Radial and nonradial oscillations of hybrid neutron stars

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Outline

- Background
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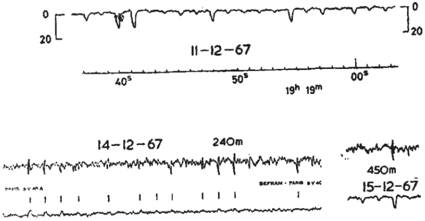
- Hydrostatic equilibrium structure
- Radial and nonradial oscillations
- Gravitational wave (GW) radiation
- Conclusions



Background

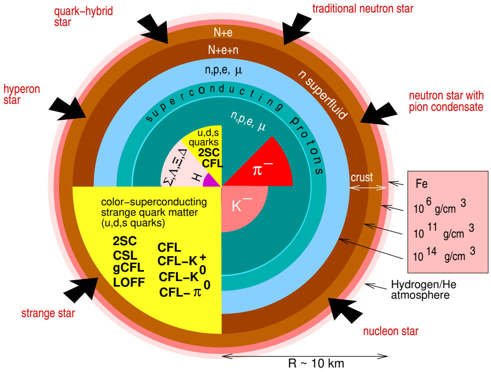
The interior of a neutron star (NS) can reach several times the nuclear saturation density.
The EOS of high-density nuclear matter (NM) remains an open theoretical problem.
Theory is highly dependent on models.

For high-density NM：

1. Kaon condensation
2. Pion condensation
3. Deconfined quark matter (hybrid star (HS))
4. Hyperon

Natural laboratory to investigate compact NM.

Great importance to both nuclear physics and astrophysics.



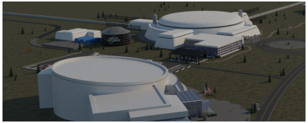
The theoretically possible structure of a compact star. (j.ppnp. 07, 001 (2001))

Background

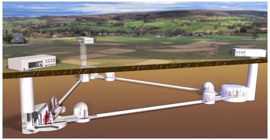
LIGO

Virgo

KAGRA



Cosmic Explorer



Einstein Telescope

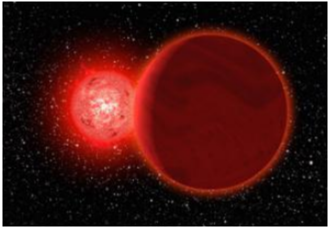
The first direct observation of GWs from a binary NS merger in 2017.

(Phys.Rev.Lett.119,161101(2017))

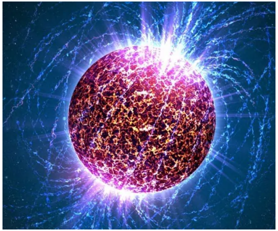
GW will become an important observables in NS astronomy!

The main source of GWs:

supernova explosion



a binary star merger



isolated neutron star

Isolated NSs primarily release GWs through nonradial oscillations.
Isolated NSs may become stable sources of GWs.

Hydrostatic equilibrium structure

Nuclear matter:

Brueckner-Hartree-Fock (BHF) theory

Phys.Rev.C 100,054335(2019)

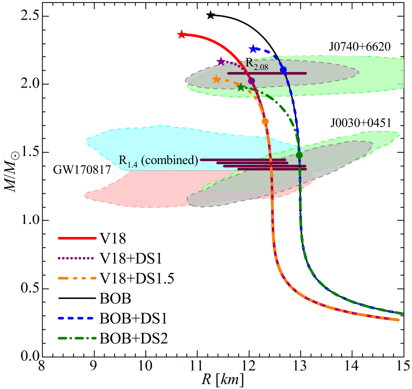
$\downarrow$ Gibbs construction

Quark matter:

Dyson-Schwinger quark model Phys.Rev.D 84,105023(2011)

EOS + TOV equations $\rightarrow$ radius and mass

It is hard to distinguish HSs and pure NSs from the mass-radius relations, since theoretically the differences between them are small, and even masked by the uncertainties of pure NSs with various models.



The mass-radius relations of NSs obtained with different EOSs.

