Constrain the solar system acceleration using Pulsar Timing Array

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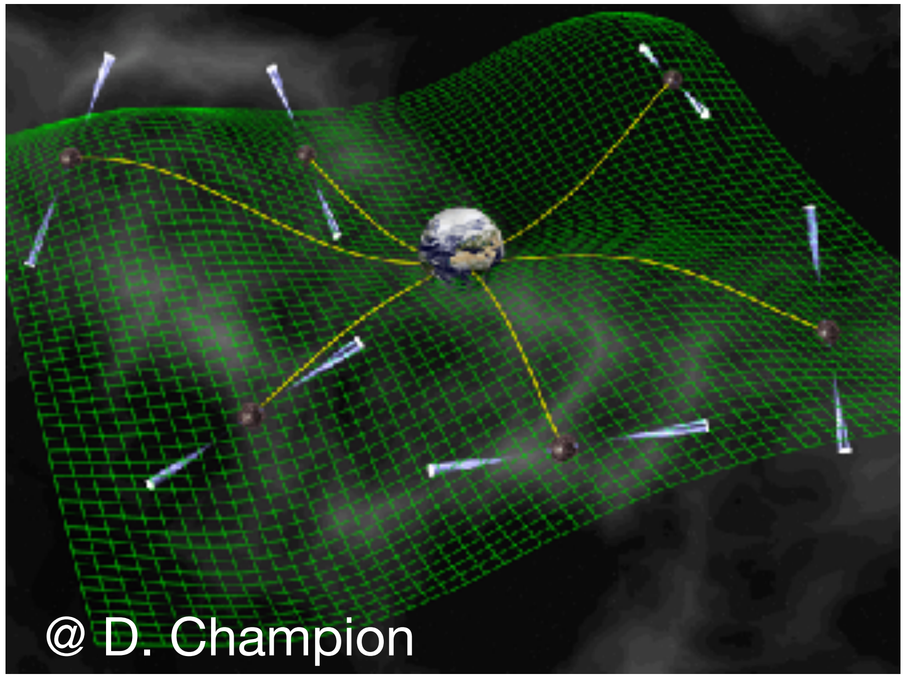
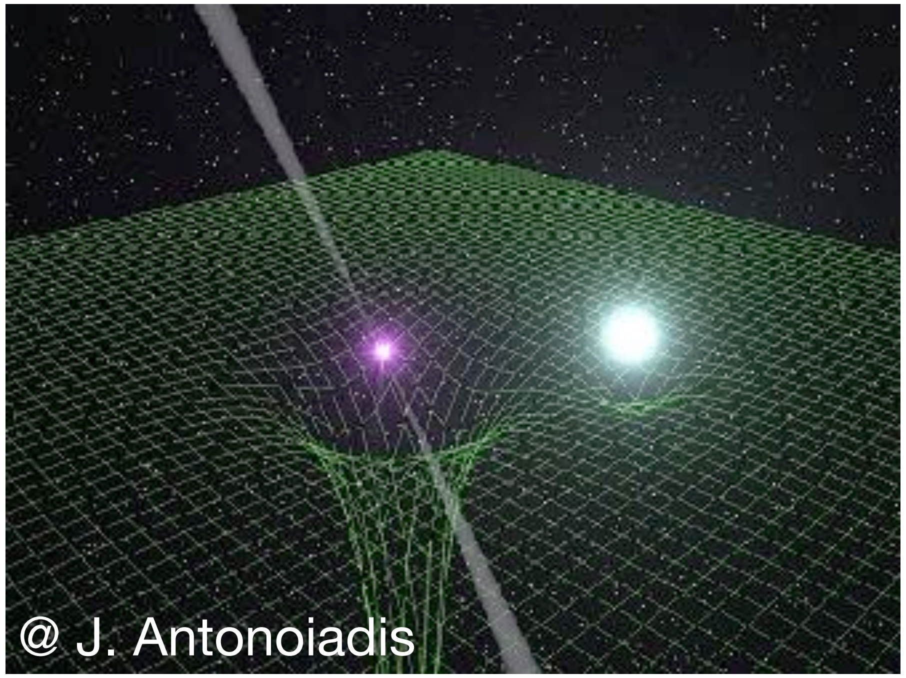
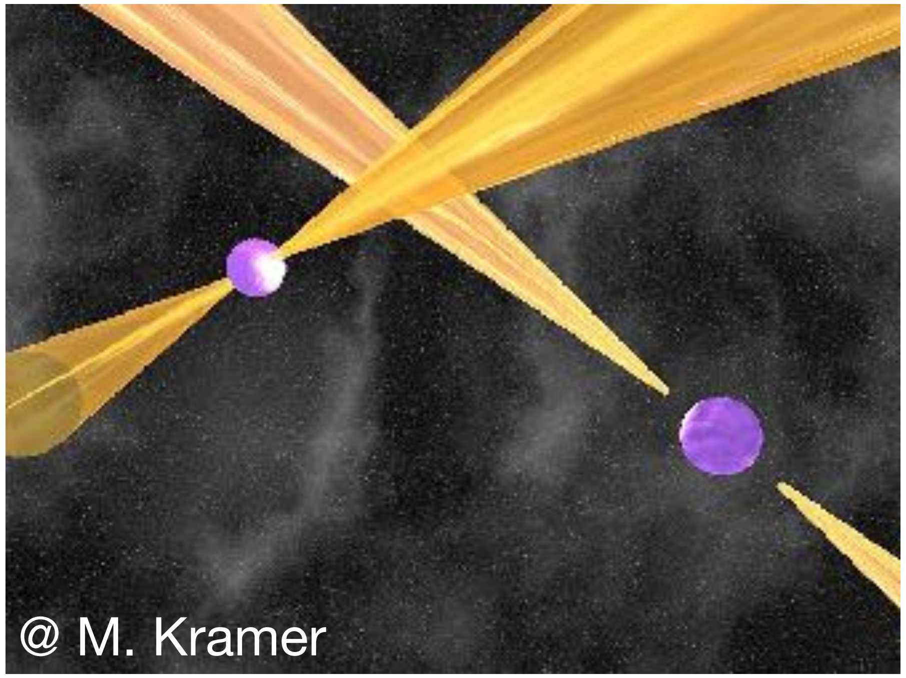






Outlines

- Introduction and motivation
- Methods and Analysis
- Results
- Discussion

Solar system dynamics

- Solar system planets, astroid and unknown objects (Champion et al, 2010; Guo et al, 2018; Caballero et al., 2018).
- Distant object in the outer solar system
  - planet Nine (Bartygin & Brown, 2016)
  - exotic objects
- Test of gravity model (Damour & Taylor, 1991 ,Weisberg & Huang, 2016).
- The choice of the inertial frame.
- Constrain the solar system barycenter (SSB) acceleration using period derivatives of pulsars (Zakamska & Tremaine, 2005).

