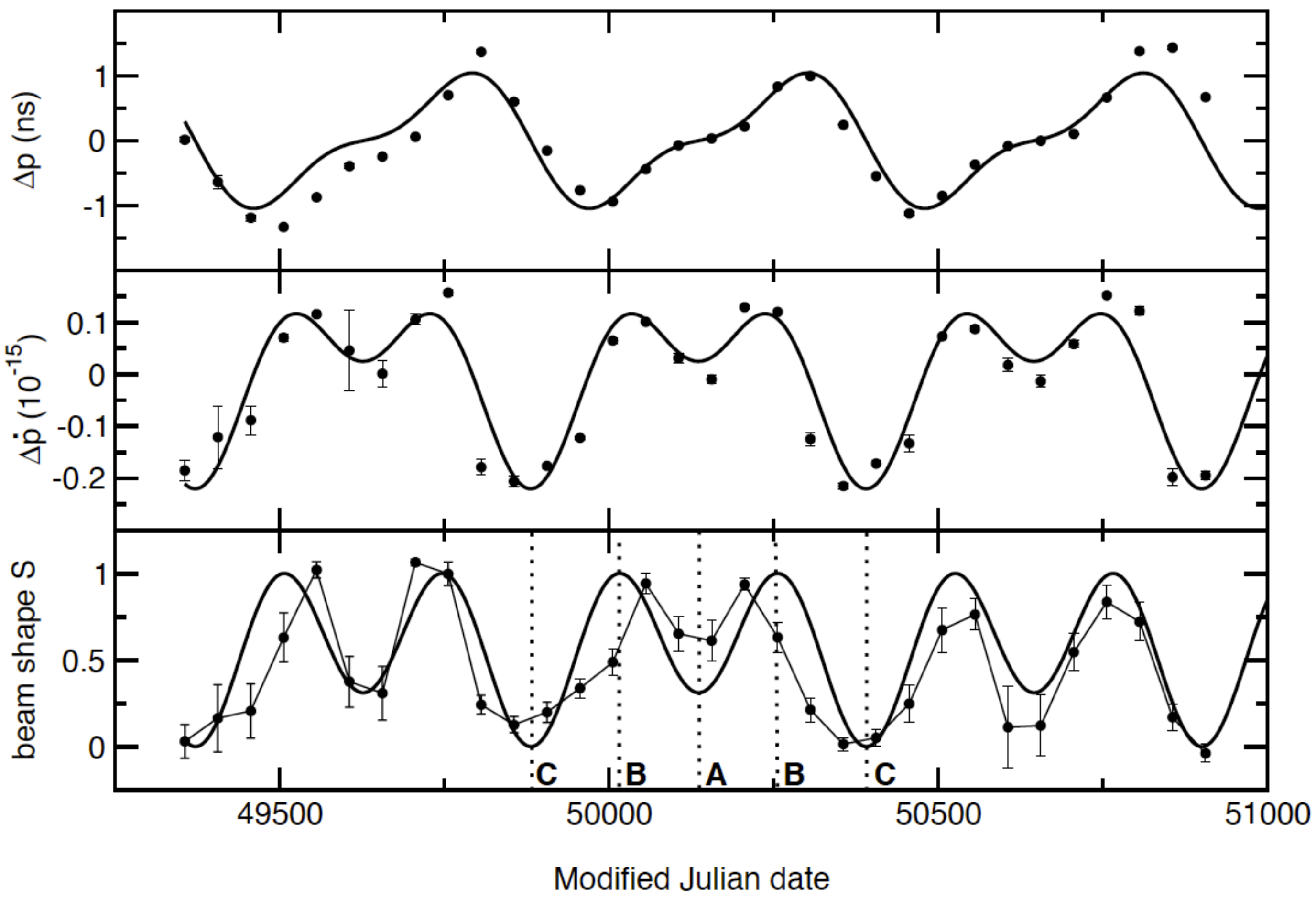
1. Elastic deformation of the crust



$$\epsilon_c = bu = 10^{-8}\left(\frac{b}{10^{-6}}\right)\left(\frac{\sigma_{\rm break}}{10^{-2}}\right)$$

Cutler et al., PRD, 2003; Gittins et al., MNRAS, 2021

Stairs, Nature, 2000; Link & Epstein, ApJ, 2001



PSR B1828-11: radio timing and beam shape

$P_{\rm f} \sim 500$ days $\quad \epsilon \sim 10^{-8}$

- Important information on NS crust physics: shear modulus & breaking strain

Precession of NSs: why we study it

Superfluid does not support long precession period without damping

- A **perfectly pinned** superfluid, the Euler equation   Shaham, ApJ, 1977

$$\boldsymbol{L}_c \dagger + \boldsymbol{\omega} \times \boldsymbol{L}_c = -\boldsymbol{\omega} \times \boldsymbol{L}_f \longrightarrow \omega_p \sim -\left(\epsilon + \frac{I_f}{I_c}\right)\omega$$

Pinning gives a precession frequency too fast!

- "Mutual friction" between superfluid and crust leads to **damping of free precession**

$$\frac{\mathrm{d}\boldsymbol{J}_{\rm shell}}{\mathrm{d}t} = K(\boldsymbol{\Omega}_{\rm fluid} - \boldsymbol{\Omega}_{\rm solid}) = -\frac{\mathrm{d}\boldsymbol{J}_{\rm fluid}}{\mathrm{d}t}$$

Alpar & Sauls, ApJ, 1988

- A glitch in PSR B1828-11 constrains moment of inertia participating into precession

$$\frac{3}{2}\frac{\delta\nu/\nu}{P/P_{\rm fp}} \leq \frac{I_{\rm prec}}{I_*} \leq 1 \Rightarrow 0.93 \leq \frac{I_{\rm prec}}{I_*} \leq 1$$

D. I. Jones et al., PRL, 2017

- Challenge our current understanding of superfluid state in NS interior

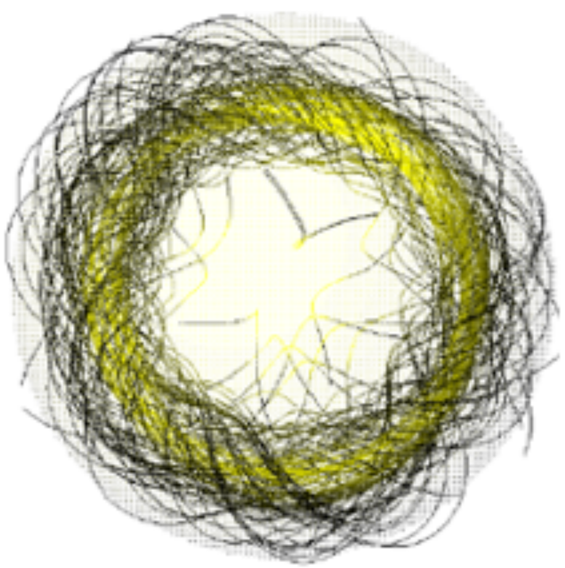
Precession of NSs: why we study it

2. Magnetic deformation due to strong internal magnetic field

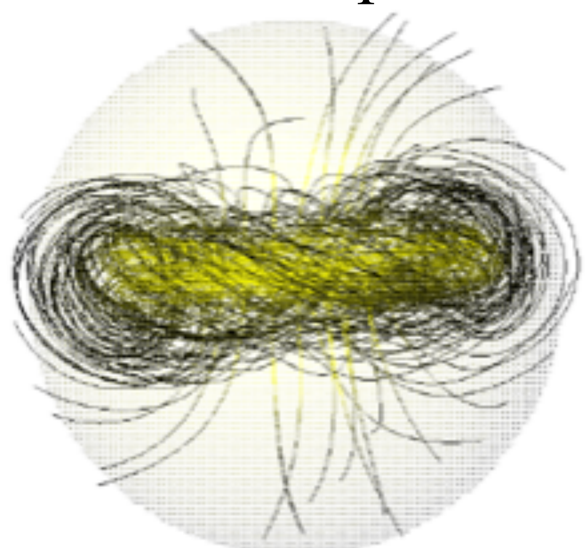
$$\epsilon_B \approx \kappa \frac{B^2 R^3}{GM^2/R} = 1.9 \times 10^{-6} \kappa B_{15}^2$$

Lander & Jones, MNRAS, 2009; Lasky & Melatos, PRD, 2014; Zanazzi & Lai, MNRAS, 2015

toroidal



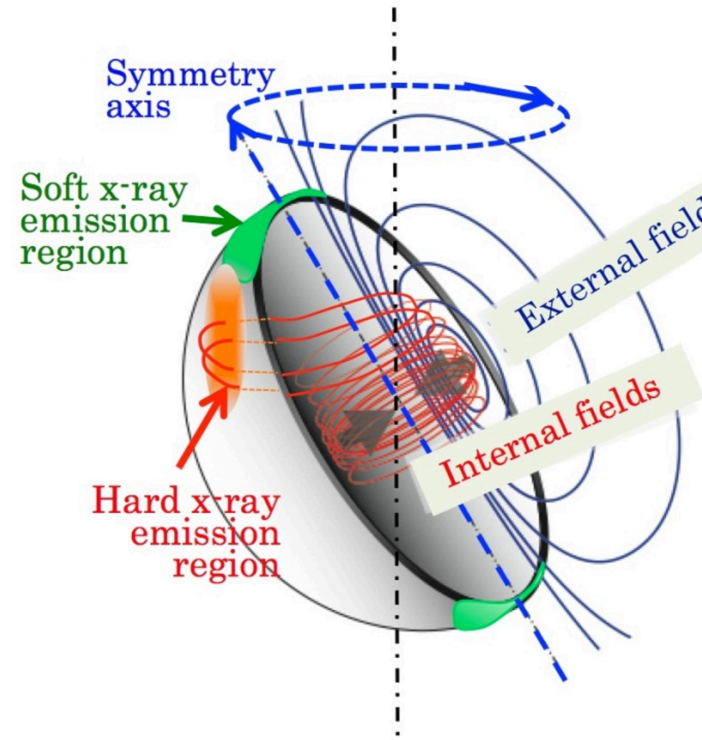
poloidal

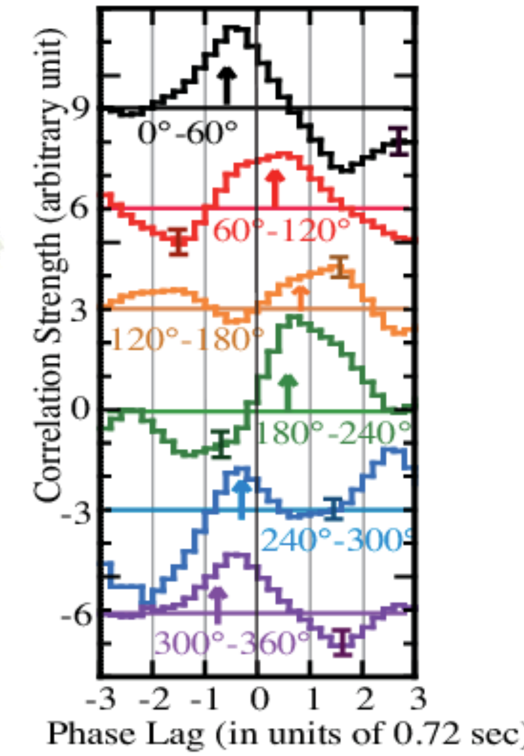


Credit: Braithwaite

Can be prolate or oblate, determined by the strength and configuration of magnetic field

Makishima, PRL, 2014





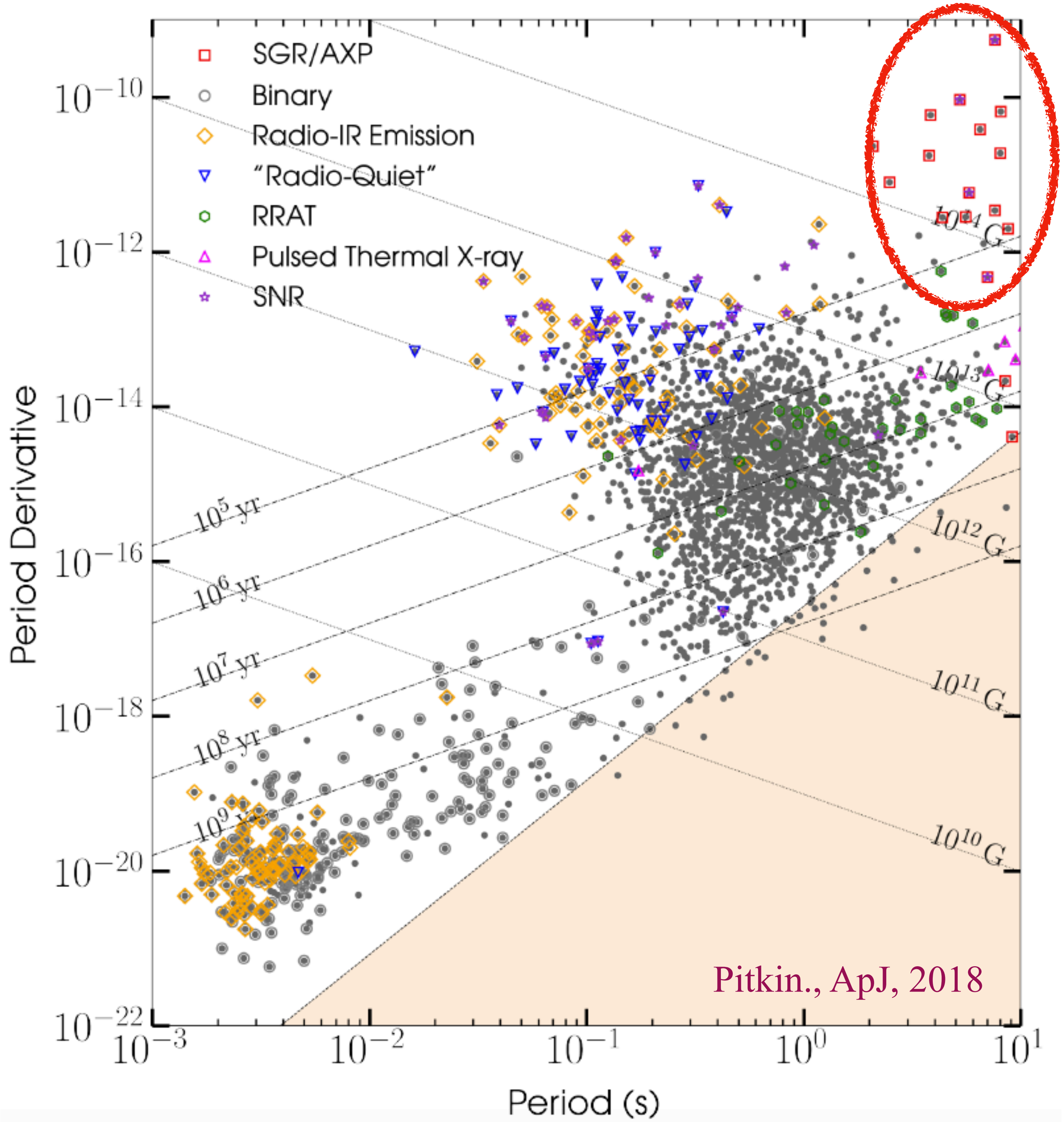
Magnetar 4U 0142+61: hard X-ray phase modulations (± 0.7 s)

$P_f \sim 15$ h $\qquad \epsilon \sim 10^{-4}$!

