

Probing Compact-star Structure with Gravitational Waves in the Hierarchical Bayesian Framework

Z. Wang, Y. Gao, D. Liang, J. Zhao and L. Shao*, JCAP 11 (2024)
Z. Zheng, **Z. Wang***, J. Deng, Y. Dong and L. Shao*, JCAP 05 (2026)

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July 4, 2026



Contents

- I. Compact Stars and Gravitational Waves
- II. Constrain the EOS of Quark Stars
- III. Constrain the Love-Q Relation
- IV. Summary



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I. Compact Stars and Gravitational Waves

II. Constrain the EOS of Quark Stars

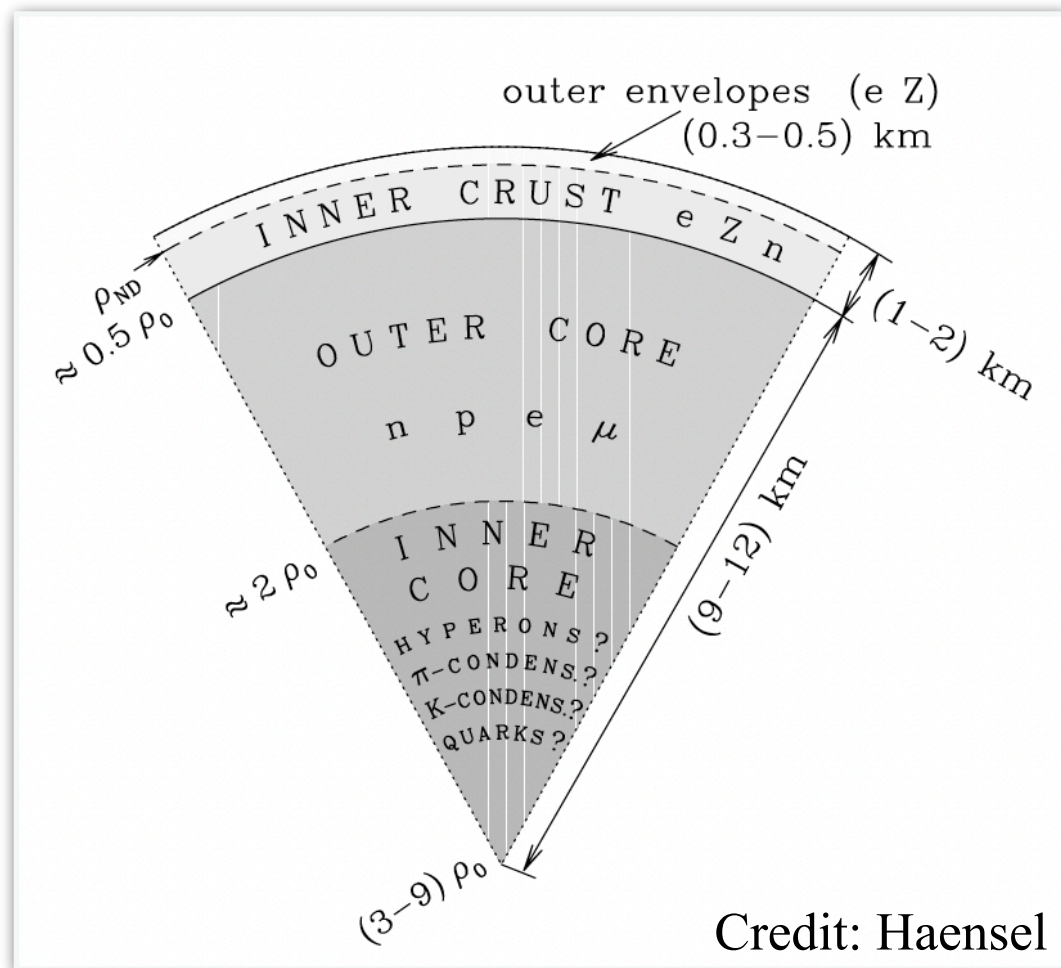
III. Constrain the Love-Q Relation

IV. Summary



State of Dense Matter in Compact Stars

Conventional neutron-star (NS) model



- Nucleon star: $npe\mu$ matter
- New freedom in the inner core: hyperons? mesons? quarks?

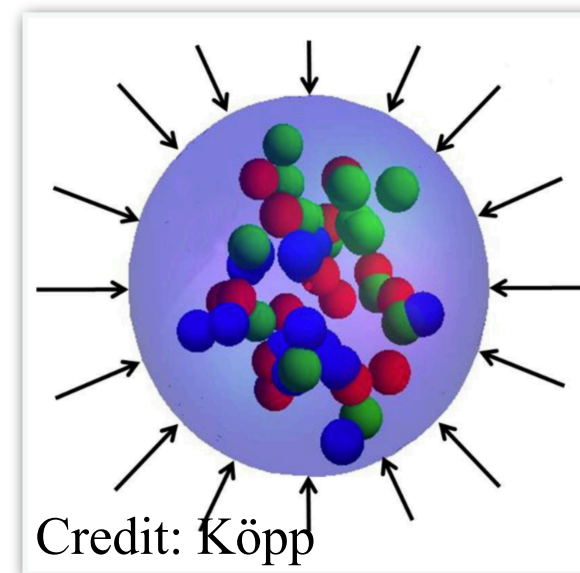
Quark-star (QS) model

Witten's conjecture:

Quark matter composed of nearly equal number of u , d , s quarks is the **ground state of strong matter**

- QSs in the MIT bag model

Alcock+ (1986); Haensel+ (1986)



Strangeon-star model

- Quarks localized

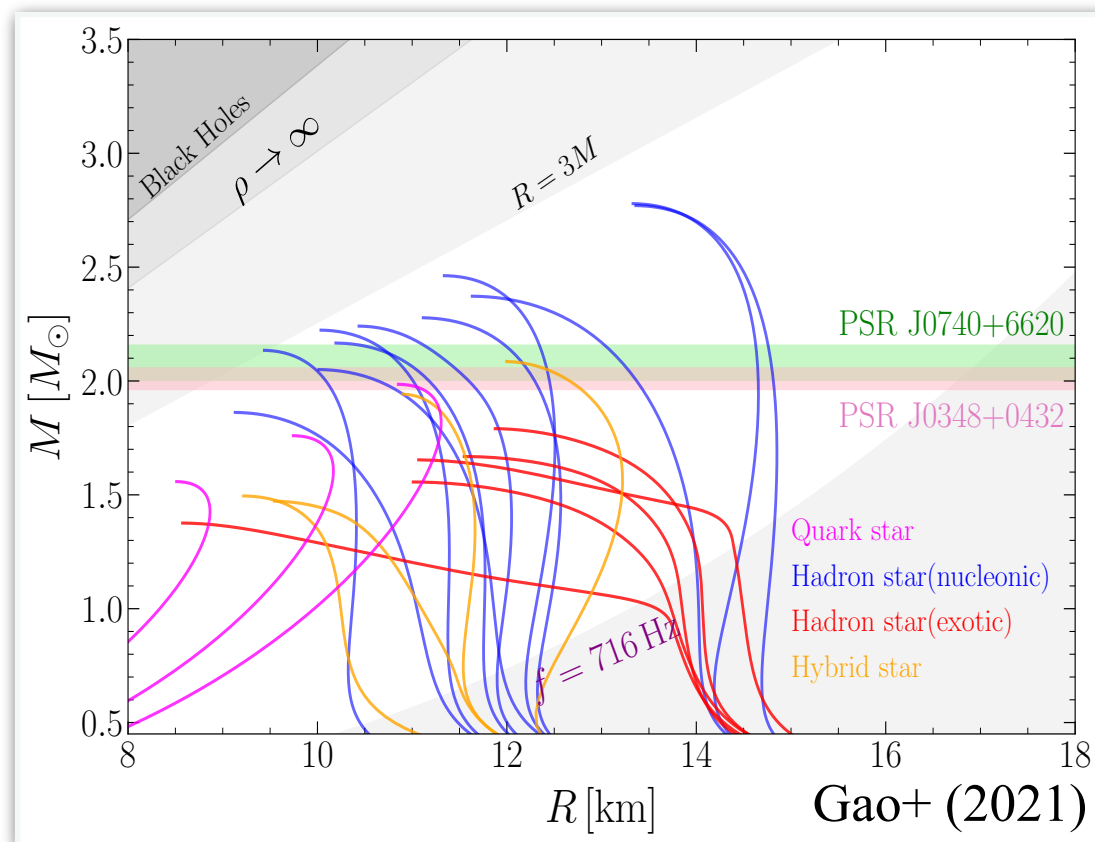
Lai & Xu (2017)



Constrain Equation of States

- Equation of State (EOS)
 - The relationship between the pressure and the energy density
 - Determine the mass-radius relation of compact stars
- Constraining the EOS

