New Results on Analysis of the Wave Summation in the Radio Emission of B1919+21

曹顺顺 Shunshun Cao
Peking University

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11 pages →

1. Intro: wave summation

(1) Coherent & Incoherent summation

Orthogonal components of quasi-monochromatic waves:

$$\tilde{E}_x(t) = E_x(t)e^{-i\omega t} = \mathcal{E}_x(t)e^{i\phi_x(t)}e^{-i\omega t}$$
$$\tilde{E}_y(t) = E_y(t)e^{-i\omega t} = \mathcal{E}_y(t)e^{i\phi_y(t)}e^{-i\omega t}$$

(Measured) Stokes parameter *I*:

$$I = \langle E_x E_x^* \rangle + \langle E_y E_y^* \rangle$$

Where the braket means average over measure time *T*:

$$\langle E_x E_x^* \rangle = \frac{1}{T}\int_0^T E_x E_x^* dt$$

Refer to e.g.
Rybicki & Lightman 1979
Radiative processes in Astrophysics

If two waves (1,2) are added together:

$$\begin{aligned} I &= \langle (E_{x1}+E_{x2})(E_{x1}^* + E_{x2}^*) \rangle + \langle (E_{y1}+E_{y2})(E_{y1}^* + E_{y2}^*) \rangle \\ &= I_1 + I_2 + \langle E_{x1}E_{x2}^* \rangle + \langle E_{x2}E_{x1}^* \rangle + \langle E_{y1}E_{y2}^* \rangle + \langle E_{y2}E_{y1}^* \rangle \end{aligned}$$

Where $\langle E_{x1}E_{x2}^{*}\rangle = \frac{1}{T}\int_0^T \mathcal{E}_{x1}(t)\mathcal{E}_{x2}(t)e^{i[\phi_{x1}(t)-\phi_{x2}(t)]}\,dt$

$\langle E_x E_x^{*}\rangle = \frac{1}{T}\int_0^T E_x E_x^{*} dt$

$E_x(t) = \mathcal{E}_x(t)e^{i\phi_x(t)}$

If the two waves are **independent**, or in other word, **incoherently summed**, their phase offset is random and uniformly distributed in [0, 2π) over time *T*.

➔$\langle E_{x1}E_{x2}^{*}\rangle = 0$

➔➔Similar for other quantities, we have

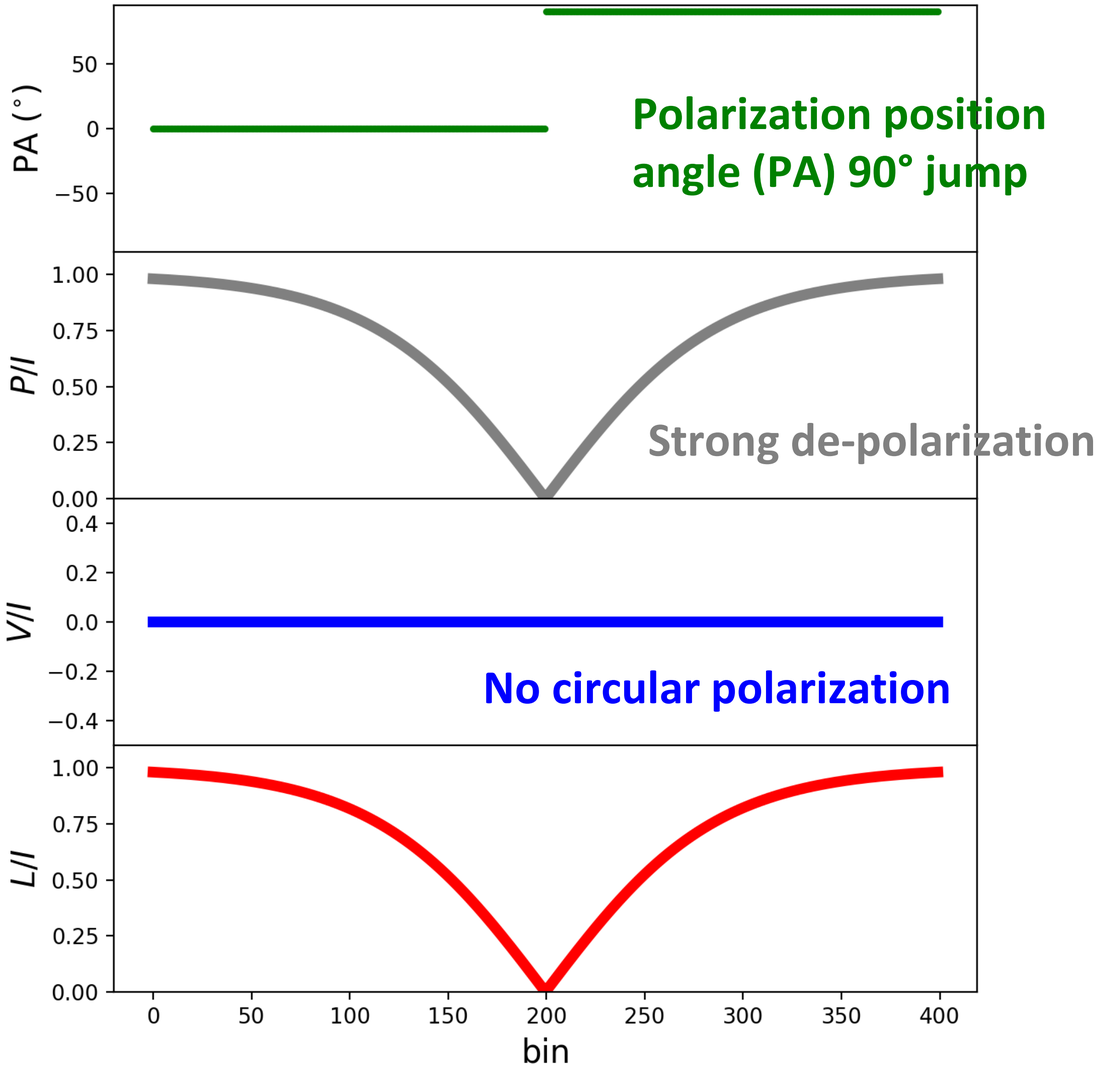
$$I = I_1 + I_2 \qquad Q = Q_1 + Q_2 \qquad U = U_1 + U_2 \qquad V = V_1 + V_2$$

