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# Could low braking-index pulsar J1734-3333 evolve into a magnetar?

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30 Jun. 2017.

# Contents

- Introduction
- Real age of PSR J1734-3333
- Spin-down evolution
- Magnetic field evolution
- Conclusions

# Abstract

In this work, we present a possible interpretation for very small braking index of PSR J1734-3333, which challenges the current theories of braking mechanisms in pulsars, and estimate some initial parameters. According to our suggestions, this pulsar could be born with a superhigh internal magnetic field ~ 10^14 -10^16 G, and could undergo a supercritical accretion soon after its formation in a supernova.

This strong magnetic field has been buried under the surface, and is relaxing out of the surface at present due to Ohmic diffusion. The increasing of surface dipole magnetic field results in the small braking index of 0.9. Keep the current field-growth index, the surface dipole field would reach a magnitude of 10^14 G within t ~50 kyrs, and would reach the maximum of the internal magnetic field strength in a few hundred kyrs , which implies that this pulsar is a potential magnetar.

# Introduction

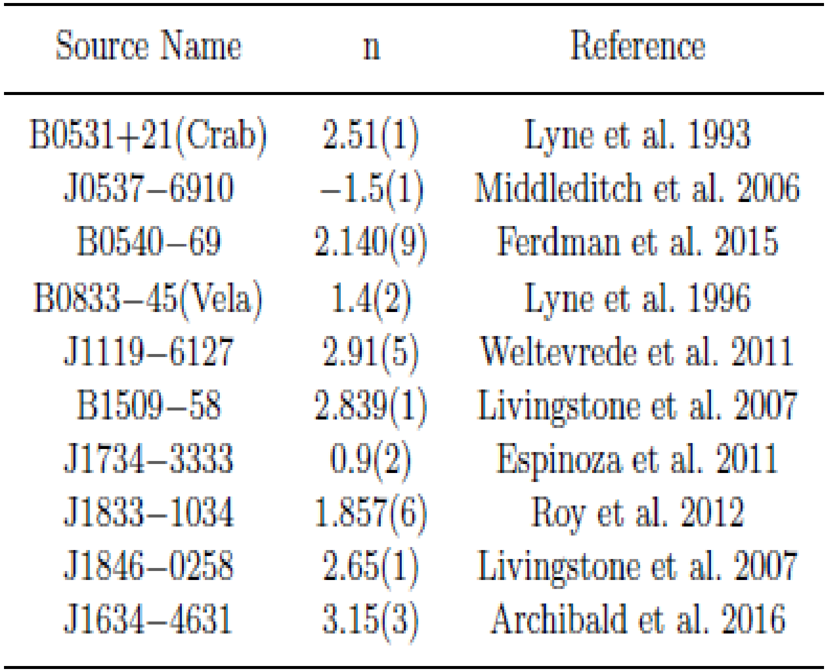
- The secular decrease in the angular velocity of a pulsar is described by

$$\dot{\Omega} \propto \Omega^n$$

- Baking index n is defined by

$$n = \frac{\Omega\ddot{\Omega}}{\dot{\Omega}^2} = \frac{\nu\ddot{\nu}}{\dot{\nu}^2} = 2 - \frac{P^2\ddot{P}}{\dot{P}^2}$$

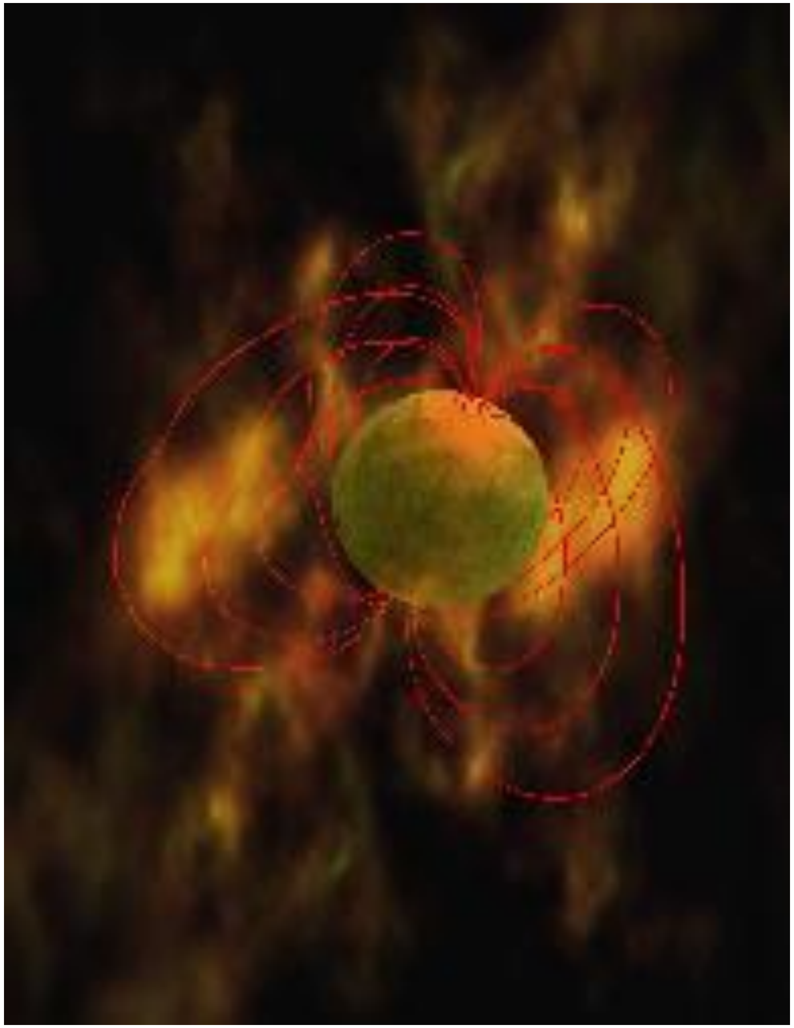
**Published braking indices of pulsars**



| Source Name | n | Reference |
|---|---|---|
| B0531+21(Crab) | 2.51(1) | Lyne et al. 1993 |
| J0537−6910 | −1.5(1) | Middleditch et al. 2006 |
| B0540−69 | 2.140(9) | Ferdman et al. 2015 |
| B0833−45(Vela) | 1.4(2) | Lyne et al. 1996 |
| J1119−6127 | 2.91(5) | Weltevrede et al. 2011 |
| B1509−58 | 2.839(1) | Livingstone et al. 2007 |
| J1734−3333 | 0.9(2) | Espinoza et al. 2011 |
| J1833−1034 | 1.857(6) | Roy et al. 2012 |
| J1846−0258 | 2.65(1) | Livingstone et al. 2007 |
| J1634−4631 | 3.15(3) | Archibald et al. 2016 |

# Introduction

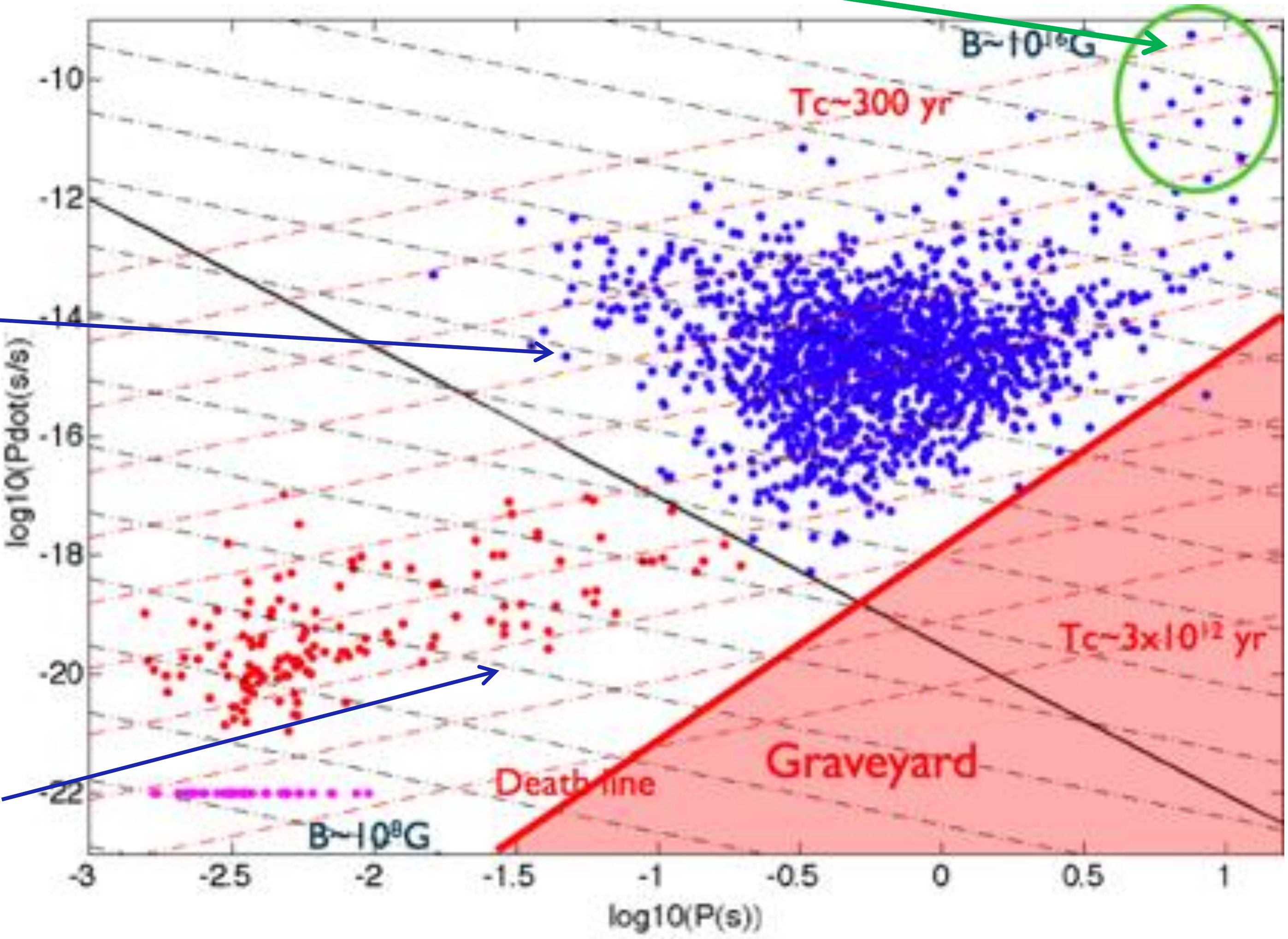
- Magnetars are neutron stars powered by magnetic field energy.
- 28 magnetar candidates.
- Classed as Anormous X-ray pulsars (AXPs) & Soft Gamma-ray repeaters (SGRs)
- Due to strong timing noise and lake of persistent emission, it is hard to measure their braking indices observationally.



