北京大學
PEKING UNIVERSITY

Free energy of strangeon stars/Free precession of neutron stars

Speaker: Yong Gao (高勇)

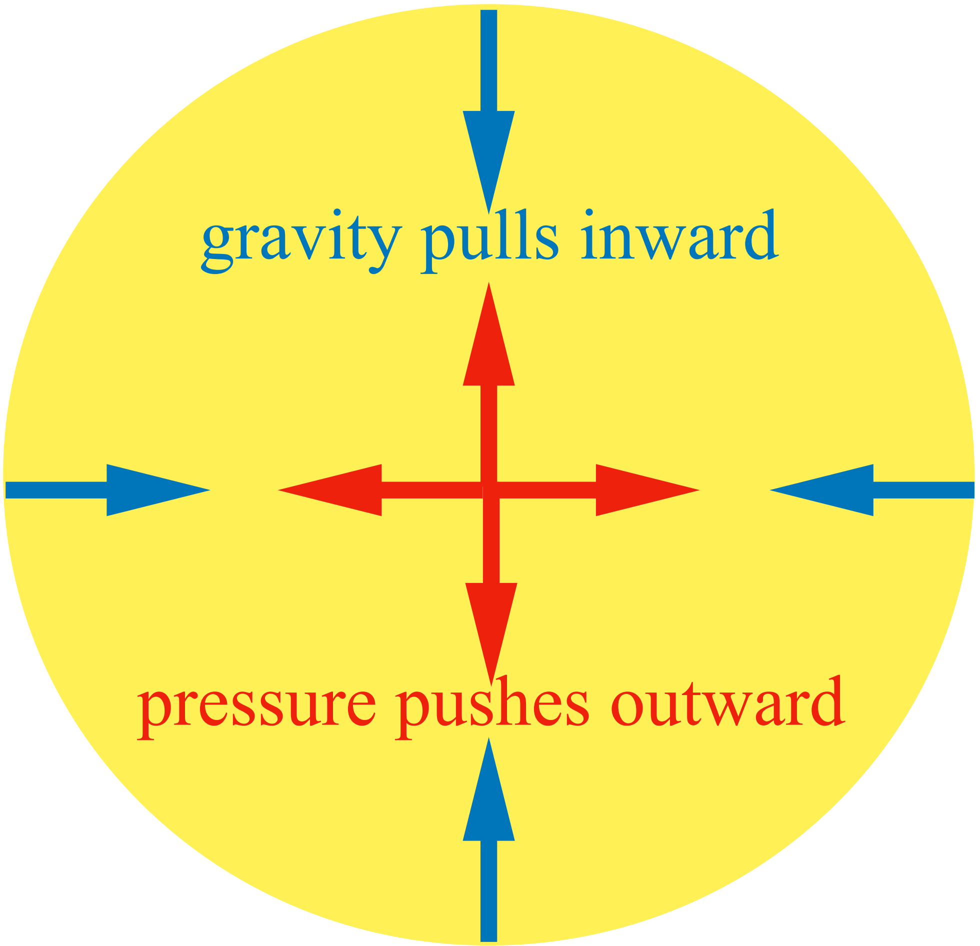
Department of Astronomy, School of Physics

Kavli Institute for Astronomy and Astrophysics

Outline

- A brief introduction to the structures of neutron stars
- Free energy of strangeon stars with anisotropic pressure
- Free precession of neutron stars
- Summary

The structure of an isotropic Newtonian star



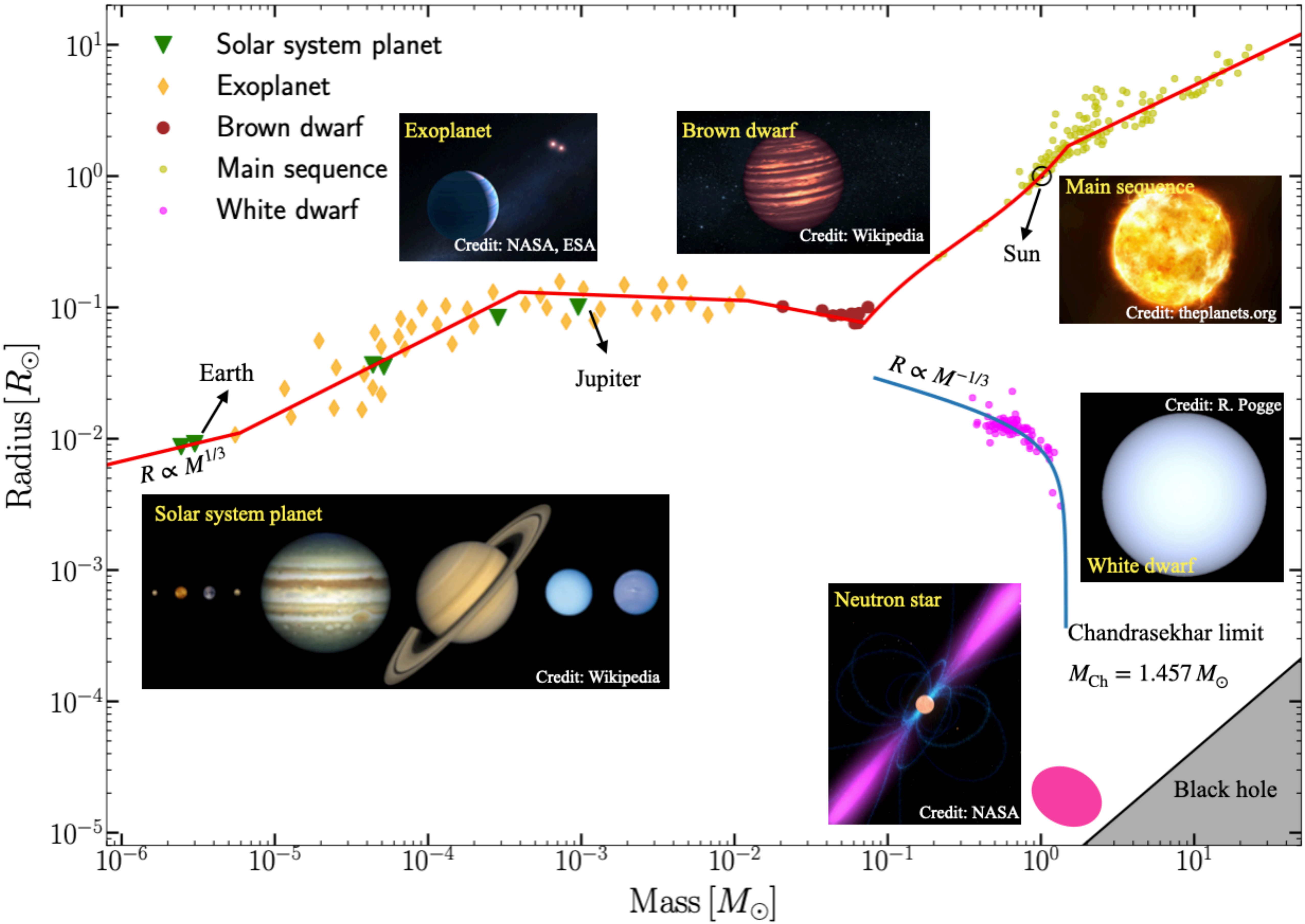
$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{Gm(r)\rho(r)}{r^2}$$

$$\frac{\mathrm{d}m(r)}{\mathrm{d}r} = 4\pi r^2 \rho(r)$$

We need the pressure-density relation

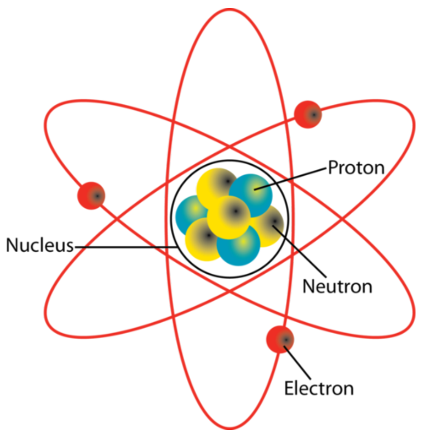
The equation of state

- Gravity is universal and global, cannot be screened (classical theory), but pressure is local, determined by the microphysics (e.g., composition and state of matter)



Solar system planet
Exoplanet
Brown dwarf
Main sequence
White dwarf
Exoplanet
Credit: NASA, ESA
Brown dwarf
Credit: Wikipedia
Main sequence
Credit: theplanets.org
Sun
Earth
Jupiter
$R \propto M^{1/3}$
$R \propto M^{-1/3}$
Solar system planet
Credit: Wikipedia
Credit: R. Pogge
White dwarf
Neutron star
Credit: NASA
Chandrasekhar limit
$M_{\rm Ch} = 1.457\, M_\odot$
Black hole
Radius $[R_\odot]$
Mass $[M_\odot]$

