Large polar caps for twisted magnetosphere of magnetars

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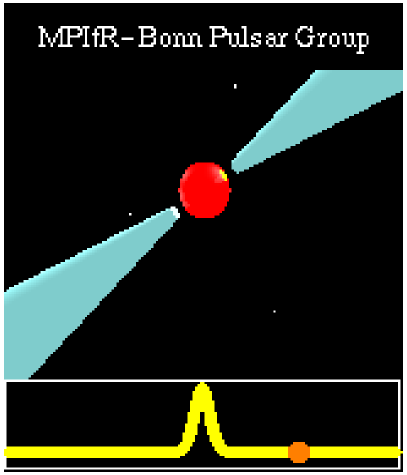
History of pulsars

- 1967: discovery of pulsars (Hewish, Bell et al.)
- ~1970: X-ray pulsars (accreting neutron stars in binary systems, Giacconi et al.)
- 1982: millisecond pulsars (Backer et al.)
  Recycled neutron stars via low mass X-ray binaries
- 1990s: magnetar (Thompson/Duncan, Kouveliotou et al.)

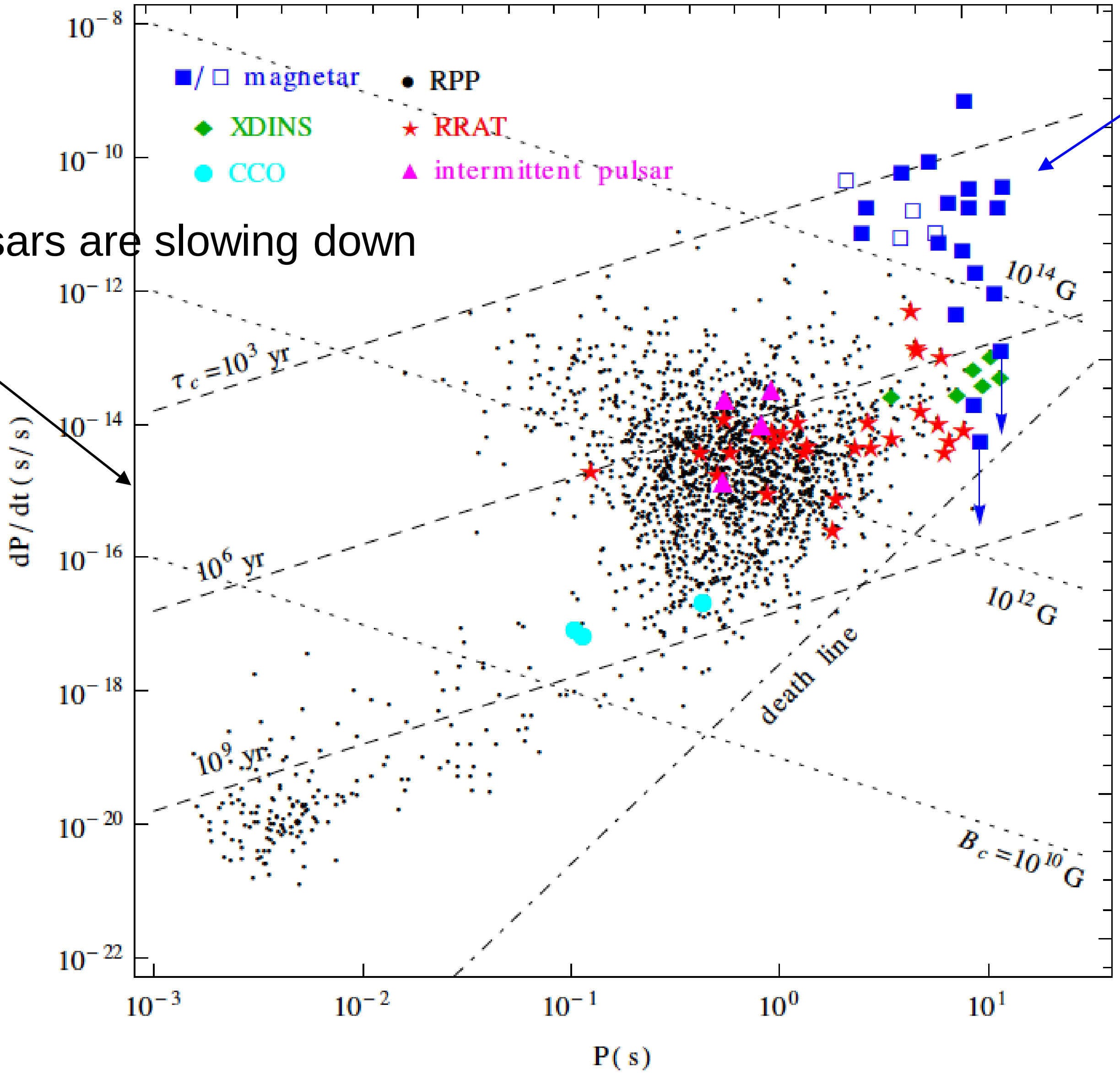
Pulsar

- Pulsar=rotating magnetized neutron star
- Pulsars are good clocks
- Why they are good clocks: (1)radiation; (2)spin down



Magnetars

Pulsars are slowing down



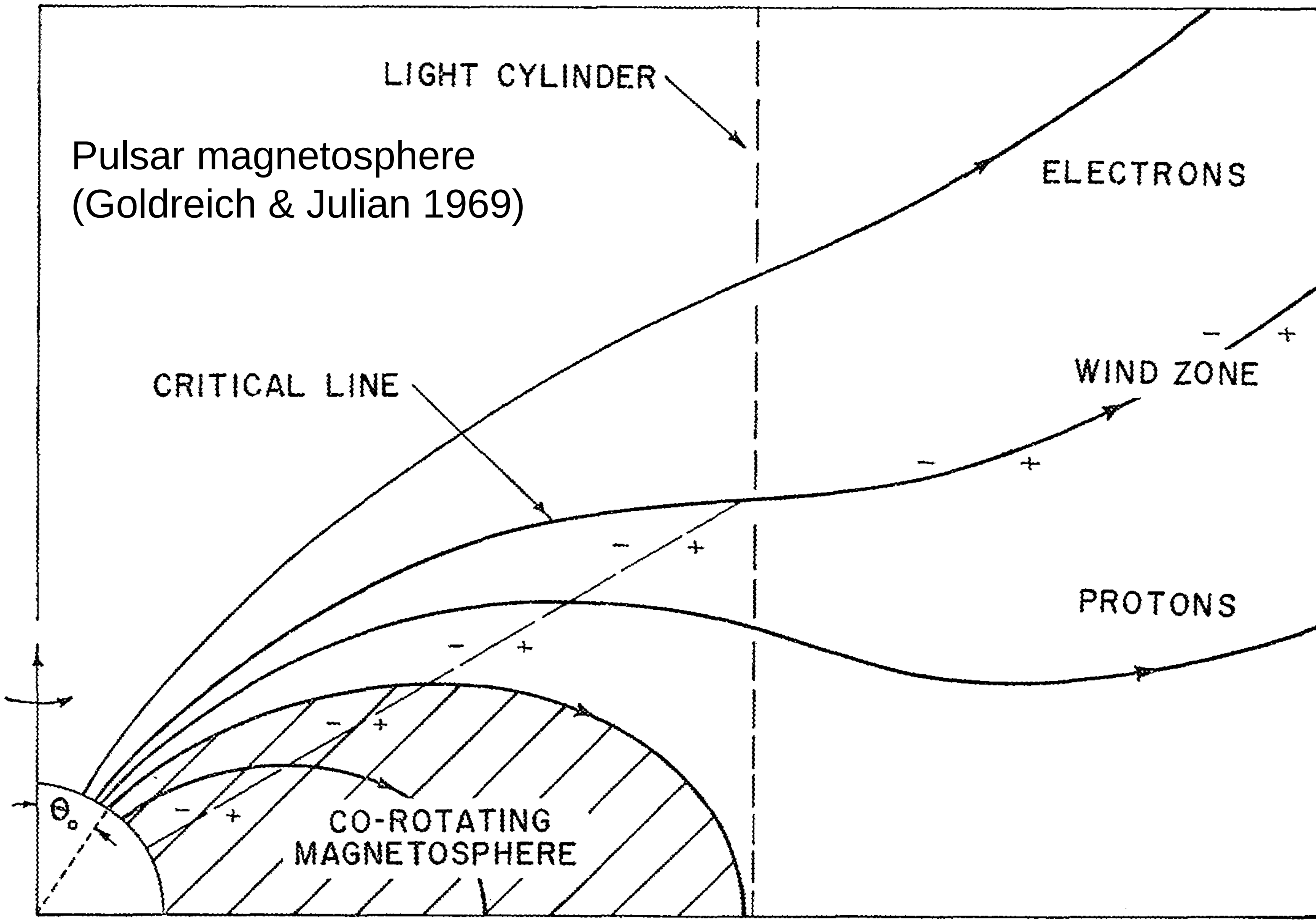
Tong & Wang (2014)

