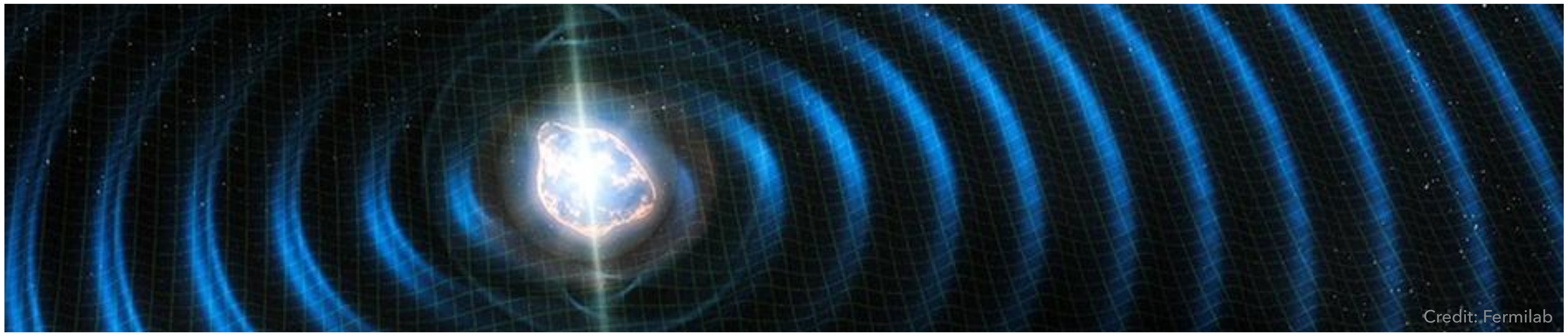


GARVIN YIM

Kavli Institute for Astronomy and Astrophysics, Peking University

g.yim@pku.edu.cn

SMALL GLITCHES AND ANTI-GLITCHES FROM NON-AXISYMMETRIC OSCILLATION MODES

"Quakes: from the Earth to the Stars" Meeting

Dream Field, FAST, Guizhou, 2023/05/22

CONTENTS

Aim: To create a model to explain how small glitches and anti-glitches arise from non-axisymmetric neutron star oscillations.

PART I – INTRODUCTION TO GRAVITATIONAL WAVES

WHAT ARE GRAVITATIONAL WAVES?

- In electrodynamics, the acceleration of charged particles gives rise to electromagnetic waves (starting from dipole radiation).
- As an *analogy*, in gravitational physics, the acceleration of masses gives rise to gravitational waves (starting from quadrupole radiation).
- More specifically, gravitational waves are emitted whenever there is a time-varying non-axisymmetric mass or current multipole moment:

$$\dot{E}_{GW} = -\sum_{l=2}^{\infty}\sum_{m=-l}^{l} N_l \left\langle \left| \overset{(l+1)}{\text{Mass multipole}} \right|_{lm}^{2} + \left| \overset{(l+1)}{\text{Current multipole}} \right|_{lm}^{2} \right\rangle$$

where (l) represents the l'th time derivative and the angled brackets represents an average over many gravitational wave cycles. [Thorne (1980); Lindblom, Owen & Morsink (1998)]

QUADRUPOLE FORMULA

$$\dot{E}_{GW} = -\sum_{l=2}^{\infty}\sum_{m=-l}^{l} N_l \left\langle \left| \overset{(l+1)}{\text{Mass multipole}} \right|_{lm}^2 + \left| \overset{(l+1)}{\text{Current multipole}} \right|_{lm}^2 \right\rangle$$

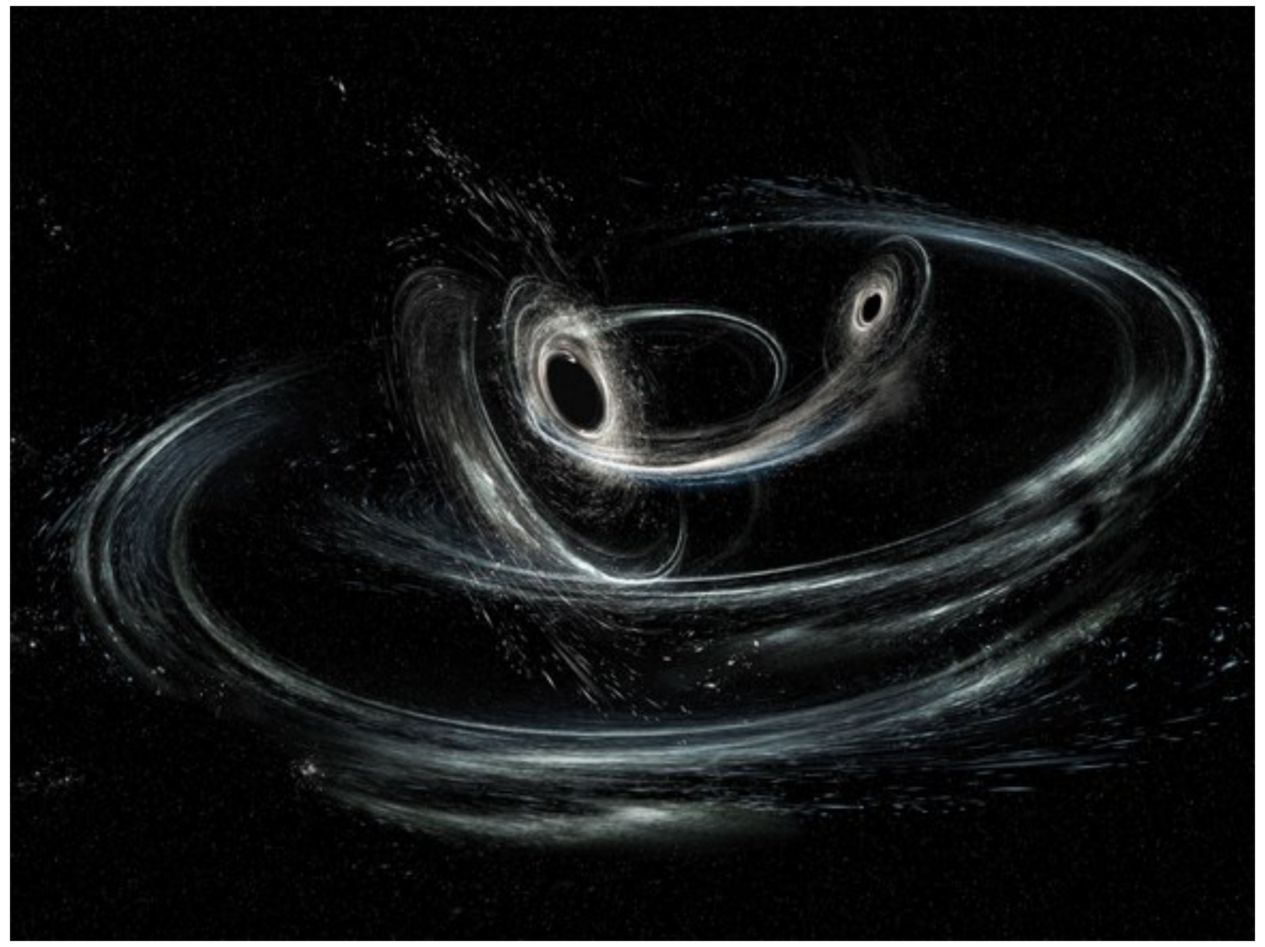
- The current multipole is a factor of c smaller than the mass multipole.
- For a given multipole l, the GW luminosity from the current multipole is a factor of c^2 weaker than the mass multipole → ignore current multipole.
- Also, for each increase in multipole l, the GW strain gets weaker by a factor of v/c, where v is some typical velocity of the system → only keep lowest multipole ($l = 2$).

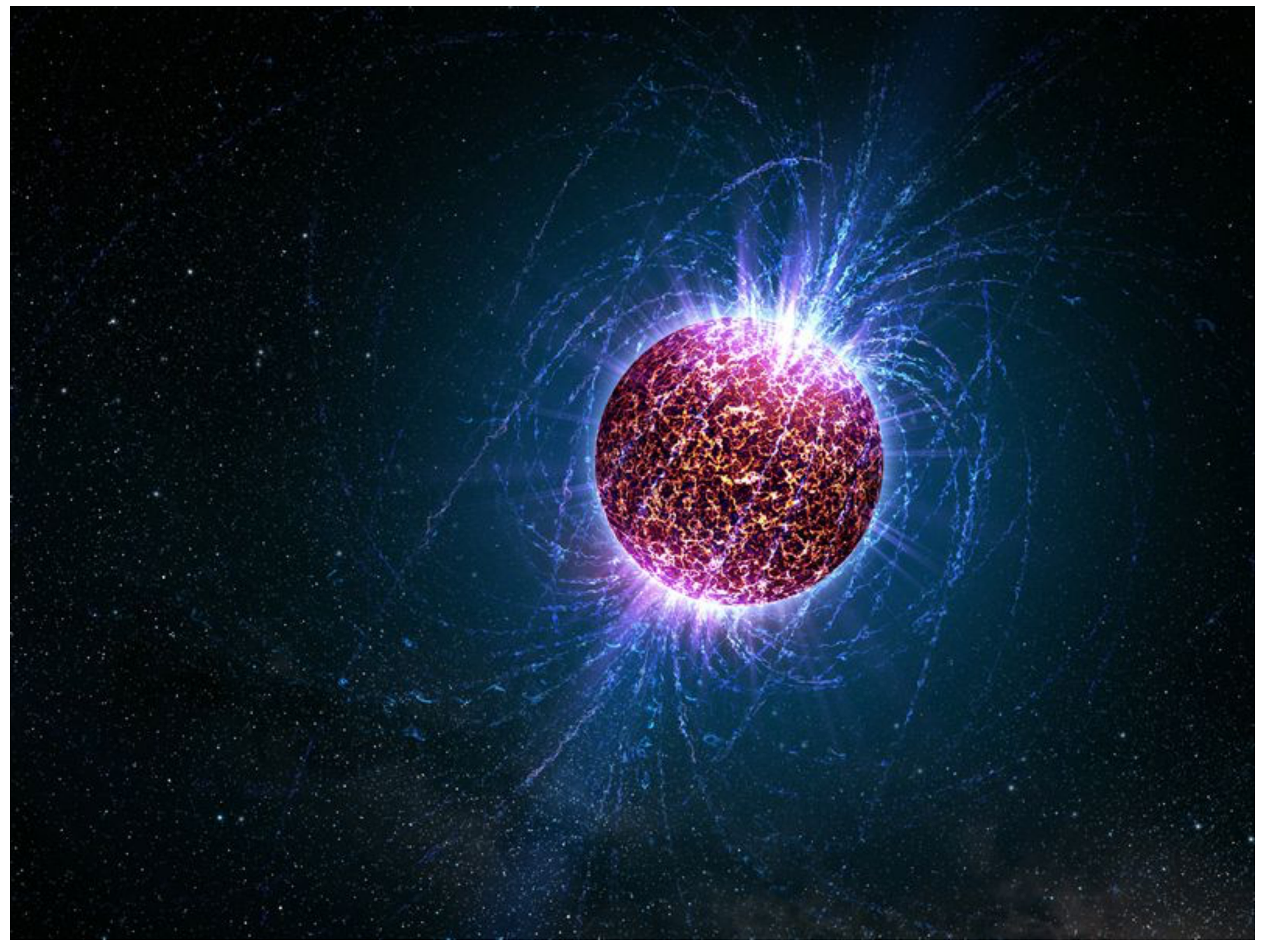
$$\dot{E}_{GW} \approx -\frac{1}{5}\frac{G}{c^5}\left\langle \dddot{I\!\!\!-}_{ij}\,\dddot{I\!\!\!-}^{ij} \right\rangle \quad \text{where} \quad I\!\!\!-_{ij} = \int_V \rho\left(x_i x_j - \frac{1}{3}x^k x_k\right) dV$$

