Fast/Future Pulsar Symposium 9

Xiamen University, Xiamen, Fujian, China

Triaxially-deformed Freely-precessing Neutron Stars

Continuous electromagnetic and gravitational radiation

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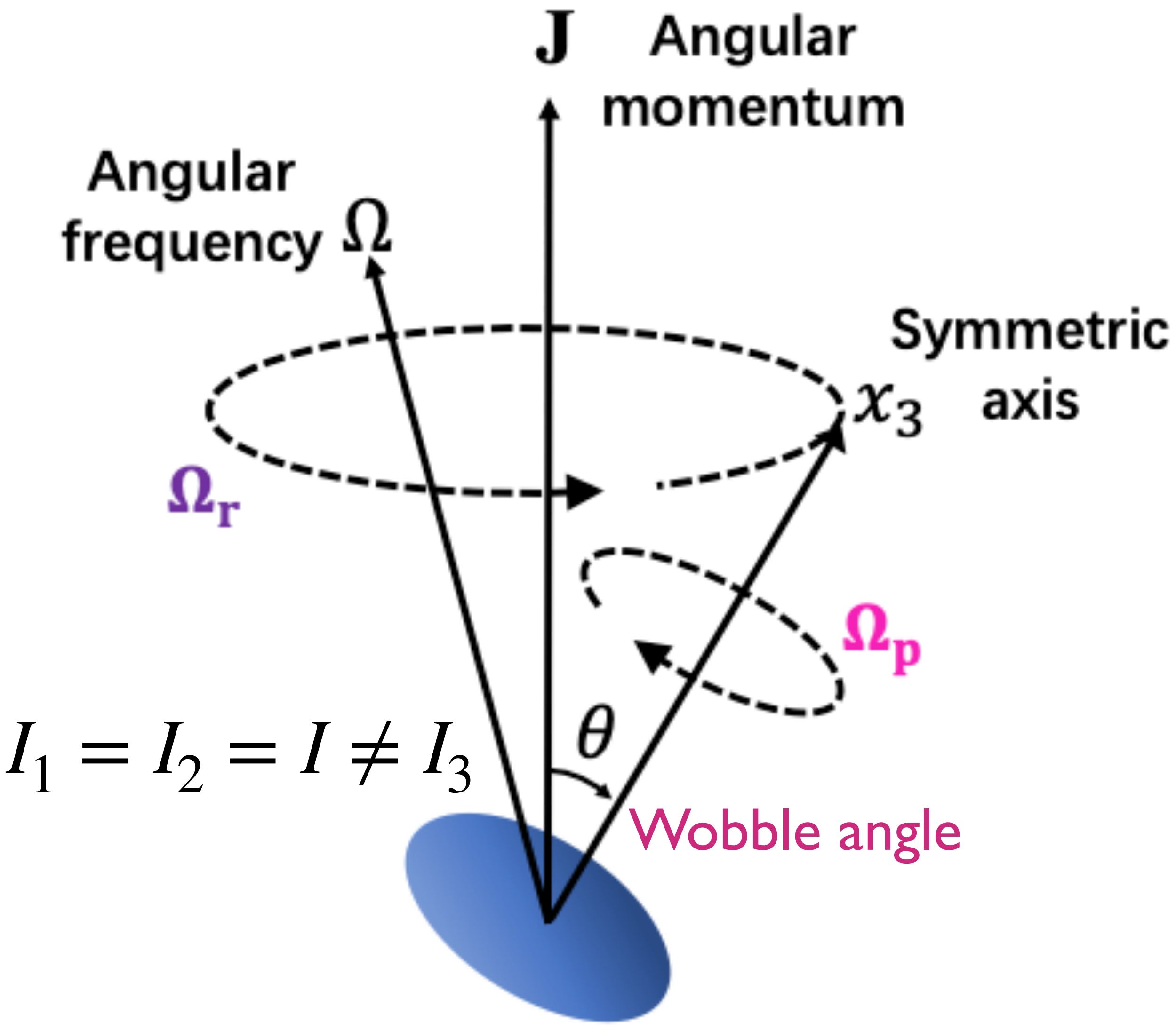
Outline

- Freely-precessing neutron stars (NSs)
- Continuous electromagnetic and gravitational radiation
- Indications for NS structures
- Summary

What is free precession?



Free precession of a biaxially-deformed body

Two superimposed rotations:

$$\mathbf{\Omega} = \Omega_r \mathbf{n}_J + \Omega_p \mathbf{n}_{x_3}$$

Rotation around $\mathbf{J}$ + Retrograde motion around x_3

Free precessional angular frequency:

$$\Omega_p = \epsilon \cos\theta \, \Omega_r$$

The oblateness $\epsilon = \dfrac{I_3 - I}{I_3}$

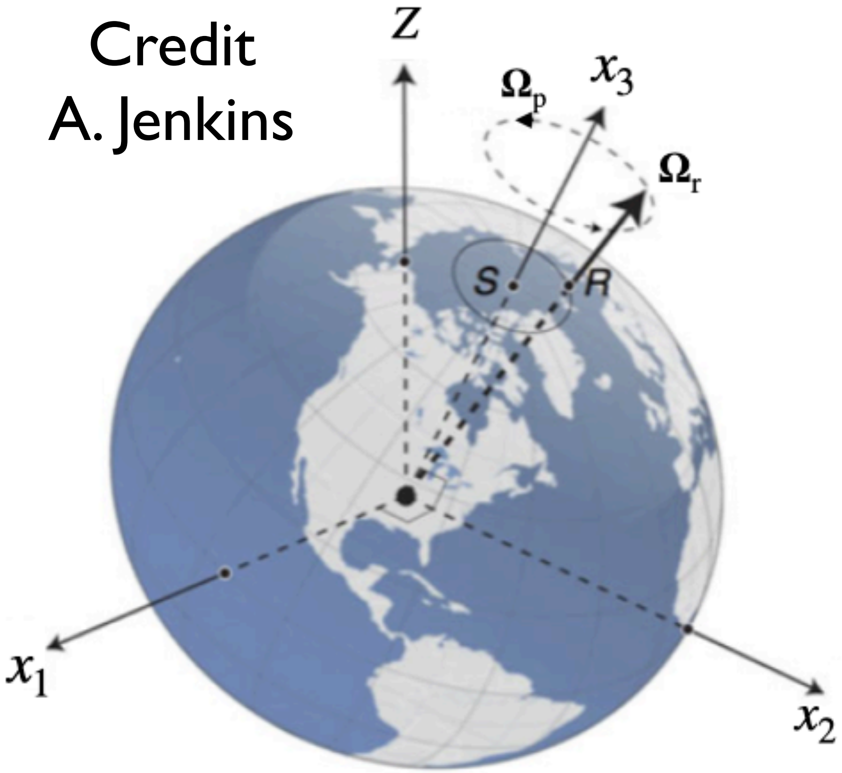
[Landau & Lifshitz, Mechanics, 1960]

Precession of elastically-deformed bodies

Property	Earth	Neutron star
Moment of inertia: Solid crust	90%	$< 5\%$
Moment of inertia: Liquid core	10%	$> 95\%$
Rigidity parameter	0.7	10^{-5}; jelly
Magnetic field	Unimportant	Maybe
Free precession observed?	Yes, 14 month 'Chandler wobble'	Handful of candidates