# Observational characteristics of magnetars

- **Spin period: 2 – 12 sec**
- **Period derivative: $10^{-14}$ – $10^{-10}$ s/s**
- **Dipolar magnetic field: $10^{13}$ – $10^{15}$ G**
- **Persistent soft X-ray luminosity ($10^{33}$-$10^{35}$ erg/s) higher than their rotational energy loss**
- **X-ray burst /flare**
  **luminosity > $10^{37}$ erg/s**
  **giant bursts Lx > $10^{42}$ erg/s**

# Distribution of pulsars



AXPs & SGRs

Radio pulsars

Millisecond pulsars

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# Constraining the braking indices of magnetars

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## ABSTRACT

Because of the lack of long-term pulsed emission in quiescence and the strong timing noise, it is impossible to directly measure the braking index $n$ of a magnetar. Based on the estimated ages of their potentially associated supernova remnants (SNRs), we estimate the values of the mean braking indices of eight magnetars with SNRs, and find that they cluster in the range of 1–42. Five magnetars have smaller mean braking indices of $1 < n < 3$, and we interpret them within a combination of magneto-dipole radiation and wind-aided braking. The larger mean braking indices of $n > 3$ for the other three magnetars are attributed to the decay of external braking torque, which might be caused by magnetic field decay. We estimate the possible wind luminosities for the magnetars with $1 < n < 3$, and the dipolar magnetic field decay rates for the magnetars with $n > 3$, within the updated magneto-thermal evolution models. Although the constrained range of the magnetars' braking indices is tentative, as a result of the uncertainties in the SNR ages due to distance uncertainties and the unknown conditions of the expanding shells, our method provides an effective way to constrain the magnetars' braking indices if the measurements of the SNR ages are reliable, which can be improved by future observations.

# Magnetar pindown evolution

**Z. F. Gao, et al. MNRAS, 456, 55-65 (2016)**

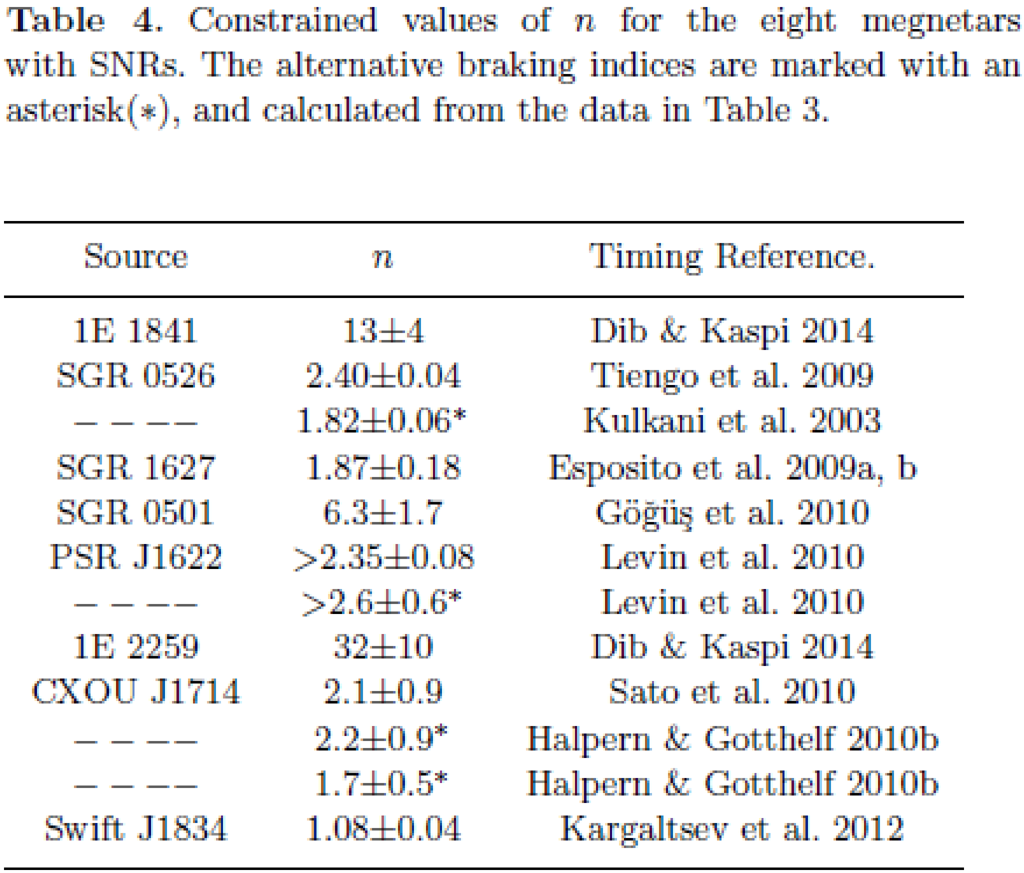
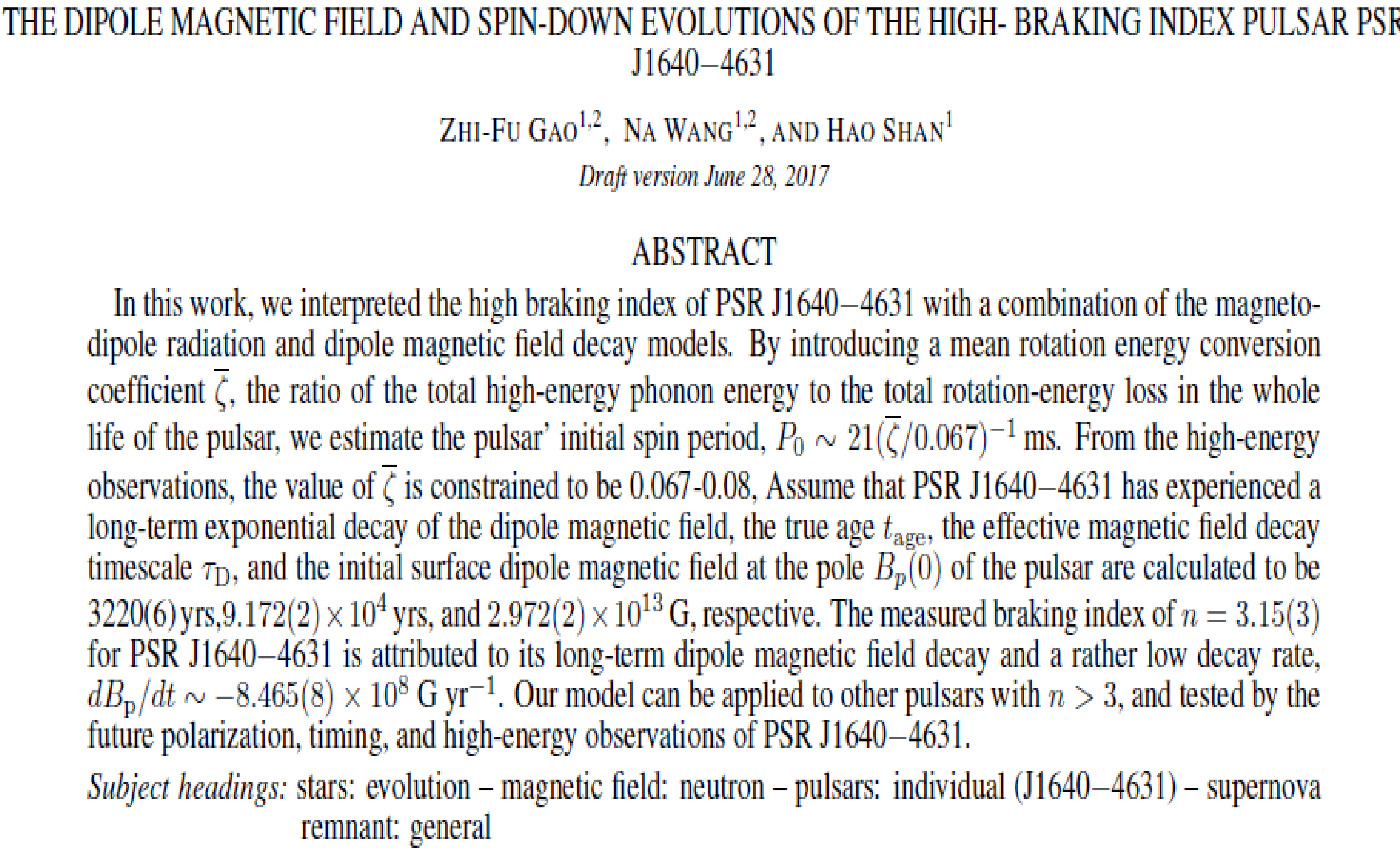