Generic picture

1. Pulsars have a magnetosphere, where there is particle acceleration and subsequent radiation process--> pulse profile
2. When flowing out, this particle component will also take away the rotational energy of the pulsar --> spin down

- Dipole radiation+particle component: wind braking model (Xu & Qiao 2001; Kou & Tong 2015)

Pulsar magnetosphere
(Goldreich & Julian 1969)



Goldreich-Julian density

- Steady state: no Lorentz force

$$\boldsymbol{E} + \boldsymbol{\beta} \times \boldsymbol{B} = 0$$

$$\rho = \frac{\nabla \cdot \boldsymbol{E}}{4\pi} = \frac{-\boldsymbol{\Omega} \cdot \boldsymbol{B}}{2\pi c} \frac{1}{[1 - (\Omega r/c)^2 \sin^2 \theta]}$$

$$n_- - n_+ = 7 \times 10^{-2} B_\mathrm{z} P^{-1} \mathrm{cm}^{-3}$$

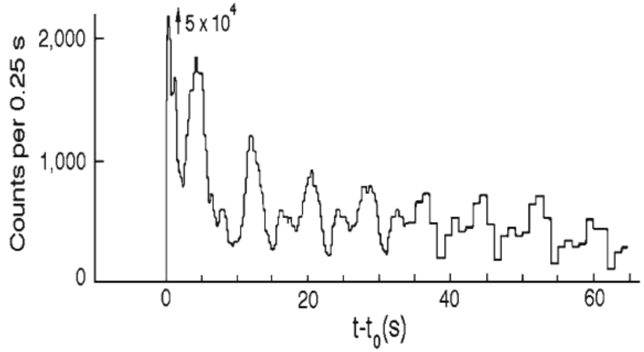
Meaning of GJ density

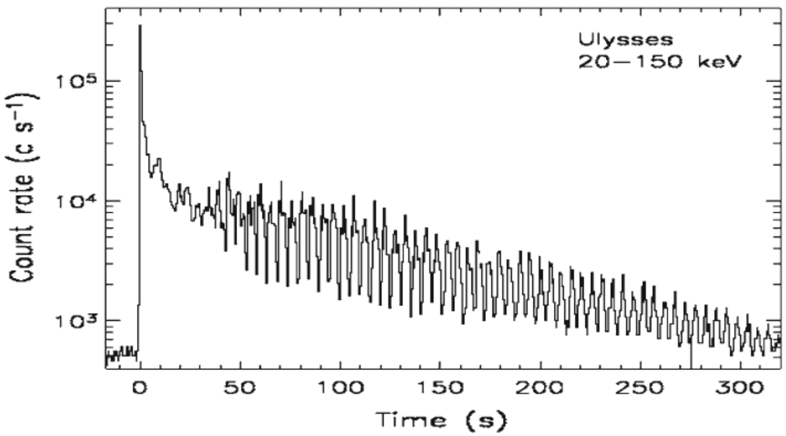
1. Difference between positive and negative charges
2. Only valid for a static magnetosphere
3. If actual charge density derivatives from GJ density: particle acceleration
   (polar gap; free flow; outer gap; annular gap etc)
4. Problem: What's the actual charge density?

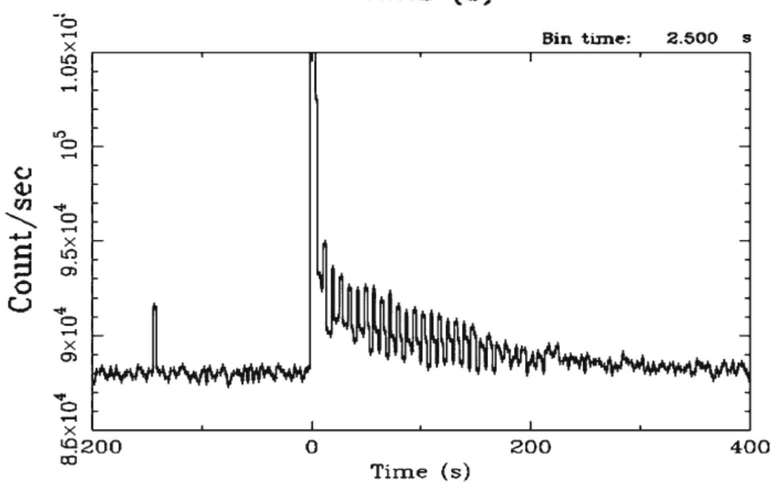
Traditional magnetar model (Mereghetti 2008)

- Magnetar =

1. young NS (SNR & MSC)
2. $B_{dip} > B_{QED} = 4.4 \times 10^{13}$ G (braking)
3. $B_{mul} = 10^{14}$ -10^{15} G (burst and super-Eddington luminosity and persistent emission)







Giant flares of magnetars (Mereghetti 2008):

1. Spike+pulsating tail (hundreds of seconds)

2. 10^4 times super-Eddington during the tail (10^42 erg s^-1)

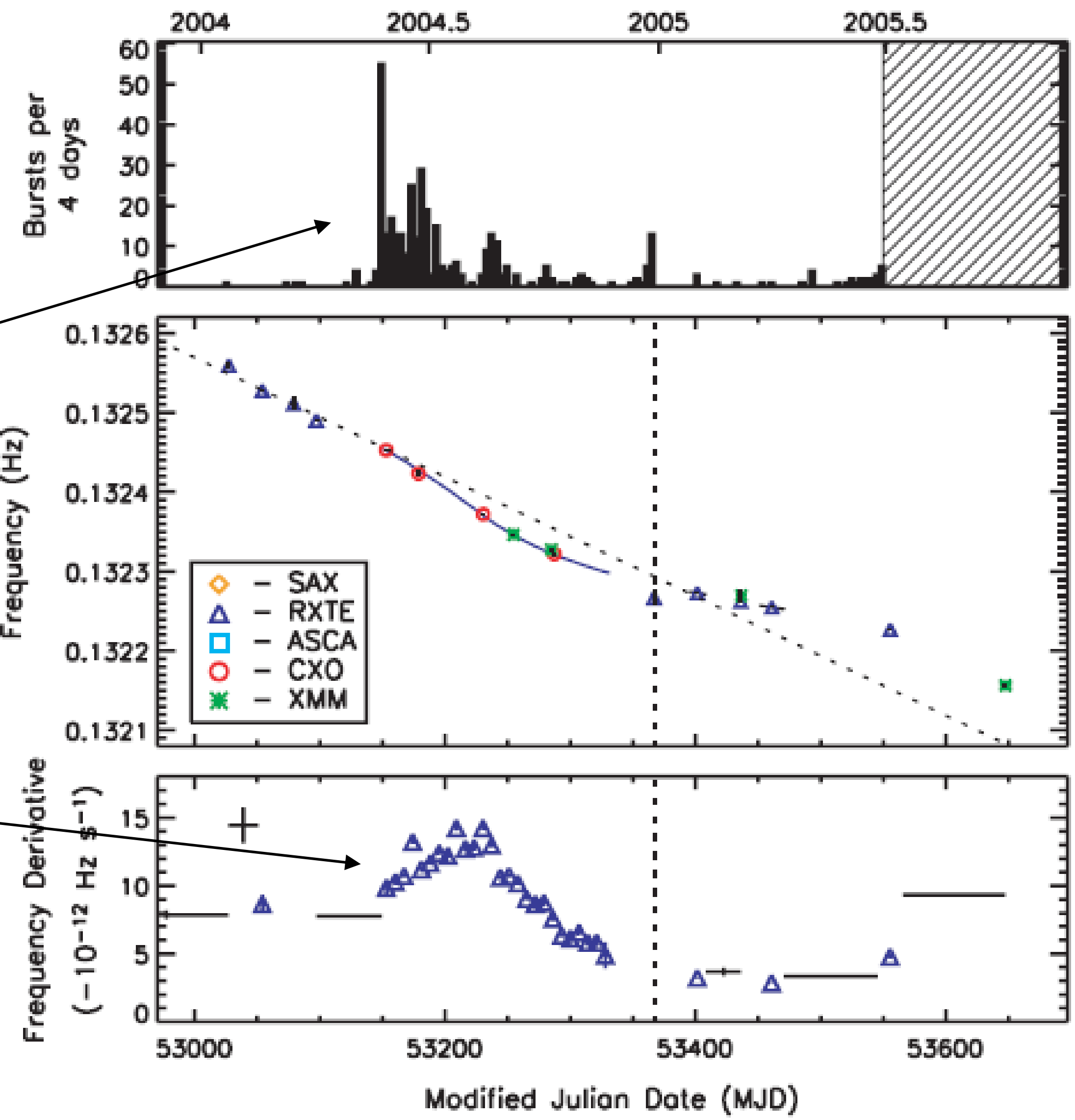
Explanation: 10^15 G magnetic field as the energy power and cause of super-Eddington luminosity

Restless magnetars:

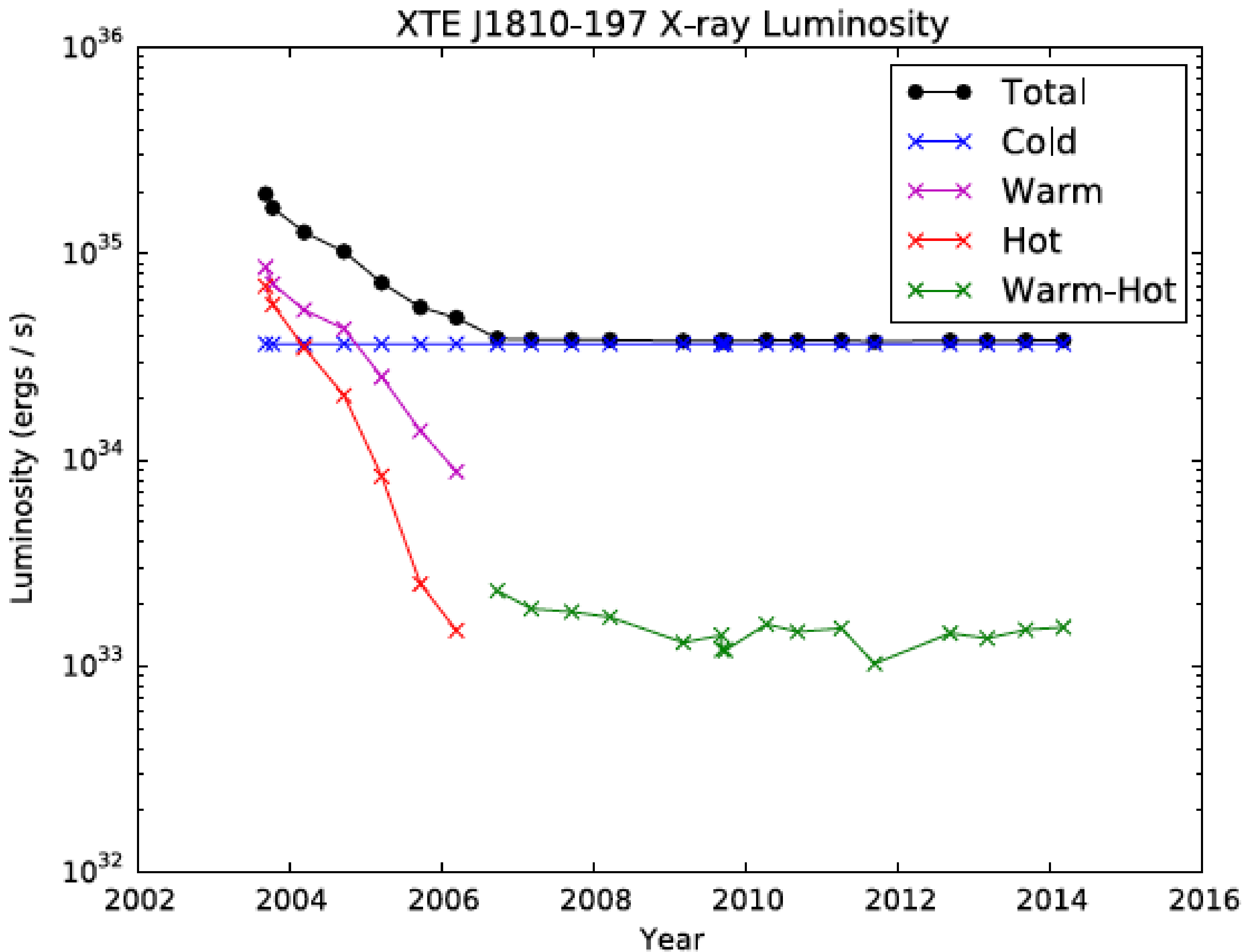
Woods+ 2007

bursts

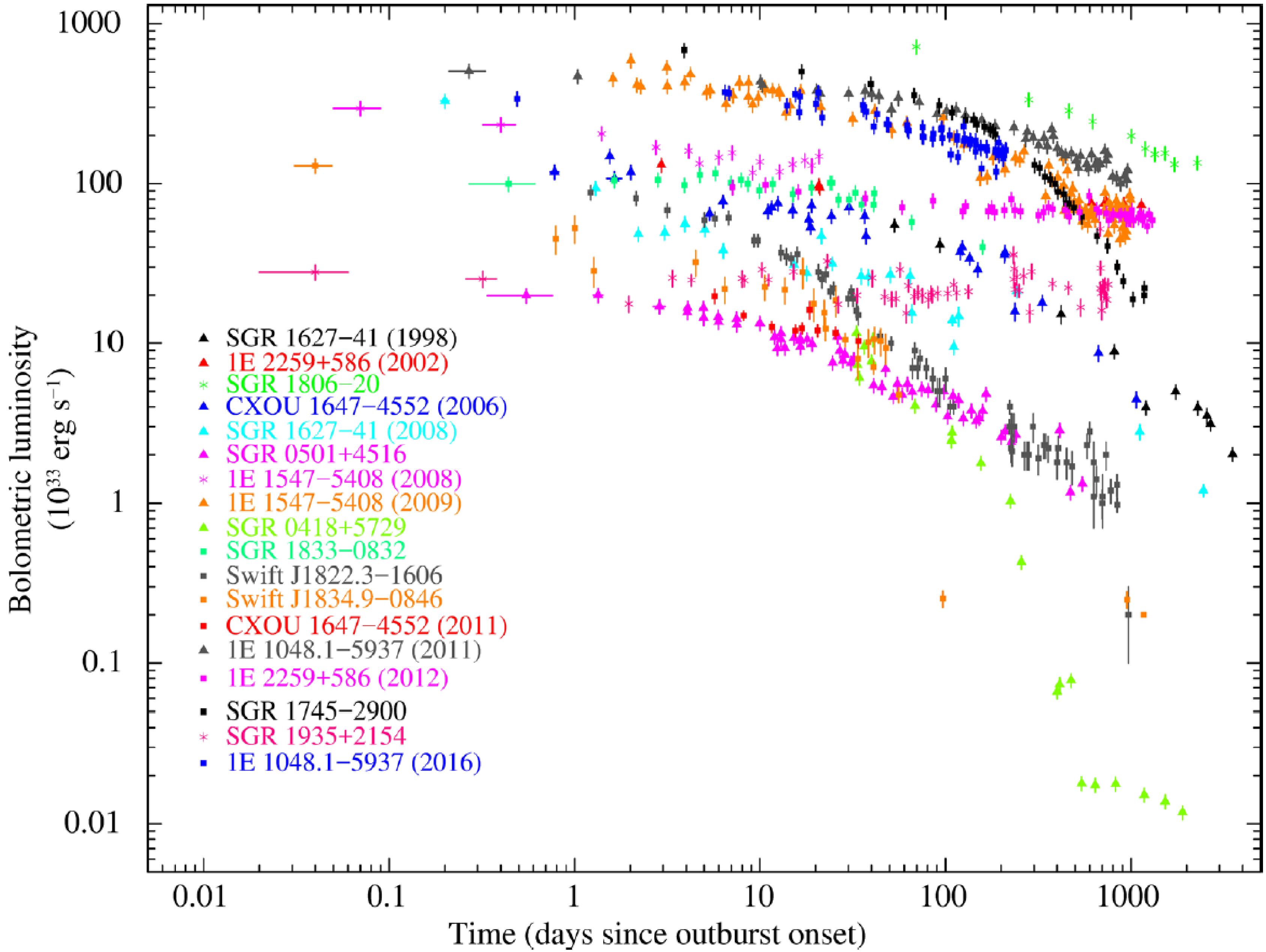
A period of enhanced spindown



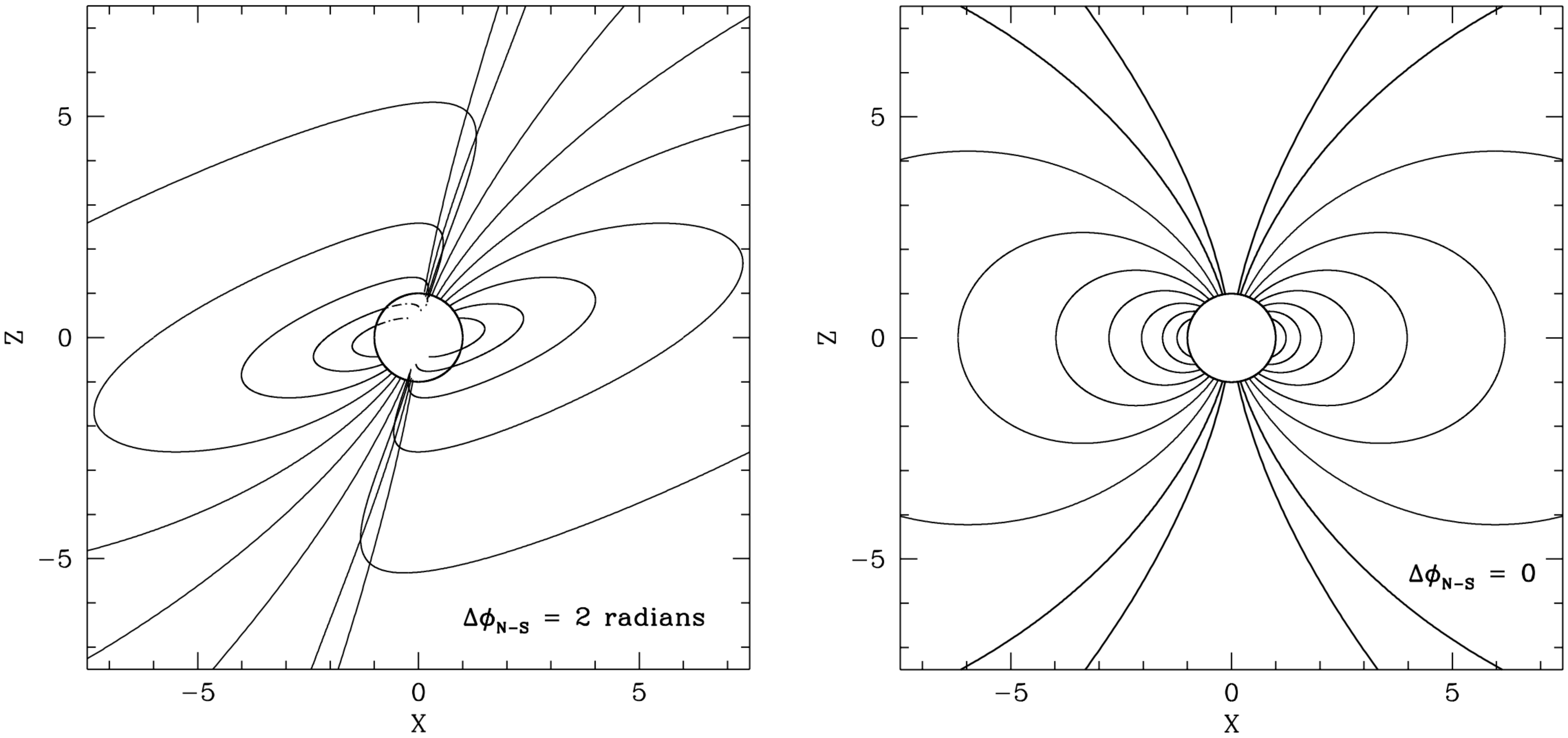
outburst decay over 10 years
Alford & Halpern 2016



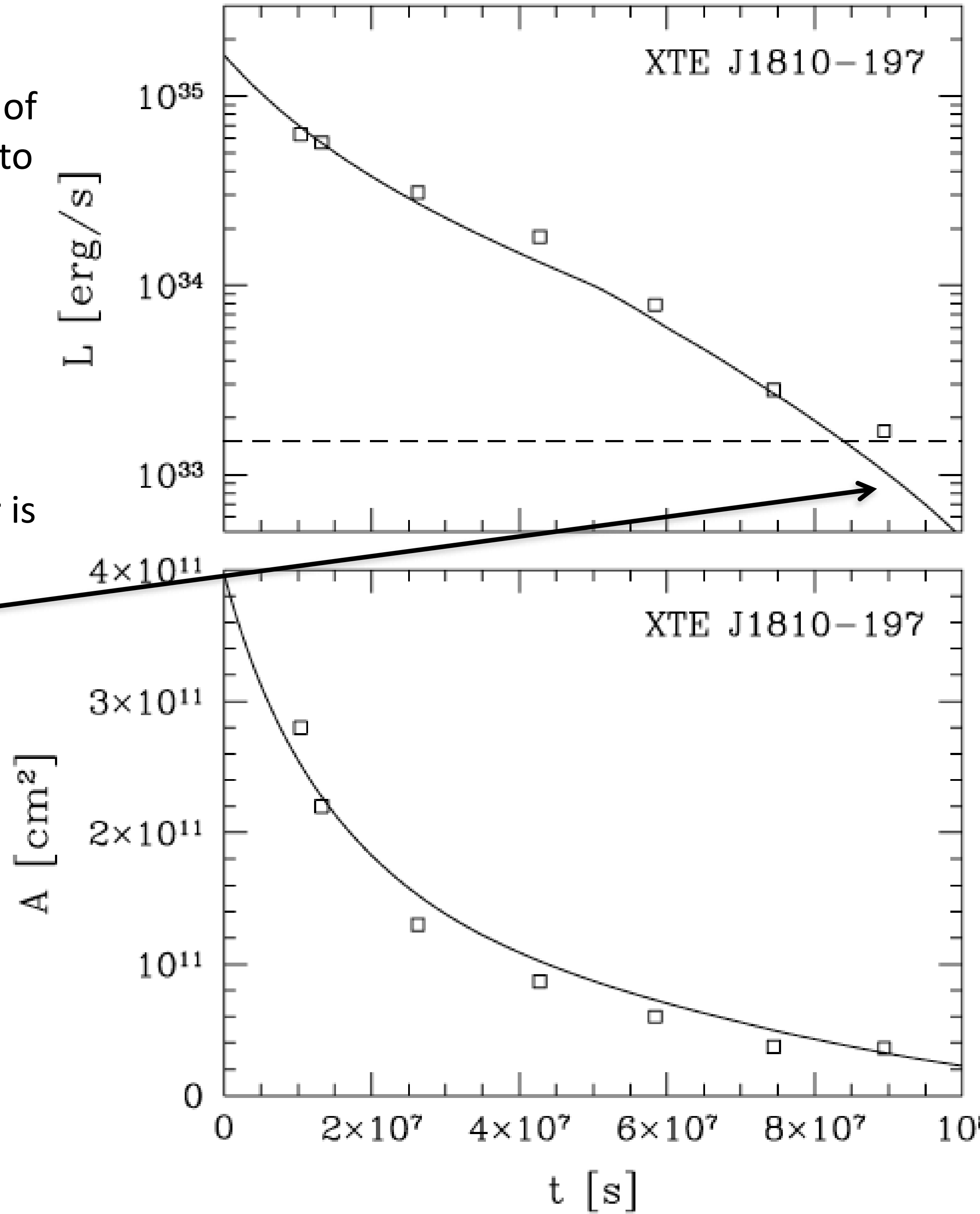
Other magnetar outburst light curve: Coti Zelati et al. (2018)



Magnetars may have twisted magnetosphere compared with that of normal pulsars (Thompson et al. 2002)



untwisting magnetosphere of magnetars and application to XTE J1810-197
Beloborodov 2009



two shortcomings:

1. the central neutron star is non-rotating
2. it decays quickly

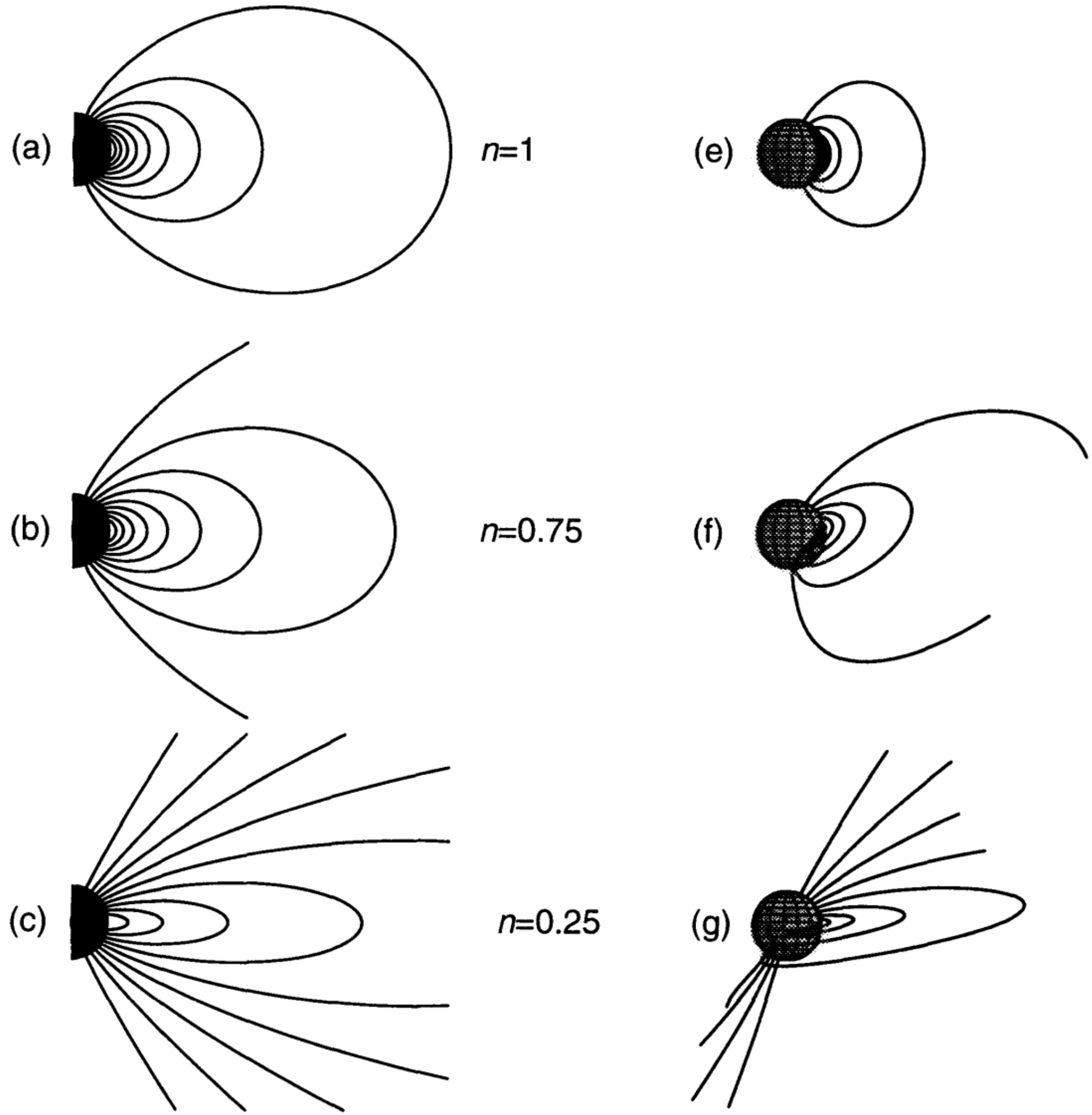
More information

- Lynden-Bell & Boily (1994), Wolfson (1995): self-similar solution
- Thompson et al. (2002), Pavan et al. (2009), Beloborodov (2009): twisted magnetosphere of magnetars (global or local)
- Glampedakis et al. (2014), Gourgouliatos (2008), Fujisawa & Kisaka (2014), Akgun et al. (2016), Pili et al. (2015), Kojima 2017 ….: magnetars with localized twist in the magnetosphere

One wrong concept

- Magnetars have long period (P ~ 10s), then their polar caps is very small (~100m)
- Therefore, the open field line regions of magnetar magnetosphere are not considered. Magnetars have no open field lines, no polar caps in these models. Magnetars (closed field line matters) and pulsars (open field line matters) are total different and can not be unified.
- This is wrong: magnetar may have twisted magnetic field. For a twisted magnetic field, magnetar may have large polar caps despite of their long period.

Wolfson 1995



Twist example: 麻花，小蛮腰

图片来源：百度

油条： fired twisted dough sticks
→ Small twist case

The magnetosphere of pulsars and magnetars may be in the force-free equilibrium state. Assuming axisymmetric condition, the magnetosphere is described by the Grad-Shafranov equation. In spherical coordinate, the Grad-Shafranov equation is:

$$\frac{\partial^2 A}{\partial r^2} + \frac{1-x^2}{r^2}\frac{\partial^2 A}{\partial x^2} + F(A)\frac{dF}{dA} = 0, \tag{1}$$

where r is the dimensionless radial coordinate (in units of the neutron star radius), $x = \cos\theta$ (θ is the polar angle), $A = A(r, \theta)$ is the flux flunction, and $F(A)$ is a yet undetermined function (Wolfson 1995 and references therein). The

For self-similar solutions, the dimensionless flux function governed by the following ODE:

$$(1-x^2)f''(x)+n(n+1)f(x)+\lambda^2\left(1+\frac{1}{n}\right)f^{1+2/n}(x)=0. \tag{4}$$

When $\lambda=0$ and $n=1$, this corresponds to the magnetic dipole field. When λ is different from zero, this means the presence of toroidal field, and the magnetic field is twisted compared with the dipole case. The polar axis should be a

Analytical solutions for the small twist case:

$$f(x) = f_0(x)\left[1 - \frac{22-5x^2}{32}x^2(1-n)\right].$$

For the polar cap region, the polar angle is relatively small, e.g., $\theta \le 0.3$, and $x = \cos\theta > 0.9$. This will corresponds to a hot spot of 3 km if observed[1]. In this case, the analytical solution can match the numerical solutions quite well, even for n as small as $n = 0.1$, see figure 2. This fea-

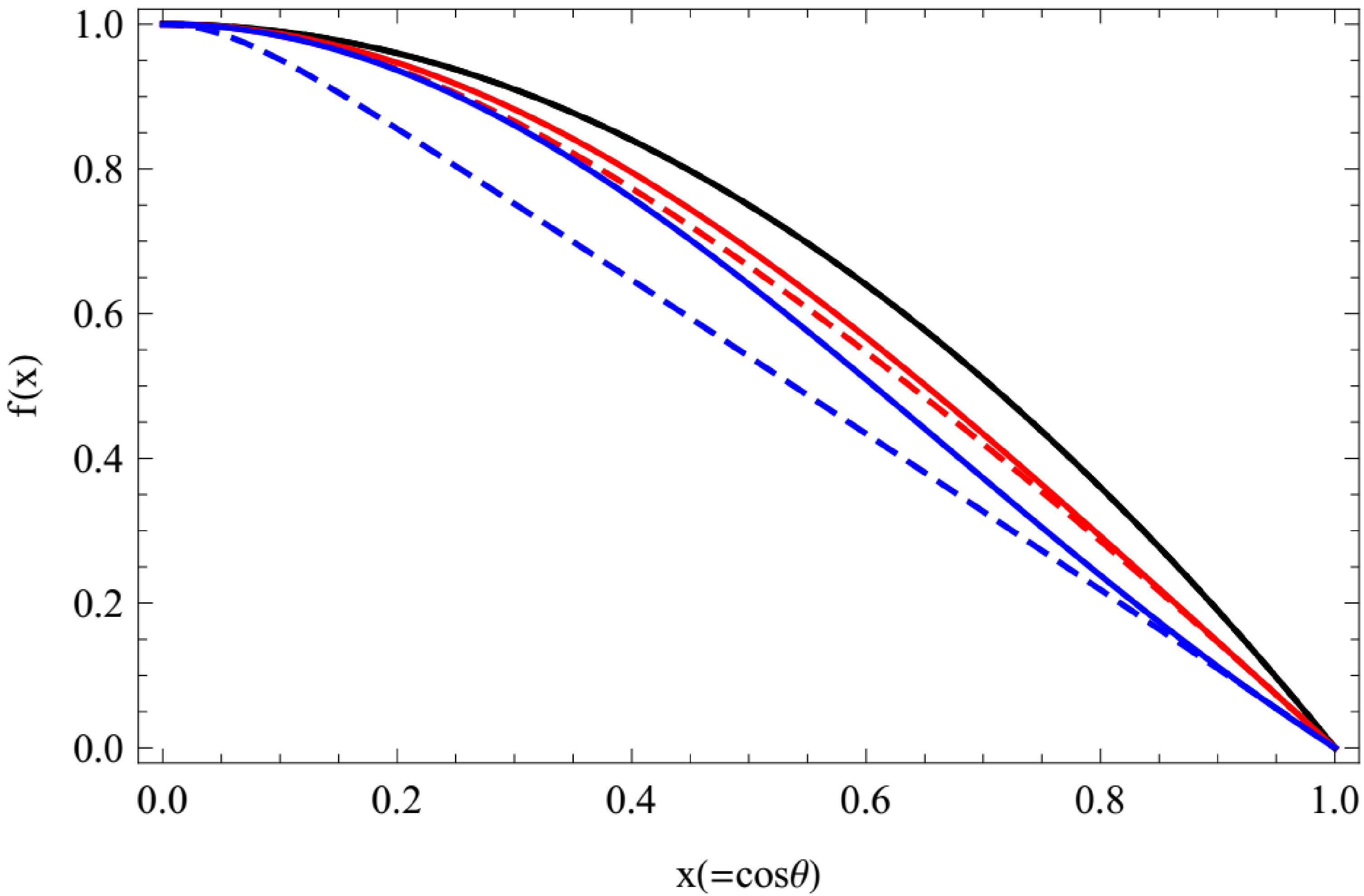


Figure 1. Dimensionless magnetic flux as a function of polar angle. The solid lines are analytical approximations. The dashed lines are numerical calculations. The black, red and blue colors are for values of $n = 1, 0.5, 0.1$, respectively.