Timing a clock in MW

- The observed period and period change rate of a clock is given by:

$$P^{\rm obs} = P^{\rm int}\left(1 + \frac{\boldsymbol{v}\cdot\boldsymbol{n}}{c}\right)$$

$$\dot{P}^{\rm obs} = \dot{P}^{\rm int}\left(1 + \frac{\boldsymbol{v}\cdot\boldsymbol{n}}{c}\right) + P^{\rm int}\left(\frac{\mu^2 d}{c} + \frac{\boldsymbol{a}\cdot\boldsymbol{n}}{c}\right)$$

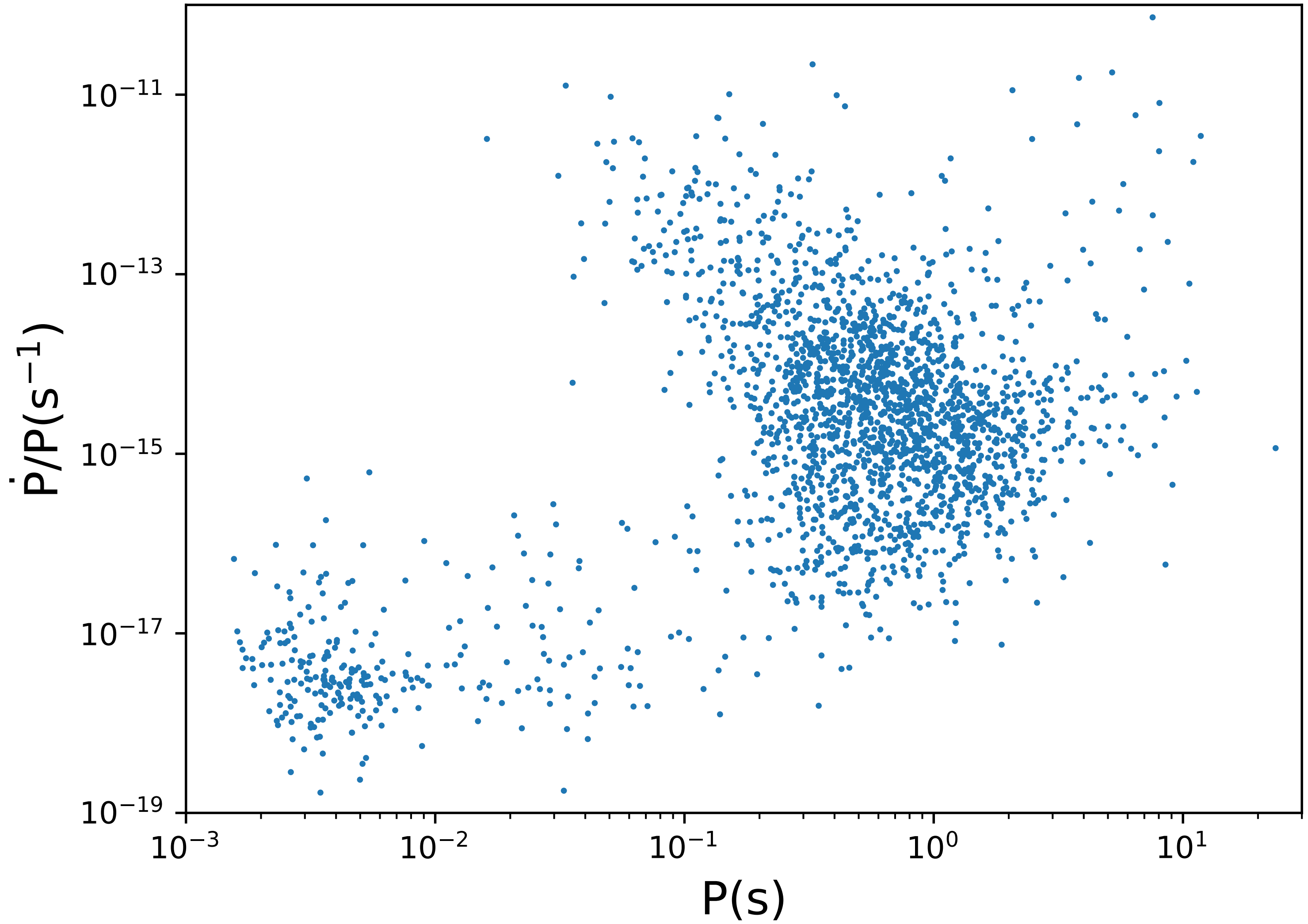
- The acceleration of SSB relative to distant clock (pulsars) could be measured by :

$$\frac{-\boldsymbol{a}_\odot\cdot\boldsymbol{n}}{c} = \frac{\dot{P}^{\rm obs} - \dot{P}^{\rm Shk} - \dot{P}^{\rm Gal} - \dot{P}^{\rm int}}{P}$$

  - observed period derivative
  - intrinsic period derivative (spin-down/up, orbital decay rate…)
  - Shklovskii effect from transverse motion
  - relative acceleration between pulsar and SSB in the Galactic potential.

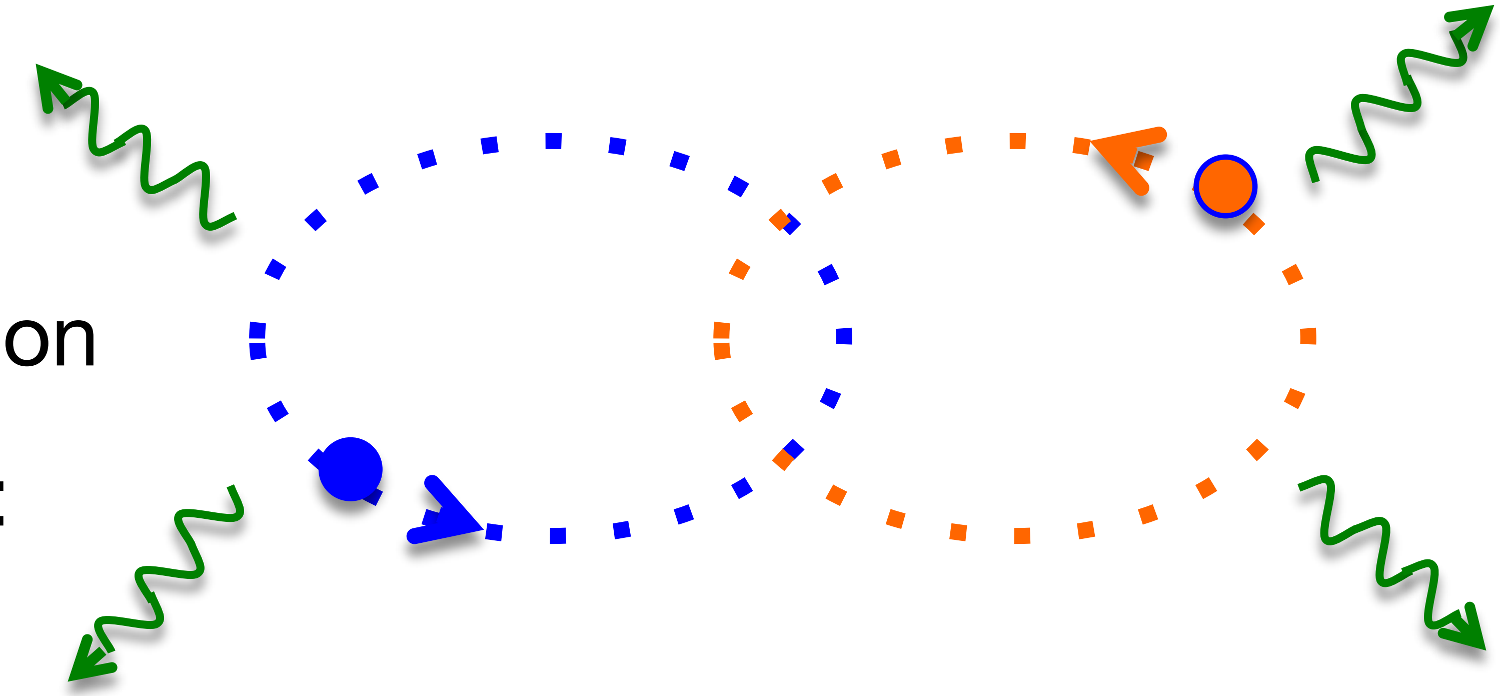
Spin period

- Intrinsic period derivative is unknown.
- Search for systematic dependence of $\dot{P}/P$ on pulsar position



Sensitivity limited by the dispersion of $\dot{P}/P$ distribution ~ 10^{-18} s^{-1}

Orbital period



- pulsar in binary system
  - gravitational wave radiation
  - orbital period decay rate:

$$\dot{P}_b^{\rm int} = \dot{P}_b^{\rm GR}$$

- For most sensitive pulsar, the error of $\dot{P}_b/P_b$ ~ 10^{-19}-10^{-20} s^{-1}, thus can constrain acceleration to a level of ~ 10^{-19} c and even lower.