Table 4. Constrained values of $n$ for the eight megnetars with SNRs. The alternative braking indices are marked with an asterisk(∗), and calculated from the data in Table 3.

| Source | $n$ | Timing Reference. |
|---|---|---|
| 1E 1841 | 13±4 | Dib & Kaspi 2014 |
| SGR 0526 | 2.40±0.04 | Tiengo et al. 2009 |
| – – –– | 1.82±0.06* | Kulkani et al. 2003 |
| SGR 1627 | 1.87±0.18 | Esposito et al. 2009a, b |
| SGR 0501 | 6.3±1.7 | Göğüş et al. 2010 |
| PSR J1622 | >2.35±0.08 | Levin et al. 2010 |
| – – –– | >2.6±0.6* | Levin et al. 2010 |
| 1E 2259 | 32±10 | Dib & Kaspi 2014 |
| CXOU J1714 | 2.1±0.9 | Sato et al. 2010 |
| – – –– | 2.2±0.9* | Halpern & Gotthelf 2010b |
| – – –– | 1.7±0.5* | Halpern & Gotthelf 2010b |
| Swift J1834 | 1.08±0.04 | Kargaltsev et al. 2012 |



# THE DIPOLE MAGNETIC FIELD AND SPIN-DOWN EVOLUTIONS OF THE HIGH- BRAKING INDEX PULSAR PSR J1640−4631

ZHI-FU GAO[1,2], NA WANG[1,2], AND HAO SHAN[1]

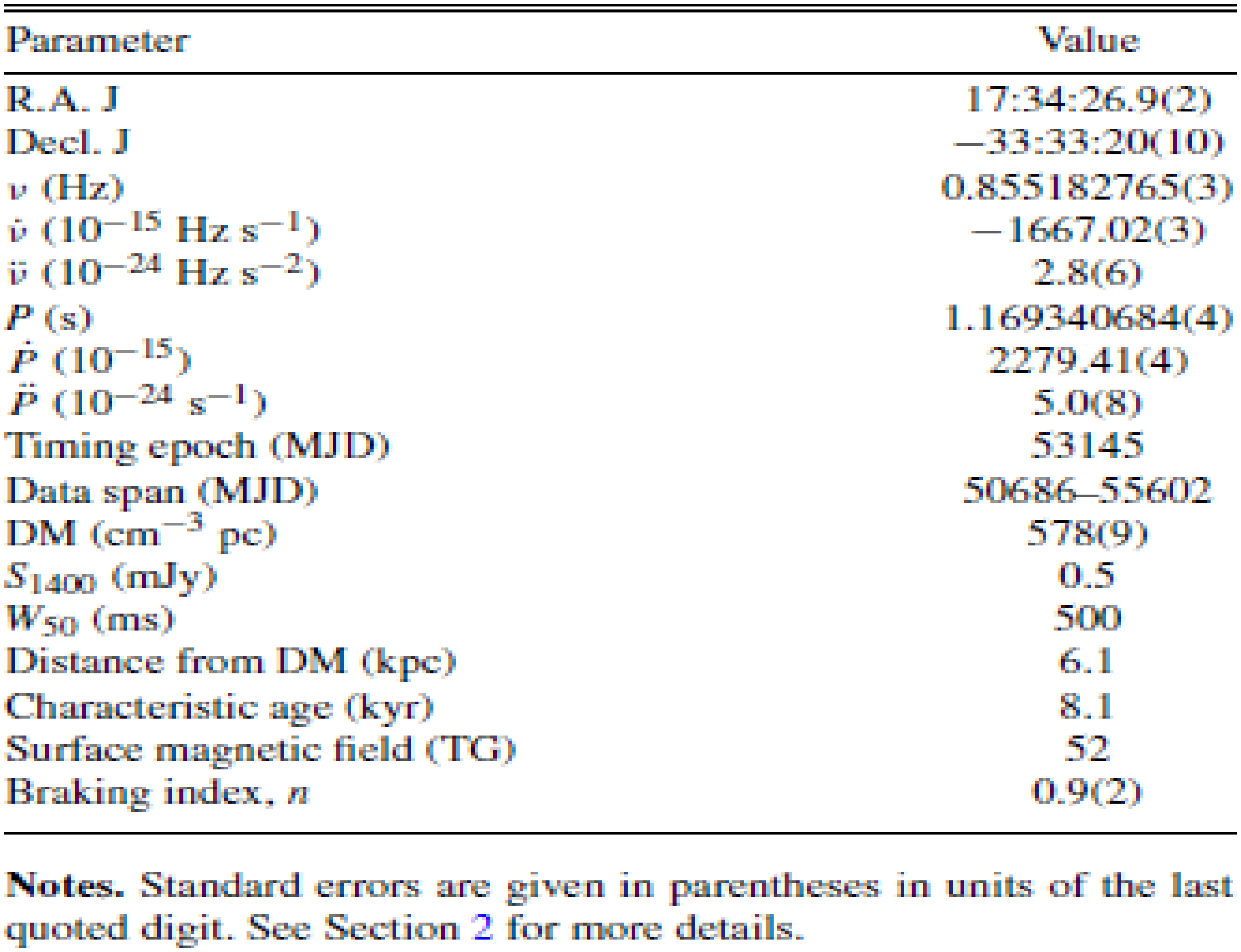
*Draft version June 28, 2017*

## ABSTRACT

In this work, we interpreted the high braking index of PSR J1640−4631 with a combination of the magneto-dipole radiation and dipole magnetic field decay models. By introducing a mean rotation energy conversion coefficient $\overline{\zeta}$, the ratio of the total high-energy phonon energy to the total rotation-energy loss in the whole life of the pulsar, we estimate the pulsar' initial spin period, $P_0 \sim 21(\overline{\zeta}/0.067)^{-1}$ ms. From the high-energy observations, the value of $\overline{\zeta}$ is constrained to be 0.067-0.08, Assume that PSR J1640−4631 has experienced a long-term exponential decay of the dipole magnetic field, the true age $t_{\rm age}$, the effective magnetic field decay timescale $\tau_{\rm D}$, and the initial surface dipole magnetic field at the pole $B_p(0)$ of the pulsar are calculated to be 3220(6) yrs,9.172(2) $\times 10^4$ yrs, and 2.972(2) $\times 10^{13}$ G, respective. The measured braking index of $n = 3.15(3)$ for PSR J1640−4631 is attributed to its long-term dipole magnetic field decay and a rather low decay rate, $dB_{\rm p}/dt \sim -8.465(8) \times 10^8$ G yr$^{-1}$. Our model can be applied to other pulsars with $n > 3$, and tested by the future polarization, timing, and high-energy observations of PSR J1640−4631.

*Subject headings:* stars: evolution – magnetic field: neutron – pulsars: individual (J1640−4631) – supernova remnant: general

# Basic information for PSRJ1734-3333



| Parameter | Value |
|---|---|
| R.A. J | 17:34:26.9(2) |
| Decl. J | −33:33:20(10) |
| $\nu$ (Hz) | 0.855182765(3) |
| $\dot{\nu}$ ($10^{-15}$ Hz s$^{-1}$) | −1667.02(3) |
| $\ddot{\nu}$ ($10^{-24}$ Hz s$^{-2}$) | 2.8(6) |
| $P$ (s) | 1.169340684(4) |
| $\dot{P}$ ($10^{-15}$) | 2279.41(4) |
| $\ddot{P}$ ($10^{-24}$ s$^{-1}$) | 5.0(8) |
| Timing epoch (MJD) | 53145 |
| Data span (MJD) | 50686–55602 |
| DM (cm$^{-3}$ pc) | 578(9) |
| $S_{1400}$ (mJy) | 0.5 |
| $W_{50}$ (ms) | 500 |
| Distance from DM (kpc) | 6.1 |
| Characteristic age (kyr) | 8.1 |
| Surface magnetic field (TG) | 52 |
| Braking index, $n$ | 0.9(2) |

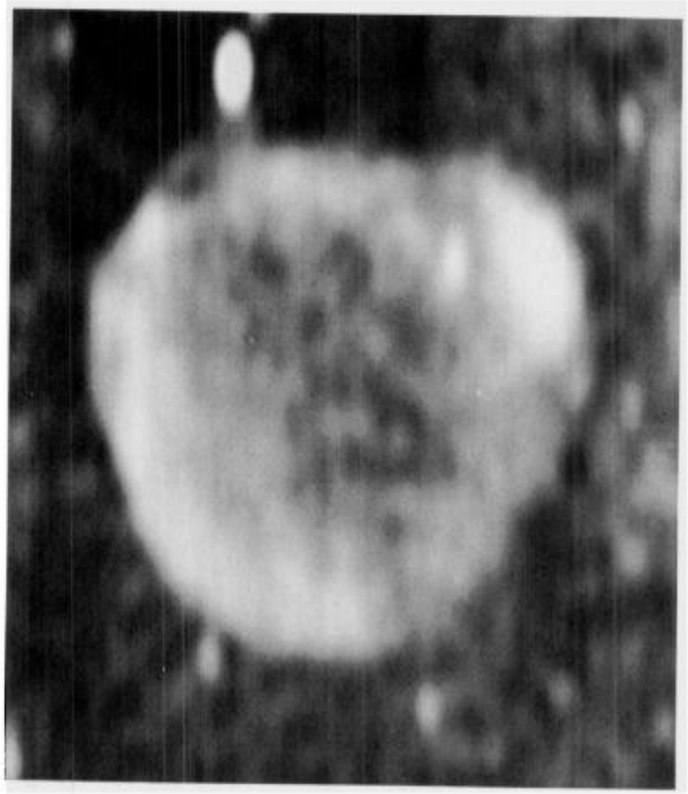
**Notes.** Standard errors are given in parentheses in units of the last quoted digit. See Section 2 for more details.