Credit
D. I. Jones

Credit
A. Jenkins



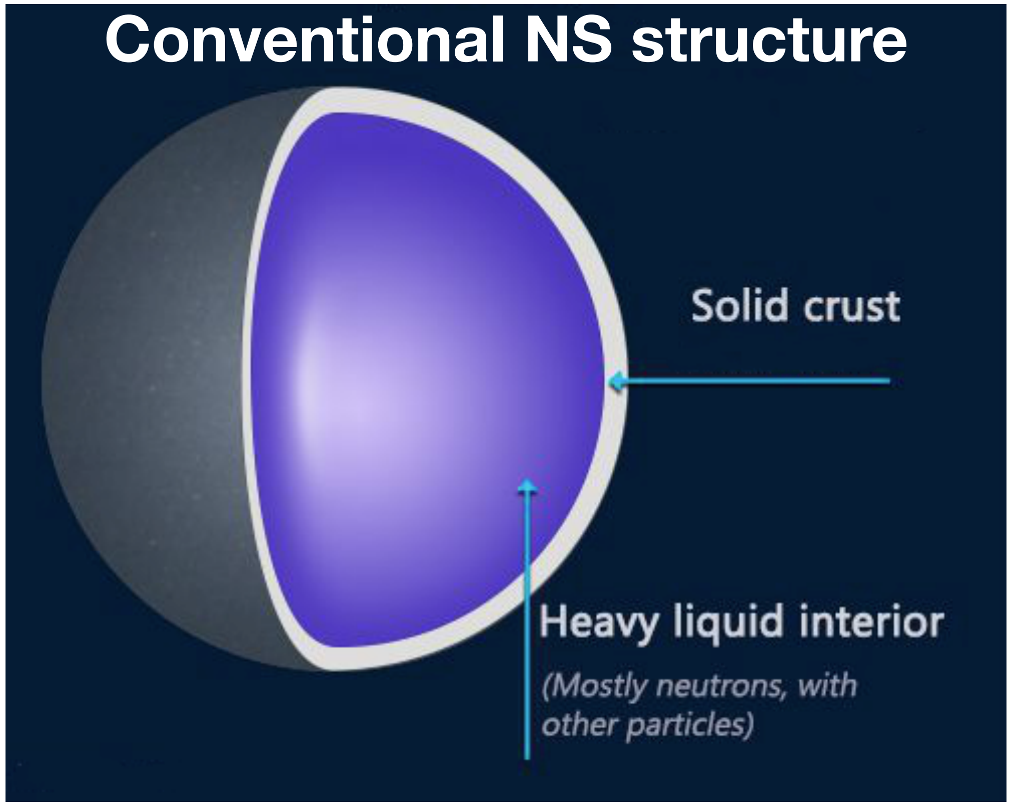
$$\frac{GM}{Rc^2} \sim 10^{-10},\ \epsilon_{\rm elast} \sim \frac{1}{300}$$

Effect: latitude modulation

Rigidity parameter

$$\epsilon_{\rm elast} = \eta \epsilon_\Omega$$

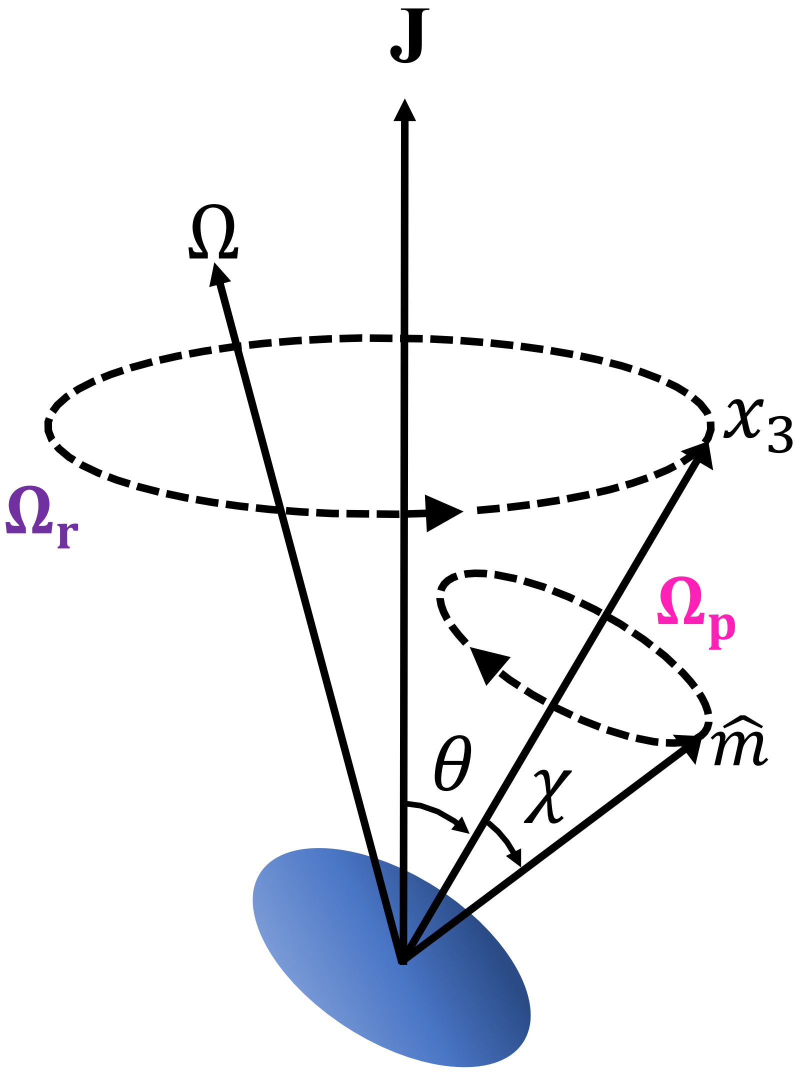
Centrifugal deformation $\simeq \dfrac{\Omega_r^2 R^3}{GM}$