Radial and nonradial oscillations

In order to distinguish the HSs and NSs

⇓

It is hard to distinguish them from the mass-radius relations

⇓

Oscillations of NSs ⟵

⇓

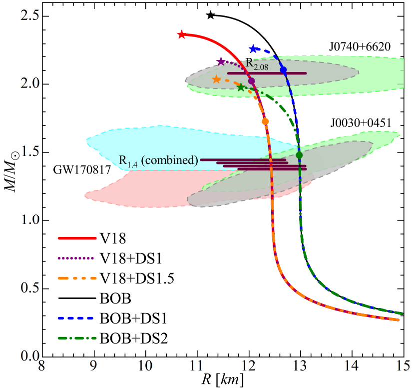
Radial oscillations of NSs

⇓

Nonradial oscillations of NSs

⇓

Gravitational wave radiation



The mass-radius relations of NSs obtained with different EOSs.

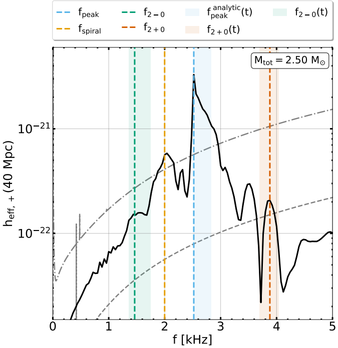
Radial and nonradial oscillations

Effective GW spectrum for the post-merger phase:

f_{peak} ⟵fundamental quadrupolar oscillation

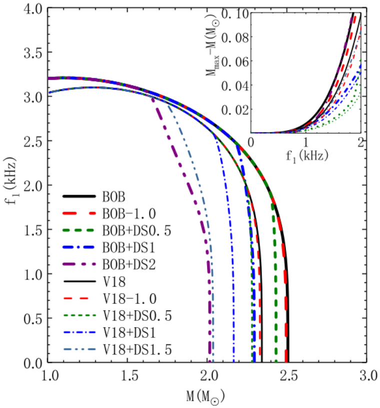
f_{2-0}
f_{2+0} ⟵coupling between the quadrupolar mode and the quasi-radial oscillation mode

Oscillations and GW emission are closely related!



Effective GW spectrum for the post-merger phase.
Phys.Rev.D 105,043020 (2022)

Radial oscillations

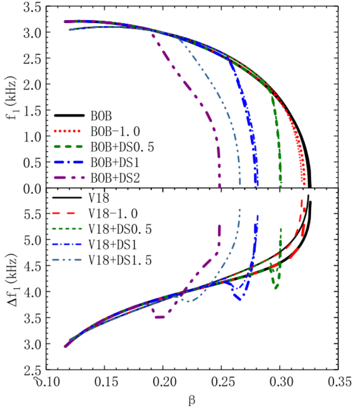


Fundamental frequency f_1 vs mass M.

1. As the masses increase, f_1 decreases to zero at the maximum mass, consistent with the stability criterion $\frac{dM}{d\rho_c} \geq 0$.

2. Small values of f_1 can provide an accurate estimate of the maximum NS mass.

Radial oscillations



The dependence of f_1 (upper panel) and $\Delta f_1 = f_2 - f_1$ (lower panel) on the compactness

3. Small values of the compactness parameter β together with small values of f_1 or large values of Δf_1 characterize HSs in our approach and allow to disentangle them from pure NSs.

Phys. Rev. D 103, 103003 (2021)

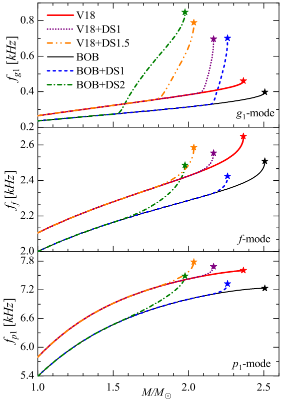
1. Quadrupole $(l = 2)$ oscillations.

