



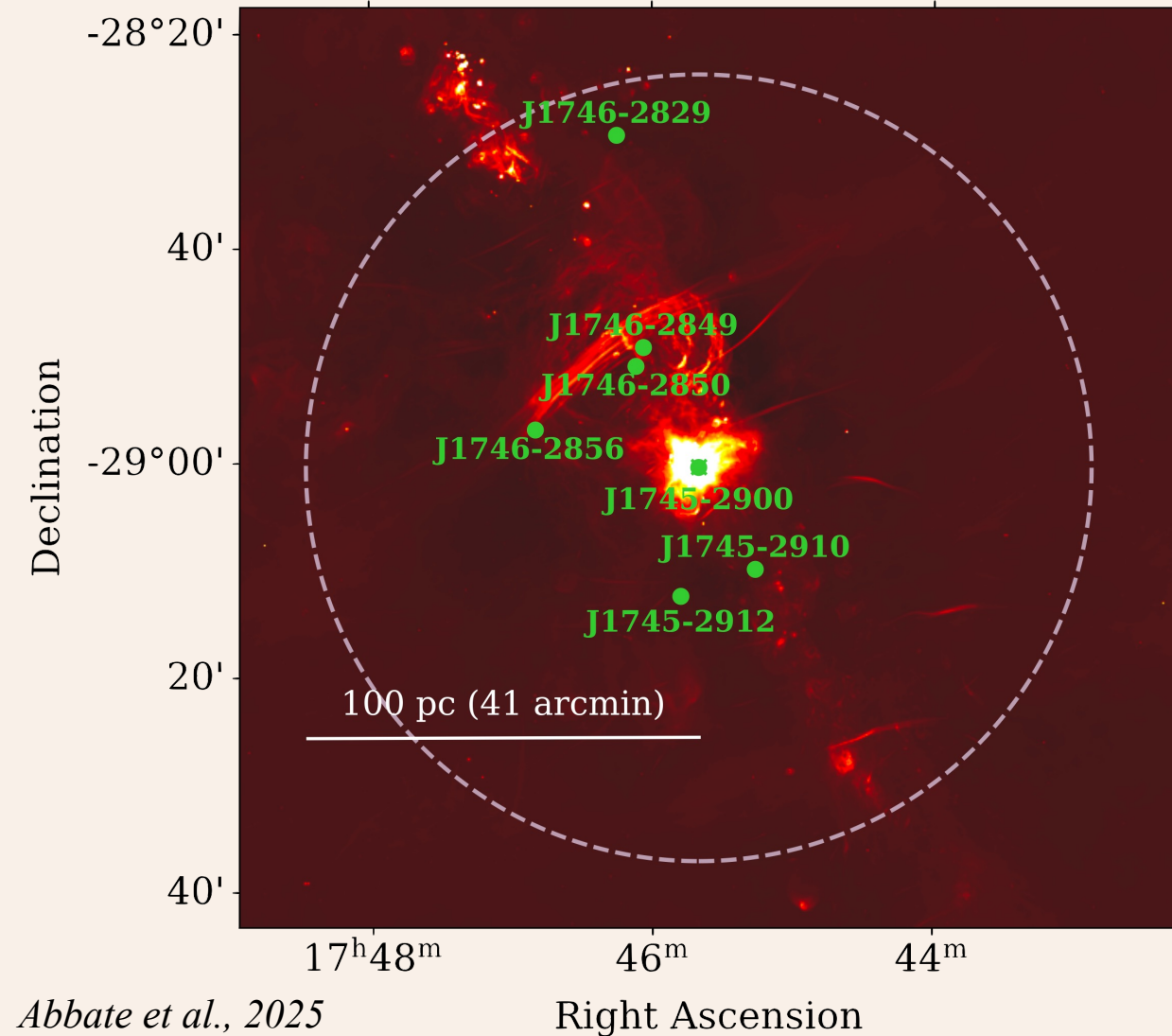
A Realistic Pulsar-Supermassive Black Hole Timing Model