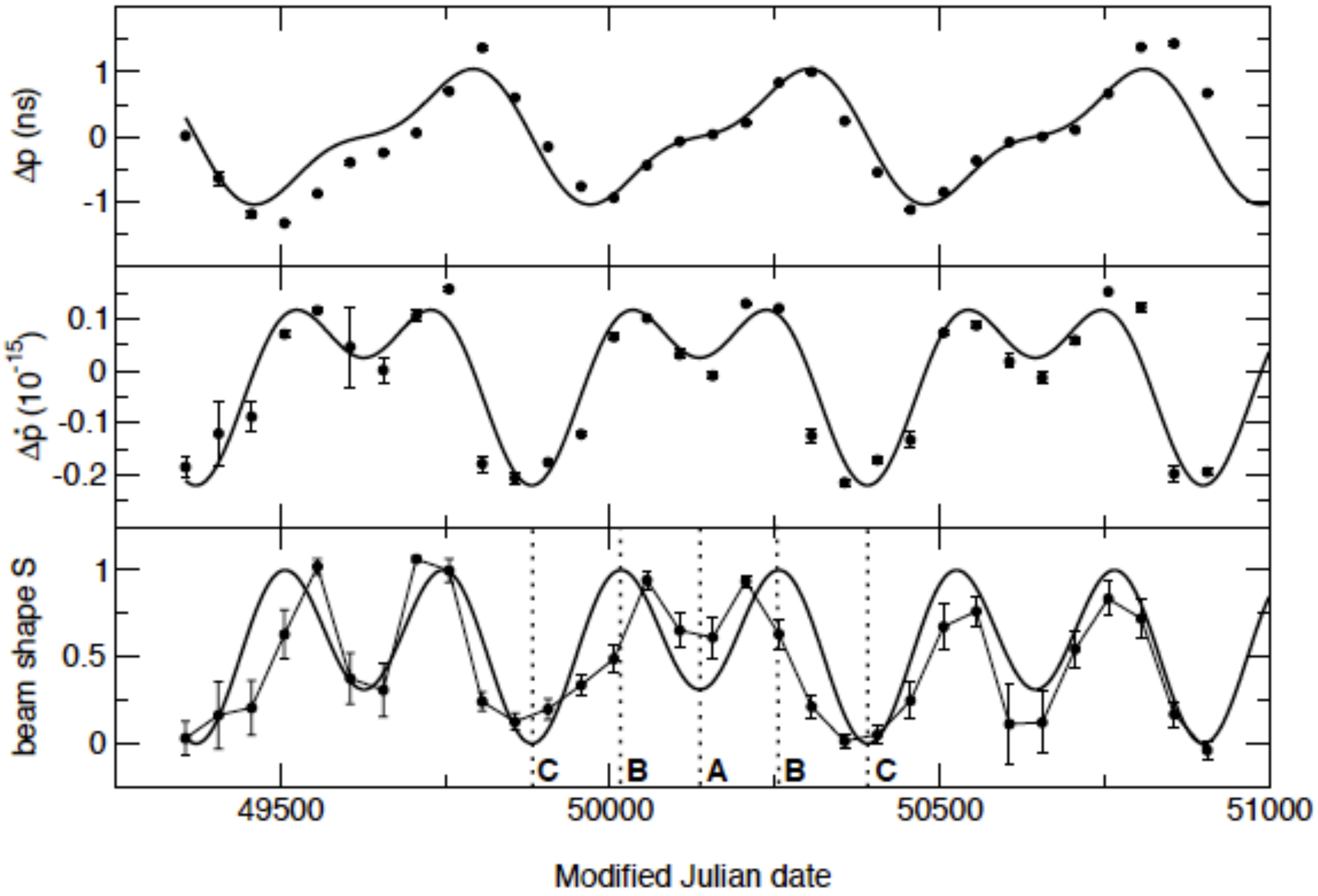


$$\frac{GM}{Rc^2} \sim 0.2,\ \epsilon_{\rm elast} < 10^{-7}$$

If exist, how to observe?

Observational effects 1: electromagnetic radiation





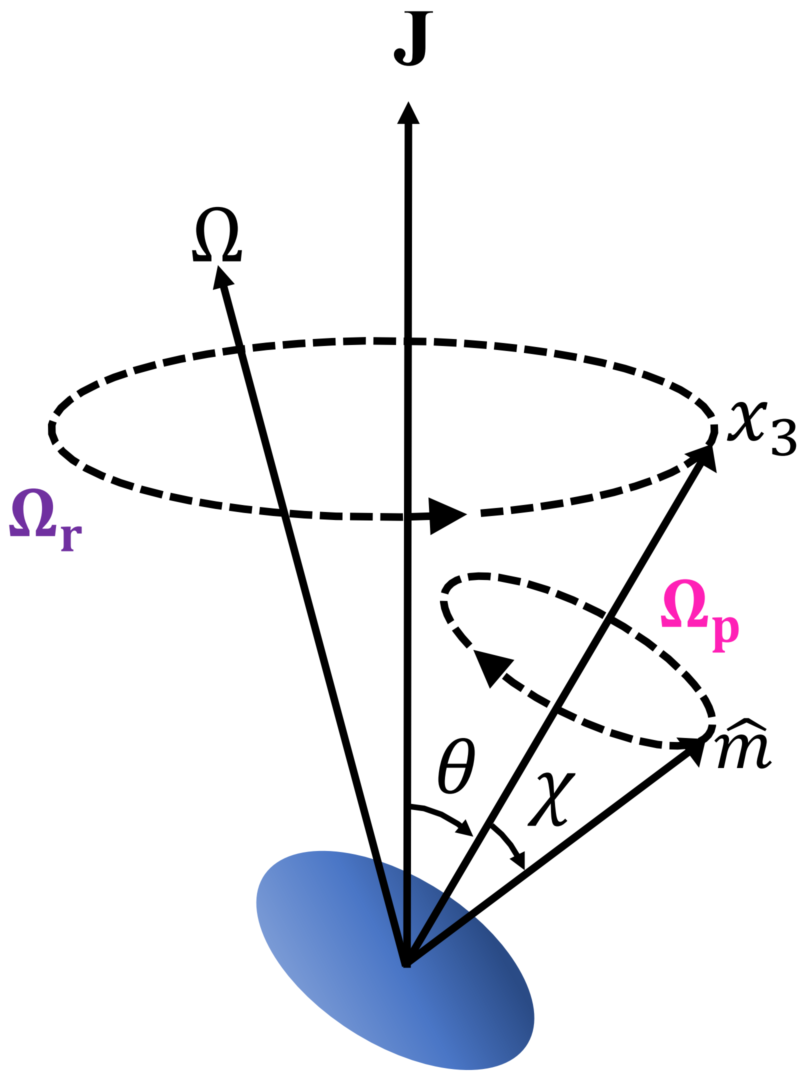
Timing and beam shape data for PSR B1828 –11
(most possible evidence for free precession)

1. Timing residuals: the magnetic dipole undergoes two superimposed motion

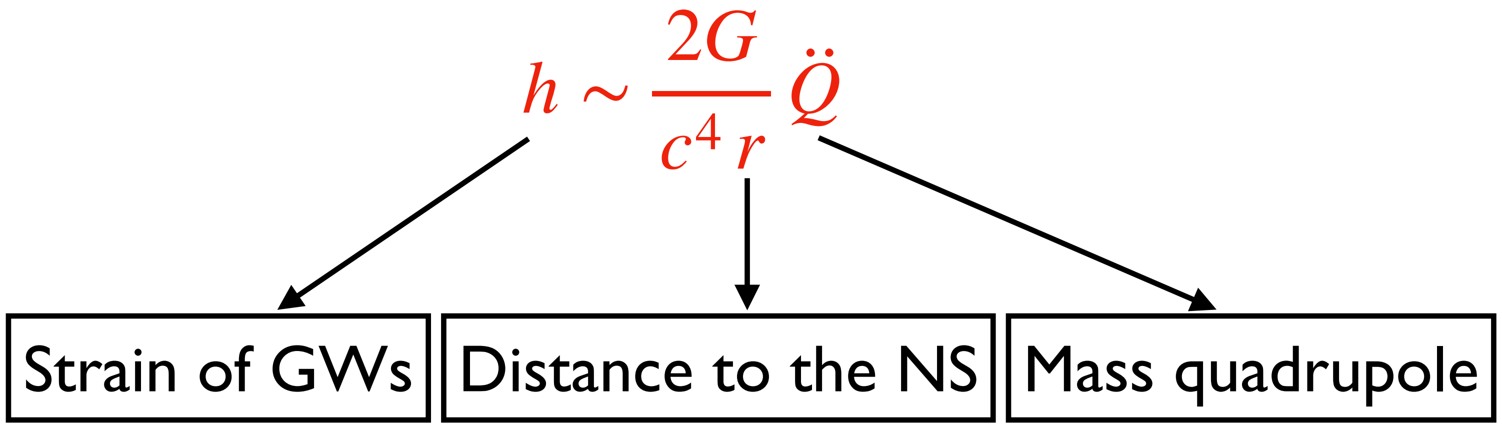
2. Pulse profile modulation: the line of sight sweeps different region of emission cone

[Stairs et al, Nature, 2000; Link& Epstein, ApJ, 2001]

Observational effects 2: continuous gravitational waves



Oblique rotator



$$h \sim \frac{2G}{c^4\, r} \ddot{Q}$$

Strain of GWs

Distance to the NS

Mass quadrupole

- Emit at two harmonics, (f, 2f)
- For small wobble angle, lower harmonic dominant

$$h \sim \frac{G I_0}{r c^4} f^2 \epsilon \, \theta$$

- Detectability limited by oblateness
- Searches are on going

[Zimmermann & Szedenits, PRD, 1979; Jones & Andersson, MNRAS, 2001, 2002]