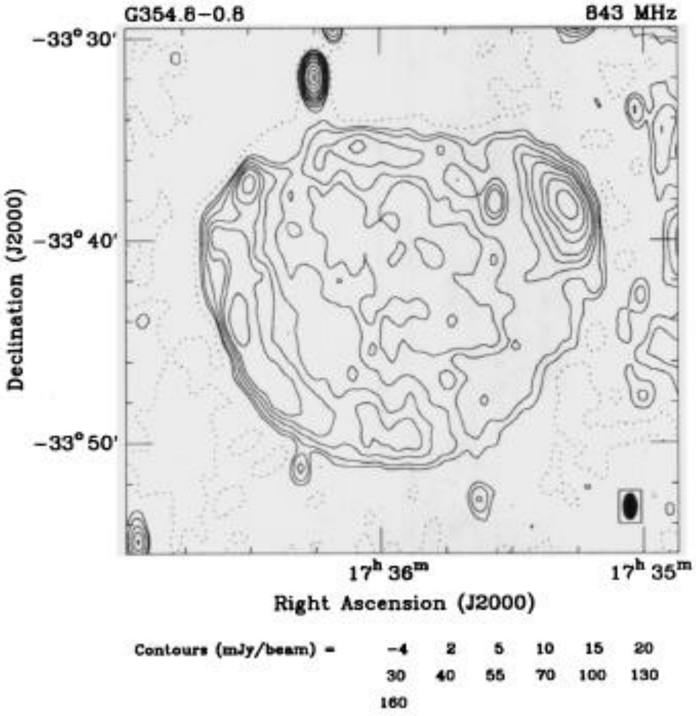
$$n = 0.9 \pm 0.2$$

# Why n < 3 ?

- Neutrino and photon radiation (Peng et al. 1982);
- Combination of dipole radiation and the propeller torque applied by debris-disk (e.g., Alpar & Baykal 2006);
- Frequent glitches, as well as magnetosphere currents (e.g., Chen 2009).
- Wind braking (Tong et al. 2013)
- Magnetic field increases (e.g.,Muslimov & Page 1996).

# PSR J1734-3333 is associated with SNR G354.8-0.8





**(From White \& Green , Astro.Astrophys. Supple. Ser.118. 329 .1996)**

# Estimating diameter of G354.8-0.8

- Pavlovic et al (2014) present new empirical radio surface-brightness-to diameter ($\sum - D$) relations for supernova remnants (SNRs) in our Galaxy.

- They select calibrators from Greens SNR catalog (Green 2009) based on literature of `A Catalogue of Galactic supernova remnants (2009 March version).

For G354.8-0.8 Diameter = 34.8 pc, Distance =6.3 kpc from flux-density 2.8 Jy.

# Estimating real age of G354.8-0.8

- ✓ **The evolution of a SNR : free-expansion, Sedov-Taylor (ST) and pressure -driven snowplow (PDS) .**

- ✓ **According the model of Cioffi ( D.F. Cioffi , et al ApJ ,334, 252, 1988) , we derive an expression**

$$t \approx 1.0\times10^{4}\cdot[(\frac{R_{PDS}}{14\mathrm{pc}})^{10/3} E_{51}^{-20/21} n_{0}^{10/7} \xi_{m}^{-10/21} + \frac{1}{3}] \xi_{m}^{-5/14} E_{51}^{3/14} n_{0}^{-4/7} \ \mathrm{yrs} \qquad (3)$$

$$R_{SNR} \sim 17.4 \ \mathrm{pc}, \qquad t_{SNR} \sim 18 \ \mathrm{kyrs}$$

1) Low latitude , 2) Core-collapse supernova

$\xi_m$ --Metallicity factor for solar abundance,s

$n_0$ Ambient hydrogen density

$$n_0 \sim 1 \ cm^{-3}, E \sim 1\times10^{51} \ \mathrm{erg/s}, \ \xi_m \sim 1;$$

# Spin-down evolution

If the dipole braking still dominates, and the magnetic field evolution can't be ignored, Blandford \& Romani (1988) re-formulate the braking law of a pulsar as

$$\dot{\nu} = -\frac{8\pi^2 R^6 sin^2\theta}{3c^3} B_{surf}^2(t)\nu^3 \quad (4)$$

Integraling Eq.(4) gives

$$\nu^{-2} = \nu_0^{-2} + 2\int_0^t \frac{8\pi^2 R^6 sin^2\theta}{3Ic^3} B_{surf}^2(t)dt'$$

$$= \nu_0^{-2} + 2\int_0^t \frac{B_{surf}^2(t)}{(3.2\times10^{19})^2}dt', \qquad (5)$$

where we assume $\mathrm{R} \sim 10^6\,\mathrm{cm},\ sin^2\theta \sim 1,\ \text{and } \mathrm{I} \sim 10^{45}\,\mathrm{g}\cdot\mathrm{cm}^2$

From Eq. (5) we get: $$P = P_0[1 + 2P_0^{-2}\int_0^t \frac{B_{surf}^2(t)}{(3.2\times10^{19})^2}dt']^{1/2} \qquad (6)$$

Then we can represent the spin-down age of the star in the form:

$$\tau_c = \frac{-\nu}{2\dot{\nu}} = \frac{P}{2\dot{P}} = \frac{K}{B_{\mathrm{surf}}^2(t)}\int_0^t B_{\mathrm{surf}}^2(t)dt', \qquad (7)$$ where $\mathrm{K} = [1-(P_0/P)^2]^{-1}$

# Spin-down evolution

Thus, assuming that in the saturation regime for PST J1734 3333,

$$B_{surf}(t) \propto t^{\varepsilon},$$

we obtain

$$\tau_c \sim \frac{K}{2\varepsilon+1} t \qquad (8)$$

·Combing with

$$n = 3 - 4\left(\frac{\dot{B}_{\rm surf}}{B_{\rm surf}}\right)\left(\frac{P}{2\dot{P}}\right) = 3 - 4\left(\frac{\dot{B}_{\rm surf}}{B_{\rm surf}}\right)\tau_c \quad ,$$

we get

$$n \sim 3 - \frac{4\varepsilon K}{2\varepsilon+1} \qquad (9),$$

# Spin-down evolution

From Eq.(8) and Eq,(9) , we find that $K \sim \frac{3-n}{2} + \frac{\tau_c}{t}$ (10),

and $\varepsilon \sim \frac{3-n}{2(\mathrm{n}-3+2\mathrm{K})}$ (11)

Inserting n=0.9(2) , $\tau_c = 8.13$ kys, and $B_{surf} = 5.22 \times 10^{13}$ G,

into Eqs.(8-10), we obtain

$$\mathrm{K} = 1.39 \pm 0.10, \ \varepsilon = 1.53 \pm 0.17$$

for a wide range of $\xi_m \sim 0.5 - 2$ and $n_0 \sim 0.5 - 2 \ cm^{-3}$

# Initial parameters

From $\mathrm{K} = [1-(P_0/P)^2]^{-1} \Rightarrow P_0 = P(1-\frac{1}{K})^{1/2}$

we get the initial spin period $\mathrm{P}_0 \approx 0.619 \pm 0.051\ \mathrm{s}$,