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Galactic Centre Pulsars

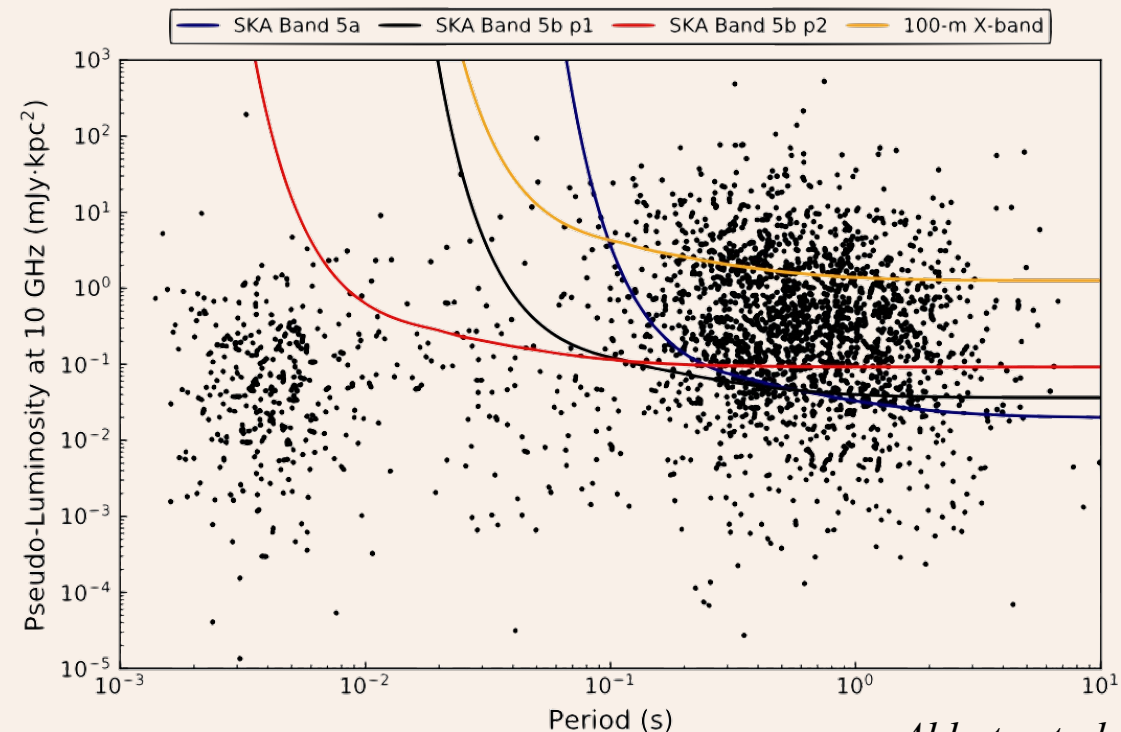
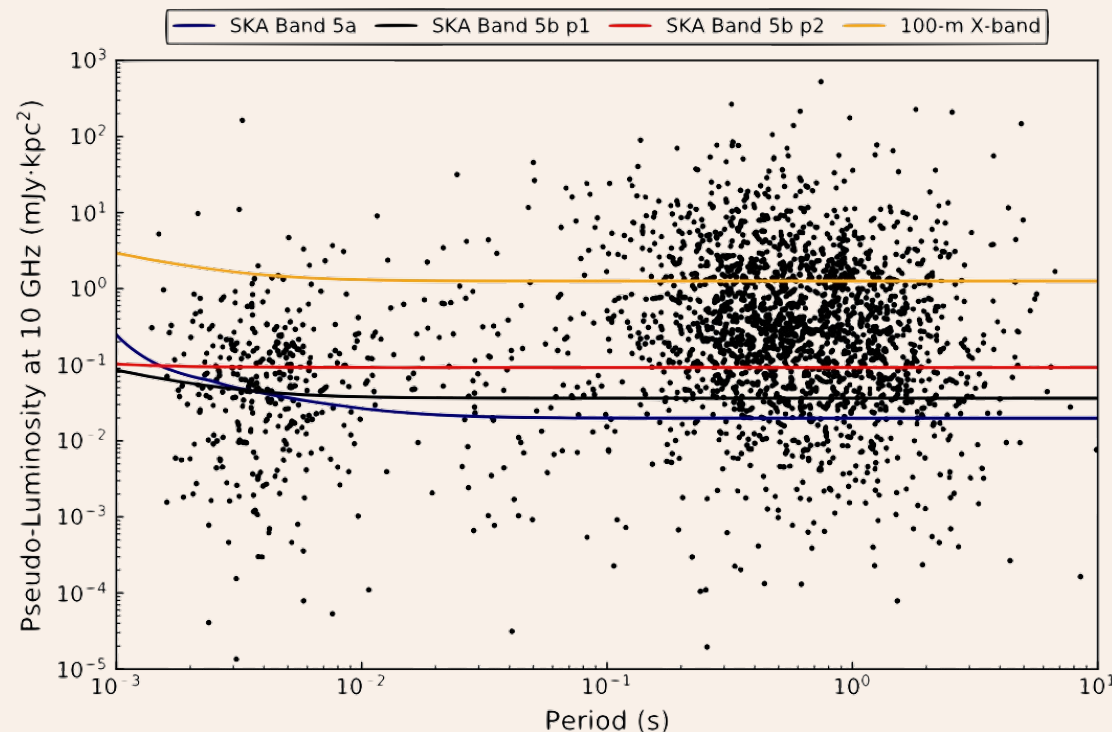