Our questions and motivation

1. Necessary to be biaxially-deformed?

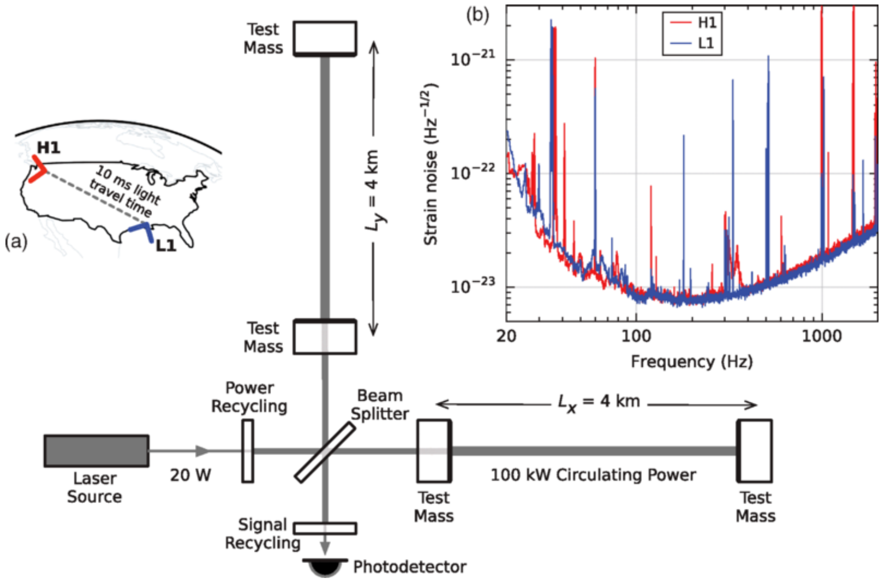
No, change of elastic field, accretion process, magnetic pressure…

→

Extend to fully triaxial case

2. What Information from multi-messenger observation of precessing NS?

LIGO



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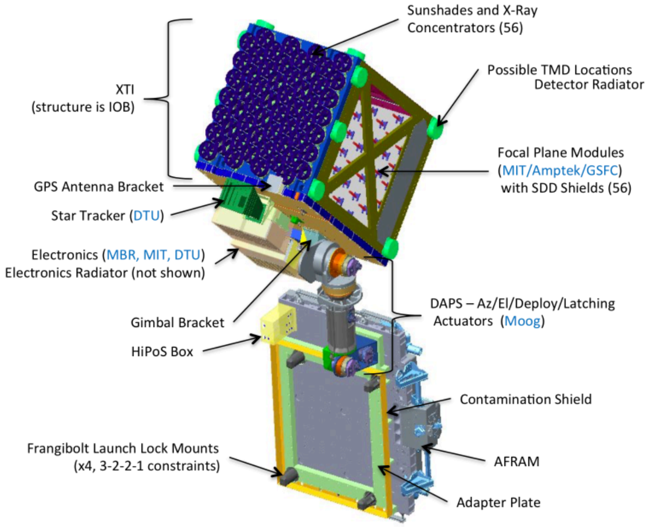
FAST

NICER



GW observation

Radio/X-ray timing

[Gao et al, MNRAS, 2020]

Triaxially-deformed freely-precessing NSs

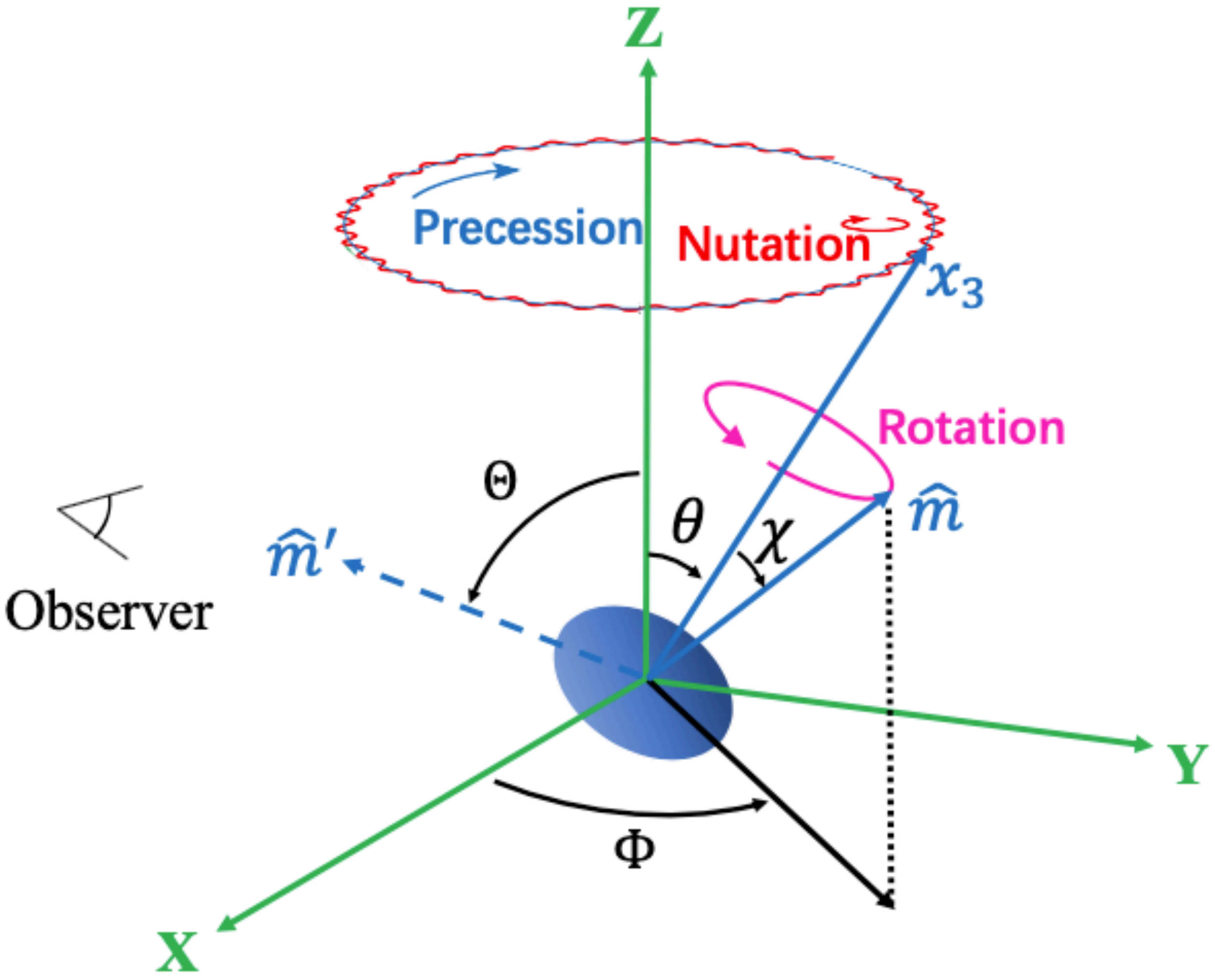
Rotation: ψ

Nutation: θ

Precession: ϕ

Magnetic dipole: $\hat{m}$

Magnetic inclination: χ



- Time evolution of the configuration

Analytical & Numerical

- Parameterized description of NS:

$$\epsilon \equiv \frac{I_3 - I_1}{I_3} \qquad \delta \equiv \frac{I_2 - I_1}{I_3 - I_2} \qquad \gamma \equiv \tan\theta_{\min}$$

Oblateness | **Nonaxisymmetry** | **Wobble**

closely linked to the structure of NSs !