IPTA DR1

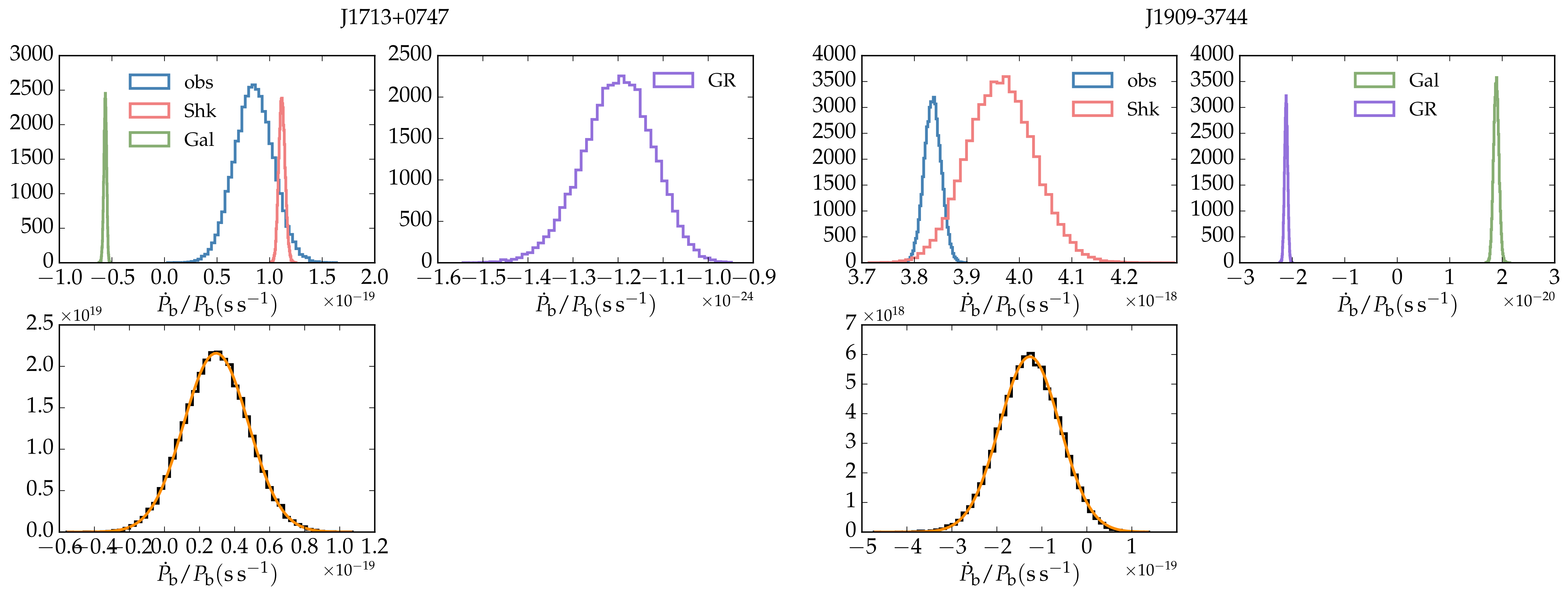
- 35 binary pulsars out of 49 MSPs
- 15 pulsars with precision of $\dot{P}_b/P_b < 10^{-18}\ s^{-1}$.

- astrometric parameters: ra, dec, pmra, pmdec, px
- binary parameters: P_b, $\dot{P}_b$, A1, e, m_c, i

- Other prior measurements:
  - px: VLBI, DM
  - m_c, i: optical observations, upper limits

Hierarchical Bayesian framework

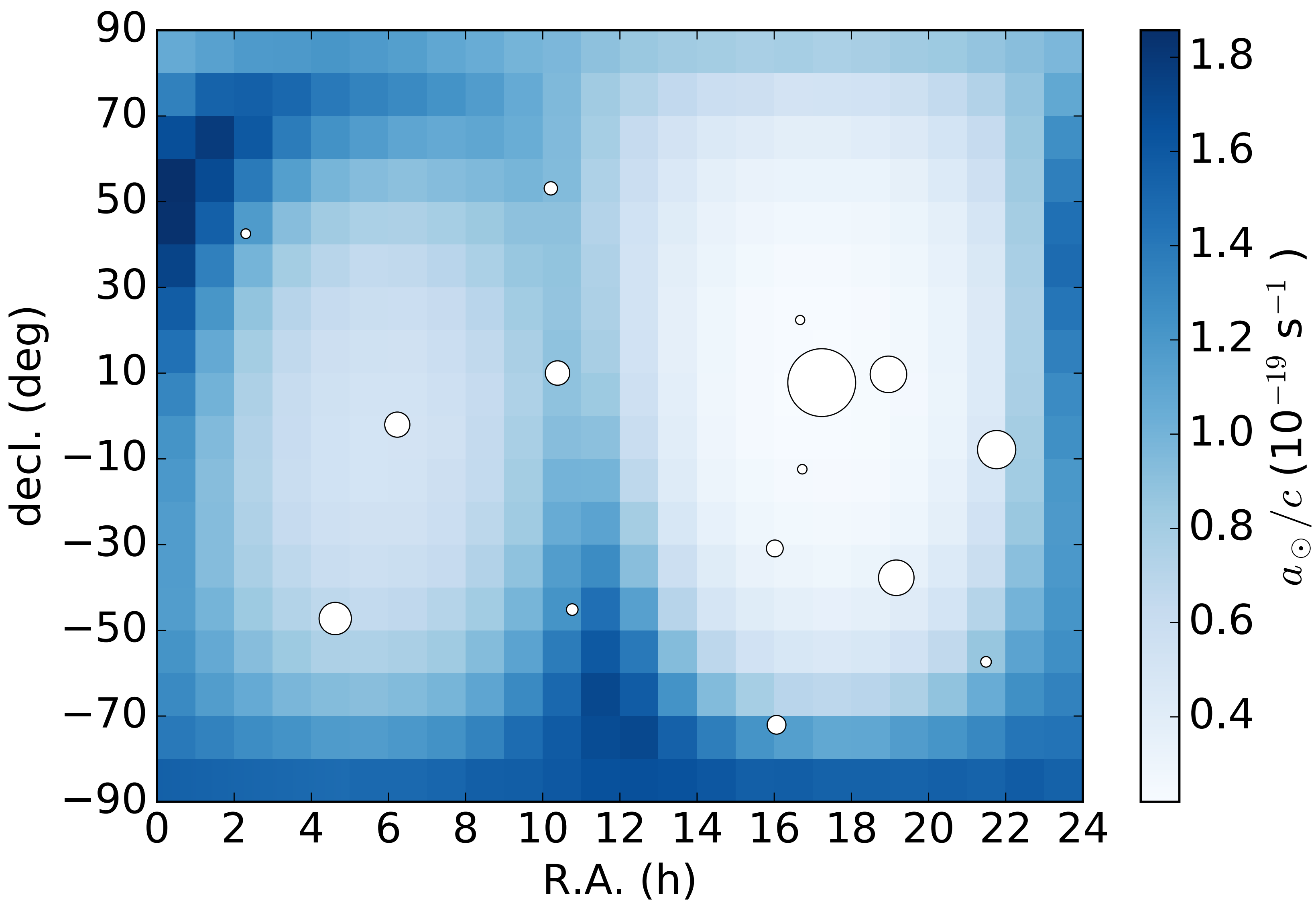
- Timing parameter inference using Temponest (Lentati et al, 2014)
- posterior of $\dot{P}_b/P_b$ component: $\dfrac{-\boldsymbol{a}_\odot \cdot \boldsymbol{n}}{c} = \dfrac{\dot{P}^{\rm obs} - \dot{P}^{\rm Shk} - \dot{P}^{\rm Gal} - \dot{P}^{\rm int}}{P}$