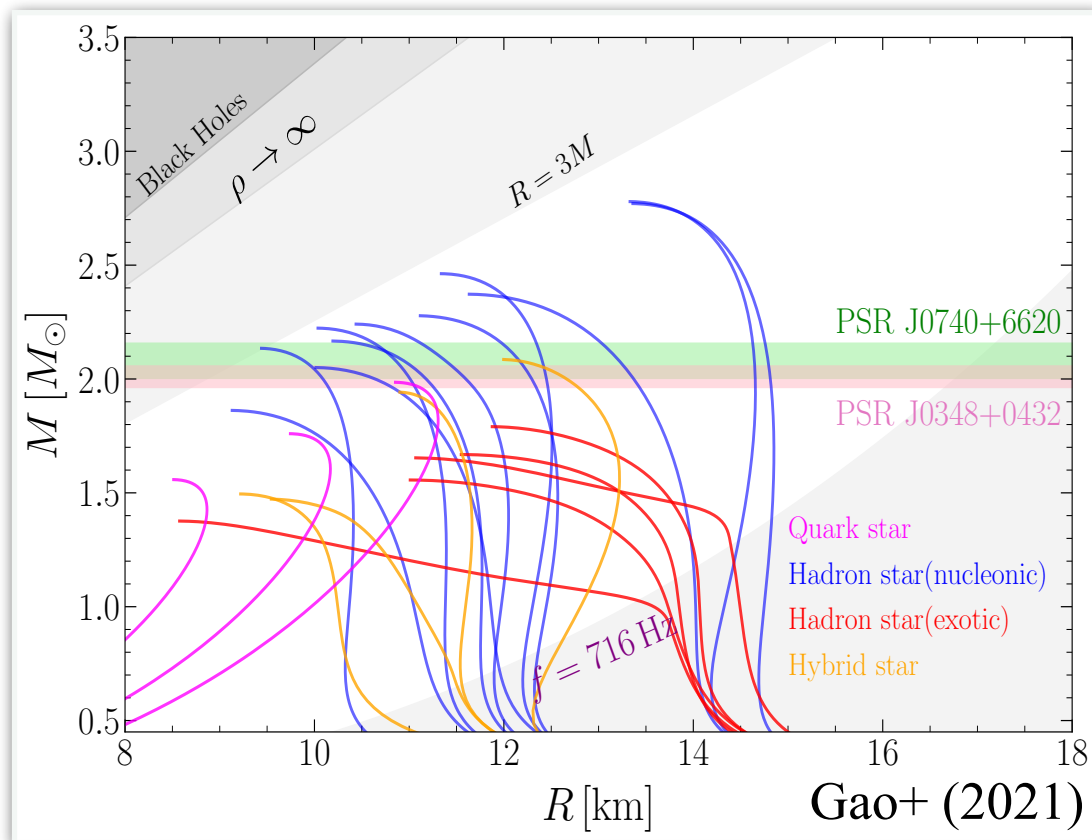
Pulsar Observations



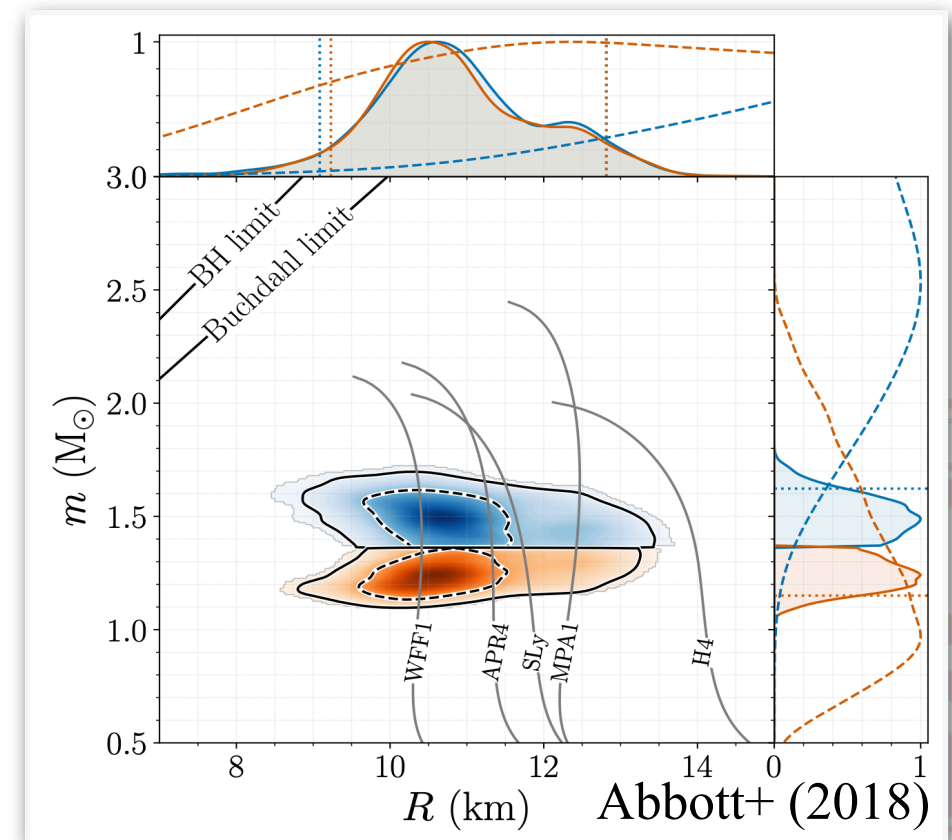
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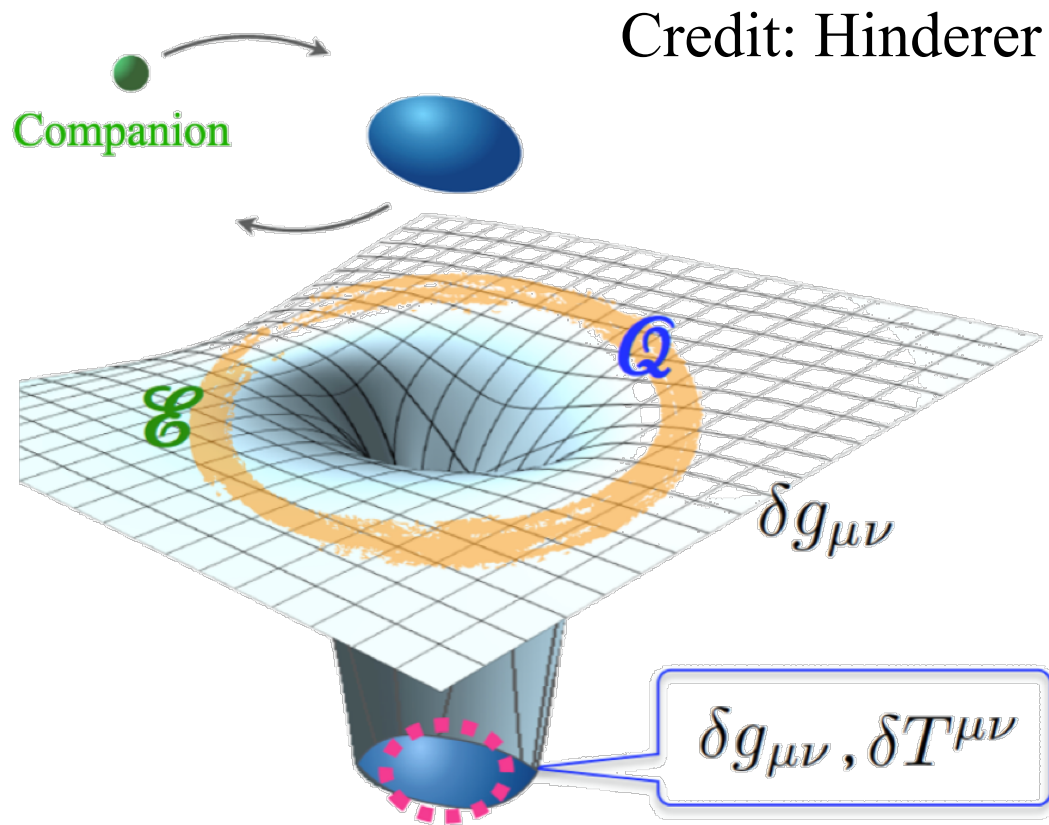
Pulsar Observations



Gravitational-wave (GW) Observations



Tidal Deformation and Gravitational Waves



$$Q_{ij} = -\lambda \mathcal{E}_{ij}$$

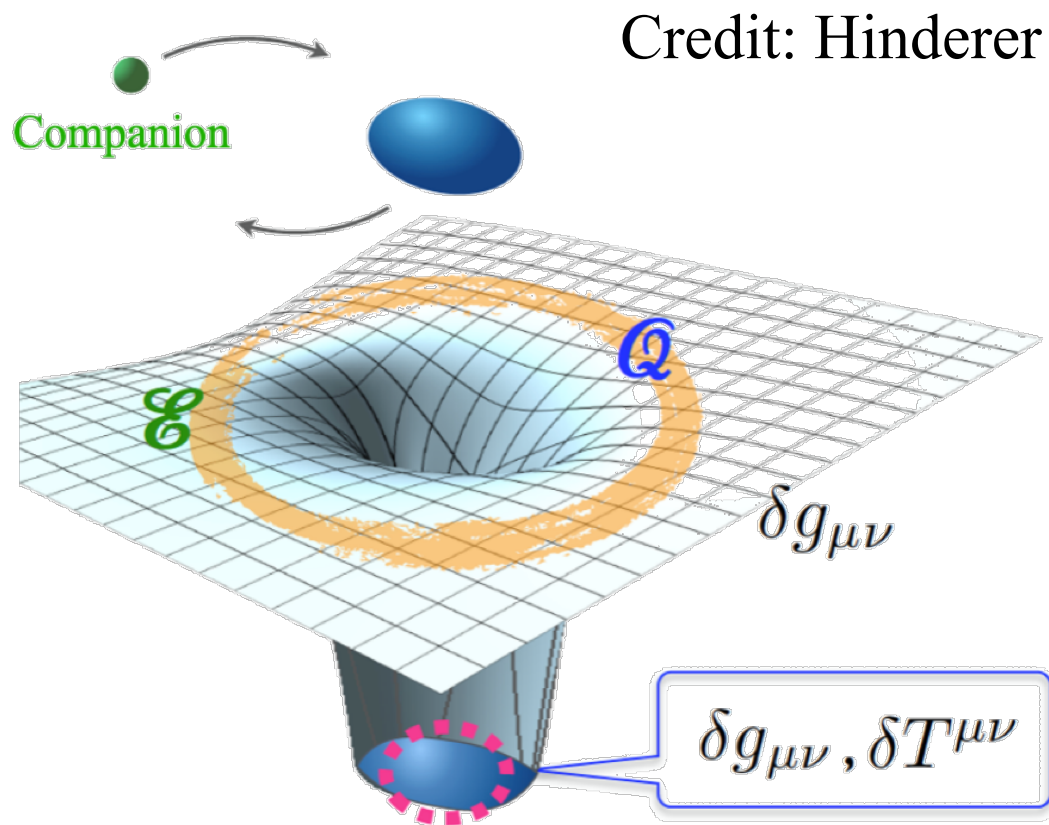
$$\Lambda = \frac{\lambda}{m^5}$$

Tidal deformability, $\lambda \propto R^5$

- m - Λ relation is also determined by EOS



Tidal Deformation and Gravitational Waves



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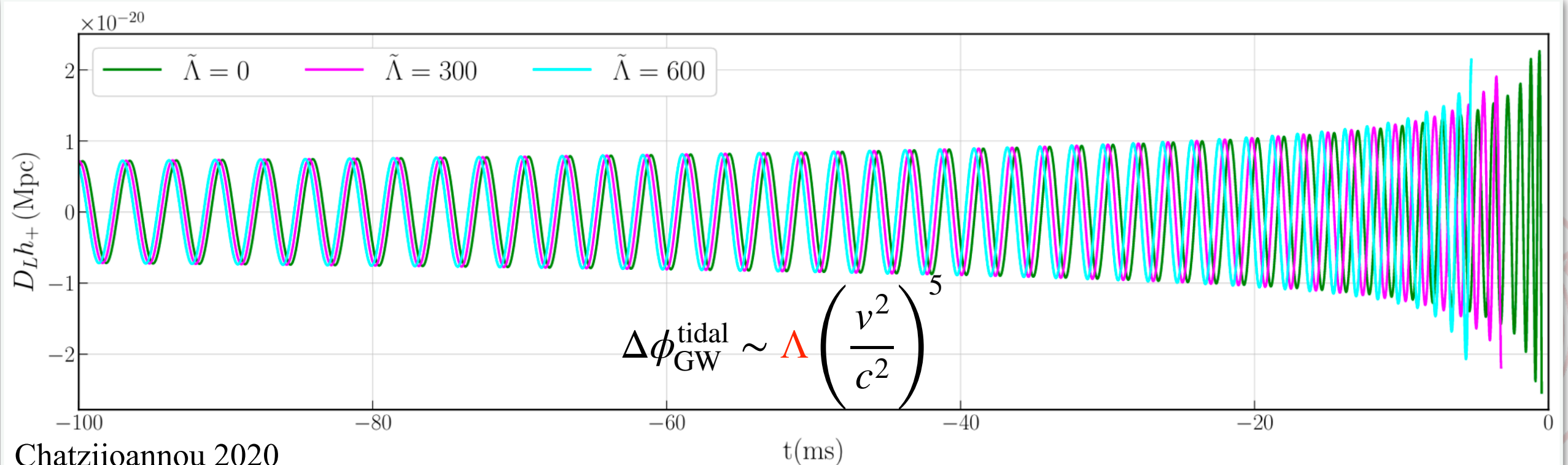
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Measure m and Λ from GWs

Constrain the EOSs



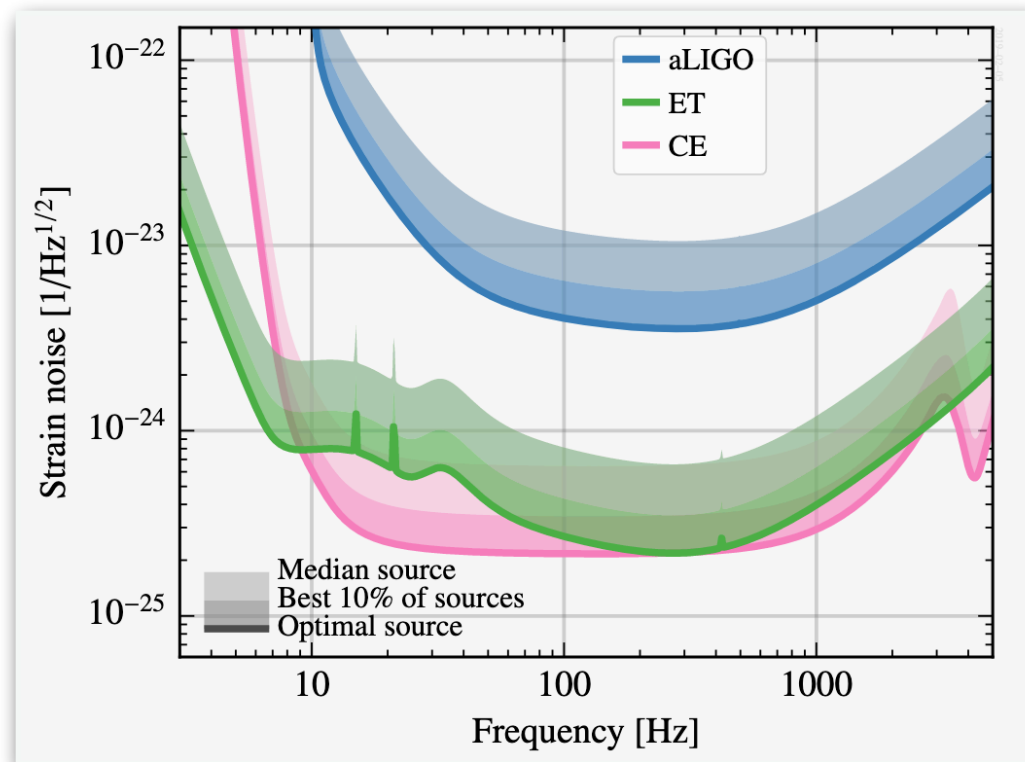
Current and Future GW Observations

- Since GW150914, over 300 GW events are detected
 - 8 compact stars: 2 BNS, 4 NSBH

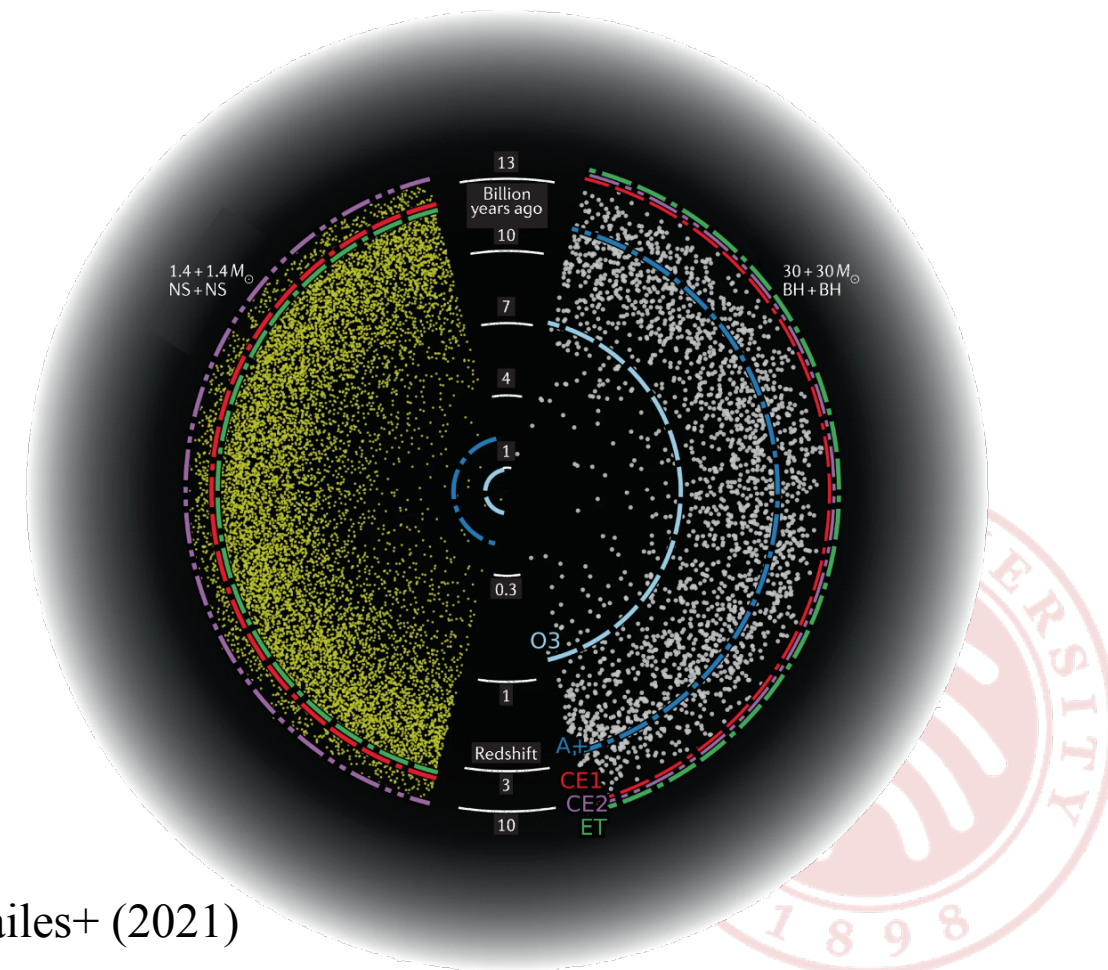


Current and Future GW Observations

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 - 8 compact stars: 2 BNS, 4 NSBH
- Future detectors: Einstein Telescope, Cosmic Explorer
 - Lots of high-precision GW observations, $10^5 \sim 10^6$ /yr