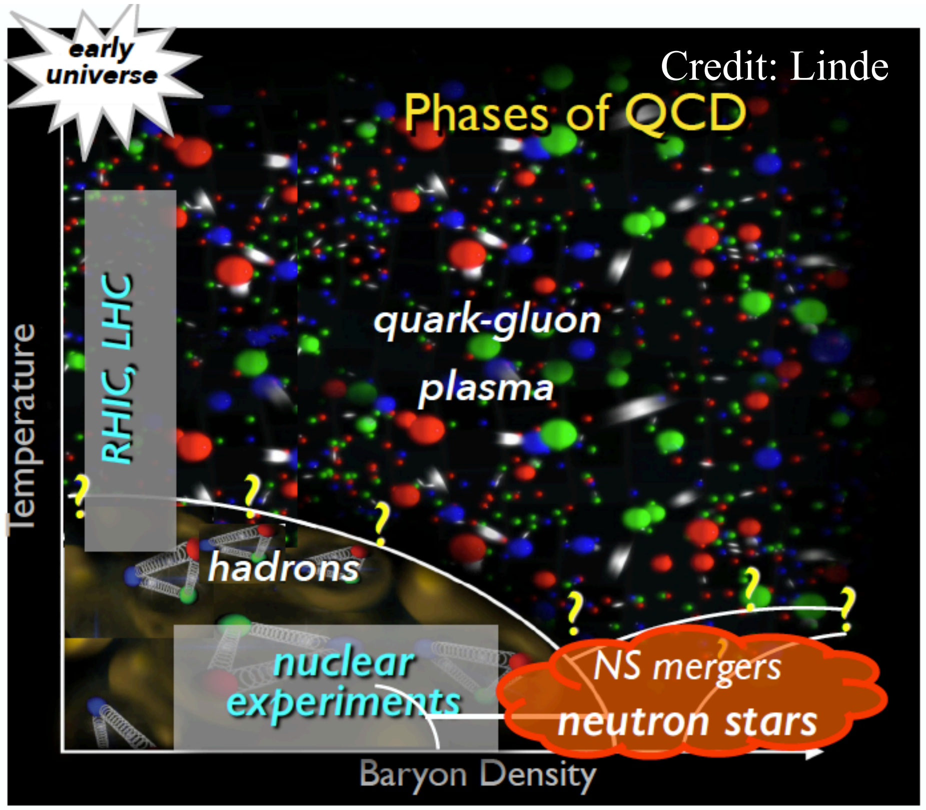
The equation of state of NSs



Compression

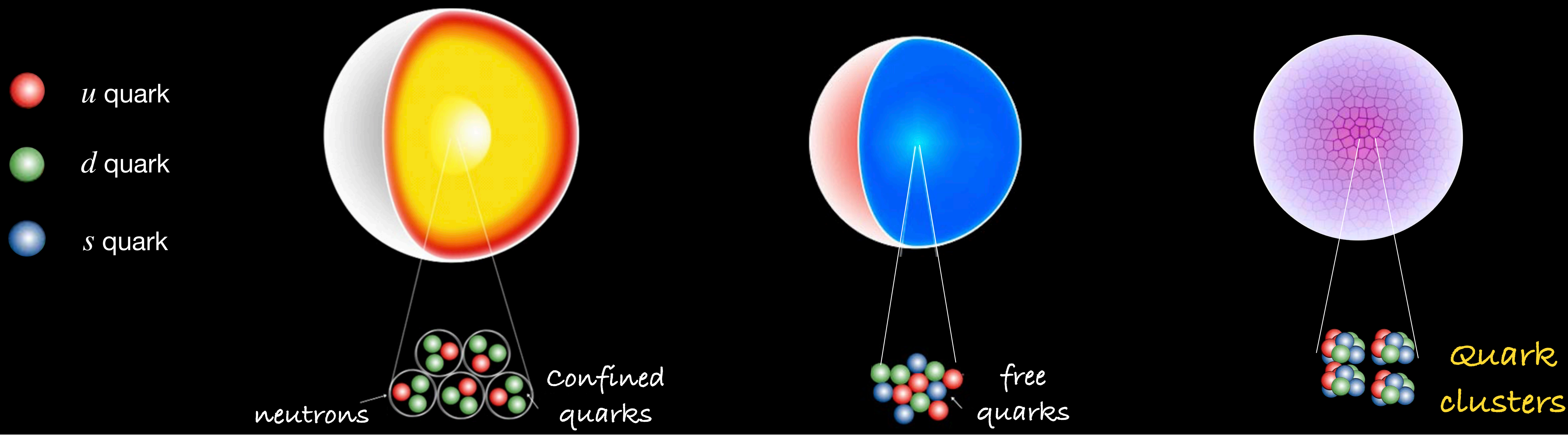
$p_{\rm F} \propto n_e^{1/3}$

$E_{\rm e} \sim 100\,{\rm MeV}$

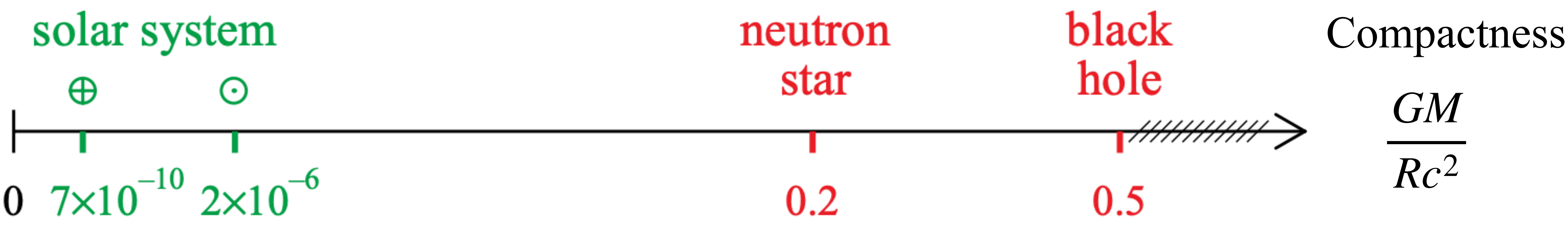


Adapted from physicsworld

Normal Neutron Stars	Strange Quark Stars	Strangeon Stars
$e^- + p \rightarrow n + \nu_e$	$2f(u,d) \rightarrow 3f(u,d,s)$	$2f(u,d) \rightarrow 3f(u,d,s)$



The hydro-equilibrium equation in GR



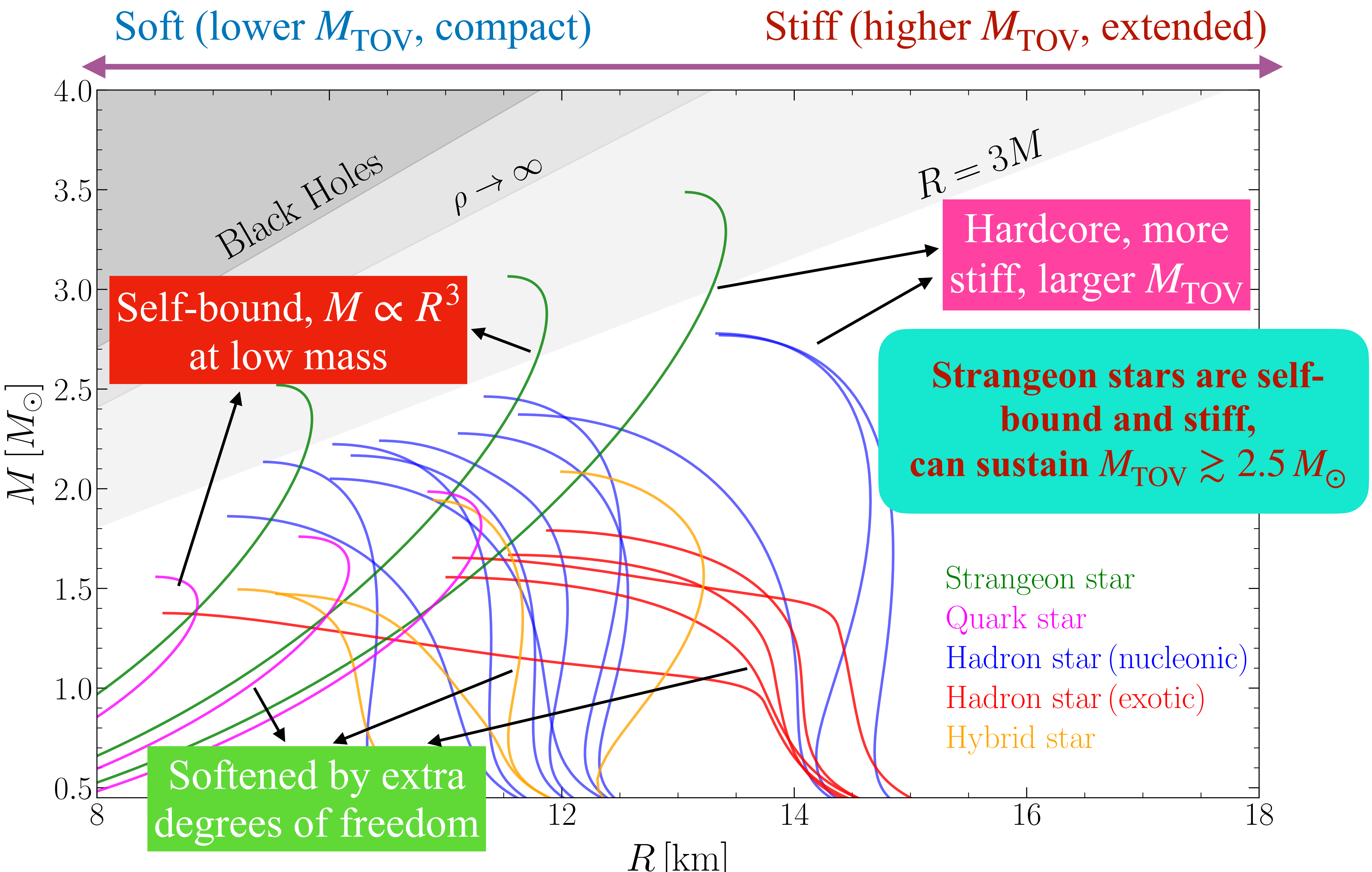
$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{Gm\epsilon}{r^2}\left(1+\frac{P}{\epsilon c^2}\right)\left(1+\frac{4\pi r^3 P}{mc^2}\right)\left(1-\frac{2Gm}{c^2 r}\right)^{-1}$$

$$\frac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2 \epsilon$$

Give equation of state, we can obtain the structure (e.g., mass and radius)

