A magnetospheric coherent radiation model for FRBs

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Work to be finished

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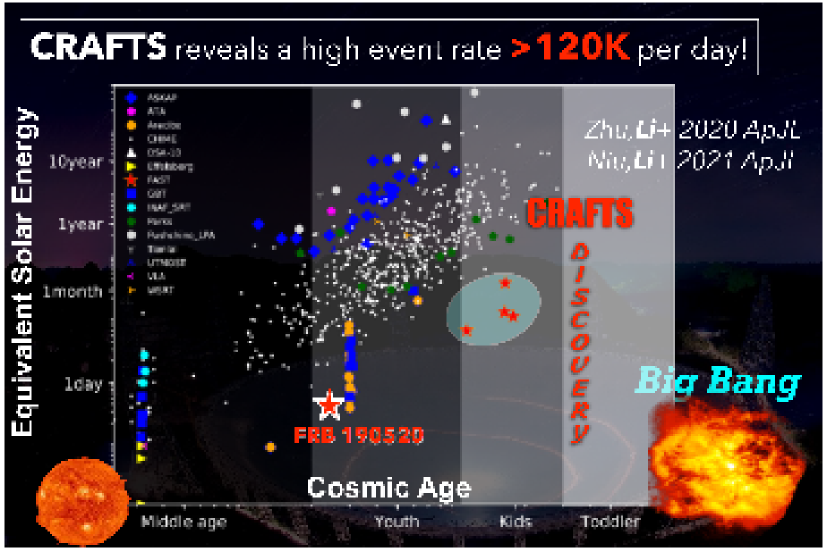
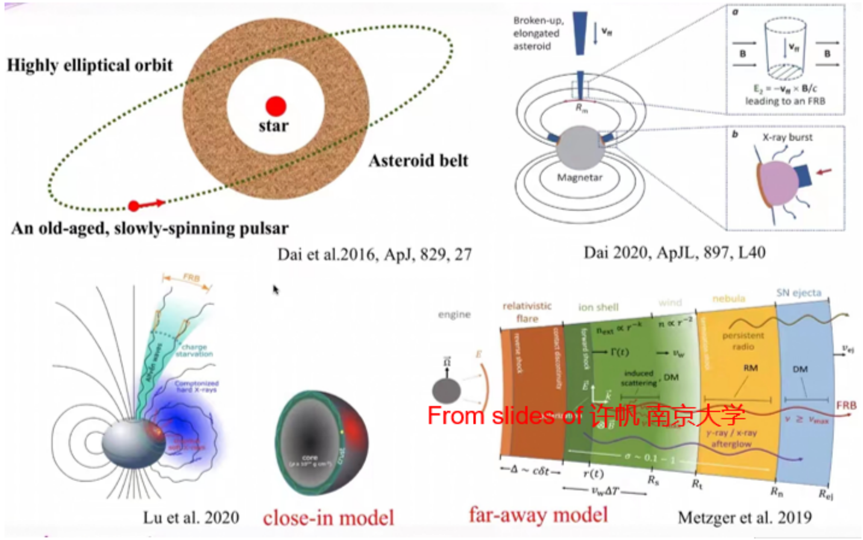
Overview

- Motivation
- Model introduction
- Simulation results

Zoo of FRBs *vs* Zoo of FRB models

- There are already many FRBs, and many FRB models





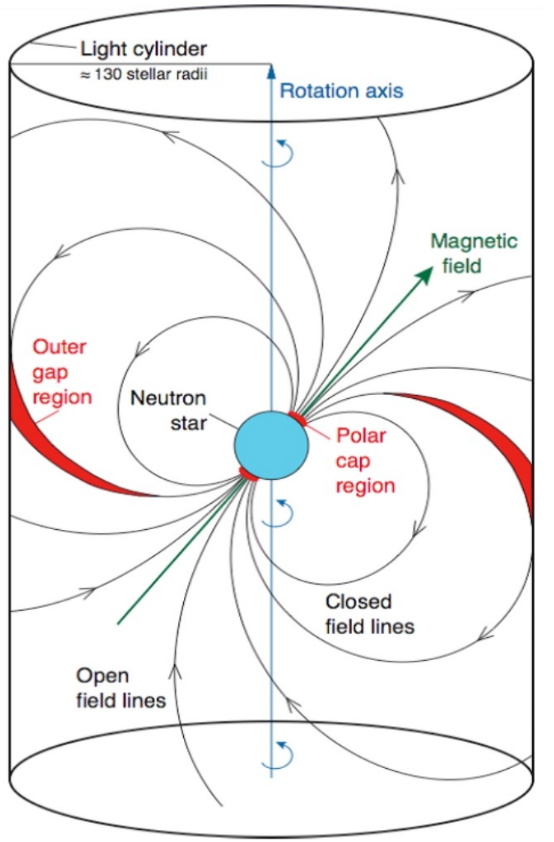
- Science: Enlarge the first zoo and hunt in the second zoo