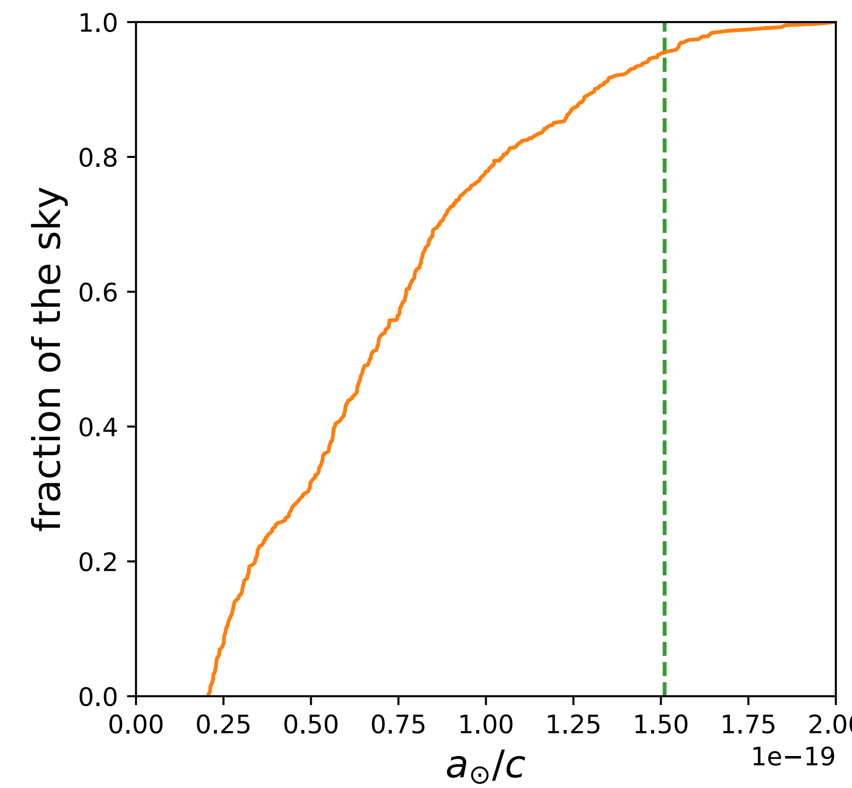


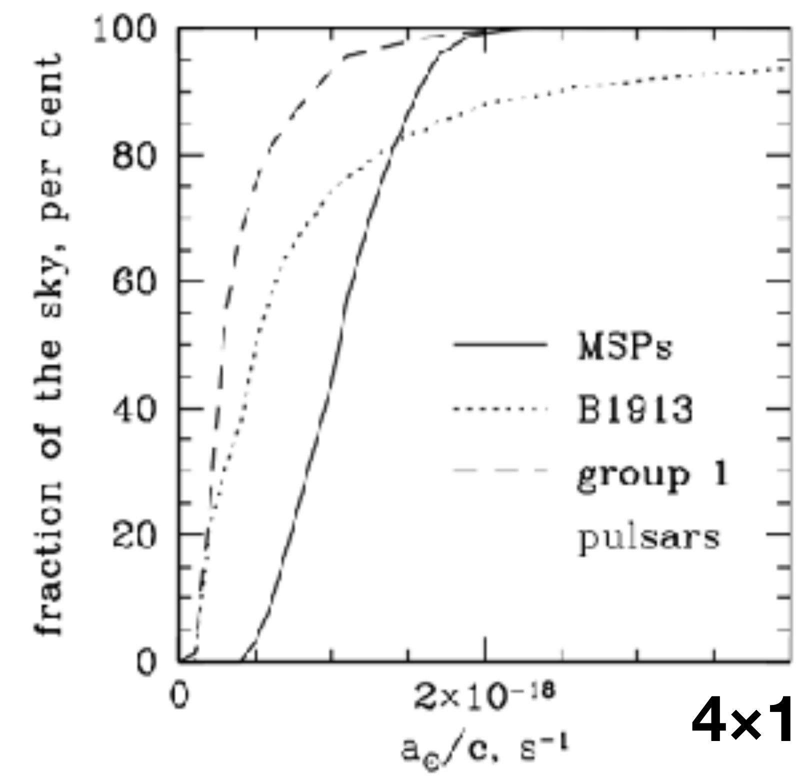
$$f(\boldsymbol{a}_\odot | \boldsymbol{r}) = \frac{f(\boldsymbol{a}_\odot)}{\sum_i f(a_i)} \sum_{i=1}^{n_{psr}} \frac{1}{\sqrt{2\pi\sigma_i}} \exp[-\frac{(\boldsymbol{a}_\odot \cdot \boldsymbol{n}_i - \bar{a}_i)^2}{2\sigma_i^2}]$$

Upper limit of solar system acceleration

- all-sky upper limit of a/c: 1.9×10^{-19} s^{-1}
- 95% sky upper limit of a/c: 1.5×10^{-19} s^{-1}
- a/c < $(2\sim19)\times10^{-20}$ s^{-1} ($10^{-20}\sim100$ μm/s /yr)







4×10^{-18}

Tremaine et al, 2005

Analytic formula

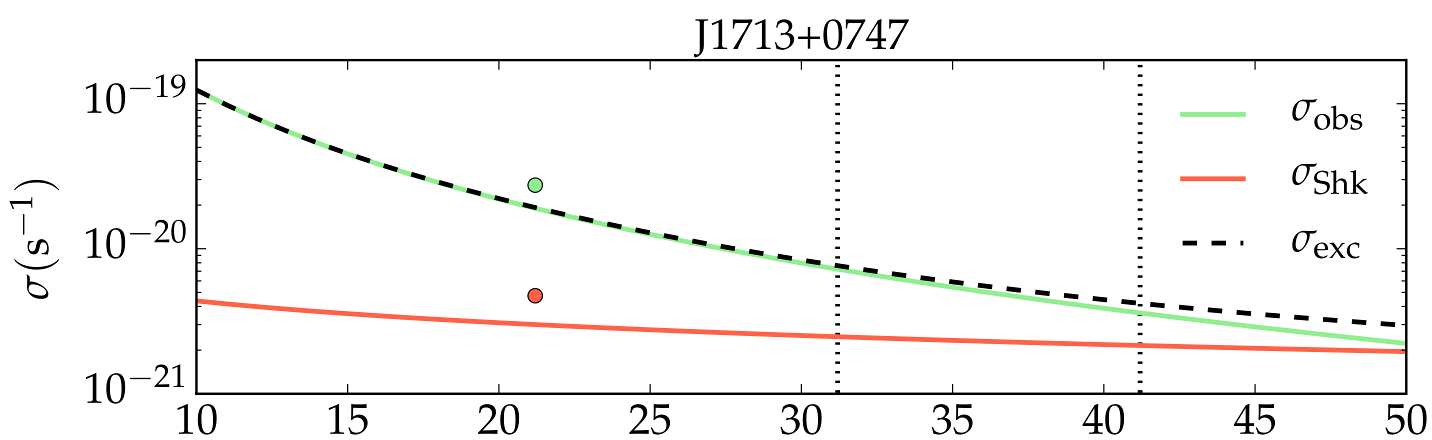
$$\frac{-\boldsymbol{a}_\odot \cdot \boldsymbol{n}}{c} = \frac{\dot{P}_b^{\rm obs} - \dot{P}_b^{\rm Shk} - \dot{P}_b^{\rm Gal} - \dot{P}_b^{\rm GR}}{P_b} = \left(\frac{\dot{P}_b}{P_b}\right)^{\rm exc} \longrightarrow \sigma_{\rm exc} \approx \sqrt{\sigma_{\rm obs}^2 + \sigma_{\rm Shk}^2}$$