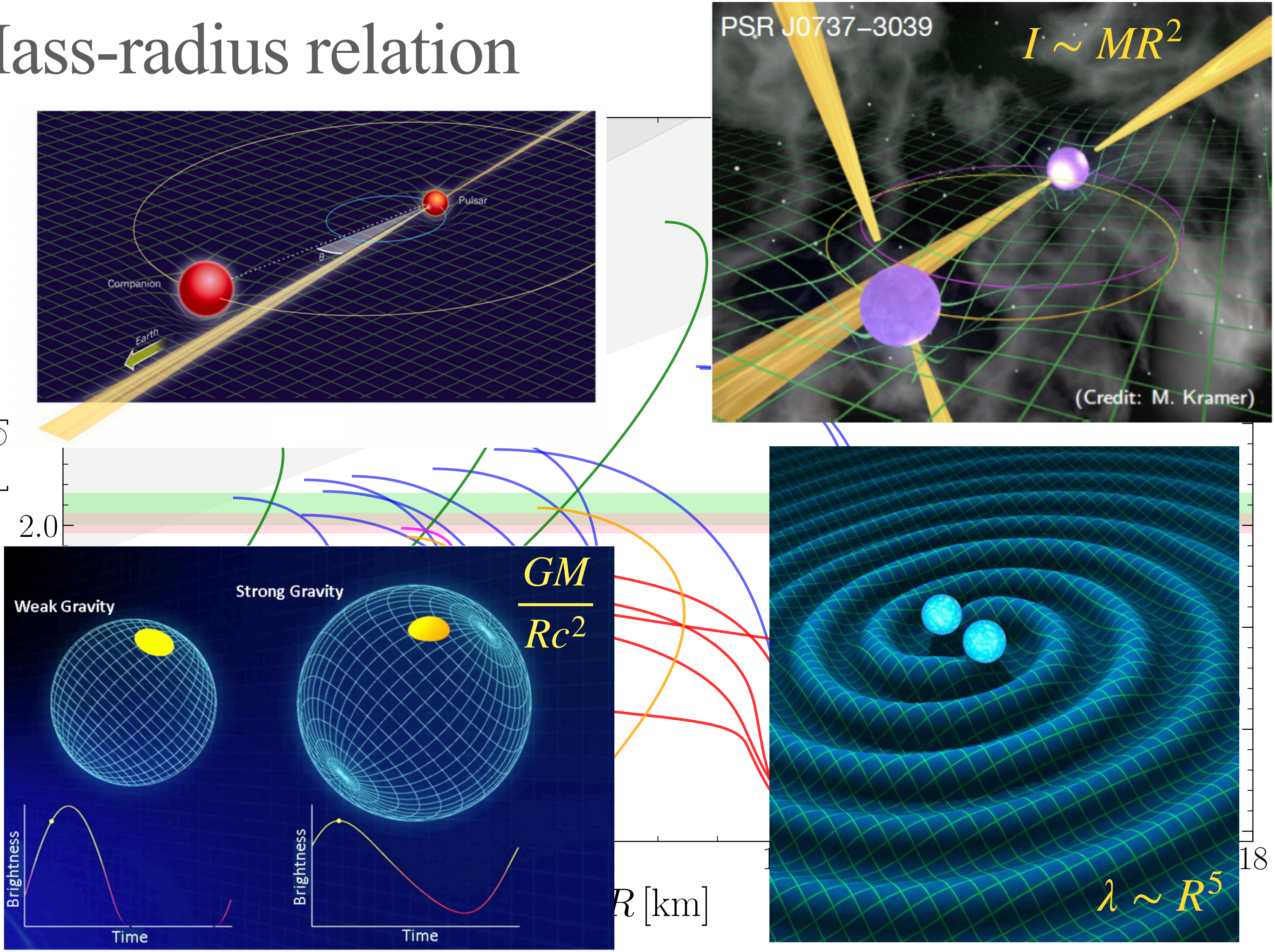
$$\epsilon = \rho(1+e)$$

Mass-radius relation

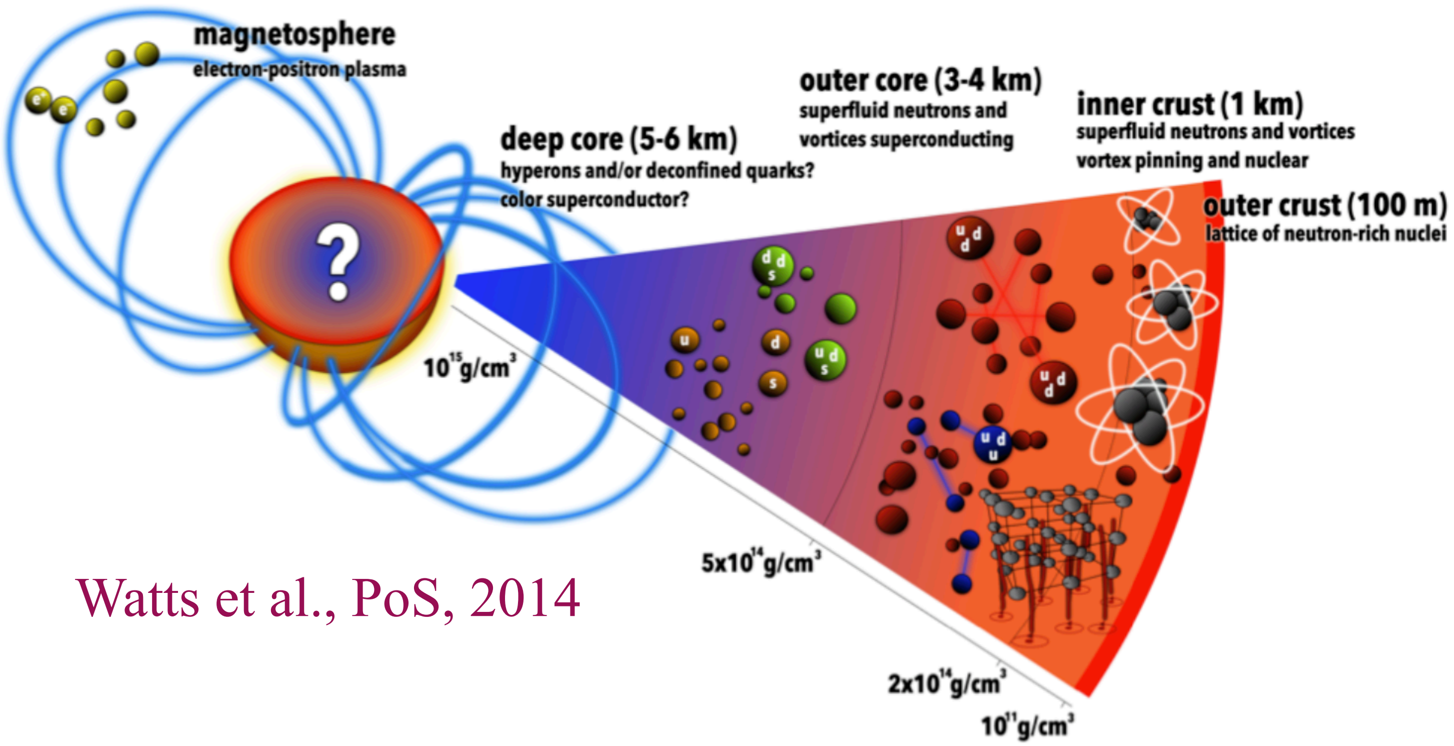


Gao et al., MNRAS, 2022

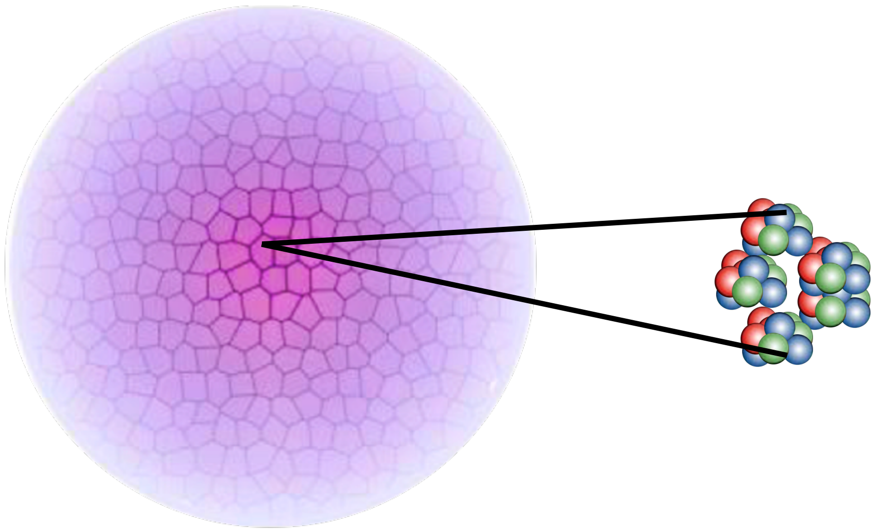
Mass-radius relation



To give a realistic matter description, we also need to consider solid state, strong magnetic field, superfluid/superconductivity (conventional neutron-star model)



Conventional neutron star

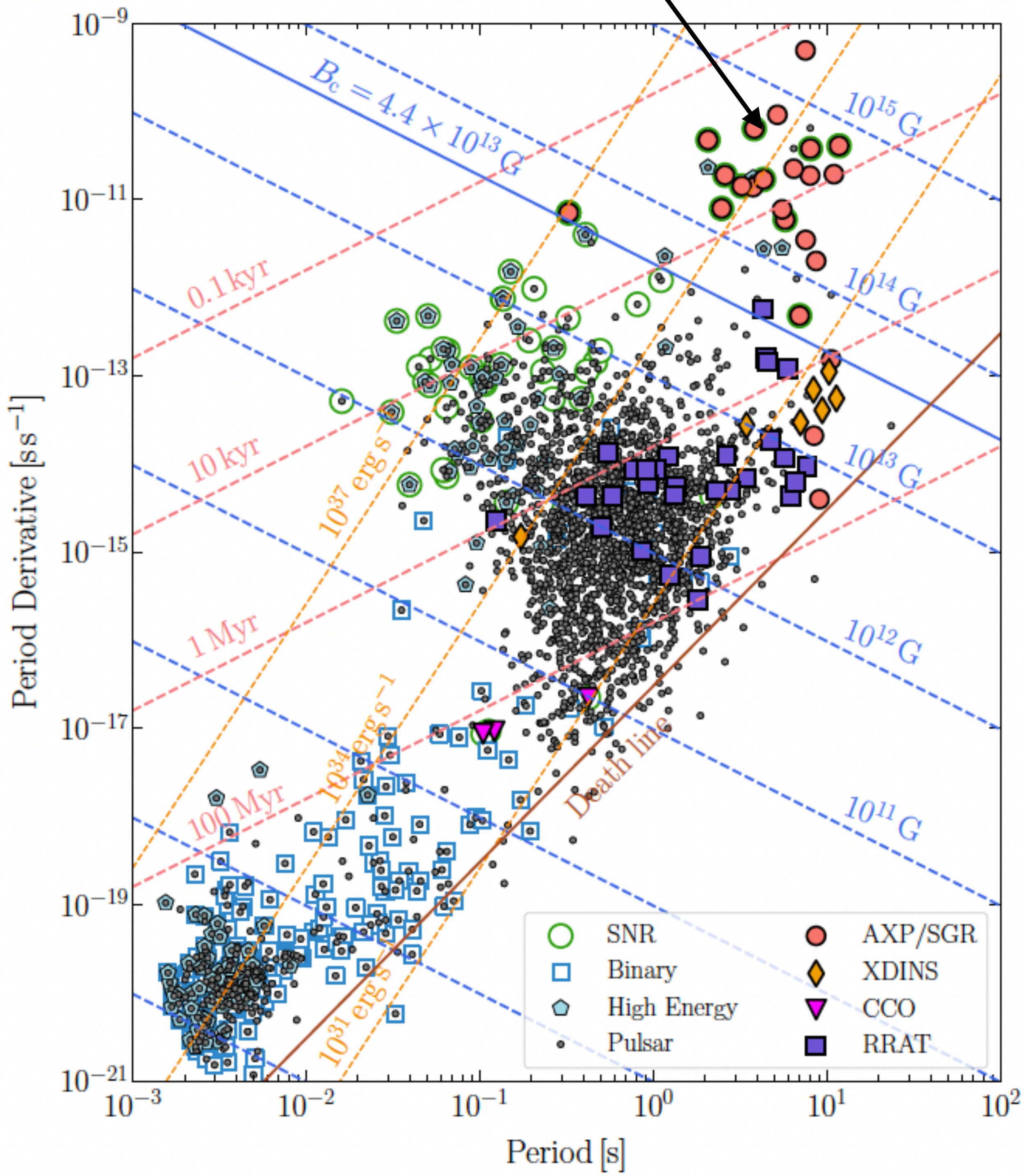


Strangeon star

Important for many phenomena: pulsars' radiation, glitch, non-hydro deformation (mountain), **giant flare**, quasi-periodic oscillation, **free precession**…