[Zimmermann, PRD, 1980; Gao et al, MNRAS, 2020]

Estimation of ϵ, γ and θ

Oblateness $\epsilon_{\rm elast} \simeq 4.9 \times 10^{-8} \left(\frac{V_c/V}{0.1}\right) \left(\frac{f_{\rm rot}}{100\text{Hz}}\right)^2 \left(\frac{\mu}{10^{30}\,\text{ergcm}^{-3}}\right) R_6^7 M_{1.4}^{-3}$

EoS dependent parameters:

Thickness Of the crust

Shear modulus of the crust

Mass - Radius

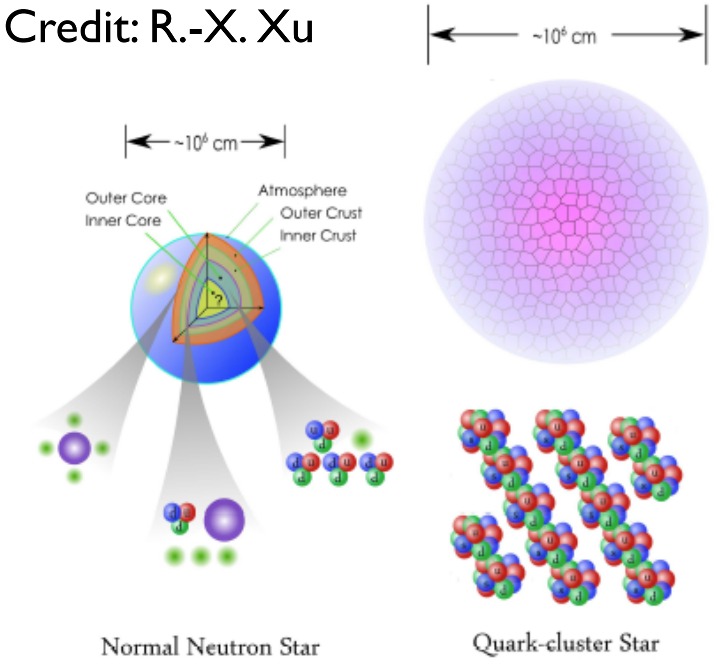
Possible maximum oblateness

$$\epsilon_{\max} \approx 10^{-6} \left(\frac{V_c/V}{0.1}\right) \left(\frac{\mu}{10^{29}\,\text{erg cm}^{-3}}\right) \left(\frac{\sigma_{\rm break}}{0.1}\right)$$

Breaking strain, highly uncertain

Recent lattice study: 0.04-0.1

Credit: R.-X. Xu



Normal Neutron Star

Quark-cluster Star

Conventional NS

$$\epsilon_{\max} \sim 10^{-6}$$

Solid quark star

$$\epsilon_{\max} \sim 10^{-4}$$

Nonaxisymmetry

$$\delta \equiv \frac{I_2 - I_1}{I_3 - I_2}$$

Possible to be any value

Wobble angle

$$\theta_{\max} \approx 0.45 \left(\frac{100\text{Hz}}{f_r}\right)^2 \left(\frac{\sigma_{\rm break}}{10^{-3}}\right) M_{1.4} R_6^{-3}$$

[Jones & Andersson, MNRAS 2002; Xu, ApJL, 2003; Owen, PRL, 2005; Horowwitz & Kadau, PRL, 2008]

Modulated timing signals

- Modulated timing signal of pulsar

$$\Delta\Phi = F(\psi, \theta, \phi)$$

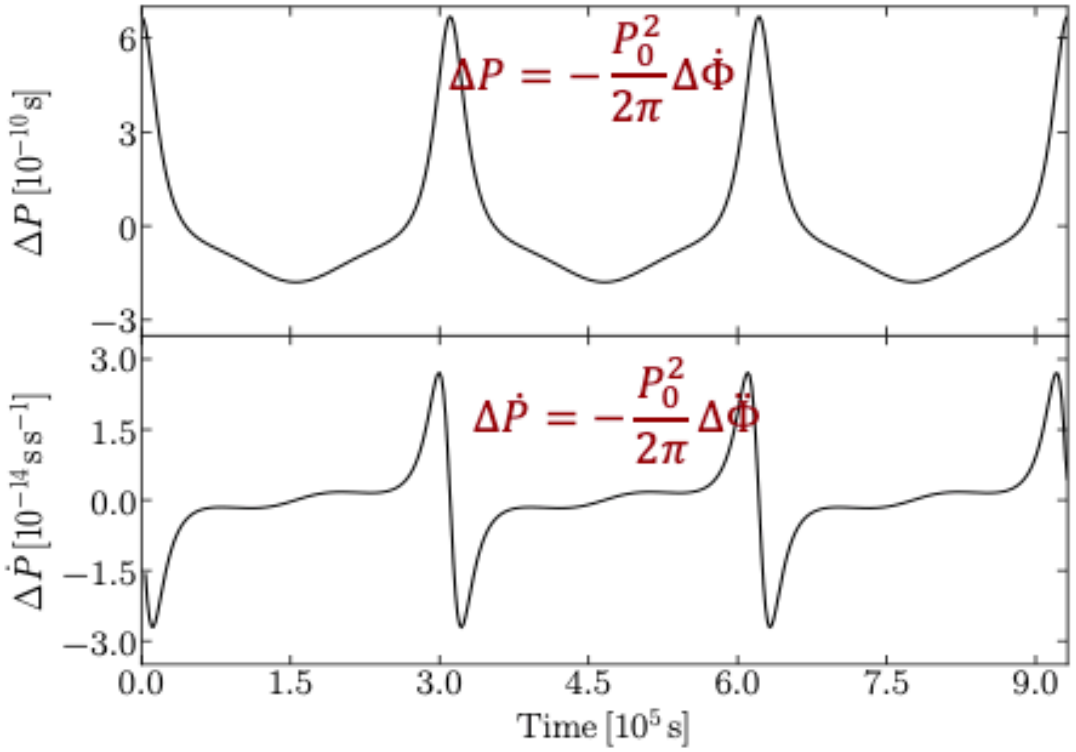
Phase residual due to precession = Function of NS configuration

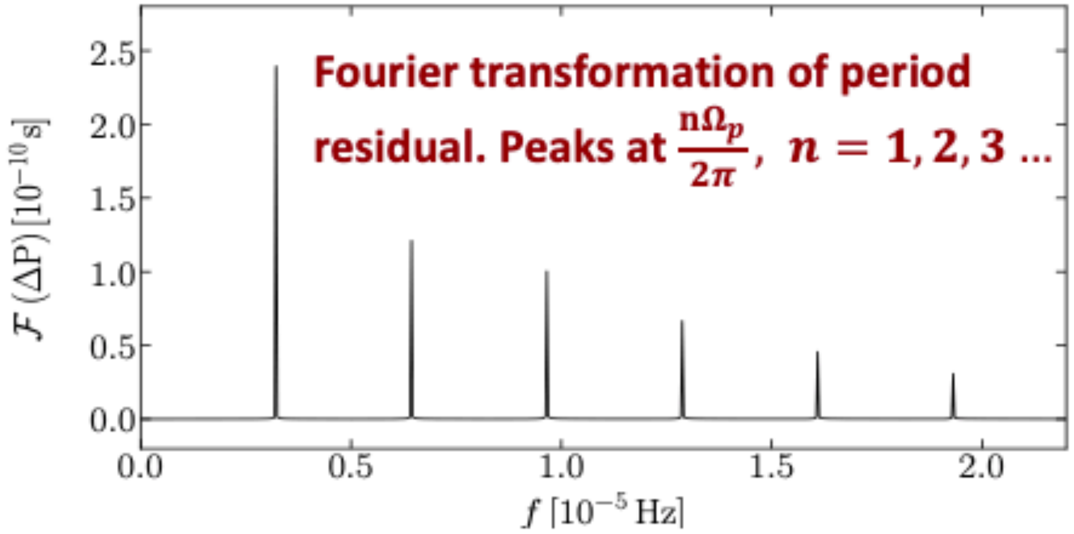
- Small wobble angle case

$$\Delta P \approx \frac{P_0^2}{2\pi}\Omega_{\rm p}\gamma(\delta+1)\cot\chi\cos\left(\Omega_{\rm p}t\right) + \frac{P_0^2}{4\pi}\Omega_{\rm p}\gamma^2\left(1+2\cot^2\chi\right)\cos\left(2\Omega_{\rm p}t\right)$$