Indication of strong internal toroidal magnetic field in the order of 10^{16} G

- Information on NS internal magnetic field configuration and strength

Magnetars as precession candidates



Why we consider magnetars?

- Large deformation due to strong internal magnetic field

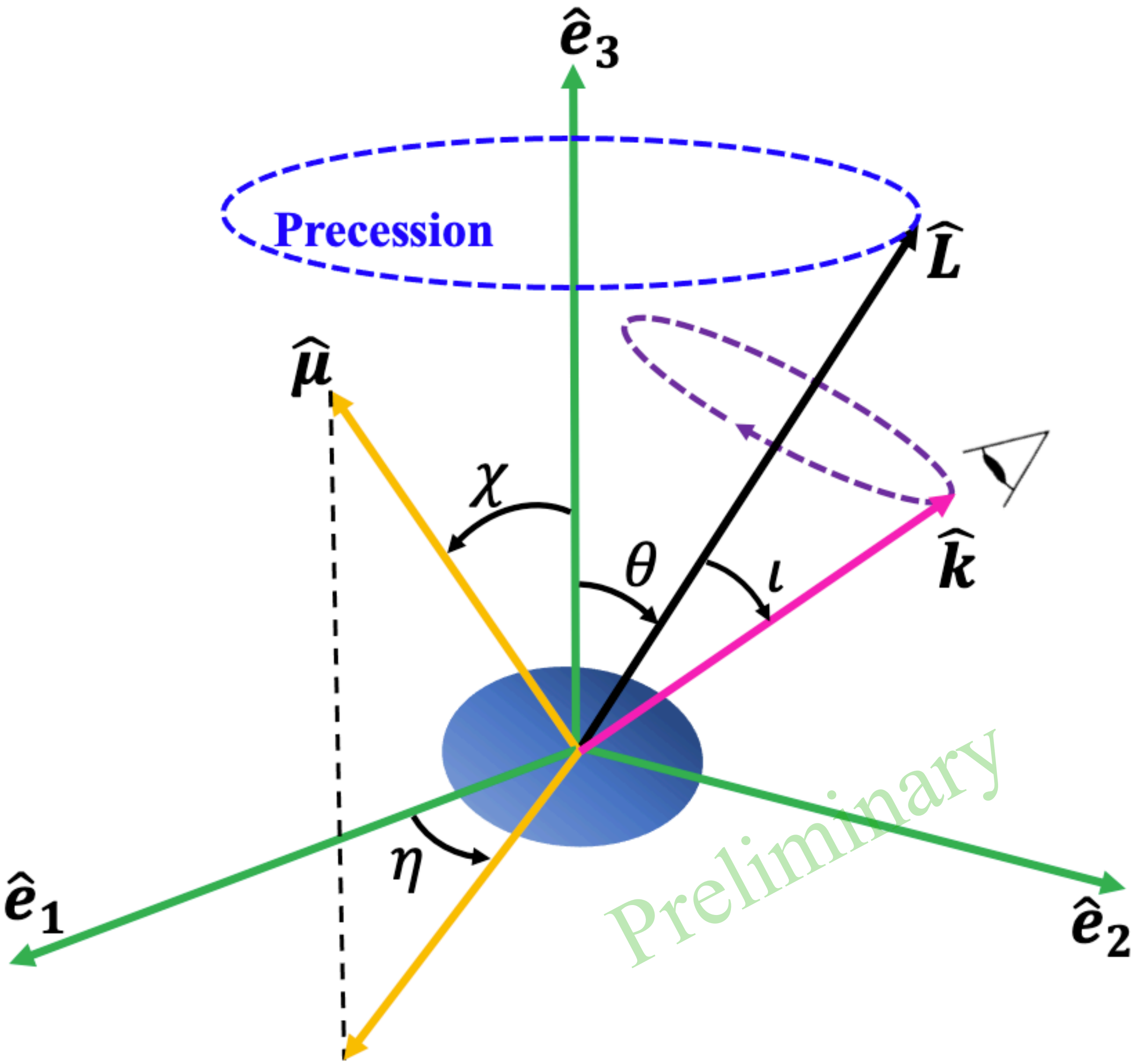
  Haskell et al., MNRAS, 2008;
  Mastrano et al., MNRAS, 2015

$$\epsilon_B \approx \kappa \frac{B^2 R^3}{GM^2/R} = 1.9 \times 10^{-6} \kappa B_{15}^2$$

- They are young and very active, energetic process may excite wobble angle and precession

  Levin et al., ApJ, 2020

Precession dynamics of magnetars: free



Precession geometry of NS

- Dynamical evolution can be obtained from the Euler equations

$$\dot{\boldsymbol{L}} + \boldsymbol{\omega} \times \boldsymbol{L} = 0$$

- General **triaxial** case and **rigid** precession, **analytical** solution

$$\epsilon \equiv \frac{I_3 - I_1}{I_1}, \quad \delta \equiv \frac{I_3\left(I_2 - I_1\right)}{I_1\left(I_3 - I_2\right)}, \quad \theta \equiv \arccos \frac{L_3}{L}$$

$$\hat{L}_1 = \sin\theta_0 \mathrm{cn}\left(\omega_{\mathrm{p}} t, m\right)$$

$$\hat{L}_2 = \sin\theta_0 \sqrt{1+\delta}\, \mathrm{sn}\left(\omega_{\mathrm{p}} t, m\right)$$

$$\hat{L}_3 = \cos\theta_0 \mathrm{dn}\left(\omega_{\mathrm{p}} t, m\right)$$

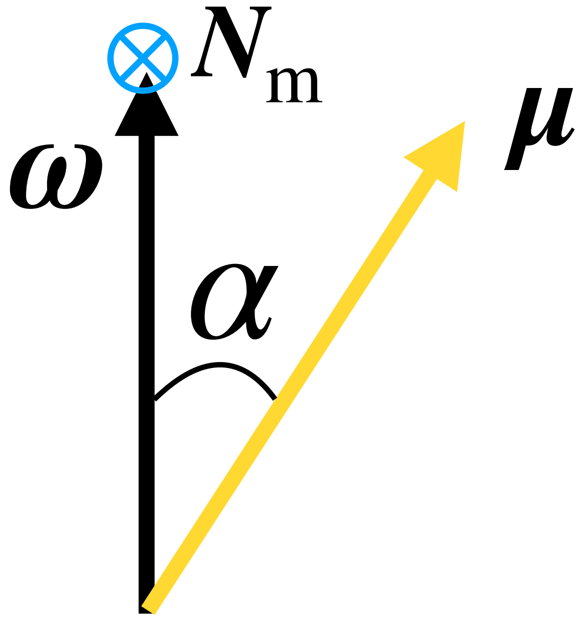
- Special biaxial case: degenerates into simple harmonic functions
- Jacobi elliptic function is not 2π periodic

Precession dynamics of magnetars: forced

- Large magnetic field indicates large electromagnetic torques

The near-field torque

$$\boldsymbol{N}_{\rm m} = \frac{3\omega^2\mu^2}{5Rc^2}(\hat{\boldsymbol{\omega}} \cdot \hat{\boldsymbol{\mu}})(\hat{\boldsymbol{\omega}} \times \hat{\boldsymbol{\mu}})$$

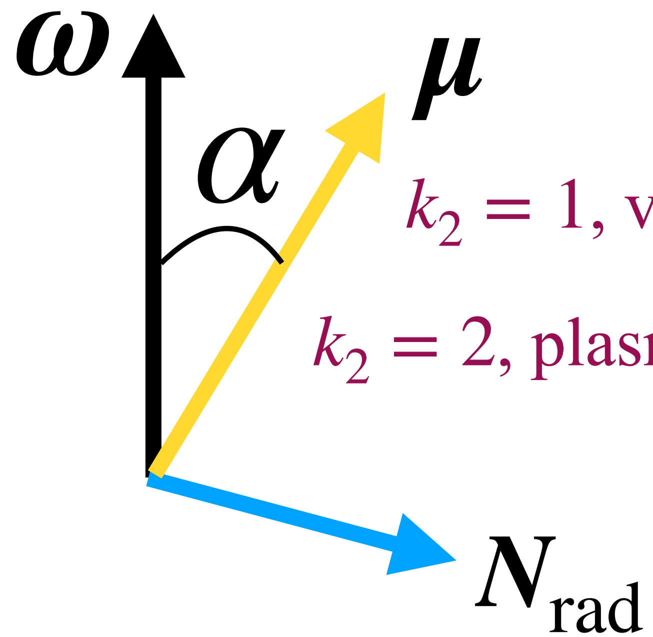


Does not dissipate energy, but change the geometry

$$\tau_{\rm m} = \frac{5RI_0c^2}{3\omega\mu^2} = 3.36\, M_{1.4}R_6P_1B_{14}^{-2}\,{\rm yr}$$

The far-field torque (spindown torque)

$$\boldsymbol{N}_{\rm rad} = \frac{k_1\mu^2\omega^3}{c^3}\left[(\hat{\boldsymbol{\omega}} \cdot \hat{\boldsymbol{\mu}})\hat{\boldsymbol{\mu}} - k_2\hat{\boldsymbol{\omega}}\right]$$



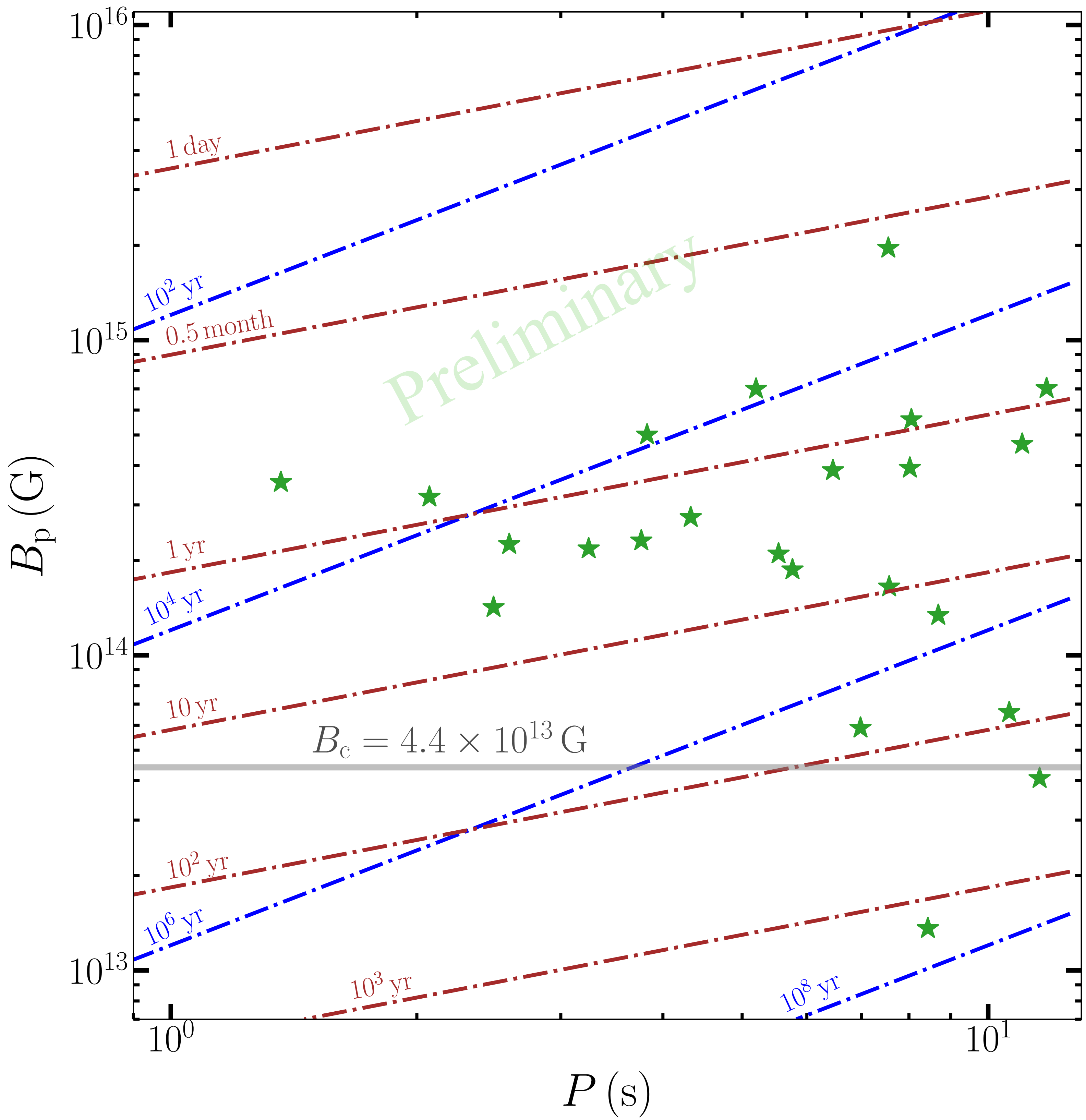
$k_2 = 1$, vacuum torque

$k_2 = 2$, plasma-filled torque

Dissipates energy (spindown) and change the geometry

$$\tau_{\rm rad} = \frac{3c^3I_0}{2\mu^2\omega^2} = 1.44 \times 10^4\, M_{1.4}P_1^2B_{14}^{-2}\,{\rm yr}$$