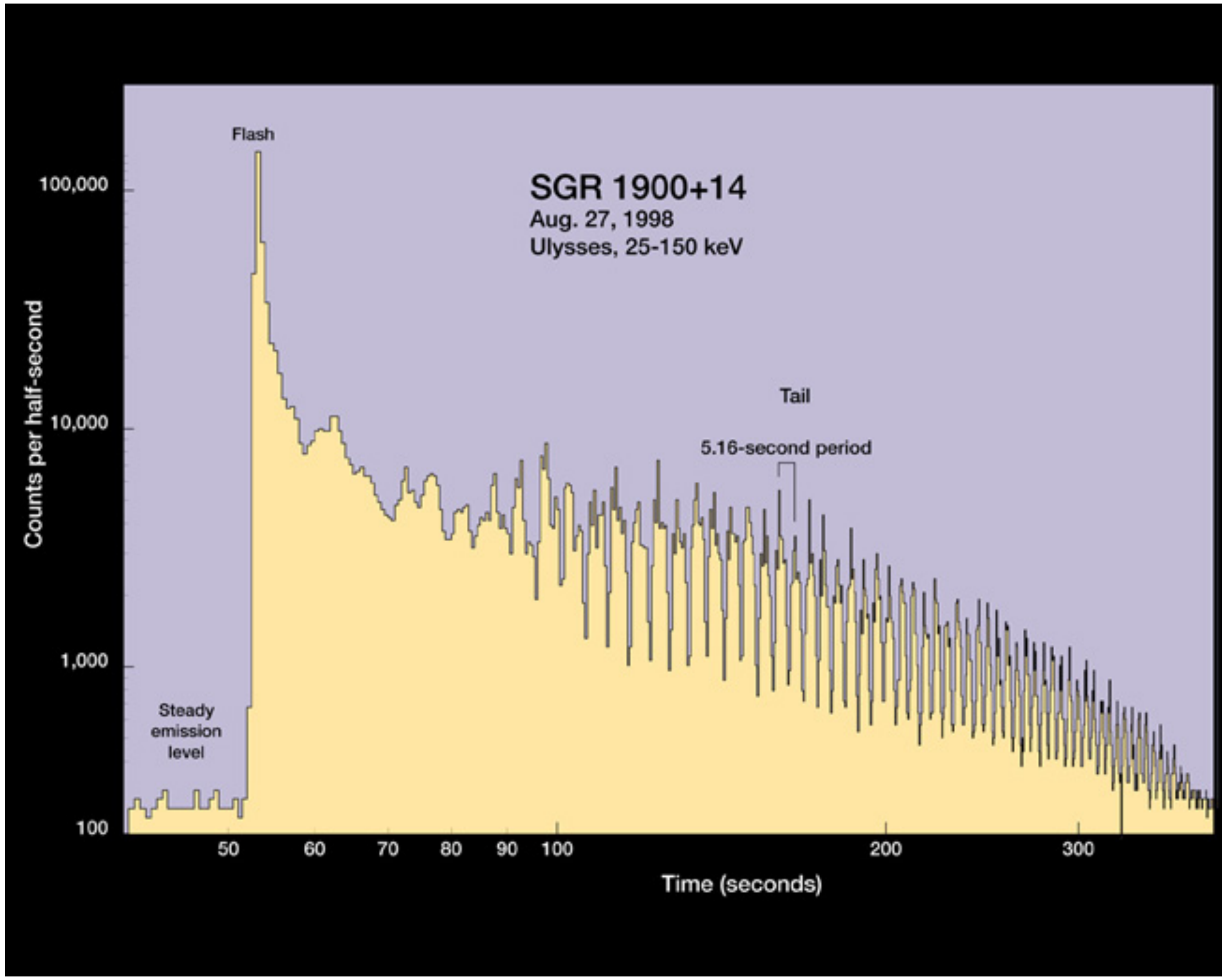
Giant flare and free energy of strangeon stars

Soft gamma repeaters and anomalous X-ray pulsars



Steadily pulsating X-rays with sporadic, bright outbursts

$\sim 10^{44-47}$ erg



Giant flare

Giant flare and free energy of strangeon stars

- Models of these giant flares usually involves **magnetars**, a kind of neutron stars with ultra high magnetic fields (over 10^{13} G) (Thompson & Duncan, MNRAS, 1995; Kouveliotou et al., MNRAS, 1998). A conventional neutron star is drop-like except for a solid crust (i.e., similar to a raw egg), the free energy could be significant if it's strongly magnetized.

- A strangeon star should be in a globally solid state (i.e., similar to a cooked egg) due to the large masses of and the strong coupling between strangeon. A huge amount of energy can be released via starquakes for anisotropic strangeon star.

Anisotropic strangeon stars

- TOV equation for a star with anisotropic pressure

$$\frac{\mathrm{d}P_{\mathrm{rad}}}{\mathrm{d}r} = -\frac{Gm\epsilon}{r^2}\left(1+\frac{P_{\mathrm{rad}}}{\epsilon c^2}\right)\left(1+\frac{4\pi r^3 P_{\mathrm{rad}}}{mc^2}\right)\left(1-\frac{2Gm}{c^2 r}\right)^{-1} + \frac{2(P_{\tan}-P_{\mathrm{rad}})}{r}$$

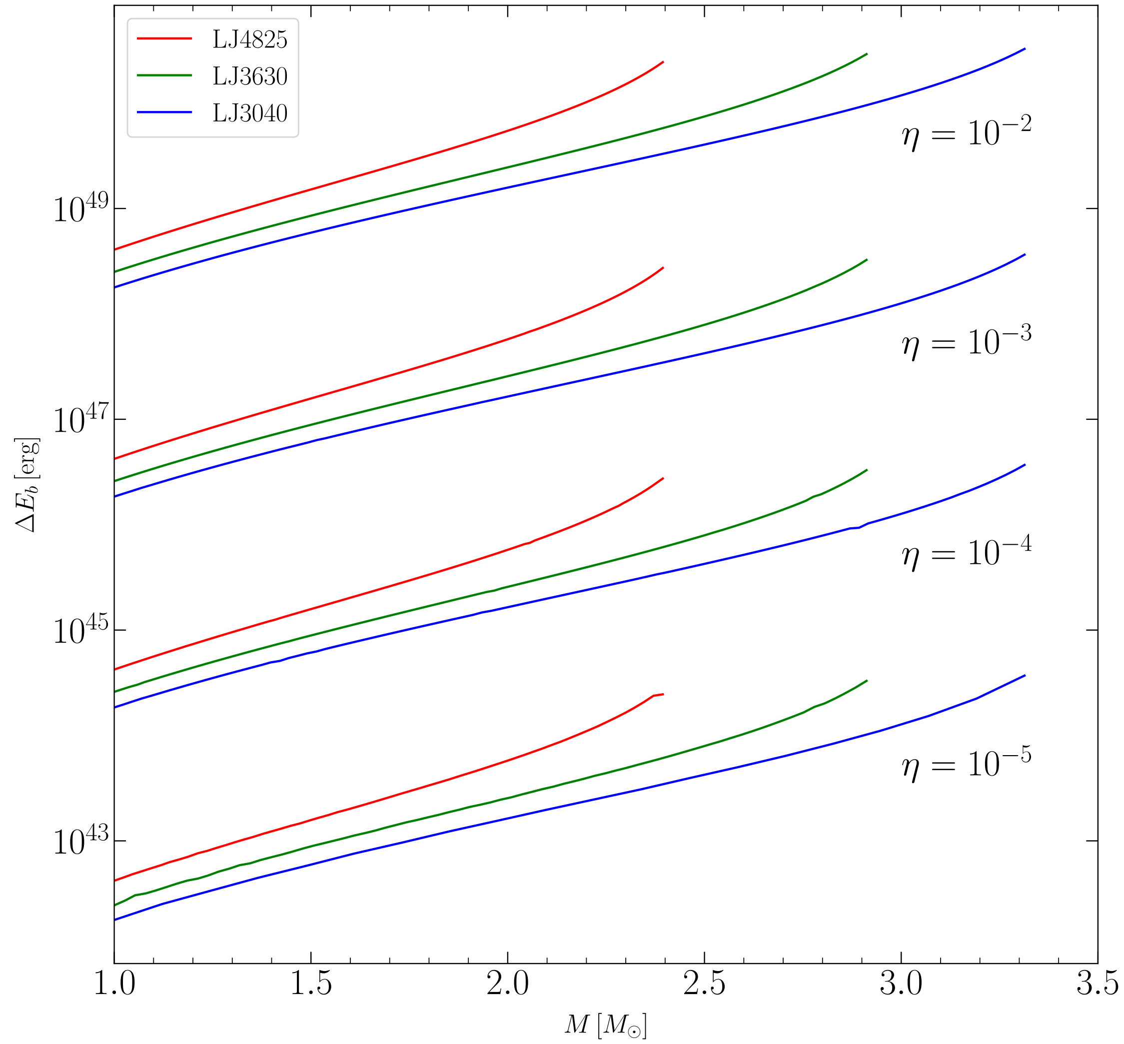
$$\sigma = P_{\tan} - P_{\mathrm{rad}}$$

- Since it's too difficult to obtain σ on physical grounds from first principle, one could only guess some heuristic models.

$$\sigma = -\eta \frac{\mathrm{d}P_{\mathrm{rad}}}{\mathrm{d}r} r$$

• Starquakes may cause the sudden change of σ, with a release of the gravitational energy as well as the strain energy. The difference of binding energy $\Delta E_b = E_b(\eta) - E_b(\eta = 0)$ implies the free energy the star may release.

$$\frac{\mathrm{d}P_{\rm rad}}{\mathrm{d}r} = -\frac{Gm\epsilon}{r^2}\left(1+\frac{P_{\rm rad}}{\epsilon c^2}\right)\left(1+\frac{4\pi r^3 P_{\rm rad}}{mc^2}\right)\left(1-\frac{2Gm}{c^2 r}\right)^{-1} + \frac{2(P_{\rm tan}-P_{\rm rad})}{r}$$