Refer to e.g.
Rybicki & Lightman 1979
Radiative processes in Astrophysics

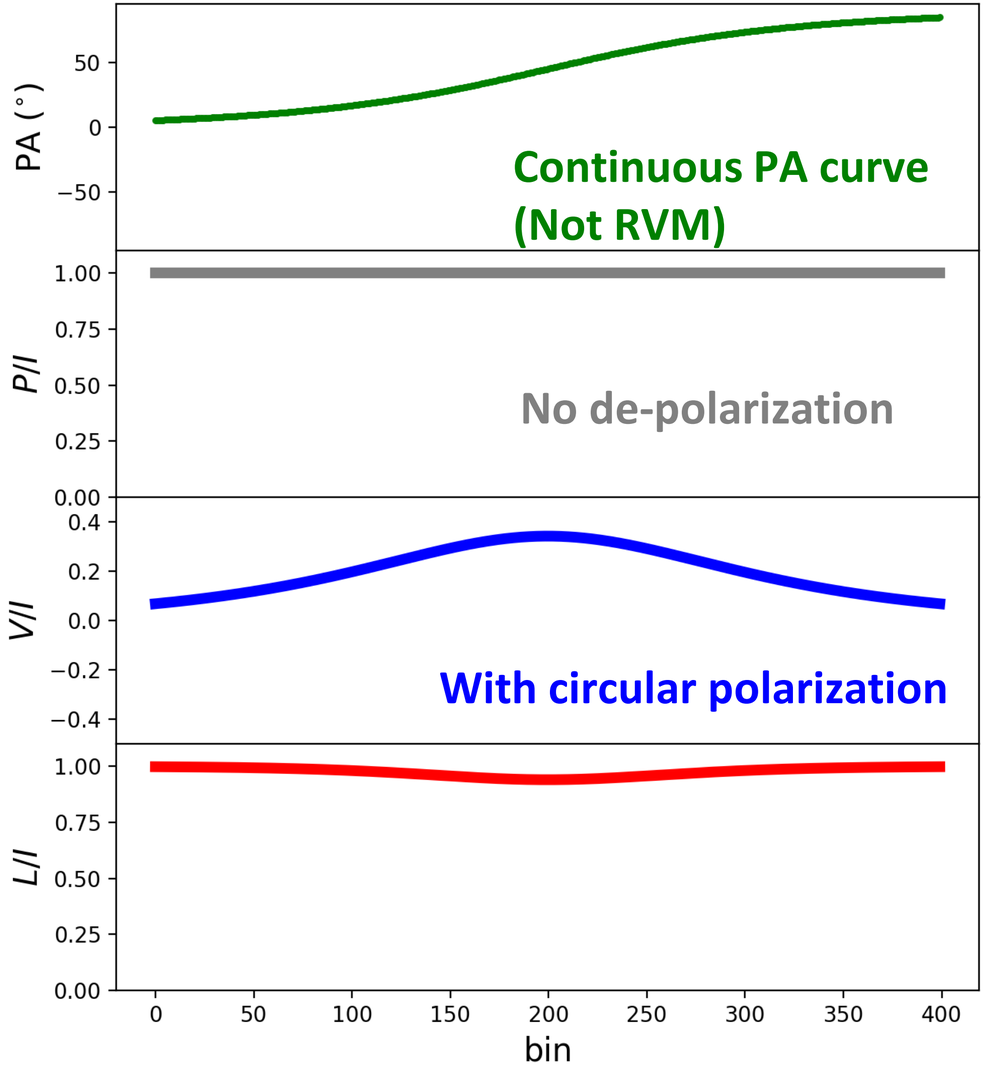
Conclusion: usually,
if incoherent summation, we add up Stokes parameters;
otherwise, we add up electric fields.

They reflect two basic types of polarization patterns:

Set two orthogonal **linearly polarized** waves with some certain phase offset, and let their mode amplitude ratio change with longitude (bin). Their mode amplitudes equal at center (bin=200).



