Prospects of testing dark matter model with pulsar around Sgr A*

Presenter: Zexin Hu
Advisor: Lijing Shao

2023.7.4 FPS12&SPSS2023

Sagittarius A* (Sgr A*)

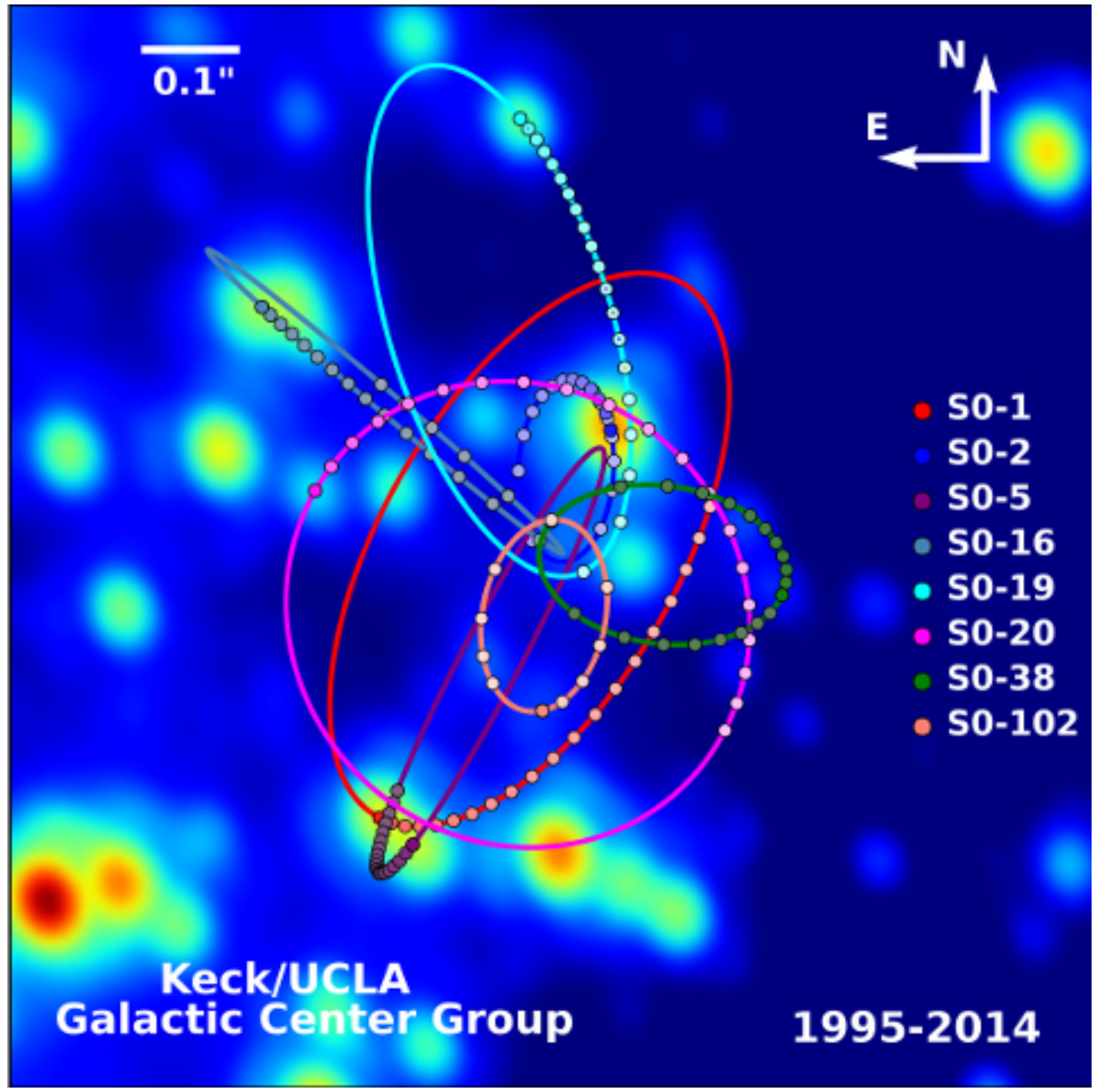
The supermassive black hole in the Galactic center

- $M_\bullet \sim 4.3 \times 10^6 \, M_\odot$, $R \sim 8\,\mathrm{kpc}$
- Observation with:

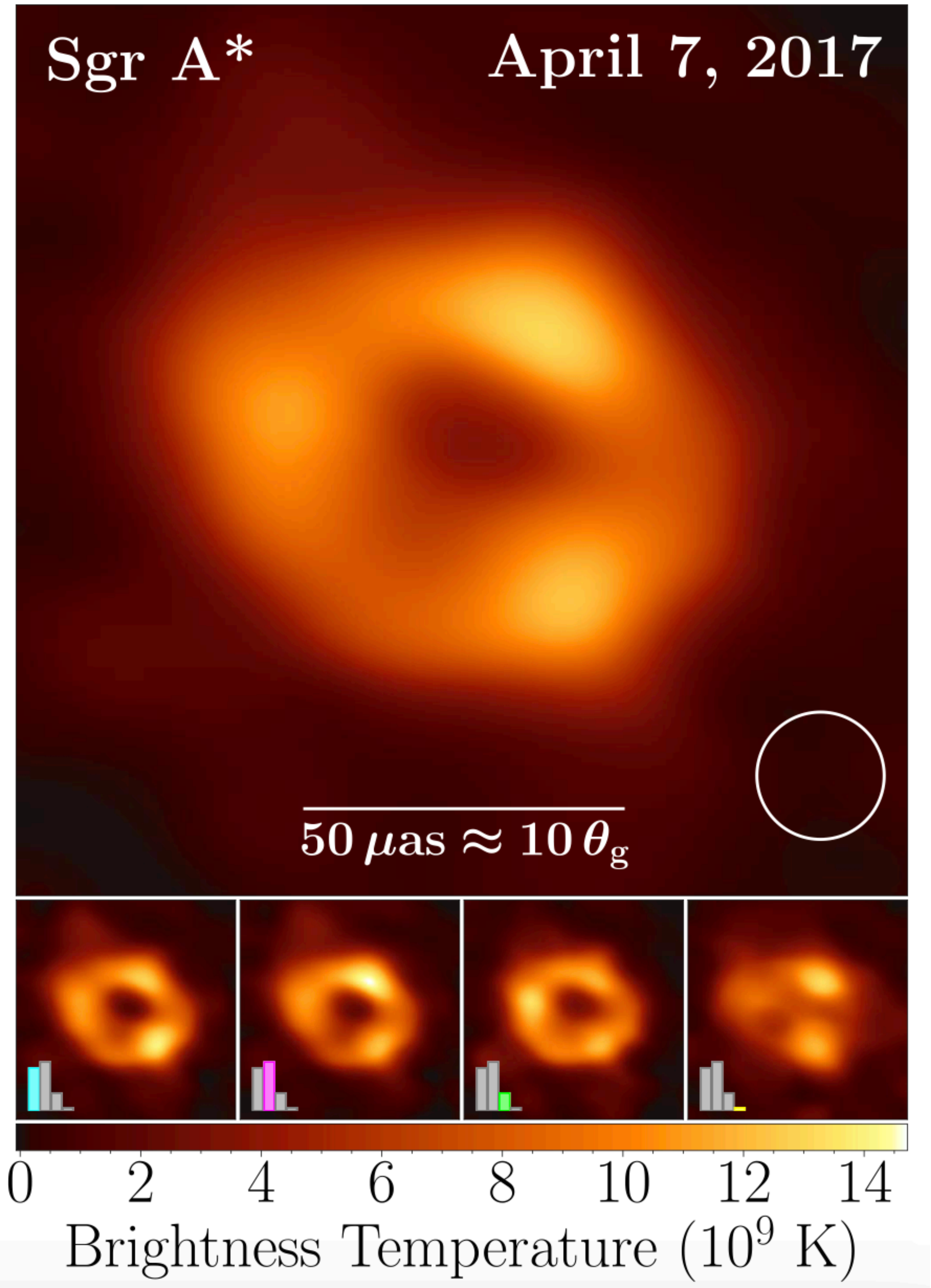
☑ S-stars

☑ BH shadow

☐ Pulsar timing

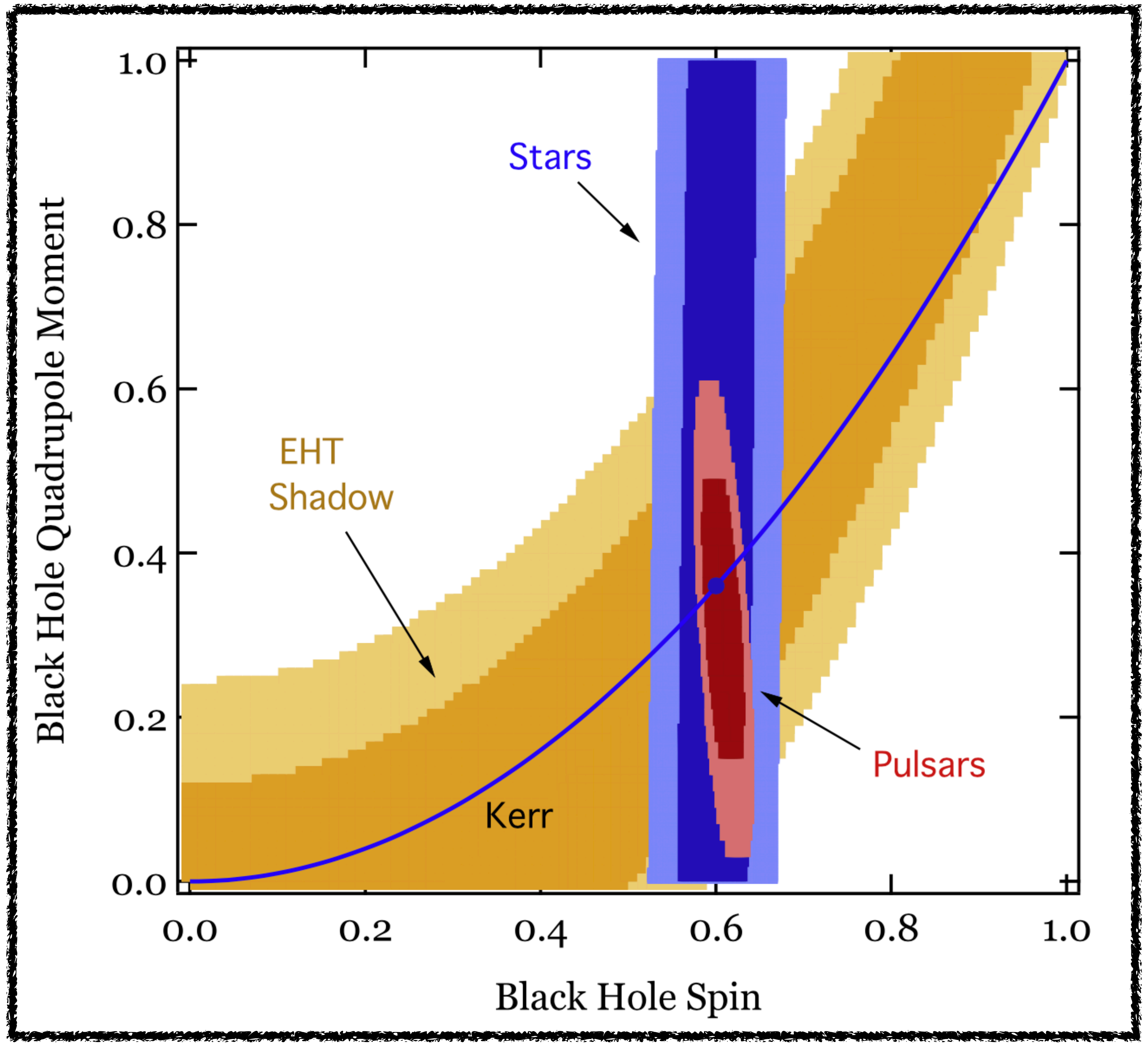


The EHT Collaboration



Timing a pulsar orbiting around Sgr A*

Testing the gravity theory



Psaltis, Wex & Kramer 2016

- The no-hair theorem:

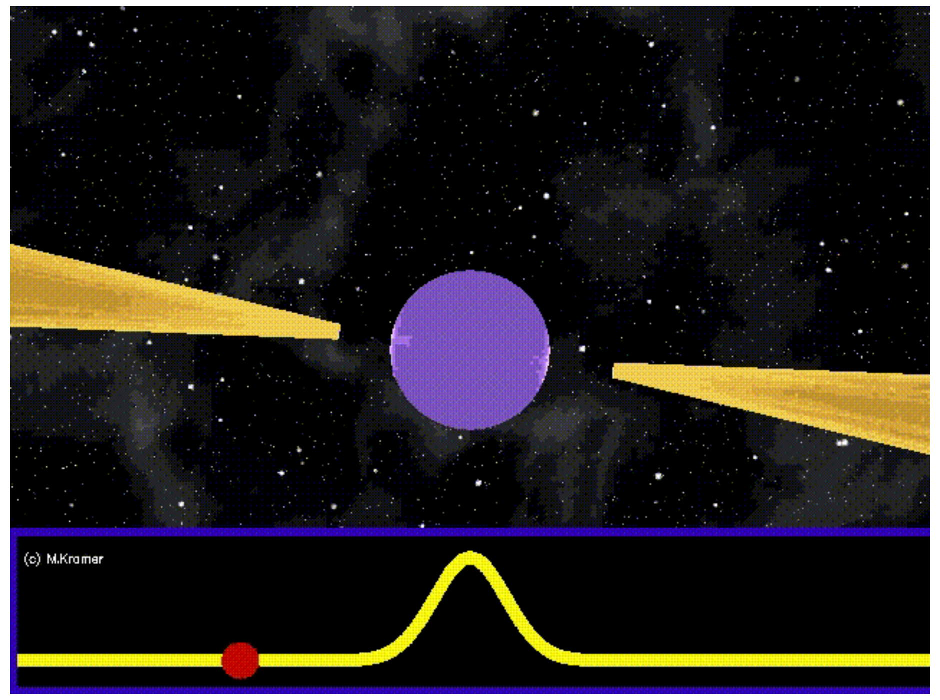
$$M_\bullet\,,\ S_\bullet\,,\ Q_\bullet$$

- Kerr BH:

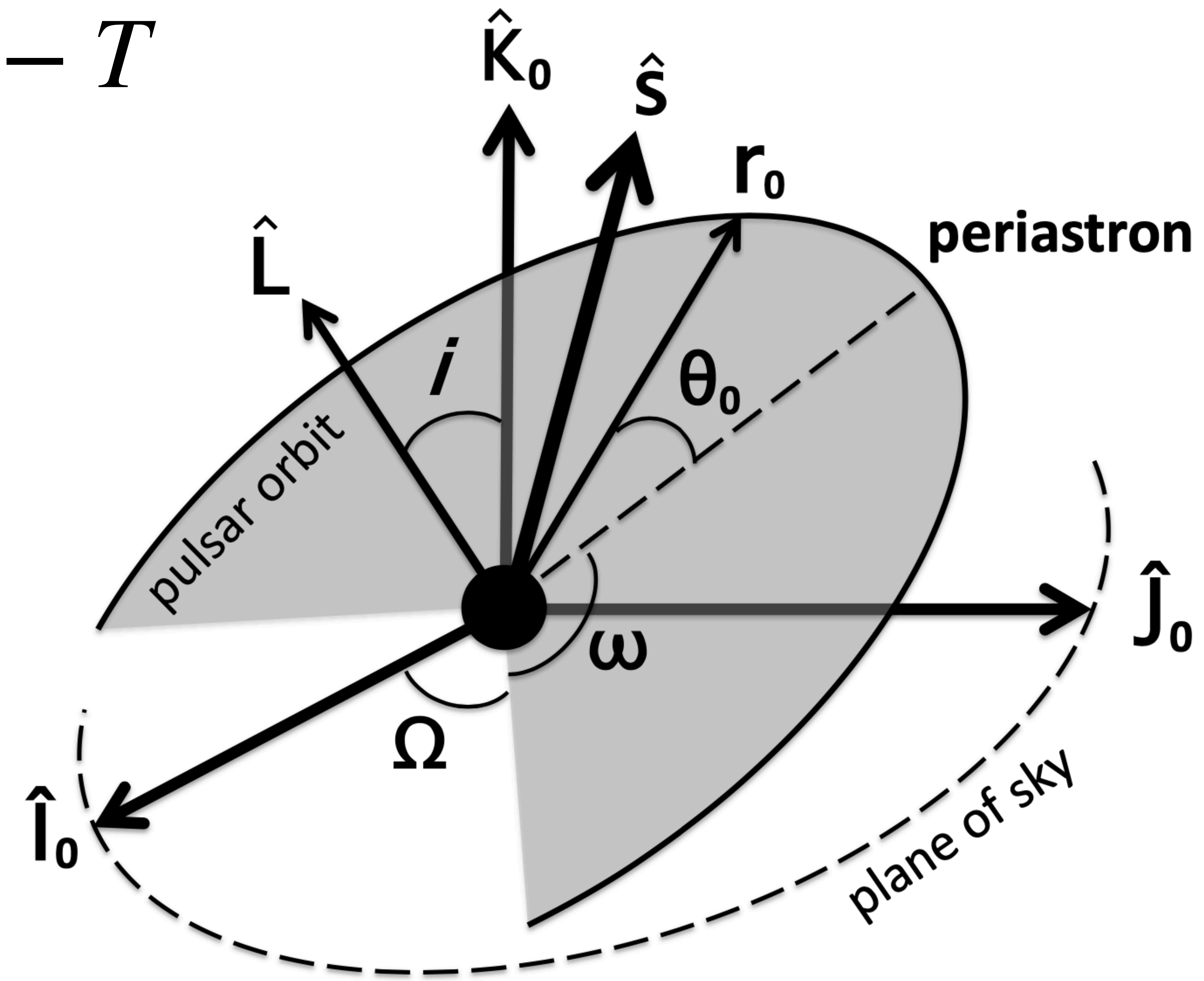
$$q_\bullet = -\chi_\bullet^2$$

Timing a pulsar orbiting around Sgr A*

Timing model



- $N(T) = N_0 + \nu T + \frac{1}{2}\dot{\nu}T^2 + \cdots$
- Einstein delay: $\frac{\mathrm{d}T}{\mathrm{d}t} = 1 - \frac{GM_\bullet}{c^2 r} - \frac{v^2}{2c^2}, \Delta_E = t - T$
- Romer delay: $\Delta_R = \hat{K}_0 \cdot \vec{r}$
- Shapiro delay: $\Delta_S = -\frac{2GM_\bullet}{c^3} \ln(r - \vec{r} \cdot \hat{K}_0)$
- $t^{\mathrm{TOA}} = t + \Delta_R + \Delta_S + \cdots$

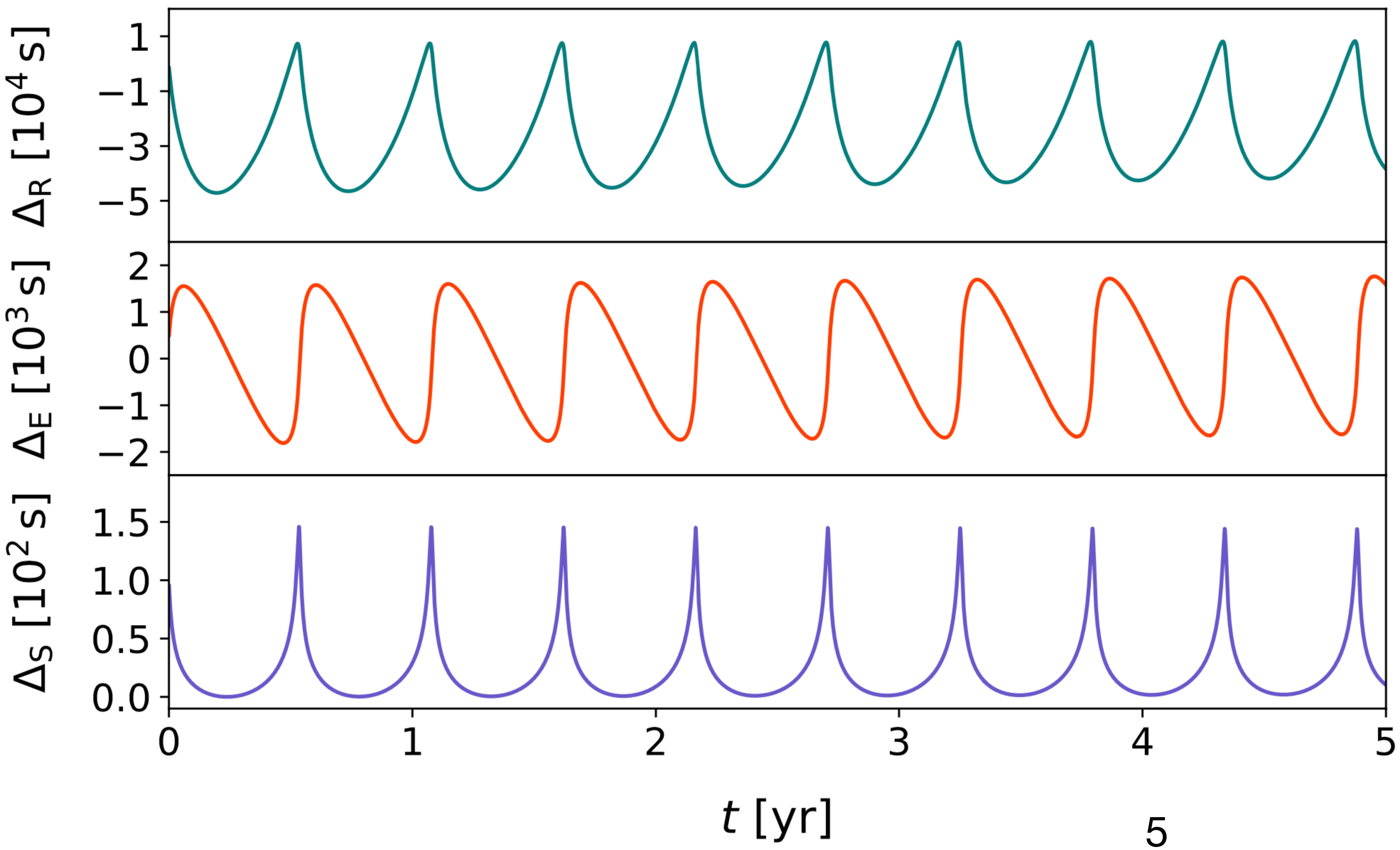


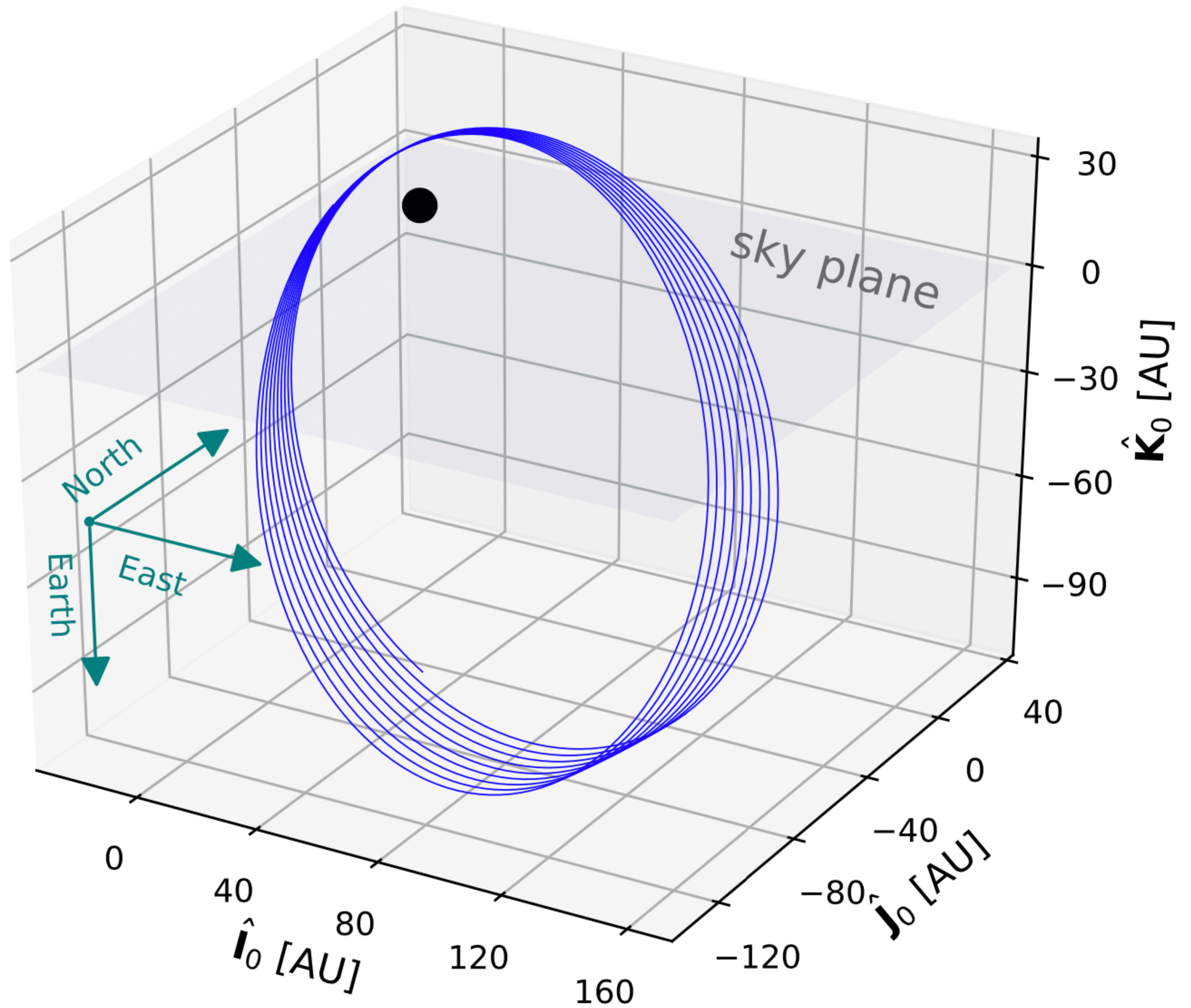
Damour and Deruelle, 1986

Orbital motion

Without DM

- $\vec{a} = \vec{a}_{\rm N} + \vec{a}_{\rm 1PN} + \vec{a}_{\rm SO} + \vec{a}_{\rm Q} + \cdots$