TYPES OF GRAVITATIONAL WAVES

Compact Binary Coalescence

- $T \sim$ Seconds - Minutes
- Modelled
- Binary black holes, binary neutron stars, neutron star-black hole binary





Continuous

- $T \sim$ Quasi-infinite
- Modelled
- Neutron star mountains, precession, r-modes, accreting systems, boson clouds

Stochastic

- Always present
- Unmodelled
- Overlapping of compact binary signals, inflation, cosmological phase transitions, cosmic strings

Bursts

- $T \sim$ Milliseconds - Seconds
- Mostly unmodelled
- Supernovae, neutron star oscillations, anything unexpected

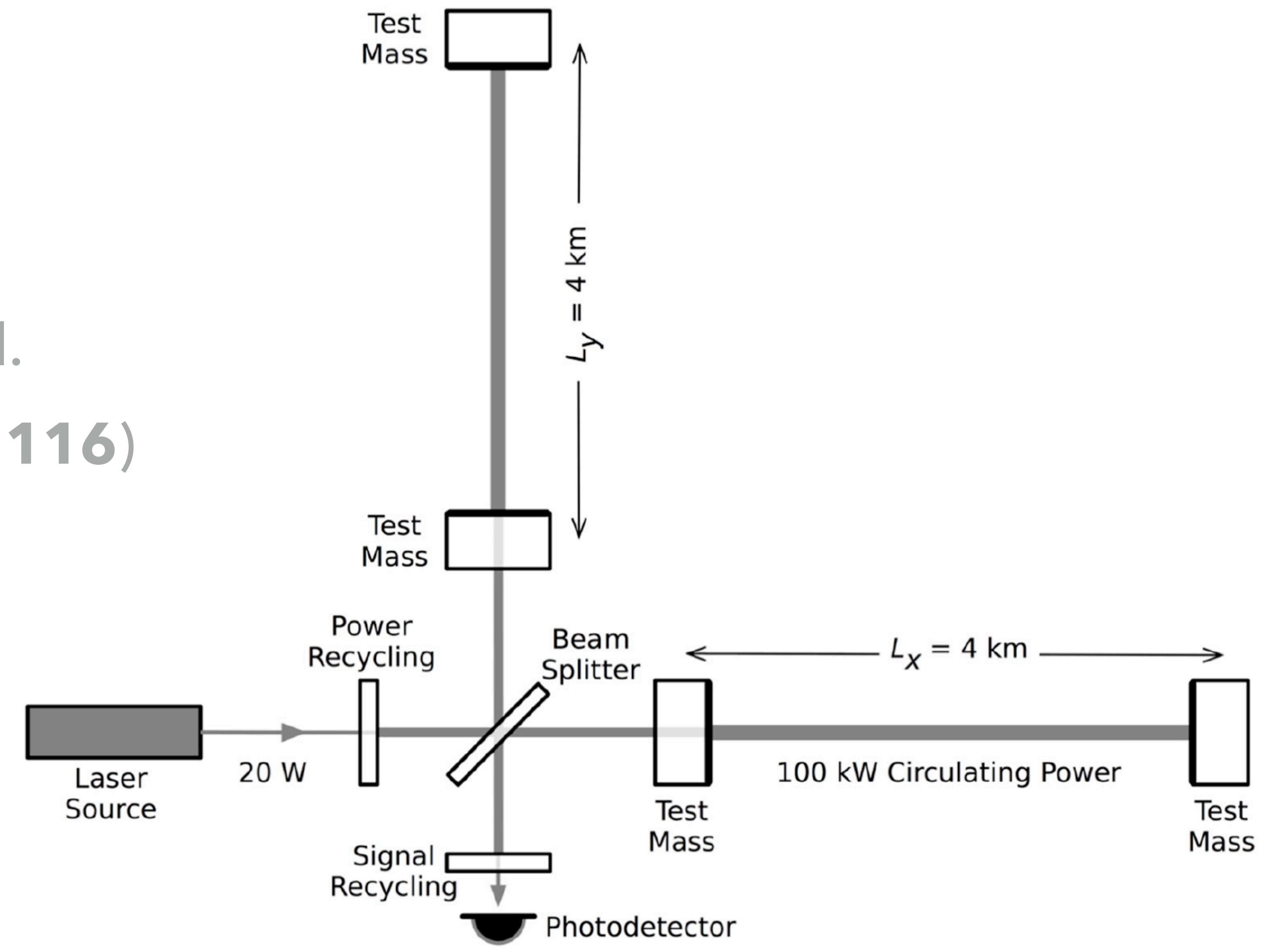
DETECTION OF GRAVITATIONAL WAVES WITH GROUND-BASED DETECTORS

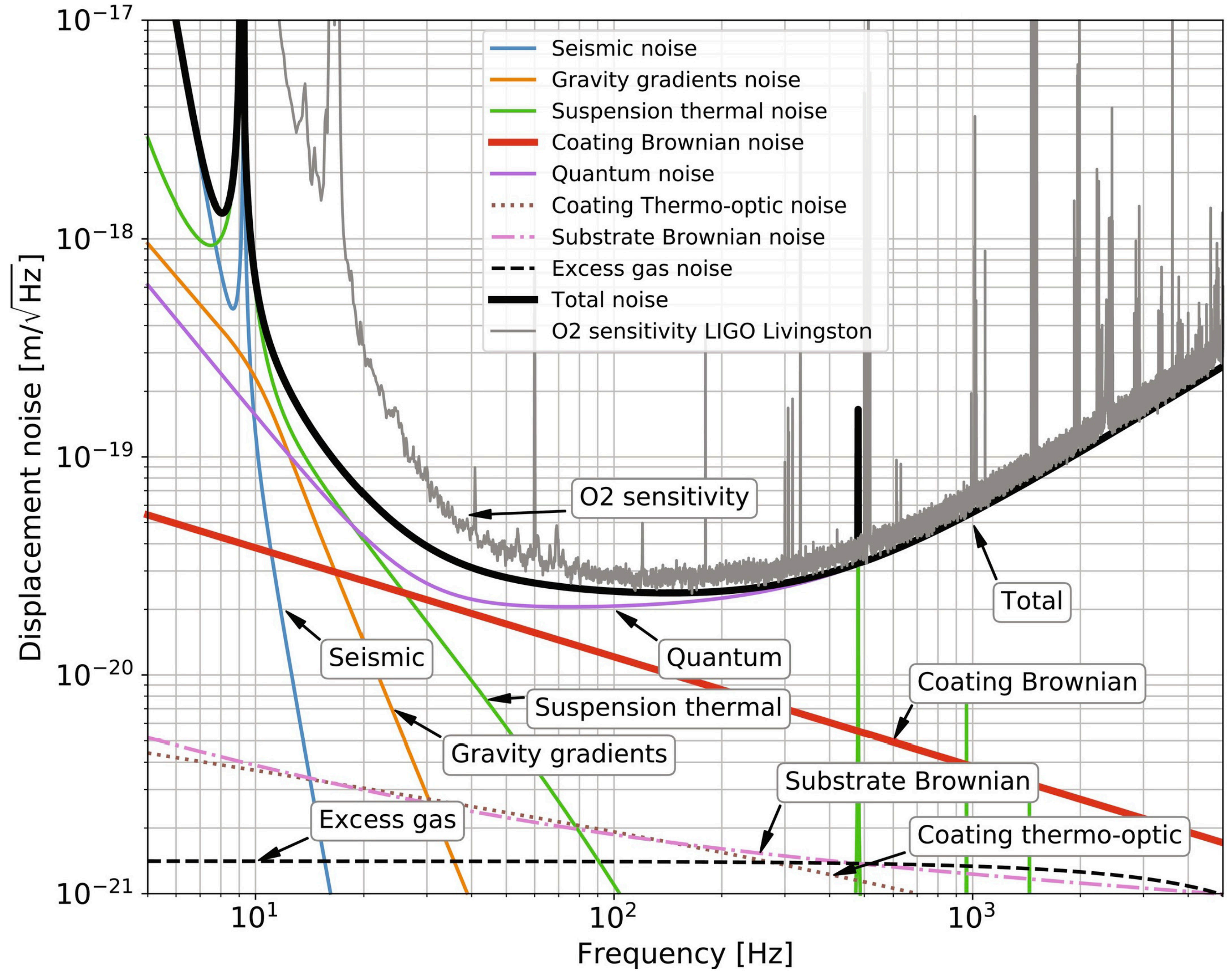
- Response of a passing GW is a tidal effect, i.e. stretches in one direction and squashes in the perpendicular direction.
- The GW strain tells us how much each arm gets stretched and squeezed.

$$h \sim \frac{\Delta L}{L}$$

- Sensitivity of this measurement depends on laser power, mirror coatings, mirror suspension, vacuum power and more.