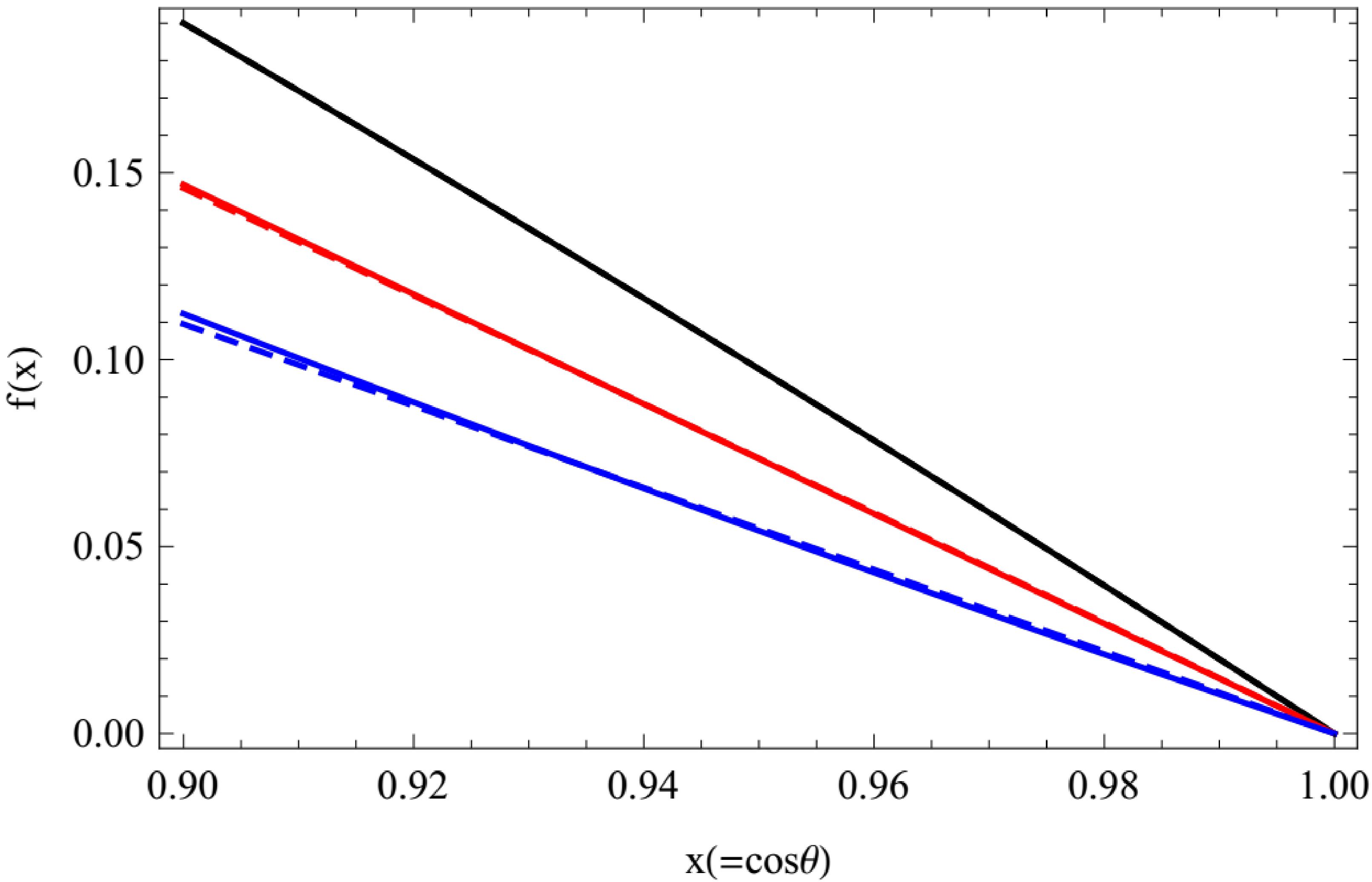


Figure 2. Same as figure 1, for magnetic flux near the polar cap region.

For a constant flux $A = r^{-n} f(x) =$ constant (equation (3)), it corresponds to the projection of magnetic field lines in the $r - \theta$ plane. The dimension flux function $f(x)$ has the largest value at the equator (figure 1). Therefore, the maximum radial extension of the magnetic field line is also reached in the equatorial plane. The last closed field line can be defined as those with maximum radial extension equal to the light cylinder radius: $r_{\mathrm{max}} = R_{\mathrm{lc}} = Pc/(2\pi)$, where P the magnetar rotational period, and c is the speed of light. The intersection of the last closed field line with the neutron star surface defines the boundary of the polar cap region. Since the flux function is a constant along a field line, then

$$\frac{f(0)}{R_{\mathrm{lc}}^n} = \frac{f(x_{\mathrm{pc}})}{R^n}, \tag{10}$$

Higher flux fraction for a twisted polar cap:

$$f(x_{\rm pc}) = \left(\frac{R}{R_{\rm lc}}\right)^n .$$

Large polar caps for twisted magnetosphere of magnetars:

$$\sin\theta_{\rm pc} = \sqrt{\frac{(R/R_{\rm lc})^n}{(15+17n)/32}}.$$

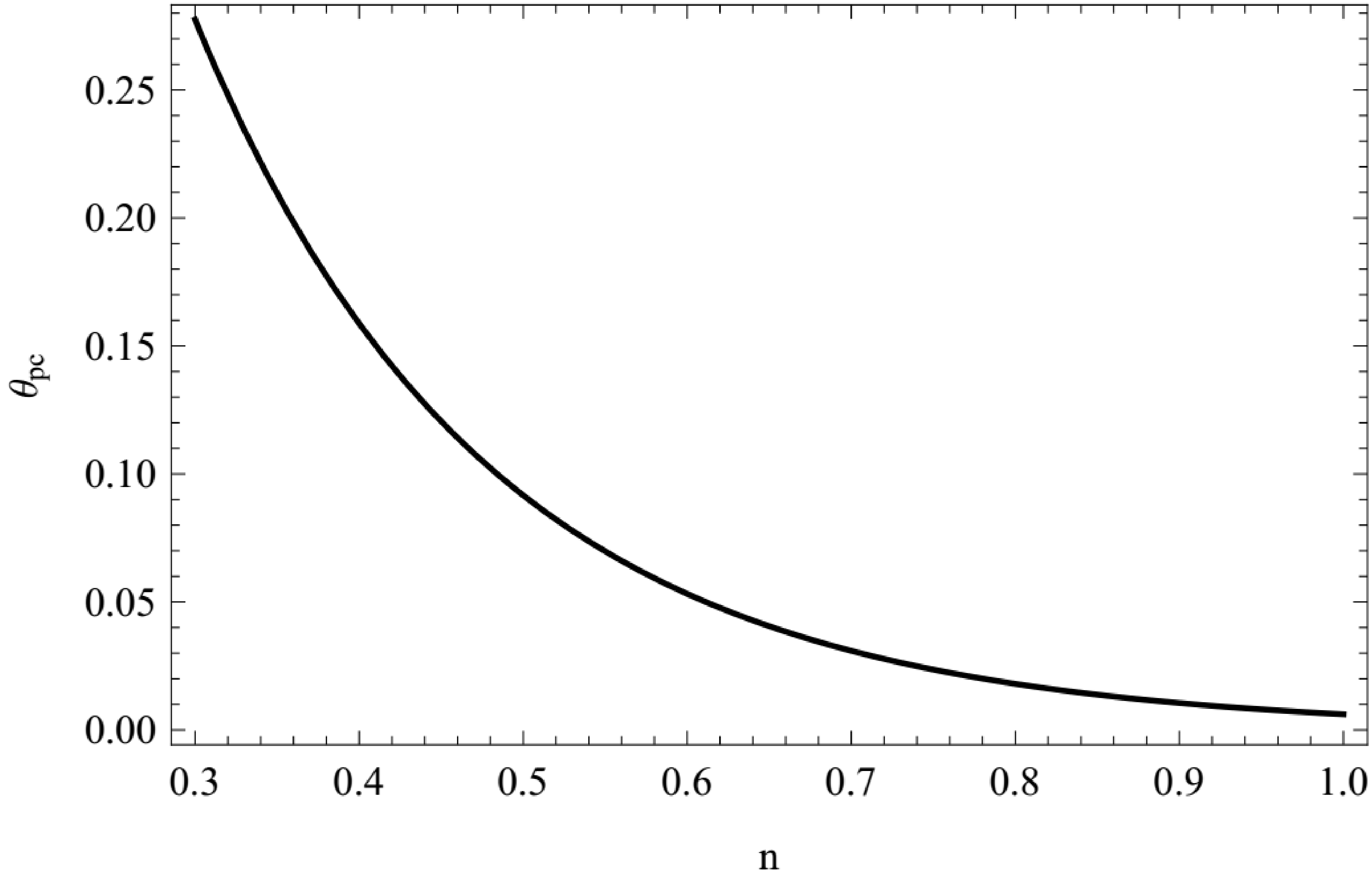


Figure 4. Polar cap angular radius of a magnetar with twisted dipole field.

More calculations

- Magnetic free energy
- Particle outflow in the open field line regions of magnetars
- Untwisting of magnetar magnetosphere (due to magnetic energy release in the open field line regions)
- Decreasing X-ray luminosity during outburst, shrinking hot spot and temperature etc
- Geometry and spin-down torque: same geometry and enhanced spin-down
- Statistical properties of magnetars:

these two cases. Therefore, a correlation between the X-ray luminosity and hot spot area will be: $L_{\mathrm{x}} \propto A^{\alpha}$, where the power law coefficient $1 < \alpha < 2$ is expected. Long term flux

A general correlation between magnetar quiescent luminosity and dipole magnetic field is: $L_{\mathrm{x,q}} \propto B_{\mathrm{p}}^{2}$. For the case of a constant acceleration potential, the correlation will be: $L_{\mathrm{x,q}} \propto B_{\mathrm{p}}$. Therefore, the power law index between one and two is expected for $L_{\mathrm{x,q}} \propto B_{\mathrm{p}}^{\beta}$, where $1 < \beta < 2$. Obser-

Enhanced particle outflow due to a large polar cap:

$$\dot{E}_{\mathrm{p,twist}} = \frac{\Omega^2 R^4 B_{\mathrm{p}}^2}{2c} \sin^4 \theta_{\mathrm{pc}}, \tag{19}$$

viewed as the strong particle flow case. When the twisted dipole field is relaxed back to the pure dipole case, the polar cap will return back to the dipole case. The corresponding particle luminosity (equation(19)) will also return back the dipole case. In this way, the magnetosphere of magnetars and normal pulsars may be unified together. This is merit of the above treatment.