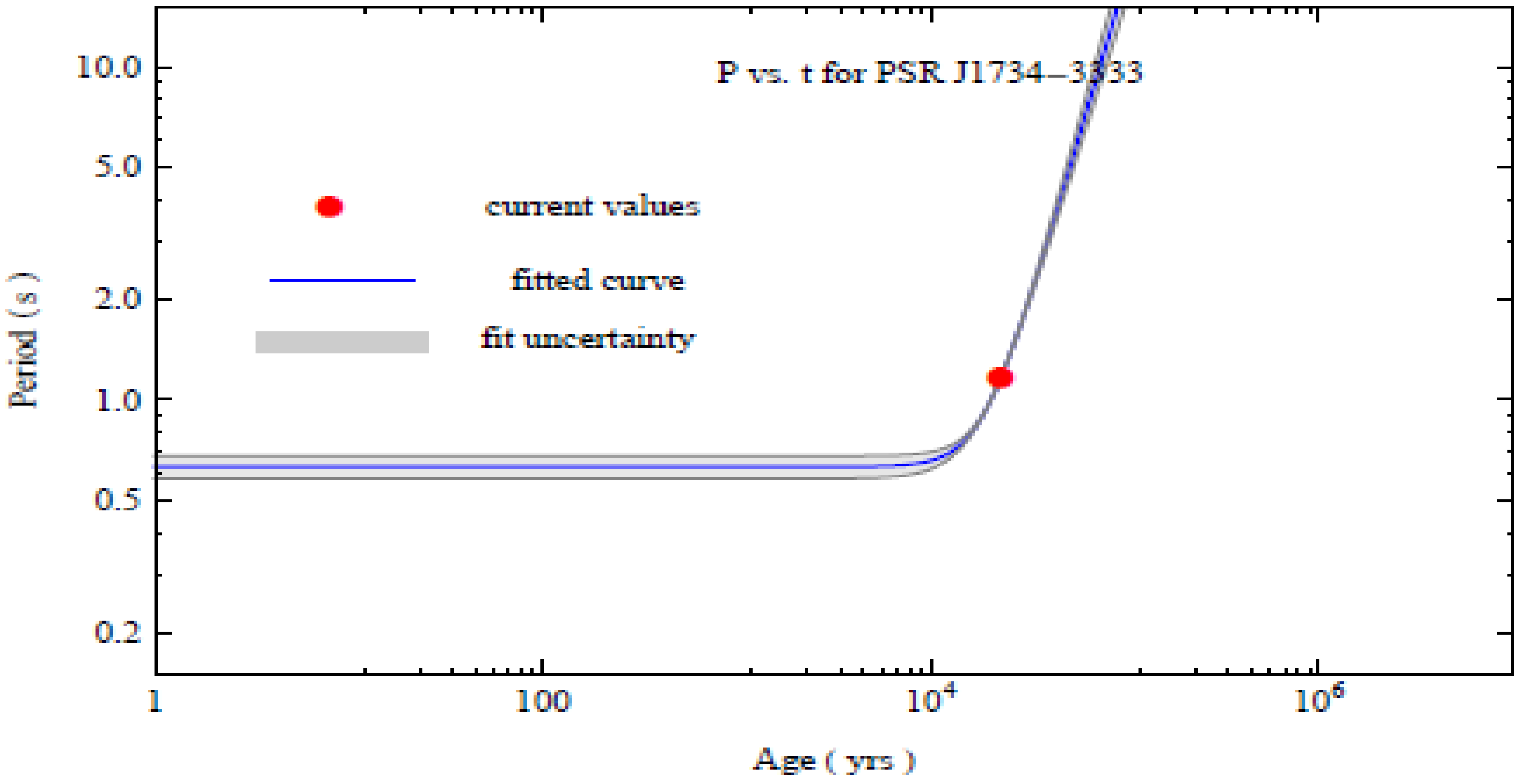
From $$B_{surf}(t) = B_{surf}(0) \times (\frac{t}{1\mathrm{yr}})^{\varepsilon},$$

we get the initial surface magnetic field ,

$$B_{surf}(0) = (1.75 \pm 0.28) \times 10^8\,\mathrm{G},$$

In Ho et al. 2015, they estimated

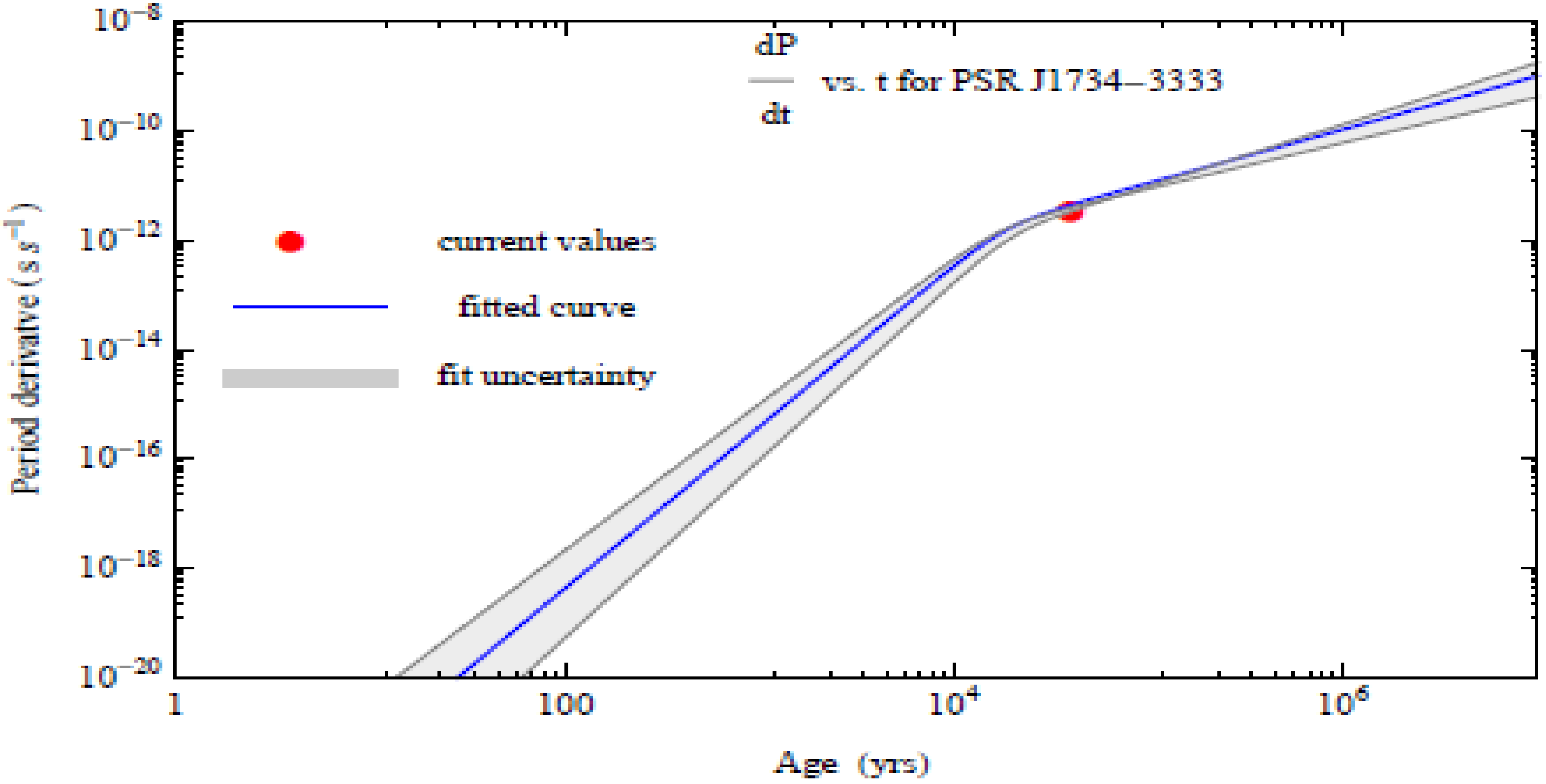
$$B_{surf}(0) = 10^{12}\,\mathrm{G}, \quad t_{SNR} \sim 1-2\ \mathrm{kys}, \quad P_0 \sim 100\ \mathrm{ms}, \mathrm{v}_{kick} \sim 10^4\ \mathrm{km/s}$$



(a)

$$P = P_0[1+2P_0^{-2}\int_0^t \frac{B_{surf}^2(t)}{(3.2\times10^{19})^2}dt^{'}]^{1/2} \approx P_0[1+\frac{2\cdot B_{surf}^2(0)\cdot(\frac{t}{\text{yr}})^{2\varepsilon}}{P_0^2\cdot(3.2\times10^{19})^2(2\varepsilon+1)}\cdot(\text{t}\times3.154\times10^7\ \text{s})]^{1/2}$$

If the evolution time t~50 kyrs, $P \sim 5.3$ s  t $\geq$100 kyrs, $P \geq 16.6$ s



(b)

$$\dot{\mathrm{P}}(\mathrm{t}) = \frac{B_{surf}^2(t)}{(3.2\times10^{19})^2 P(t)} \approx \frac{B_{surf}^2(0)\cdot(\frac{t}{\mathrm{yr}})^{2\varepsilon}}{P_0\cdot(3.2\times10^{19})^2}\cdot[1+\frac{2\cdot B_{surf}^2(0)\cdot(\frac{t}{\mathrm{yr}})^{2\varepsilon}}{P_0^2\cdot(3.2\times10^{19})^2(2\varepsilon+1)}\cdot(\mathrm{t}\times3.154\times10^7\ \mathrm{s})]^{-1/2}$$