- Theorists: Always enlarge the second zoo ~

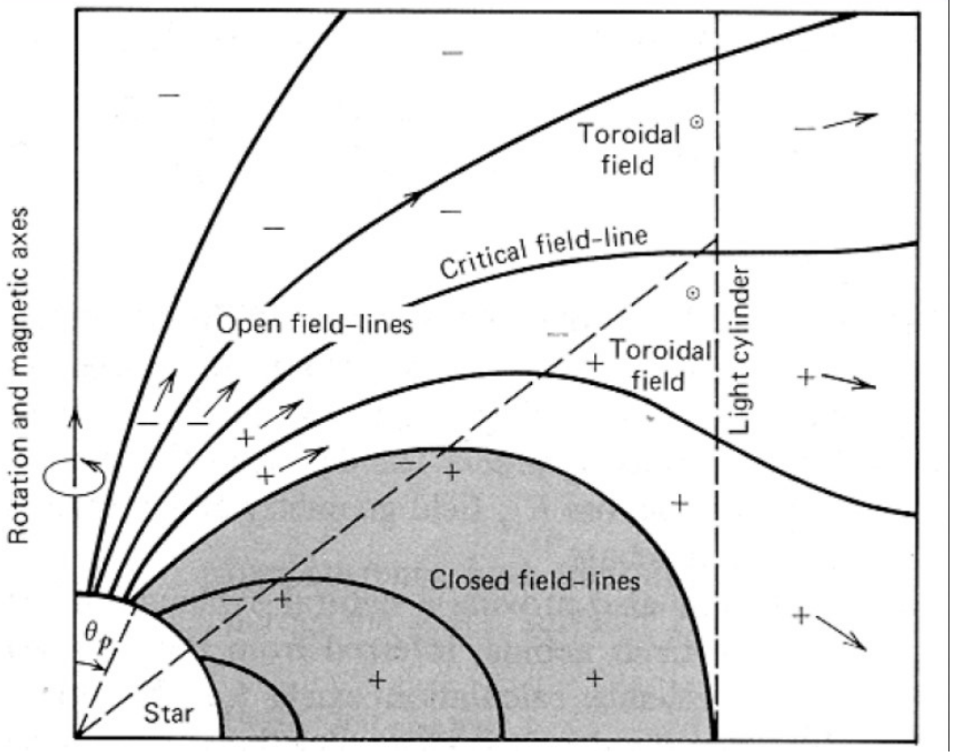
Basic conditions for getting a FRB

- Magnetic field
- Free energy & trigger event
- Coherent radiation mechanism

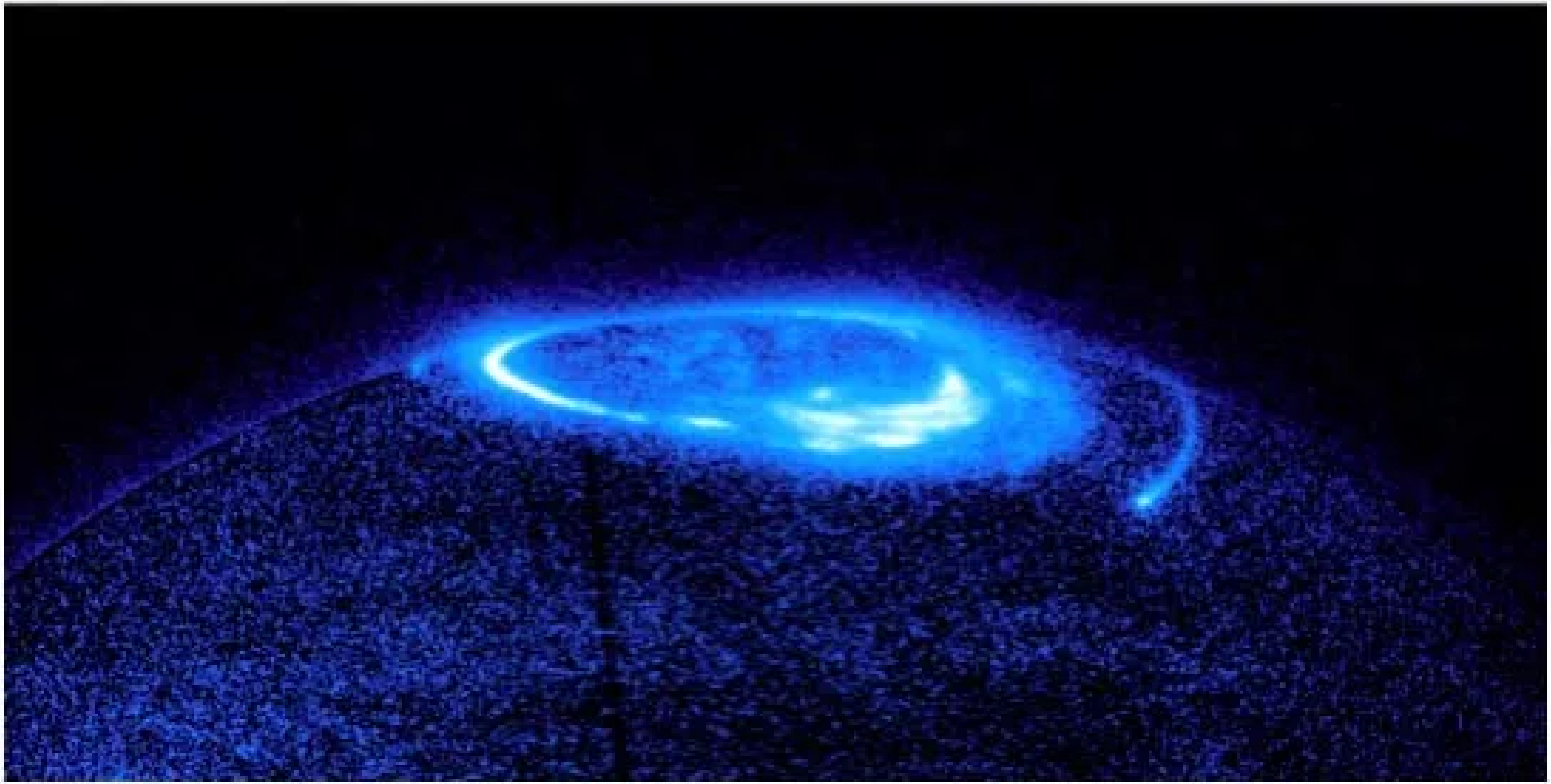
Magnetic field (magnetar)