Credit:
Abbott et al.
(2016, PRL, **116**)





Low freq. limitation
= Seismic noise
(See Han Yue and Li Zhao's talks)

High freq. limitation
= Photon shot noise
+ radiation pressure noise

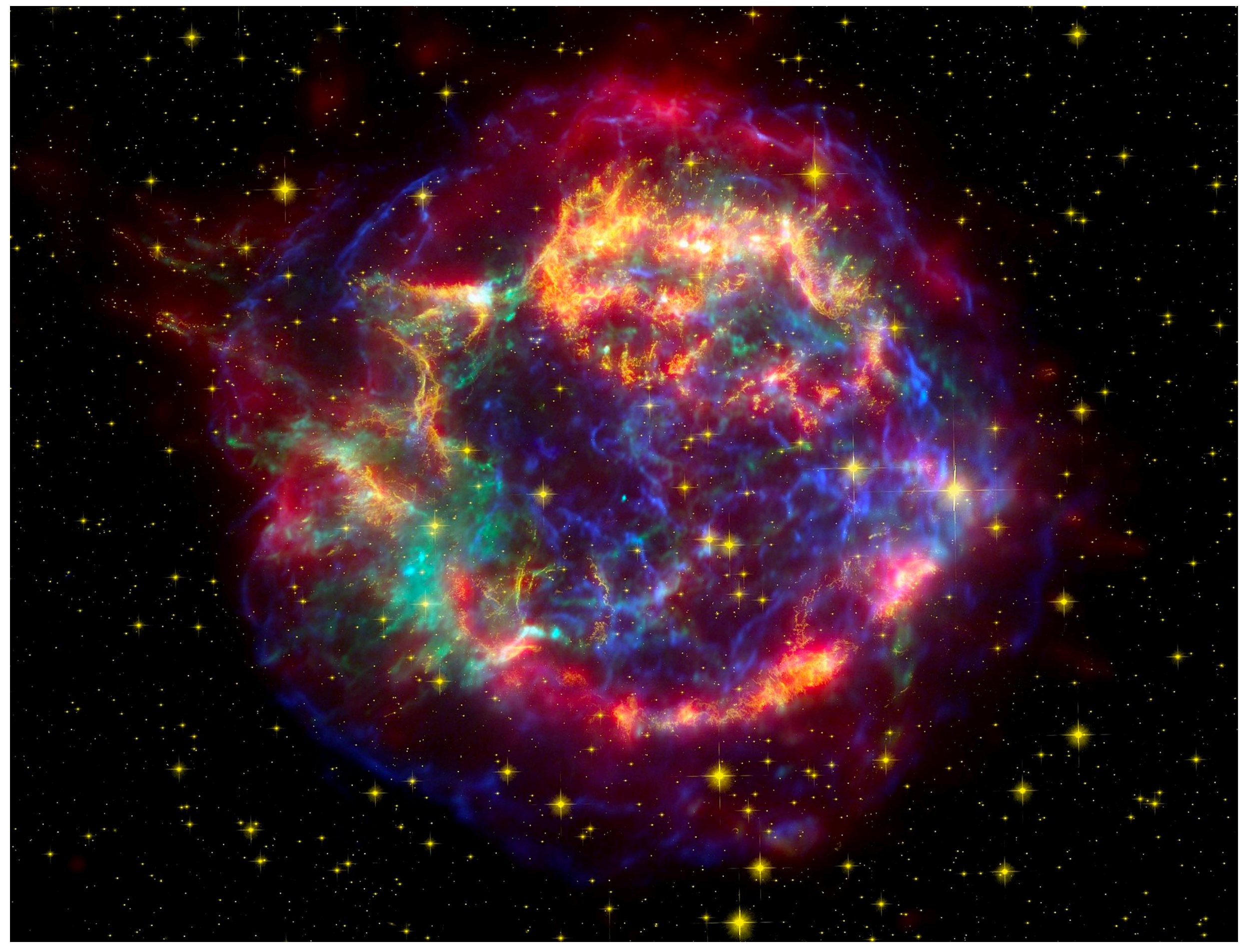
Credit: Vajente et al. (2019)

GRAVITATIONAL WAVE SEARCHES

- There are modelled and unmodelled searches for GWs.
- Modelled searches use a bank of templates/waveforms to compare to, can get a signal-to-noise.
- Unmodelled searches use algorithms to accumulate GW power in frequency-time space.

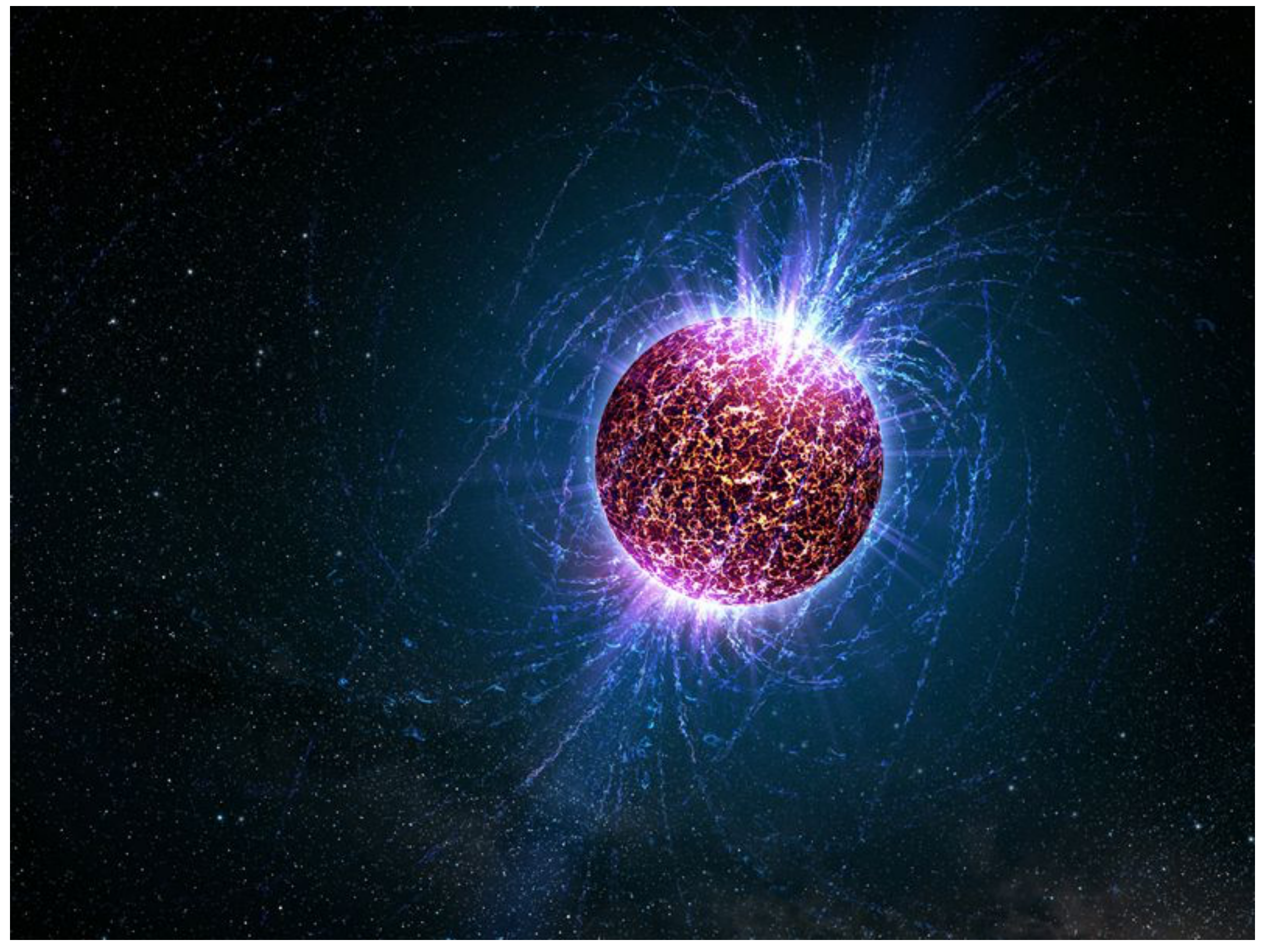
All-sky

(unknown location, unknown frequency)



Directed

(known location, unknown frequency)



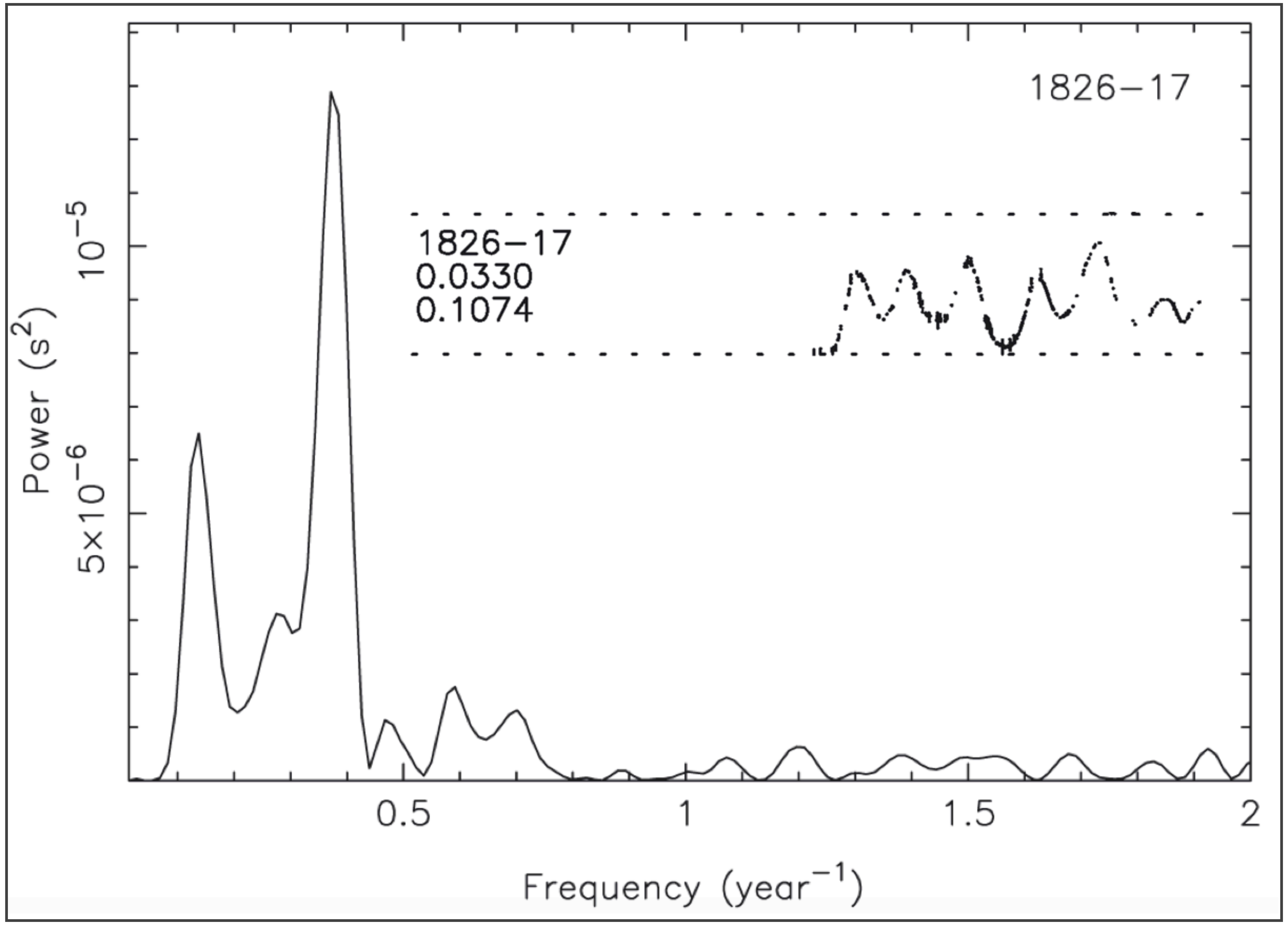
Targeted

(known location, known frequency)