**For convenience, we denote**

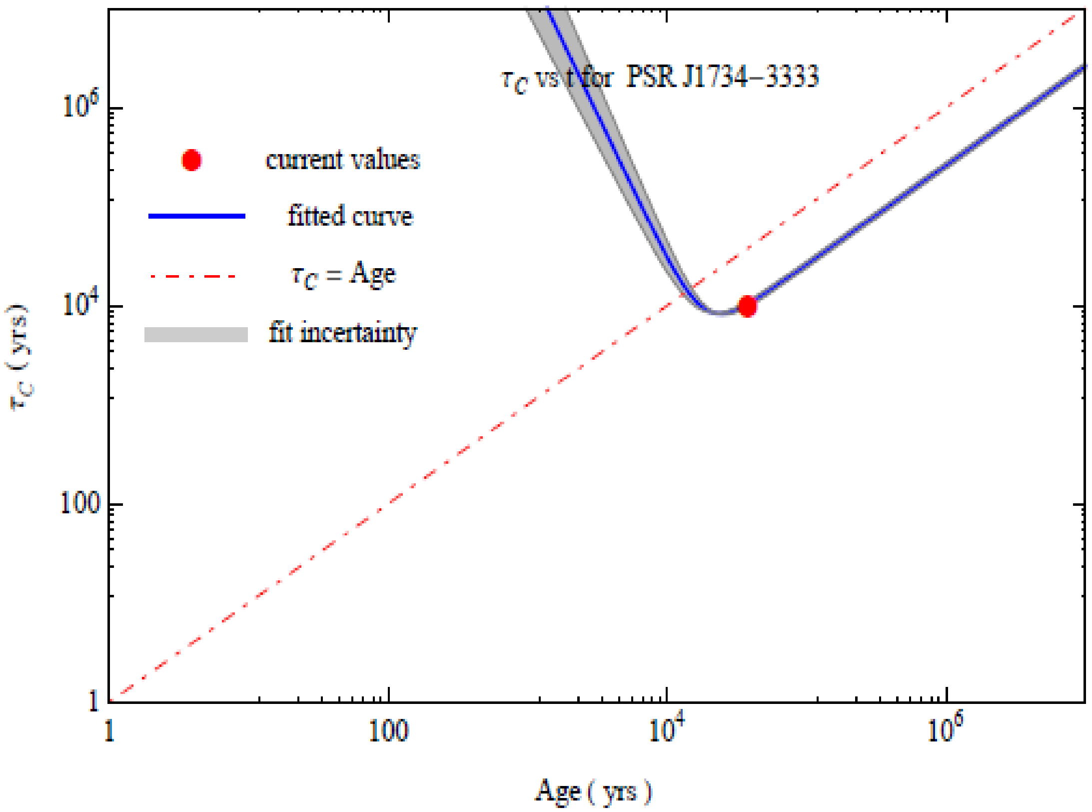
$$f(t) = [1+\frac{2\cdot B_{surf}^2(0)\cdot(\frac{t}{\mathrm{yr}})^{2\varepsilon}}{P_0^2\cdot(3.2\times10^{19})^2(2\varepsilon+1)}\cdot(\mathrm{t}\times3.154\times10^7\ \mathrm{s})]$$

# Spin-down evolution

In the early stage of field evolution, the pulsar appears older, $\tau_c >> t$, in the late evolution stage, the pulsar appears younger, $\tau_c < t$.

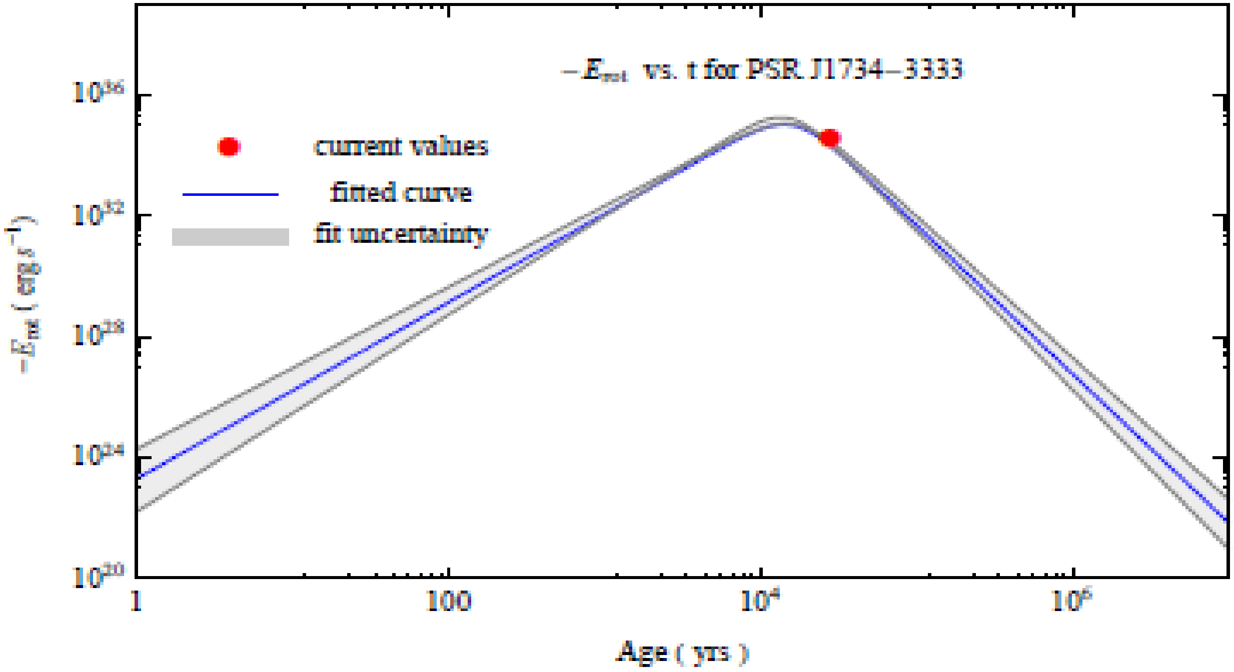
$$\tau_c = \frac{P}{2\dot{P}} = \frac{P_0^2 \cdot (3.2\times10^{19})^2}{2B_0^2 \cdot (\frac{t}{\mathrm{yr}})^{2\varepsilon}} \cdot f(t) \quad \mathrm{s}$$

$$= \frac{P_0^2 \cdot (3.2\times10^{19})^2}{2B_0^2 \cdot (\frac{t}{\mathrm{yr}})^{2\varepsilon} \cdot 3.154\times10^7} \cdot f(t) \quad \mathrm{yrs}$$



# Rotation energy loss rate

$$\dot{\mathrm{E}}_{rot} \approx -4\pi^2 I\dot{P}P^{-3} \approx \frac{-4\pi^2 I B_{surf}^2(0)\cdot(\frac{t}{\mathrm{yr}})^{2\varepsilon}}{P_0^4\cdot(3.2\times10^{19})^2}\cdot f^2(t) \qquad \mathrm{I}\sim10^{45}\ \mathrm{g}\cdot\mathrm{cm}^2.$$

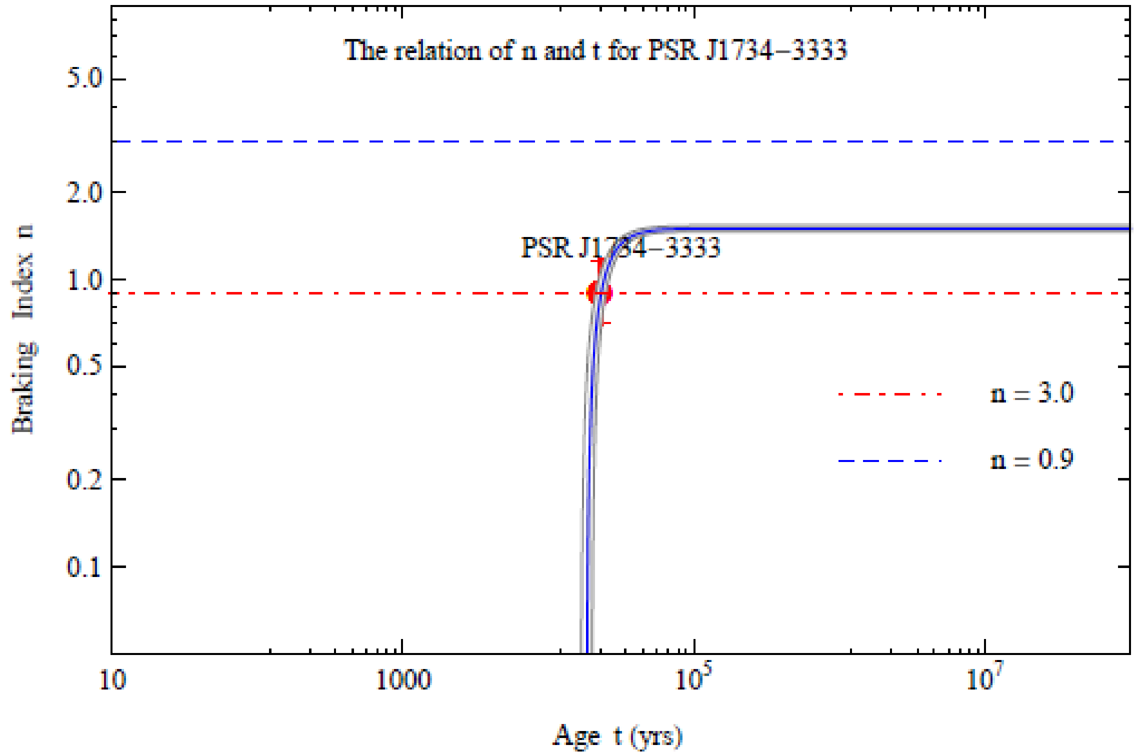


Initial value of $\dot{\mathrm{E}}_{rot}(0)\sim2.58\times10^{23}$ erg·s$^{-1}$; Current value of $\dot{\mathrm{E}}_{rot}\sim5.63\times10^{34}$ erg·s$^{-1}$

# Braking index evolution

- The increase of the dipole magnetic field causes a low braking index n < 3

- Braking index n increases (faster, then slower ), and finally approaches a limited value n~1.625



$$n = 3 - \frac{4\varepsilon\tau_c}{t} \approx 3 - \frac{4\varepsilon}{t} \times \frac{P_0^2 \cdot (3.2\times10^{19})^2}{2B_0^2 \cdot (\frac{t}{\mathrm{yr}})^{2\varepsilon} \cdot (3.154\times10^7)} \cdot f(t)$$

# Neutron star re-magnetization

- If the accreting matter is weakly, or non, magnetized, this implies that NSs produced by supernovae are born with weak, or even vanishing, surface magnetic field.
- In addition, this accreting matter has suffered a turbulent episode during which the plasma behaved as a diamagnet and its field could have been severely reduced, which would means that the final surface field of the NS could be weak.
- Later diffusion of the field back to surface could produce a delayed switch-on of a pulsar.