Sturrock, Astrophys. J. 164, 529 (1971)



Emissions from the sun & the planets



Particles in magnetic field: quantum view

- Landau energy levels

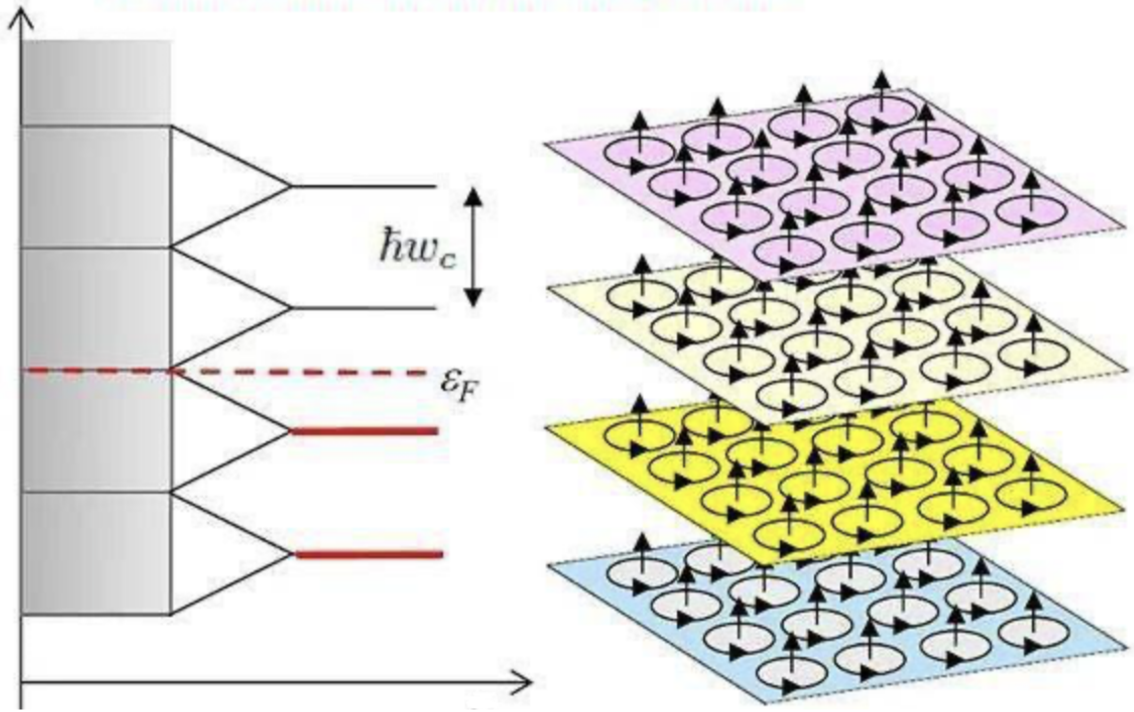
$$E_n = (\frac{1}{2} + n)h\nu,\ \nu = \frac{qB}{2\pi m}$$