Two kinds of torques



$$\tau_f \sim \frac{P}{\epsilon} = 1.58 P_5 \epsilon_7^{-1}\ \mathrm{yr}$$

- For magnetars:

$$\tau_f\ ,\ \tau_m \ll \tau_{rad}$$

- **The forced precession under the far-field torque can be obtained by perturbation method**
- **In some cases, $\tau_f \sim \tau_m$, couples to precession on precession timescale, cannot use perturbation**

Precession dynamics under the near-field torque

$$\dot{\boldsymbol{L}} + \boldsymbol{\omega} \times \boldsymbol{L} = \frac{3\omega^2\mu^2}{5Rc^2}(\hat{\boldsymbol{\omega}} \cdot \hat{\boldsymbol{\mu}})(\hat{\boldsymbol{\omega}} \times \hat{\boldsymbol{\mu}})$$

Originating from the MoI of EM field itself

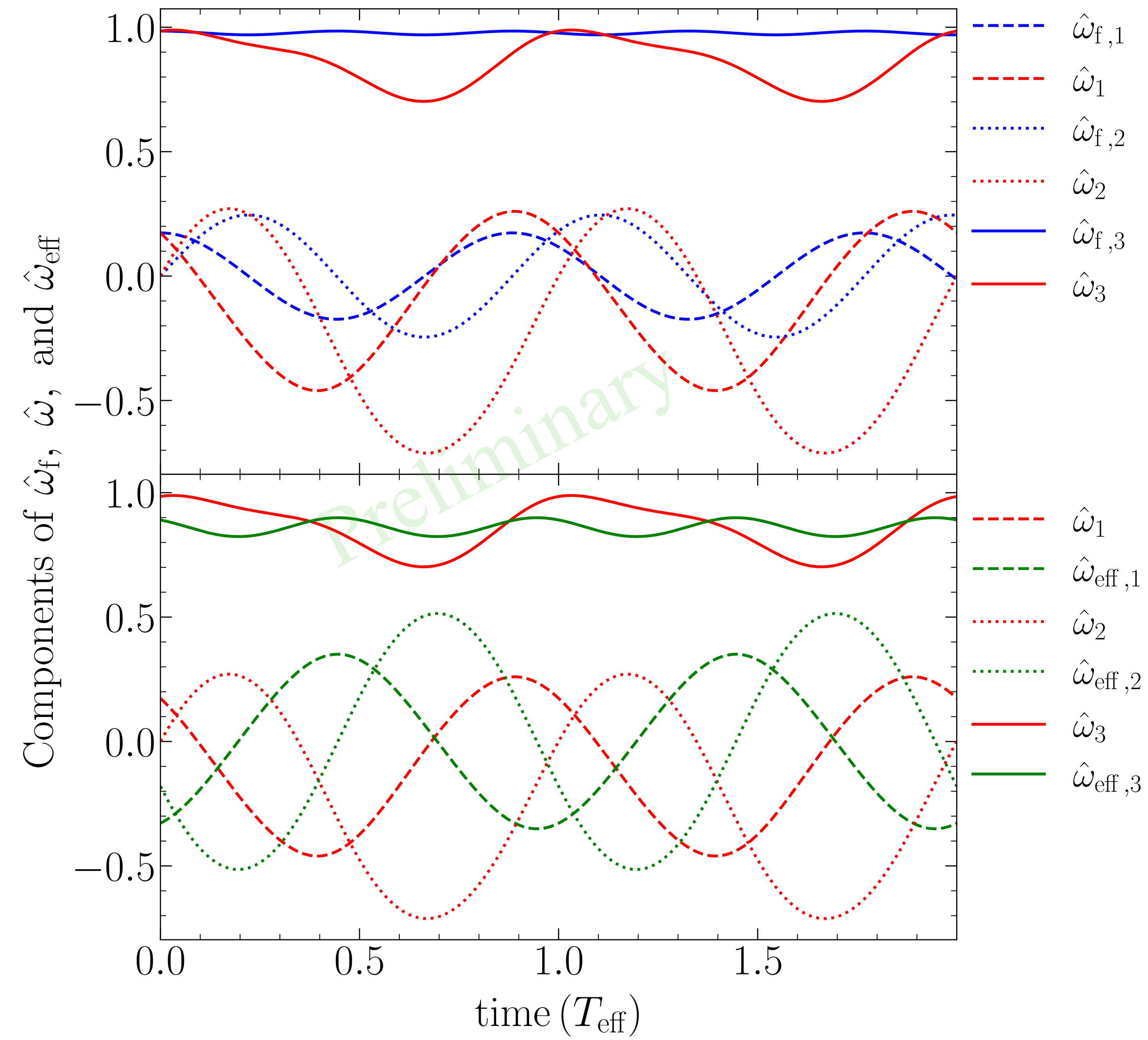
$$\dot{\boldsymbol{L}} + \boldsymbol{\omega} \times (\boldsymbol{L} + \boldsymbol{\omega} \cdot \boldsymbol{M}) = 0$$

$$\boldsymbol{M} = -I_0\epsilon_{\rm m}(\hat{\boldsymbol{\mu}} \otimes \hat{\boldsymbol{\mu}})$$

$$\epsilon_{\rm m} = \frac{3\mu^2}{5I_0Rc^2} = 1.5 \times 10^{-9} M_{1.4}^{-1} B_{14}^2 R_6^3$$

Transform into an effective free precession problem

$$\dot{\boldsymbol{L}}_{\rm eff} + \boldsymbol{\omega} \times \boldsymbol{L}_{\rm eff} = 0$$

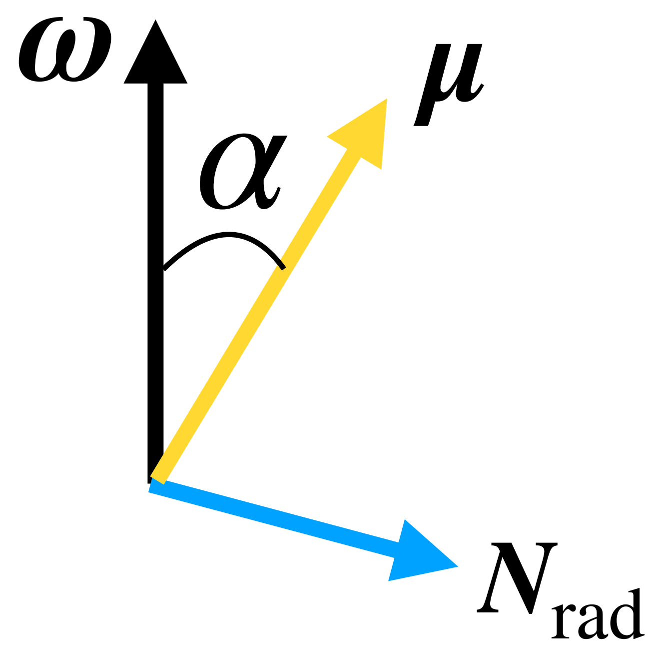


A triaxial case

$P = 5\,{\rm s}$, $\epsilon = 10^{-7}$, $\delta = 1$, $\theta_0 = 15^\circ$, $T_{\rm eff} = 2.59\,{\rm yr}$

Precession dynamics under the far-field torque

$$\dot{\boldsymbol{L}} + \boldsymbol{\omega} \times \boldsymbol{L} = \frac{k_1 \mu^2 \omega^3}{c^3} \left[(\hat{\boldsymbol{\omega}} \cdot \hat{\boldsymbol{\mu}})\hat{\boldsymbol{\mu}} - k_2 \hat{\boldsymbol{\omega}} \right]$$



Taking the dot product

$$\dot{L} = \boldsymbol{N}^{\parallel}_{\rm rad} \cdot \hat{\boldsymbol{L}} \simeq \frac{3 k_1 I_0 \omega}{2\tau_{\rm rad}} \left(\cos^2 \alpha - k_2 \right)$$