- error of observed $\dot{P}_b/P_b$

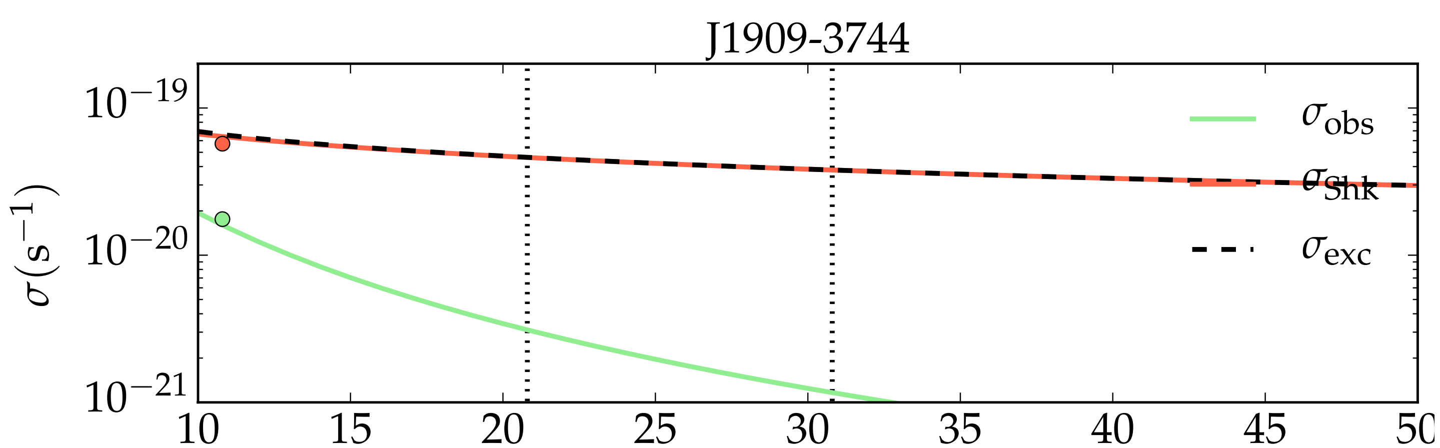
$$\sigma_{\rm obs} \approx \frac{6\sqrt{10}P_b\sigma}{a_1\pi^2\sqrt{\dot{n}T^5}} \propto \frac{1}{\sqrt{T^5}}$$

- error of Shklovskii effect induced $\dot{P}_b/P_b$

$$\sigma_{\rm Shk} \approx \frac{4\sqrt{2}\mu^2 d^2\sigma}{c a_\oplus^2 \cos^2\beta\sqrt{\dot{n}T}} \propto \frac{1}{\sqrt{T}}$$