$M_\bullet = 4.3 \times 10^6\, M_\odot$

$\chi_\bullet = 0.6\,, \lambda = \frac{1}{3}\pi\,, \eta = \frac{5}{9}\pi$

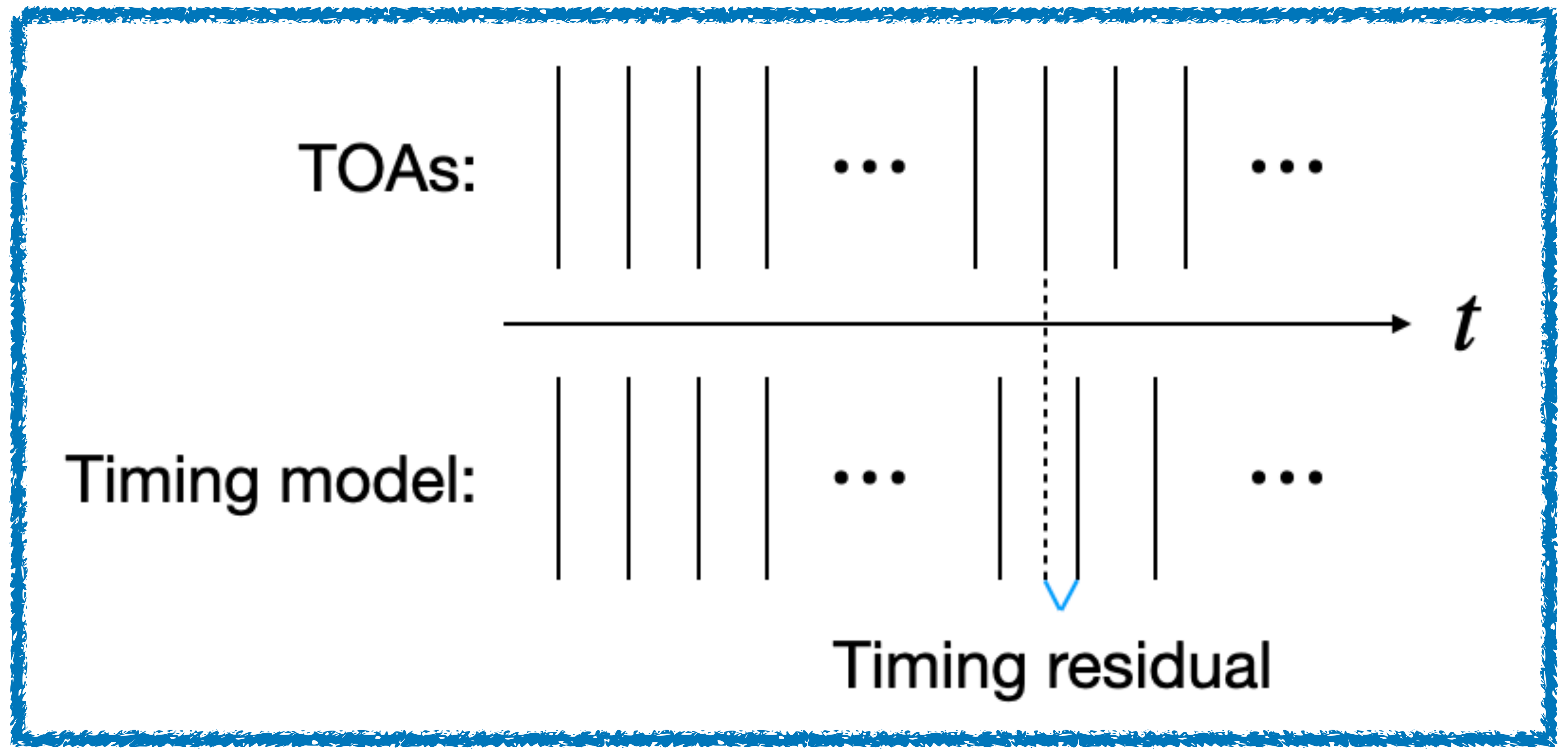
$q_\bullet = -0.36$

$P_b = 0.5\,\mathrm{year}\,, e = 0.8$

The inverse timing formula

Residuals

- $R(t_a^{\mathrm{TOA}}) \equiv (\mathcal{N}(t_a^{\mathrm{TOA}}, \xi^\mu) - N_a)/\nu$



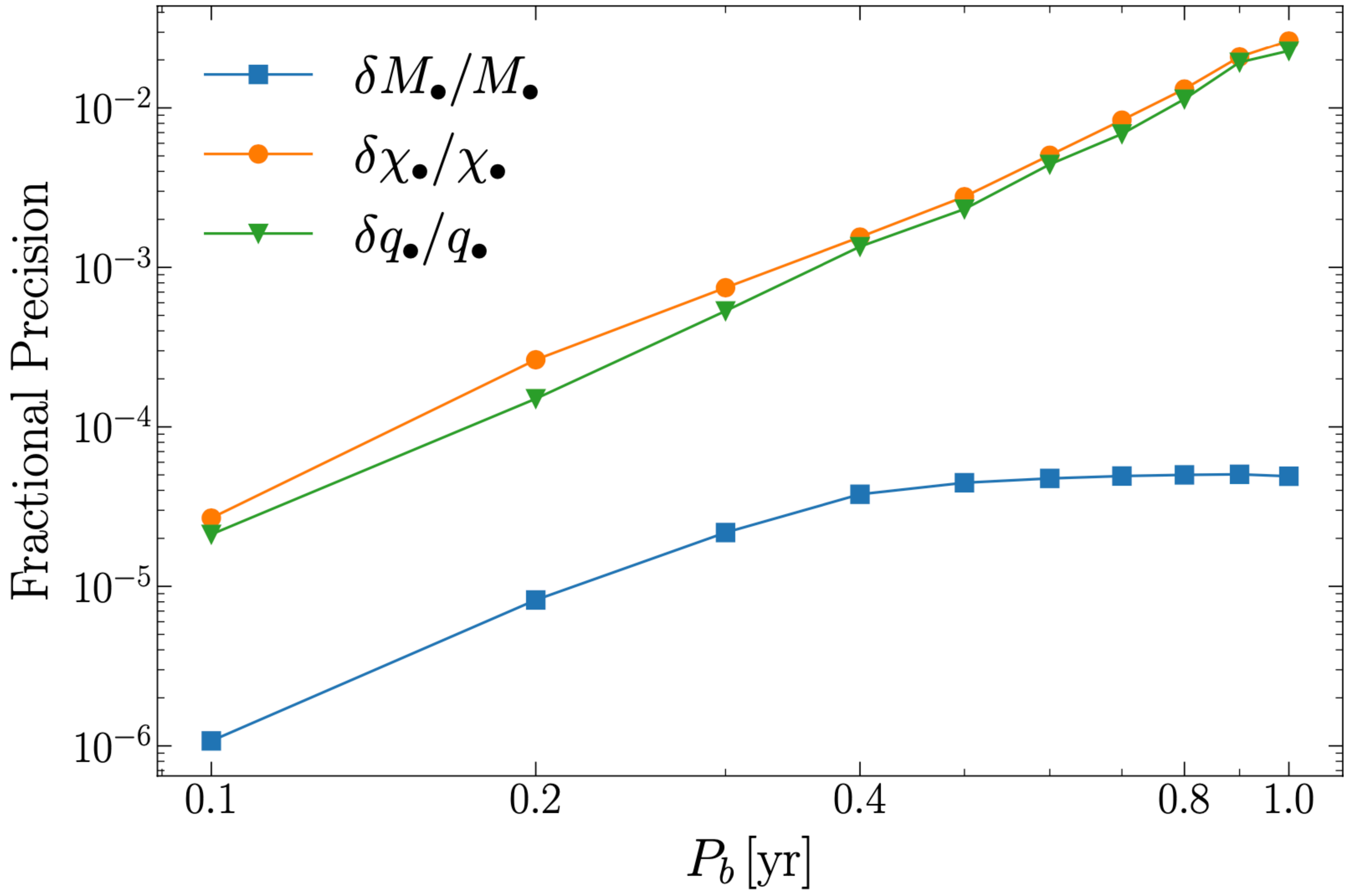
- N_a is the integer nearest to $\mathcal{N}(t_a, \xi^\mu)$