$$\omega(t) = \omega_0 (1 + \ell(t))$$

$k_2 = 1$, vacuum torque

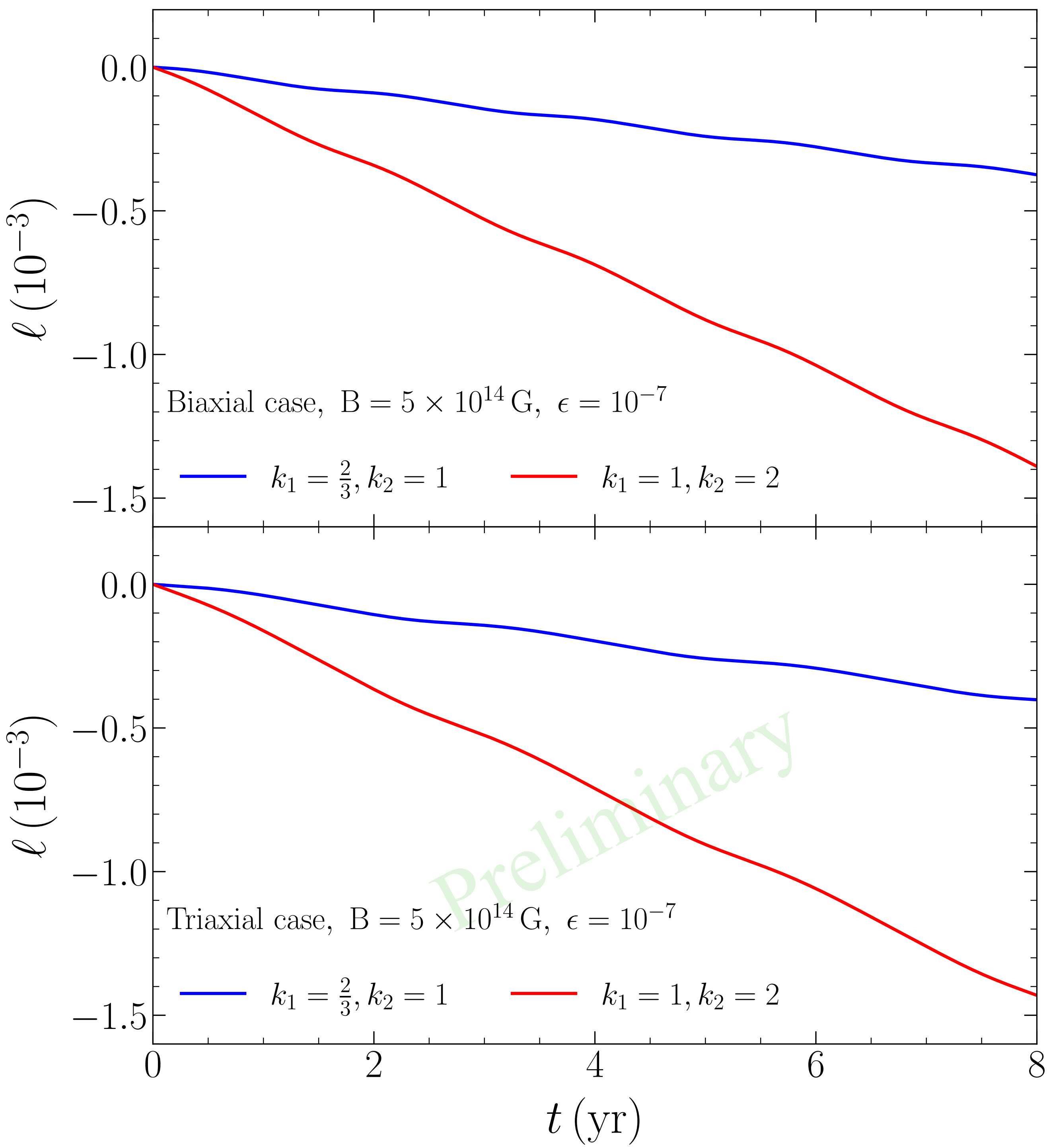
$k_2 = 2$, plasma-filled torque

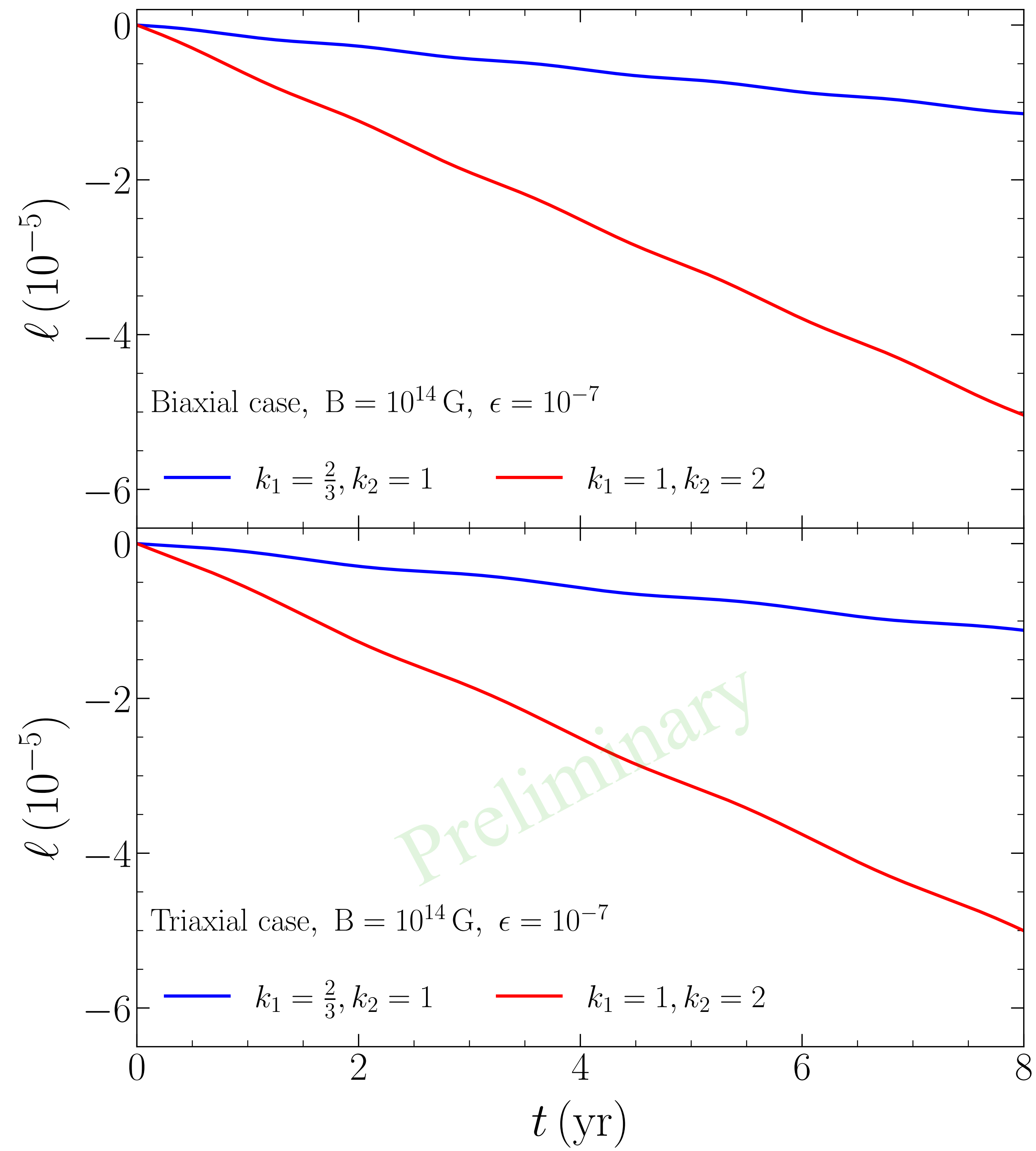
$$\ell = -\frac{3k_1}{2\tau_{\rm rad}} \left(k_2 t - \int_0^t \cos^2 \alpha \, {\rm d}t \right)$$

↓

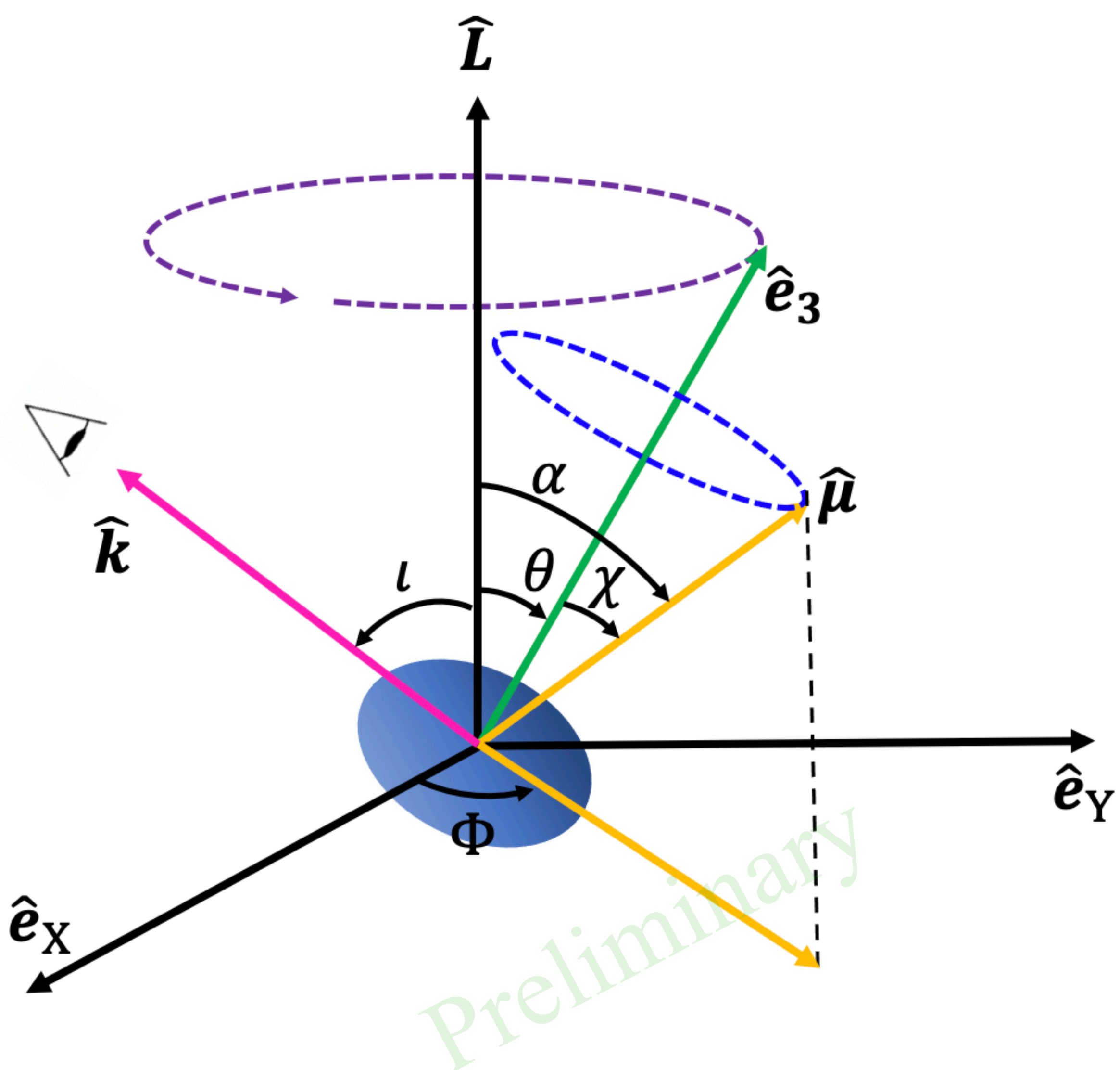
Can be integrated analytically, α changes periodically, leading to periodical modulations on the angular frequency

Precession dynamics under the far-field torque





Precession modulates emissions—key points



Precession geometry in inertial frame

- The rotation phase Φ are different in different precession epoch

Phase modulations and timing residuals

- The angle α changes periodically with precession period P_f

$$\left[\left|\theta_{\min}-\chi\right|, \left|\theta_{\max}-\chi\right|\right]$$

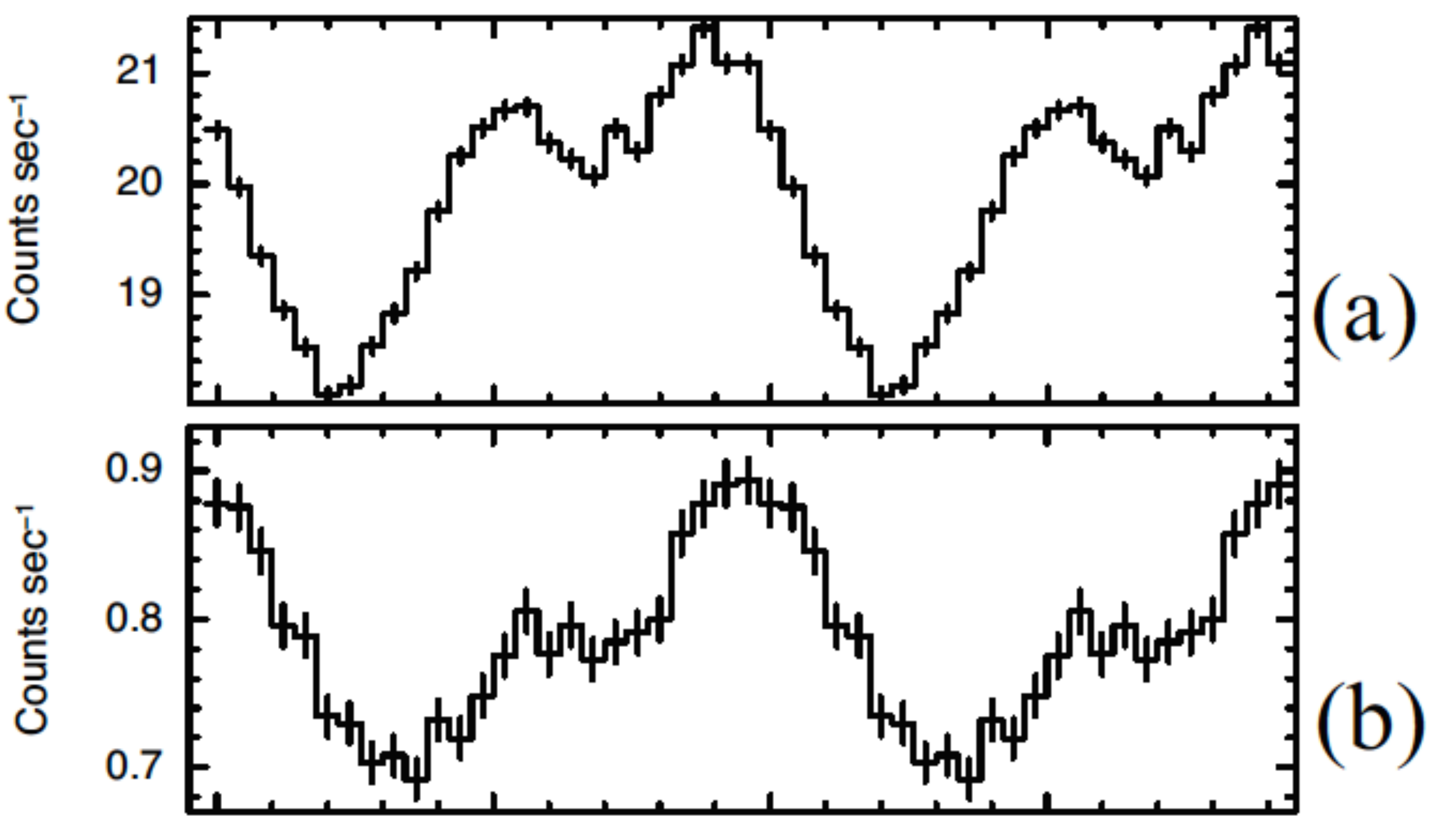
Swing of the emission region

Modulate flux, profile, polarization,…

X-ray pulsations of magnetars

- Some magnetars are **persistent X-ray sources** with a luminosity $L_x = 10^{33} - 10^{36}\ \mathrm{erg\ s^{-1}}$

Pulse of 4U 0142+61



Enoto et al., ApJ, 2011

- Show clear X-ray pulsations due to their spin
- Timing has been obtained for most magnetars

Timing residuals from X-ray pulsations

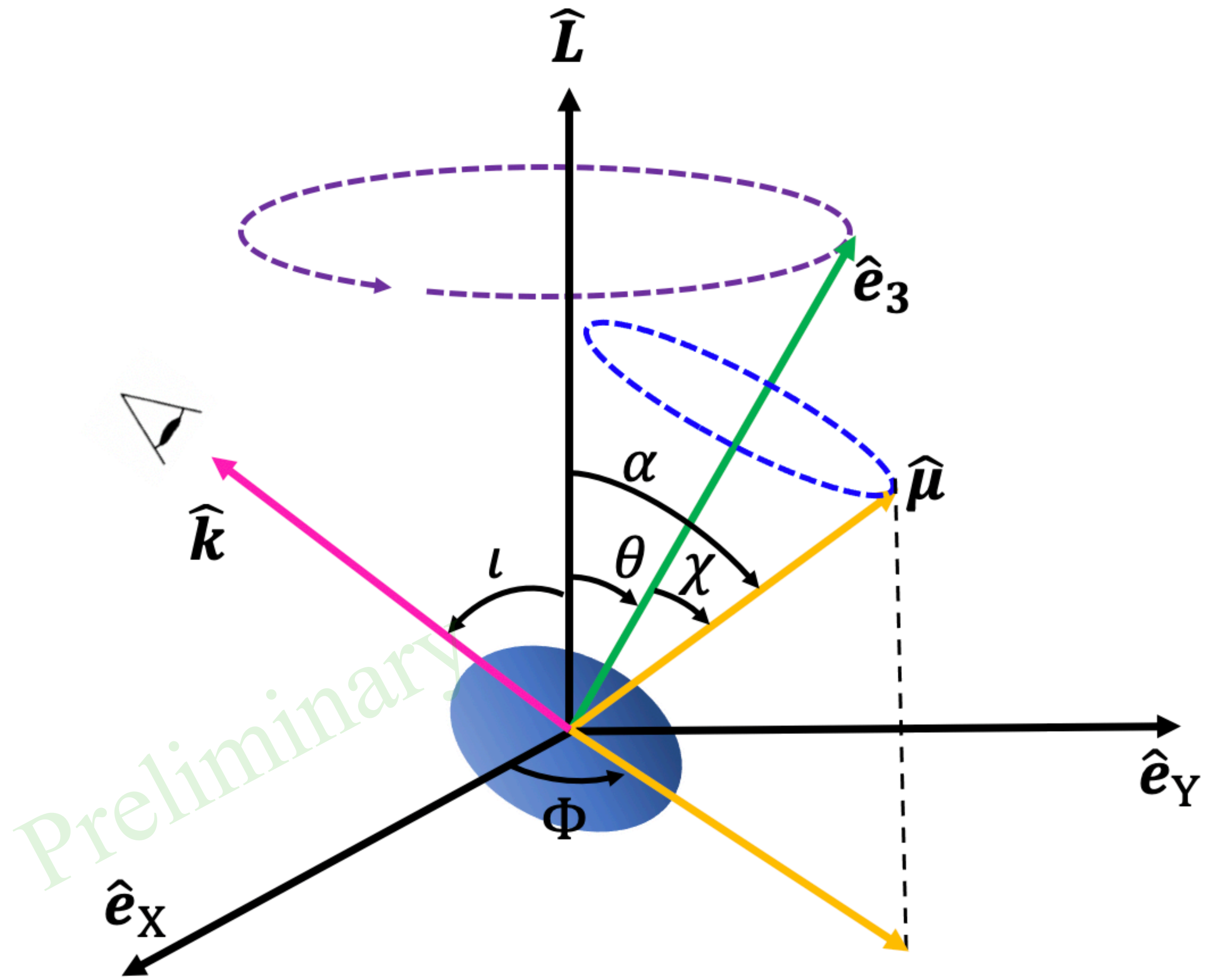
$$\Delta P_{\rm fp} = \left(\frac{\mathrm{d}\arctan\phi_1}{\mathrm{d}\tau} - \frac{\sqrt{1+\delta}/\cos\theta_0}{1+\delta\,\mathrm{sn}^2\,\tau} \right) \frac{\epsilon\cos\theta_0 P_0}{\sqrt{1+\delta}}$$