Lifetime $T = \frac{3mc^3}{16\pi^2\nu^2 q^2} \propto B^{-2}$

- *For* $B = 100G$ *,*

cyclotron frequency $\nu \sim 300MHz$

lifetme $T \sim 26000s$



Coherence $\Rightarrow$ ***Lifetime shrinks***

Coherent radiation mechanism（laser & maser）

- **Gain Medium**: high engery particles in the magnetosphere

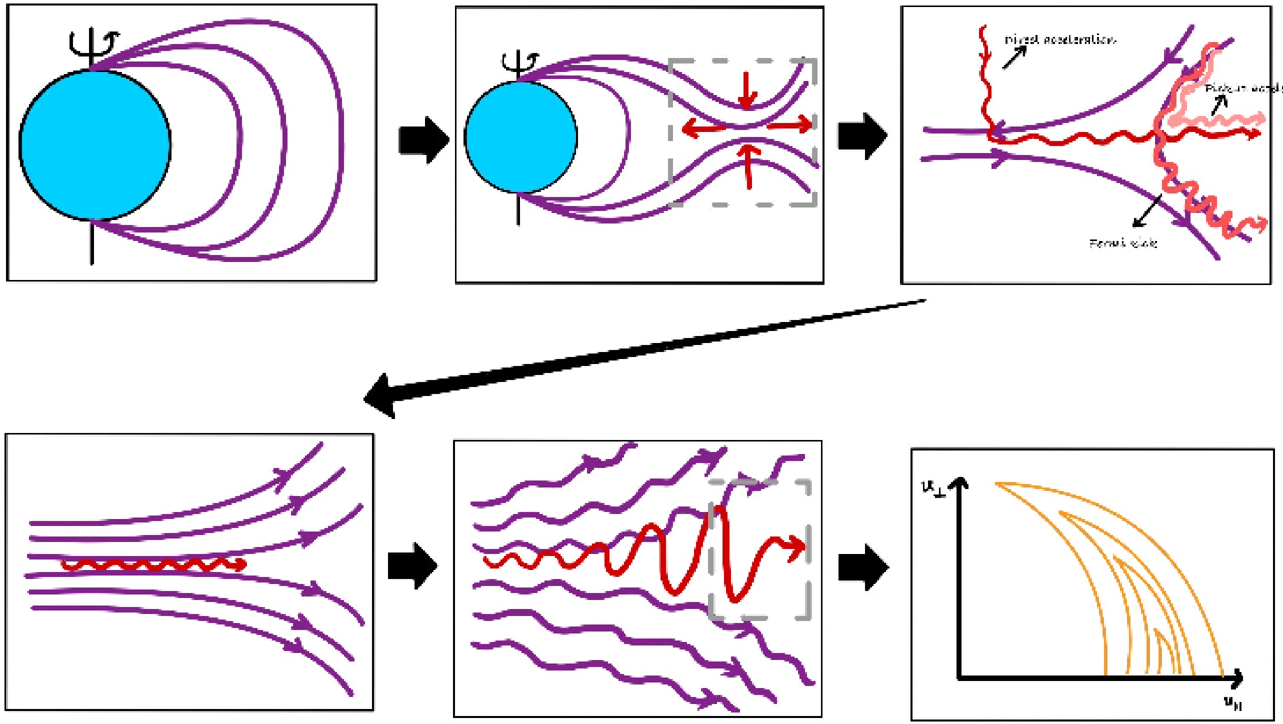
- **Nonthermal equilibrium**：particle population inversion

- **Geometric or kinematic conditions**: play the role of resonant cavity

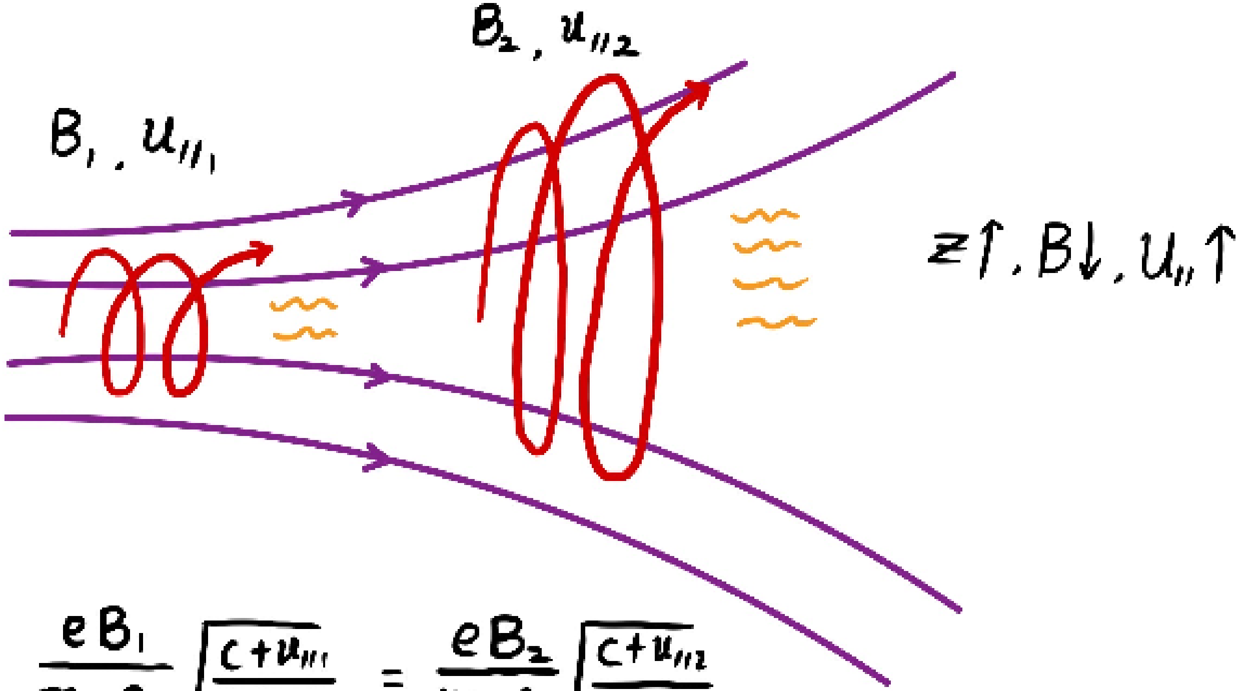
Burst trigger event

- Magnetic reconnection triggered by magnetospheric disturbances

  (accretion of matter, glitch, asteroid impacts, etc.)