1. $\Omega_{\rm p}$ is the free precession angular frequency
2. To second order expansion: $\Omega_{\rm p}$ and $2\Omega_{\rm p}$

Large wobble angle case





Fourier transformation of period residual. Peaks at $\frac{n\Omega_p}{2\pi}$, $n = 1, 2, 3 \ldots$

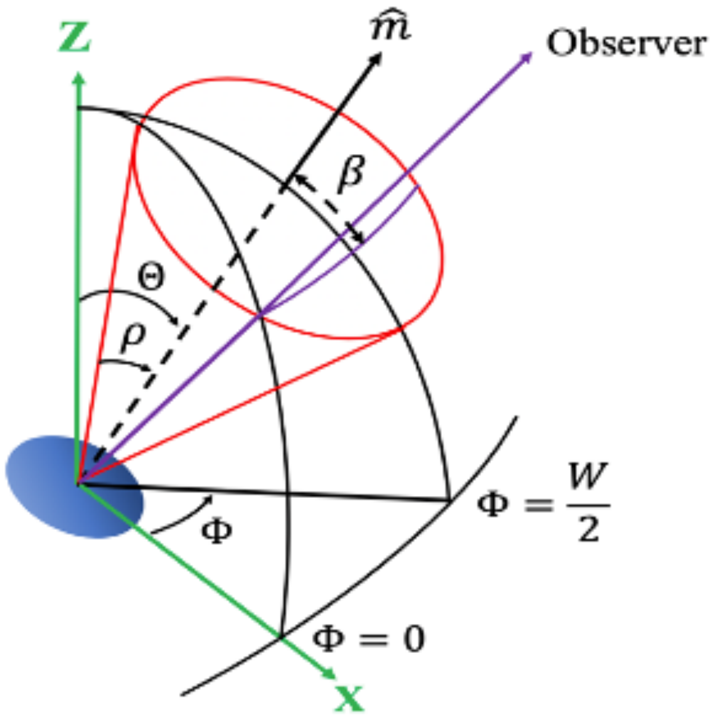
$P_0 = 10\,\mathrm{ms}$ $P_{\rm fp} = 3.1\times10^5\,\mathrm{s}$

$\epsilon = 4.9\times10^{-8}$ $\delta = 0.1$ $\theta \in (0.79, 0.84)$

[Jones & Andersson, MNRAS, 2001; Gao et al, MNRAS, 2020]

Modulated pulse signals

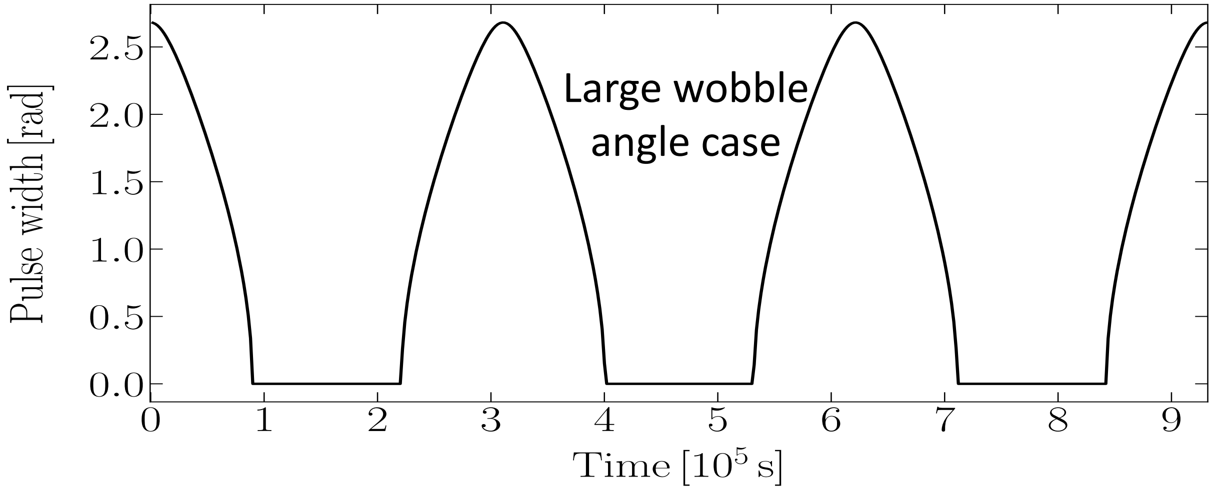
- Simple Cone model

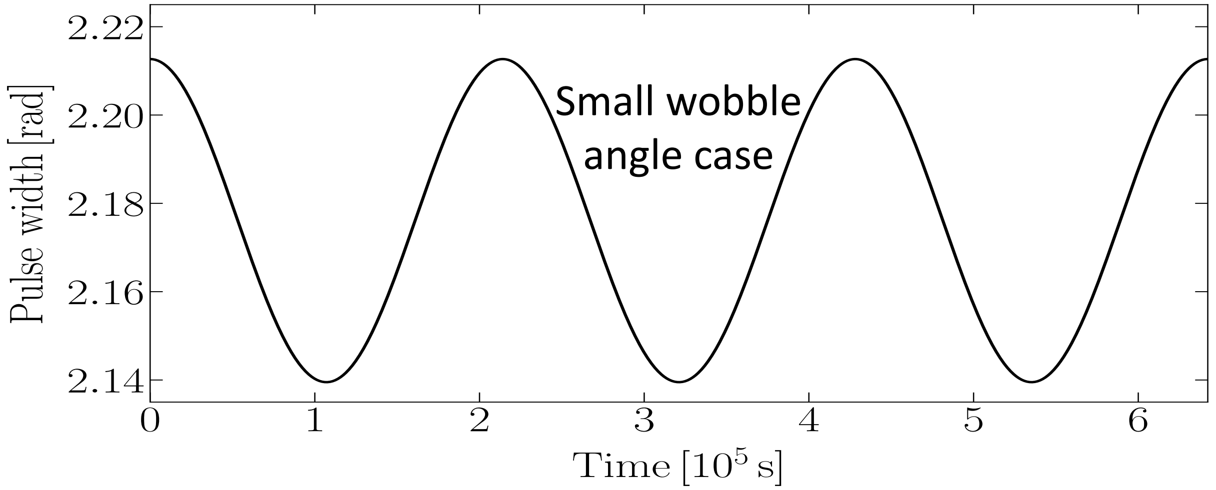


- Modulated pulse widths

$$W = G(\Theta, \rho), \quad \Theta = H(\psi, \theta, \phi, \chi, \iota)$$

Pulse-width due to precession = Function of NS configuration, opening angle of emission cone, line of sight