Probe GC ISM & DM

Measure SMBH properties

Test Strong Field Gravity

Galactic Centre Pulsars

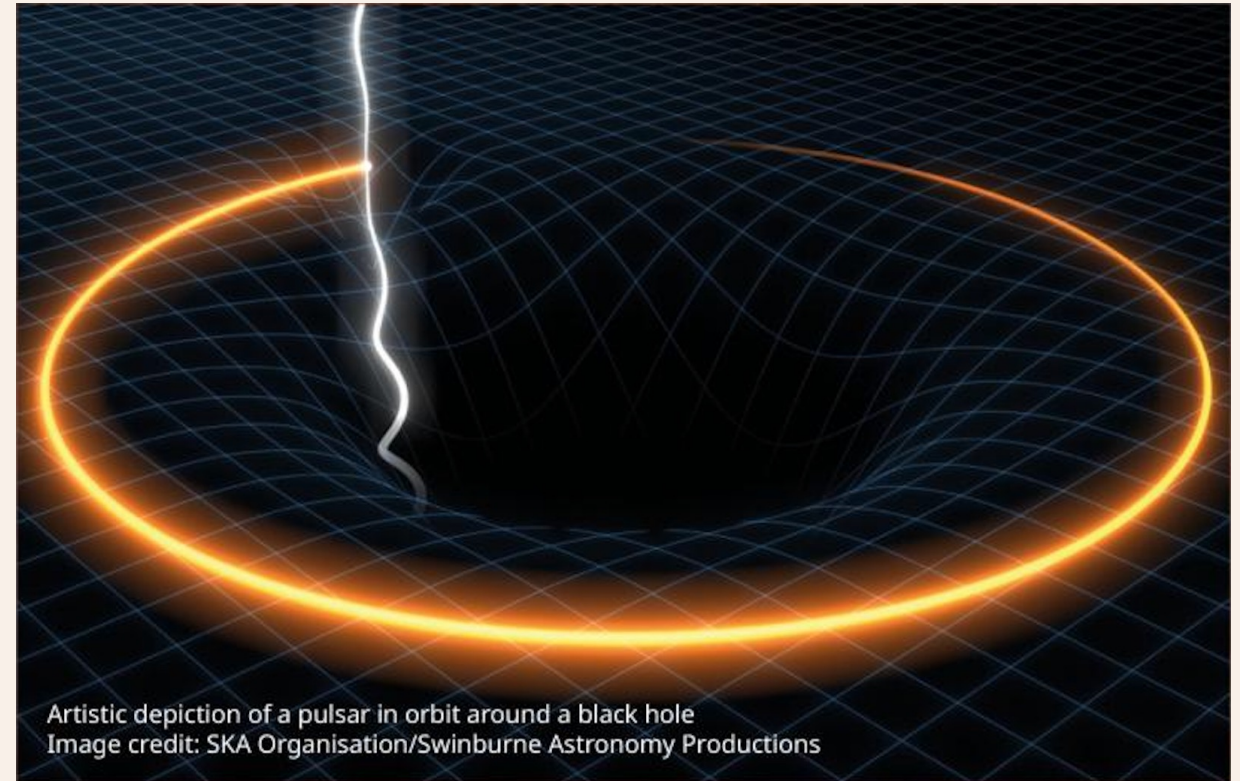


Abbate et al., 2025

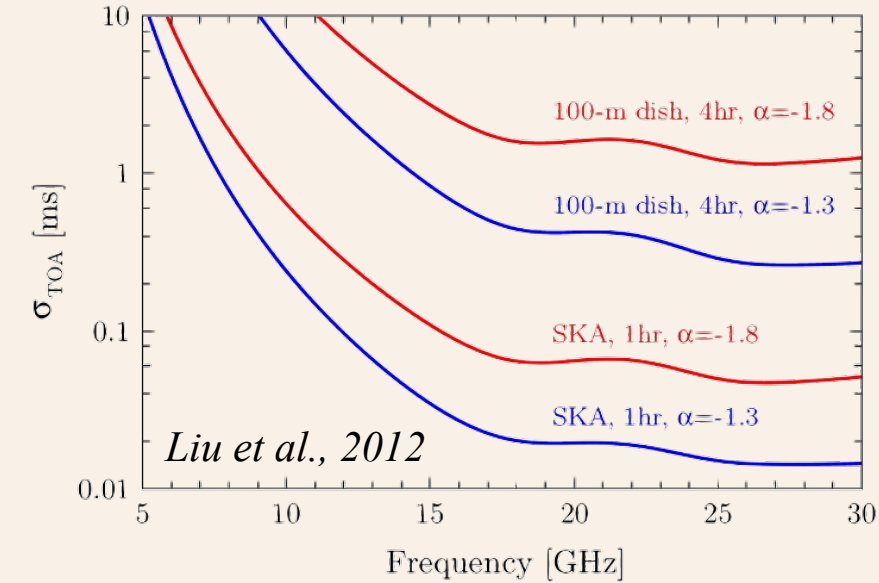
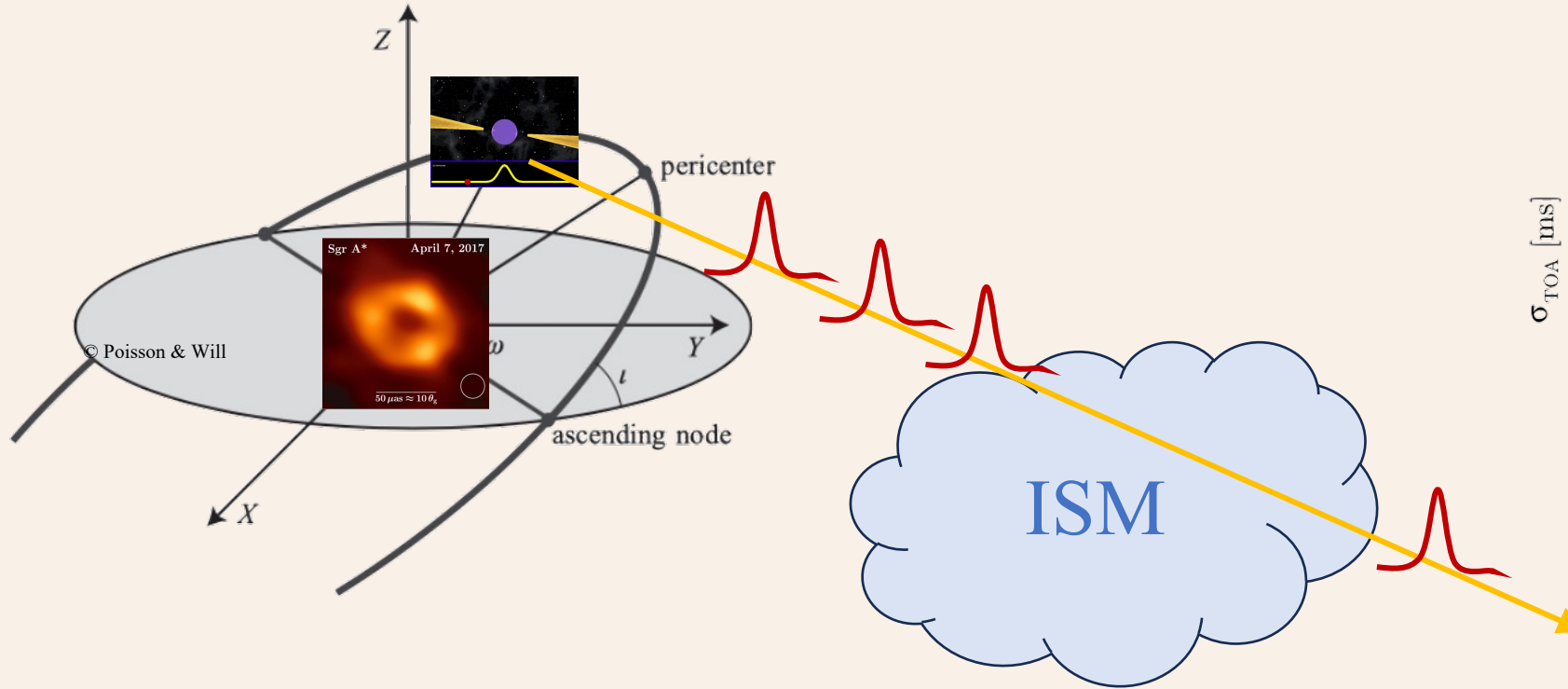
- Within the central parsecs: **hundreds** of young pulsars + **thousands** of MSPs
- A large fraction of pulsars can be detected under the AA4 or AA* array stages