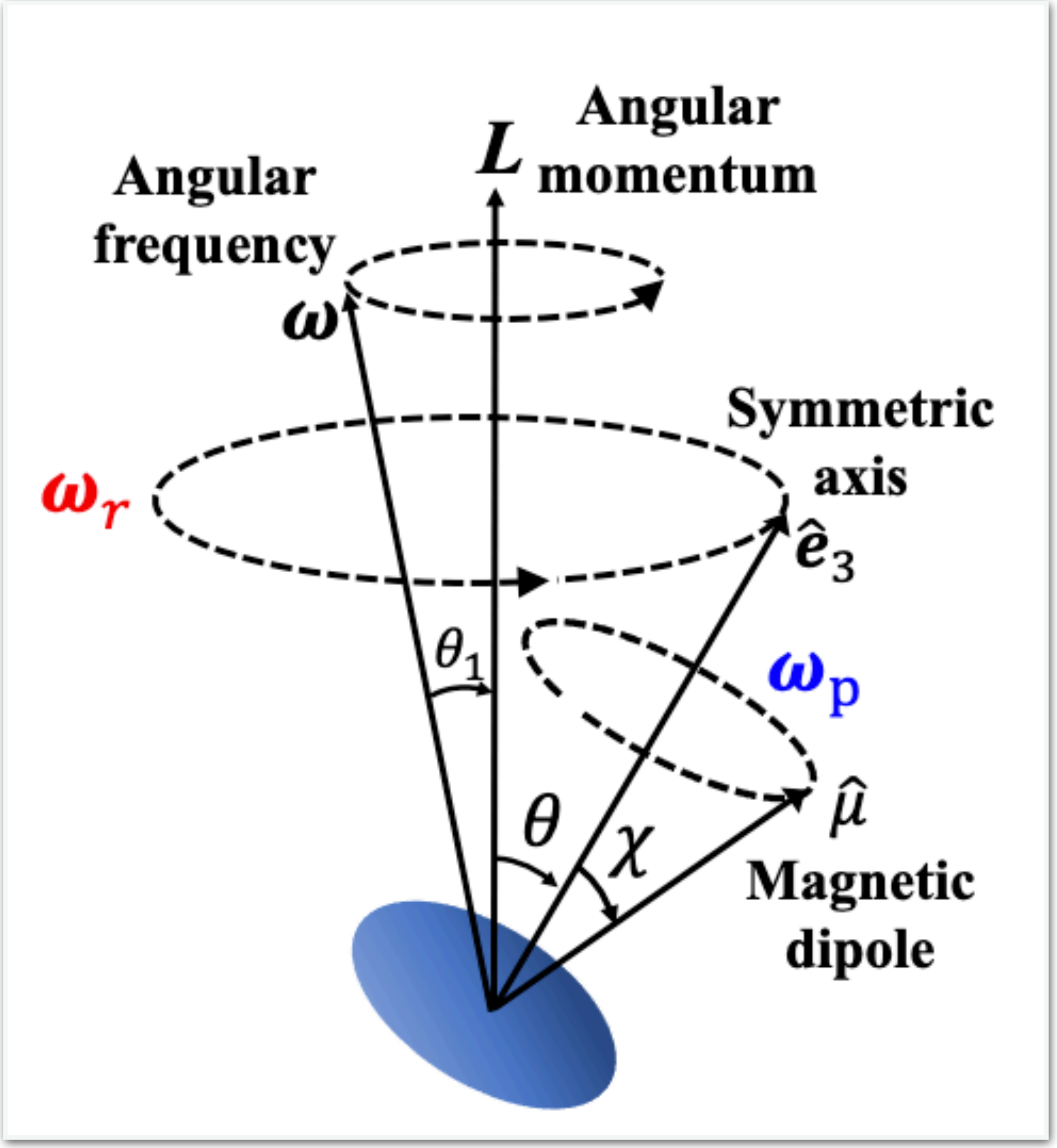


$$\sigma = -\eta \frac{\mathrm{d}P_{\rm rad}}{\mathrm{d}r} r$$

- Argument from Newtonian physics

$$\Delta E_b \sim \frac{GM^2}{R}\frac{\Delta R}{R} \sim 10^{53}\left(\frac{\Delta R}{R}\right)\ \mathrm{erg}$$

Free precession



- Precession happens if some deformation pieces are not aligned with the rotation bulges

ellipticity $\epsilon = \dfrac{\Delta I_d}{I_0}$ wobble angle: θ

- **Two superimposed motion:**

$$\boldsymbol{\omega} = \omega_r \hat{\boldsymbol{L}} - \omega_p \hat{\boldsymbol{e}}_3 \qquad \omega_p = \epsilon \cos\theta\, \omega_r$$

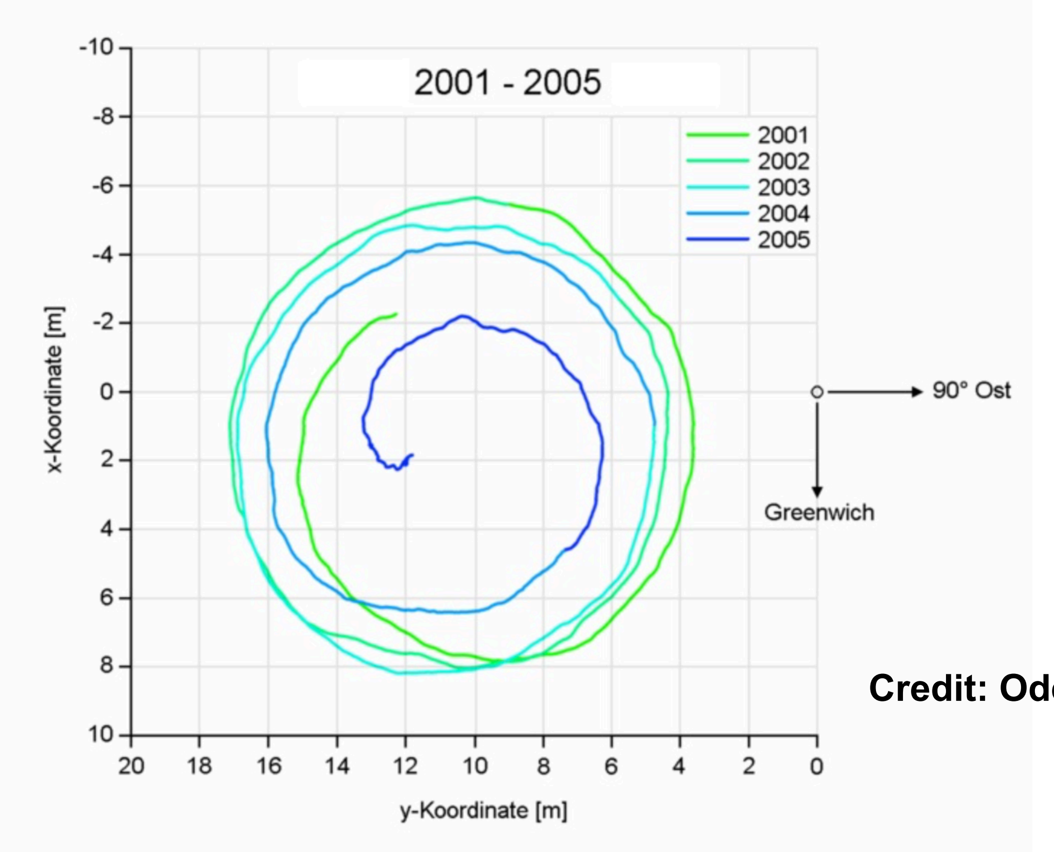
Precession period $P_f = \dfrac{P}{\epsilon \cos\theta}$

Free precession of the earth—Chandler wobble

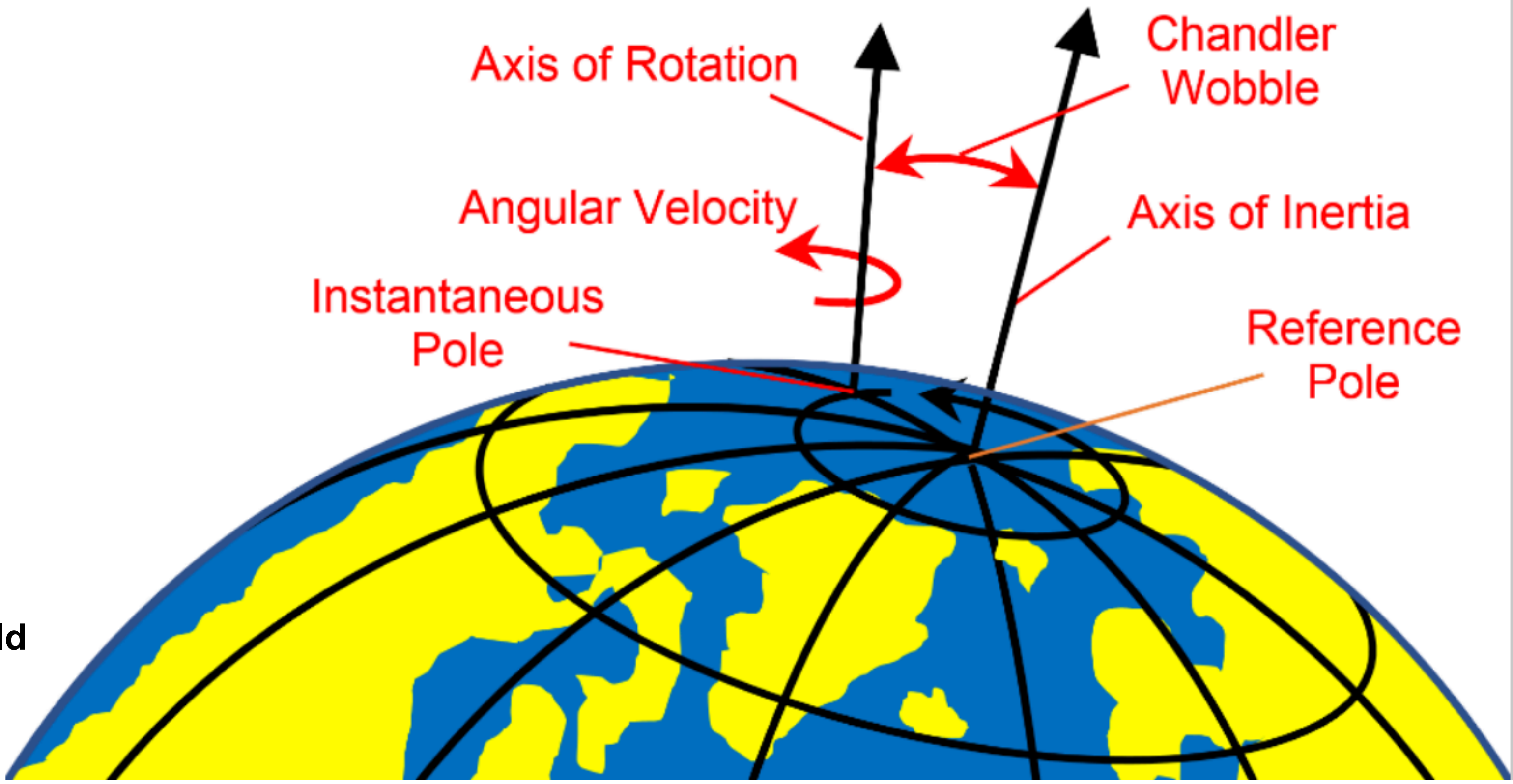
- The Chandler wobble or Chandler variation of latitude is a small deviation in the Earth's axis of rotation relative to the solid earth