2. For a nonrotating NS, the eigenmodes of oscillation are divided into g mode, f mode and p mode. They can be ordered as

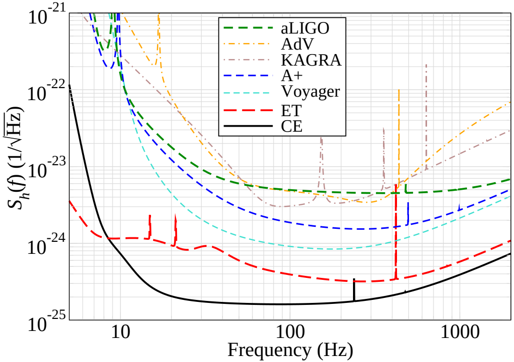
$$\omega_{g_n} < \cdots < \omega_{g_1} < \omega_f < \omega_{p_1} < \cdots < \omega_{p_n},$$

and

$$\lim_{n\to\infty} \omega_{g_n} = 0, \lim_{n\to\infty} \omega_{p_n} = \infty .$$



g_1-, f-, and p_1-mode frequencies vs M for various EOSs.



The detection capabilities of various detectors.
(Phys.Rev.Lett. 118,151105 (2017))

Nonradial oscillations

The total energy of an eigenmode in Cowling approximation:

$$E = \int \frac{\omega^2}{2}(p+\varepsilon)e^{(\lambda-\nu)/2}\left[W^2r^4 + l(l+1)V^2r^4\right]dr\,.$$

The power of the GW radiation released by the mode can be estimated as:

$$P_{GW} = \frac{G(l+1)(l+2)}{8\pi(l-1)l}\left[\frac{4\pi\omega^{l+1}}{(2l+1)!!}\int_0^R \delta\varepsilon r^{l+2}dr\right]^2 .$$

The damping time of oscillations through GW emission:

$$\tau_{GW} = \frac{2E}{P_{GW}}.$$

The GW strain can be calculated by

$$h_+ = \frac{3G\sin^2\alpha}{2D}\ddot{Q}_{33},$$

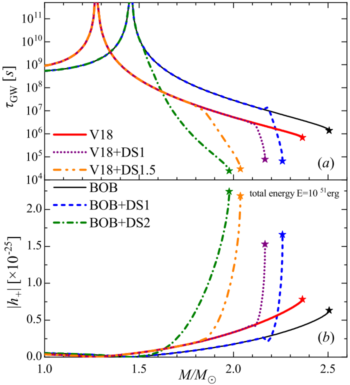
we choose $\sin\alpha = 1$, D is the distance to the source and $\ddot{Q}_{33}$ can be written as

$$\ddot{Q}_{33} = \frac{4\sqrt{\pi/5}}{3}\omega^2 \int drr^4\delta\varepsilon(r).$$

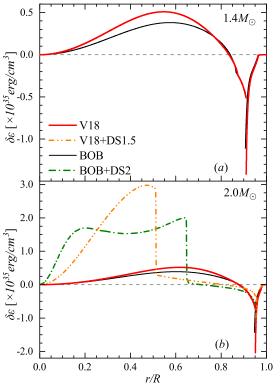
Choosing a typical energy scale $E \sim 10^{51}$ erg and a typical distance $D \sim 15\,\mathrm{Mpc}$ (star in the Virgo cluster).

The detected GW strain is currently approximately $h \sim 10^{-22}$ (Phys. Rev. D 101, 084002).

Gravitational wave radiation



Properties of g_1-mode oscillations vs NS mass M for various EOSs: (a) The damping time and (b) the amplitude of the GW strain $|h_+|$.



Eulerian perturbation of the energy density in NSs with $1.4\ M_\odot$ (a) and $2.0\ M_\odot$ (b) for various EOSs.

Gravitational wave radiation

The minimum energy that should be released to be detectable in present and planned GW observatories can be estimated as: (Mon. Not. R. Astron. Soc. 320, 307)

$$\frac{E_{\mathrm{GW}}}{M_\odot} = 3.5\times 10^{36}\frac{1+4Q^2}{4Q^2}\frac{S_n}{1\mathrm{s}}\left(\frac{S}{N}\frac{D}{10\mathrm{kpc}}\frac{f}{1\mathrm{kHz}}\right)^2 ,$$

Table: The minimum detectable energy E_{GW} (in units of erg) of $M = 1.4, 2.0\,M_\odot$ NSs at two representative distances for various EOSs.