Pulsar-Supermassive Black Hole System

- **Novel probe** of the theories of gravity and the GC environment
- Measuring the properties of the GC SMBH at **unprecedented precision**
- *Pfahl & Loeb, 2004; Liu et al., 2012; Psaltis et al., 2016; Hu et al., 2023; Hu et al., 2026...*

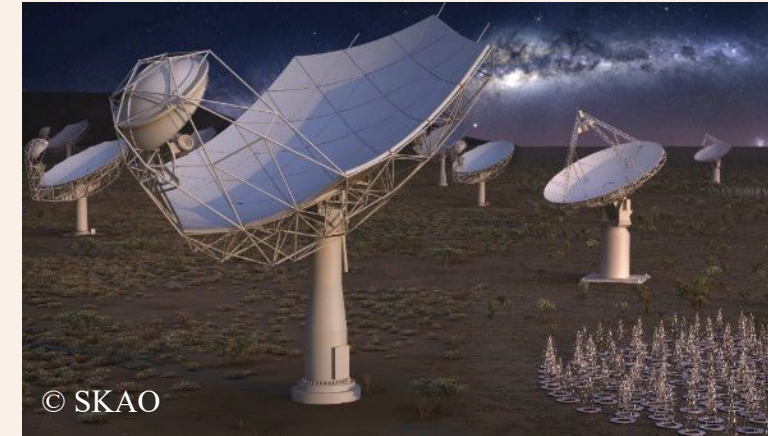


Pulsar Timing and Timing Model



- Orbital motion & light propagation
- **Realistic** timing model: all effects above timing precision

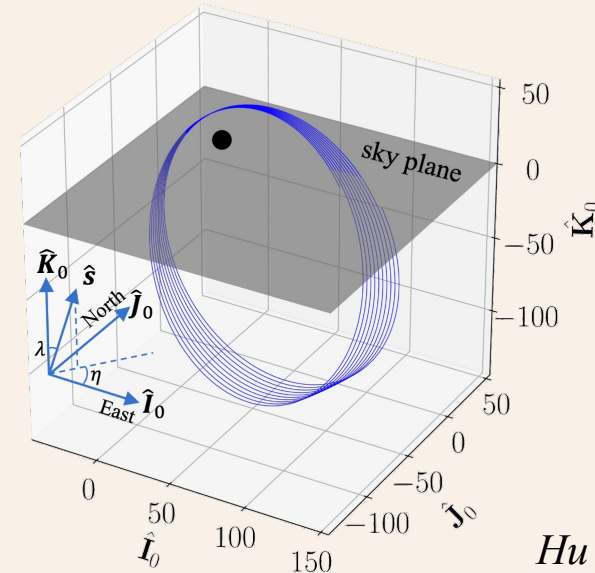
$$\sigma_{\text{TOA}} \approx 1 \text{ ms}$$



Orbital Motion

- $\ddot{\mathbf{r}} = \ddot{\mathbf{r}}_{\text{N}} + \ddot{\mathbf{r}}_{\text{1PN}} + \ddot{\mathbf{r}}_{\text{SO}} + \ddot{\mathbf{r}}_{\text{Q}} + \ddot{\mathbf{r}}_{\text{2PN}} + \dots$
- $\ddot{\mathbf{r}}_{\text{N}} = -GM\hat{\mathbf{n}}/r^2$
- $\ddot{\mathbf{r}}_{\text{1PN}} = -\frac{GM}{c^2 r^2} \left[\left(-\frac{4GM}{r} + v^2 \right) \hat{\mathbf{n}} - 4\dot{r}\mathbf{v} \right]$
- $\ddot{\mathbf{r}}_{\text{SO}} = \chi \frac{6G^2 M^2}{c^3 r^3} \left[\hat{\mathbf{s}} \cdot (\hat{\mathbf{n}} \times \mathbf{v}) \hat{\mathbf{n}} + \dot{r}(\hat{\mathbf{n}} \times \hat{\mathbf{s}}) - \frac{2}{3}(\mathbf{v} \times \hat{\mathbf{s}}) \right]$
- $\ddot{\mathbf{r}}_{\text{Q}} = -q \frac{3G^3 M^3}{2c^4 r^4} \{ [5(\hat{\mathbf{n}} \cdot \hat{\mathbf{s}})^2 - 1] \hat{\mathbf{n}} - 2(\hat{\mathbf{n}} \cdot \hat{\mathbf{s}}) \hat{\mathbf{s}} \}$
- $\ddot{\mathbf{r}}_{\text{2PN}} = \frac{G^2 M^2}{c^4 r^3} \left[\left(2\dot{r}^2 - \frac{9GM}{r} \right) \hat{\mathbf{n}} - 2\dot{r}\mathbf{v} \right]$