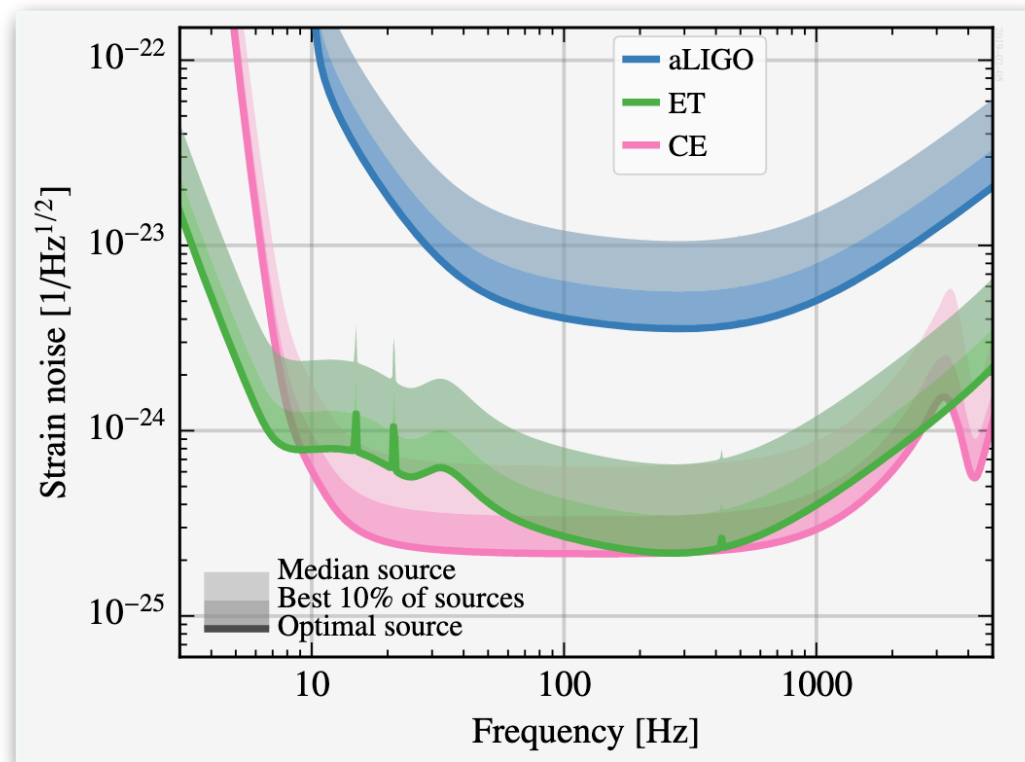
GWIC 3G Science Book



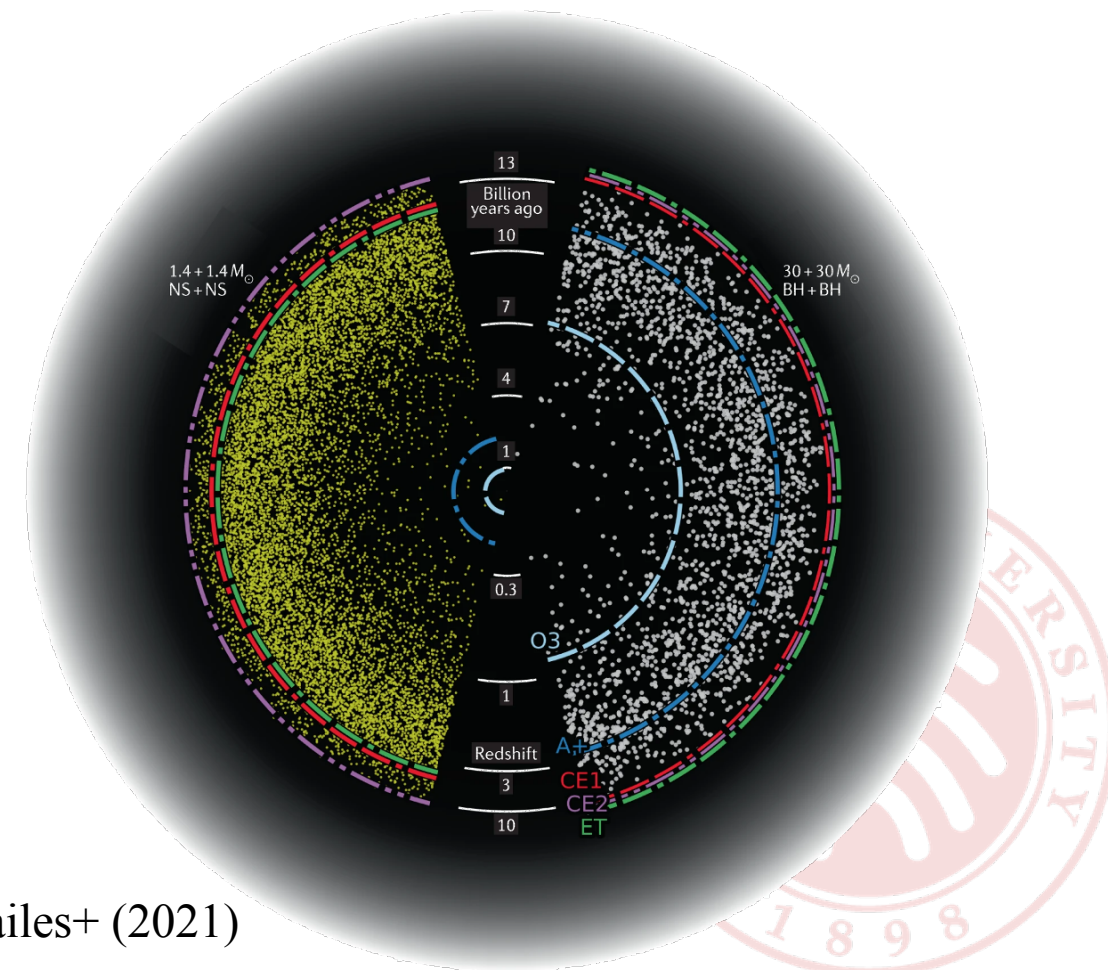
Bailes+ (2021)

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 - 8 compact stars: 2 BNS, 4 NSBH
- Future detectors: Einstein Telescope, Cosmic Explorer
 - Lots of high-precision GW observations, $10^5 \sim 10^6$ /yr
 - New era for probing compact-star structure!



GWIC 3G Science Book



Bailes+ (2021)

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Constrain EOS of Quark Stars (QSs)

Vetting quark-star models with gravitational waves in the hierarchical Bayesian framework

Z. Wang, Y. Gao, D. Liang, J. Zhao and L. Shao*, JCAP 11 (2024)

- Grand canonical potential density in the bag model

Alford+ (2005)

$$\Omega = \sum_{i=u, d, s, e^-} \Omega_i^0 + \frac{3(1-a_4)}{4\pi^2} \mu^4 + B_{\text{eff}} + \frac{3m_s^4 - 48\Delta^2 \mu^2}{16\pi^2}$$

Potential of non-interacting fermions

- B_{eff} : effective bag constant
- m_s : mass of s quark
- a_4 : QCD correction
- Δ : pairing energy gap



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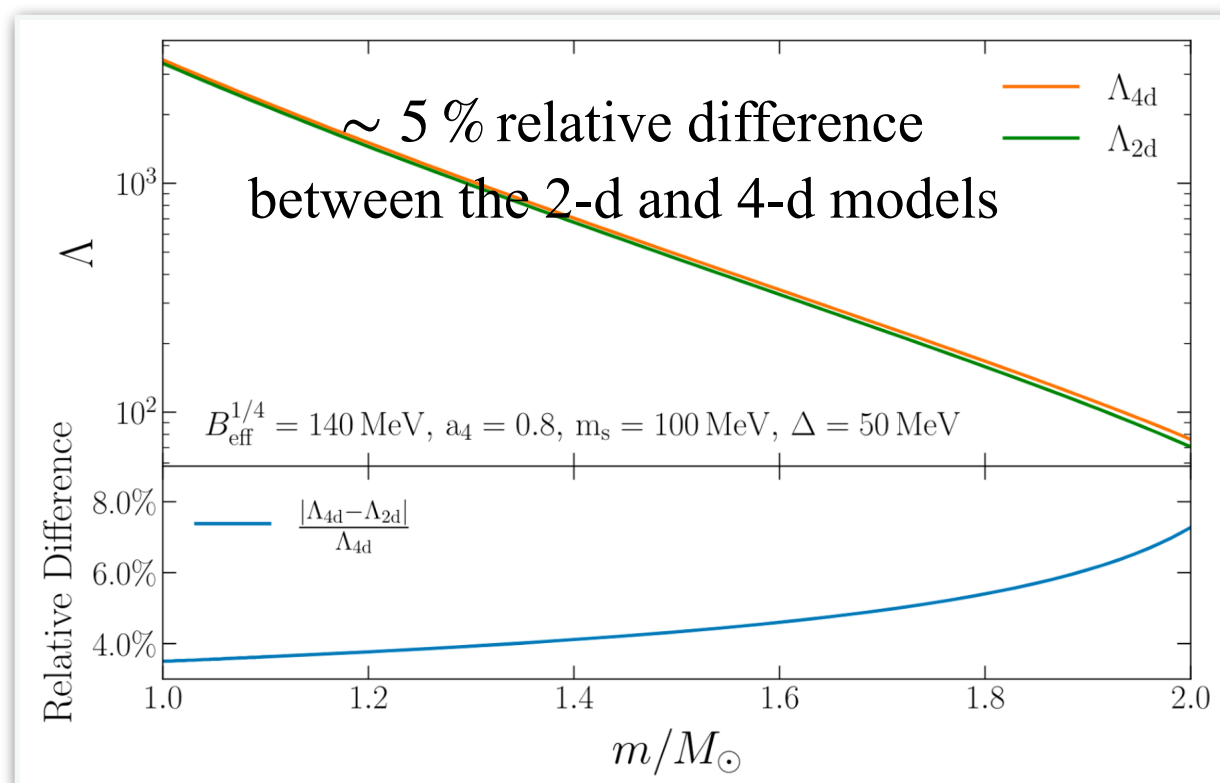
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Zhang & Mann (2021)

$$\beta = \frac{3\Delta^2 - 3m_s^2/4}{\sqrt{3a_4}}$$



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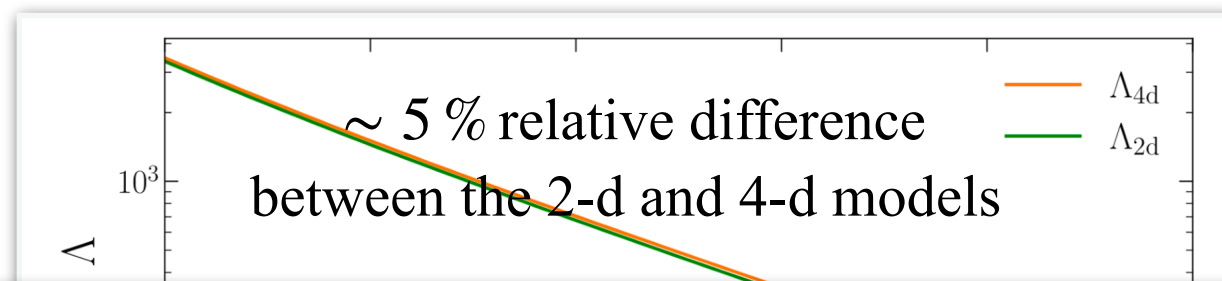
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Zhang & Mann (2021)

This work

- Investigates the potential of constraining QS EOSs with GWs
- Compares the full 4-d and simplified 2-d models in the EOS inference

Data Generation and Inference Implementation

EOS parameters: $H = \{B_{\text{eff}}^{1/4}, a_4, m_s, \Delta\}$ & $H = \{B_{\text{eff}}^{1/4}, \beta\}$

Detectors: Two Cosmic Explorer detectors & one Einstein Telescope detector

Single-event parameters: $\theta = \{\mathcal{M}, \eta, \tilde{\Lambda}, \delta\tilde{\Lambda}, d_L, t_c, \alpha, \delta, \psi, \iota\}$ (Omit spins)

Data: 1000 events within 500 Mpc uniformly distributed in volume

- Select 25 loudest signals
- Mass distribution from Farrow+ (2019)
- Corresponds to a few years' observation
- Inject Λ with the 4-d model: $B_{\text{eff}}^{1/4} = 140 \text{ MeV}$
 $a_4 = 0.8$, $m_s = 100 \text{ MeV}$, $\Delta = 50 \text{ MeV}$