		$1.4\,M_\odot$		$2.0\,M_\odot$			
Detector	Distance	V18	BOB	V18	V18+DS1.5	BOB	BOB +DS2
LIGO/Virgo	10 kpc	1.55×10^{46}	1.13×10^{46}	2.33×10^{46}	6.76×10^{46}	1.57×10^{46}	1.14×10^{47}
LIGO/Virgo	15 Mpc	3.48×10^{52}	2.54×10^{52}	5.23×10^{52}	1.52×10^{53}	3.53×10^{52}	2.56×10^{53}
Einstein	10 kpc	6.19×10^{44}	4.52×10^{44}	9.30×10^{44}	2.71×10^{45}	6.28×10^{44}	4.56×10^{45}
Einstein	15 Mpc	1.39×10^{51}	1.02×10^{51}	2.09×10^{51}	6.09×10^{51}	1.41×10^{51}	1.03×10^{52}

Conclusions

1. Eigenfrequencies $f_{g_1} \sim 300\,\mathrm{Hz}$ for pure NSs. For HSs, reaching above $700\,\mathrm{Hz}$. All these frequencies are in the sensitivity range of current ground-based GW detectors. This shows a clear difference of the g-mode frequency between pure NSs and HSs.

2. The concurrent shorter g_1 damping times of HSs correspond to larger GW strain and radiation power, and thus easier detection than for pure NSs.

3. Estimates of the GW strain h_+ and minimum detectable energy E_{GW} suggest that the GWs from the g_1 mode of NSs/HSs in our galaxy could be detected by present and planned detectors.

4. To sum up, the g_1 mode is the most suitable mode to provide a window on the internal composition of the compact object.

Phys. Rev. D 107, 103048 (2023)

Thanks for your listening!

Consider a spherically symmetric system with only radial motion, one can obtain：

$$\frac{d\xi}{dr} = -\frac{1}{r}\left(3\xi + \frac{\eta}{\Gamma}\right) - \frac{dp}{dr}\frac{\xi}{p+\varepsilon}\,,$$

$$\frac{d\eta}{dr} = \frac{\xi}{p}\left[\omega^2 e^{\lambda-\nu}(p+\varepsilon)r - 4\frac{dp}{dr} + \left(\frac{dp}{dr}\right)^2 \frac{r}{p+\varepsilon} - 8\pi G e^{\lambda}(p+\varepsilon)pr\right] - \eta\left[\frac{dp}{dr}\frac{\varepsilon}{p(p+\varepsilon)} + 4\pi G e^{\lambda}(p+\varepsilon)r\right]\,,$$

where

$$\xi = \frac{\Delta r}{r}\,, \quad \eta = \frac{\Delta p}{p}\,, \quad \Gamma = \frac{p+\varepsilon}{p}v_s^2 = \frac{p+\varepsilon}{p}\frac{dp}{d\varepsilon}\,.$$

Boundary conditions：

$$[\eta + 3\Gamma\xi](0) = 0\,, \quad \eta(R) = 0\,.$$

The solutions provide the discrete eigenvalues ω_i^2

$$\omega_1^2 < \omega_2^2 < \ldots < \omega_n^2$$

where n is the number of nodes. Negative ω^2 indicate unstable oscillations and thus $\omega^2 = 0$ critical condition for the stability of NSs under radial perturbations.

	$2.0M_\odot$			
n	BOB	BOB + DS2	V18	V18+ DS1.5
1	2.73	1.25	2.64	1.59
2	6.80	5.90	6.85	6.11
3	10.08	8.57	10.05	9.16
4	12.24	11.44	12.93	11.78
5	13.45	13.39	14.72	14.36
6	15.08	14.51	16.04	15.64

Table: The radial oscillation frequencies f_n[kHz] of $M = 2.0M_\odot$ NSs using different EOSs.

Nonradial oscillations

The perturbation of the fluid in the star is described by the Lagrangian displacement vector ξ^α in terms of the dimensionless perturbation functions $W(r)$ and $V(r)$,

$$\begin{aligned}
\xi^r &= r^{l-1} e^{-\lambda/2} W Y_m^l e^{i\omega t} \ , \\
\xi^\theta &= -r^{l-2} V \partial_\theta Y_m^l e^{i\omega t} \ , \\
\xi^\phi &= -r^{l-2} (\sin\theta)^{-2} V \partial_\phi Y_m^l e^{i\omega t} \ .
\end{aligned}$$