Barker & O'Connell, 1975; Damour & Schaefer, 1988

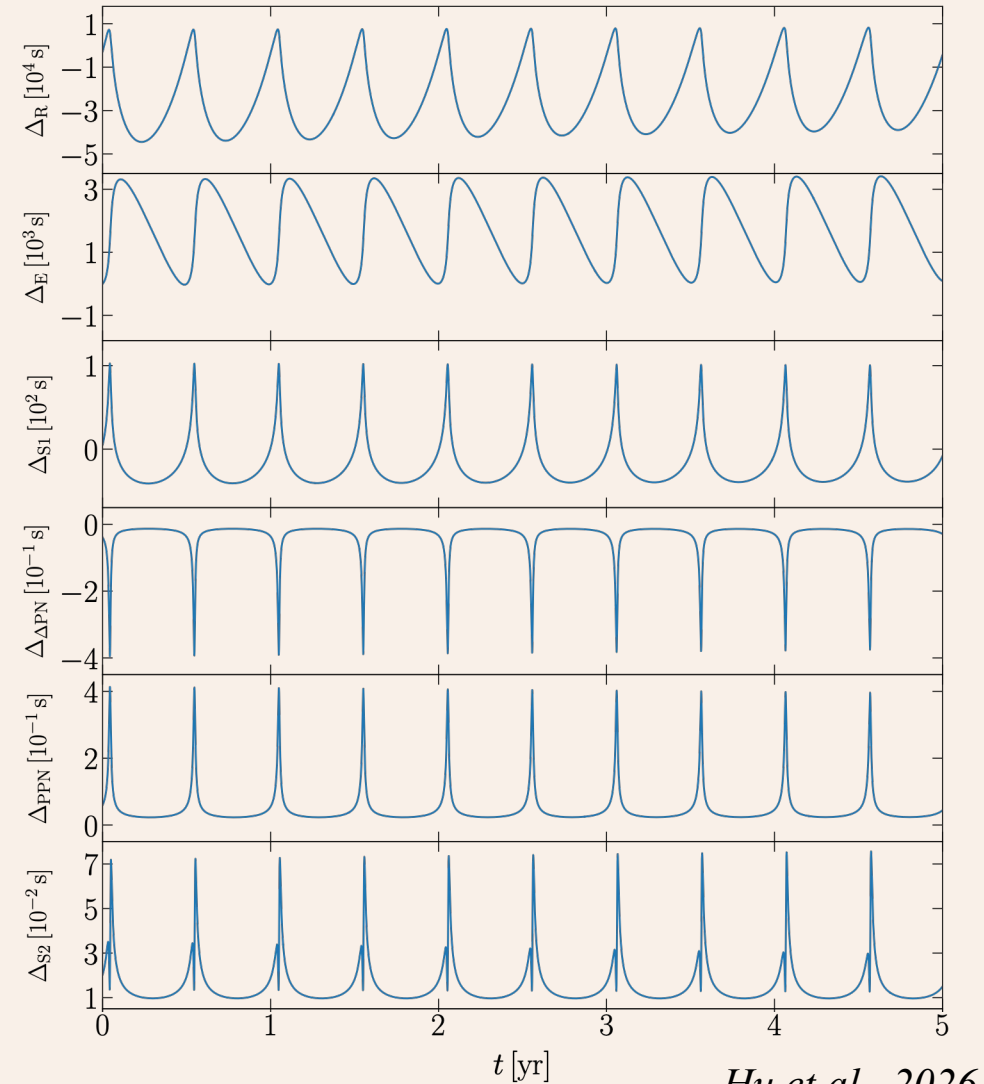


Hu et al., 2026

Effects	Time Delay [s]	Time Delay (fiducial) [s]	Included
Newtonian	$8 \times 10^4 P_{b1}^{2/3} \sin i$	3×10^4	Yes
1PN	$4 \times 10^2 P_{b1}^{-1} T_{\text{obs}1} (1 - e^2)^{-1} \sin i$	7×10^3	Yes
Spin-orbit coupling	$4 \chi P_{b1}^{-4/3} T_{\text{obs}1} (1 - e^2)^{-3/2} \sin i$	80	Yes
2PN	$0.7 P_{b1}^{-5/3} T_{\text{obs}1} (1 - e^2)^{-2} \sin i$	50	Yes
Quadrupole	$0.03 q P_{b1}^{-5/3} T_{\text{obs}1} (1 - e^2)^{-2} \sin i$	1	Yes
Mass ratio	$3 \times 10^{-2} P_{b1}^{2/3} \sin i$	1×10^{-2}	No
Radiation reaction	$6 \times 10^{-9} P_{b1}^{-3} T_{\text{obs}1}^2 (1 - e^2)^{-7/2} f(e) \sin i$	1×10^{-4}	No
Higher PN	$2 \times 10^{-4} P_{b1}^{-7/3} T_{\text{obs}1} \sin i$	3×10^{-3}	No
SSC	$2 \times 10^{-7} P_{b1}^{-1/3}$	2×10^{-7}	No

Light Propagation

- $\Delta_R = z/c + \text{proper motion } (\mu_\alpha, \mu_\delta)$
Reid & Brunthaler, 2020
- $\Delta_E = t - T$
Blandford & Teukolsky, 1976
- $\Delta_{S1} = -\frac{2GM}{c^3} \ln(r - z)$
Shapiro, 1964
- $\Delta_{S2} = \Delta_{\Delta\text{PN}} + \Delta_{\text{PPN}}$
Klioner & Zschocke, 2010
- $\Delta_{\Delta\text{PN}} = -\frac{4G^2M^2}{c^5r} \left(1 - \frac{z}{r}\right)^{-1}$
- $\Delta_{\text{PPN}} = -\frac{G^2M^2}{4C^5r} \left(\frac{z}{r} - 15 \frac{\arccos(-z/r)}{\sqrt{1-z^2/r^2}}\right)$



Light Propagation

- $\Delta_{\text{FD}} = -\chi \frac{2G^2 M^2}{c^5} \frac{\hat{s} \cdot (\hat{K}_0 \times \hat{n})}{r-z}$

Wex & Kopeikin, 1999

- $\Delta_{\text{A1}} = -\frac{1}{2\pi\nu} \frac{\vec{v} \cdot (\hat{K}_0 \times \hat{e}_3)}{c |\hat{K}_0 \times \hat{e}_3|^2}$