PART II – TIMING OBSERVATIONS

See also Heng Xu's talk

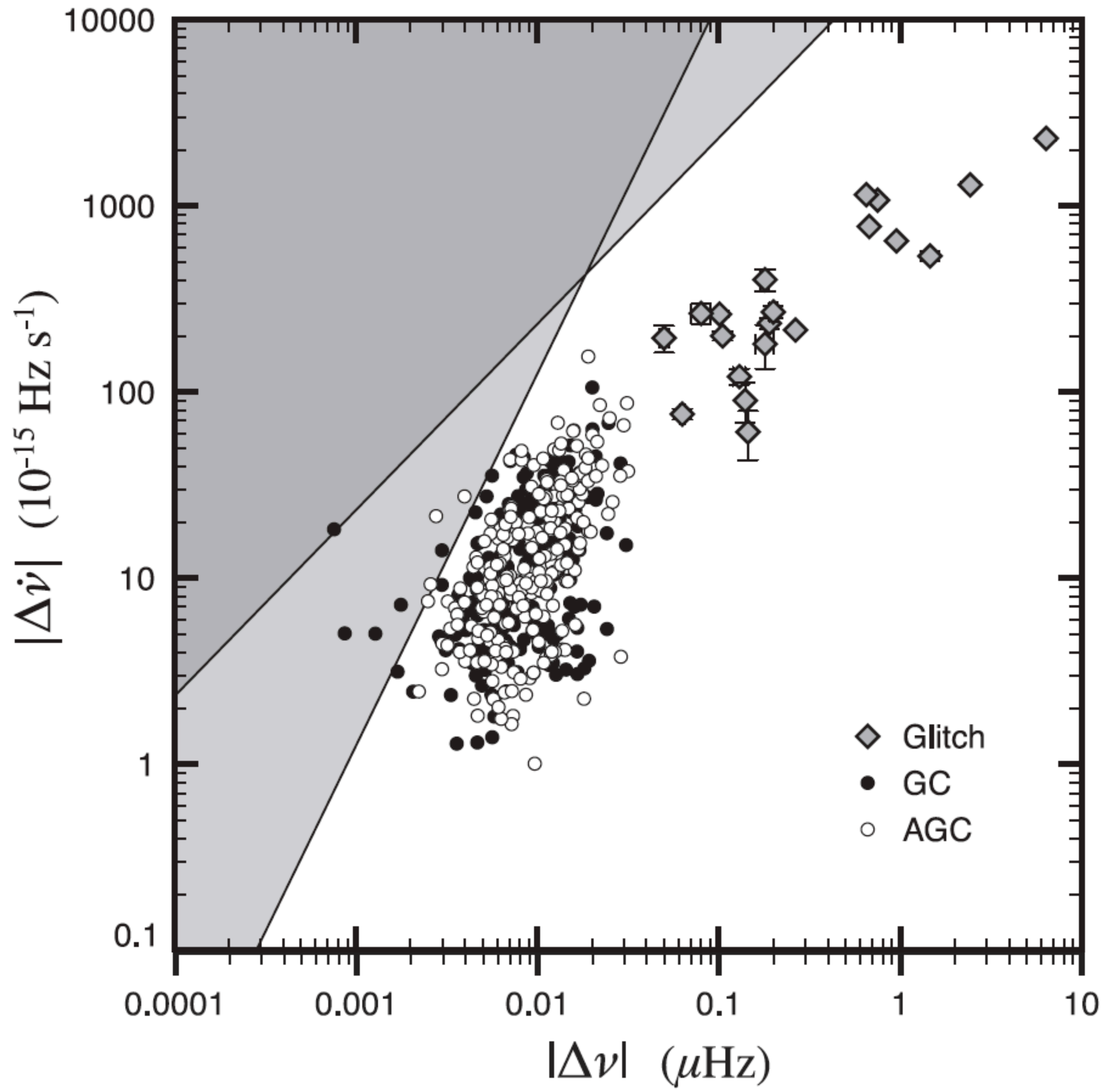
OVERVIEW OF PULSAR TIMING NOISE



An example of pulsar timing noise. *Taken from Hobbs et al. (2010).*

- Refers to any unmodelled residuals left over after known effects have been considered.
- Typically "red noise".
- Period > 1 year.
- Idea: Timing noise caused by consecutive small spin-ups and spin-downs.

OVERVIEW OF SMALL SPIN-UPS AND SPIN-DOWNS



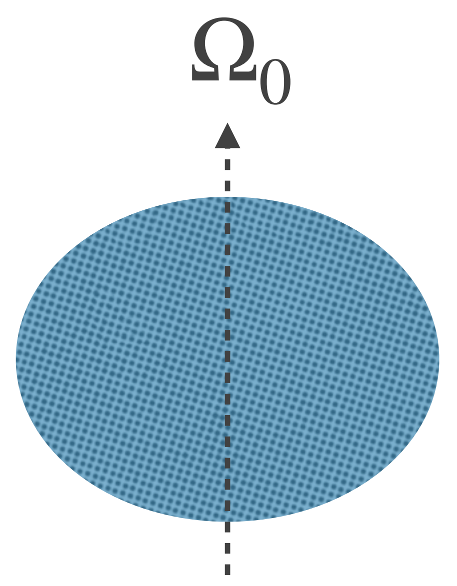
Taken from Espinoza et al. (2014).

- Espinoza et al. (2014, 2021) used an automated glitch detector on Crab and Vela data.
- Glitch candidates (GCs) are like glitches but smaller in magnitude and show no signs of recovery
  $\rightarrow$ GC = $\Delta\nu > 0, \Delta\dot{\nu} < 0$
- Anti-glitch candidates (AGCs) are the same, but have an opposite signature
  $\rightarrow$ AGC = $\Delta\nu < 0, \Delta\dot{\nu} > 0$