The metric becomes:

$$\begin{aligned}
ds'^2 =& ds^2 + h_{\mu\nu} dx^\mu dx^\nu \\
=& - e^v \left(1 + r^l H_0 Y_m^l e^{i\omega t}\right) dt^2 - 2i\omega r^{l+1} H_1 Y_m^l e^{i\omega t} dt dr \\
& + e^\lambda \left(1 - r^l H_0 Y_m^l e^{i\omega t}\right) dr^2 + r^2 \left(1 - r^l K Y_m^l e^{i\omega t}\right) \left(d\theta^2 + \sin^2\theta d\phi^2\right)
\end{aligned}$$

The oscillation equations can be obtained from the Einstein field equation and the equation of motion:

$$c^4 \delta G_\mu^\nu = 8\pi \delta T_\mu^\nu \ , \ \delta\left(T_{\mu;\nu}^\nu\right) = 0$$

The oscillation equations in Cowling approximation:

$$\frac{dW}{dr} = \left(\frac{gr}{c_s^2} - 3\right)\frac{W}{r} + e^{\lambda/2}\left[\frac{\omega^2 r^2}{c_s^2 e^{\nu}} - l(l+1)\right]\frac{V}{r},$$
$$\frac{dV}{dr} = e^{\lambda/2}\left(\frac{N^2}{\omega^2} - 1\right)\frac{W}{r} + \left(2gr + \frac{N^2 r}{g e^{\nu-\lambda}} - 2\right)\frac{V}{r},$$

where

$$c_e^2 = \frac{dp}{d\varepsilon}, \quad c_s^2 = \left(\frac{\partial p}{\partial \varepsilon}\right)_{S,x}, \quad g = -(dp/dr)/(p+\varepsilon), \quad N^2 \equiv g^2\left(\frac{1}{c_e^2} - \frac{1}{c_s^2}\right)\mathrm{e}^{\nu-\lambda}.$$

Boundary conditions:

$$W(0) + lV(0) = 0, \quad \Delta p(R) = \gamma p\left[-e^{-\lambda/2}(r\frac{dW}{dr} + 3W) - l(l+1)V\right] = 0\,.$$

Nonradial oscillations

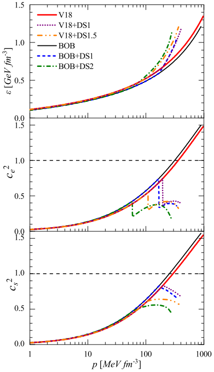
1. $c_e^2 = \frac{dp}{d\varepsilon}$: the equilibrium speed of sound.

2. $c_s^2 = \left(\frac{\partial p}{\partial \varepsilon}\right)_{S,x}$: the adiabatic speed of sound.

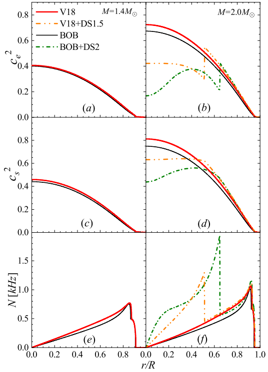
- Consider three timescales: weak τ_{weak}、strong τ_{strong} and oscillation τ_{os}
- In pure nuclear phase： $\tau_{os} < \tau_{weak}$， without β equilibrium. Fixed particles fractions.
- In mixed phase： $\tau_{os} < \tau_{weak}, \tau_{os} < \tau_{strong}$， without β equilibrium and the strong chemical equilibrium between NM and QM. keeping all particles fractions constant in NM and QM separately. Universe. 7, 493 (2021). Phys. Lett. B 729 (2014) 79–84.

3. $N^2 \equiv g^2 \left(\frac{1}{c_e^2} - \frac{1}{c_s^2}\right) \mathrm{e}^{\nu-\lambda}$: Brunt-Väisälä (BV) frequency. In the Newtonian approximation N is the frequency of the perturbed fluid elements forced by buoyancy to perform harmonic oscillations.

Nonradial oscillations



The energy density (upper panel), squared equilibrium speed of sound (central panel), and squared adiabatic speed of sound (lower panel) of NS matter as functions of pressure with different EOSs.



Squared equilibrium speed of sound (upper panels), squared adiabatic speed of sound (central panels), and BV frequency N (lower panels) in NSs with $1.4\,M_\odot$ (left panels) and $2.0\,M_\odot$ (right panels), for various EOSs.