Damour & Deruelle, 1986

- $\Delta_{\text{A2}} = \Delta_{\text{A1}} \left(\frac{\vec{v} \cdot \hat{K}_0}{2c} + \frac{\vec{v} \cdot [\hat{K}_0 - (\hat{K}_0 \cdot \hat{e}_3) \hat{e}_3]}{c |\hat{K}_0 \times \hat{e}_3|^2} \right)$

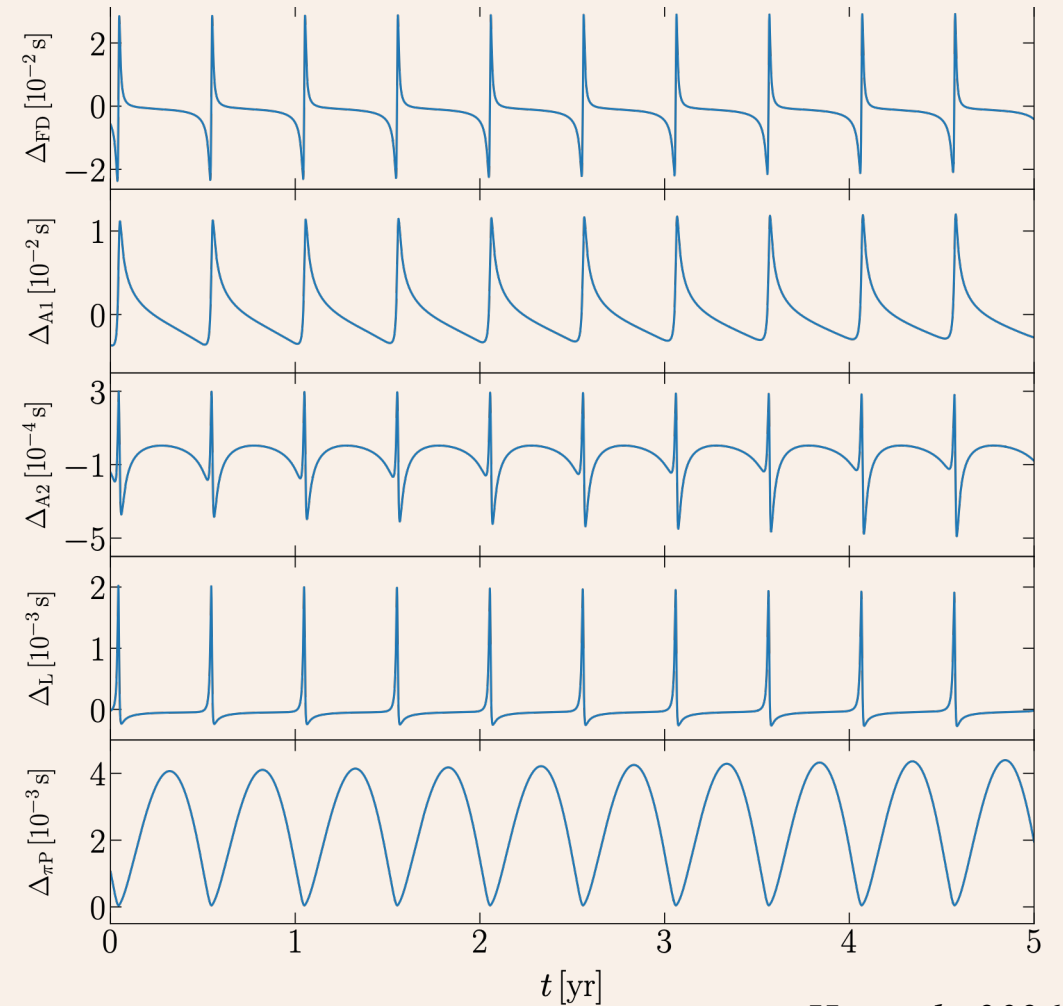
Klioner & Kopeikin, 1992

- $\Delta_{\text{L}} = \frac{1}{2\pi\nu} \frac{2GM}{c^2} \frac{\hat{n}}{r-z} \frac{\hat{K}_0 \times \hat{e}_3}{|\hat{K}_0 \times \hat{e}_3|^2}$