Purely incoherently summed



Purely coherently summed

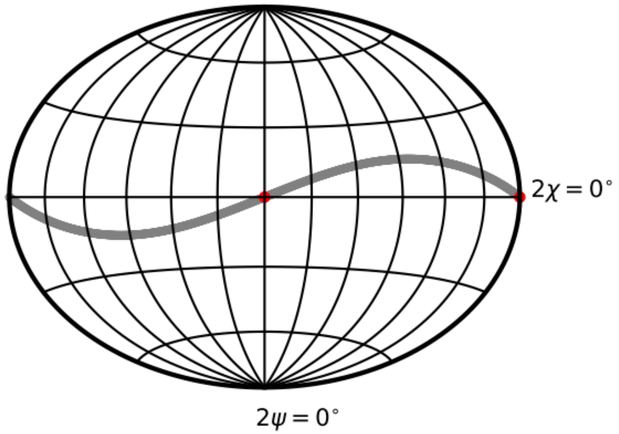
Real case: Partially coherent

(2) Mode-ratio-driven v.s. phase-offset-driven

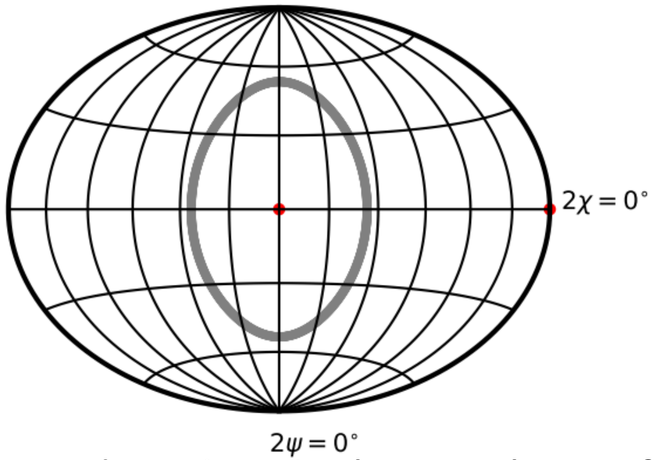
If we add two orthogonally (linearly) polarized waves coherently, the resulting polarization will have different properties, when we change the mode amplitude ratio E_X/E_O, or change the phase offset η.

Simple case: $E_1 = E_O$ $E_2 = E_X e^{i\eta}$

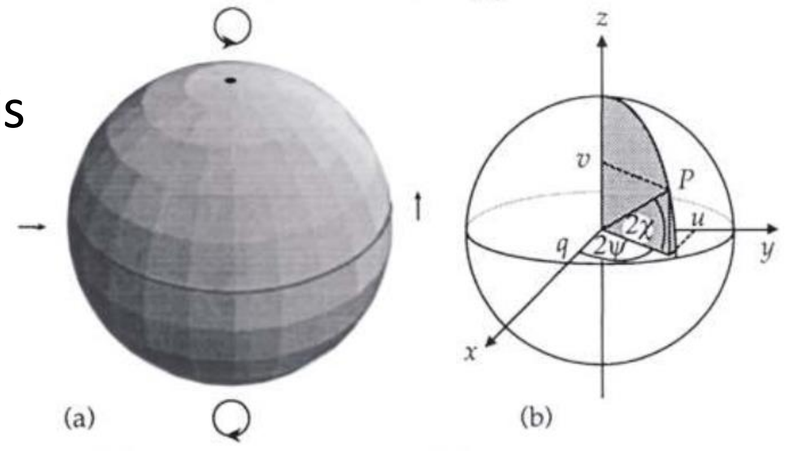
The difference will be most clear when the resulting polarization is presented in the form of **Poincaré sphere**:



Fix phase lag, change mode ratio



Fix mode ratio ≠ 1, change phase offset



Melrose & McPhedran 1991
Electromagnetic processes in Dispersive Media

Mode-ratio-driven: always leads to a **great circle** on the Poincaré sphere.

Phase-offset-driven: leads to a **small circle** on the Poincaré sphere when mode ratio ≠ 1.

(3) Phenomenological models for radio pulsar polarization based on wave summations

Pulsar radio emission is rich in polarization characteristics:

· PA swing due to magnetic field configuration (e.g. RVM), and due to wave summations

· Circular polarization

· Orthogonal polarization modes (OPMs)

