Tests of conservation laws in post-Newtonian gravity with binary pulsars

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Introduction

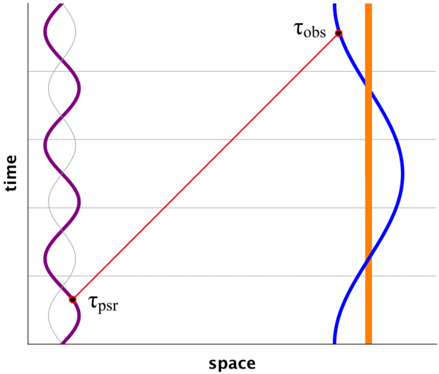
- Post-Newtonian (PN) limit: theory-independent.

At the 1PN order, the degree of violation in conservation laws is expressed via some specific parameterized post-Newtonian (PPN) parameters.

- Fully conservative theories: $\alpha_1 = \alpha_2 = \alpha_3 = \zeta_1 = \zeta_2 = \zeta_3 = \zeta_4 = 0$.
- Semi-conservative theories: $\alpha_3 = \zeta_1 = \zeta_2 = \zeta_3 = \zeta_4 = 0$.
  (Preferred frame,α_1, α_2, Shao & Wex (2012),Shao et al. (2013))
- Non-conservative theories: one or more of $\{\alpha_3, \zeta_1, \zeta_2, \zeta_3, \zeta_4\}$ will be zero.

Will, C. M. 2018, Theory and Experiment in Gravitational Physics

Introduction



- Pulsar timing

$$t_{\mathrm{SSB}} = t_{\mathrm{topo}} + t_{\mathrm{corr}} - \Delta D/f^2 + \Delta_{\mathrm{R}\odot} + \Delta_{\mathrm{S}\odot} + \Delta_{\mathrm{E}\odot} + \Delta_{\mathrm{RB}} + \Delta_{\mathrm{SB}} + \Delta_{\mathrm{EB}} + \Delta_{\mathrm{AB}}$$

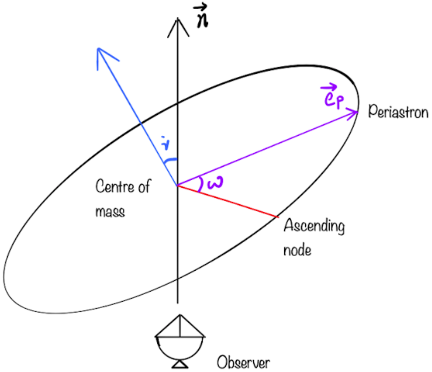
Wex, N. 2014, arxiv:1402.5594

Lorimer, D. R.,& Kramer, M. 2005, Handbook of Pulsar Astronomy

For nonconservative gravity theories with the PPN parameter ζ_2, there exists a self-acceleration for the center of mass of a binary system

$$a_{\rm cm}(t) = \frac{\zeta_2}{2} c T_\odot m_c \frac{q(q-1)}{(1+q)^2} \left(\frac{2\pi}{P_b}\right)^2 \frac{e}{(1-e^2)^{3/2}} \hat{\mathbf{e}}_{\rm p}(t) \,,$$

where $T_\odot = GM_\odot/c^3 \simeq 4.9254909 \mu s$.



Will, C. M. 1992, ApJ, 205, 861

The relation between $\ddot{\nu}$ and ζ_2 is,

$$\frac{1}{\nu}\frac{\mathrm{d}^2\nu}{\mathrm{d}t^2} = -\frac{\mathcal{A}_2}{c}\cos\omega\frac{\mathrm{d}\omega}{\mathrm{d}t}$$
$$\frac{1}{\nu}\frac{\mathrm{d}^3\nu}{\mathrm{d}t^3} = \frac{\mathcal{A}_2}{c}\sin\omega\left(\frac{\mathrm{d}\omega}{\mathrm{d}t}\right)^2 ,$$

where

$$\mathcal{A}_2 \equiv -\frac{\zeta_2}{2}cT_\odot\left(\frac{2\pi}{P_b}\right)^2\frac{q(q-1)}{(1+q)^2}\frac{e}{(1-e^2)^{3/2}}m_c\sin i\,,$$

The latest bound on ζ_2 was obtained by Will in 1992. He used the second time derivative of the spin period ($\ddot{P}$) of PSR B1913+16, and $\zeta_2 < 4\times10^{-5}$ at 95% confidence level (C.L.).($\ddot{P} = 4\times10^{-30}\,\mathrm{s}^{-1}$)