Doroshenko & Kopeikin, 1995

- $\Delta_{\pi P} = \frac{1}{2cd} (\hat{K}_0 \times \vec{r})^2$

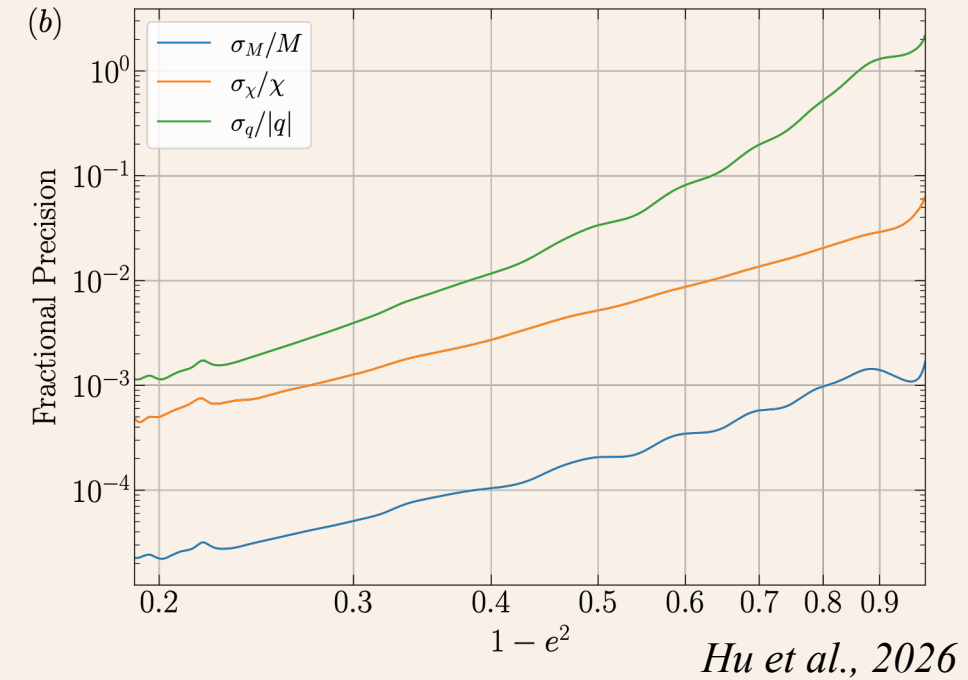
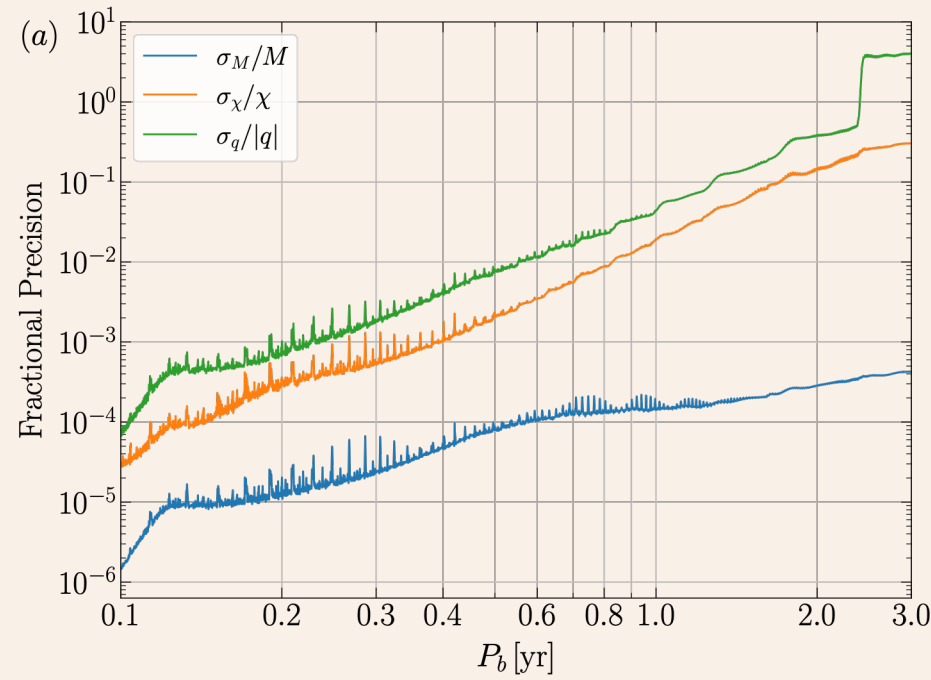
Kopeikin, 1995



Hu et al., 2026

Forecast the measurement precision

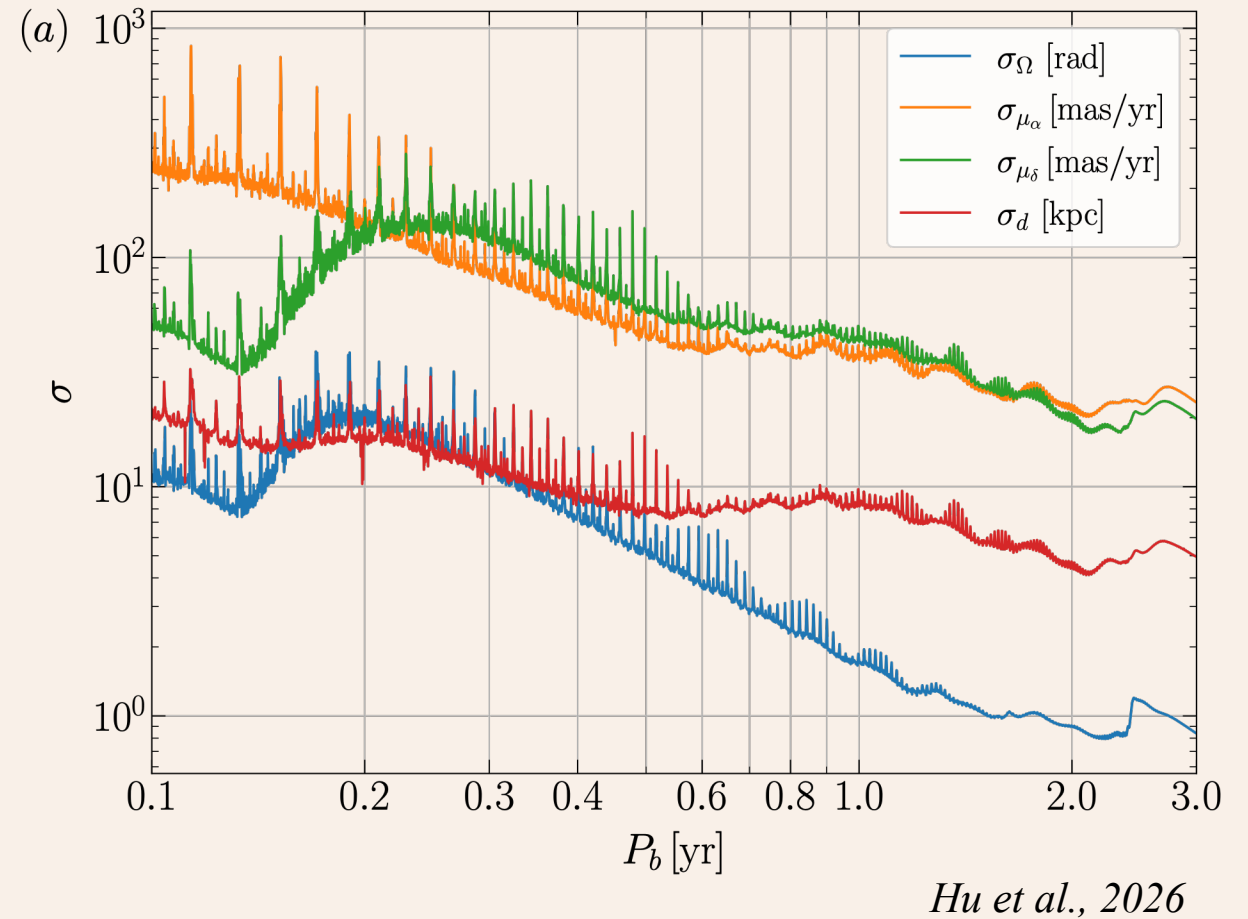
- $T_{\text{tot}} = 5 \text{ yr}$
- $\Delta t = 1 \text{ week}$
- $N_{\text{TOA}} = 260$
- $\sigma_{\text{TOA}} = 1 \text{ ms}$