- We need the mapping: $(t_a^{\mathrm{TOA}}, \xi^\mu) \to \mathcal{N}$ which is so-called the inverse timing formula, where $\{\xi^\mu\} = \{N_0, \nu, \dot{\nu}, M_\bullet, \chi_\bullet, q_\bullet, \lambda, \eta, P_b, e, i, \omega, T_0; \xi^{\mathrm{DM}}\}$

- $t^{\mathrm{TOA}} = t + \Delta_R + \Delta_S \to \dfrac{dt^{\mathrm{TOA}}}{dt} = 1 - \dfrac{1}{c}\vec{n} \cdot \vec{v} - \dfrac{2GM_\bullet}{c^3} \dfrac{\hat{r} \cdot \vec{v} + \vec{n} \cdot \vec{v}}{r + \vec{n} \cdot \vec{r}}$

Parameter estimation

Fractional precision as function of P_b

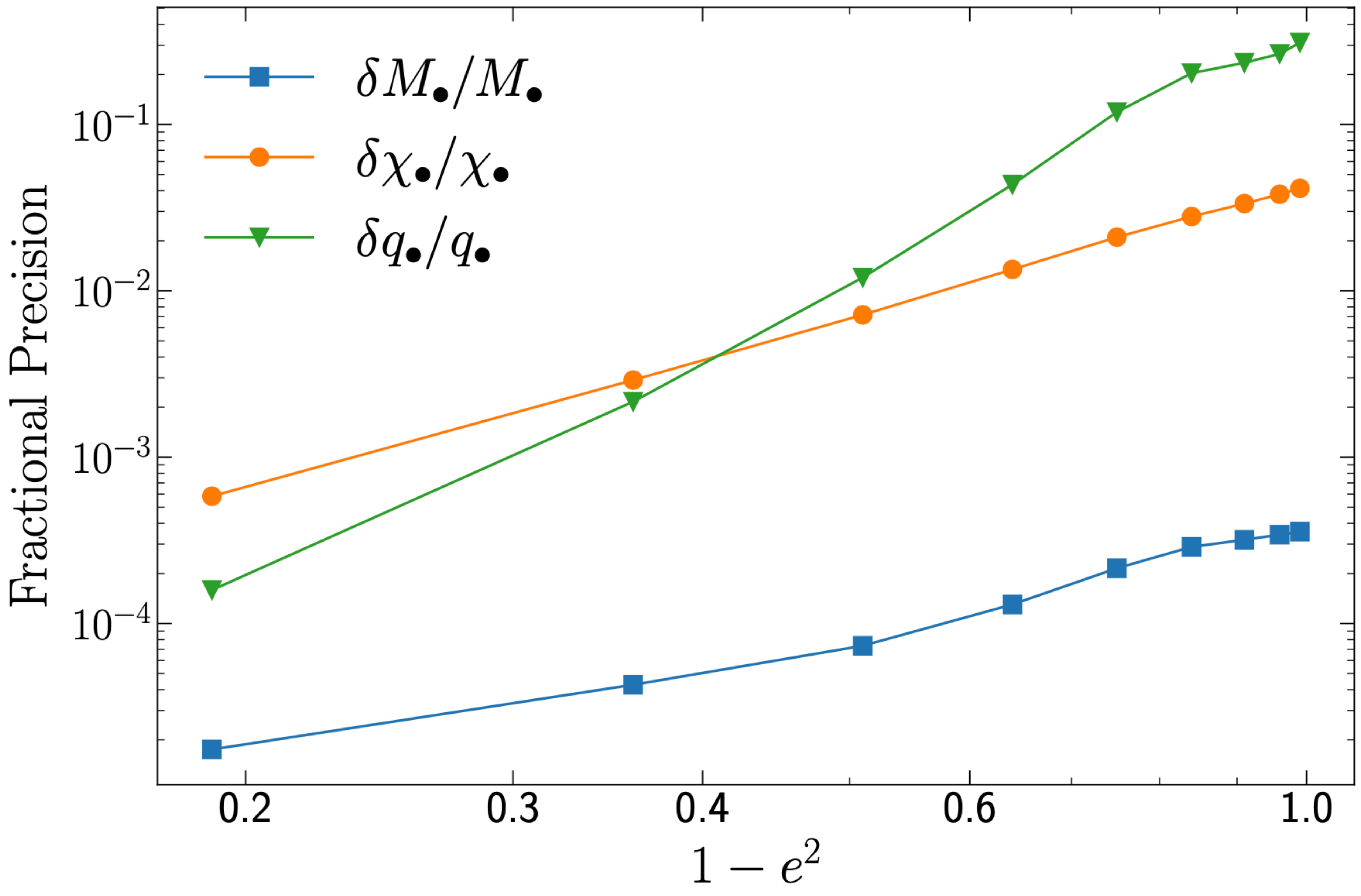


- Fisher matrix
- 5-yr observation
- Once per week
- $\sigma_{\rm TOA} = 1\,{\rm ms}$
- $e = 0.8$

Parameter estimation

Fractional precision as function of e

- $\dot{\omega}_0^{(1\mathrm{PN})} = \dfrac{6\pi}{P_b c^2} \dfrac{GM}{(1-e^2)a}$
- $\dot{\omega}_0^{(\mathrm{S})} \propto -\dfrac{4\pi\chi}{P_b c^3} \left[\dfrac{GM}{(1-e^2)a}\right]^{3/2}$
- $\dot{\omega}_0^{(\mathrm{Q})} \propto -\dfrac{3\pi q}{2P_b c^4} \left[\dfrac{GM}{(1-e^2)a}\right]^{2}$