· De-polarization

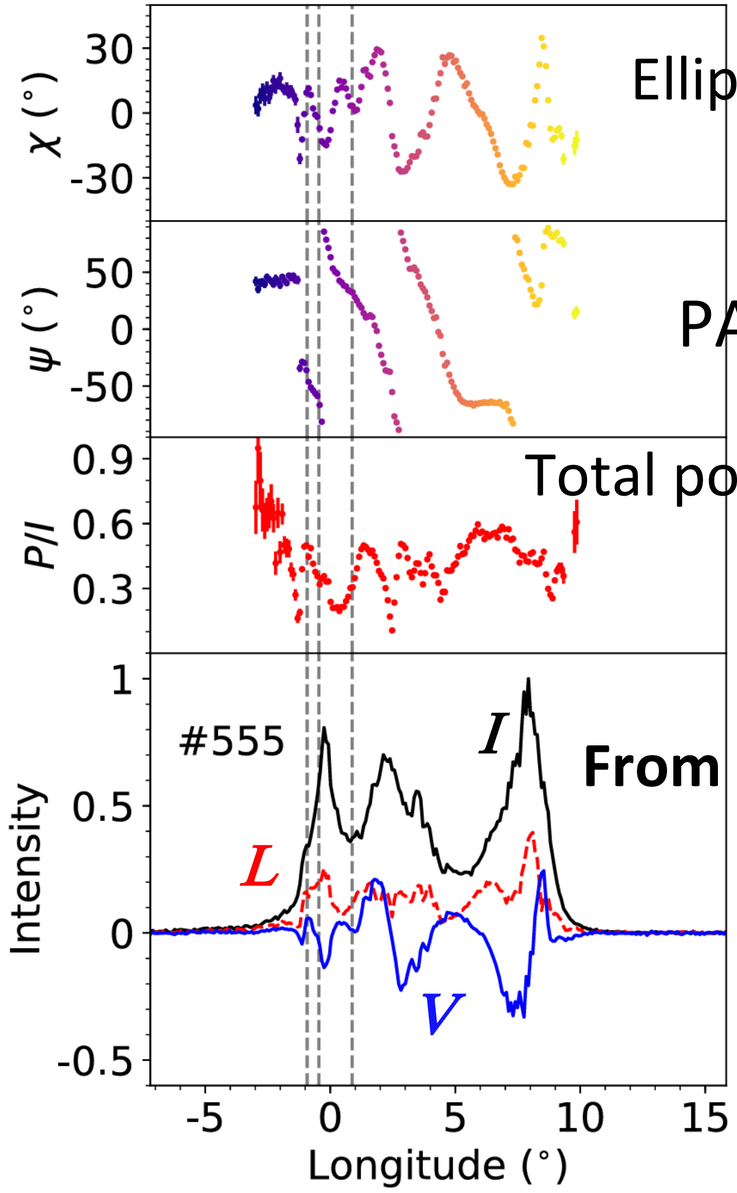
· Spectral evolution of above things

…

Phenomenological models based on wave summations help us figure out properties of parameters like the mode amplitude ratio E_X/E_O, and the phase offset η, by fitting to data. The parameters are related to real physical processes in pulsar magnetospheric plasma.

e.g., Oswald, Karastergiou & Johnston 2023 MNRAS:
They use mode ratio, phase lag, and coherence to describe pulsar polarization behaviors.

2. Wave summations in PSR B1919+21 (Bell pulsar)



>1/3 pulses of B1919+21 have polarization position angle (PA, ψ) **quasi-monotonically rotating** over 180° or even 360°. Oscillations of circular polarization (or EA) are accompanied.

The largest rotation is about 720°.

Such large rotation could not be produced by RVM. Therefore, we seek for answers from wave summation.

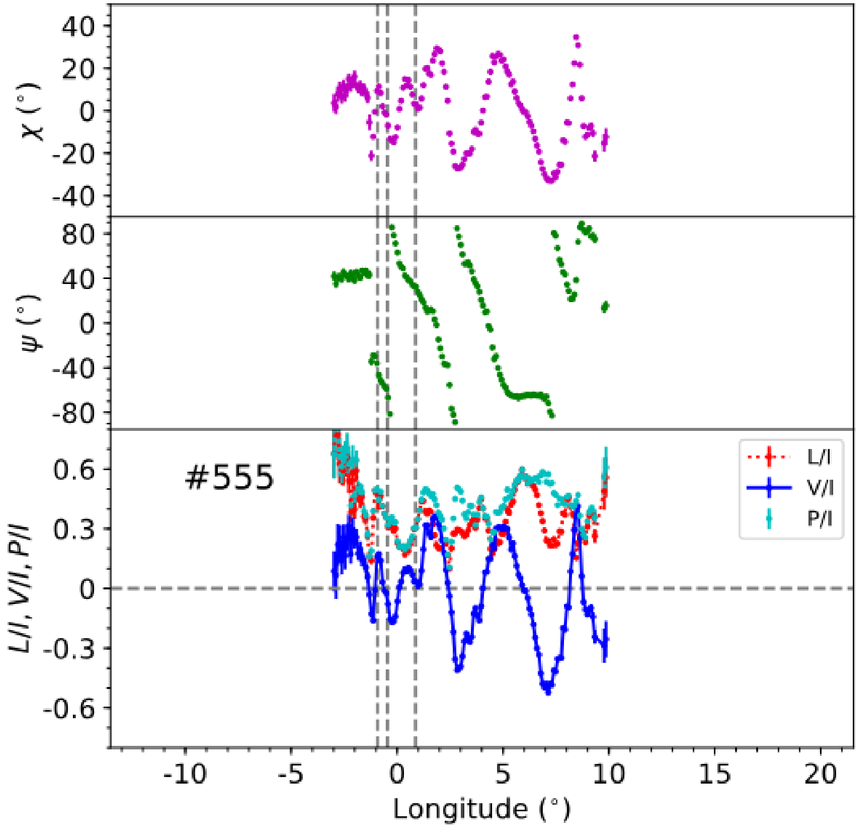
Cao SS, Jiang JC, Dyks J, Lee KJ, Lu JG, Oswald LS, Wang WY, Xu RX 2025 *ApJ*.