- The free precession period $P_{\text{fp}} \approx 433$ days

$$\epsilon \approx \frac{1}{433} = 0.002$$

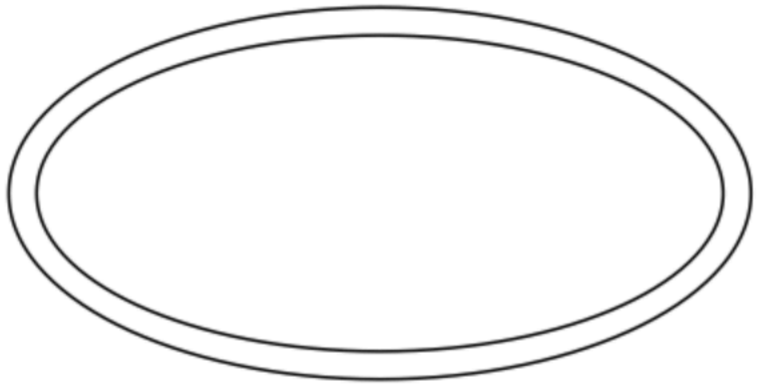


Credit: Odenwald

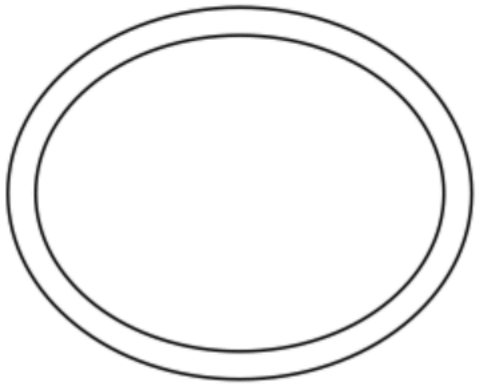


Comparison between the earth and NSs

Initial oblateness ϵ_0



Relaxed ($\epsilon < \epsilon_0$)



$$E(\epsilon) = E_{\text{spherical}} + A\epsilon^2 + B\left(\epsilon - \epsilon_0\right)^2 + \frac{J^2}{2I_0(1+\epsilon)}$$

$$\epsilon = \frac{I_{\text{star}}\Omega^2}{4(A+B)} + \frac{B}{A+B}\epsilon_0 \equiv \epsilon_\Omega + b\epsilon_0$$ Rigidity parameter b

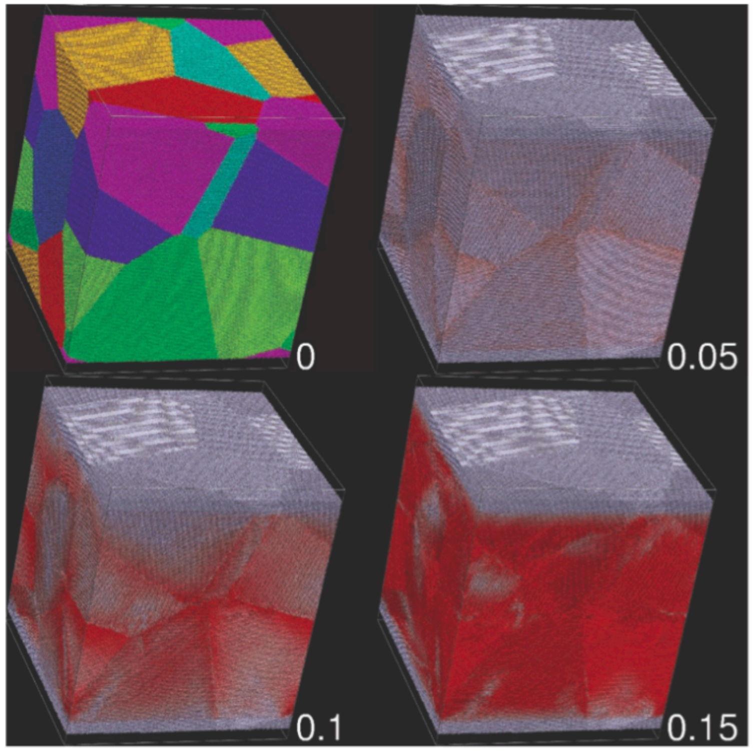
Property	Earth	Neutron star
Moment of inertia: Solid crust	90%	$< 5\%$
Moment of inertia: Liquid core	10%	$> 95\%$
Rigidity parameter	0.7	10^{-5}; jelly
Magnetic field	Unimportant	Maybe
Free precession observed?	Yes, 14 month 'Chandler wobble'	Handful of candidates

Maximal elastic mountain for NSs

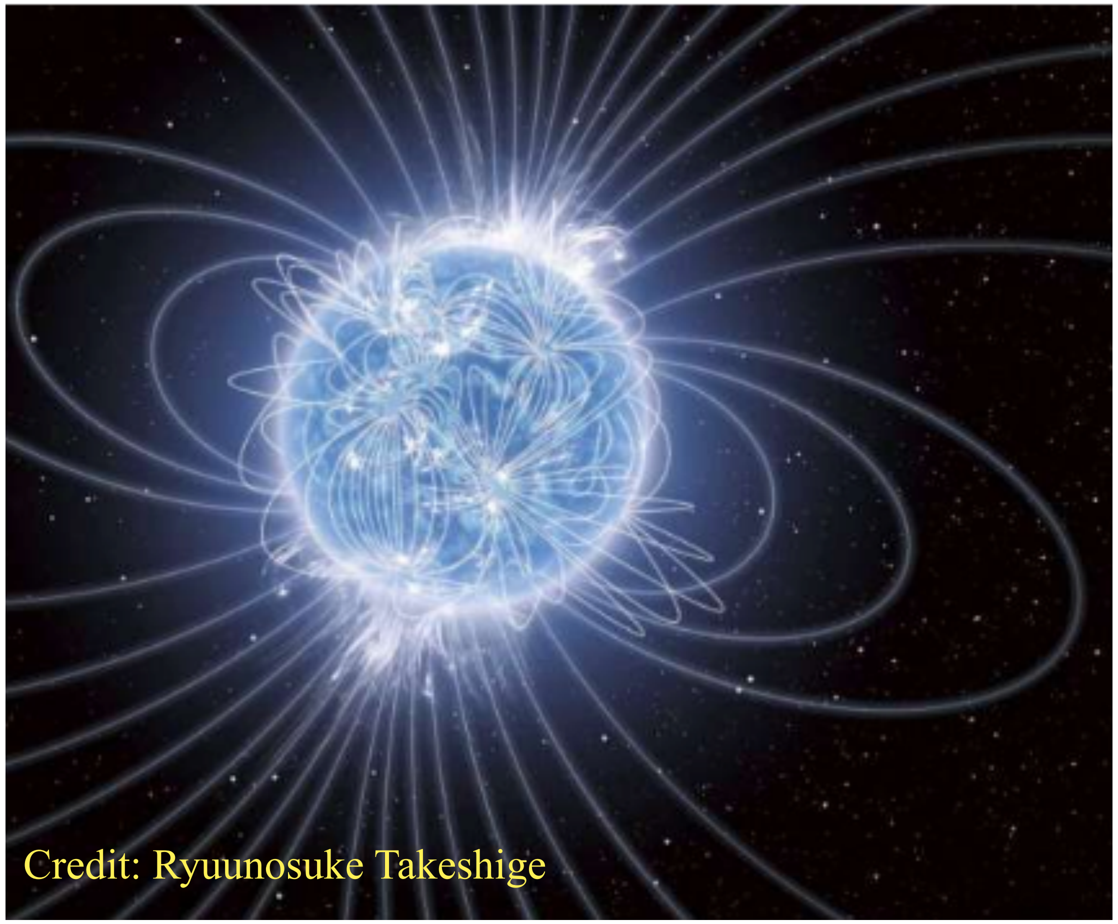
- Crust strain $\sim \left|\epsilon - \epsilon_0\right|$ cannot larger than the breaking strain $\sigma_{\rm break}$ of star

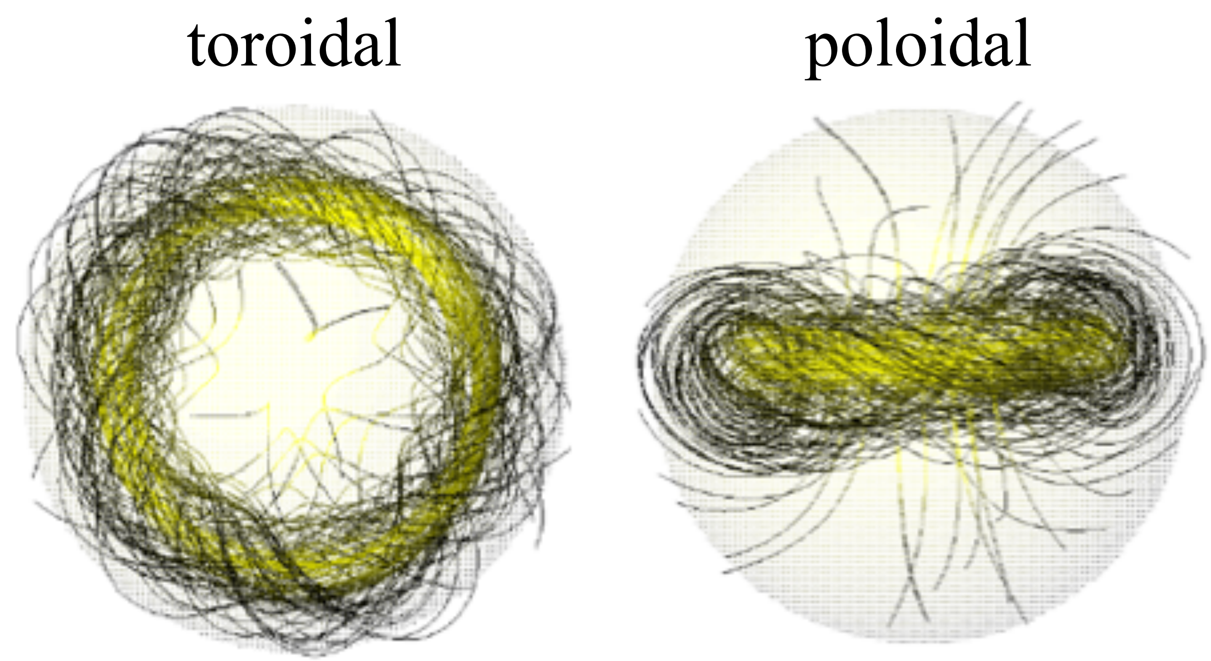
$$\epsilon \approx \frac{\mu V_{\rm crust}}{GM^2/R} \times \sigma_{\rm break} \approx 10^{-6} \left(\frac{\sigma_{\rm break}}{10^{-1}}\right)\left(\frac{\mu}{10^{29}\,{\rm erg\,cm^{-3}}}\right)$$

- Large-scale molecular dynamics of Horowitz & Kadau, PRL, 2009 indicate very high breaking strain, ~ 0.1 (see Figure), for some parts of crust at least