**If the field is buried in the inner crust** $(\rho \sim 4.3\times10^{11} - 1.4\times10^{14} \quad \mathrm{g\cdot cm^{-3}})$

$$\sigma_{\mathrm{ph}} \sim 7\times10^{21}\rho_9^{2/3}T_8^{-1} \quad \mathrm{s}^{-1}$$

$$T_8 \sim 1-5, \quad \rho_9 \sim 10^3 - 10^5,$$

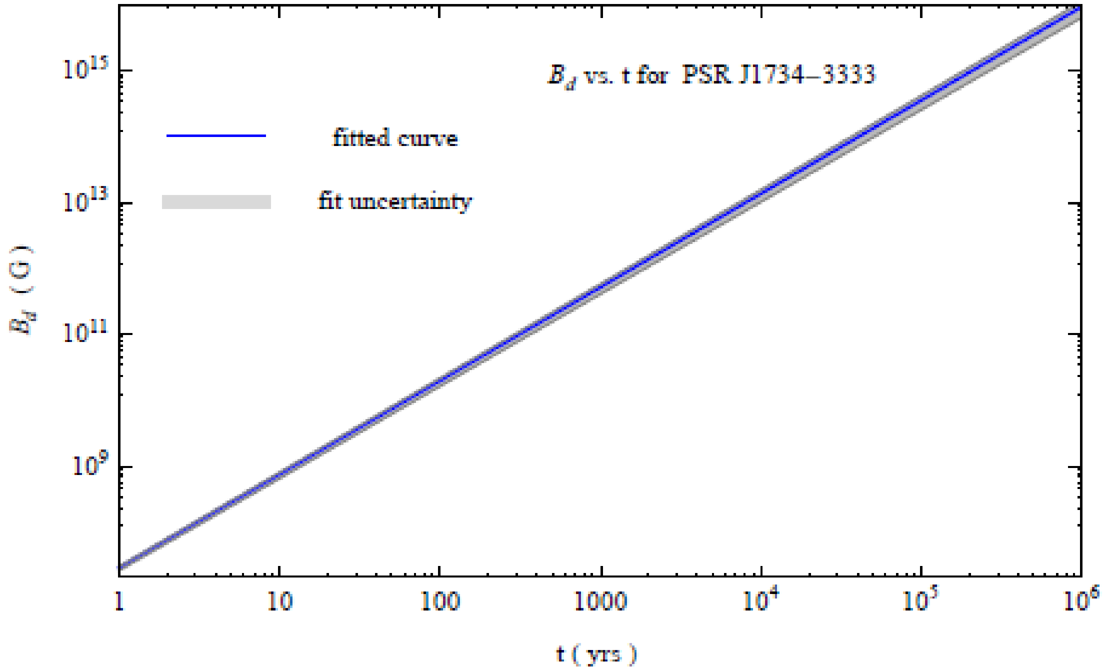
$$t_{\mathrm{d}} = \frac{4\pi\sigma H^2}{c^2}$$

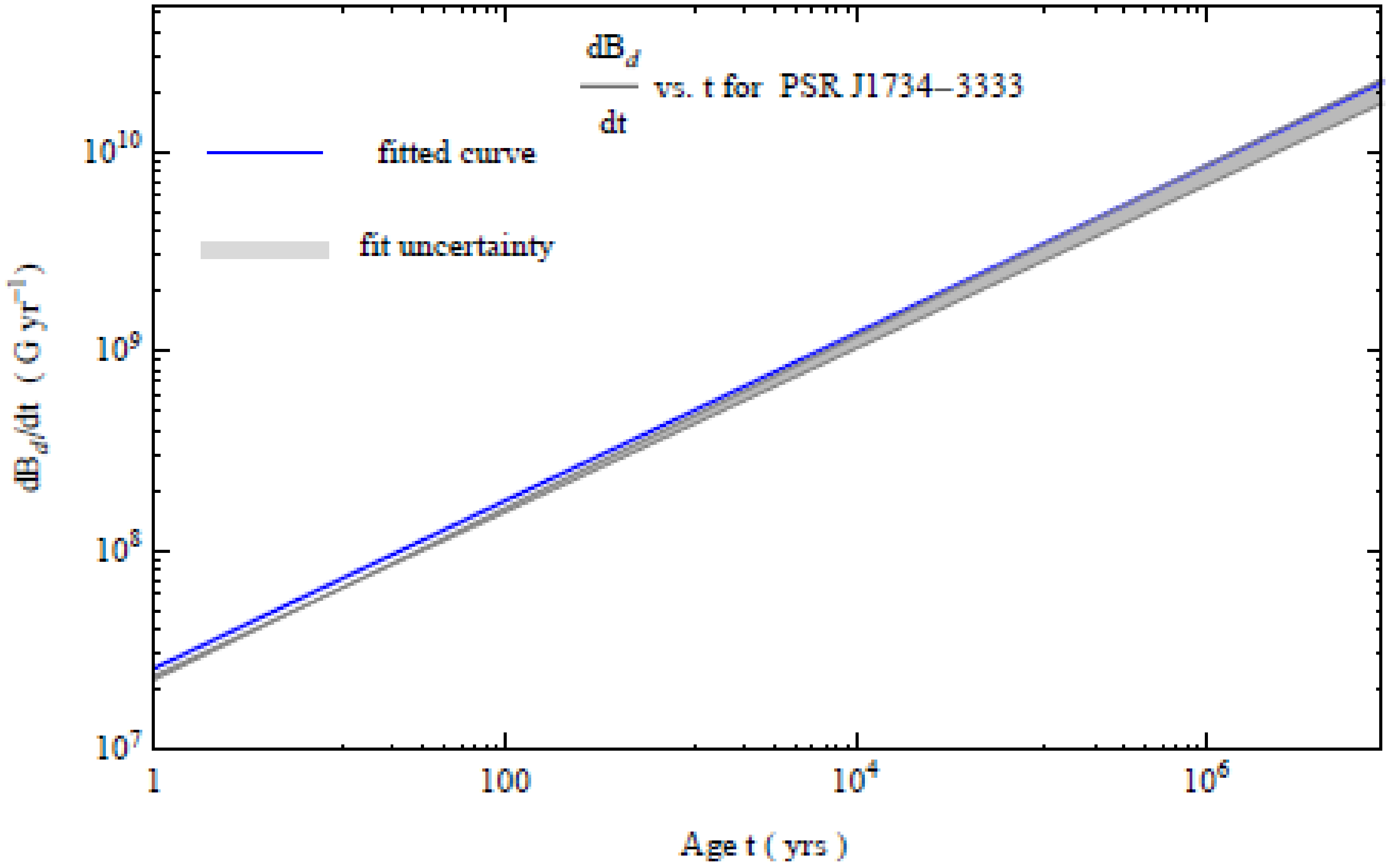
$$\sigma \approx \sigma_{\mathrm{ph}} \sim 10^{24} - 10^{25} \quad \mathrm{s}^{-1}$$

$$H \sim 10^2 - 10^3 \quad \mathrm{m}, \quad t_d \sim 10^2 - 10^3 \quad \mathrm{kyrs}$$

$$B_{surf}(t) = B_{surf}(0)\cdot\left(\frac{t}{\mathrm{yr}}\right)^{\varepsilon}$$

**If the surface dipole magnetic field of PSR J1734-3333 increase with current power -index of 1.50, this pulsar will become a magnetar with** $B_{surf} \sim 2\times10^{14}$ G, **when the evolution time** $t \sim 50$ kyrs.