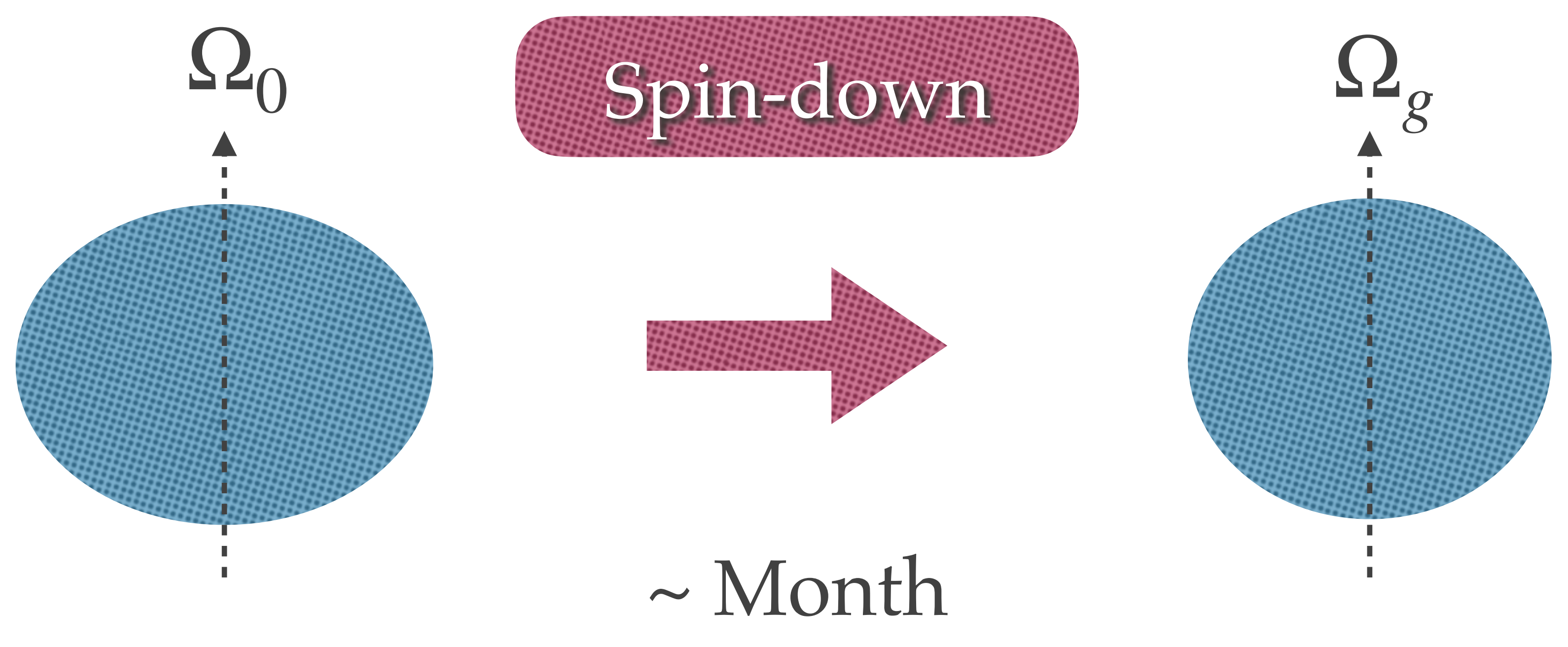
PART III – THE MODEL

See also Hongbo Li’s talk

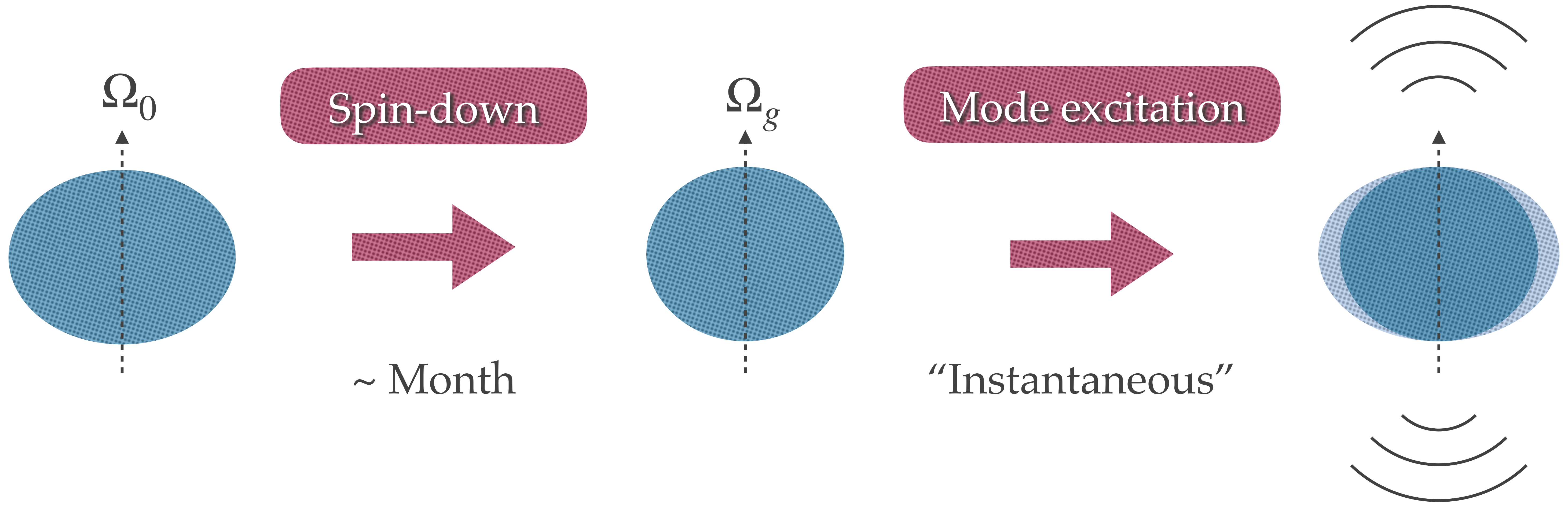
NEUTRON STAR OSCILLATION MODEL FOR SMALL SPIN-UPS AND SPIN-DOWNS



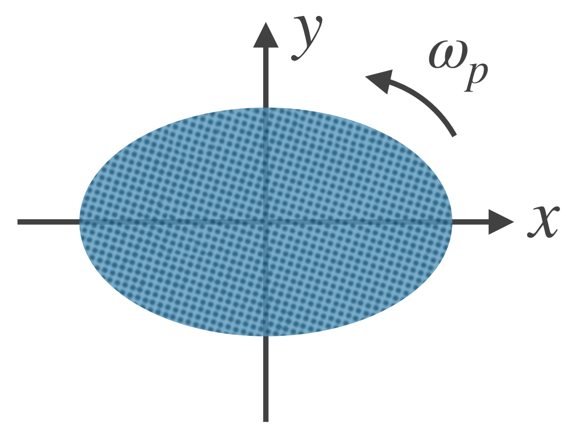
NEUTRON STAR OSCILLATION MODEL FOR SMALL SPIN-UPS AND SPIN-DOWNS



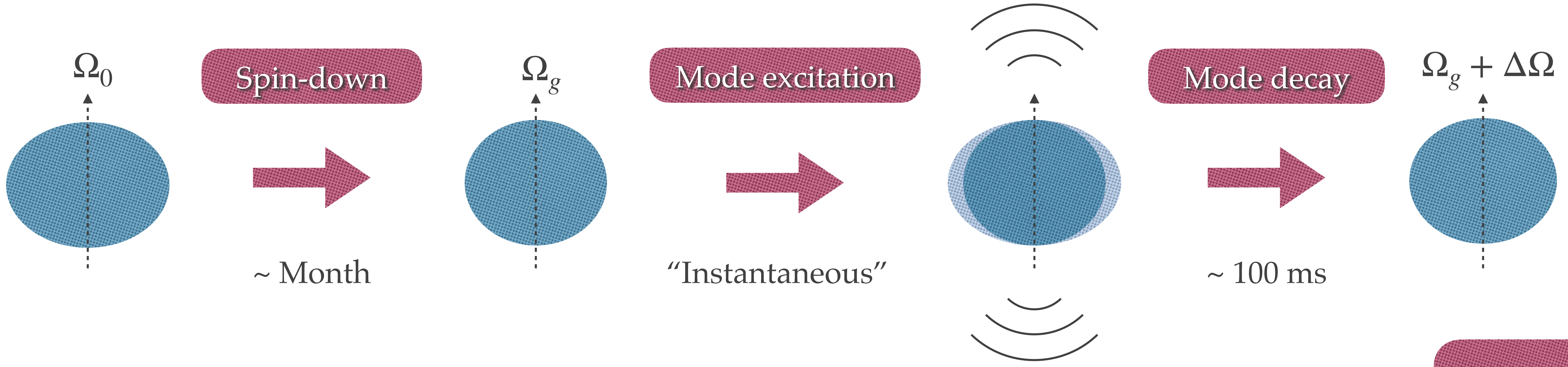
NEUTRON STAR OSCILLATION MODEL FOR SMALL SPIN-UPS AND SPIN-DOWNS



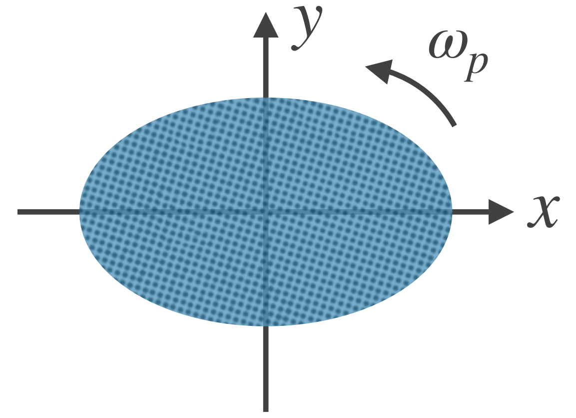
$l = 2,$
$m = \pm 2$
f-modes