According to energy conservation, the untwisting of the twisted dipole field is governed by the following equation

$$\frac{\mathrm{d}E_{\mathrm{mf}}}{\mathrm{d}t} = -\dot{E}_{\mathrm{p,twist}}. \tag{20}$$

It should be noted that the magnetic free energy is expressed in units of the magnetic energy of a pure dipole field.

The particle luminosity should be of magnetic origin (instead of rotational origin, Lyutikov 2013). This is one basic assumption of our model. Justification of equation (20) from the energy conservation point of view is presented in the appendix. Different modeling of the particle luminosity will only result in quantitative differences.

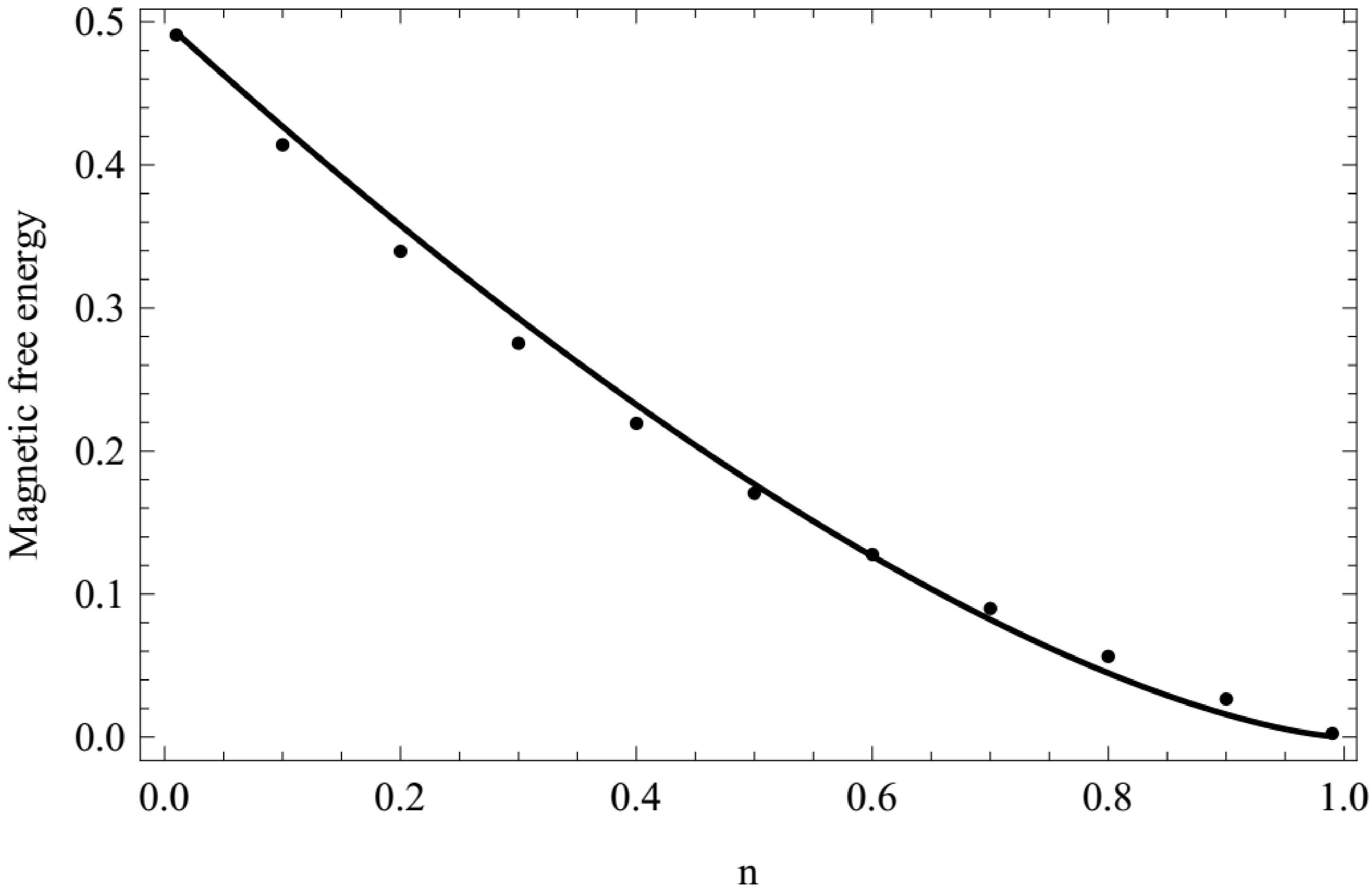


Figure 5. Magnetic free energy of a twisted dipole field as a function of n. The black points are the numerical calculations. The solid line is the analytical fitting.

the open field lines regions. For a typical parameter $n = 0.5$, the total magnetic free energy is about $E_{\rm mf} \approx 10^{44}$ $\rm B_{14}^2$ erg, the particle luminosity is about $\dot{E}_{\rm p,twist} \approx 10^{37} B_{14}^2$ erg $\rm s^{-1}$. From equation (20), the magnetic energy decays with a typical timescale (i.e. untwisting timescale)

$$\tau(n) \equiv E_{\rm mf}/\dot{E}_{\rm p,twist} \sim 0.3 \text{ yr (for n=0.5)}. \tag{21}$$

Difference between magnetosphere and high-B pulsar?

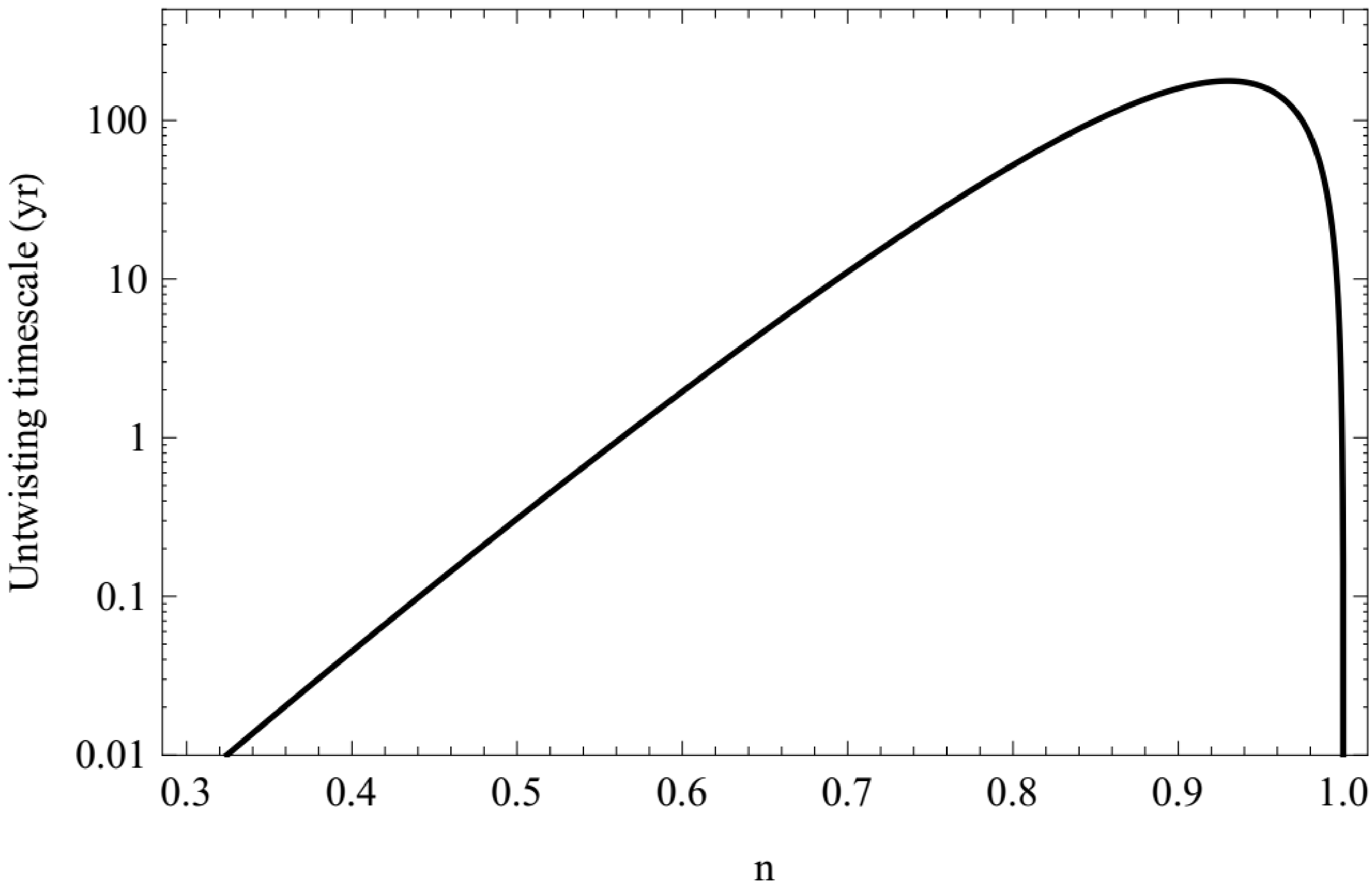


Figure 6. Magnetic field untwisting timescale as a function of n. The untwisting timescale peaks at about $n = 0.93$.