$$\tau = \omega_{\rm p} t + \psi_0$$

$$\tan\phi_1 = \frac{\hat{\mu}_1\cos\psi - \hat{\mu}_2\sin\psi}{\hat{\mu}_2\cos\theta\cos\psi - \hat{\mu}_3\sin\theta + \hat{\mu}_1\cos\theta\sin\psi}$$

$$\Delta P_{\rm sd} = -\frac{3k_1P_0}{2\tau_{\rm rad}} \left(\int_0^t \cos^2\alpha\,\mathrm{d}t - \left\langle \int_0^t \cos^2\alpha\,\mathrm{d}t \right\rangle t \right)$$

$$\approx \frac{3k_1P_0}{2\tau_{\rm rad}\omega_{\rm p}} \left\{ a_1\mathrm{cn}\tau + a_2\mathrm{sn}\tau + a_3\mathrm{dn}\tau + a_4\left[\frac{E(m)}{K(m)}\tau - E(\mathrm{am}\tau)\right] + B_{\rm c} \right\}$$



1. Geometric term

$$\frac{\Delta P_{\rm fp}}{P} = \text{Geometric factor} \times \frac{P}{\tau_{\rm f}}$$

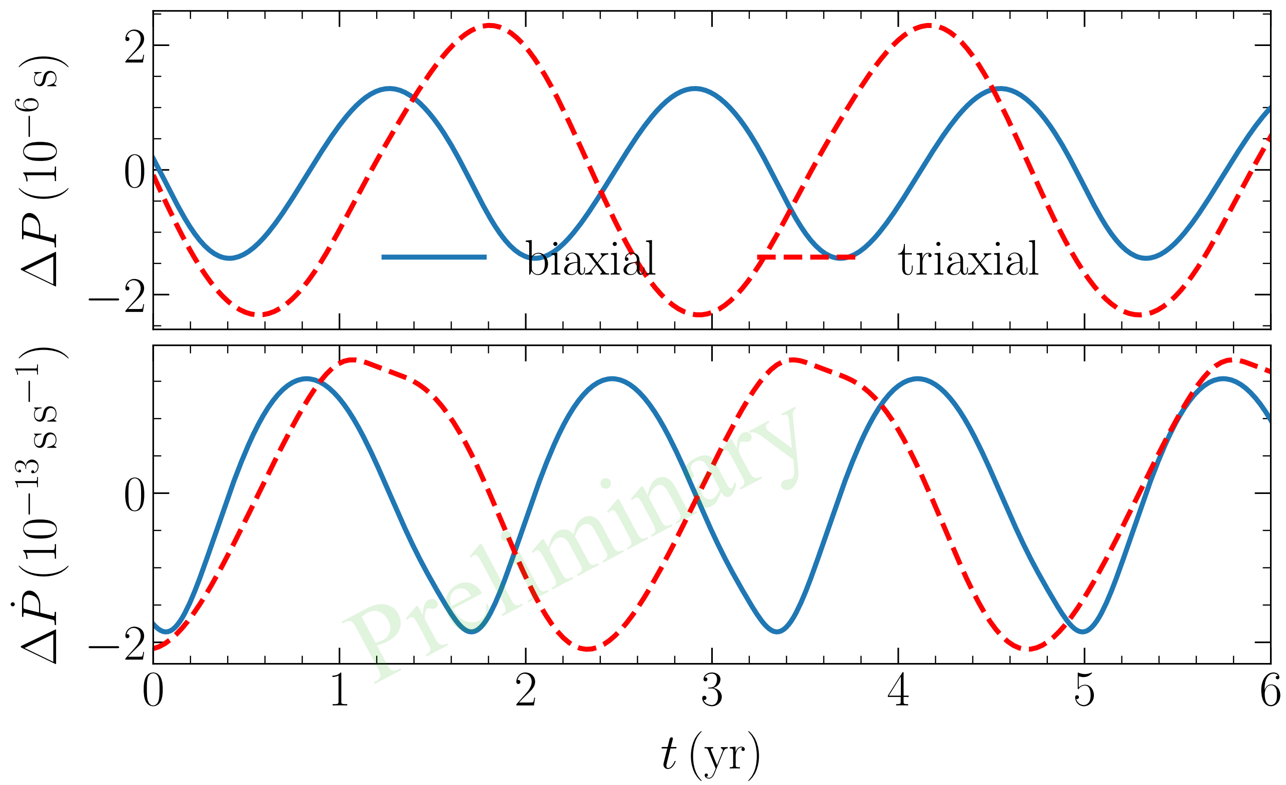
2. Spindown term

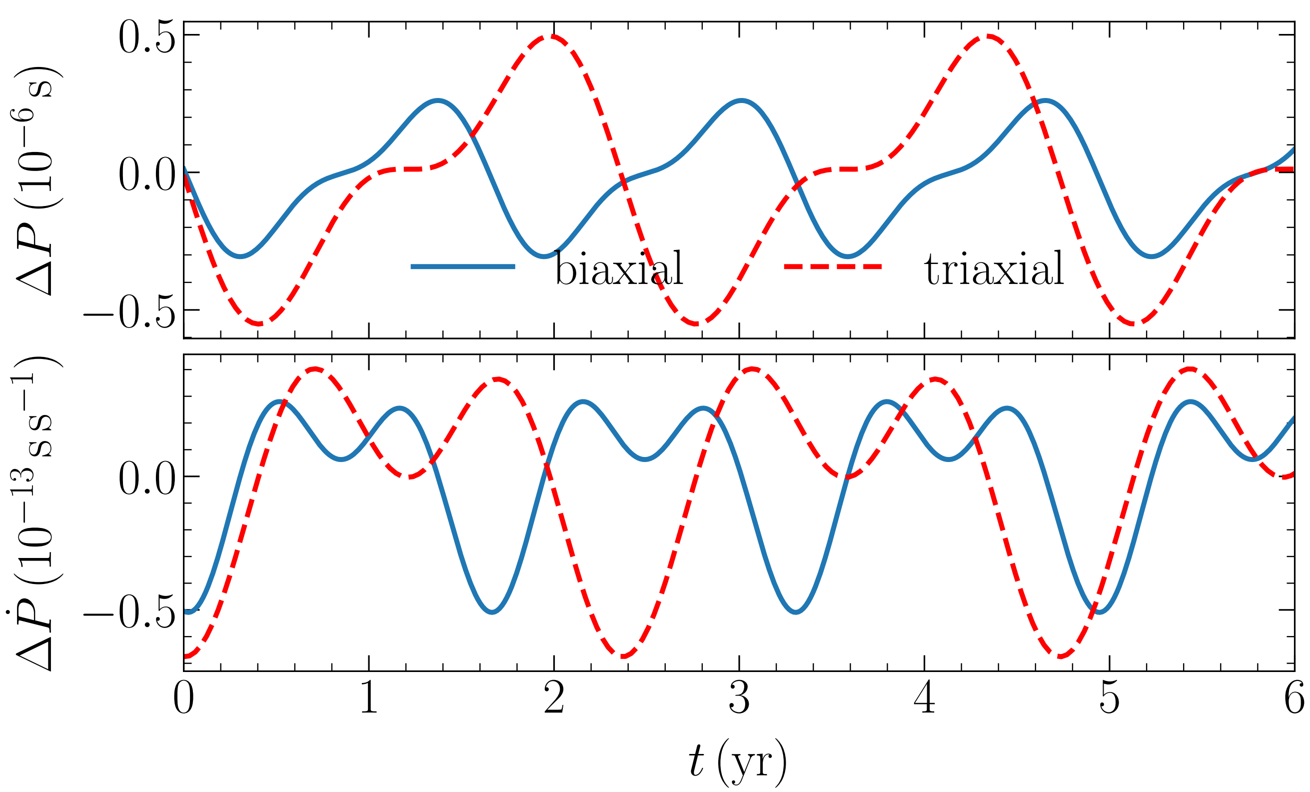
$$\frac{\Delta P_{\rm sd}}{P} = \text{Geometric factor} \times \frac{\tau_{\rm f}}{\tau_{\rm rad}}$$

Timing residuals from X-ray pulsations

Spindown term dominated cases

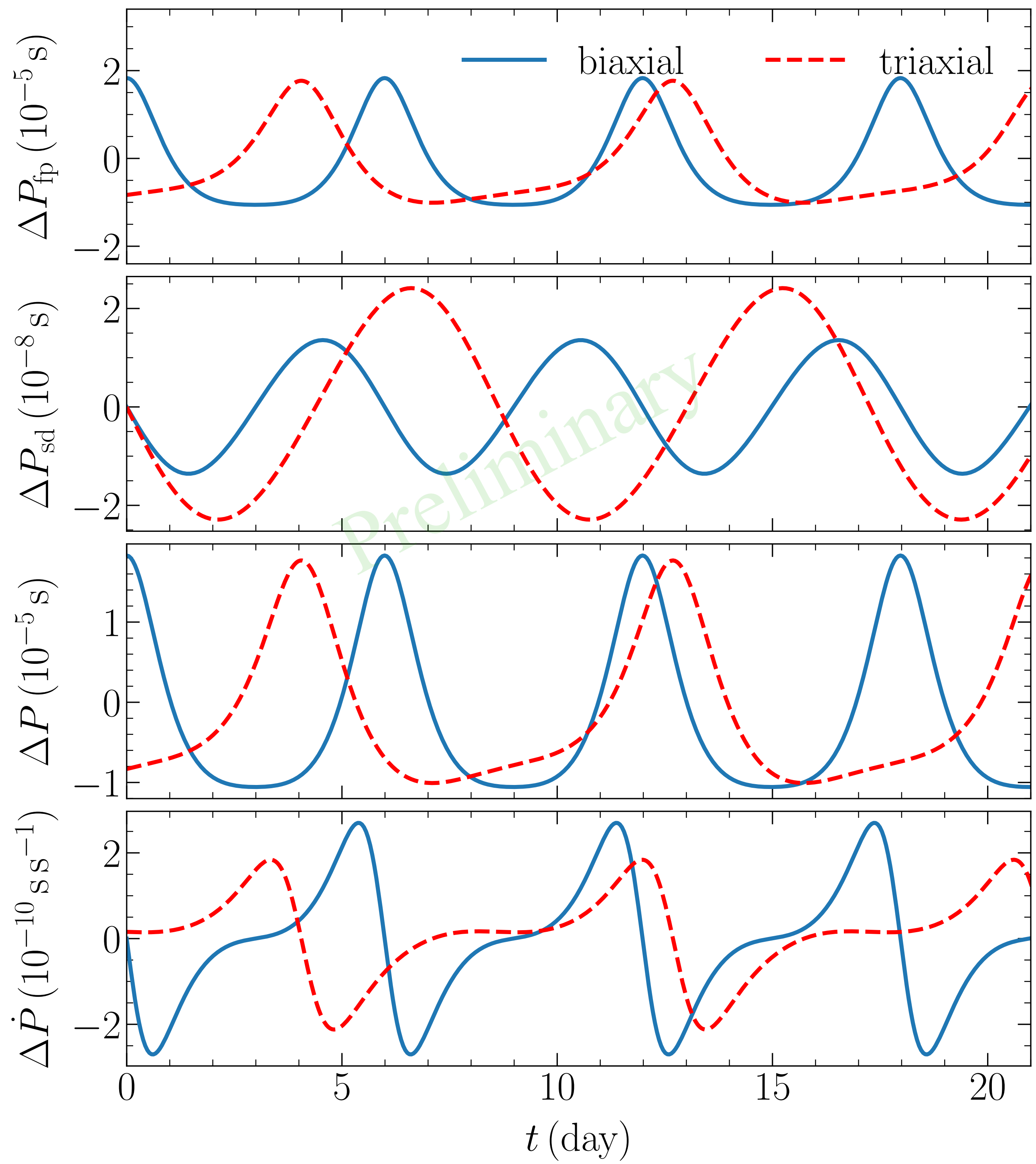
$\tau_\mathrm{f}/\tau_\mathrm{rad} \gg P/\tau_\mathrm{f} \quad \epsilon = 10^{-7}$