$\frac{dB_d}{dt}$ vs. t for PSR J1734−3333
fitted curve
fit uncertainty
$10^{10}$
$10^{9}$
$10^{8}$
$10^{7}$
1
100
$10^{4}$
$10^{6}$
Age t ( yrs )
$dB_d/dt$ ( G $yr^{-1}$ )

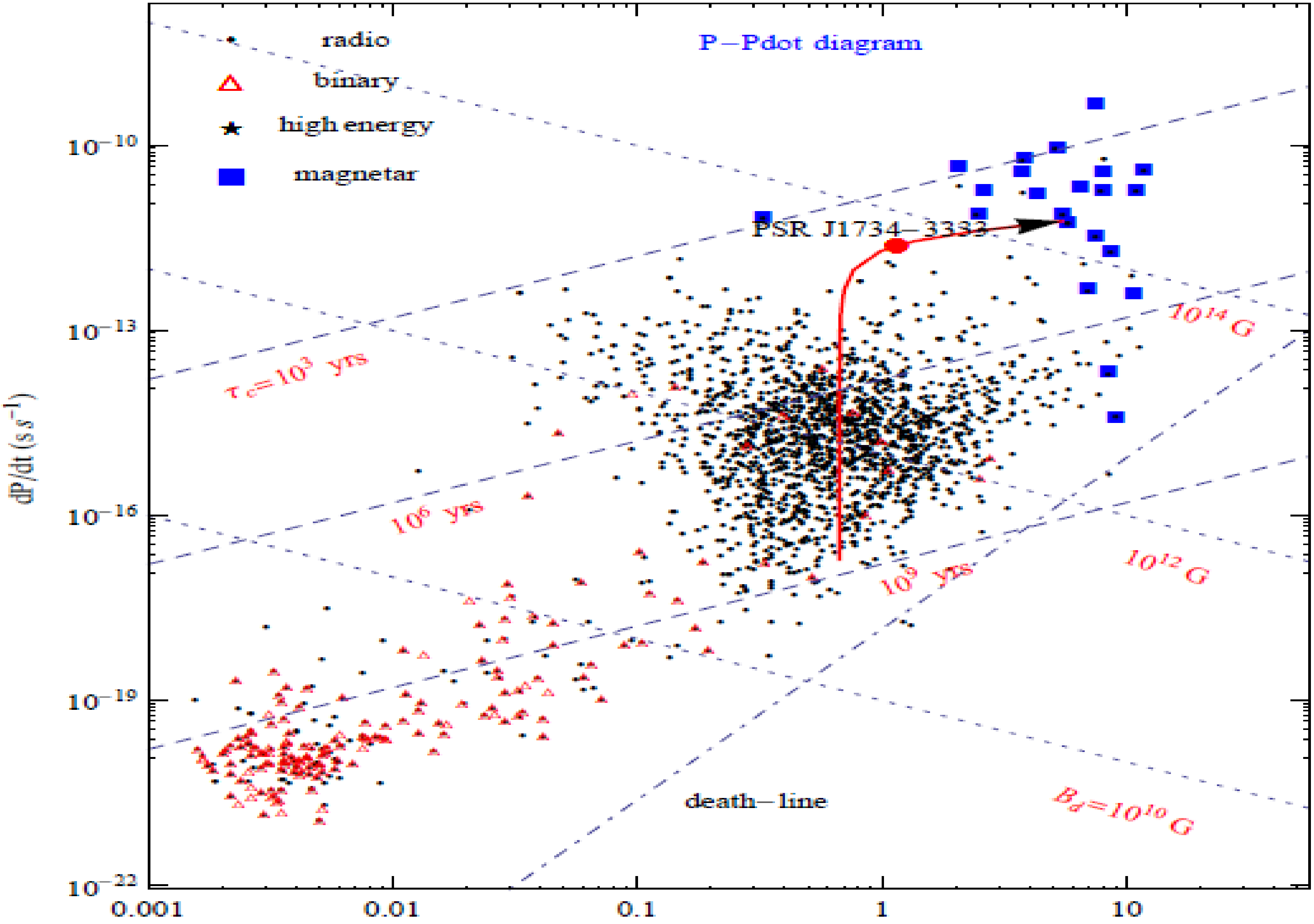
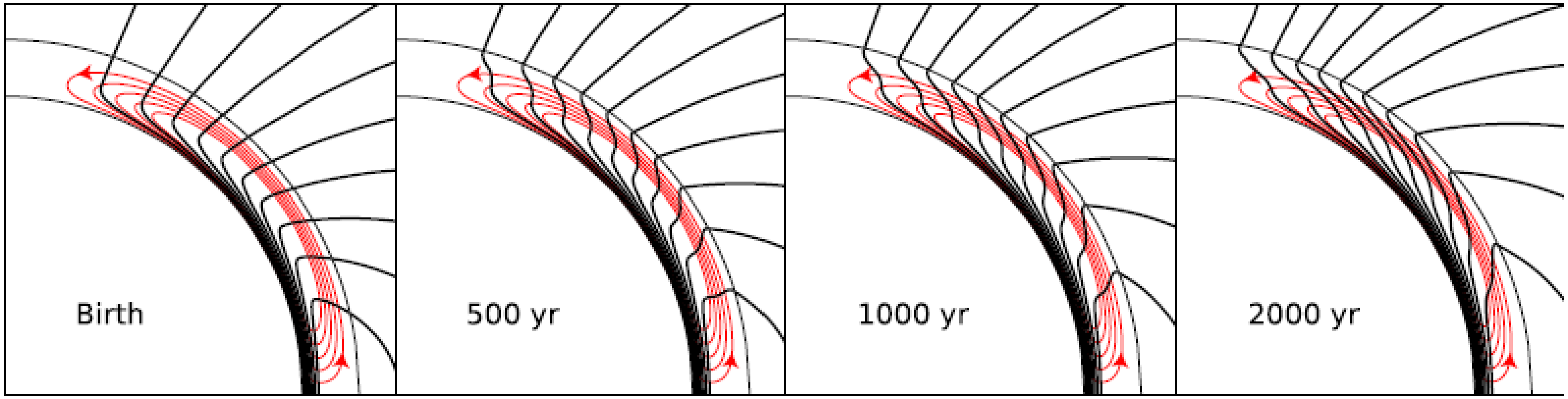


FIG. 6.— Long term rotational evolution of PSR J1734−3333 dominated by the dipole magnetic field increase. Radio, binary and magnetars are defined by black dot, red triangle, and blue square, respectively. The red solid circle is the observations of PSR J1734−3333.

# Comparison (1)



(Gourgouliatos \& Cumming 2014)

Recently, Gourgouliatos \& Cumming (2014) investigated magnetic field evolution in the NS crust due to Hall drift as an explanation for observed braking indices of NSs, they pointed that rapid interior cooling after 100 kyrs stops the field growth

# Comparison (2)

- Caliskan et al. (2013) presented fall-back disk solution for n of PSR J1734 -3333. They Need a small disk whose mass is much smaller than the mass of the disk around AXP 4U 0142+61 (Wang et al., 2006) to fit the observational parameters of J1734-3333.

- Liu et al. (2014) presented another fall-back disk solution for n of PSR J1734 -3333. The propeller torque of a fall-back disk would modify the period derivative, which makes the current dipole magnetic field strength much stronger than the real field strength.

**The above two models didn’t give the relation of spin parameters and secular evolution time t for this source !**

# Discussion (1)

- In this work, we assume that the surface dipole field always increase with a constant index of $\varepsilon$. Here, $\varepsilon$ can be served as an average quantity of $\overline{\varepsilon}$ , since the magnetic flux density always vary with a non-liner decay-rate.

- Because the strong temperature dependence of electric conductivity, when a NS's crust cools below 10^7 K, typically 1Myrs after birth, the Ohmic dissipation time increase insignificantly, and no rapid field decay can be expected after that age.

- The maximum uncertainty of field-growth index could be from the age estimation of G354.8-0.8. Due to lack of X-ray emission and the accurate measurements of distance and radius, we can not present an accurate estimate of true age for the SNR. We expect that future observations will provide us an appropriate age range. Thus, the initial parameters in this work will be modified substantially, according to the observations

# Discussion (2)

- The origin of magnetic field of magnetars is an open issue. If the proto-NS has a rotational period of the order of a millisecond, then an efficient dynamo is expected to occur within it, and generate a magnetic field of the order of 10^15−16 G (Duncan \& Thompson 1992): combined with angular momentum, this is likely to impede accretion onto this source.

- For the purpose of explaining bursts, magnetars should possess a superhigh internal multipolar field, and a large dipole field seems to be unnecessary (e.g., 7.2 * 10^12 G for SGR 0527). Note, here we have introduced a surperhigh internal dipole magnetic field. The increase of surface dipole field, due to Ohmic diffusion of internal (burred) dipole field, will result in an increase of magnetic stress on the crust, which will break the crust. If so, the bursts (or flares) of the pulsar will be expected.

- In addition, due to the poloidal current induced in the superconducting core and penetrating the crust. We don’t expel the possibility of internal toroidal magnetic fields, though the intensity of the current is unknown.

# Discussion (3)

- Here we mainly focus on the scenario of magnetic field evolution for PSR J1734 -3333 within one~several hundred kyrs . When the surface dipole field reach a saturate value, and the pulsar becomes a magnetar, its surface dipole field will decrease , surface neutrino and photon emission cause the NS's cooling , which in turn speeds up the decay of surface dipole field , mainly through Ohmic diffusion.

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- The maximum internal dipole magnetic field buried under surface, is unknown, but could be $B_{bury} \geq 10^{14} \sim 10^{16}$ G, large enough to transport magnetic flux to the surface through Ohmic diffusion , which makes this low braking-index pulsar evolve into a potential magnetar . The estimated real age of 18(4) kyrs, based on supernova explosion, and inferred field growth index of ~1.53 can provide an explanation for too small braking index of n=0.9.

# Thank you very much!