Will, C. M. 1992, ApJ, 205, 861
Xueli Miao, et al. 2020, ApJ, 898, 69

Our results

- Individual bound on ζ_2

Pulsar		Method A	Method B
B2127+11C	$\ddot{\nu}$	3.1×10^{-5}	2.9×10^{-5}
J1756−2251	$\ddot{\nu}$	1.7×10^{-4}	1.8×10^{-4}
B1534+12	$\ddot{\nu}$	4.5×10^{-4}	8.1×10^{-5}
B1913+16	$\dddot{\nu}$	1.2×10^{-3}	8.4×10^{-4}
B1534+12	$\dddot{\nu}$	1.9×10^{-3}	1.9×10^{-3}
B1913+16	$\ddot{\nu}$	4.1×10^{-3}	1.5×10^{-3}

Figure: The bounds on the absolute value of ζ_2 from individual binary pulsar systems at 95% C.L.. For PSR B1913+16, the $\ddot{P} = 5.6 \times 10^{-29}\,\mathrm{s}^{-1}$

- Combined bound on ζ_2

$$|\zeta_2| < 1.3 \times 10^{-5}, \text{ with flat prior in } \log_{10}|\zeta_2| \ (95\%\,\mathrm{C.L.})\,.$$

Xueli Miao, et al. 2020, ApJ, 898, 69

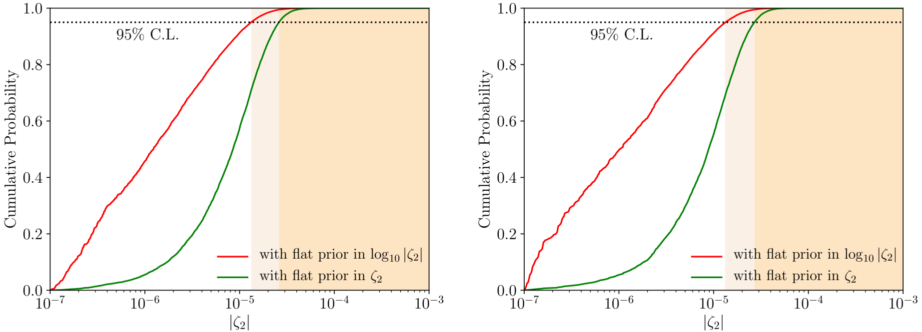


Figure: Cumulative posterior distributions with two different choices of priors. Left is using the Method A, and right is using the Method B for $\omega(t)$.

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A full timing model with simulation data

Extending the Damour-Deruelle (DD) timing model we have

$$t = T + \Delta_{\rm R}(T) + \Delta_{\rm E}(T) + \Delta_{\rm S}(T) + \Delta_{\rm A}(T) + \Delta_{\zeta_2}(T)\,.$$

The acceleration along the line of sight is

$$a_r(t) \equiv \hat{\mathbf{n}} \cdot \boldsymbol{a}_{\rm cm}(t) = \mathcal{A}_2 \sin\left[\omega_0 + \dot{\omega}\left(t - T_0\right)\right]\,.$$

The displacement $z(t)$ along the line of sight is determined via the relation $\ddot{z} = a_r(t)$. So after integration, we obtain

$$z(t) = \frac{\mathcal{A}_2}{\dot{\omega}^2}\left[\sin\omega_0 + \delta\omega\cos\omega_0 - \sin\left(\omega_0 + \delta\omega\right)\right]\,,$$

where $\delta\omega \equiv \dot{\omega}(t - t_0)$. The extra time delay of arrival of pulses can be described by,

$$\Delta_{\zeta_2} = z(t)/c \simeq z(T)/c\,.$$

Xueli Miao, et al. 2020, ApJ, 898, 69

We apply Taylor expansion to Δ_{ζ_2} with respect to $T - t_0$,

$$\Delta_{\zeta_2}(T) = \frac{1}{2}\frac{\mathcal{A}_2}{c}\sin\omega_0\,(T - t_0)^2 + \frac{1}{6}\frac{\mathcal{A}_2\dot{\omega}}{c}\cos\omega_0\,(T - t_0)^3 - \frac{1}{24}\frac{\mathcal{A}_2\dot{\omega}^2}{c}\sin\omega_0\,(T - t_0)^4 + \ldots$$