Geometric term dominated cases

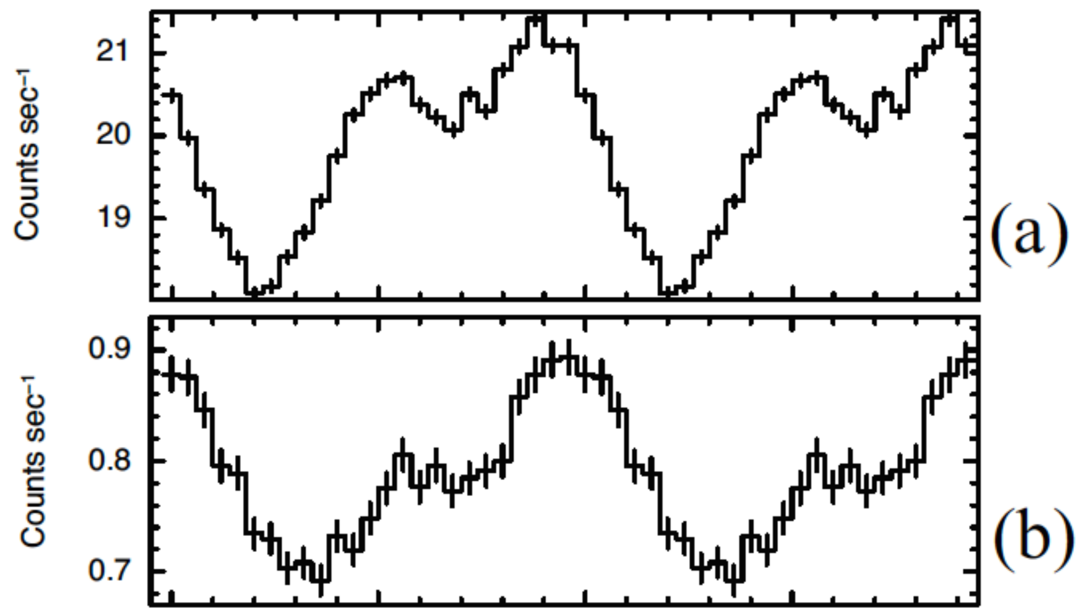
$\tau_\mathrm{f}/\tau_\mathrm{rad} \ll P/\tau_\mathrm{f} \quad \epsilon = 10^{-5}$



X-ray emission from magnetars

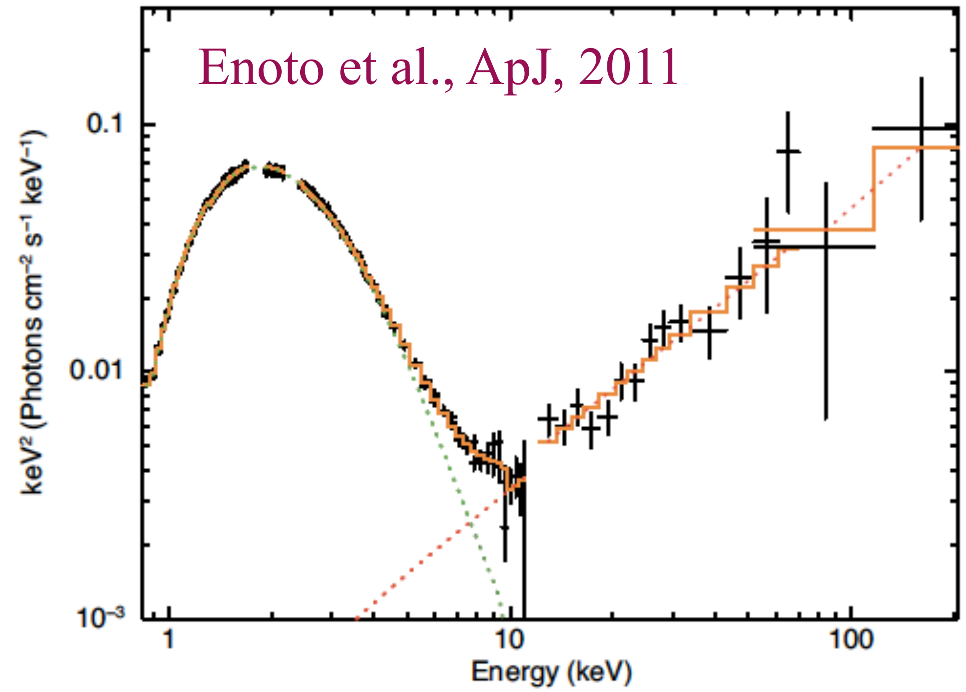
- Some magnetars are **persistent X-ray sources** with a luminosity $L_x = 10^{33} - 10^{36}\ \mathrm{erg\ s^{-1}}$

Pulse of 4U 0142+61



Enoto et al., ApJ, 2011

X-ray spectra of 4U 0142+61



- Show clear X-ray pulsations due to their spin

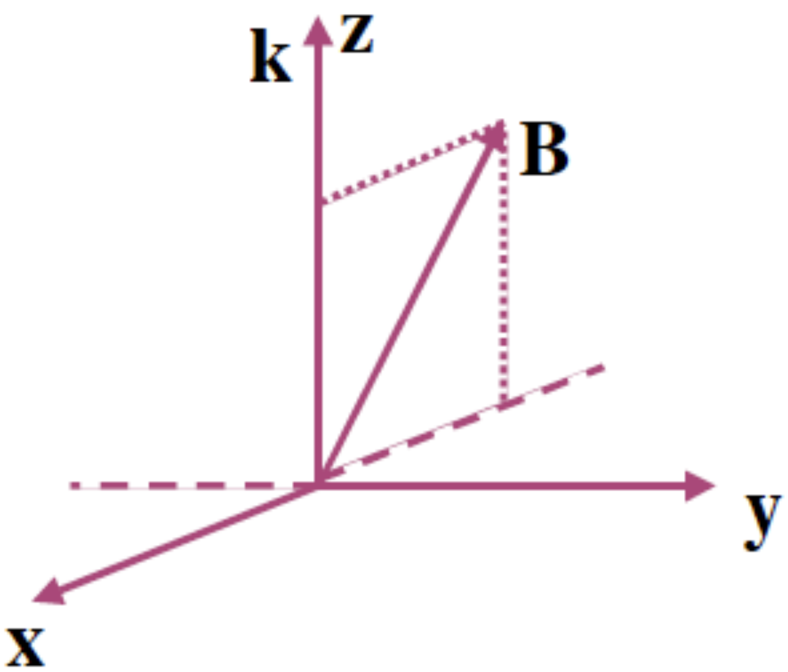
- Soft component at $0.5 - 10\,\mathrm{keV}$:

  1. Well described by either multiple blackbodies or a blackbody plus power law
  2. Thought as thermal emission from magnetar surface, reprocessed by the magnetosphere

- The emission is highly polarized according current emission models

Polarized X-ray from magnetized NSs

1. Surface emission from magnetar is thought as highly polarized (up to 100%)



O mode — $\mathbf{E}$ nearly in the $\mathbf{k}-\mathbf{B}$ plane

X-mode — $\mathbf{E}$ nearly $\perp$ $\mathbf{k}-\mathbf{B}$ plane

- The two modes have different opacities (scattering, absorption):

$$\kappa_{\mathrm{O}} \sim \kappa_{(B=0)} \quad \kappa_{\mathrm{X}} \sim \kappa_{(B=0)} \left(\omega/\omega_{ce}\right)^2$$

- X-mode photons are the main carrier of X-ray flux (two photospheres), the emergent radiation is highly polarized

Gnedin & Sunyaev, A&A, 1974

Pavlov & Zavlin, 2000

Heyl et al., PRD, 2003

Lai & Ho, PRL, 2003

Thermal X-ray polarization from magnetized NSs

2. Including vacuum polarization in strong B

Dielectric tensor of magnetized plasma including vacuum polarization

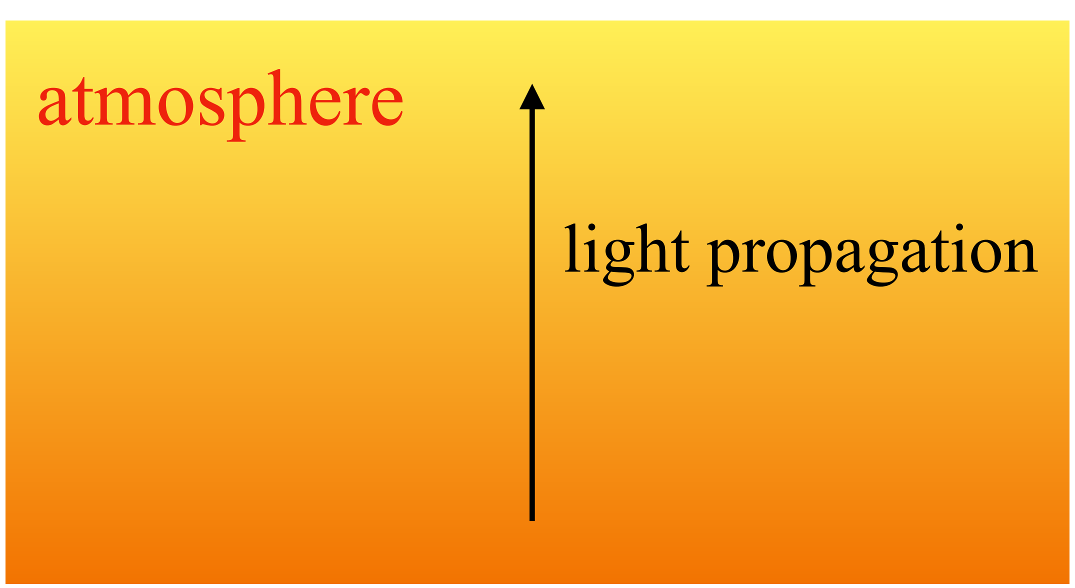
$$\boldsymbol{\varepsilon} = \mathbf{I} + \Delta\boldsymbol{\varepsilon}^{(\mathrm{plasma})} + \Delta\boldsymbol{\varepsilon}^{(\mathrm{vac})}$$

Vacuum resonance and mode conversion

$$\Delta\boldsymbol{\varepsilon}^{(\mathrm{plasma})} + \Delta\boldsymbol{\varepsilon}^{(\mathrm{vac})} \sim 0$$

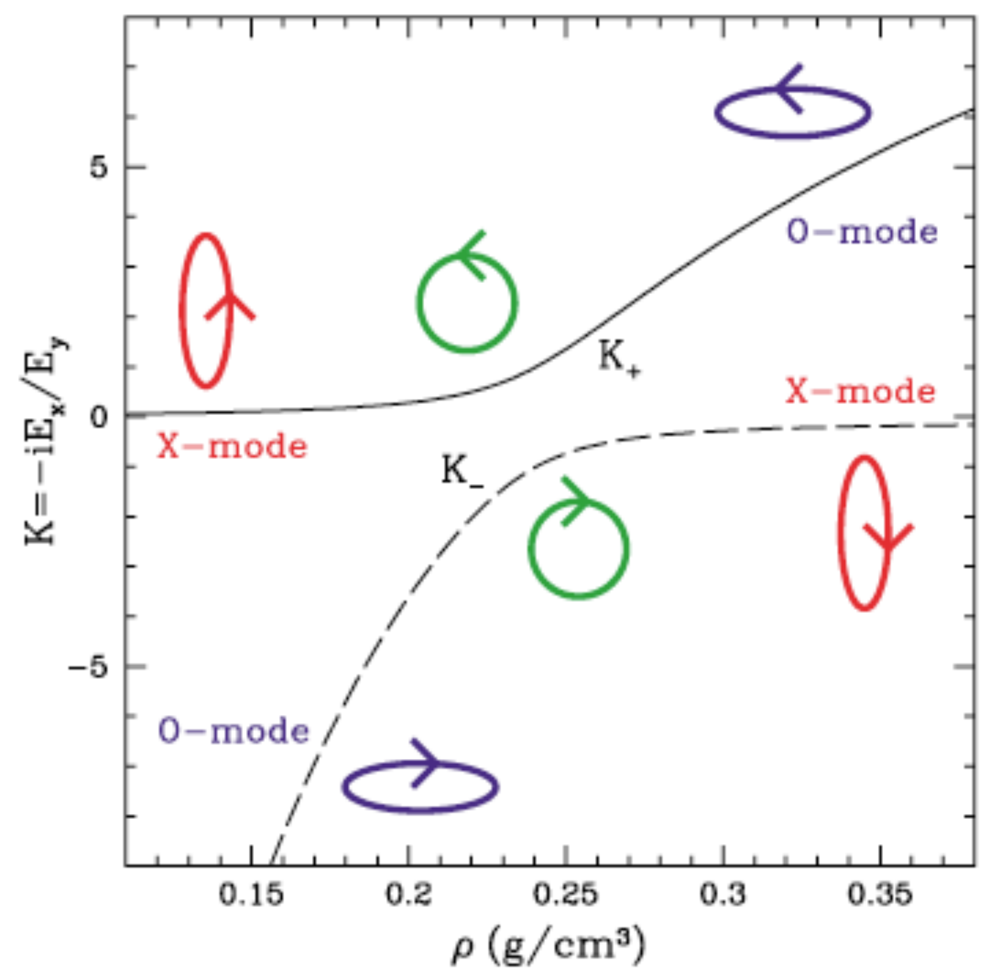
↓

depends on $-\left(\omega_{\mathrm{p}}/\omega\right)^2 \propto \rho/\mathrm{E}^2$