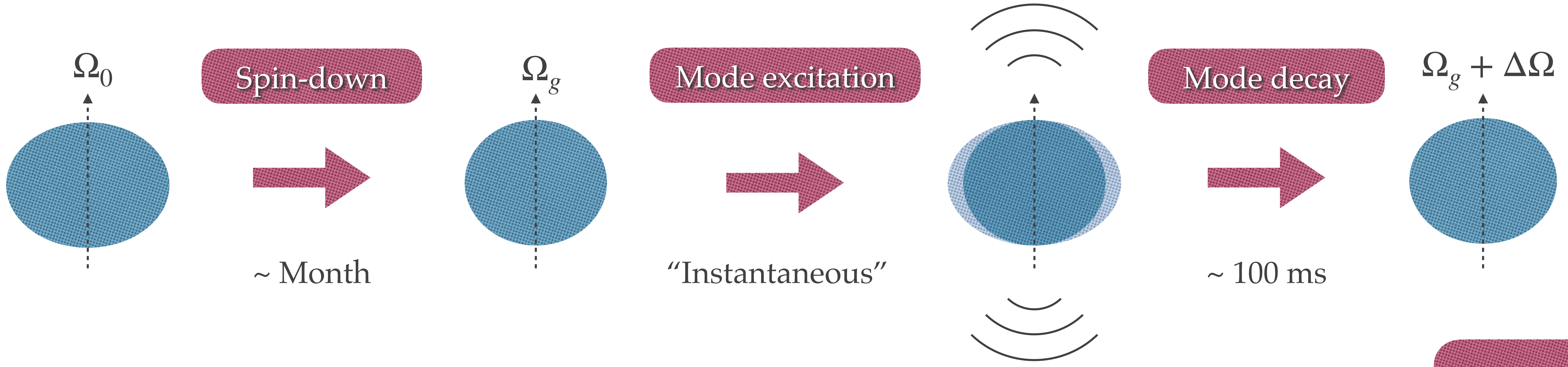
NEUTRON STAR OSCILLATION MODEL FOR SMALL SPIN-UPS AND SPIN-DOWNS



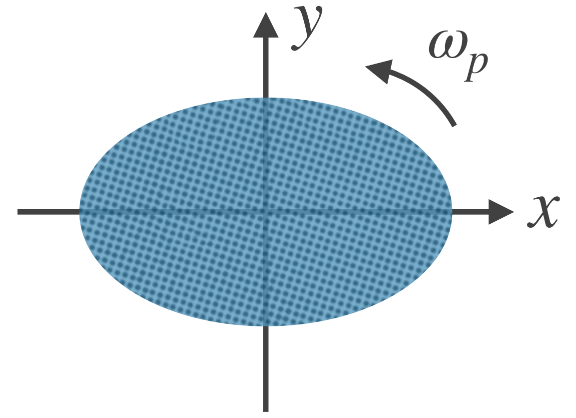
$l = 2$,
$m = \pm 2$
f-modes



NEUTRON STAR OSCILLATION MODEL FOR SMALL SPIN-UPS AND SPIN-DOWNS



$l = 2,$
$m = \pm 2$
f-modes

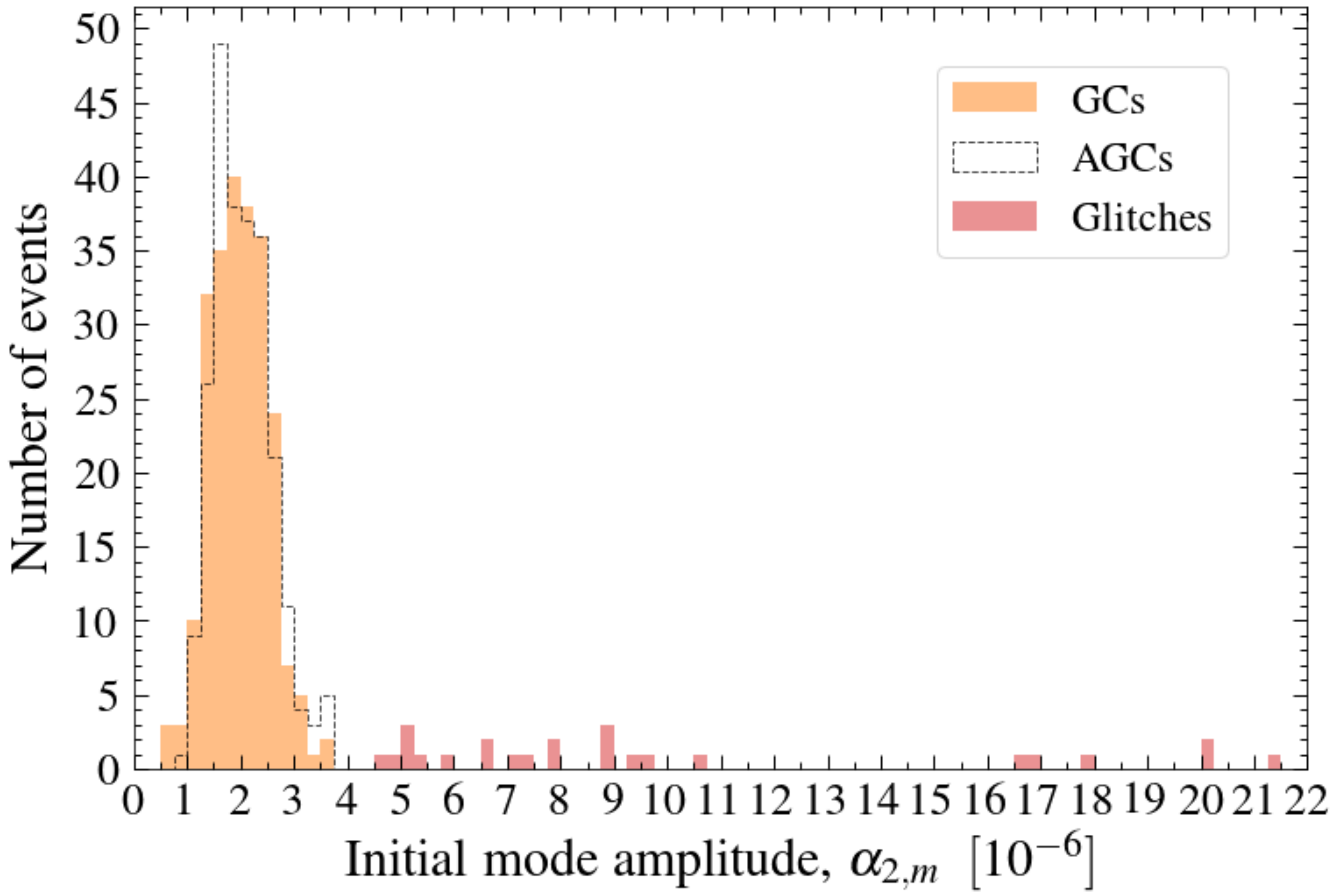


Cumulative change in angular momentum:

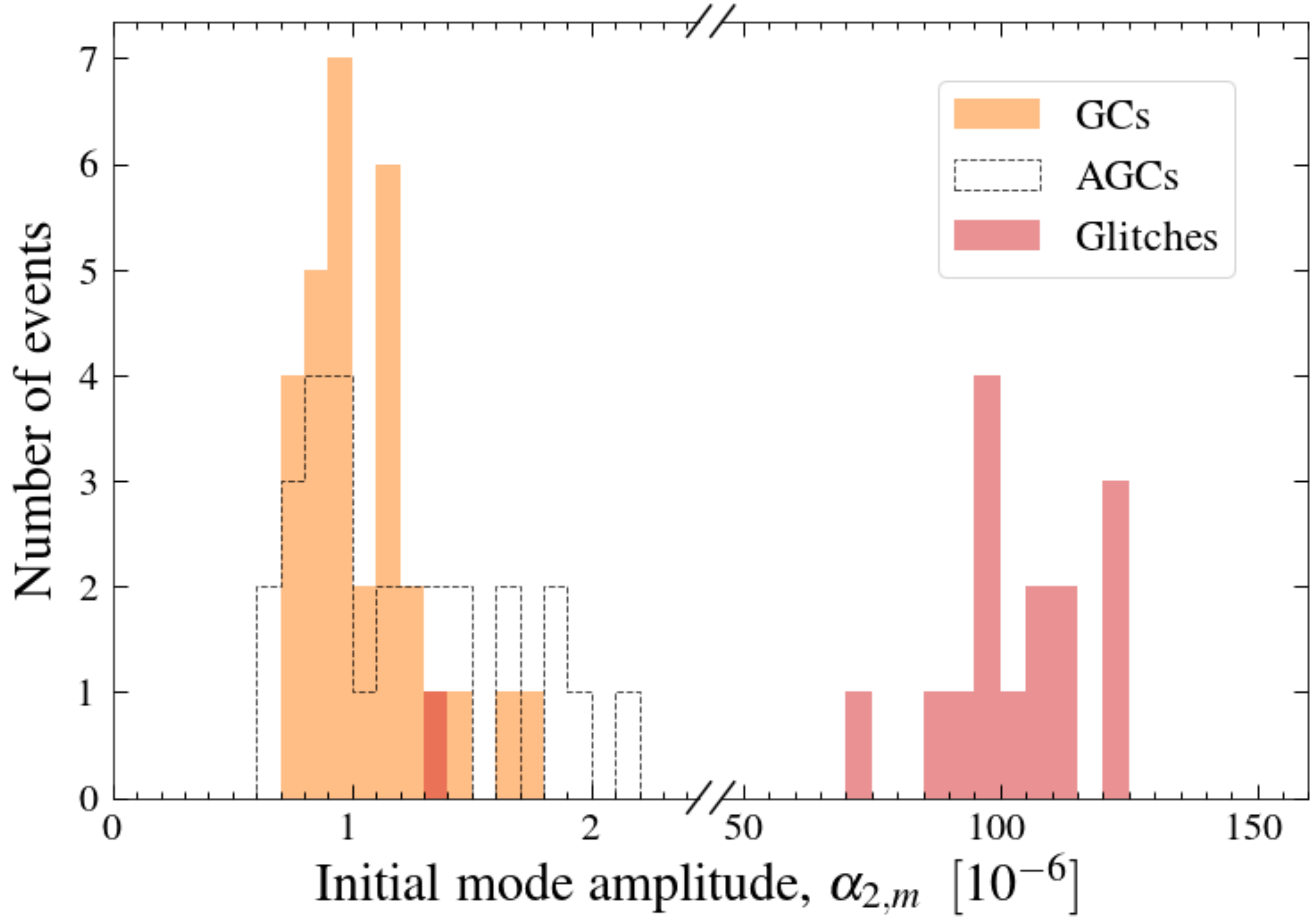
Bulk:	~ 0	$-\delta J$	$-2\delta J$
Modes:	0	$+\delta J$	0
GWs:	0	0	$+2\delta J$

(Yim & Jones, MNRAS, 2022)