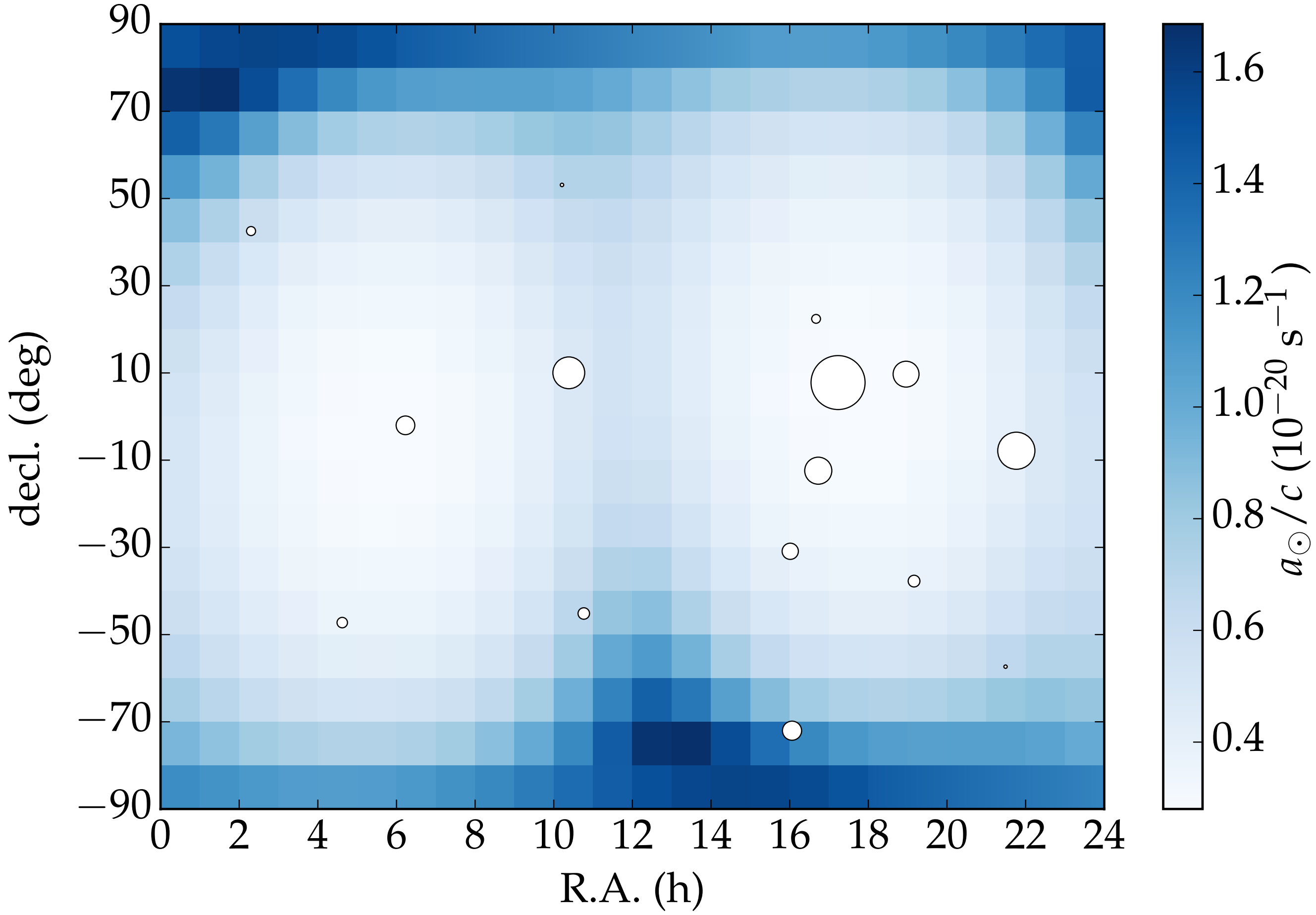
7×10^{-21} in 10yr



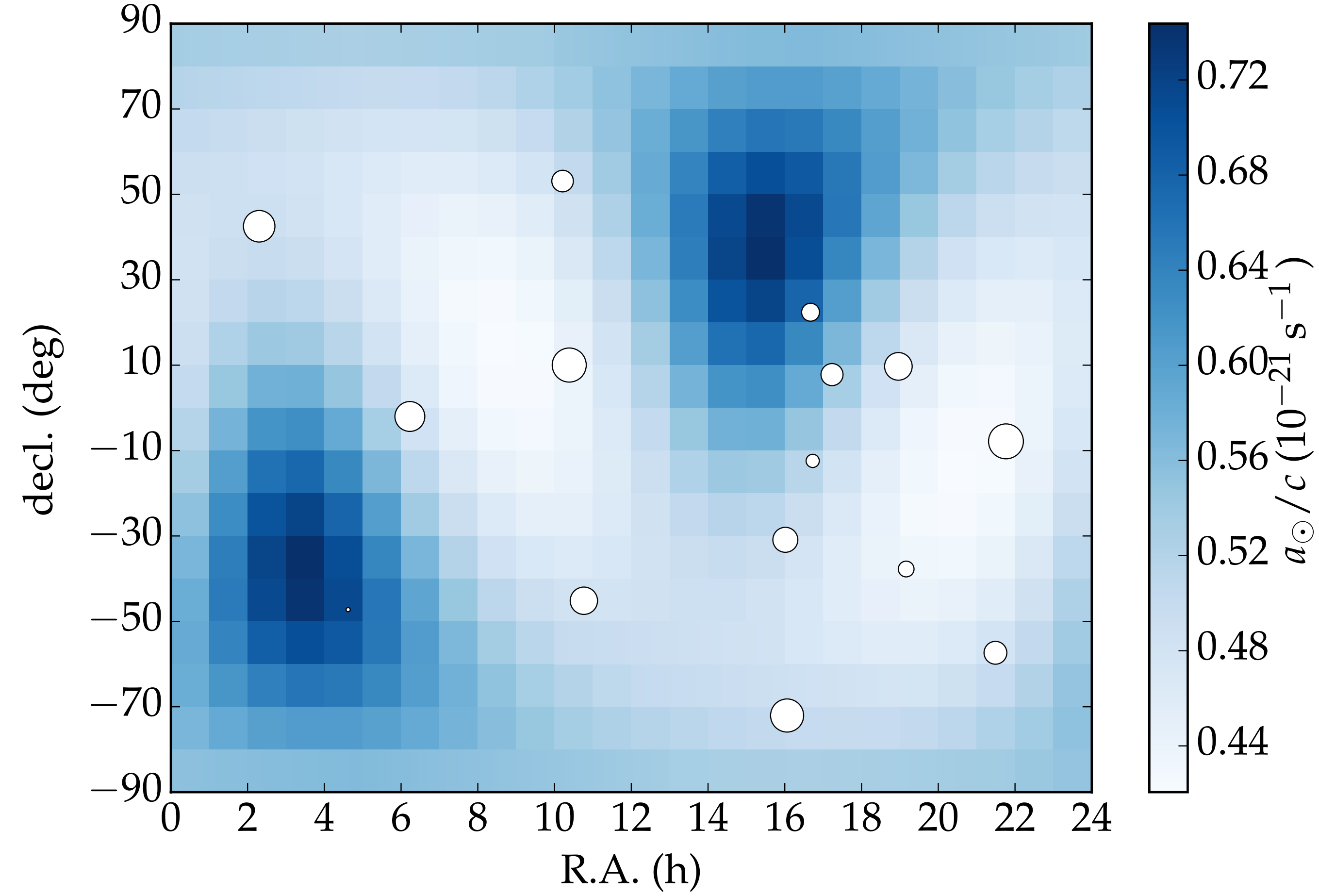
$$f(\vec{a}_\odot\,|\,\vec{r}) = \frac{f(\vec{a}_\odot)}{\prod_i f(a_i)} \prod_{i=1}^{n_{\rm psr}} f(a_i\,|\,\vec{r}_i)\,|_{a_i=\vec{a}_\odot\cdot\vec{n}_i}$$

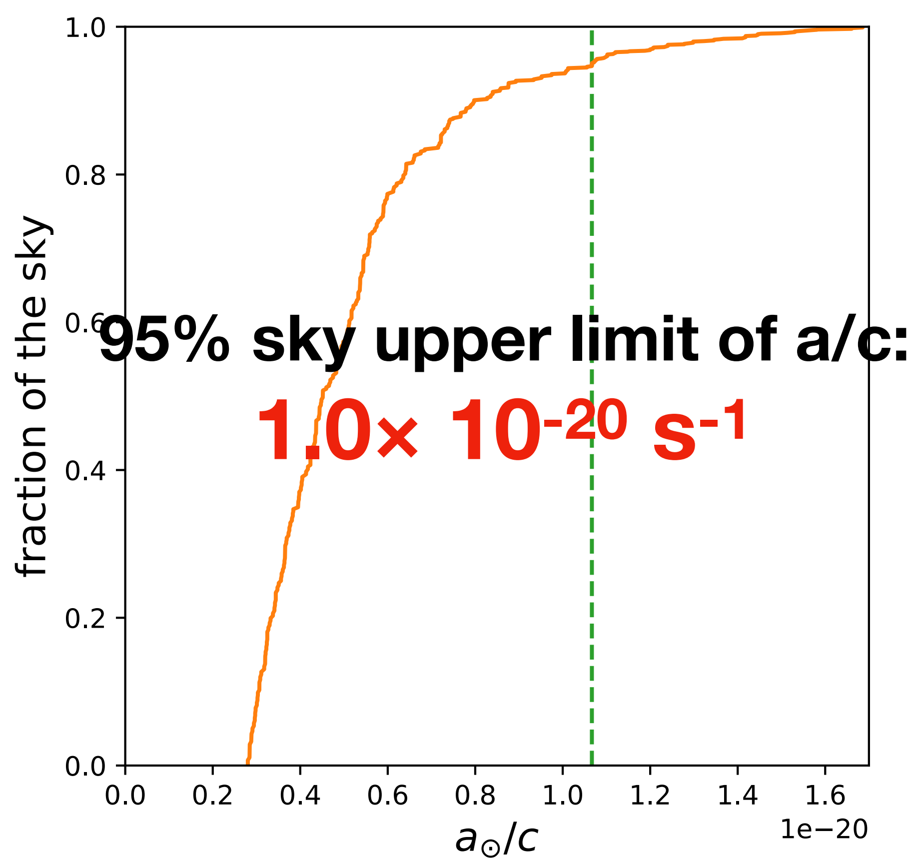
Prediction

Prediction1



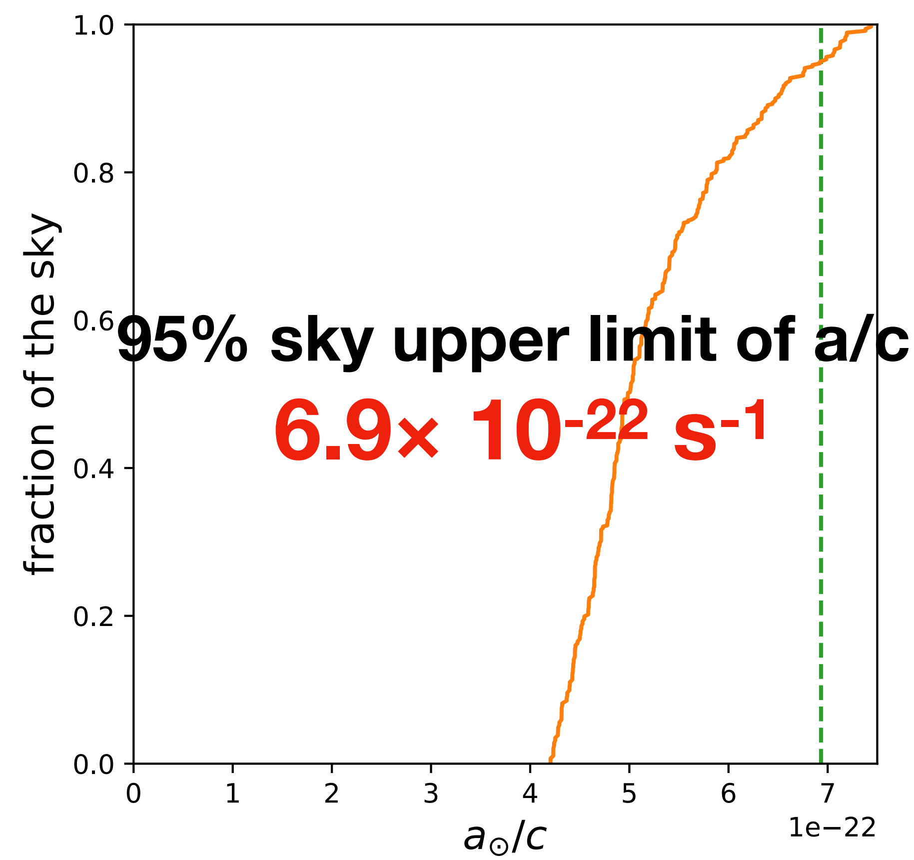
Prediction2





95% sky upper limit of a/c:

$1.0\times 10^{-20}\ \mathrm{s}^{-1}$



95% sky upper limit of a/c:

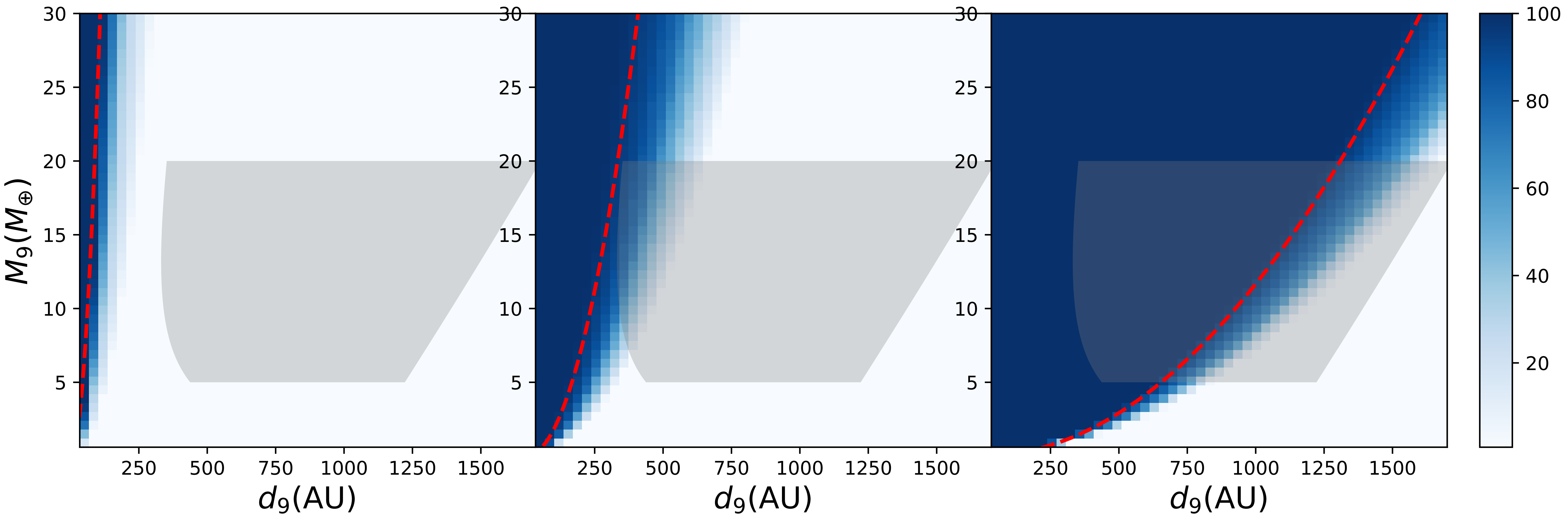
$6.9\times 10^{-22}\ \mathrm{s}^{-1}$

Applications

- point mass in the outer solar system
- constraint on $\dot{G}/G$
- Galactic acceleration of SSB and that of the Milky Way

Point mass in the outer solar system

- point mass induced acceleration to SSB: a=GM/r²
- Planet Nine: $M_9 = 11.2\, M_{\rm E}(\frac{a_\odot/c}{1.5\times10^{-19}\,{\rm s}^{-1}})(\frac{d_9}{100\,{\rm au}})^2,$



Grey area is the parameter space estimated by Brown & Bartygin, 2016

Constraint on $\dot{G}/G$

- The orbital period derivative could be used to set limit on $\dot{G}/G$ (Damour 1988; Damour & Taylor, 1991; Will 1993):

$$\left(\frac{\dot{P}_b}{P_b}\right)^{exc} \simeq -\frac{\dot{G}}{G}\left\{2 - 2\left[\frac{m_p s_p + m_c s_c}{m_p + m_c}\right] - 3\left[\frac{m_p s_p + m_c s_c}{m_p + m_c}\right]\right\}$$

- using all the pulsars: $\dot{G}/G=(-4.4\pm6.0)\times10^{-13}$ yr^{-1} at 95% certainty.
  - Deller et al, 2008: $\dot{G}/G=(-5\pm26)\times10^{-13}$ yr^{-1}
  - Zhu et al, 2018: $\dot{G}/G=(-1\pm9)\times10^{-13}$ yr^{-1}
  - Lunar Laser Ranging (Hofmann et al, 2010): $\dot{G}/G=(-0.7\pm3.8)\times10^{-13}$ yr^{-1}

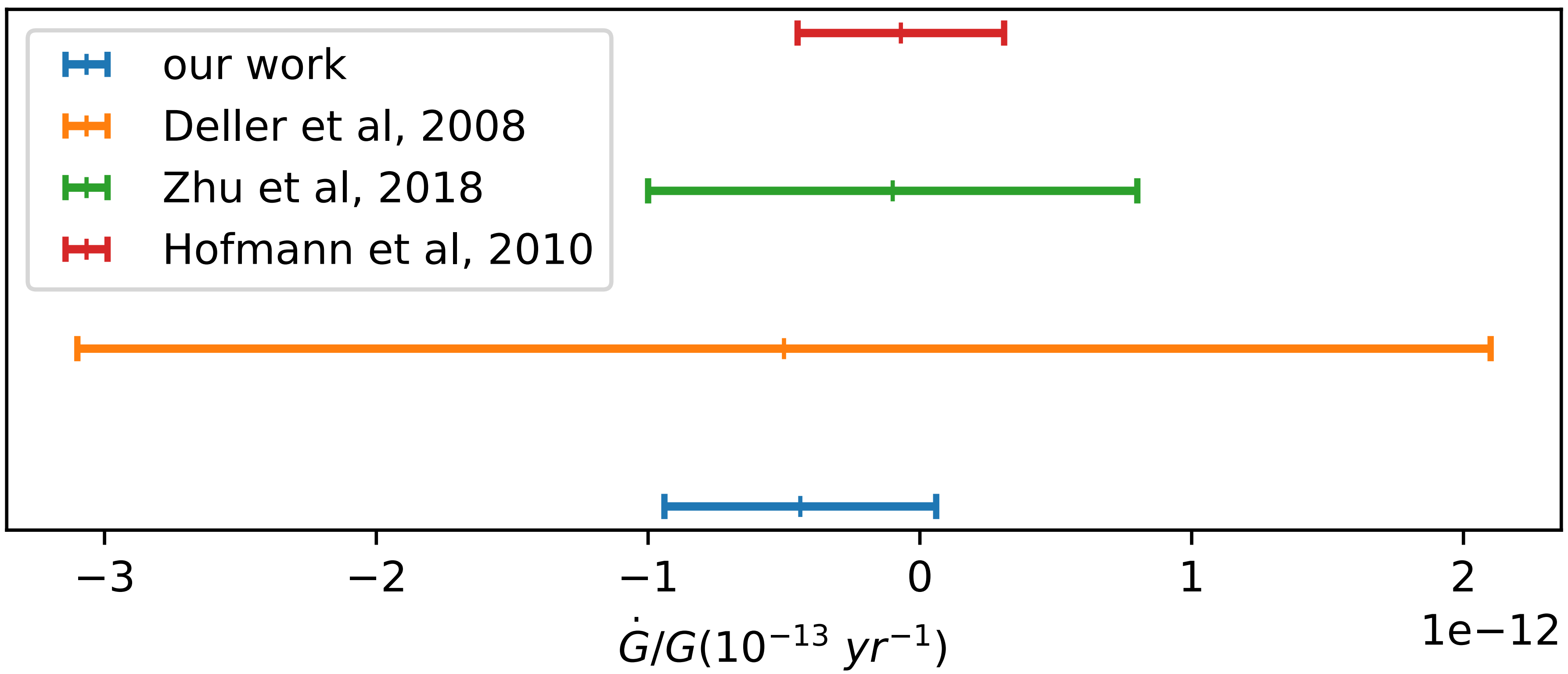
- Difference of $\dot{G}/G$ above and below the Galactic plane: $\dot{G}/G=(-8.5\pm15.8)\times10^{-13}$ yr^{-1} at 95% certainty.