Coherent radiation condition: $\Delta f_{Doppler} = \Delta f_{Larmor}$

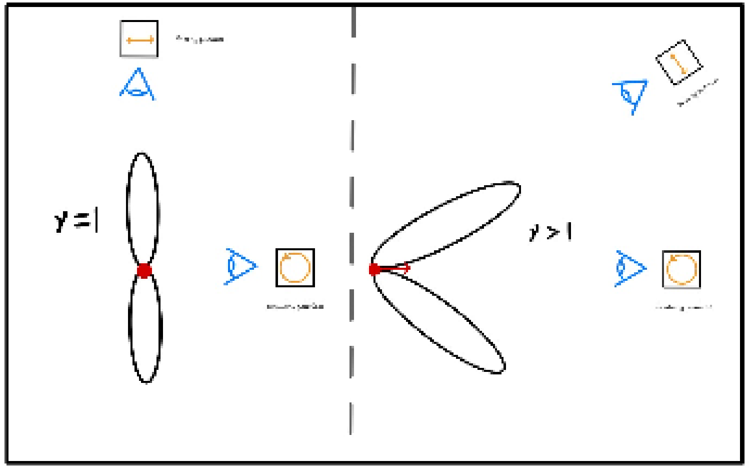


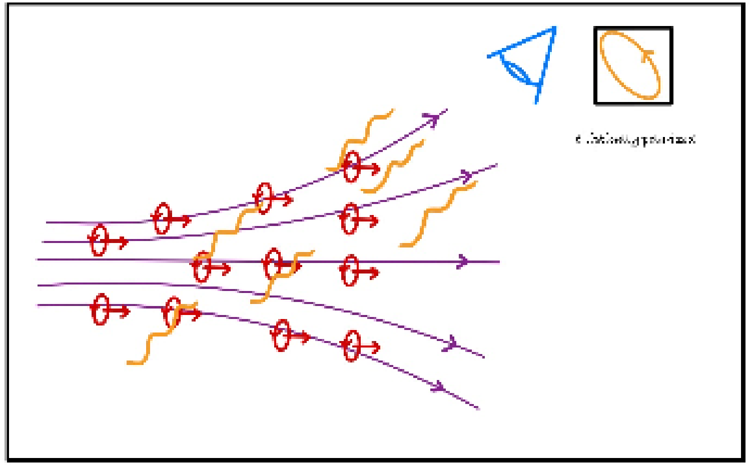
$$\frac{eB_1}{m_e c}\sqrt{\frac{c+u_{\parallel 1}}{c-u_{\parallel 1}}} = \frac{eB_2}{m_e c}\sqrt{\frac{c+u_{\parallel 2}}{c-u_{\parallel 2}}}$$