NSs also sustain magnetic mountains



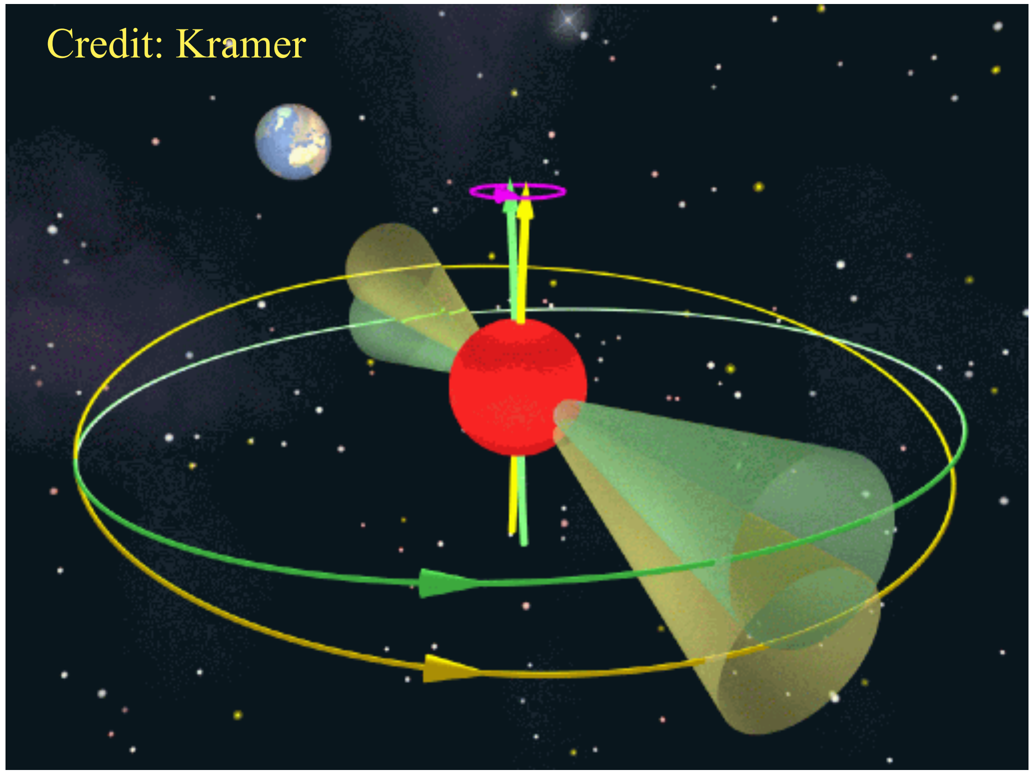


Credit: Braithwaite

$$\epsilon_{\rm B} \approx \kappa \frac{B^2 R^3}{GM^2/R} = 1.9 \times 10^{-6} \kappa B_{15}^2$$

Lander & Jones, MNRAS, 2009; Lasky & Melatos, PRD, 2014; Zanazzi & Lai, MNRAS, 2015

How to find free precession of NSs?



- The angle α changes periodically with precession period P_{f}

Swing of the emission region

Modulate period, profile, polarization,…

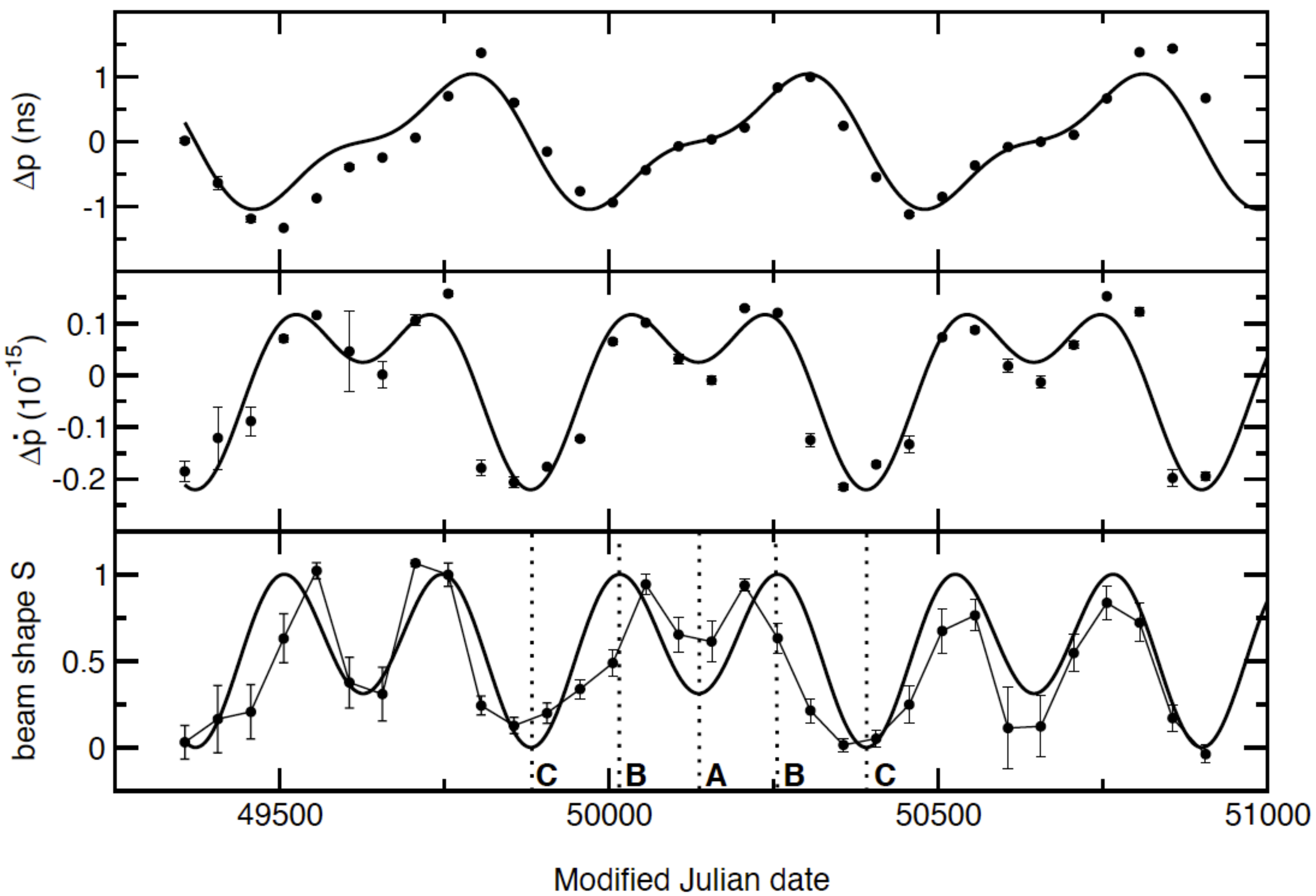
- The mass distribution changes, source for gravitational waves

The gravitational-wave frequency: spin frequency and twice of spin frequency

Small wobble angle limit: $h = \frac{G}{c^4}\frac{\dot{\phi}^2}{r}I_0\epsilon\theta$ at spin frequency

Do we observe free precession of NSs?

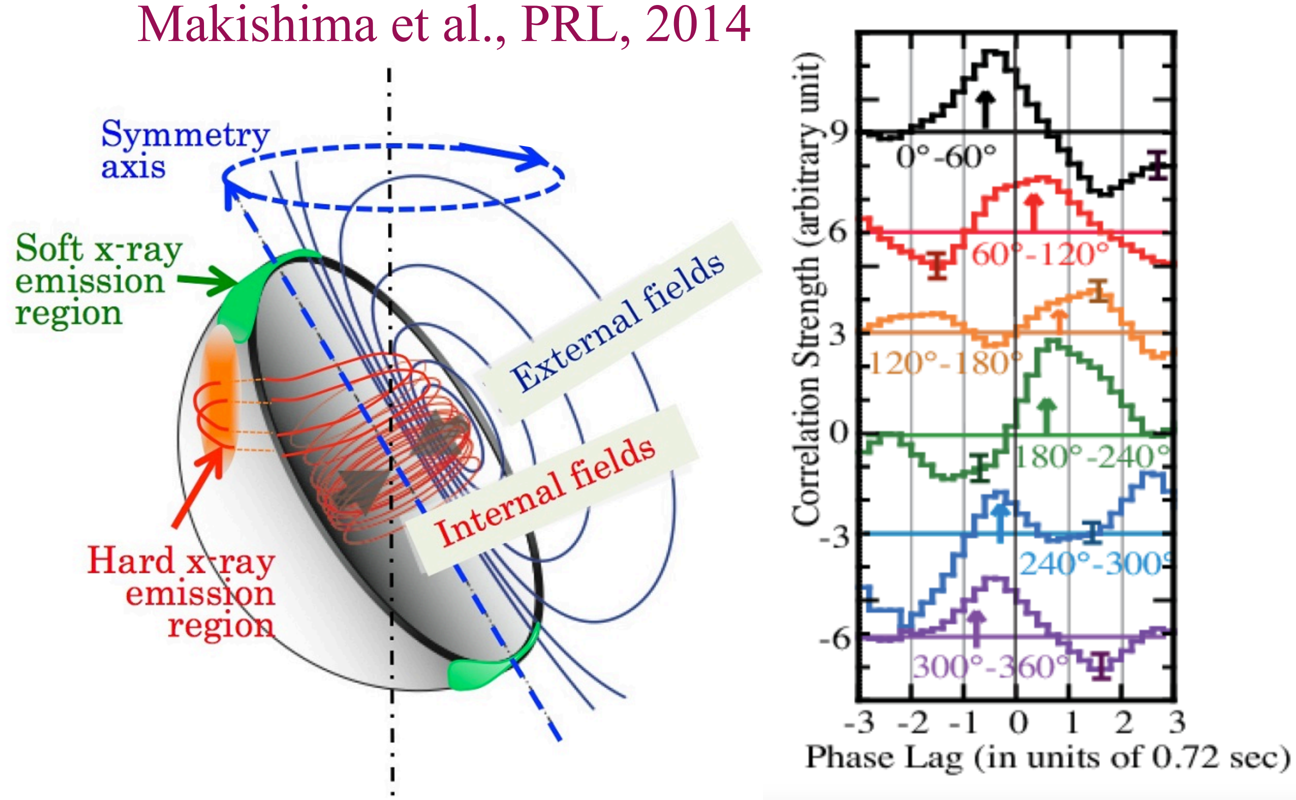
Stairs et al., Nature, 2000; Link & Epstein, ApJ, 2001