RESULTS – MODE AMPLITUDE



Crab

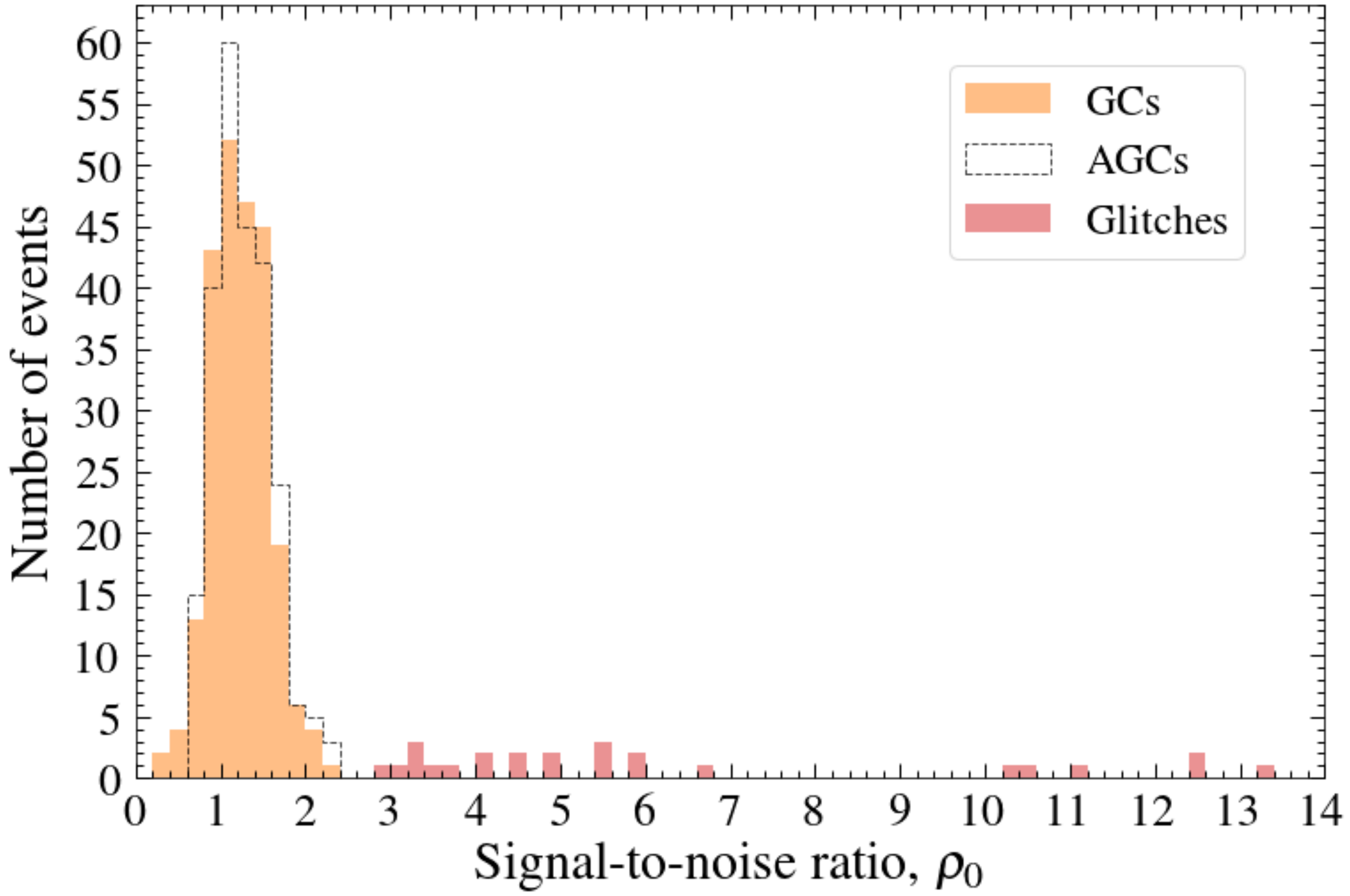


Vela

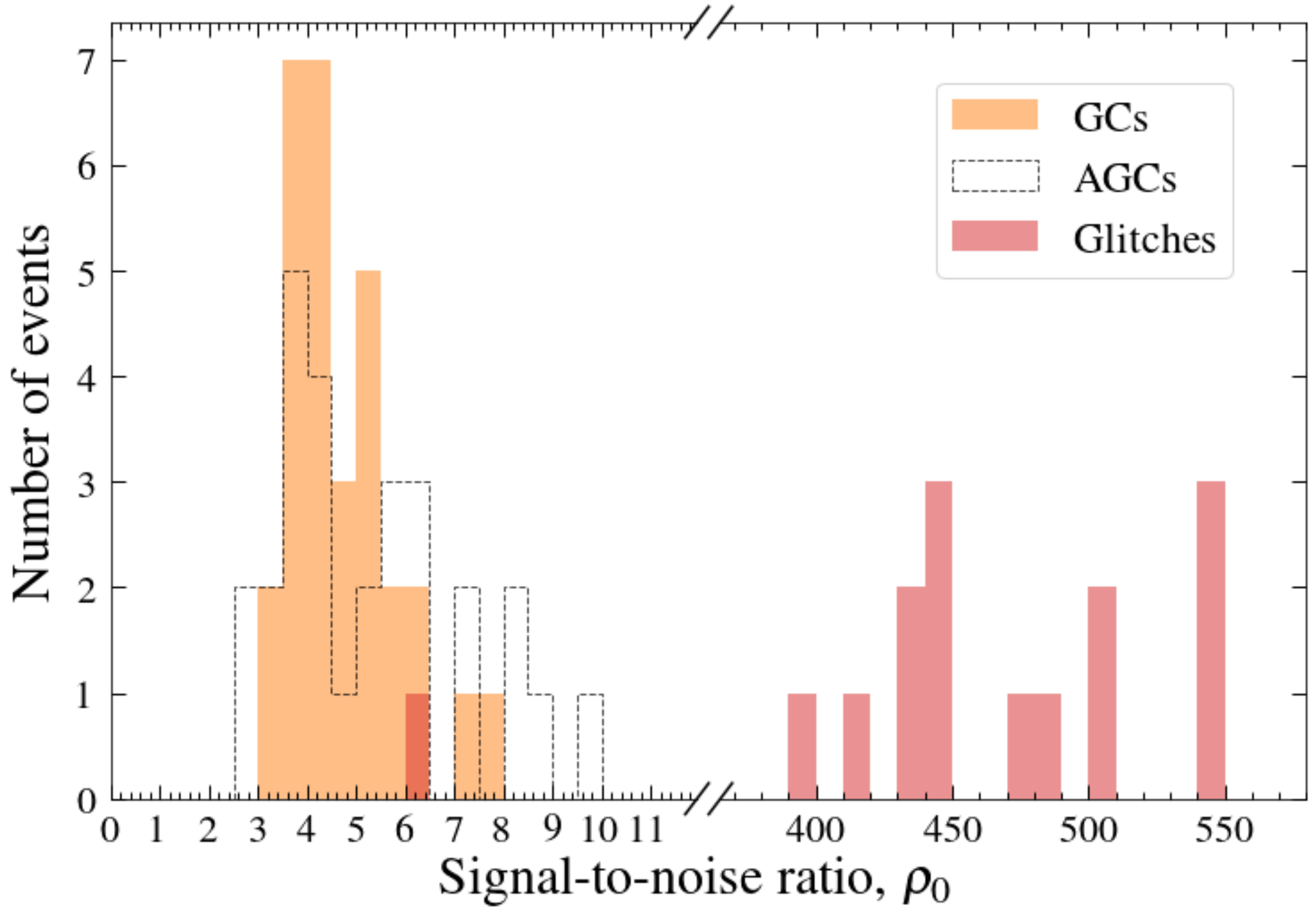
$$\alpha \propto \Delta r / R$$

$$\delta J \propto \alpha^2 \propto \Delta \nu$$

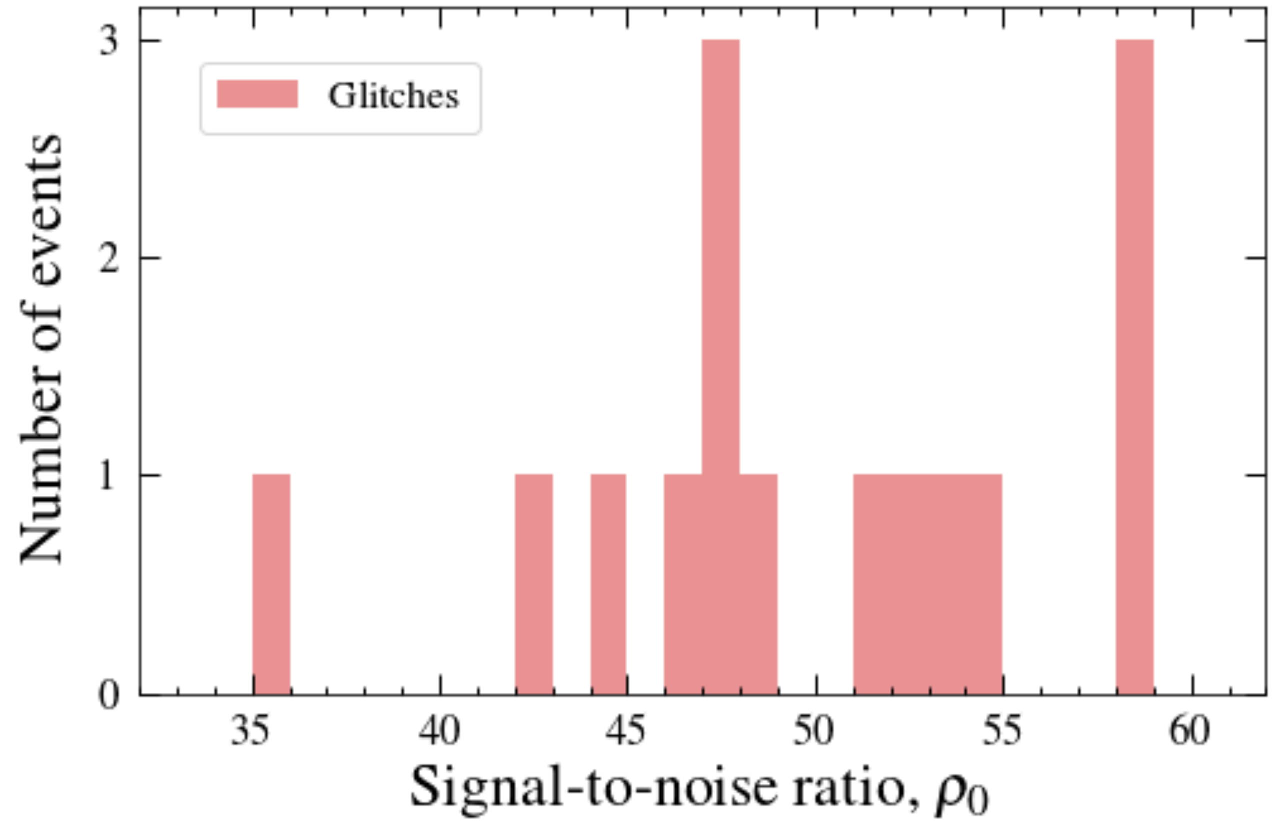
RESULTS – GRAVITATIONAL WAVE DETECTABILITY



Crab, ET or CE



Vela, ET or CE



Vela, aLIGO

$f_{GW} \sim 2$ kHz

$\tau_{GW} \sim 100$ ms

$$\rho_0 \propto \alpha \propto \sqrt{\Delta\nu}$$

PART IV – POWERING THE OSCILLATION MODES

HOW MUCH POWER IS REQUIRED TO SUSTAIN THE MODES?

- Time-averaged approach: $\langle \dot{E}_{mode} \rangle = F \langle \delta E \rangle$

where F is the rate of mode excitation (~once per month) and $\langle \delta E \rangle$ is the average mode energy.

HOW MUCH POWER IS REQUIRED TO SUSTAIN THE MODES?

- Time-averaged approach: $\langle \dot{E}_{mode} \rangle = F \langle \delta E \rangle$

where F is the rate of mode excitation (~once per month) and $\langle \delta E \rangle$ is the average mode energy.

$$\rightarrow \quad \langle \dot{E}_{mode} \rangle \approx 3.9 \times 10^{34} \left[\frac{\sqrt{\langle \alpha_{2,2}^2 \rangle}}{1 \times 10^{-6}} \right]^2 \left(\frac{M}{1.4 \ \mathrm{M}_\odot} \right)^2 \left(\frac{R}{10 \ \mathrm{km}} \right)^{-1} \left(\frac{F}{1/(30 \ \mathrm{d})} \right) \ \mathrm{erg\ s}^{-1}$$

HOW MUCH POWER IS REQUIRED TO SUSTAIN THE MODES?