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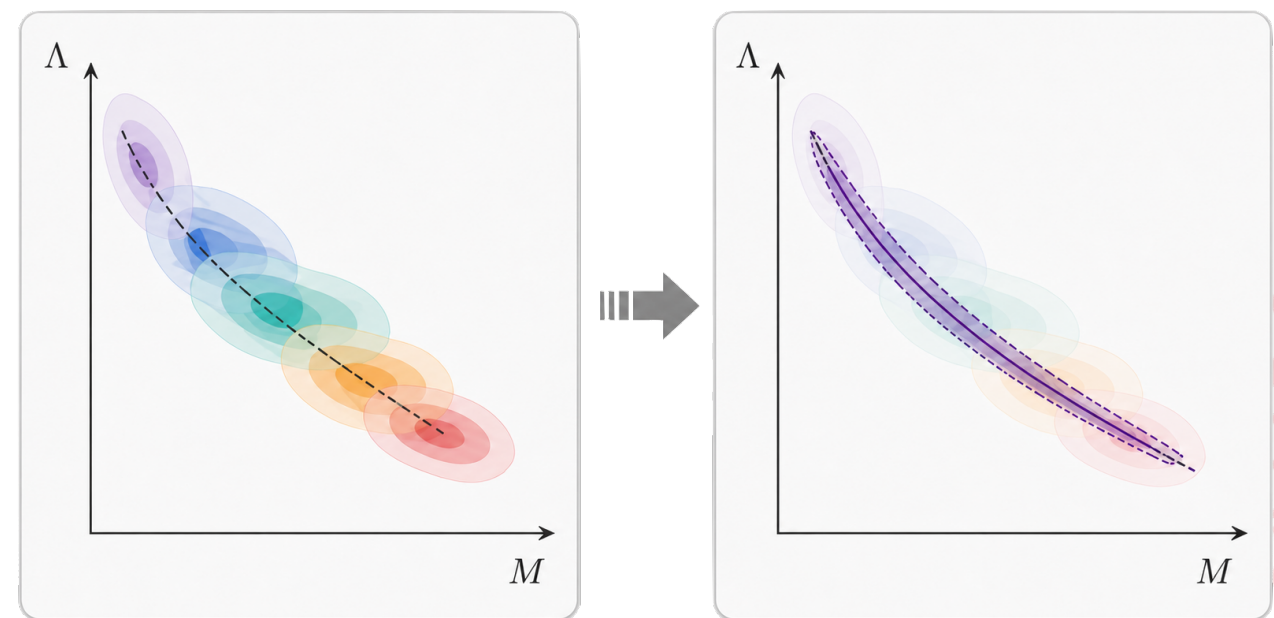
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Method:

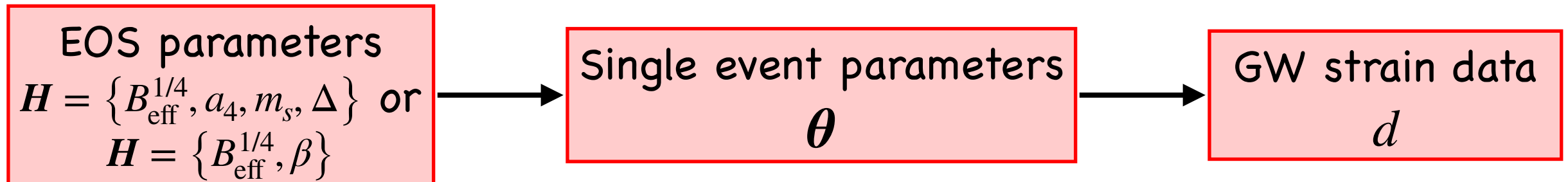
Hierarchical Bayesian inference

- Consistently combine multiple $m - \Lambda$ measurements



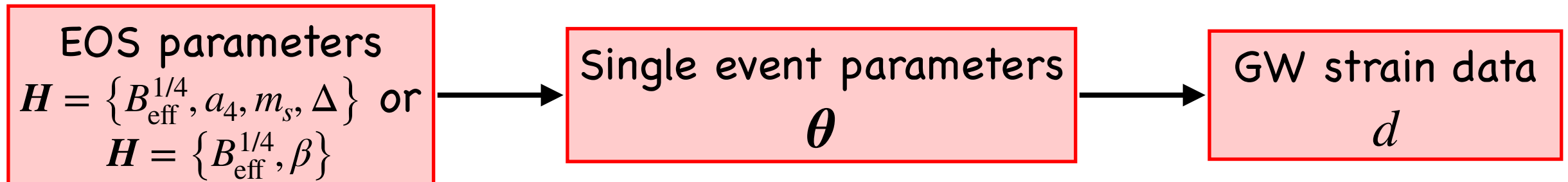
Hierarchical Inference of EOS Parameters

- The EOS parameters are not GW-waveform parameters
 - EOS parameters are **hyper-parameters**



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- Bayes' theorem

Nuisance parameters ($d_L, t_c, \dots$)

Marginal likelihood of i -th event

$$\begin{aligned}
 P(H | D) &\propto \pi(H) \prod_{i=1}^n \int P(\theta_i | H) P(d_i | \theta_i, H) d\theta_i \\
 &= \pi(H) \prod_{i=1}^n \int P(m_i | H) P(\Lambda_i | m_i, H) P(\xi_i | m_i, \Lambda_i, H) P(d_i | \theta_i) dm_i d\Lambda_i d\xi_i \\
 &= \pi(H) \prod_{i=1}^n \int P(m_i | H) \delta(\Lambda_i - \lambda(m_i, H)) dm_i d\Lambda_i \int P(\xi_i) P(d_i | \theta_i) d\xi_i \\
 &= \pi(H) \prod_{i=1}^n \int P(m_i | H) L_i^q(m_i, \lambda(m_i, H)) dm_i
 \end{aligned}$$

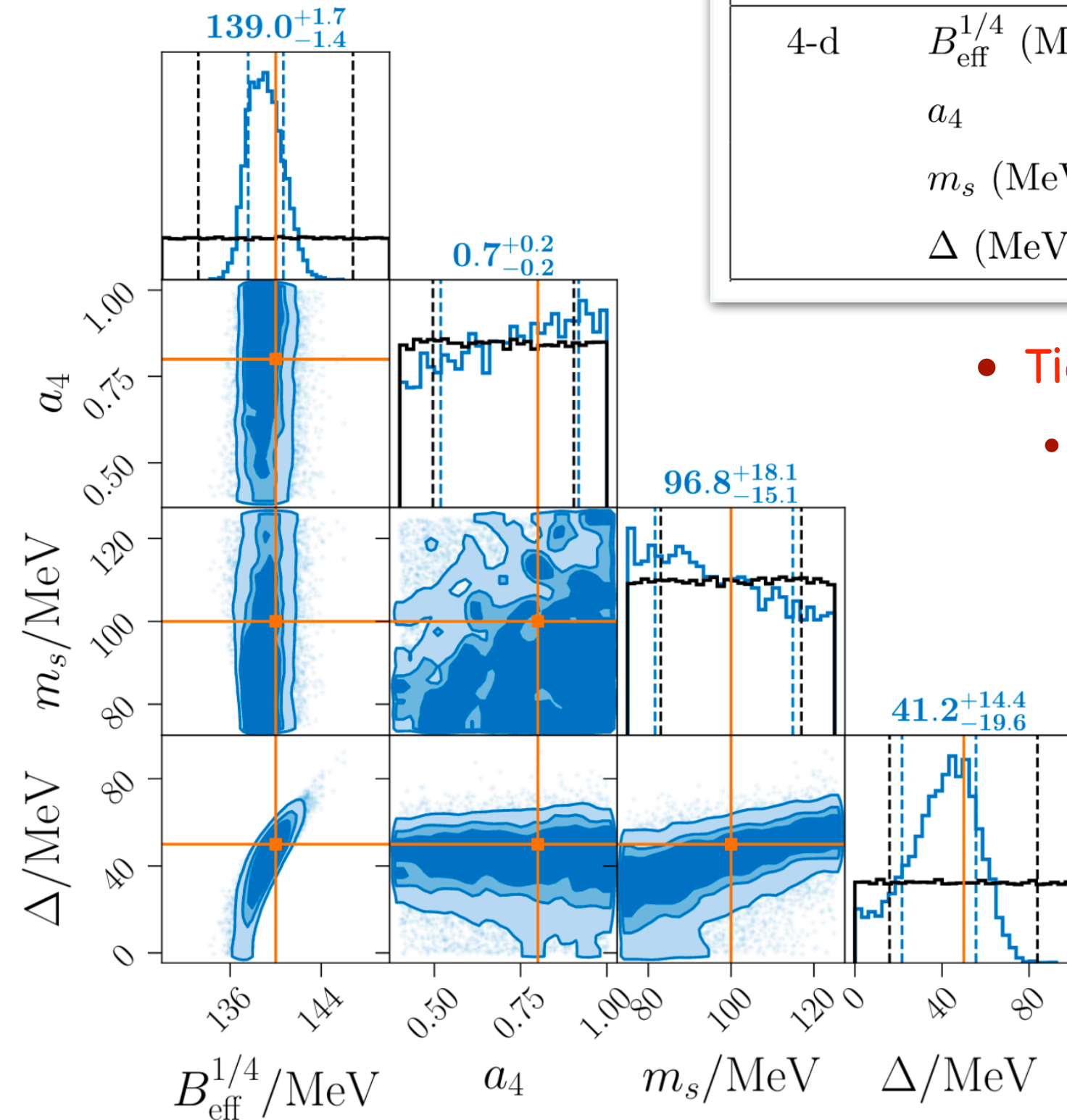
Independent of H
“Posterior” of m & Λ

m-Λ relation given by EOS



Posterior of hyper-parameters in 4-d Model

Model	Parameter	True Value	Prior	Posterior
4-d	$B_{\text{eff}}^{1/4}$ (MeV)	140	$\mathcal{U}(130, 150)$	$139.0^{+1.7}_{-1.4}$
	a_4	0.8	$\mathcal{U}(0.4, 1.0)$	$0.7^{+0.2}_{-0.2}$
	m_s (MeV)	100	$\mathcal{U}(75, 125)$	$96.8^{+18.1}_{-15.1}$
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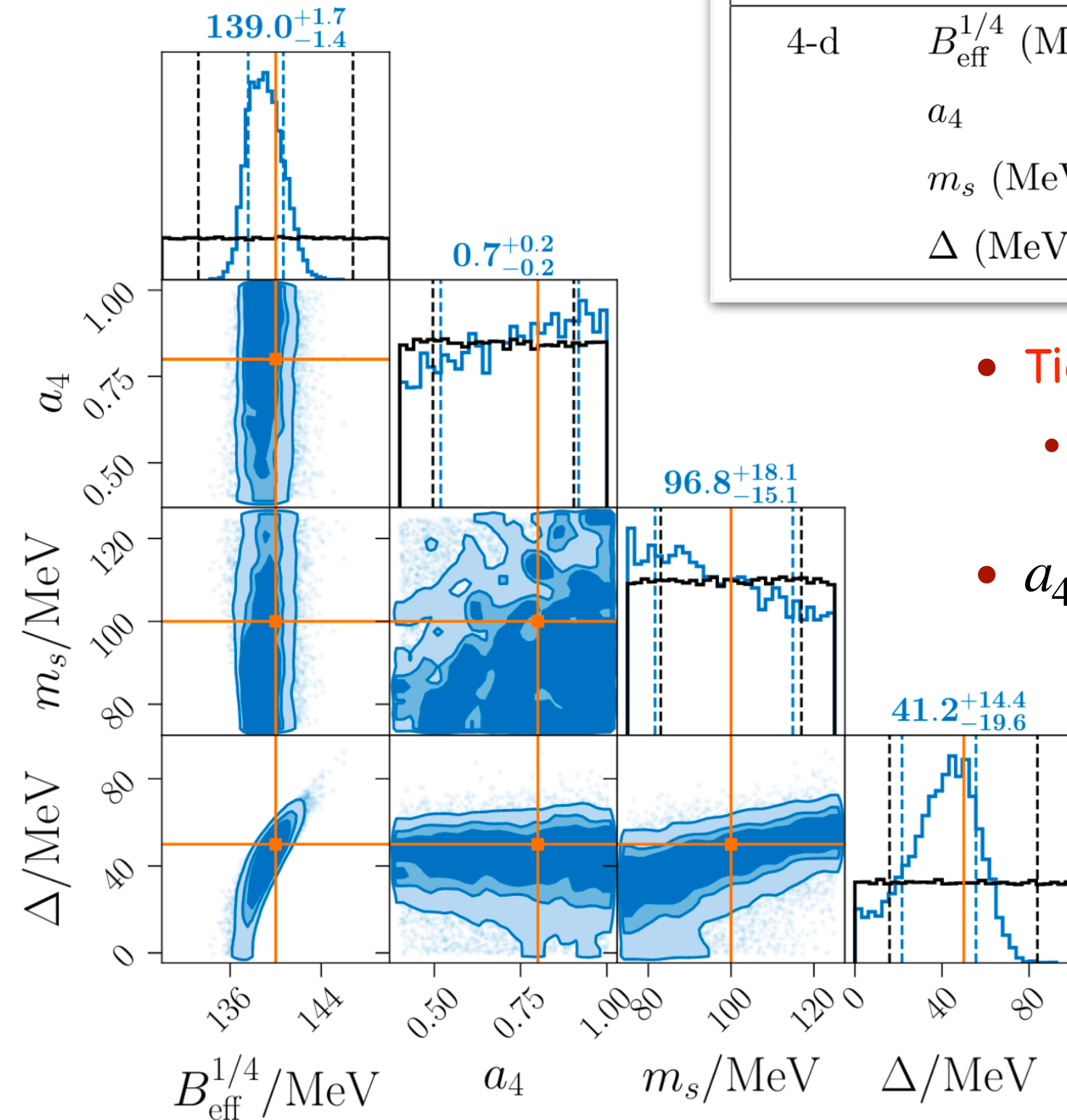