Modeling: rotation induced by phase offset changing

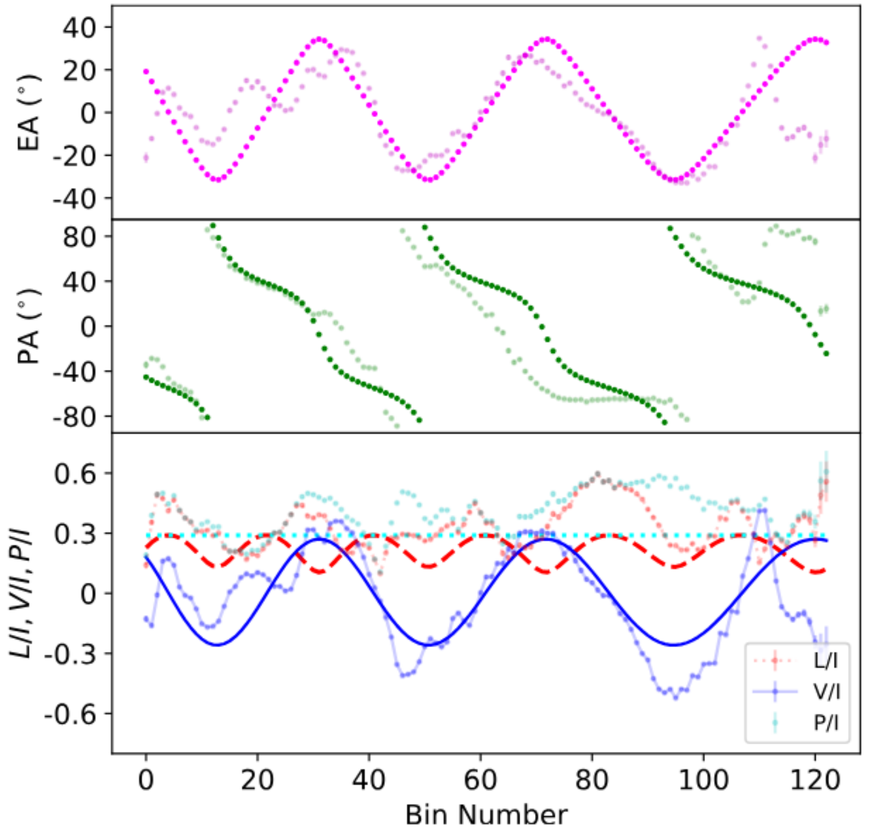
Assume that the wave summation is purely phase-offset-driven, and we fix the mode amplitude ratio between the orthogonal wave modes.

We can roughly reproduce the polarization pattern.

Cao SS, Jiang JC, Dyks J, Lee KJ, Lu JG, Oswald LS, Wang WY, Xu RX 2025 *ApJ*.



Actual pulse polarization



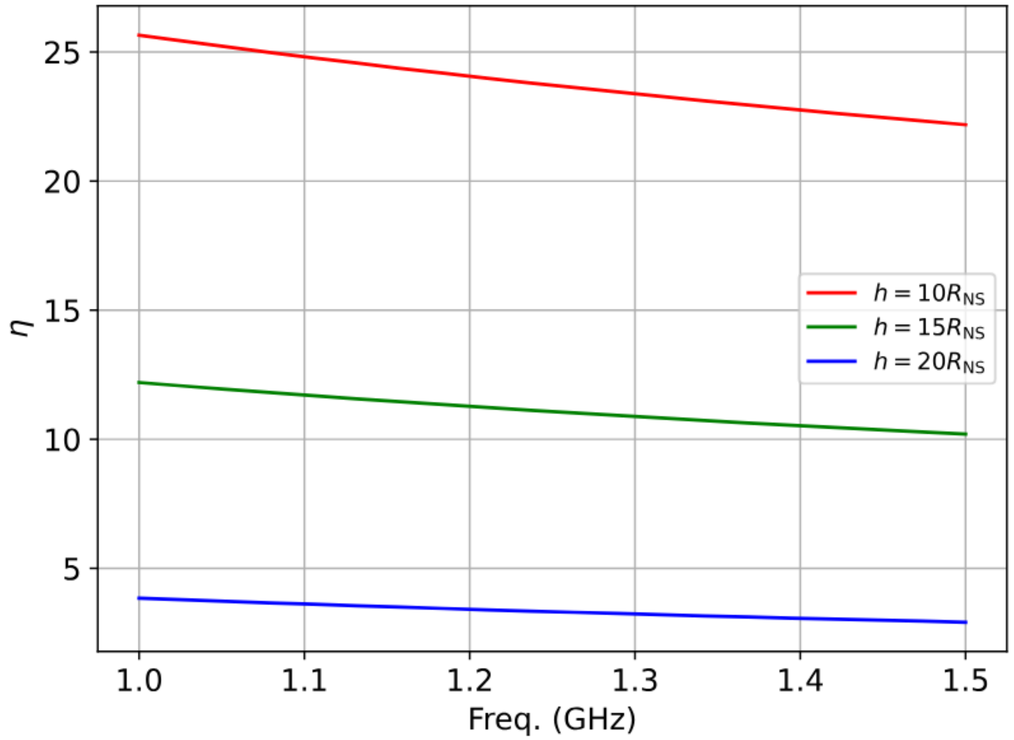
Modeled pulse polarization (darker lines & dots)