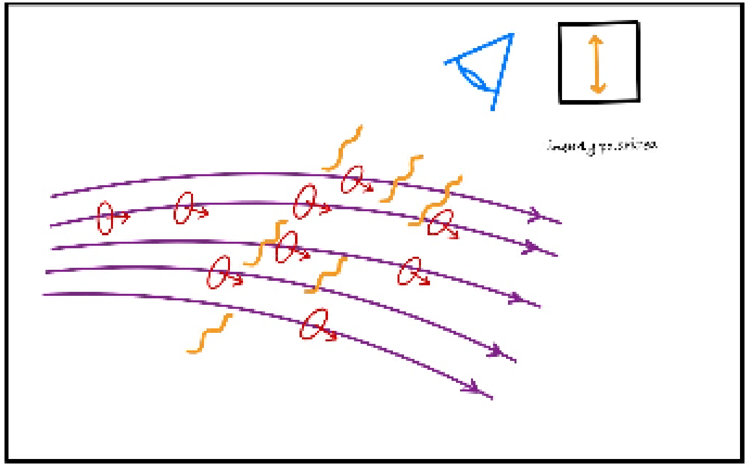
⬇

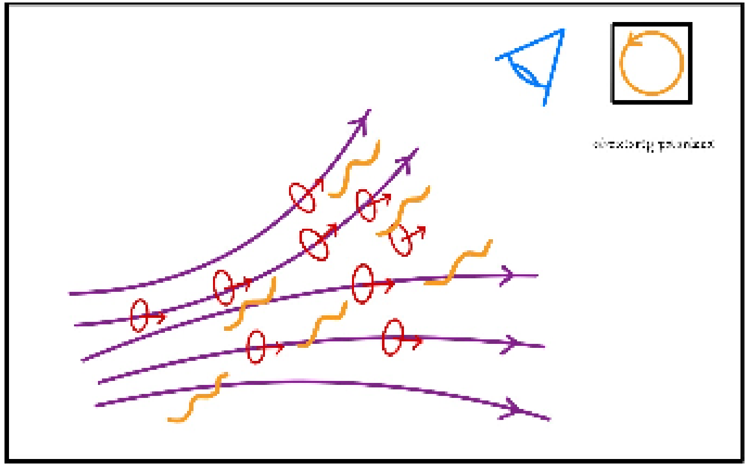
coherent

Polarization

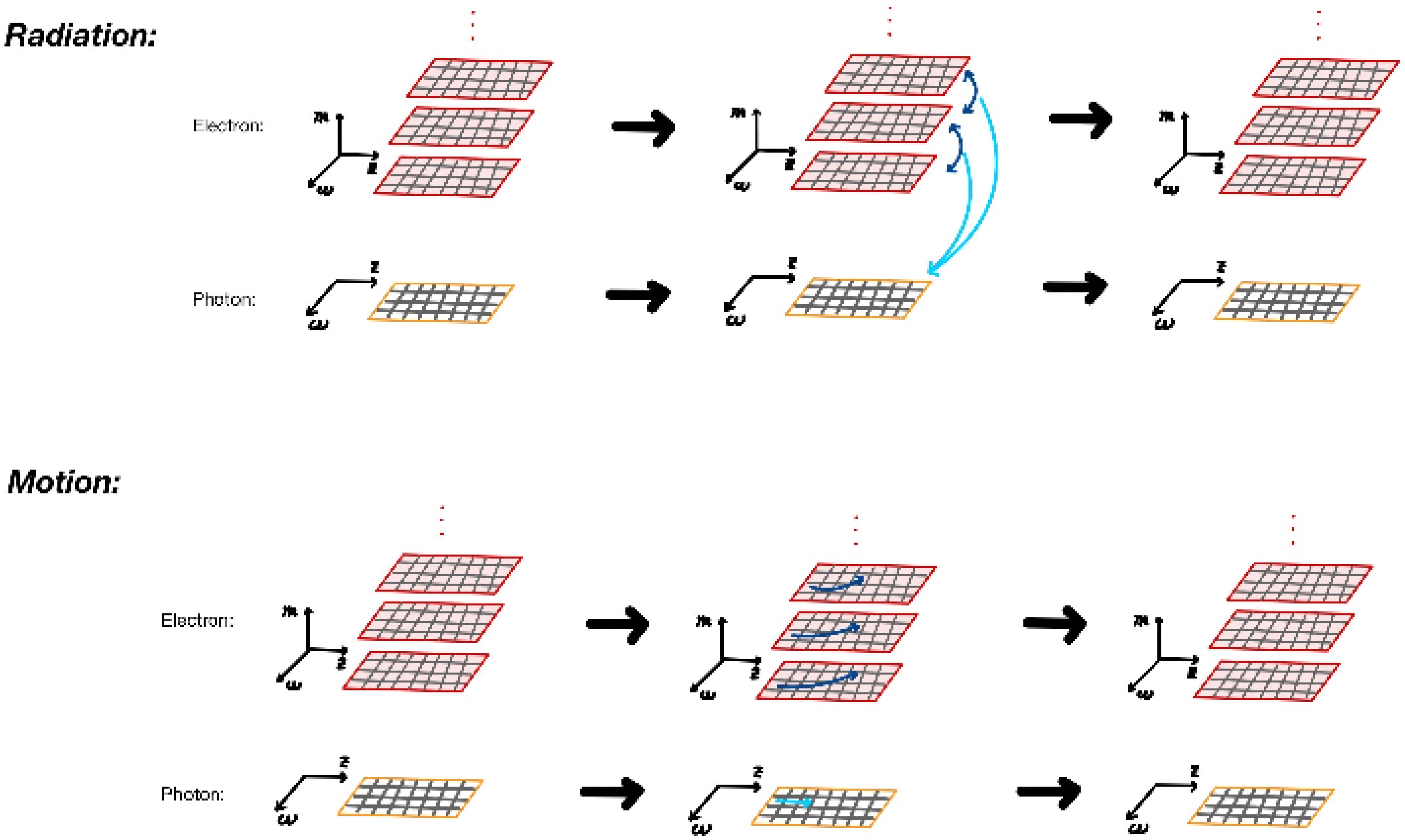








Coherent radiation: simulation



Initial conditions:

- Photon： $n_{initial} = 0$
- Electron:

$$N_e(v_\perp, v_z, z)_{initial} = A \bullet n_e(z) \bullet \tanh(\frac{v}{v_b})^{2\delta} \bullet \frac{1}{v^{2\alpha+1}} \bullet \exp(-\frac{(v-v_b)^2}{v_T^2} - \frac{v_\perp^2}{v^2\theta_\perp^2})$$

Kinetic equations:

- Transition probability:

$$\begin{cases} W_{m-1,m} = \frac{1}{\gamma}\frac{1}{(2\pi)^3}\int_{\omega_0^{'}}^{\omega_0^{'}+\Delta\omega^{'}}\int_{\Omega_0^{'}}^{\Omega_0^{'}+\Delta\Omega^{'}} \frac{4\pi^2 e^2}{\hbar^2\omega^{'} c^3}\frac{\hbar^2 e^2 B^2(z)}{m_e^2 c^2}(n^{'}+1)\,|<m-1|\overrightarrow{R}|m>\overrightarrow{e_{\lambda}^{'}}|^2\,\delta(\hbar\omega^{'}-\frac{\hbar e B(z)}{m_e c})\omega^{'2}d\omega^{'}d\Omega^{'} \\ W_{m+1,m} = \frac{1}{\gamma}\frac{1}{(2\pi)^3}\int_{\omega_0^{'}}^{\omega_0^{'}+\Delta\omega^{'}}\int_{\Omega_0^{'}}^{\Omega_0^{'}+\Delta\Omega^{'}} \frac{4\pi^2 e^2}{\hbar^2\omega^{'} c^3}\frac{\hbar^2 e^2 B^2(z)}{m_e^2 c^2}n^{'}\,|<m+1|\overrightarrow{R}|m>\overrightarrow{e_{\lambda}^{'}}|^2\,\delta(\hbar\omega^{'}-\frac{\hbar e B(z)}{m_e c})\omega^{'2}d\omega^{'}d\Omega^{'} \end{cases}$$