- Tight constraints on $B_{\text{eff}}^{1/4}$ and Δ
- Several-times improvement compared to those from GW170817 & GW190425 (Miao+ 2021)



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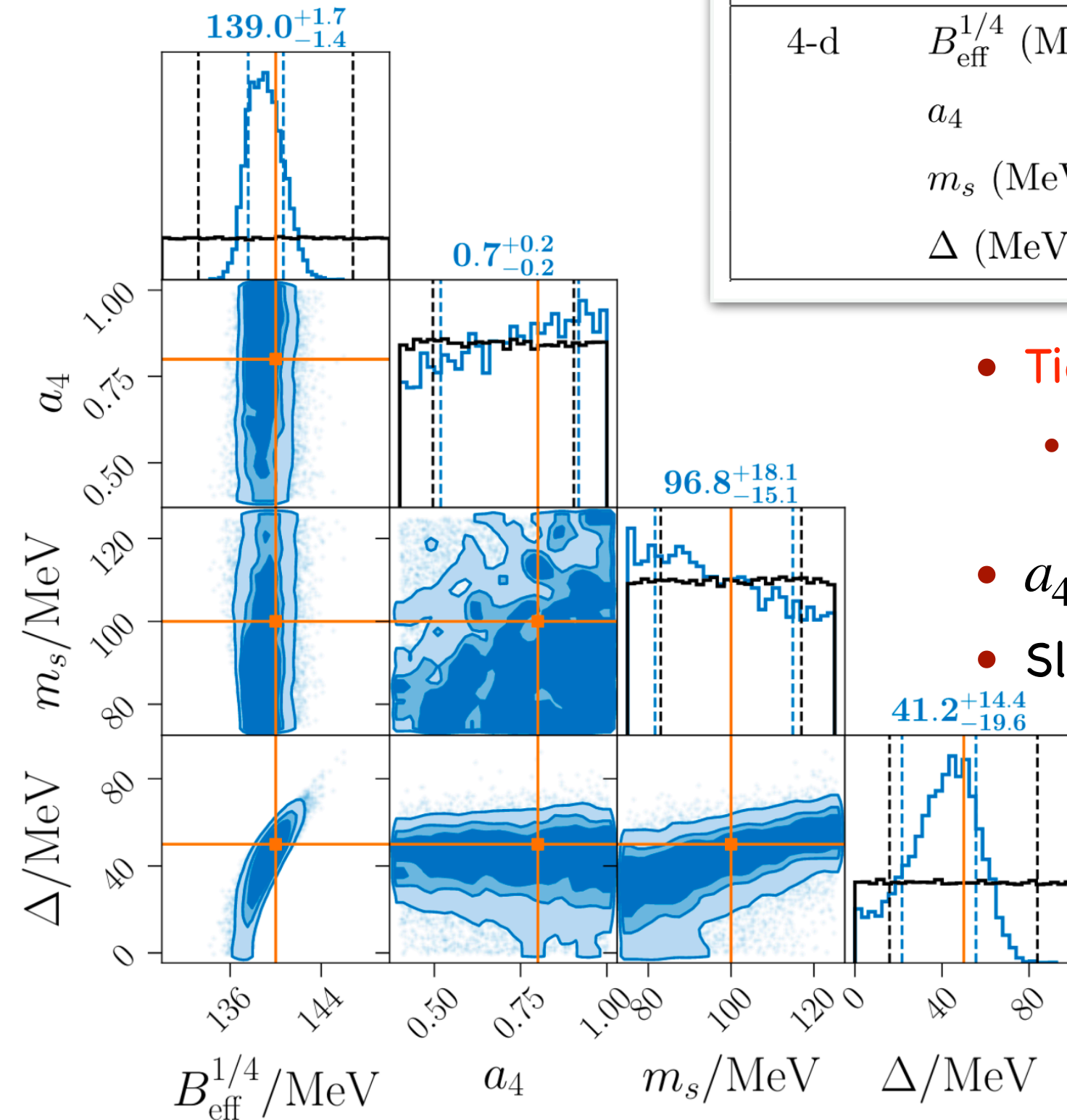


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- Tight constraints on $B_{\text{eff}}^{1/4}$ and Δ
 - Several-times improvement compared to those from GW170817 & GW190425 (Miao+ 2021)
- a_4 and m_s are not well constrained
- Slight degeneracy between m_s and Δ

$$\beta = \frac{3\Delta^2 - 3m_s^2/4}{\sqrt{3}a_4}$$



Comparison between 2-d and 4-d Models

- In the results of 4-d model
 - Unconstrained parameters
 - Correlation and degeneracy



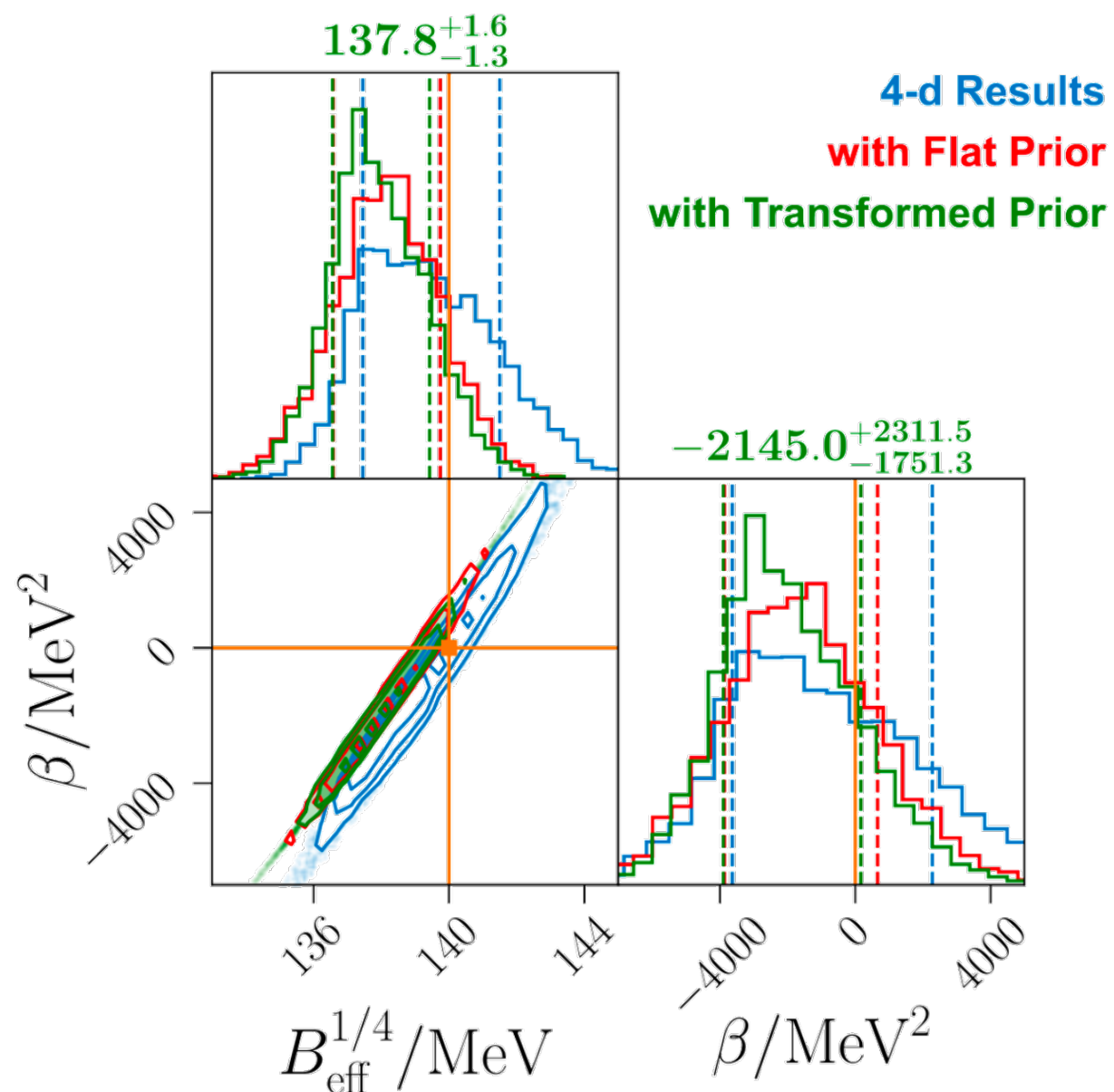
Possible redundancy in the
EOS parameterization



Comparison between 2-d and 4-d Models

- In the results of 4-d model
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Possible redundancy in the EOS parameterization



Flat prior: uniform in the allowed range

→ Test the unbiasedness

Transformed prior: transform from the 4-d prior

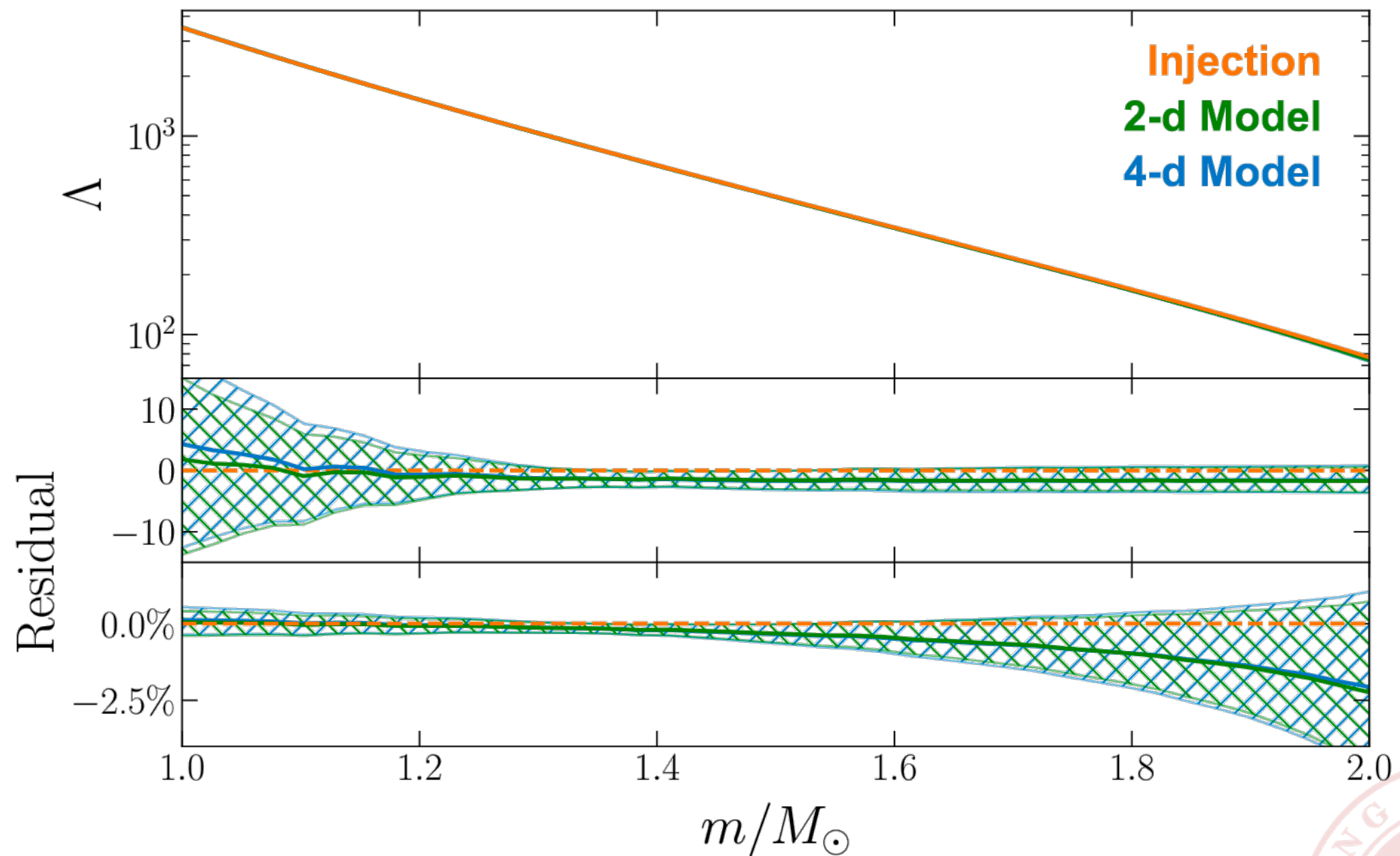
→ Compare the 2-d and 4-d models consistently

Both $B_{\text{eff}}^{1/4}$ and β are tightly constrained...

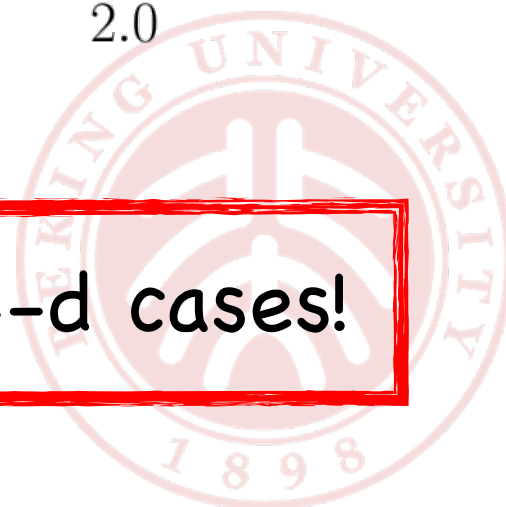
But biased!