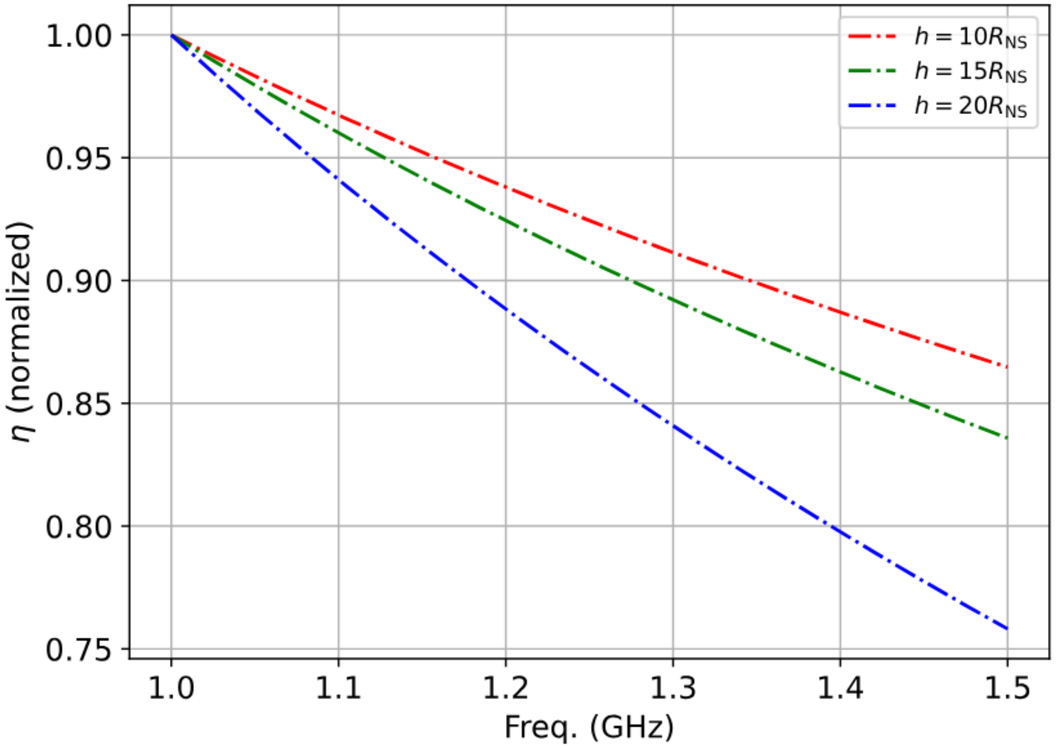
Phase offset: $\eta = \int_L (k_{\rm X} - k_{\rm O}) dl = \int_L \frac{\omega}{c}(n_{\rm X} - n_{\rm O}) dl$

Related to dispersion relations: $(1 - n_{\rm O}^2 \cos^2\theta)\left[1 - \frac{\omega_{\rm p}^2}{\omega^2\gamma^3(1 - n_{\rm O}\beta\cos\theta)^2}\right] - n_{\rm O}^2 \sin^2\theta = 0, \qquad n_{\rm X} = 1$

$\omega_{\rm p} = \sqrt{e^2 n_e / \epsilon_0 m_e}$

Phase offset versus frequency, for different emission altitudes:





A 540° rotation corresponds to pulsar surface $n_e/\gamma^3 \sim 10^3$ cm^{-3} .

Cao SS, Jiang JC, Dyks J, Lee KJ, Lu JG, Oswald LS, Wang WY, Xu RX 2025 *ApJ*.

3. New results: fit the circles on the Poincaré sphere

Our present modeling is far from precision. One simple reason is that we fix the mode ratio.
Pure phase-offset-driven is inadequate.