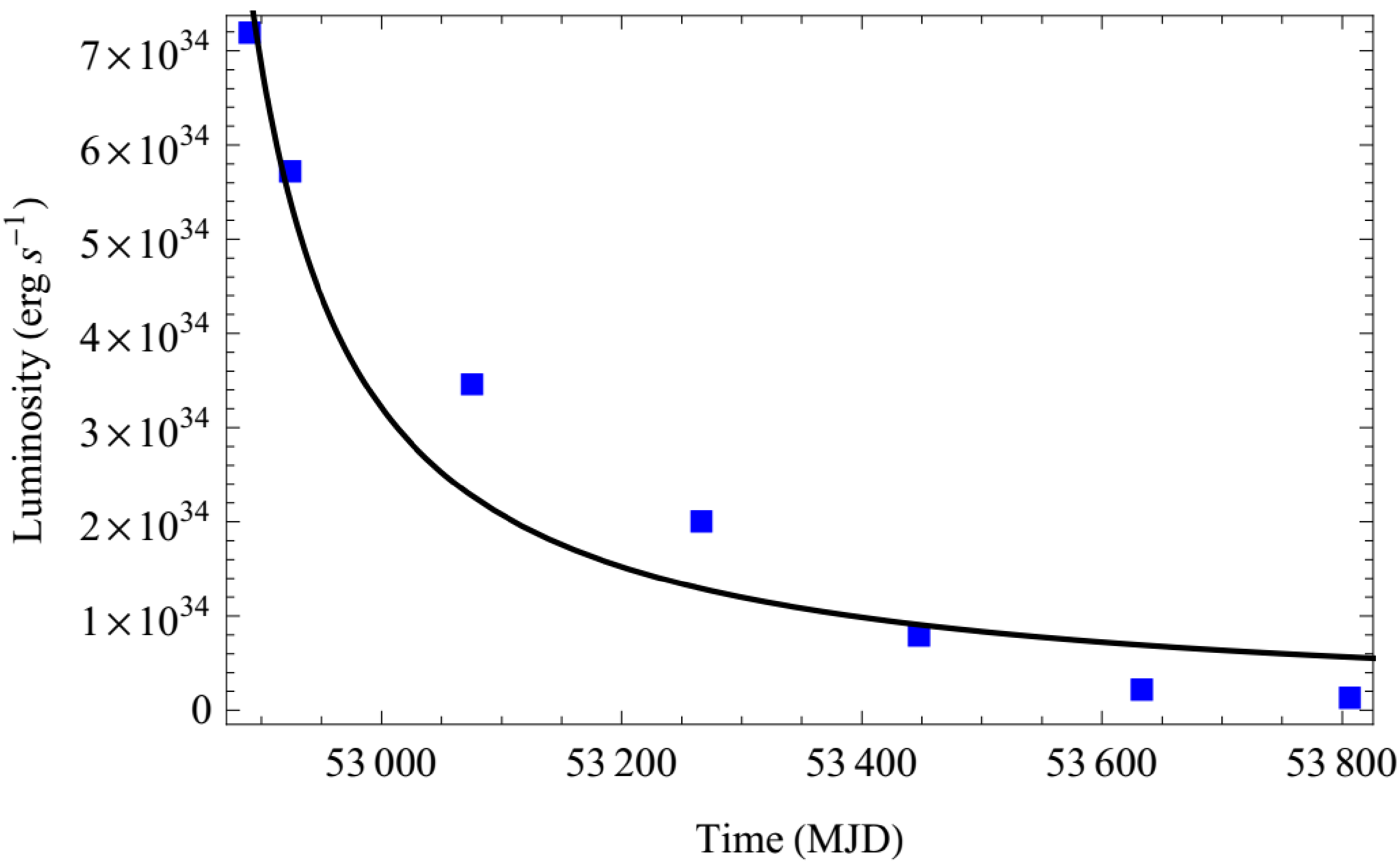


Figure 7. Luminosity of the hot spot component of XTE J1810−197. The blue squares are observations (Alford & Halpern 2016), the solid line is the model calculation.

Long term magnetic energy release?

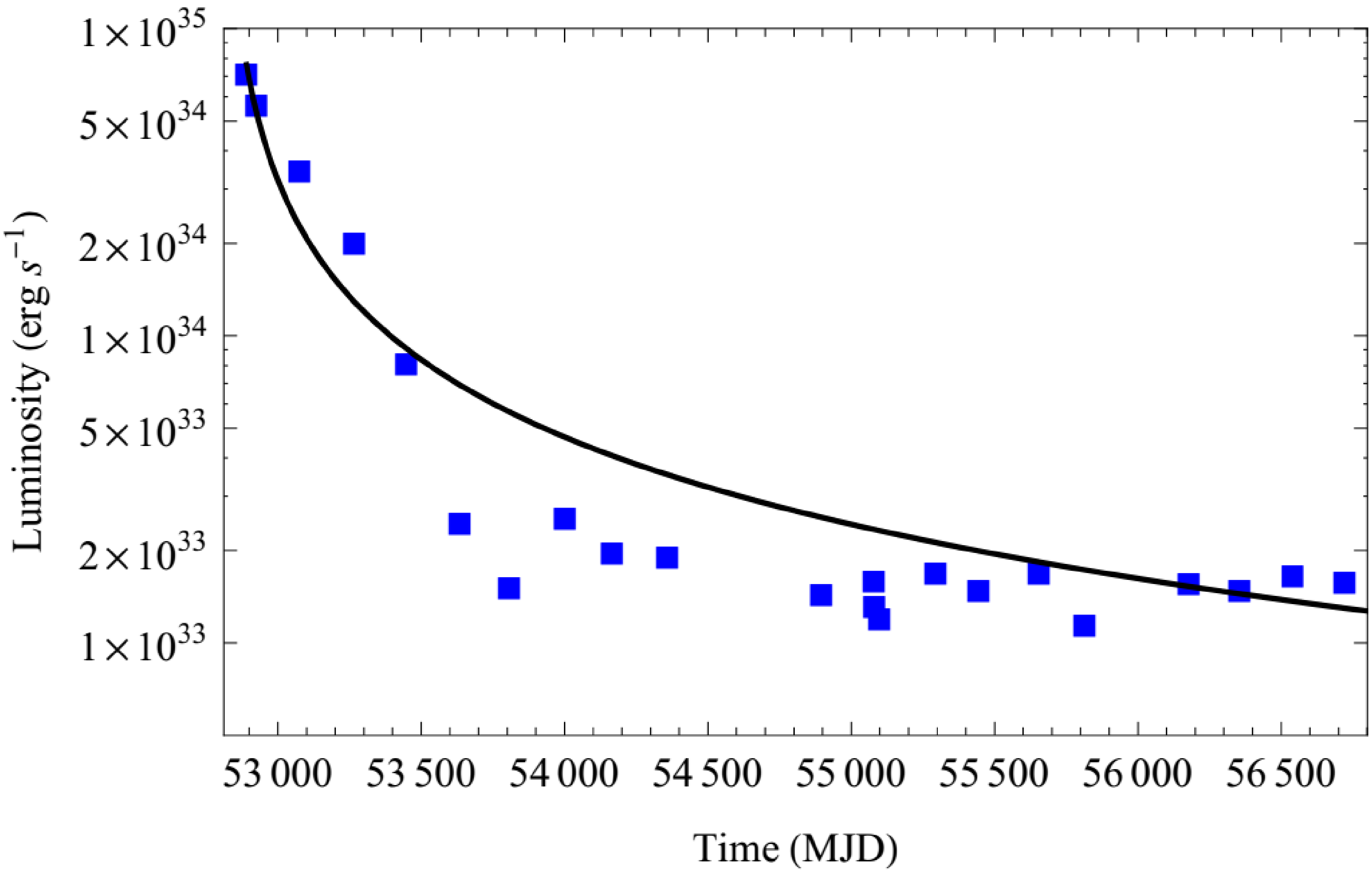


Figure 8. Long term luminosity decay of XTE J1810−197. Similar to Figure 7, the blue squares are observations (Alford & Halpern 2016), the solid line is the model calculation.

Conclusions

- Magnetars may have large polar caps, in spite of their long period
- Due to the radial inflation of the twisted magnetic field
- May result in the untwisting of the magnetic field
- May explain the general behavior of magnetars during outbursts (e.g., long term magnetic energy release)

Discussions

- Both global twisted magnetic field (our model) and local twist may occur
- Solving the “pulsar equation” with a twisted dipole boundary condition?
- Physical modeling of the particle density and acceleration potential?
- …