- Photon:

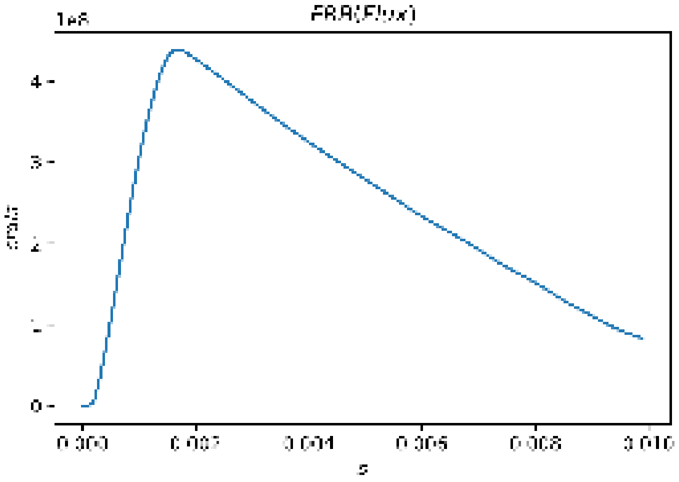
$$\int_{z_0}^{z_0+\Delta z}\int_{\omega_0}^{\omega_0+\Delta\omega}\int_{\Omega_0}^{\Omega_0+\Delta\Omega}\frac{\partial n}{\partial t}\frac{\omega^2}{8\pi^3 c^3}A_c\,dz\,d\omega\,d\Omega + \int_{z_0}^{z_0+\Delta z}\int_{\omega_0}^{\omega_0+\Delta\omega}\int_{\Omega_0}^{\Omega_0+\Delta\Omega}\frac{\partial n}{\partial z}\frac{\omega^2}{8\pi^3 c^2}A_c\,dz\,d\omega\,d\Omega = \int_{z_0}^{z_0+\Delta z}A_c\,dz\int_{-\infty}^{+\infty}dv_z\sum_{m=0}^{m=+\infty}N_e(m, v_z, z)(W_{m-1,m}-W_{m+1,m})$$

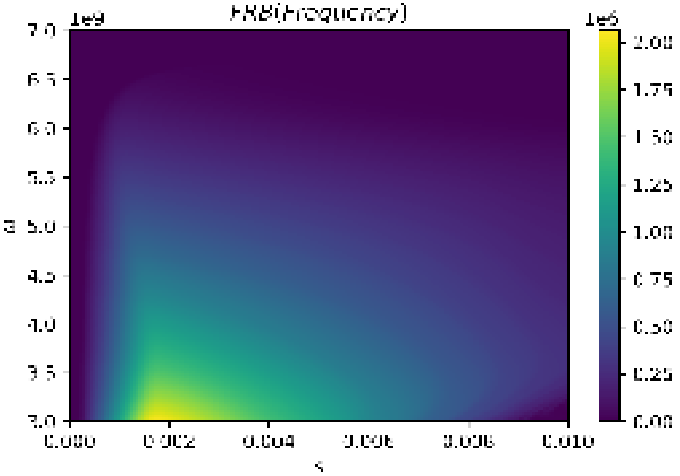
- Electron:

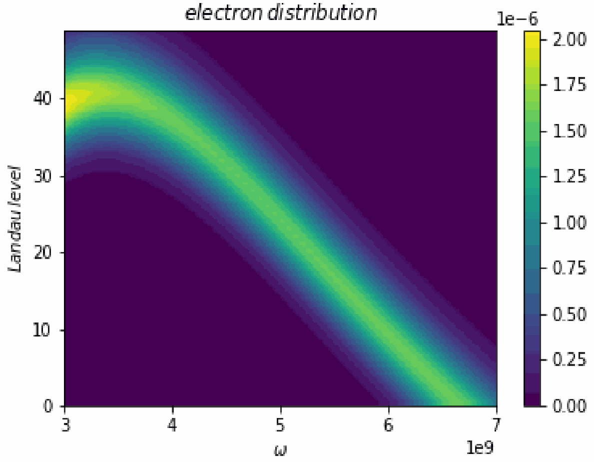
$$\frac{\partial N_e(m, v_z, z, t)}{\partial t} + \frac{\partial N_e(m, v_z, z, t)}{\partial v_z}a_z + \frac{\partial N_e(m, v_z, z, t)}{\partial z}v_z = W_{m,m+1}N_e(m+1) + W_{m,m-1}N_e(m-1) - W_{m+1,m}N_e(m) - W_{m-1,m}N_e(m)$$

A numerical simulation show

$$
\begin{cases}
n_e = 10^{-2} cm^{-3} \\
m_{max} = 50 \\
\Delta\theta = 0.001 \\
2 \bullet 10^9 cm \le z \le 2.3 \bullet 10^9 cm \\
190G \le B \le 245G
\end{cases}
$$