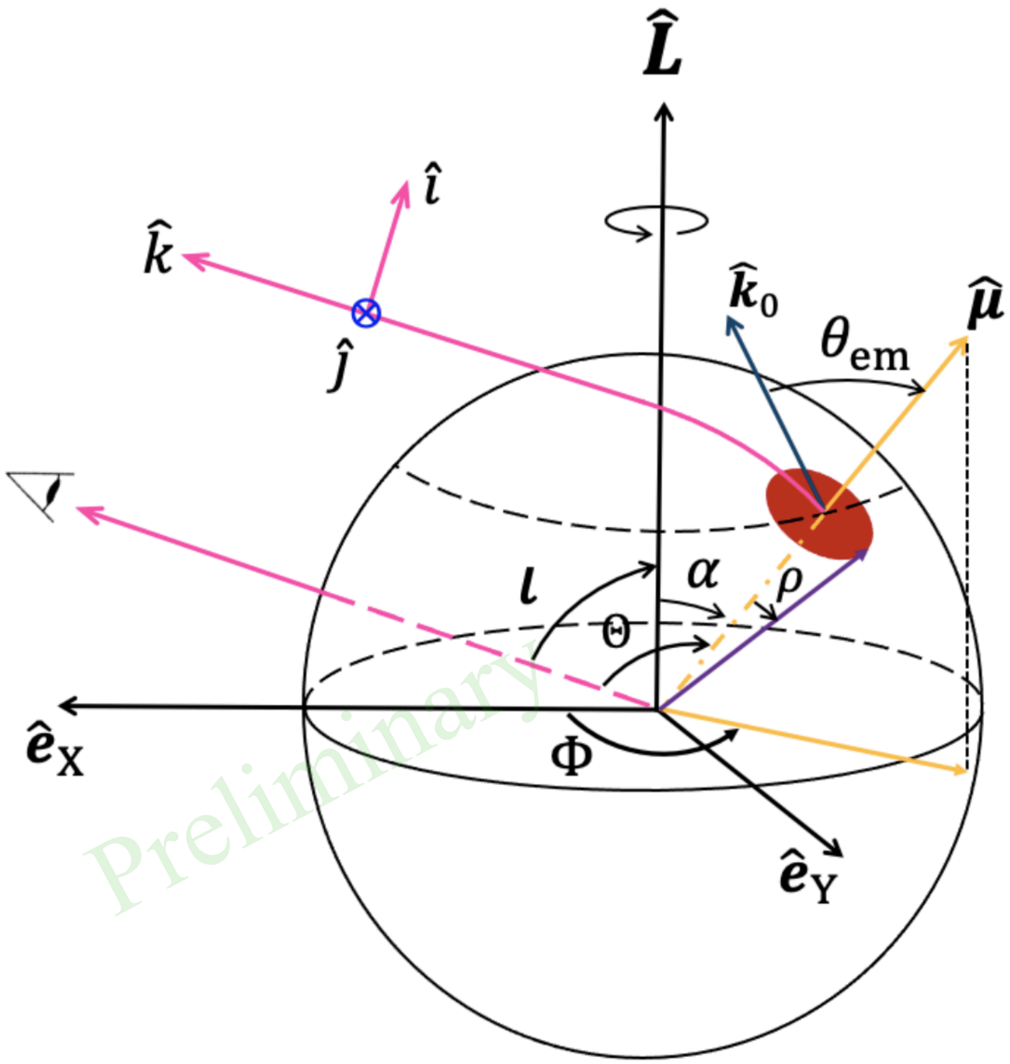


NS surface

Lai & Ho, PRL, 2003; Adelsberg & Lai, MNRAS, 2006



Thermal X-ray emission model



- Propagation of the polarized emission to the observer

1. bending of light and gravitational redshift

$$F_{\rm I} = F_{\rm O} + F_{\rm X}$$

2. Polarization state evolution: QED effect

Not parallel transport, but evolve adiabatically along the direction of the magnetic field up to the "polarization limiting radius" $r_{\rm pl}$

$$\hat{e}^{\rm p}_1 = \frac{(\hat{k}\times\hat{\mu})\times\hat{k}}{\sin\Theta}, \quad \hat{e}^{\rm p}_2 = \frac{\hat{k}\times\hat{\mu}}{\sin\Theta}$$

$$\cos\Psi = \hat{e}^{\rm p}_1\cdot\hat{\imath} = \frac{\sin\iota\cos\alpha - \cos\iota\sin\alpha\cos\Phi}{\sin\Theta}$$

$$\sin\Psi = \hat{e}^{\rm p}_1\cdot\hat{\jmath} = -\frac{\sin\alpha\sin\Phi}{\sin\Theta}$$

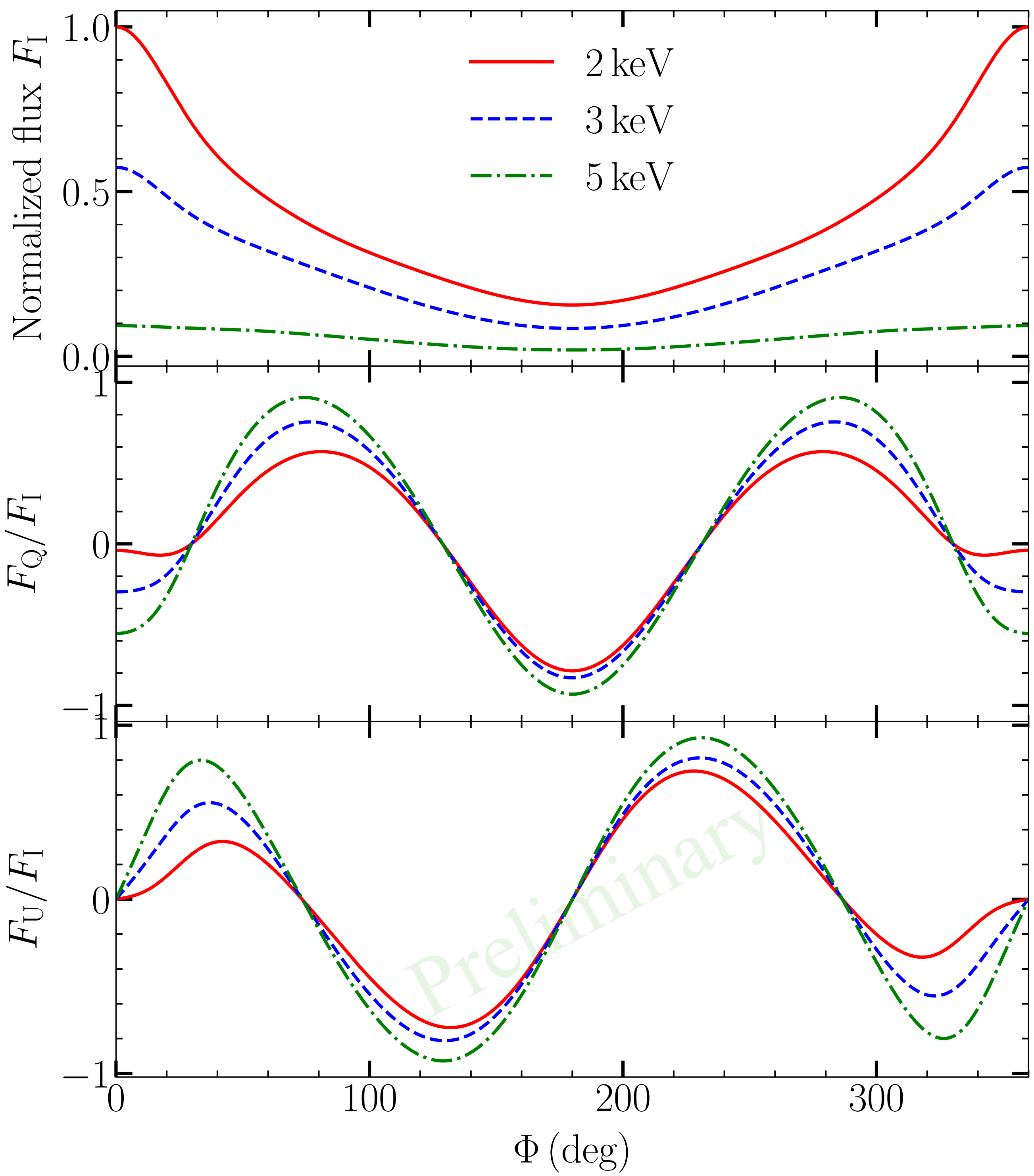
$$F_{\rm Q} = F_{\rm I}\Pi_{\rm em}\cos 2\Psi\left(r_{\rm pl}\right)$$

$$F_{\rm U} = F_{\rm I}\Pi_{\rm em}\sin 2\Psi\left(r_{\rm pl}\right)$$

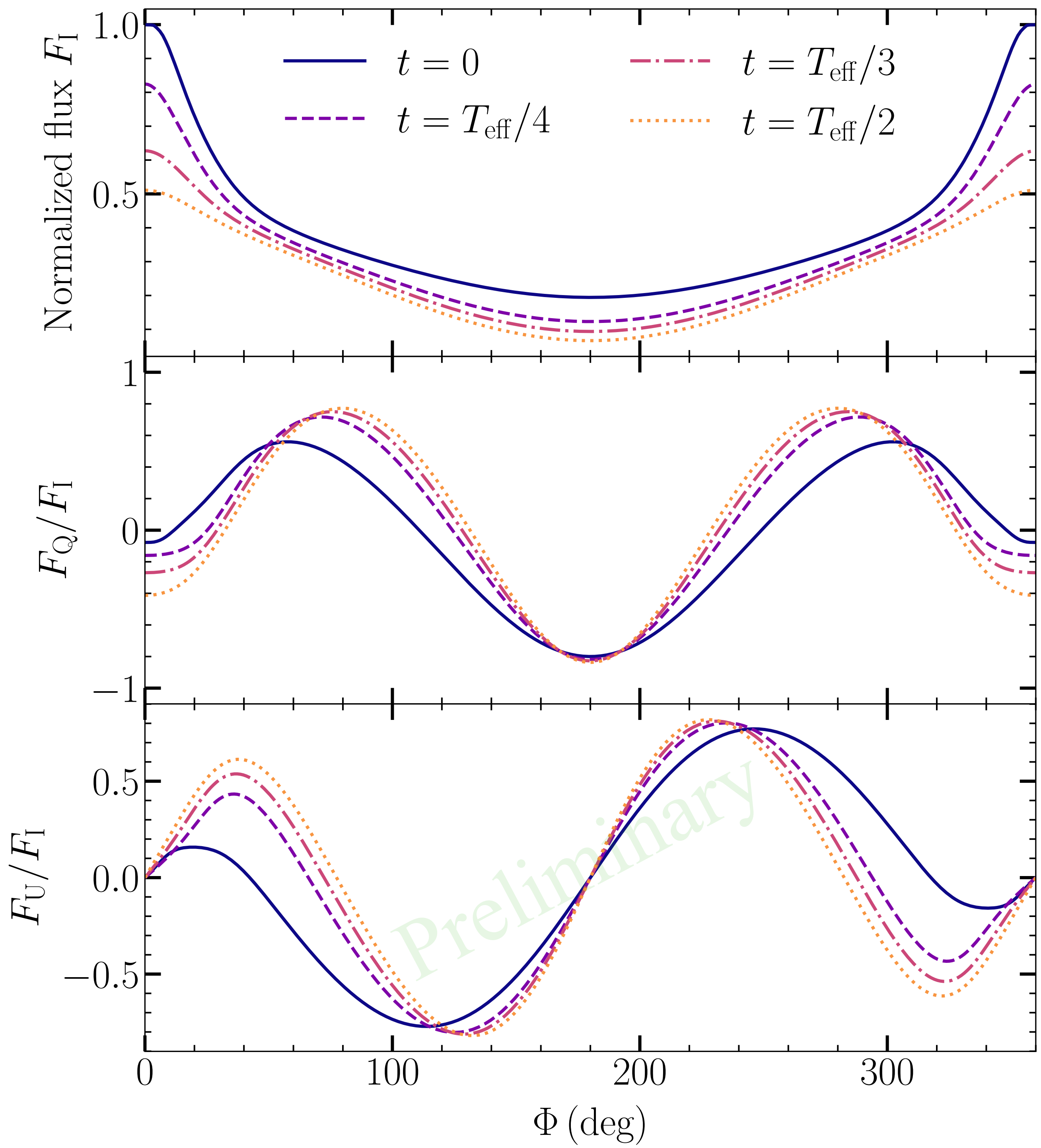
$$\Pi_{\rm L} = \frac{\left(F_{\rm Q}^2 + F_{\rm U}^2\right)^{1/2}}{F_{\rm I}} = \left|\Pi_{\rm em}\right|$$