Recover the m - Λ relations



Consist with the injection relation in both 2-d and 4-d cases!



Contents

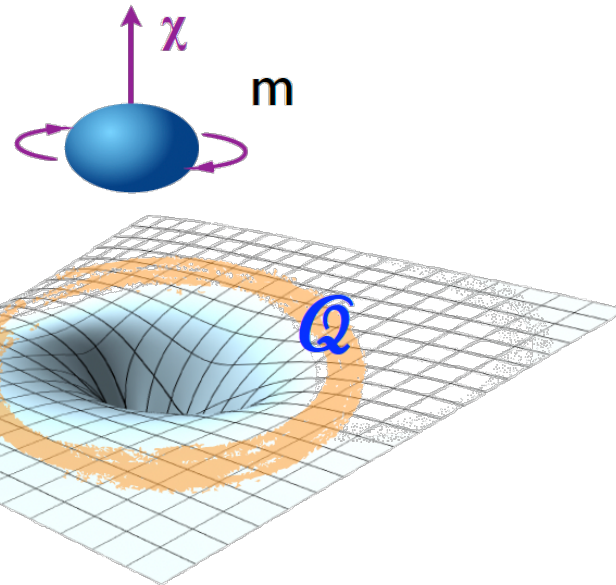
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Spin-induced Quadrupole

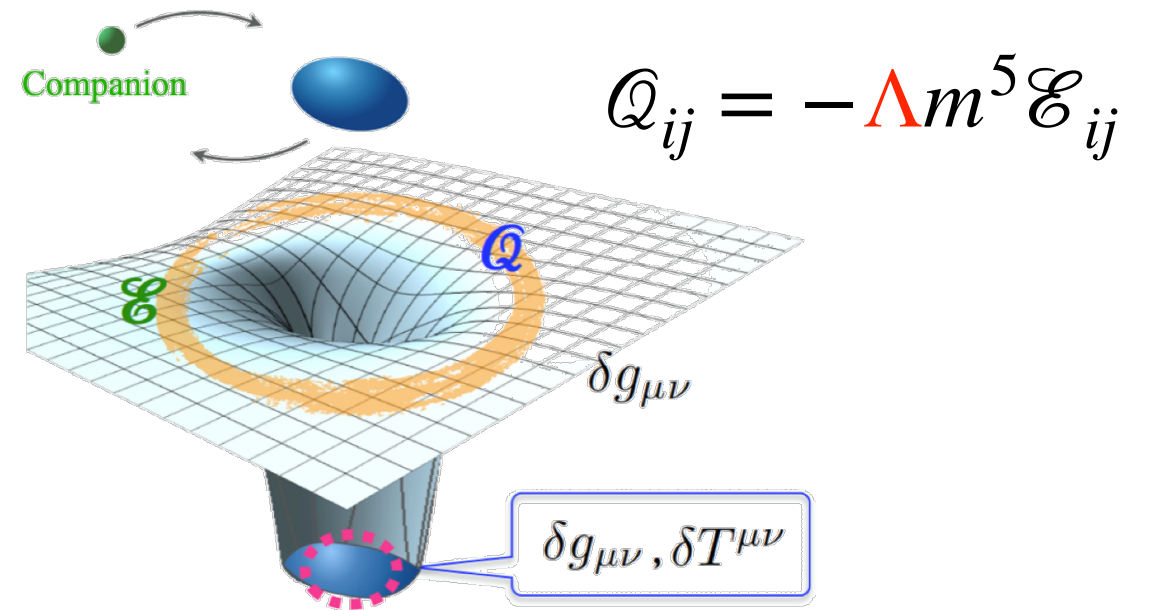
- The spin of the compact star also induces quadrupole

$$Q = -Q m^3 \chi^2$$



Credit: Hinderer

Spin-induced deformation



$$Q_{ij} = -\Lambda m^5 \mathcal{E}_{ij}$$

Tidal-induced deformation

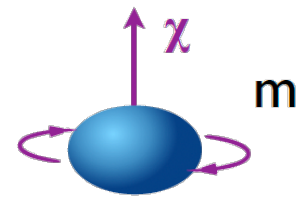
Q : Quadrupole moment



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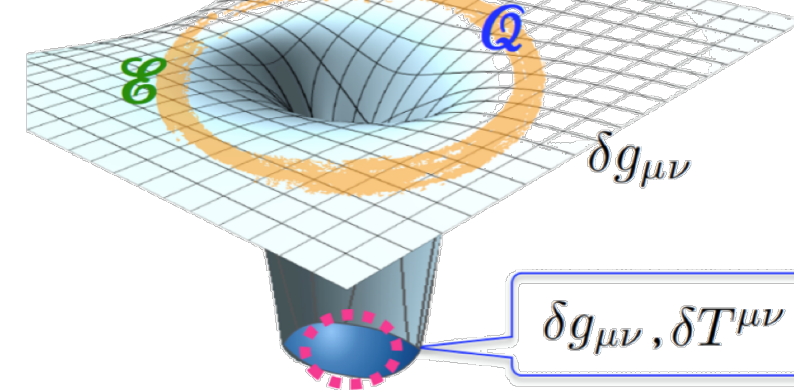


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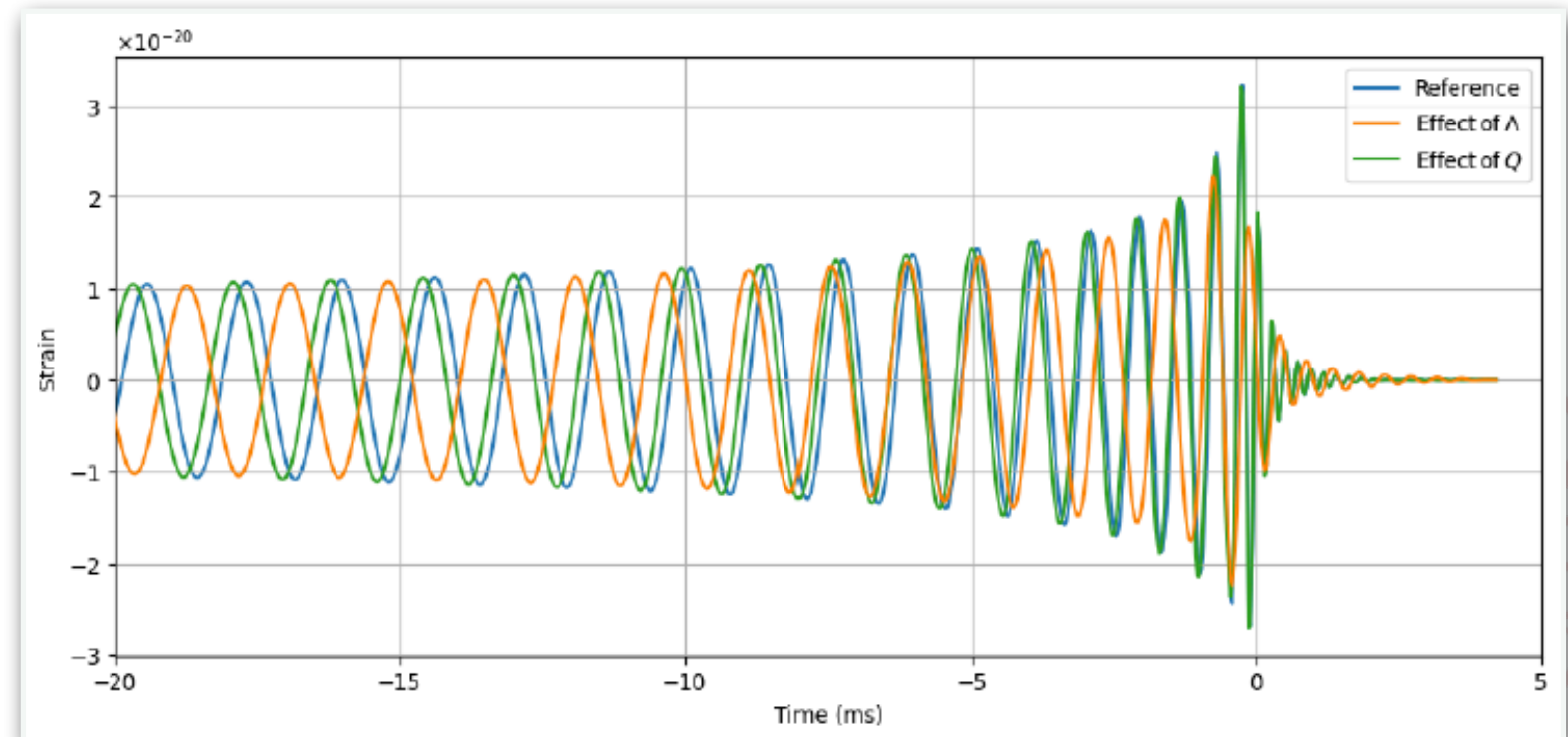
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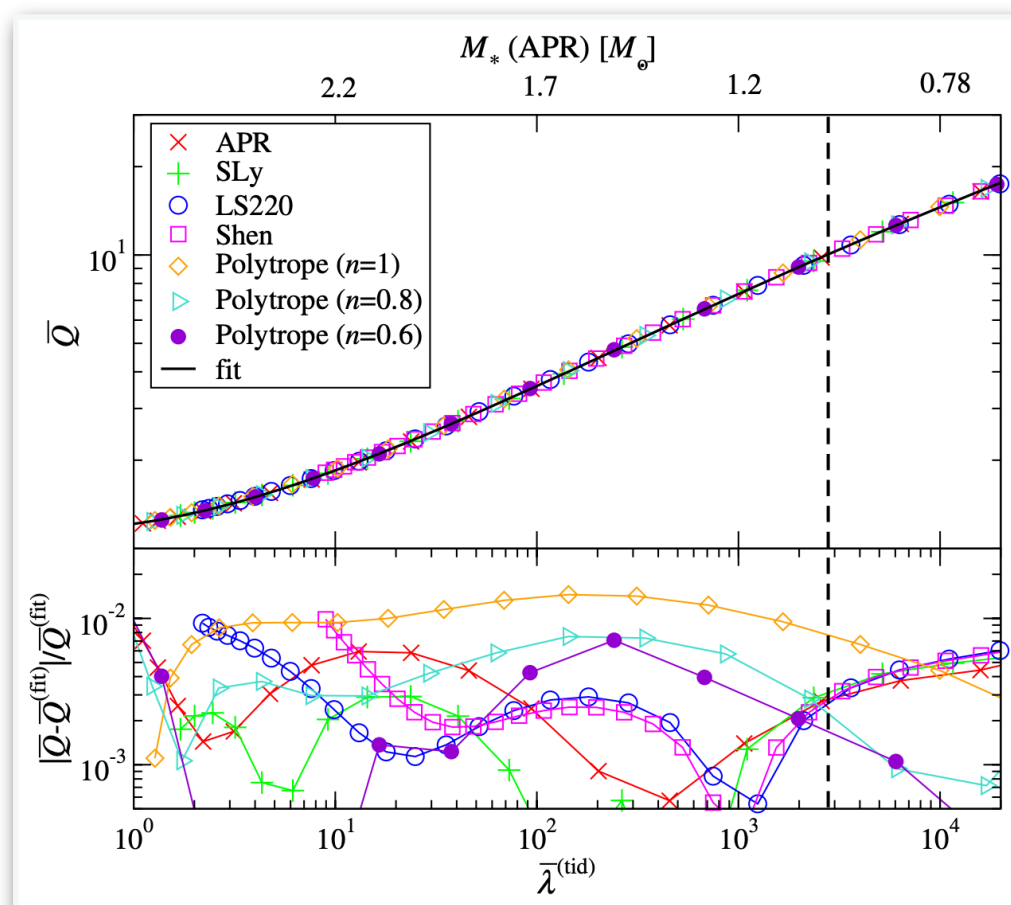
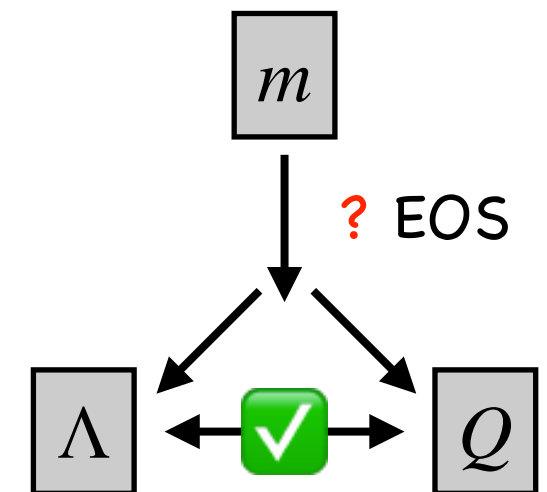
- m - Q relation is also determined by EOS
- Also affects GW waveform