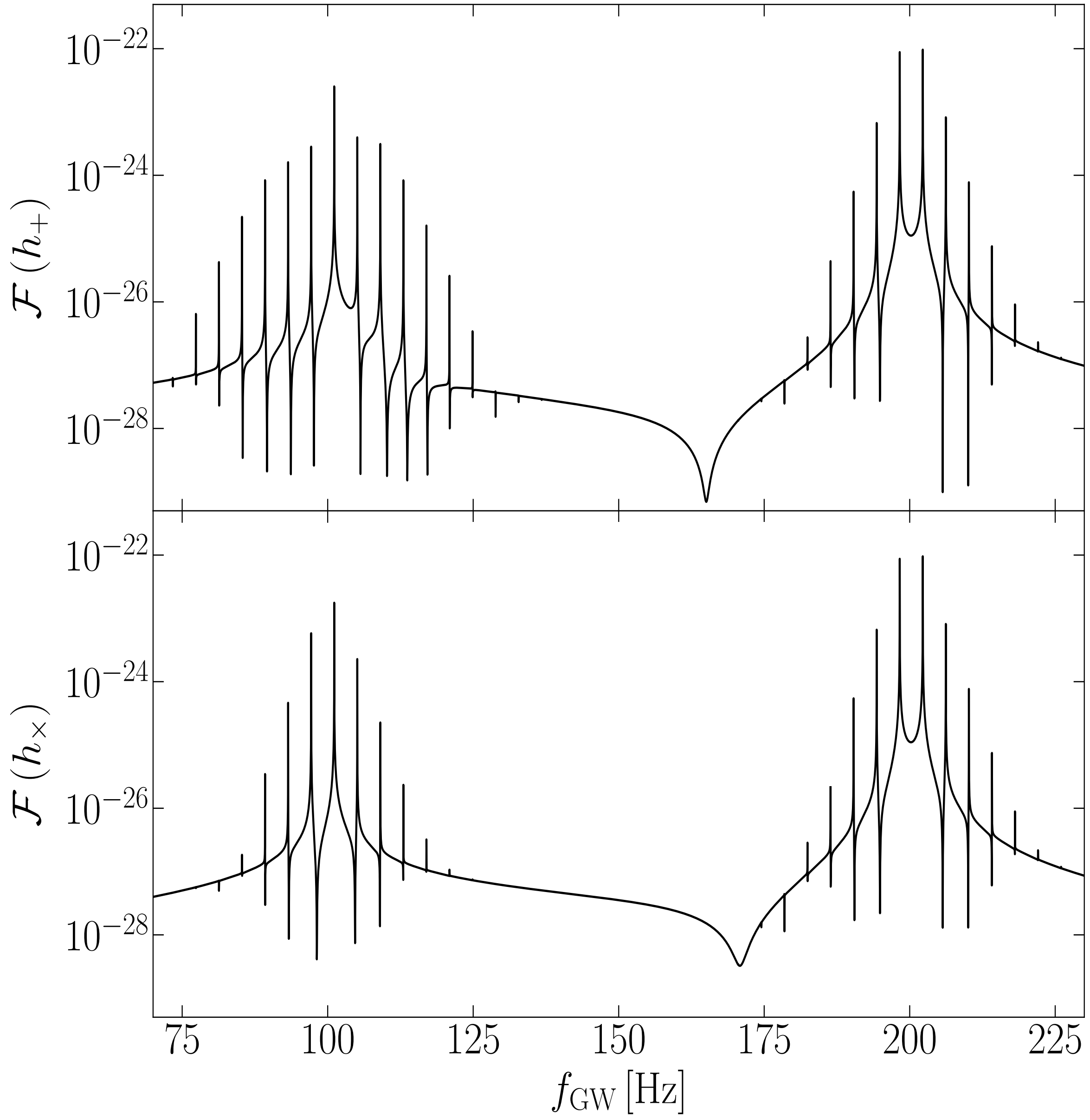
1. Large wobble angle: may lose the emission during precession

2. Small wobble angle: modulation is much weaker

[Jones & Andersson, MNRAS, 2001; Gao et al, MNRAS, 2020]

Continuous gravitational waves

- Generic waveform



$$h_{ij}^{\rm TT} = \frac{2G}{c^4 r}\frac{\mathrm{d}^2 I_{ij}}{\mathrm{d}t^2} = -\frac{2G}{rc^4}\mathcal{R}_{ik}\mathcal{R}_{jl}A_{kl}$$

- Small γ and δ case (to second order expansion)

First order lines at $\Omega_{\rm r} + \Omega_{\rm p}$ and $2\Omega_{\rm r}$

$$A_\times^1 = 1.0\times 10^{-28}\,\gamma\sin\iota\left(\frac{\epsilon}{4.9\times 10^{-8}}\right)\left(\frac{f_{\rm r}}{100\,\rm Hz}\right)^2\left(\frac{10\,\rm kpc}{r}\right)$$

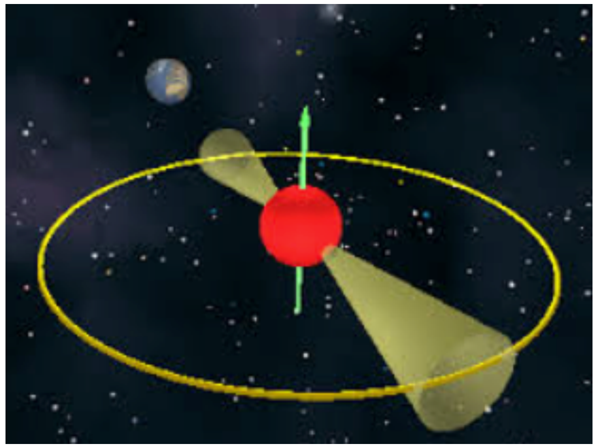
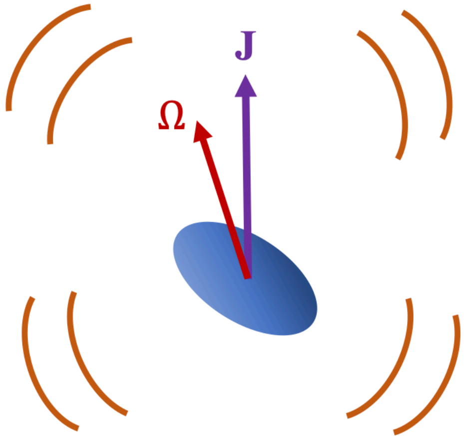
$$A_\times^2 = 2.1\times 10^{-28}\,\delta\cos\iota\left(\frac{\epsilon}{4.9\times 10^{-8}}\right)\left(\frac{f_{\rm r}}{100\,\rm Hz}\right)^2\left(\frac{10\,\rm kpc}{r}\right)$$

Second order lines at

$2\left(\Omega_{\rm r} + \Omega_{\rm p}\right)$, $\Omega_{\rm r} - \Omega_{\rm p}$, $\Omega_{\rm r} - 3\Omega_{\rm p}$, and $2\left(\Omega_{\rm r} - \Omega_{\rm p}\right)$

[Zimmermann, PRD, 1980; Chris, CQG, 2005; Gao et al, MNRAS, 2020]

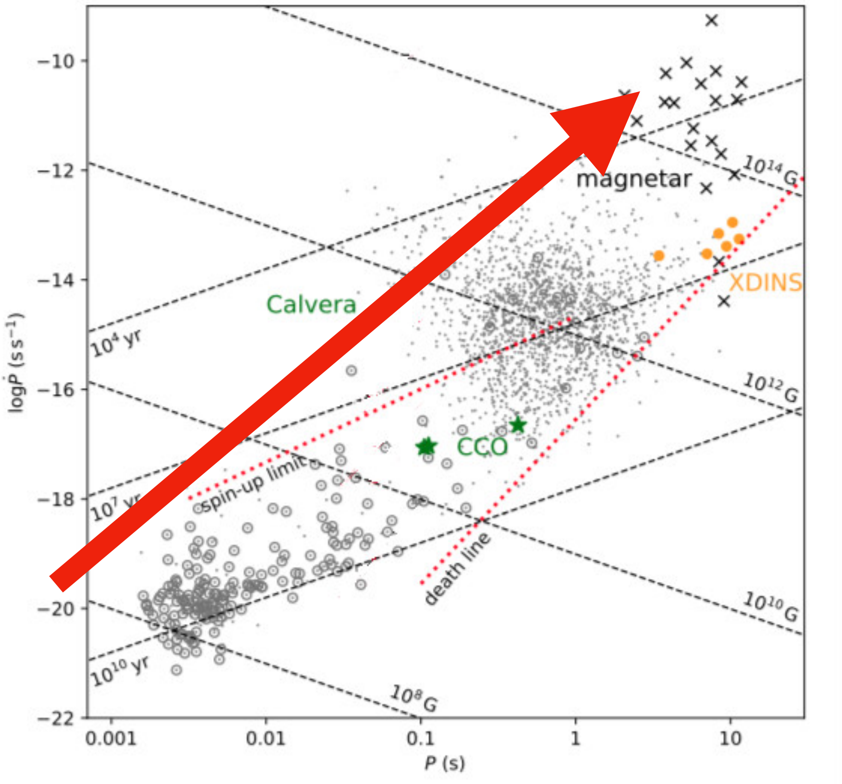
Multimessenger observation: Extraction of physical parameters