PSR B1828-11: radio timing and beam shape

$P_\mathrm{f} \sim 500\,\mathrm{days} \qquad \epsilon \sim 10^{-8}$

Makishima et al., PRL, 2014



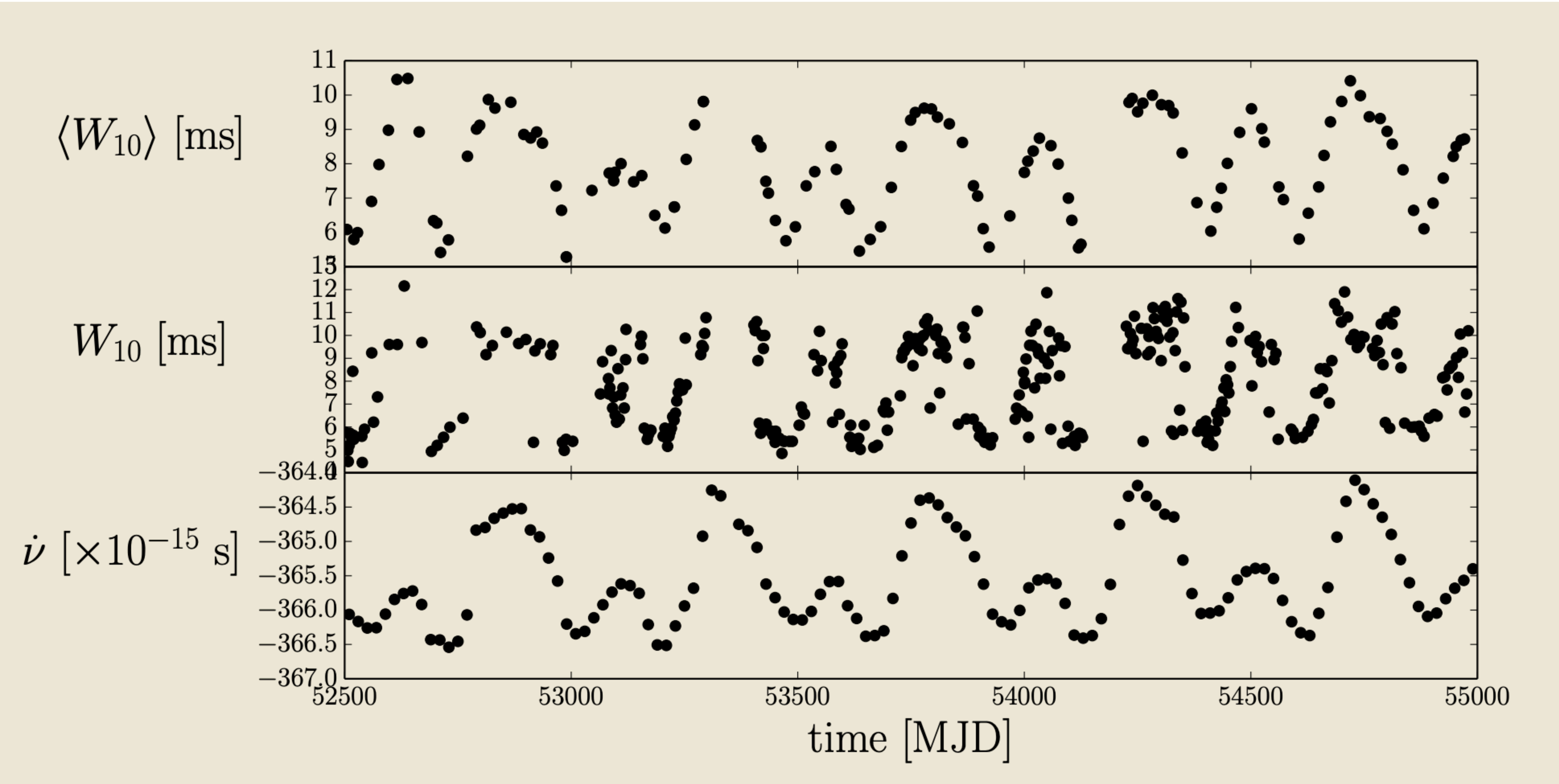
Magnetar 4U 0142+61: hard X-ray phase modulations (±0.7 s)

$P_\mathrm{f} \sim 15\,\mathrm{h} \qquad \epsilon \sim 10^{-4}\,!$

Indication of strong internal toroidal magnetic field in the order of 10^{16} G

Possible evidence, but there are different voices

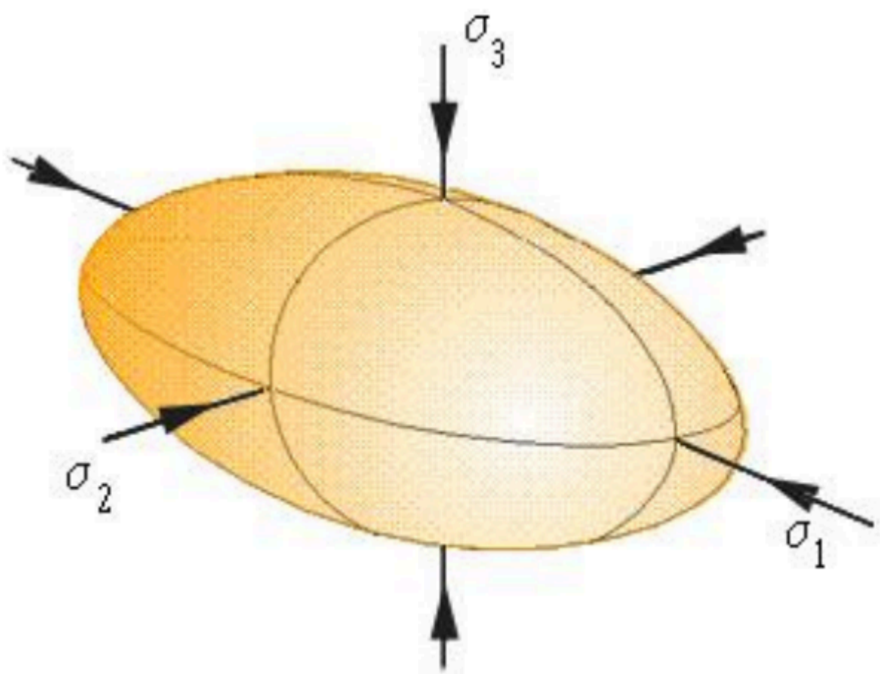
Do we observe free precession of NSs?



Data courtesy of Lyne at al. Science, 2010: Switched Magnetospheric Regulation of Pulsar Spin-Down

Gravitational waves: Set upper limit for ϵ

We study a simpler case: deformation at the equator of the NS



$$\epsilon = \frac{|I_{yy} - I_{xx}|}{I_{zz}}$$

$$h = 3 \times 10^{-28} \left(\frac{\epsilon}{10^{-6}}\right) \left(\frac{f_{\text{spin}}}{10\ \text{Hz}}\right)^2 \left(\frac{1\text{kpc}}{r}\right)$$

For pulsars with measured period P and spin-down rate P-dot can obtain upper limit on ellipticity by assuming 100% conversion of kinetic energy into GW energy:

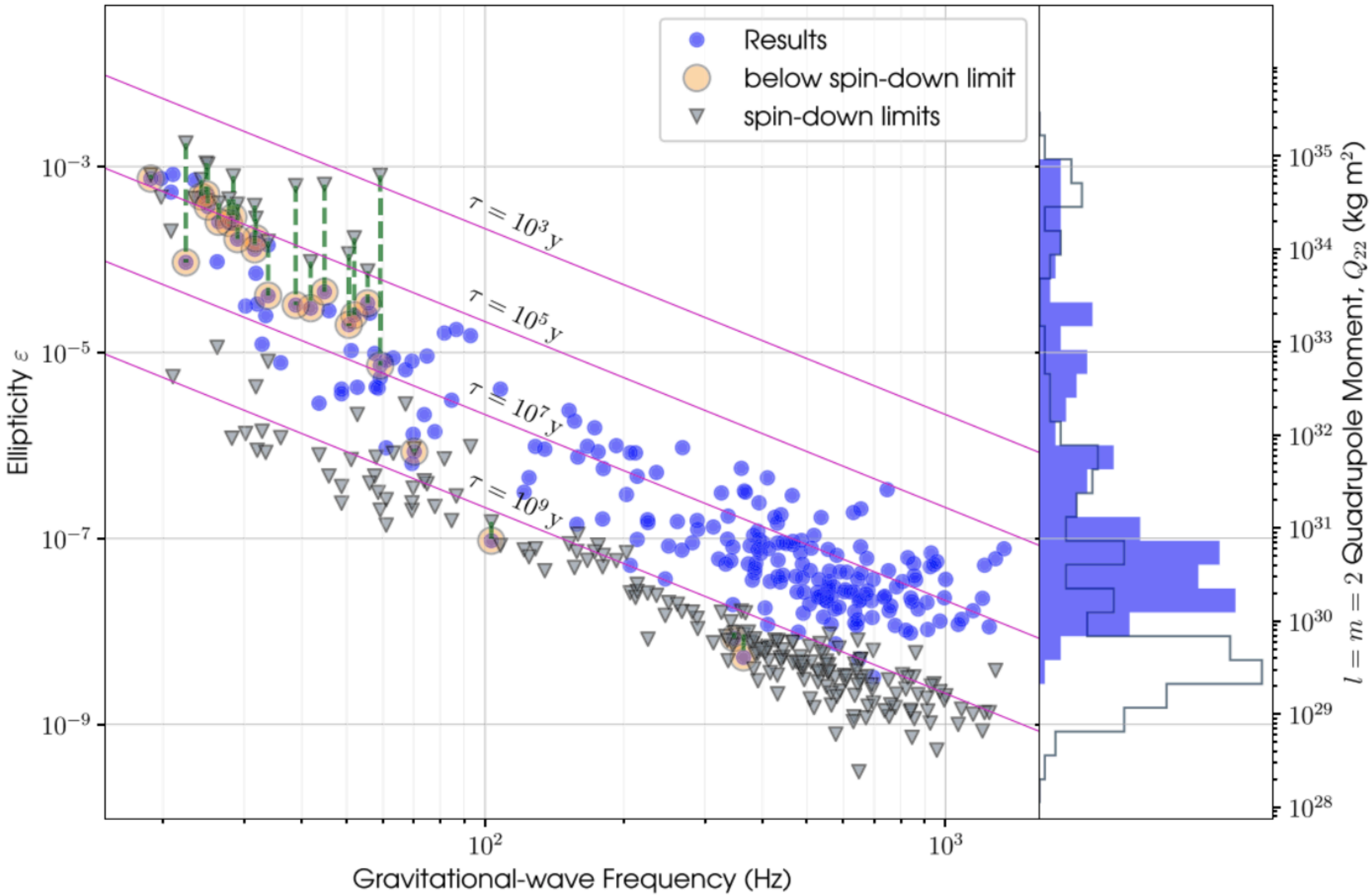
$$\epsilon_{\text{spindown}} = \left[\frac{5\dot{P}P^3}{32(2\pi)^4 I_{zz}}\right]^{1/2}$$

Crab: $\varepsilon_{\text{spindown}} \approx 7.6 \times 10^{-4}$

If the distance is known, can be converted to strain   Crab: $h_0 \approx 1.4 \times 10^{-24}$

Gravitational waves: Set upper limit for ϵ

Abbott et al., ApJ, 2022



What can we learn from free precession?

- Important information on NS crust physics: shear modulus & breaking strain
- Information on NS internal magnetic field configuration and strength
- **Superfluid does not support long precession period without damping**

Attempt to explain post-glitch behavior, the neutron vortices that coexist with the inner crust become pinned

Shaham, ApJ, 1977

$$\boldsymbol{J} = \boldsymbol{I} \cdot \boldsymbol{\Omega} + \boldsymbol{J}_{\mathrm{SF}}$$

$$\omega_{\mathrm{p}} \sim -\left(\epsilon + \frac{I_{\mathrm{f}}}{I_{\mathrm{c}}}\right)\omega$$

Motivated in part by the need to model glitches, predict a frictional type coupling between the crust and core.

$$\boldsymbol{T} = K\left(\boldsymbol{\Omega}_{\mathrm{fluid}} - \boldsymbol{\Omega}_{\mathrm{solid}}\right)$$

Bondi & Gold, MNRAS, 1955

Damping of the precession

Summary

- Solid strangeon star with anisotropic pressure can supply the energy budget for magnetar giant flare

- Precession of NSs can give information on elasticity, magnetic field, superfluid interior

Thank you for listening!