The dark matter model

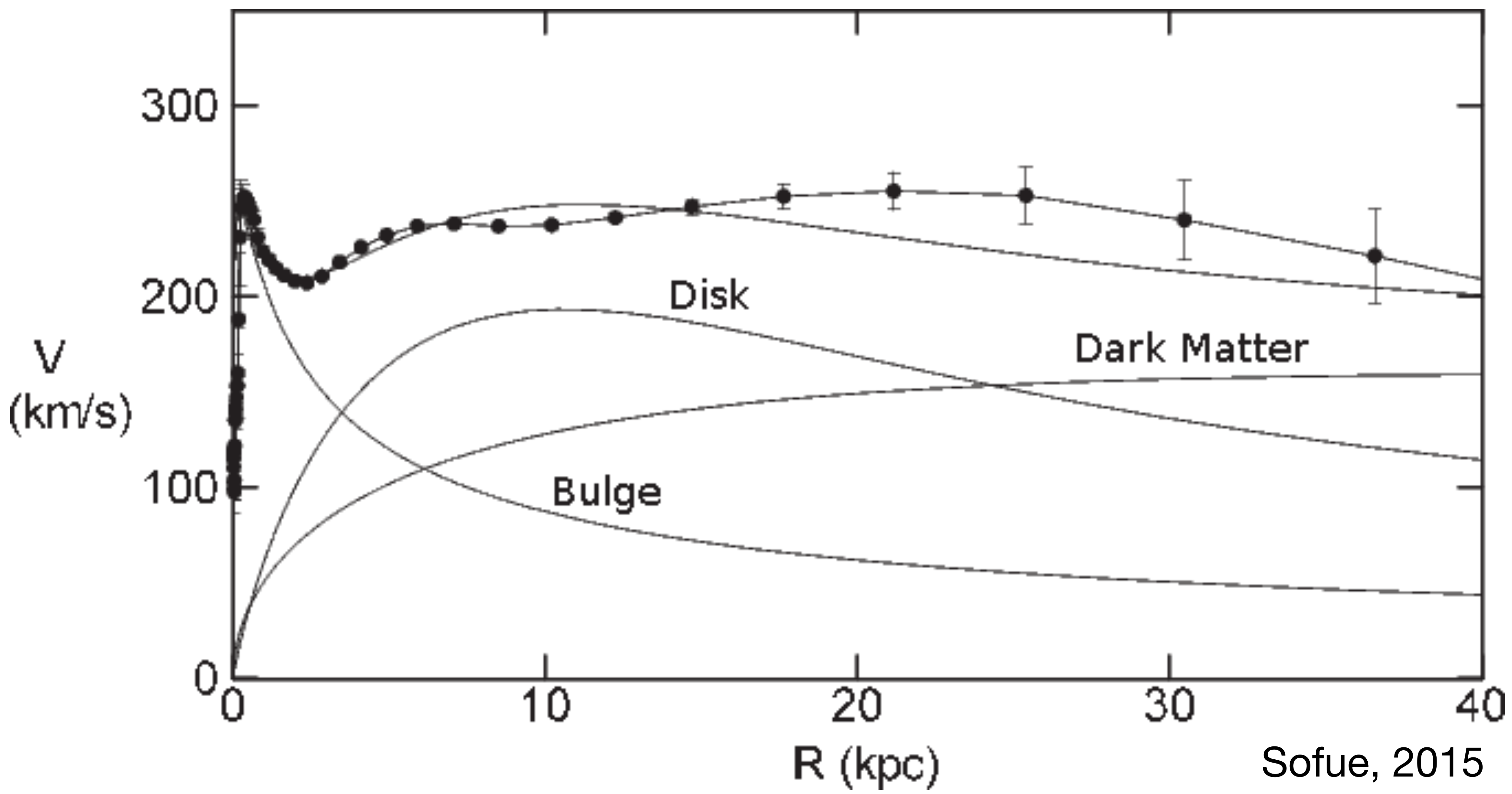
The NFW profile

- The Navarro-Frenk-White (NFW) profile ($\gamma = 1$)

$$\rho(r) = \frac{\rho_0}{(r/R_s)^\gamma (1 + r/R_s)^{3-\gamma}}$$

Navarro, Frenk & White, 1997

The rotation curve of the MW



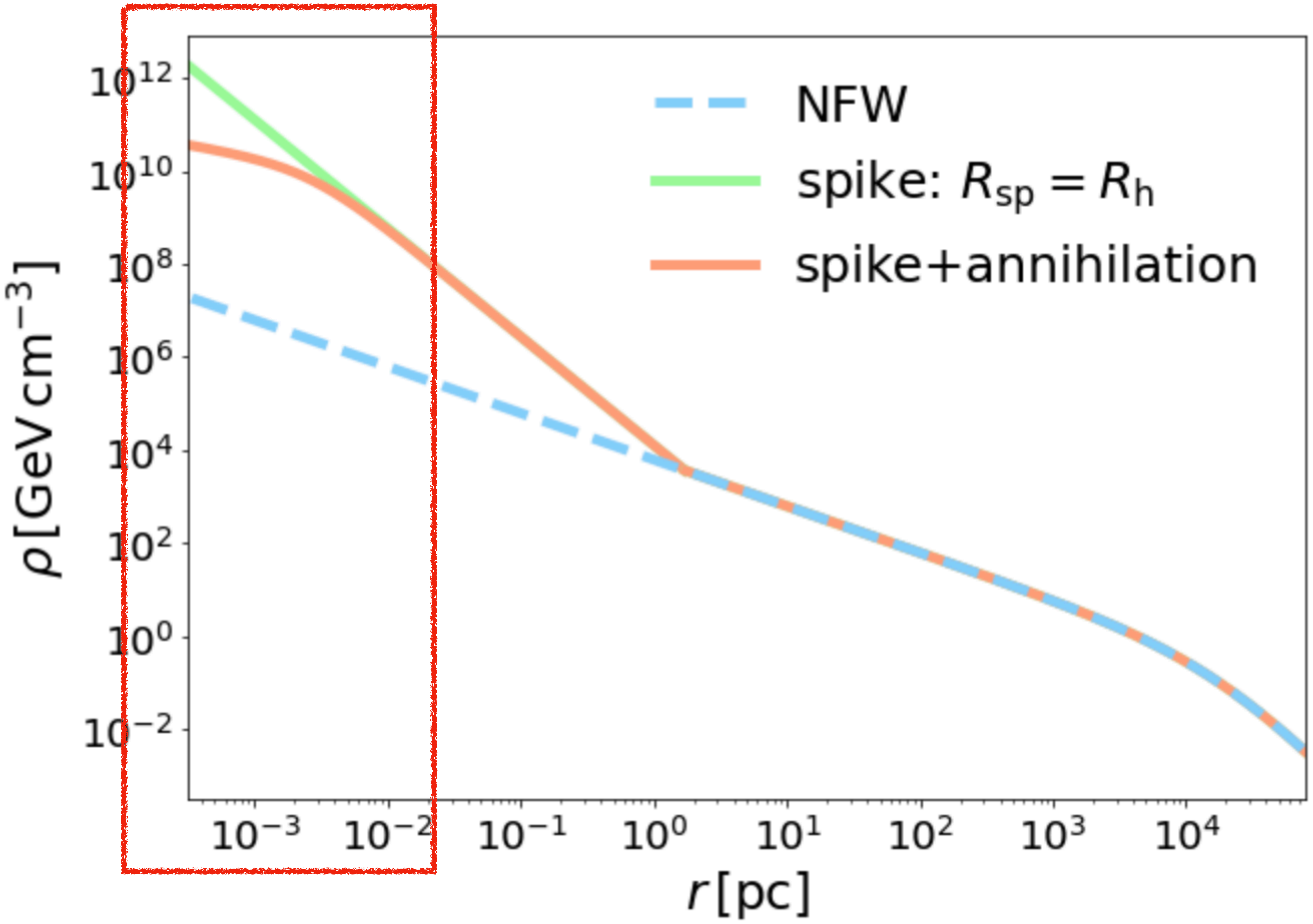
Sofue, 2015

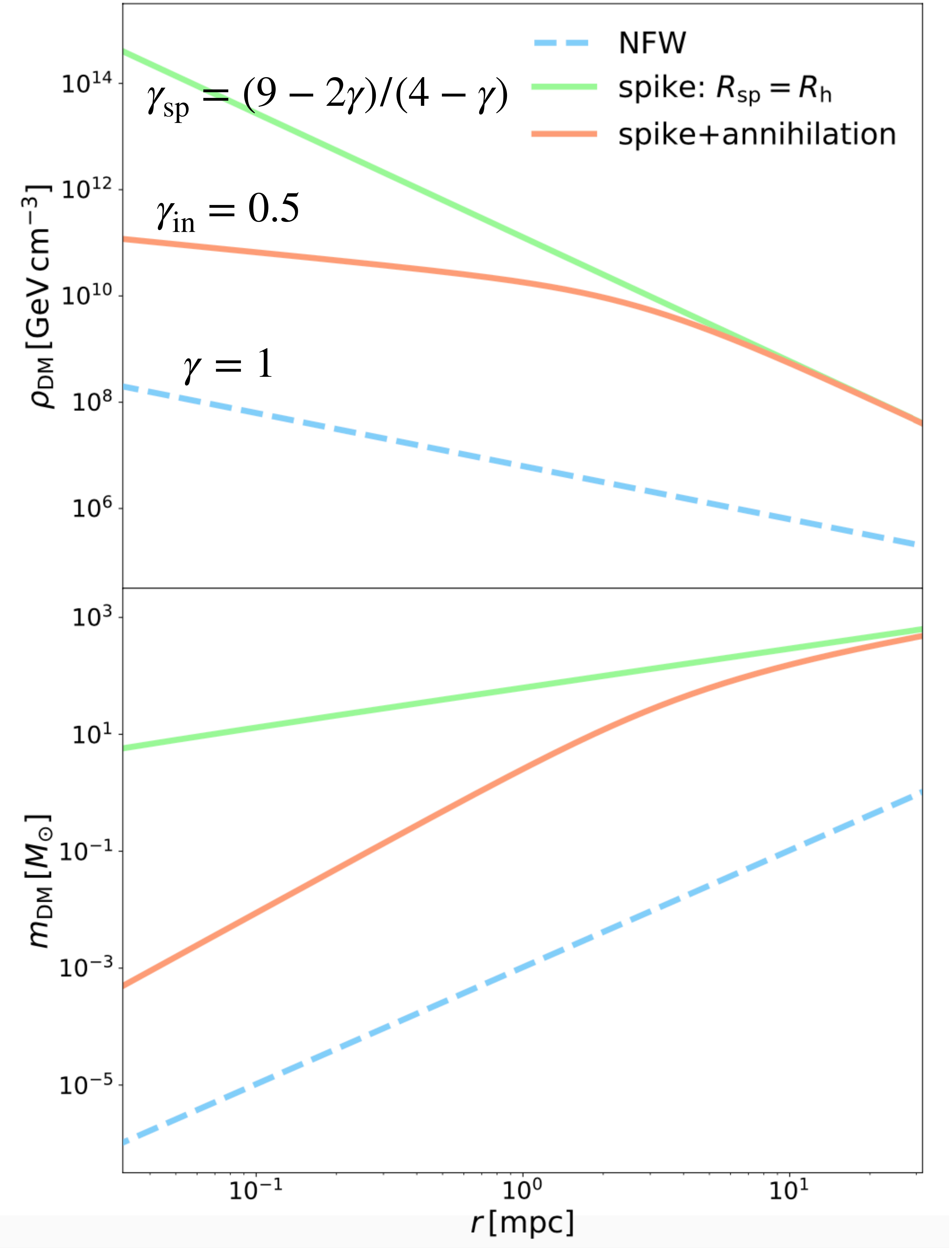
Difference from main model	$R_{d,\,thin}$ (kpc)	R_0 (kpc)	v_0 (km s^{-1})	$\rho_{h,\odot}$ (M$_\odot$ pc^{-3})	r_h (kpc)	M_* (10^9 M$_\odot$)	M_v (10^{12} M$_\odot$)
Main model: $\gamma = 1.00$	2.53 ± 0.14	8.20 ± 0.09	232.8 ± 3.0	0.0101 ± 0.0010	$18.6^{+5.3}_{-4.4}$	54.3 ± 5.7	1.32 ± 0.29
γ free ($\gamma = 0.79 \pm 0.32$)	2.51 ± 0.15	8.20 ± 0.09	232.5 ± 3.0	0.0098 ± 0.0009	$15.4^{+8.0}_{-3.8}$	56.6 ± 6.2	1.34 ± 0.28