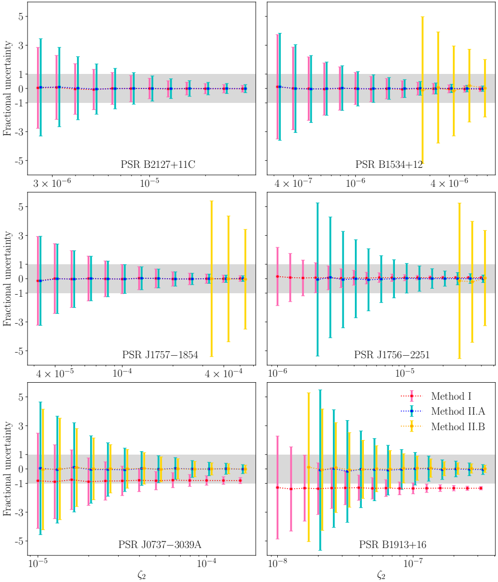
For the extra time delay that is caused by an apparent change in the spin frequency. we have $-\Delta_{\zeta_2} = \delta\phi P = \delta\phi/\nu$, and

$$\delta\phi = \frac{1}{2}\delta\dot{\nu}\,(T - t_0)^2 + \frac{1}{6}\delta\ddot{\nu}\,(T - t_0)^3 + \frac{1}{24}\delta\dddot{\nu}\,(T - t_0)^4 + \ldots.$$

We get the following relations

$$-\frac{\mathcal{A}_2\dot{\omega}}{c}\cos\omega_0 = \delta\ddot{\nu}/\nu$$
$$\frac{\mathcal{A}_2\dot{\omega}^2}{c}\sin\omega_0 = \delta\dddot{\nu}/\nu$$

Xueli Miao, et al. 2020, ApJ, 898, 69



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	Method I		Method II.A		Method II.B	
	$\zeta_2^{\rm crit}$	$\ddot{\nu}$ (Hz^3)	$\zeta_2^{\rm crit}$	$\ddot{\nu}$ (Hz^3)	$\zeta_2^{\rm crit}$	$\dddot{\nu}$ (Hz^4)
B1534+12	1.2×10^{-6}	-4.9×10^{-31}	1.2×10^{-6}	-4.9×10^{-31}	1.2×10^{-5}	-2.0×10^{-38}
J0737−3039A	–	–	4.4×10^{-5}	2.3×10^{-27}	3.4×10^{-5}	-3.2×10^{-34}
J1756−2251	2.0×10^{-6}	5.3×10^{-29}	1.1×10^{-5}	2.9×10^{-28}	1.3×10^{-4}	3.1×10^{-36}
J1757−1854	1.0×10^{-4}	-2.8×10^{-26}	1.0×10^{-4}	-2.8×10^{-26}	1.7×10^{-3}	-1.7×10^{-32}
B2127+11C	7.0×10^{-6}	8.5×10^{-29}	8.0×10^{-6}	9.7×10^{-29}	7.0×10^{-3}	5.5×10^{-35}
B1913+16	–	–	1.0×10^{-7}	2.1×10^{-30}	8.0×10^{-8}	9.5×10^{-39}

Figure: Critical values of ζ_2 for six binary pulsars with three different methods.

Xueli Miao, et al. 2020, ApJ, 898, 69

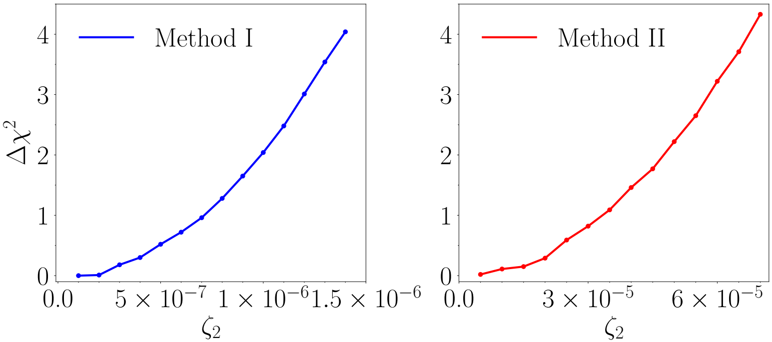

Figure: Changes in the χ^2 as a function of ζ_2 for PSR B1913+16.

Xueli Miao, et al. 2020, ApJ, 898, 69

Summary

- We test conservation laws and give the parameter ζ_2 of PPN a new bound, $|\zeta_2| < 1.3 \times 10^{-5}$.
- For some binary pulsar systems, $\dddot{\nu}$ can give a stronger limit than $\ddot{\nu}$ for ζ_2.
- We provide an extended timing model to test the effect of ζ_2.

Xueli Miao, et al. 2020, ApJ, 898, 69

Thank you!