Modulations on phase-resolved Stokes parameters

Phase-resolved Stokes parameters

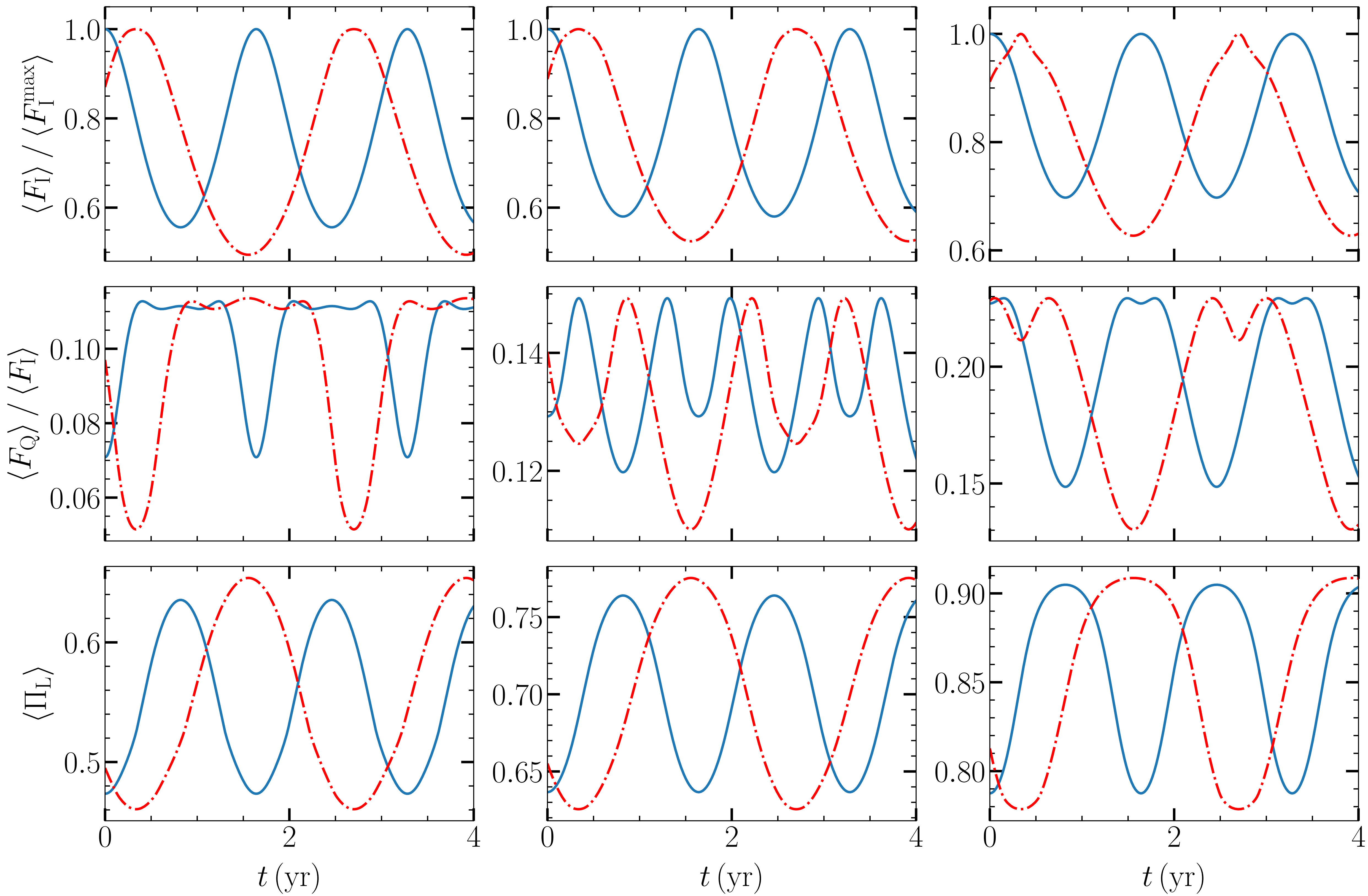


Modulations at 3 kev



Most sensitive to phase near 0°

Modulations on phase-averaged Stokes parameters



Modulations on polarized radio emissions

Radio emission

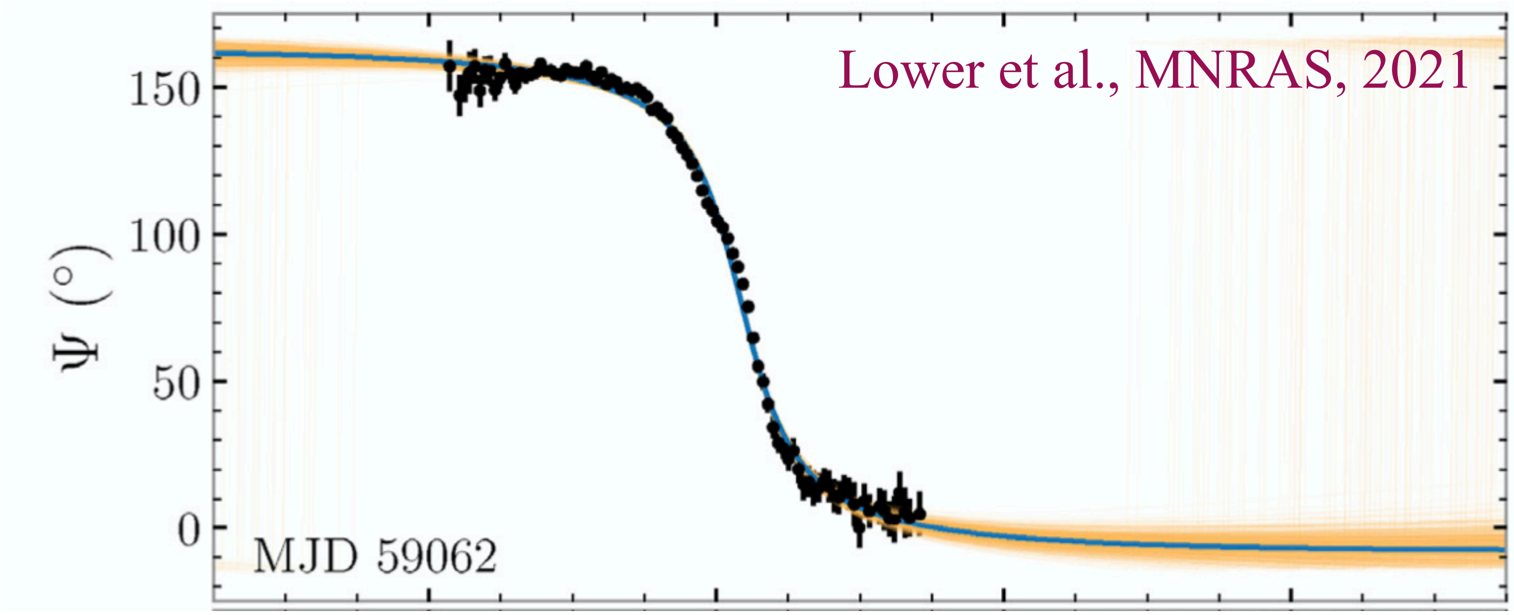
(1) Detected in transient magnetars, the emission is also transient (associated with X-ray bursts)

Kaspi & Beloborodov, ARAA, 2017

(2) Bright, show large pulse-to-pulse variability and flat spectrum

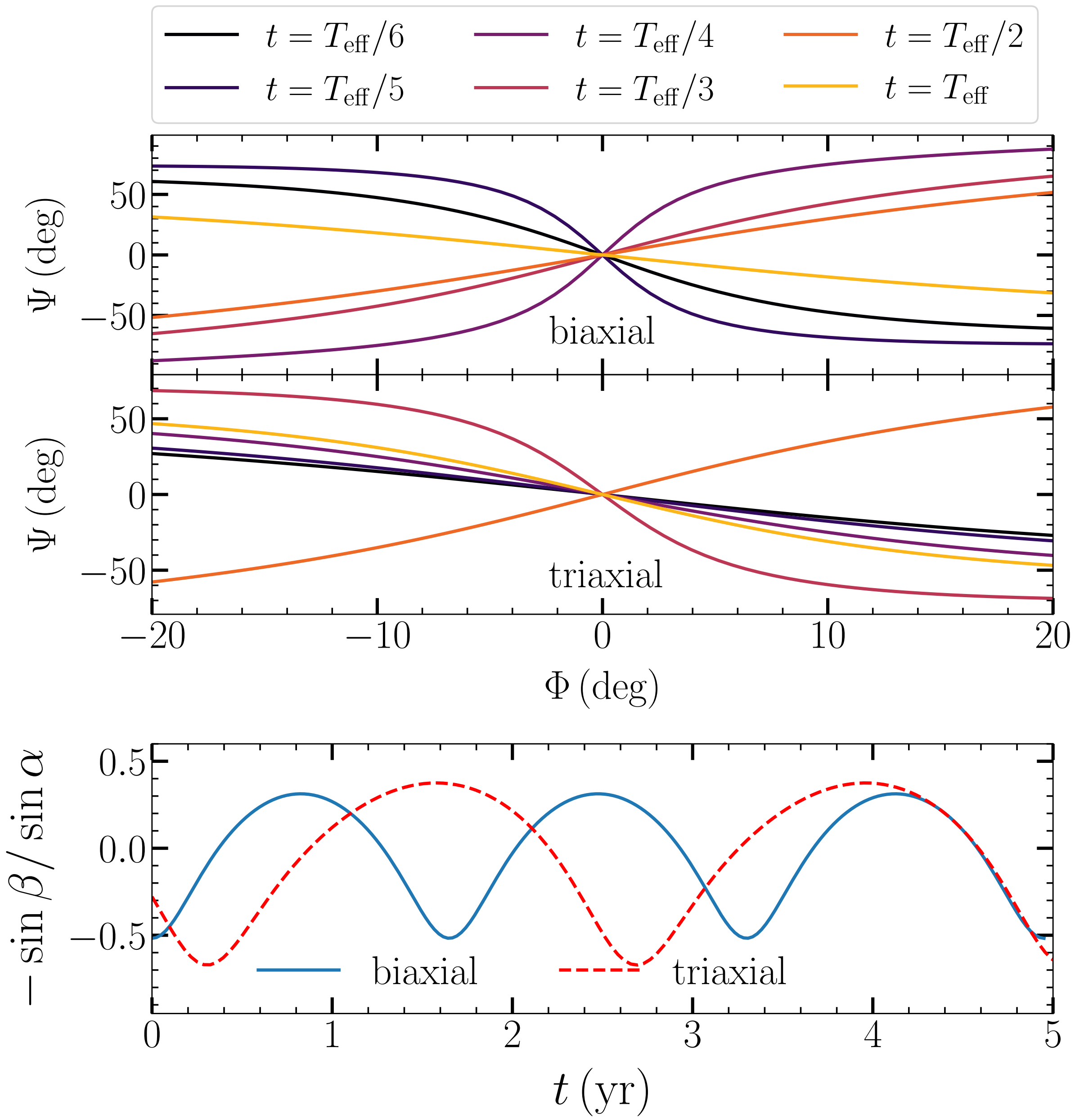
(3) **Highly linearly polarized, with polarization fractions of 60%-100%**

- The direction of polarization (PA) reflects the emission geometry (α and ι)
- The PA can be fitted with the rotating vector model (RVM) **in some cases**



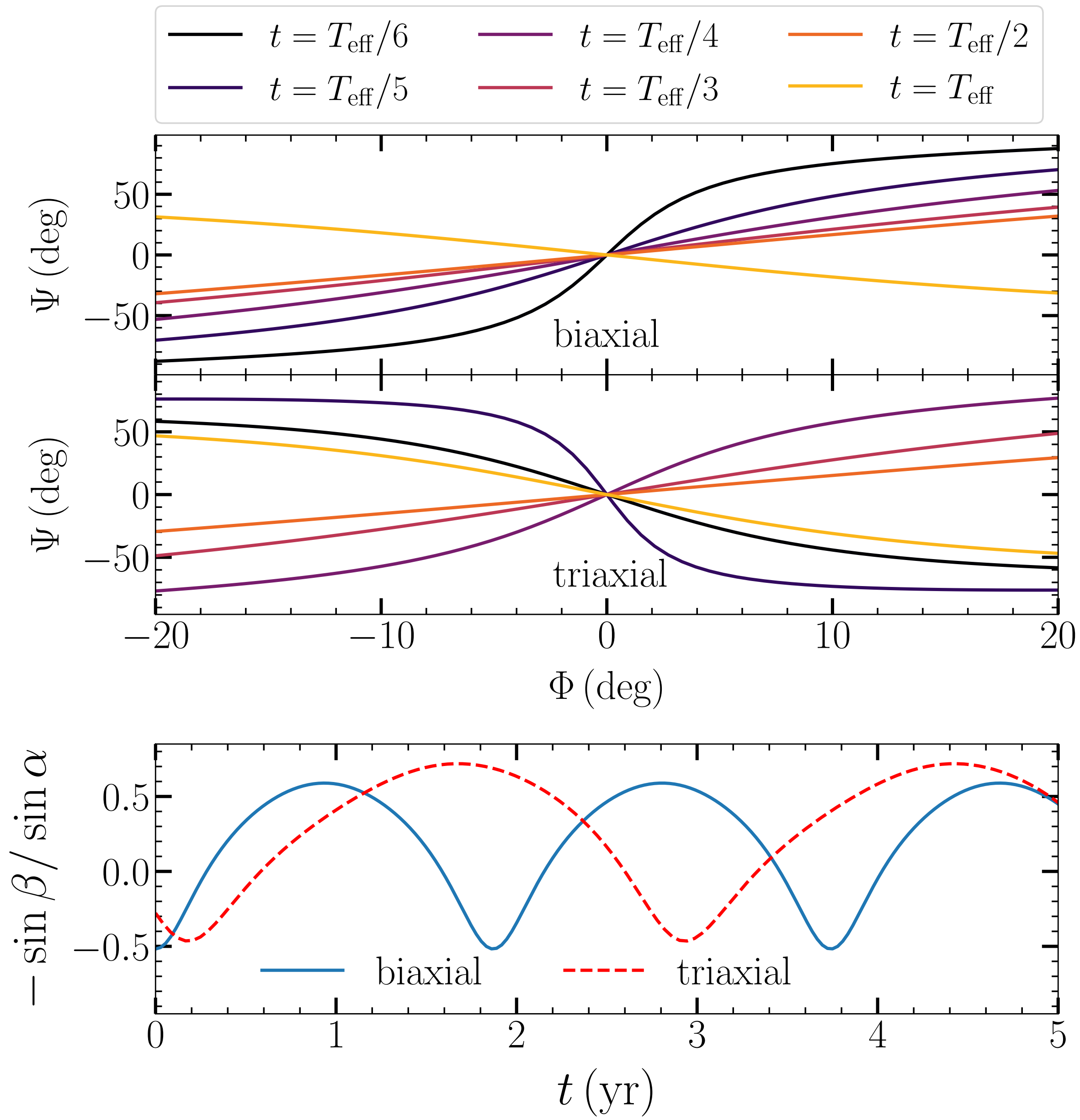
Swift J1818.0–1607

Modulations on polarized radio emissions



Neglected near-field torque

($\epsilon = 10^{-7}$, $B = 10^{14}$ G)



Large near-field torque

($\epsilon = 10^{-7}$, $B = 5 \times 10^{14}$ G)

Summary

- A analytical precession model for magnetars including complex deformation and EM torques
- Modelling the timing residuals (searching template)
- Detect precession with X-ray/radio emission is promising (Fast, IXPE, eXTP)
- More work needs to be done on timing searches and emission modelling