McMillan, 2017

The dark matter model

Dark matter distribution



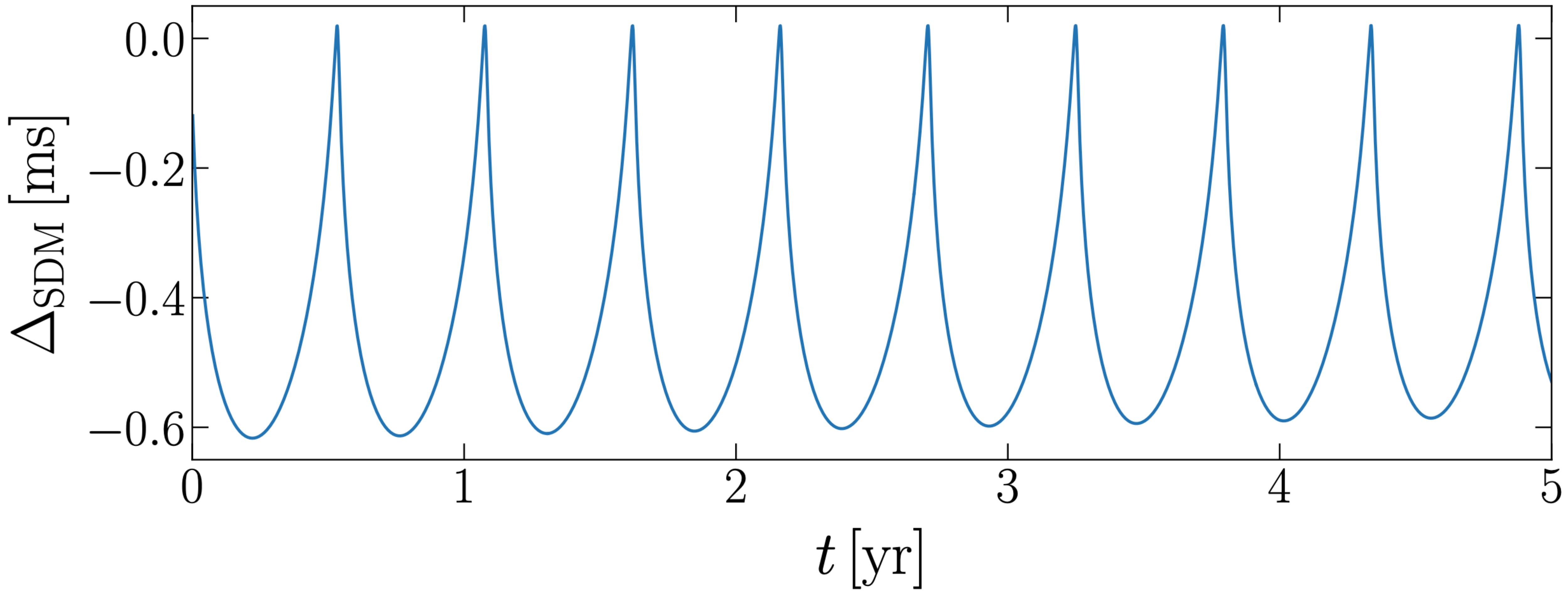


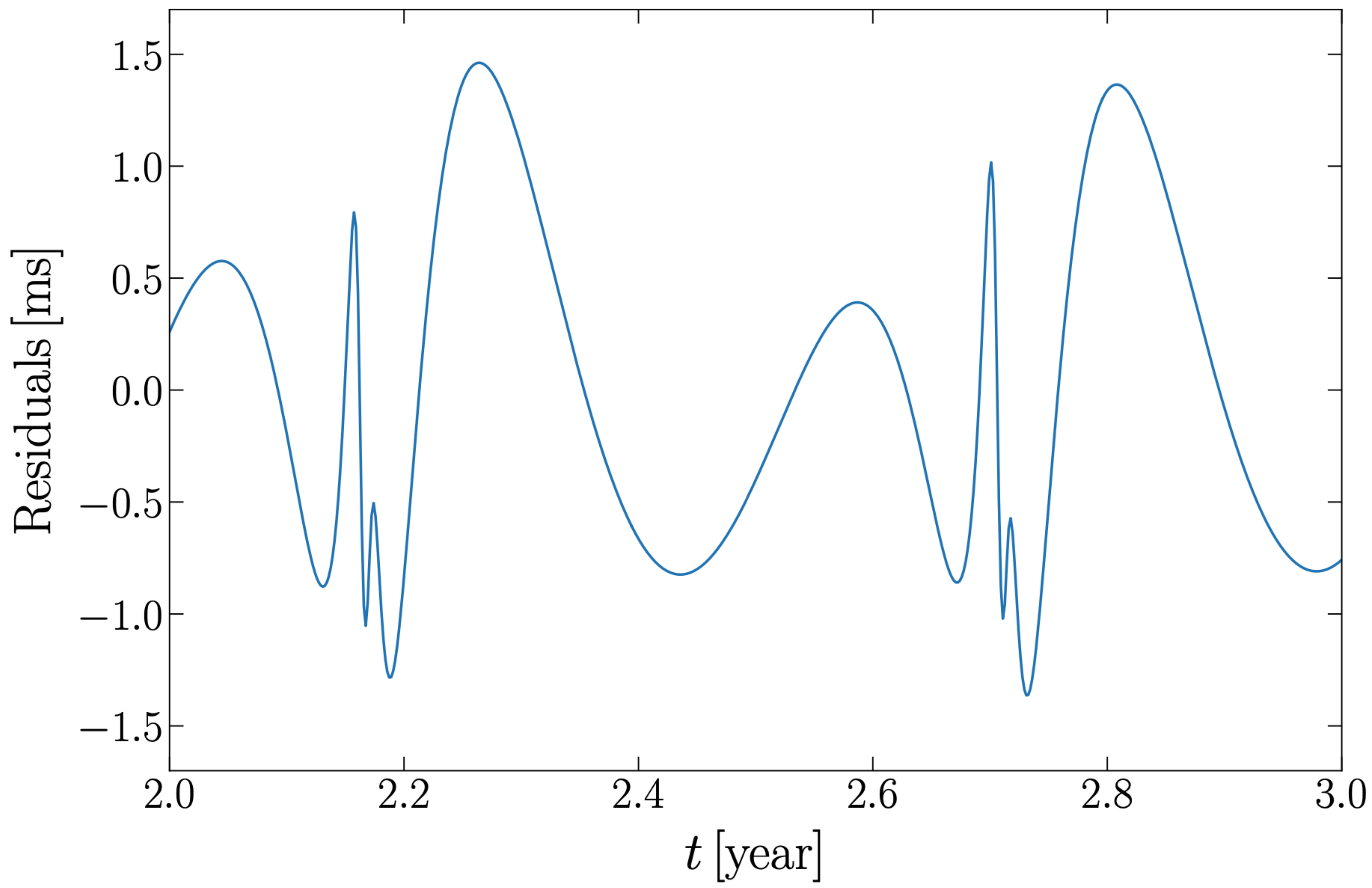
Gondolo & Silk, 1999

Timing residuals of dark matter

The detectability of the dark matter

- Consider the Newtonian gravity of the dark matter distribution
- $\rho(r) = \rho_0 \left(\frac{r}{4GM/c^2} \right)^{\gamma_{\rm sp}}, \; r > 4GM/c^2$