Love-Q Universal Relation

Yagi & Yunes (2013): Universal I-Love-Q relation for various EOSs

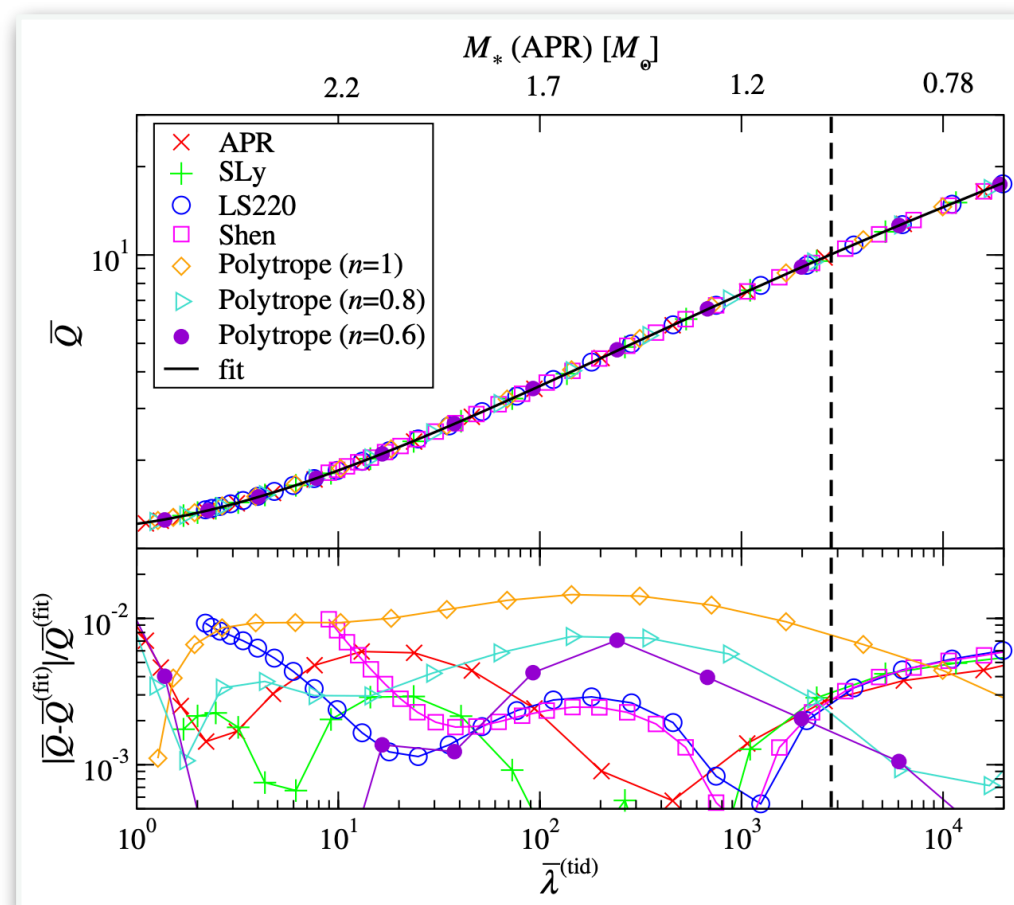
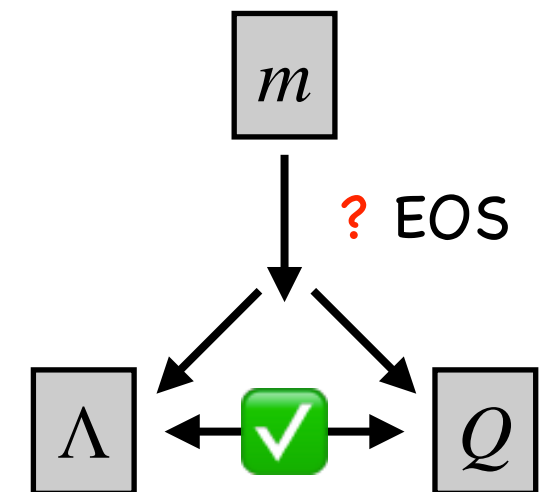
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- Similar to black hole's no-hair theorem
- Dependent on gravity theories (test GR)



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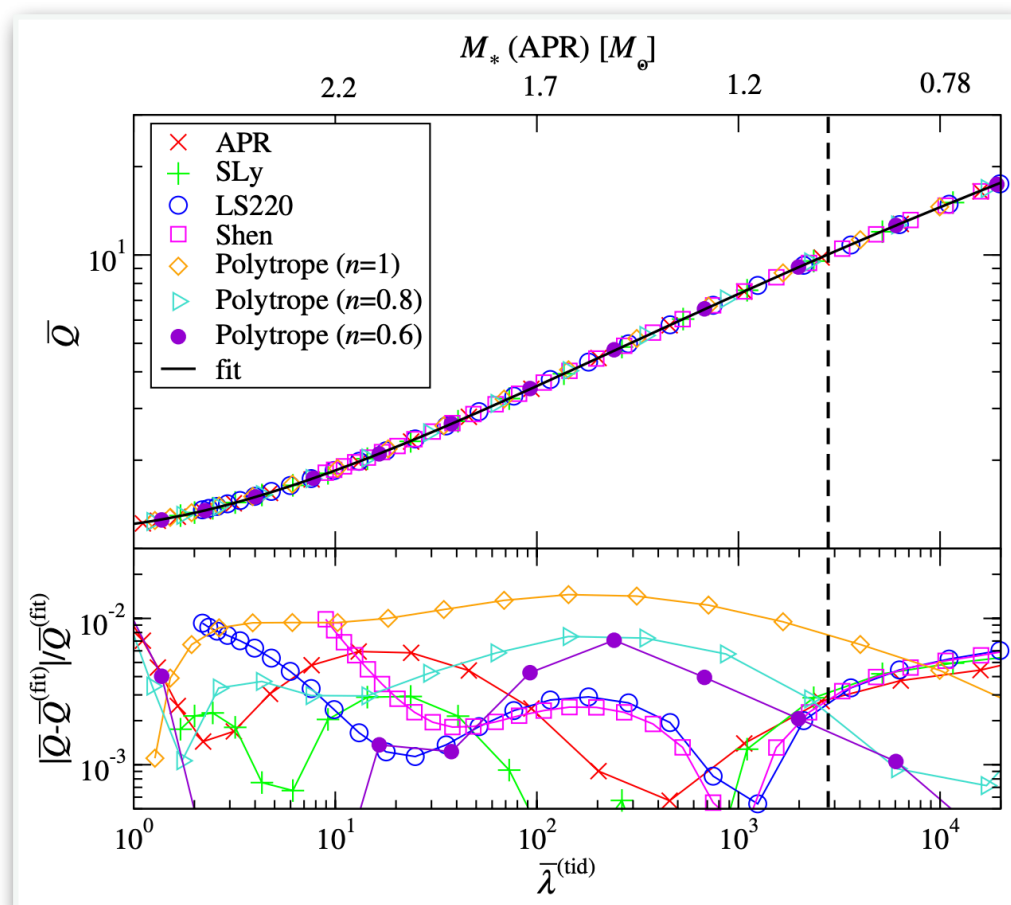
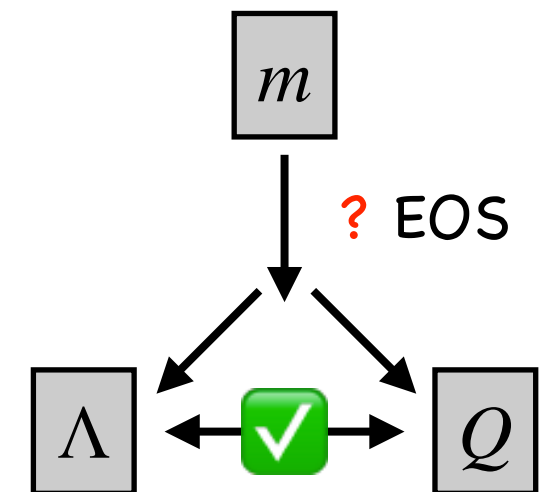
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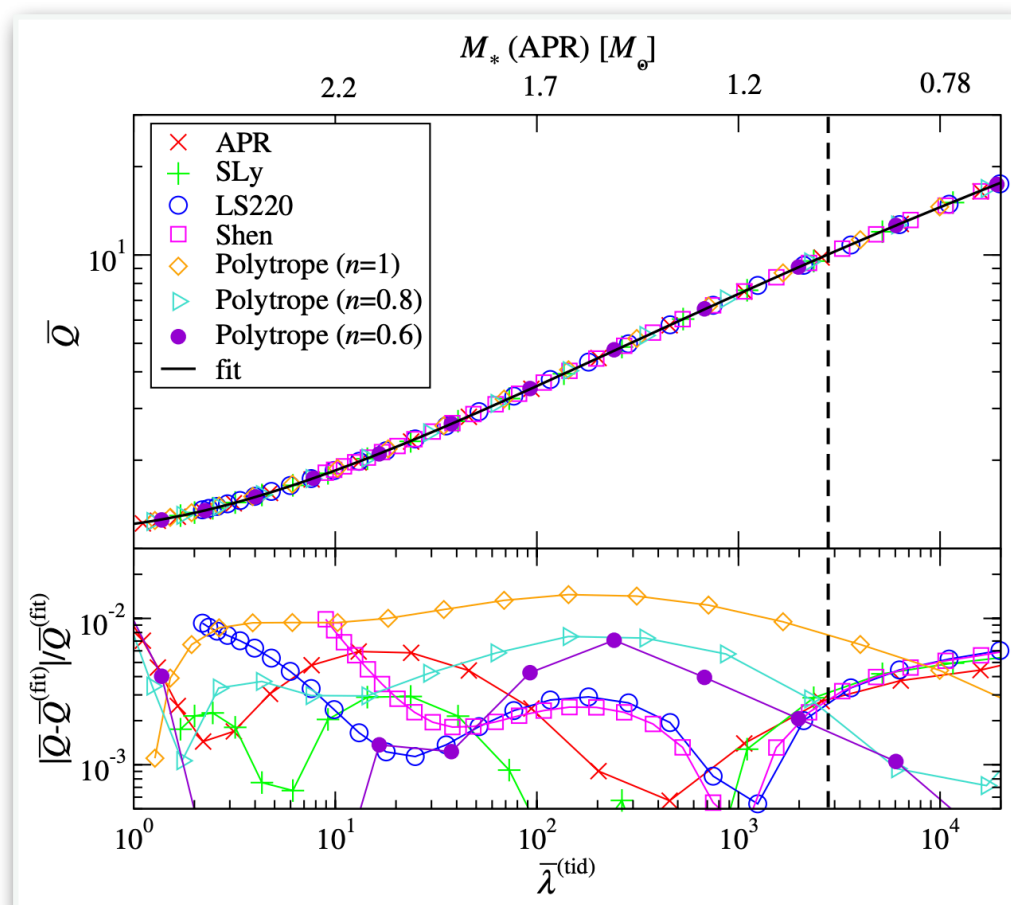
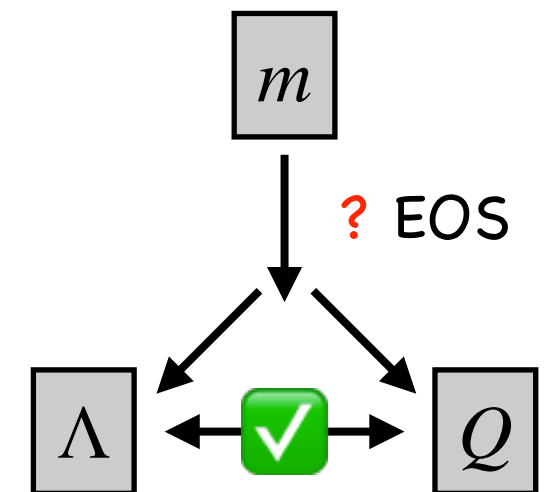
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- Constrain the Love-Q Relation!



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- Theoretically predicted
- Experimentally unconfirmed
- Measure Λ & Q independently from GW
→ Constrain the Love-Q Relation!

This work

- Prospects with future GW observations

Data Generation and Inference Implementation

Parameterization: $\ln \Lambda = \text{Polynomial}(\ln Q; \mathbf{H})$

Detectors: Two Cosmic Explorer detectors & one Einstein Telescope detector

Single-event parameters: $\theta = \{\mathcal{M}, \eta, \chi_1, \chi_2, \Lambda_1, \Lambda_2, Q_1, Q_2, d_L, t_c, \alpha, \delta, \psi, \iota\}$

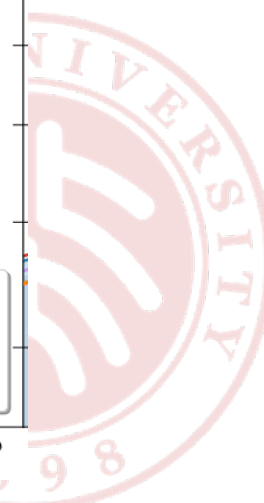
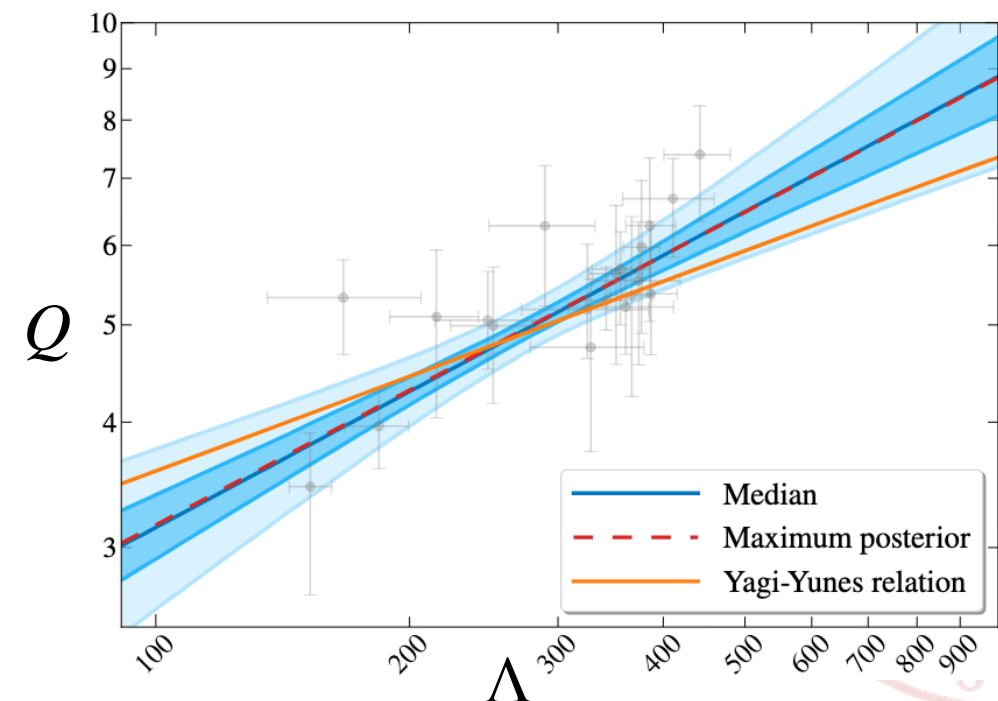
Data: 1000 events within 500 Mpc uniformly distributed in volume