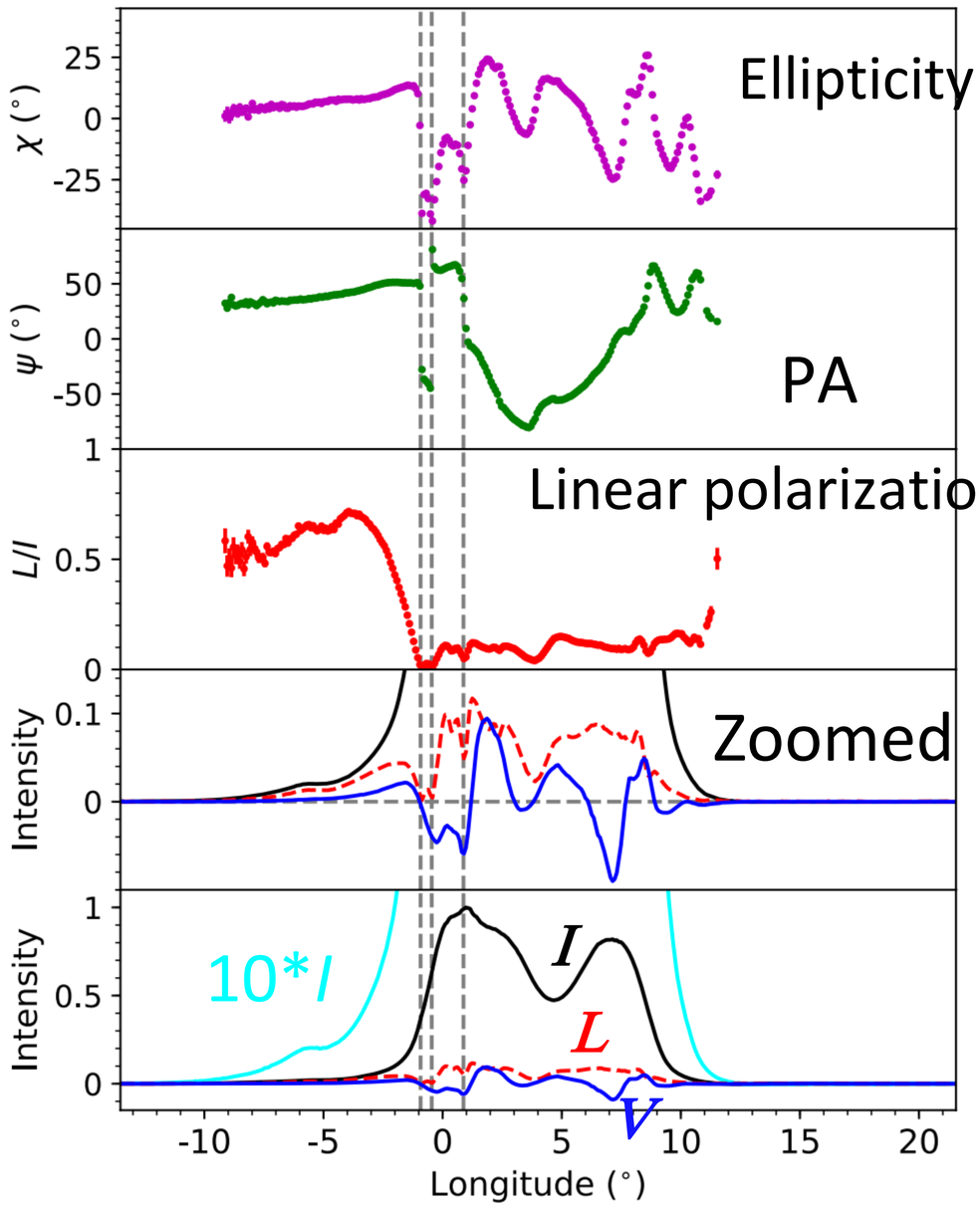
This is an online version of slides (20250708):

This version is different from the talked version (20250704).
The results of Poincaré sphere fitting is not shown here…
Because the paper is still in preparation…
Sorry for that.

A paper led by Dr. Jaroslaw Dyks is under preparation.

Important further question:
What determines the mode amplitude ratio?
➔ Origin of orthogonal modes, from linear conversion in plasma (Petrova 2001 *A&A*)…?

Thanks for your attention.



B1919+21 is the first pulsar ever discovered, by Bell & Hewish.

The polarized integrated profile observed by FAST is shown left.

· Low linear polarization degree

· Non-RVM type PA curve

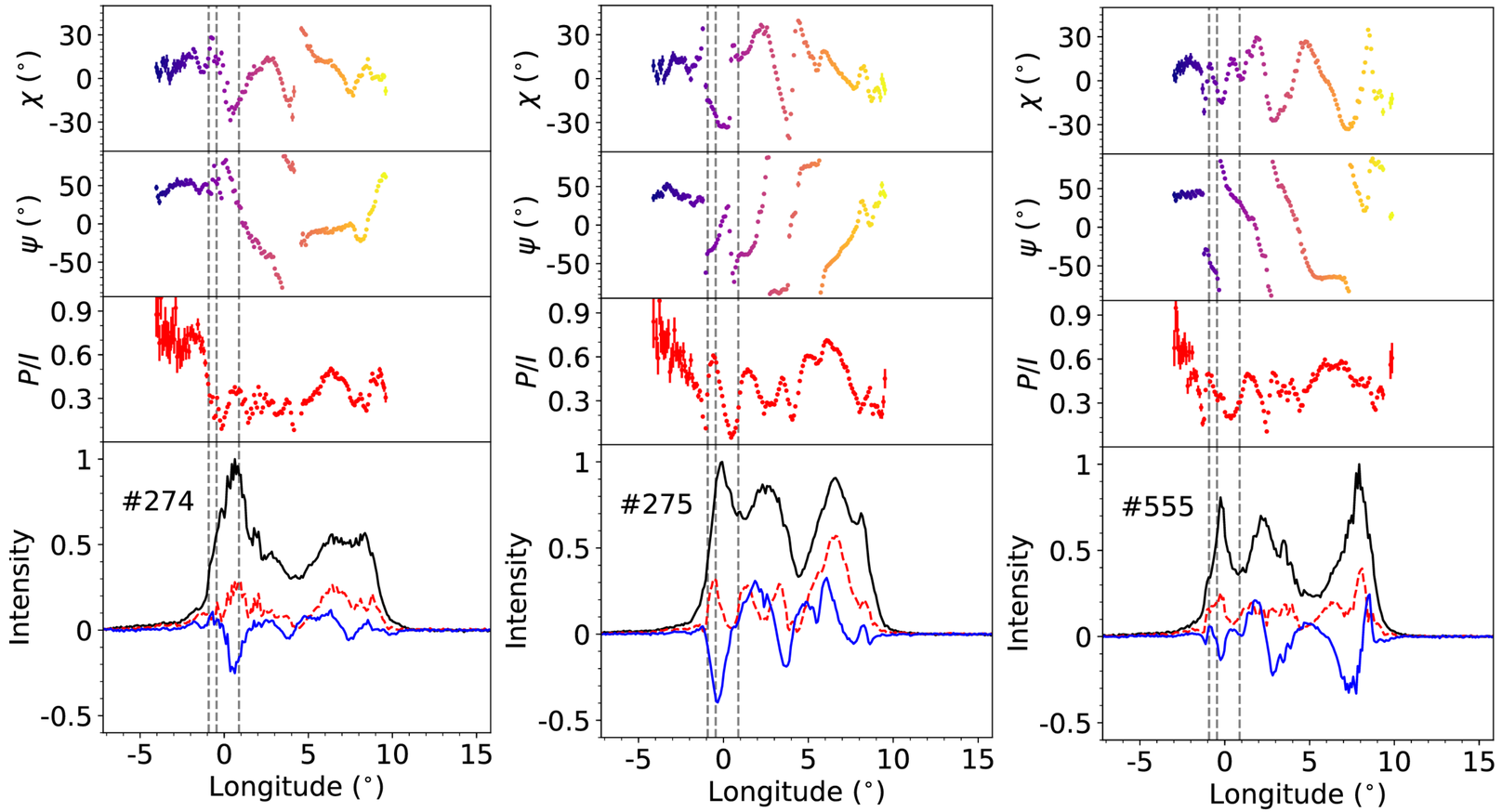
· EA seems to oscillate along longitudes