	First order	Second order
 **EM**	$\Delta P_1 \sim \gamma \cot\chi \, P_0^2 \, \Omega_\mathrm{p}$	$\Delta P_2 \sim \gamma\delta \cot\chi \, P_0^2 \, \Omega_\mathrm{p}$ $\Delta P_3 \sim \gamma^2 \left(1 + 2\cot^2\chi\right) \, P_0^2 \, \Omega_\mathrm{p}$
 **GW**	$A^1_\times \sim \dfrac{I_0 \gamma \epsilon f_\mathrm{r}^2 \sin\iota}{r}$ $A^1_\times \sim \dfrac{I_0 \delta \epsilon f_\mathrm{r}^2 \cos\iota}{r}$	Dependent on γ^2, δ^2, and $\gamma\delta$

Multi-messenger observation: extract ϵ, δ, γ , χ, ι, and Ω_p

Future possible work

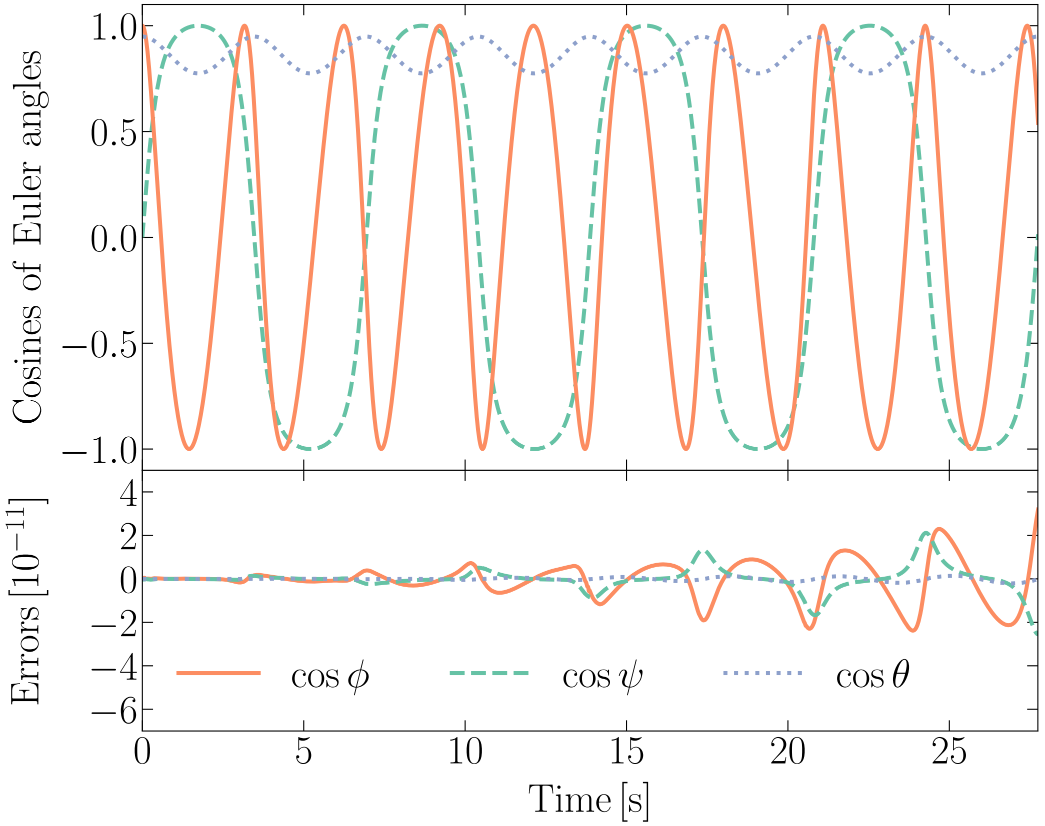
Credit: Yunyang Li



Numerical approach using quaternions

$$I_1/I_3 = 1/3$$

$$I_2/I_3 = 2/3$$



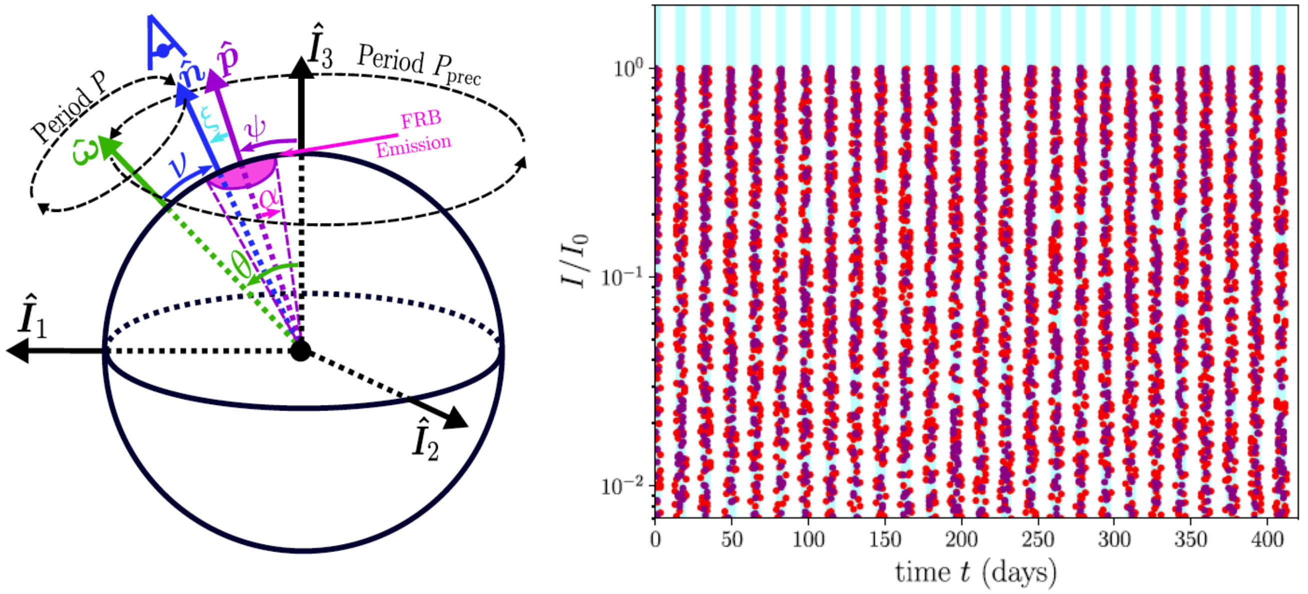
$$\Delta\Phi \sim \frac{1}{\pi}\cot\chi\,\frac{\theta}{\epsilon^2}\,\frac{P_0}{\tau_e} \longrightarrow$$ Spin down time scale

Millisecond pulsars: unimportant

Young pulsars: greatly amplify residuals

Magnetar: must consider and

$$\epsilon_{\rm mag} = \beta\frac{R^4B_\star^2}{GM^2} = 1.9\times10^{-6}\beta\left(\frac{B_\star}{10^{15}\mathrm{G}}\right)^2\frac{R_6^4}{M_{1.4}^2}$$



Precession of magnetar to explain ~16 day periodicity of FRB 180916.J0158+65 (Zanazzi & Lai, 2020)

[Jones & Andersson, MNRAS, 2002; Zanazzi & Lai, ApJ, 2020; Levin et al, ApJ, 2020]

Summary

- Triaxially-deformed NSs: new features
- Multi-messenger observation: valuable information on equation of state
- Radio/X-ray timing and GWs searches: on going

Thanks!