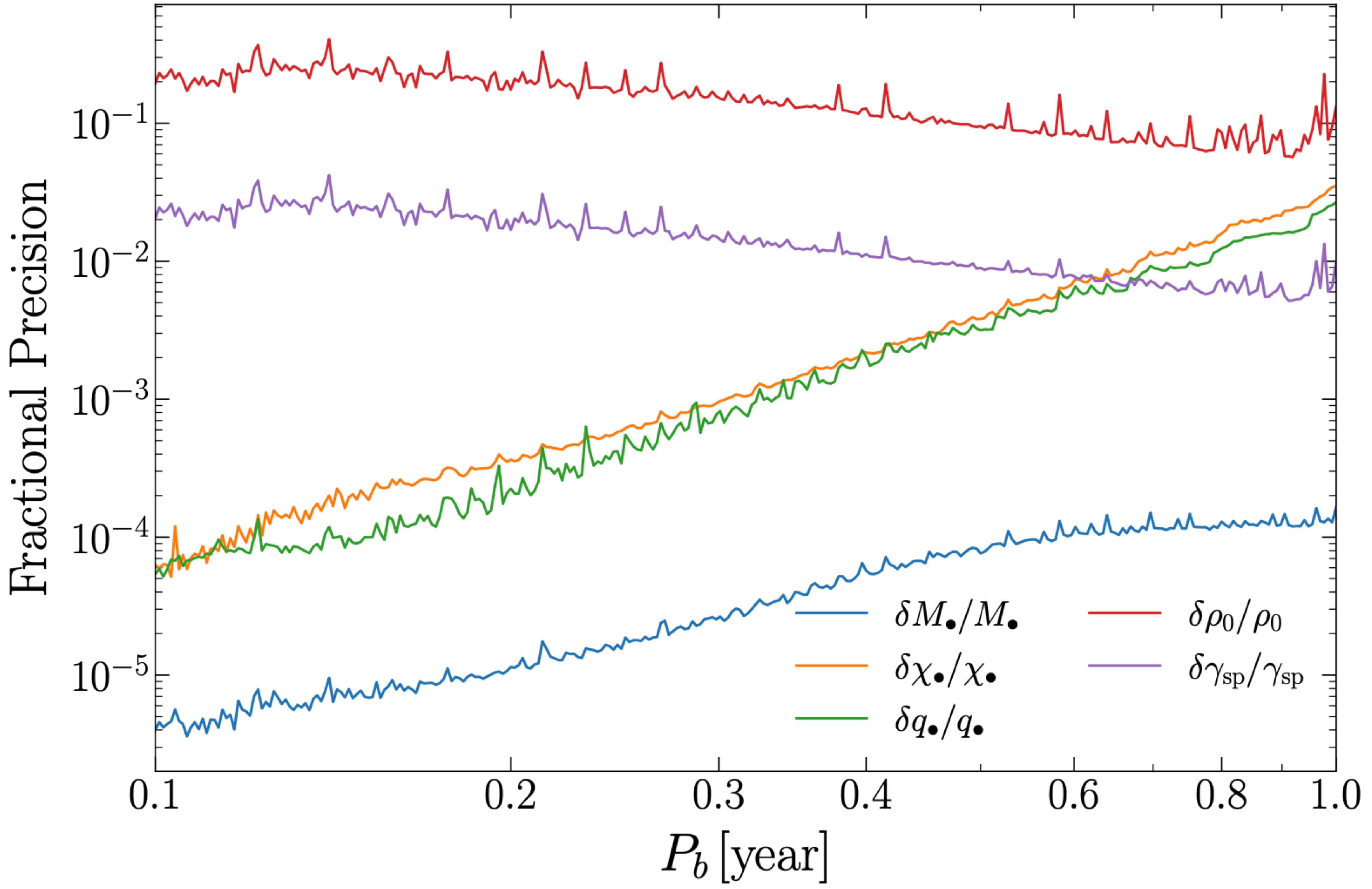


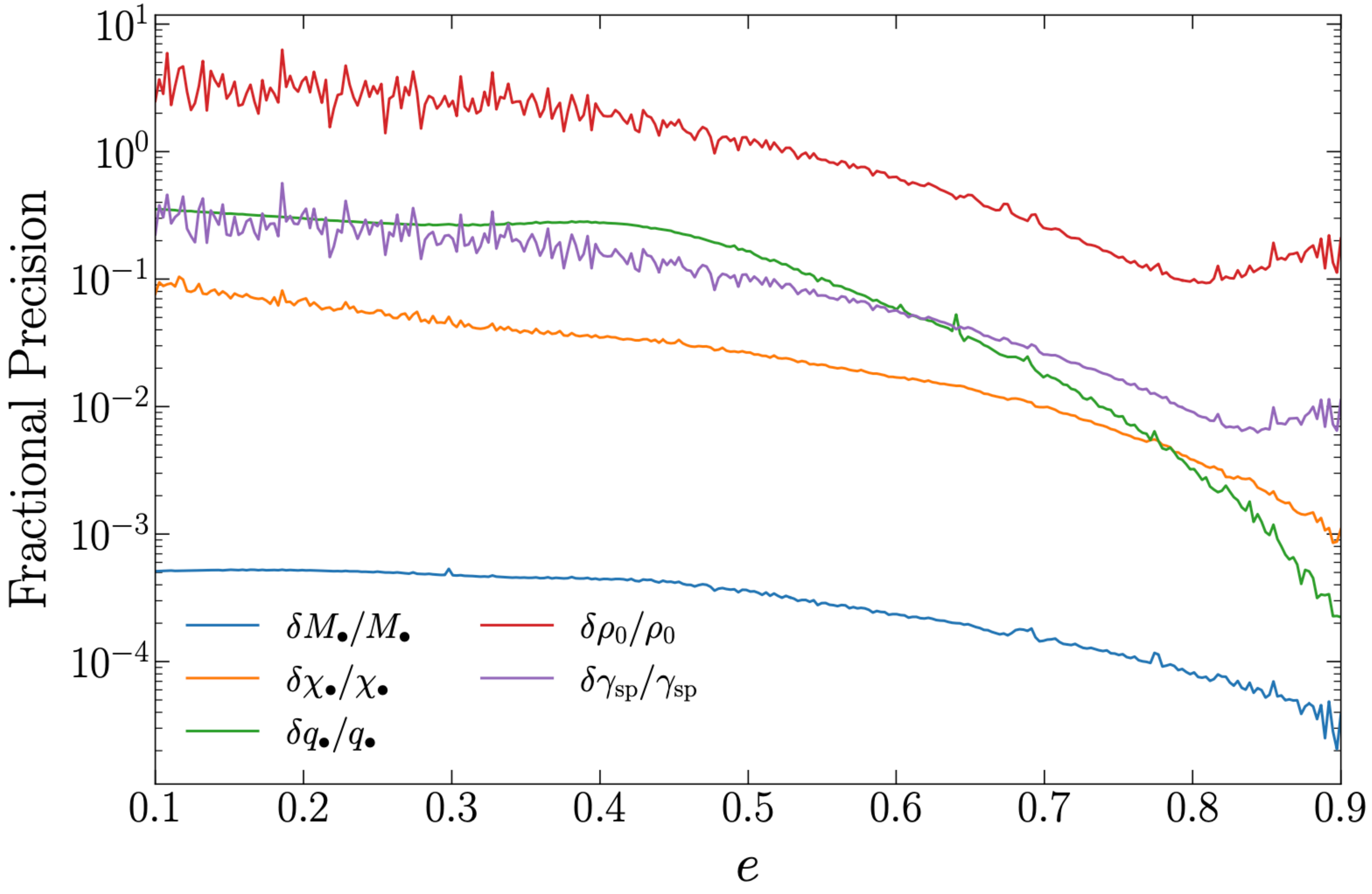


- Secular effect can be absorbed by other parameters

Parameter estimation

Fractional precision as function of P_b and e





Summary

- We develop the numerical timing/inverse timing model of the pulsar-supermassive black hole system based on the post-Newtonian equations of motion

- A 5-yr observation with weekly TOA and $\sigma_{\rm TOA} = 1\,{\rm ms}$ of a pulsar in orbit with $P_b \gtrsim 0.5\,{\rm yr}$ and $e \sim 0.8$ can give a measurement of ~1% fractional precision in $\gamma_{\rm sp}$ for the spike model, related to ~20% fraction precision in γ. Which is comparable to the result from fitting kinematic data (McMillan, 2017: $\gamma = 0.79 \pm 0.32$)