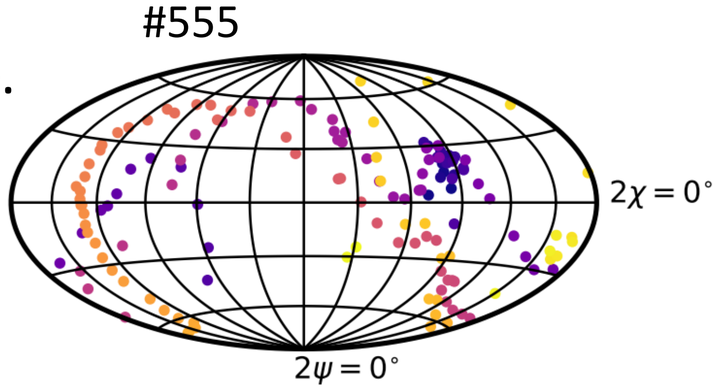
· PA jumps on the leading edge

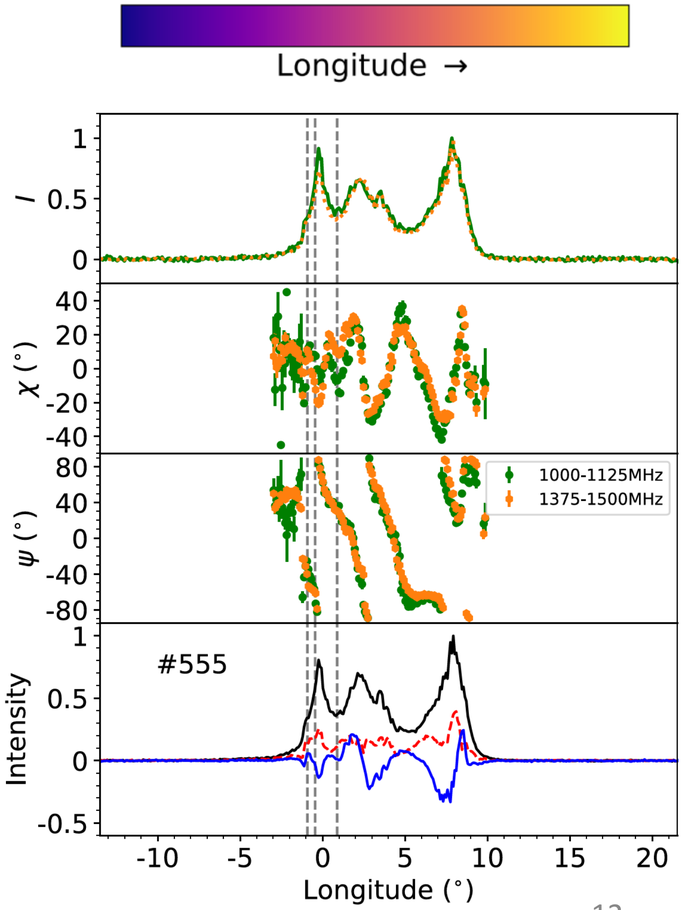
Cao SS, Jiang JC, Dyks J, Lee KJ, Lu JG, Oswald LS, Wang WY, Xu RX 2025 *ApJ*.

· Most quasi-monotonic PA curves have negative slopes versus longitude. Very few have positive slope (**asymmetry in rotation**).

· The PA curve depends only **weakly on frequency**.







3 single pulses of B1919+21 observed by FAST