- Precise measurement of the **spin** and **quadrupole** of the SMBH
- **Direct** test of the no-hair theorem: $q = -\chi^2$

Proper Motion & Finite Distance

- The **timing-only** measurement precision of the proper motion and distance is **much lower** than the current measurement
Reid & Brunthaler, 2020
- Break the rotation symmetry around the line of sight → measurement of the longitude of the ascending node

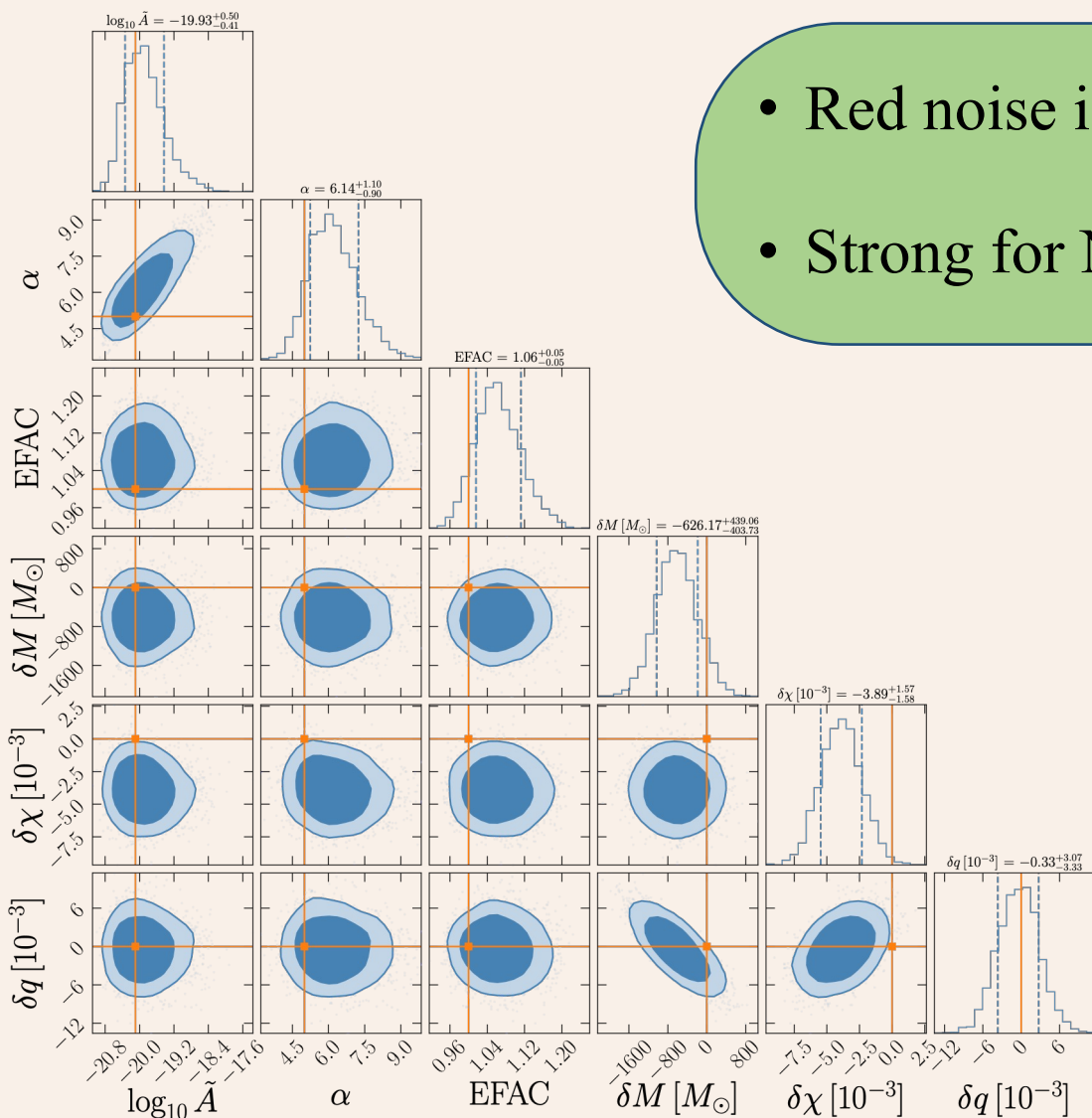


Red Noise Effects

- Red noise is **important** in the pulsar-SMBH system
- Strong for NP & GC environment + similar time scale

- $P_r(f) = \tilde{A} T_{\text{obs}}^3 (f T_{\text{obs}})^{-\alpha}$
- For strong red noise, **improper treatment** can lead to **significant biases** for parameter estimation, while **proper treatment** only **mildly increases** the parameter uncertainties

Hu et al., 2026



Summary

- We construct a realistic pulsar-SMBH timing model
- We first discuss the effects of the proper motion of Sgr A*
- We study the effects of red noise in the pulsar-SMBH system: strong red noise needs to be treated carefully

Thank you!