- Select 100 loudest signals
- Then top 20 with largest spins
- Mass distribution from Farrow+ (2019)
 $\chi_r \sim \mathcal{U}(-0.5, 0.5)$
 $\chi_s \sim \mathcal{U}(-0.1, 0.1)$
- Uniform spin distribution
- Injected with APR4 EOS

Method:

Hierarchical Bayesian inference

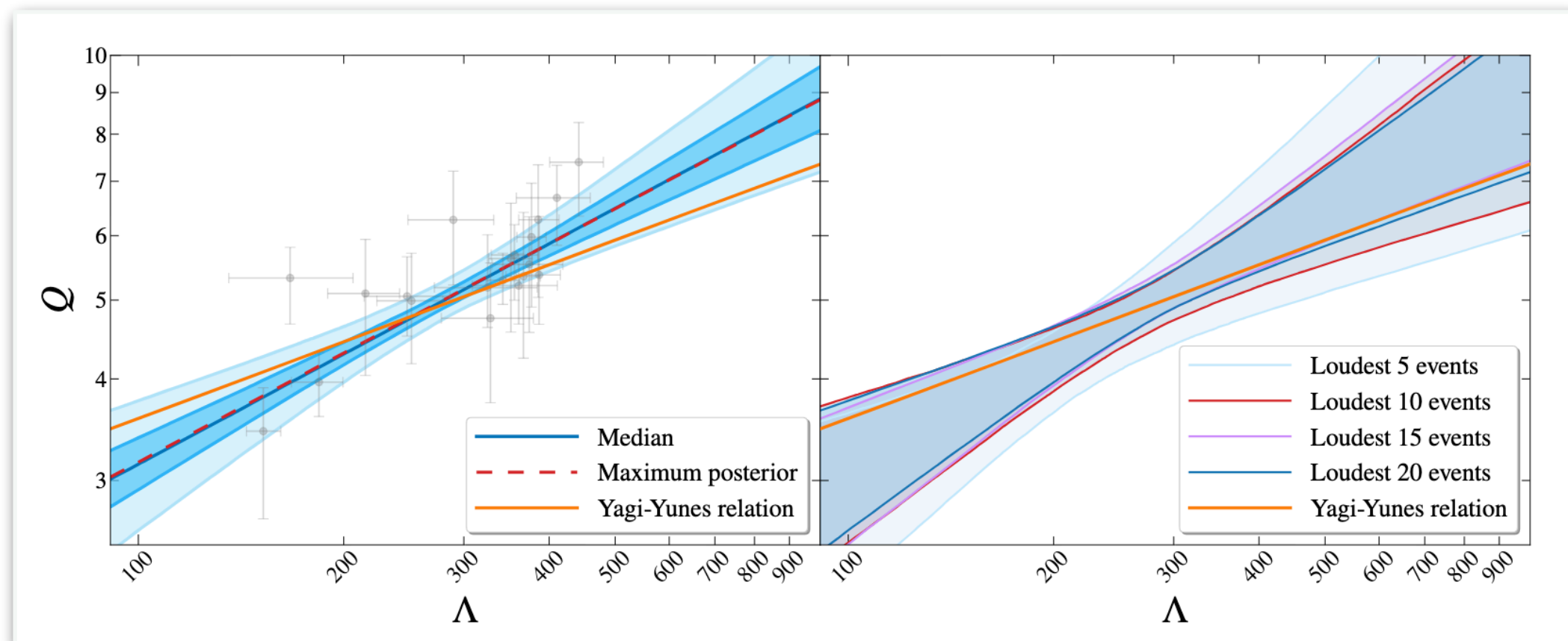
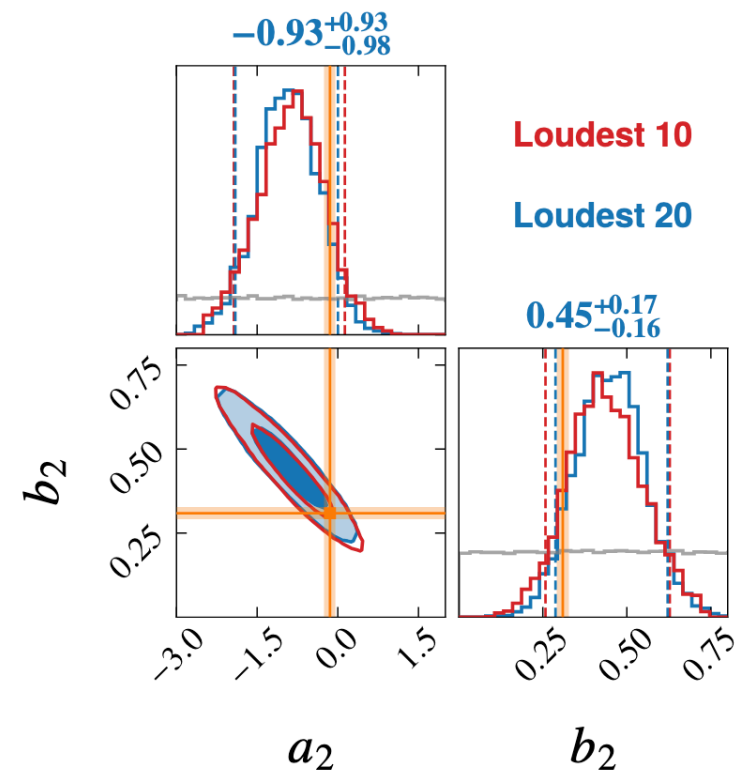
- Consistently combine multiple $\Lambda - Q$ measurements



Inference Results: Linear Model

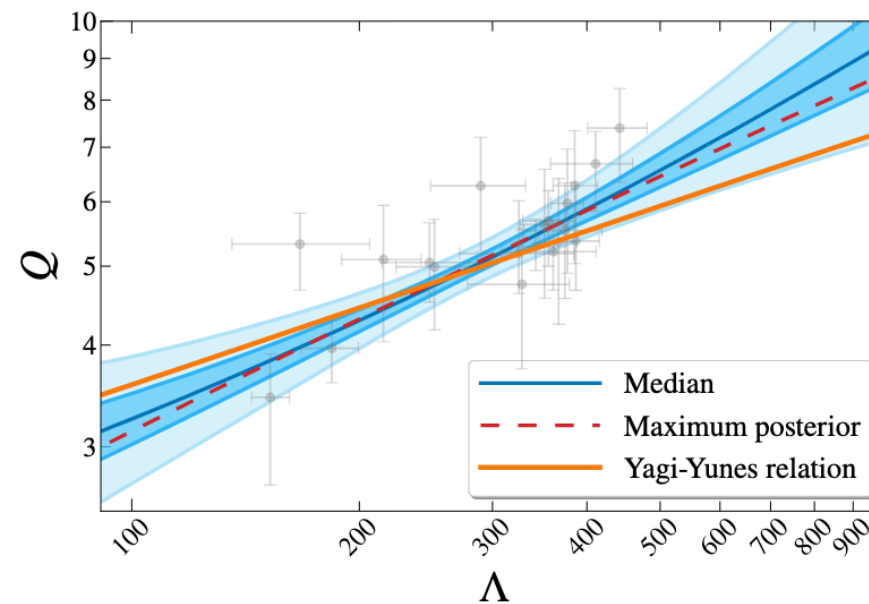
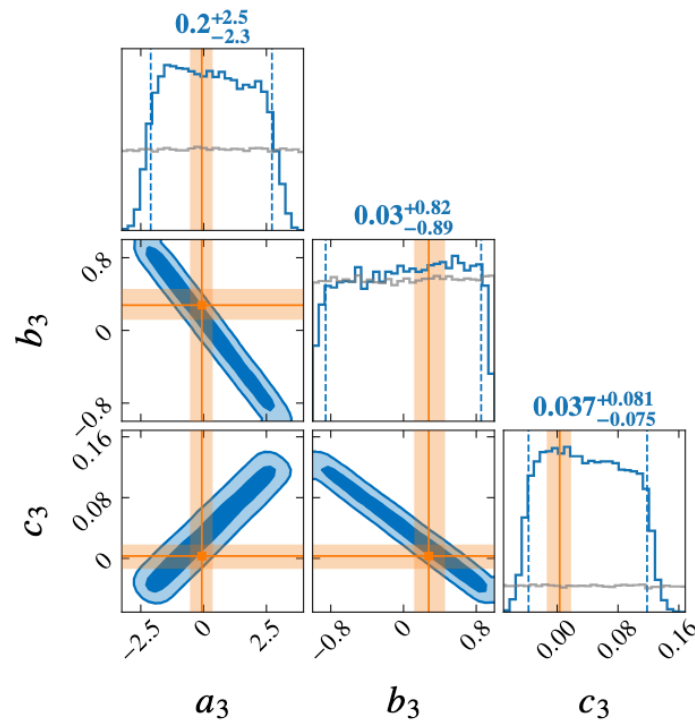
$$\ln Q = a_2 + b_2 \ln \Lambda \quad \text{Also adopted in Samajdar \& Dietrich (2020)}$$

- Tight constraints on a_2 and b_2
- Yagi-Yunes relation covered by 90% credible region
- Major constraint information from loudest 10 events

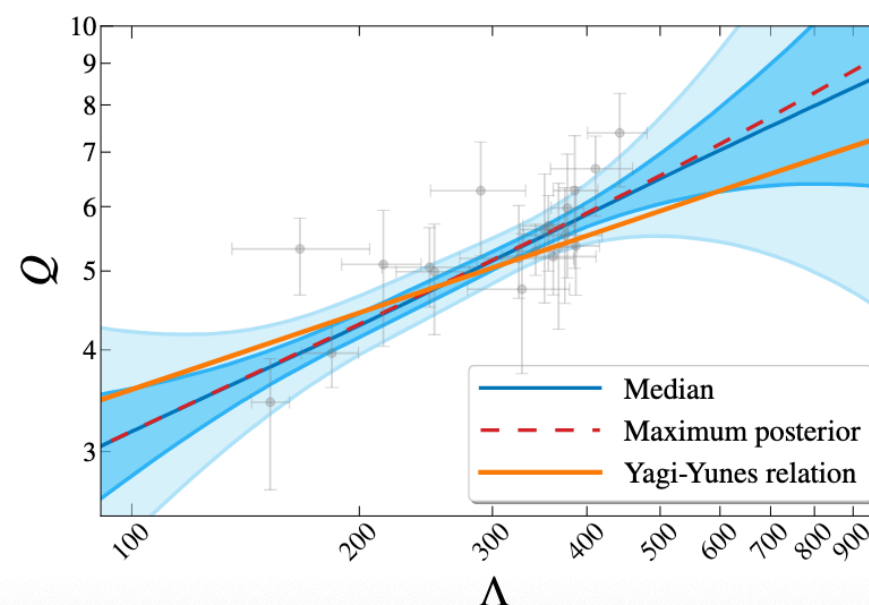
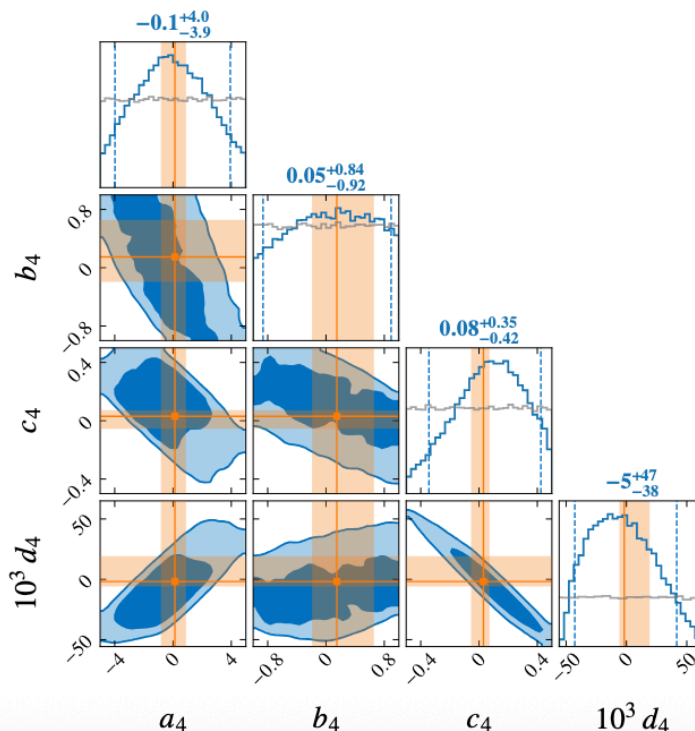


Inference Results: Polynomial Models

Cubic and quartic model



- Wide credible regions
- Strong correlations
- Linear model is enough!



Summary and Outlook

Future GW observations have remarkable potential for probing the structure of compact stars

- For quark-star EOSs
 - Constraints on $B_{\text{eff}}^{1/4}$ and Δ are tightened by several times (a_4 and m_s still not well measured)
 - Approximated 2-d model exhibits biases as more GW events are analyzed (Recovered m - Λ relation remains consistent)
- For Love-Q relation
 - Possible to constrain the Love-Q relation from GW observation
 - Linear model is precise enough
- Future work
 - Other observations: m - R observations of pulsars, the post-merger remnants
 - Infer the mass and spin distribution simultaneously



Thanks for listening!

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