Constraint on Ġ/G

- The orbital period derivative could be used to set limit on Ġ/G (Damour 1988; Damour & Taylor, 1991; Will 1993):

$$\left(\frac{\dot{P}_b}{P_b}\right)^{exc} \simeq -\frac{\dot{G}}{G}\left\{2+2\left[\frac{m_p s_p + m_c s_c}{m_p + m_c}\right]+3\left[\frac{m_p s_p + m_c s_c}{m_p + m_c}\right]\right\}$$

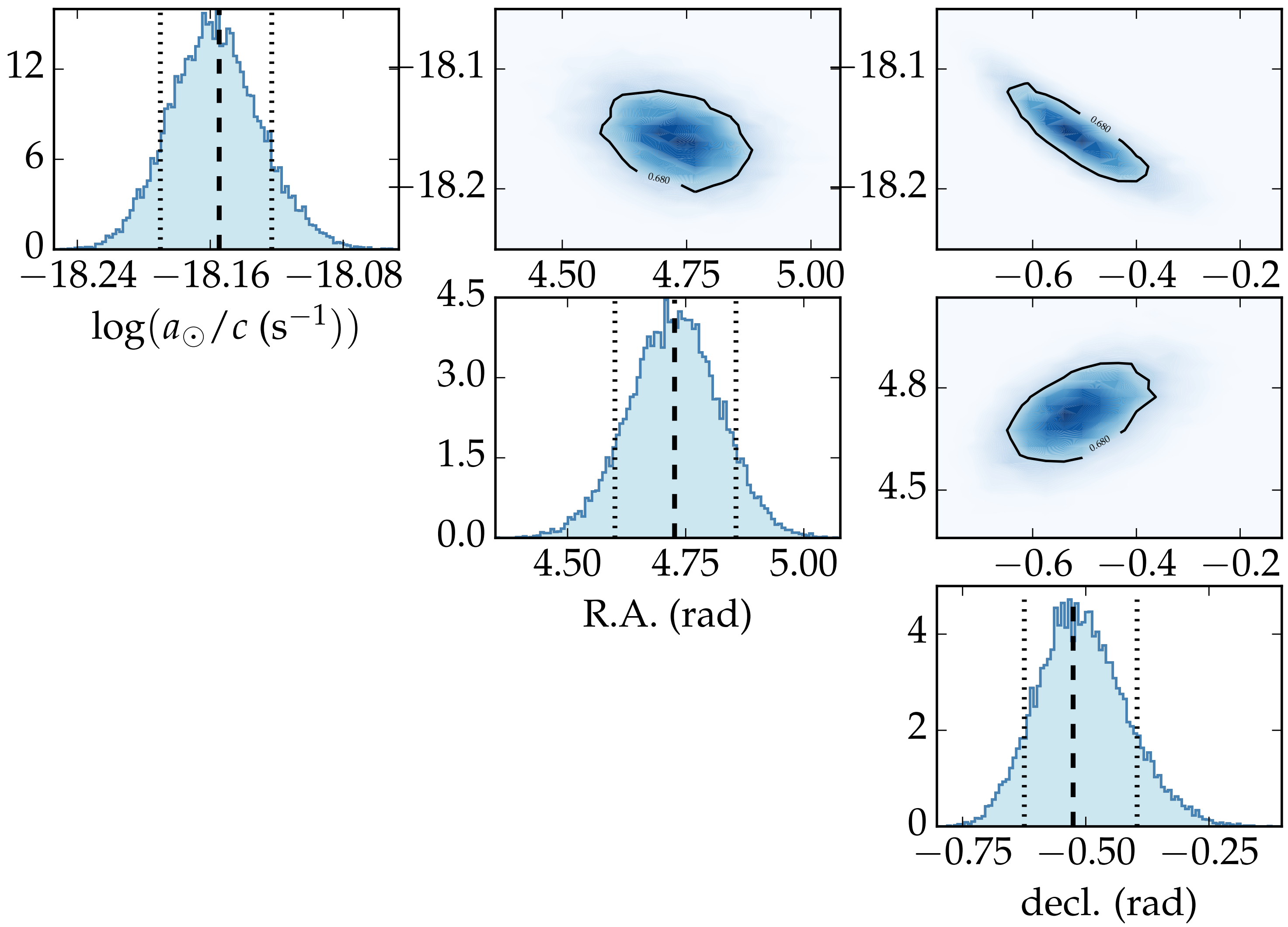
- using all the pulsars: $\dot{G}/G=(-4.4\pm6.0)\times10^{-13}\ yr^{-1}$ at 95% certainty.
  - Dell
  - Zhu
  - Lun $\times10^{-13}\ yr^{-1}$



- Difference of Ġ/G above and below the Galactic plane: $\dot{G}/G=(-8.5\pm15.8)\times10^{-13}\ yr^{-1}$ at 95% certainty.

Galactic acceleration of SSB

	$a_X/c[10^{-18}\ \mathrm{s}^{-1}]$	$a_Y/c[10^{-18}\ \mathrm{s}^{-1}]$	$a_Z/c[10^{-18}\ \mathrm{s}^{-1}]$	$a/c[10^{-18}\ \mathrm{s}^{-1}]$	$\alpha_G[°]$	$\delta_G[°]$
Our work	0.70 ± 0.05	0.01 ± 0.08	-0.05 ± 0.08	0.70 ± 0.05	271 ± 7	-30 ± 7
Titov & Krásná (2018)	0.78 ± 0.03	0.01 ± 0.04	-0.19 ± 0.04	0.80 ± 0.03	281 ± 3	-35 ± 3
Titov & Lambert (2016)	0.81 ± 0.16	-0.32 ± 0.19	-0.27 ± 0.20	0.91 ± 0.15	273 ± 13	-56 ± 9
MacMillan (2014)	0.82 ± 0.06	-0.32 ± 0.17	-0.27 ± 0.20	0.86 ± 0.06	267 ± 3	-11 ± 3
Titov & Lambert (2013)	0.98 ± 0.17	0.04 ± 0.12	0.03 ± 0.12	0.98 ± 0.17	266 ± 7	-26 ± 7
Xu et al. (2012)	0.79 ± 0.05	0.02 ± 0.06	0.41 ± 0.04	0.89 ± 0.06	243 ± 4	-11 ± 4
Titov et al. (2011)	0.97 ± 0.23	0.10 ± 0.20	0.12 ± 0.19	0.98 ± 0.23	263 ± 11	-20 ± 12



Pulsar: acceleration of SSB in the Milky Way (MW)

VLBI: acceleration of SSB relative to distant radio sources.

acceleration (**a**/c) of MW relative to distant radio sources:

$$[0.08 \pm 0.06, 0.0 \pm 0.09, -0.14 \pm 0.09] \times 10^{-18}\ \mathrm{s}^{-1}$$

Summary

- We have constructed a hierarchical Bayesian framework to combine the timing data of and ensemble of pulsars and infer the SSB acceleration.
- We derive analytic formula for the sensitivity of $(\dot{P}_b/P_b)^{obs}$ and $(\dot{P}_b/P_b)^{Shk}$ using the Cramér-Rao bound, and make predictions to our method in the future use.
- We also discuss possible applications of the SSB acceleration, or the orbital derivative of binary pulsar, including: constraints on point mass around the solar system, study of the gravity theory, ….

Thanks!