Analytical solution in quantum optics: an example

$$\rho_E = \rho_{[0]}(T_{[0]}) \otimes \rho_{[1]}(T_{[1]}) \ldots \otimes \rho_{[n]}(T_{[n]})$$

$$\rho(T) = \sum_n (1 - \mathrm{e}^{-\frac{\hbar\omega}{k_B T}})\mathrm{e}^{-\frac{n\hbar\omega}{k_B T}} \left|n\right\rangle \left\langle n\right|$$

$$|\Psi\rangle_S = |\Phi(\alpha_{[0]})\rangle_{[0]} \otimes |\Phi(\alpha_{[1]})\rangle_{[1]} \ldots \otimes |\Phi(\alpha_{[n]})\rangle_{[n]}$$

$$|\Phi(\alpha_{[n]})\rangle = \mathrm{e}^{-\frac{|\alpha_{[n]}|^2}{2}} \sum_{R=0}^{+\infty} \frac{\alpha_{[n]}^R}{\sqrt{R!}} |\overrightarrow{e}_0, \Phi_0^R\rangle$$

$$|\overrightarrow{e}_0, \Phi_0^R\rangle = \sum_{n_1, n_2, \ldots, n_{N_{[n]}}} \frac{1}{\sqrt{R! n_1! n_2! \ldots n_{N_{[n]}}!}} |n_1, n_2, \ldots, n_{N_{[n]}}\rangle\,,\ \sum_{i=1}^{N_{[n]}} n_i = R$$

$$\begin{aligned}
\hat{a}_{\overrightarrow{k}[n]}(t) =& \hat{a}_{\overrightarrow{k}[n]}(0)\mathrm{e}^{-\mathrm{i}\omega_{\overrightarrow{k}} t} + \hat{a}_{\overrightarrow{k}[n-1]}(t - \frac{\overrightarrow{k}\Delta\overrightarrow{z}_{[n][n-1]}}{\omega_{\overrightarrow{k}}}) \\
& - \mathrm{i}\mathcal{G}^*_{\overrightarrow{k}[n]} \hat{C}_{N_{[n]}}(0) \int_0^t \mathrm{e}^{-(\mathrm{i}\omega_0 + \gamma_{[n]})t'} \mathrm{e}^{-\mathrm{i}\omega_{\overrightarrow{k}}(t-t')}\, dt' \\
& - \mathcal{G}^*_{\overrightarrow{k}[n]} \int_0^t \{\int_0^{t'} \hat{a}_{in[n]}(t'')\mathrm{e}^{-(\mathrm{i}\omega_0+\gamma_{[n]})(t'-t'')}\, dt''\}\mathrm{e}^{-\mathrm{i}\omega_{\overrightarrow{k}}(t-t')}\, dt'
\end{aligned}$$

$$\hat{a}_{in[n]}(t) = \sum_{\overrightarrow{k}} \mathcal{G}_{\overrightarrow{k}[n]} \{\hat{a}_{\overrightarrow{k}[n]}(0)\mathrm{e}^{-\mathrm{i}\omega_{\overrightarrow{k}} t} + \hat{a}_{\overrightarrow{k}[n-1]}(t - \frac{\overrightarrow{k}\Delta\overrightarrow{z}_{[n][n-1]}}{\omega_{\overrightarrow{k}}})\}$$

$$\begin{aligned}
\frac{dI(\overrightarrow{z}, t)}{d\Omega} &\propto < \hat{E}^{(-)}(\overrightarrow{z}, t)\hat{E}^{(+)}(\overrightarrow{z}, t) > \\
&= \frac{\hbar}{2\varepsilon_0 V} \sum_{\overrightarrow{k}, \overrightarrow{k'}} \sqrt{\omega_{\overrightarrow{k}}\omega_{\overrightarrow{k'}}} < \hat{a}^\dagger_{\overrightarrow{k}[n]}(t - \frac{\overrightarrow{k}\Delta\overrightarrow{z}}{\omega_{\overrightarrow{k}}})\hat{a}_{\overrightarrow{k'}[n]}(t - \frac{\overrightarrow{k'}\Delta\overrightarrow{z}}{\omega_{\overrightarrow{k'}}}) >
\end{aligned}$$

$$\overrightarrow{E}(\overrightarrow{z}, t) = \mathrm{i} \sum_{\overrightarrow{k}.\sigma} \overrightarrow{e}_\sigma \sqrt{\frac{\hbar\omega_{\overrightarrow{k}}}{2\varepsilon_0 V}} (< \hat{a}_{\overrightarrow{k}[n]}(t - \frac{\overrightarrow{k}\Delta\overrightarrow{z}}{\omega_{\overrightarrow{k}}}) > - < \hat{a}^\dagger_{\overrightarrow{k}[n]}(t - \frac{\overrightarrow{k}\Delta\overrightarrow{z}}{\omega_{\overrightarrow{k}}}) >)$$

The advantages of quantum view over classical view

- A much more natural, direct and rapid way to trigger radio bursts
- A more natural way to get pulses with right duration and polarization
- Perhaps more importantly, a more efficient way to release the free energy of the magnetosphere

Thanks