- Time-averaged approach: $\langle \dot{E}_{mode} \rangle = F \langle \delta E \rangle$

where F is the rate of mode excitation (~once per month) and $\langle \delta E \rangle$ is the average mode energy.

$$\rightarrow \quad \langle \dot{E}_{mode} \rangle \approx 3.9 \times 10^{34} \left(\frac{\sqrt{\langle \alpha_{2,2}^2 \rangle}}{1 \times 10^{-6}} \right)^2 \left(\frac{M}{1.4\ \mathrm{M_\odot}} \right)^2 \left(\frac{R}{10\ \mathrm{km}} \right)^{-1} \left(\frac{F}{1/(30\ \mathrm{d})} \right) \ \mathrm{erg\ s^{-1}}$$

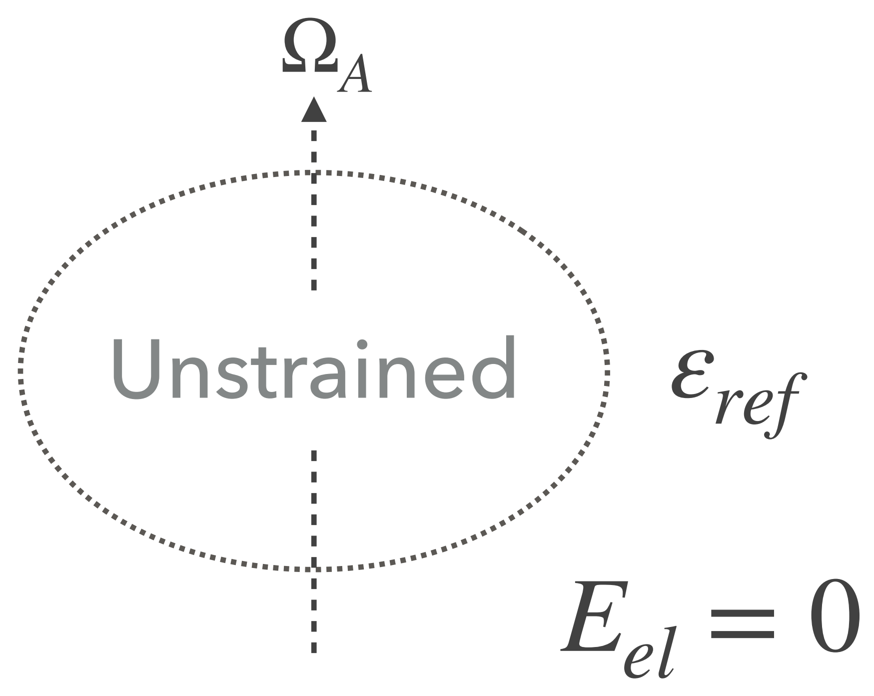
- Compare to spin-down power $\dot{E}_{spin-down} = I \Omega \dot{\Omega}$

Crab: $\langle \dot{E}_{mode} \rangle = 7 \times 10^{-4}\ \dot{E}_{spin-down}$ $\rightarrow$ $0.07\ \%$ of spin-down power required

Vela: $\langle \dot{E}_{mode} \rangle = 4 \times 10^{-3}\ \dot{E}_{spin-down}$ $\rightarrow$ $0.4\ \%$ of spin-down power required

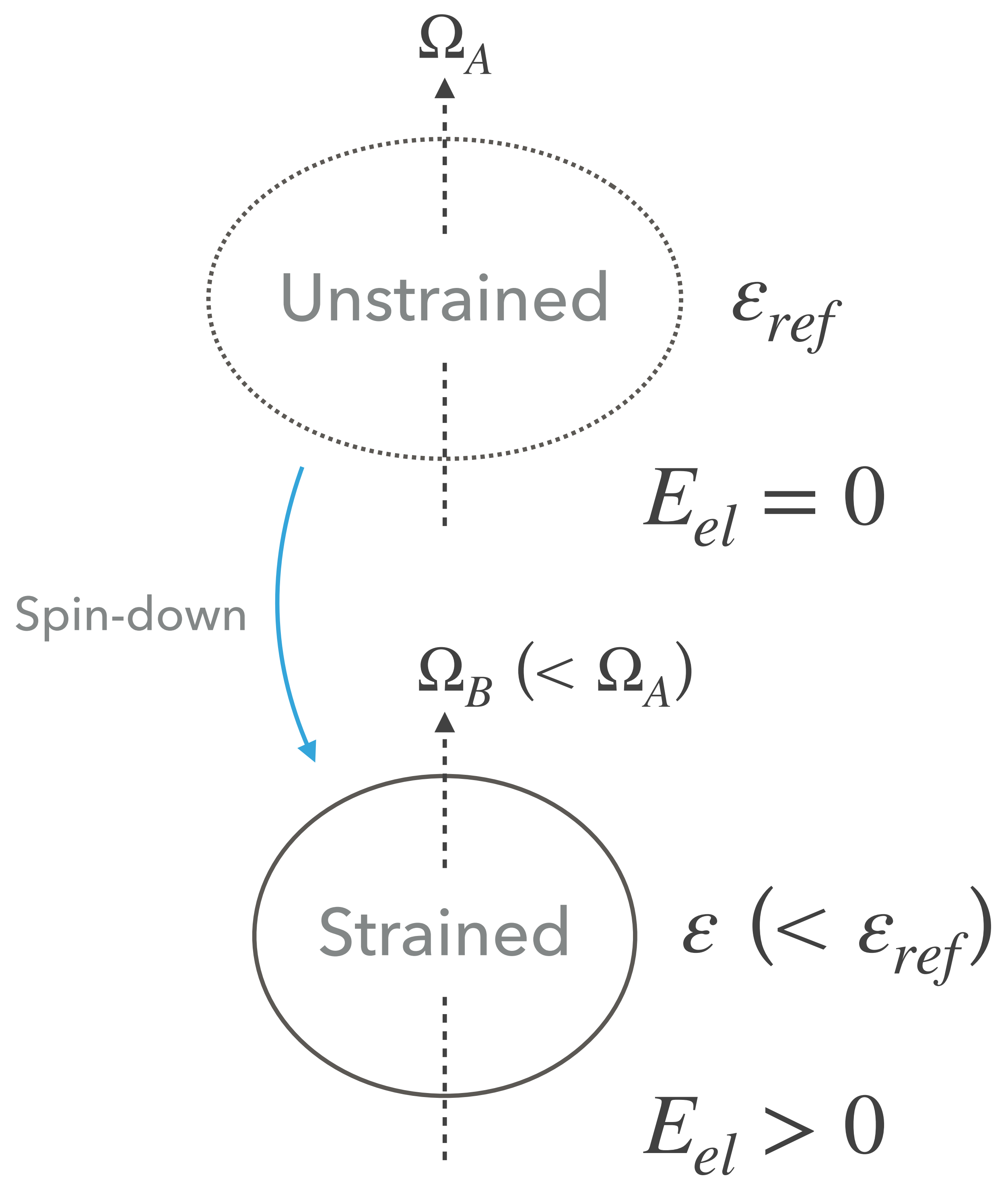
CAN ELASTICITY POWER THESE MODES?

- Back-of-the-envelope calculation using Baym & Pines (1971) → axisymmetric spin-down



CAN ELASTICITY POWER THESE MODES?

- Back-of-the-envelope calculation using Baym & Pines (1971) → axisymmetric spin-down



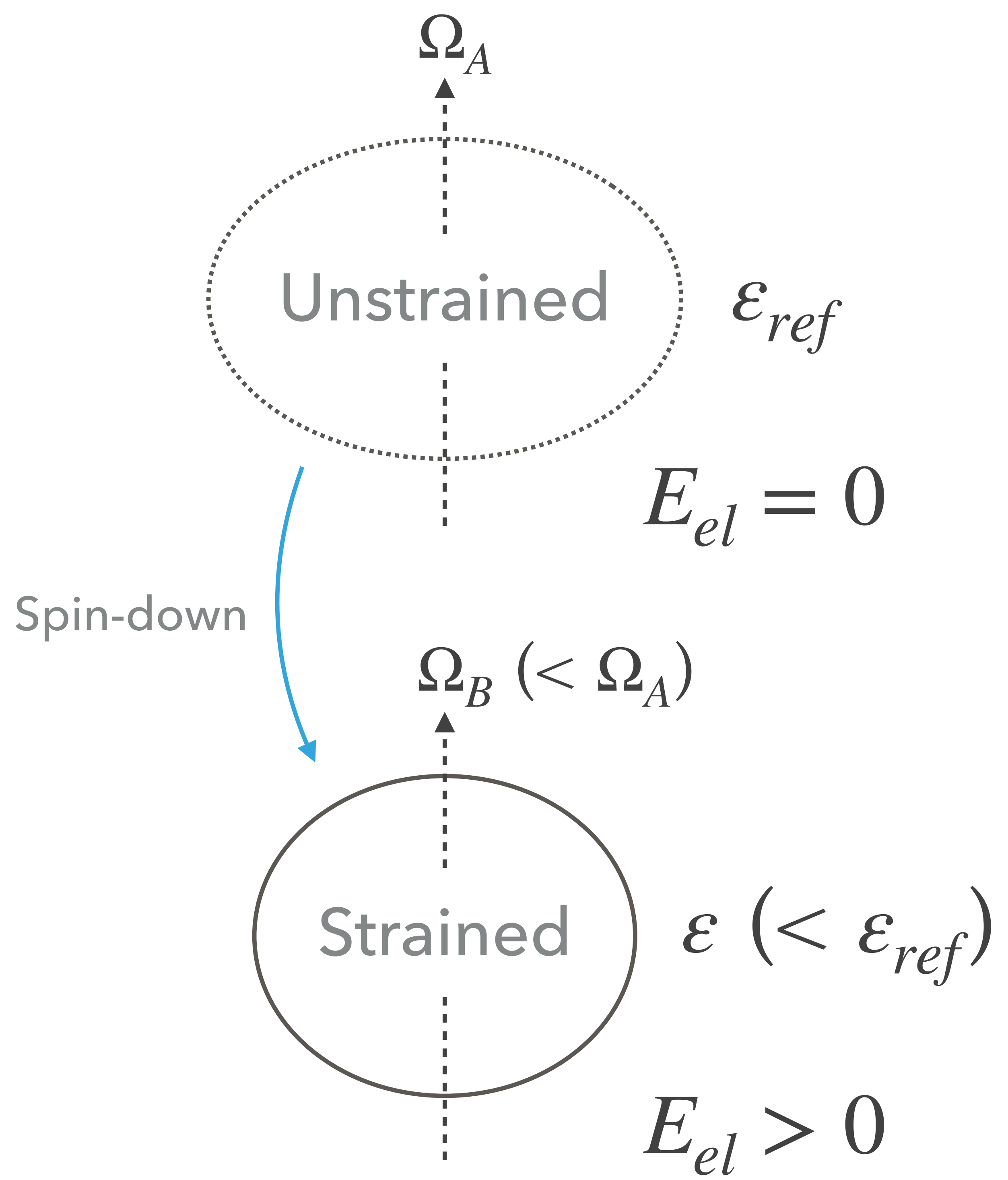
CAN ELASTICITY POWER THESE MODES?

- Back-of-the-envelope calculation using Baym & Pines (1971) → axisymmetric spin-down

$$E_{el} = B(\varepsilon_{ref} - \varepsilon)^2$$

$$\varepsilon = \frac{I_{sph}\Omega^2}{4(A+B)} + \frac{B}{A+B}\varepsilon_{ref}$$

where E_{el} is the elastic energy, ε is the oblateness, and A and B are constants due to gravitational and elastic energy corrections.



CAN ELASTICITY POWER THESE MODES?

- Back-of-the-envelope calculation using Baym & Pines (1971) → axisymmetric spin-down

$$E_{el} = B(\varepsilon_{ref} - \varepsilon)^2$$

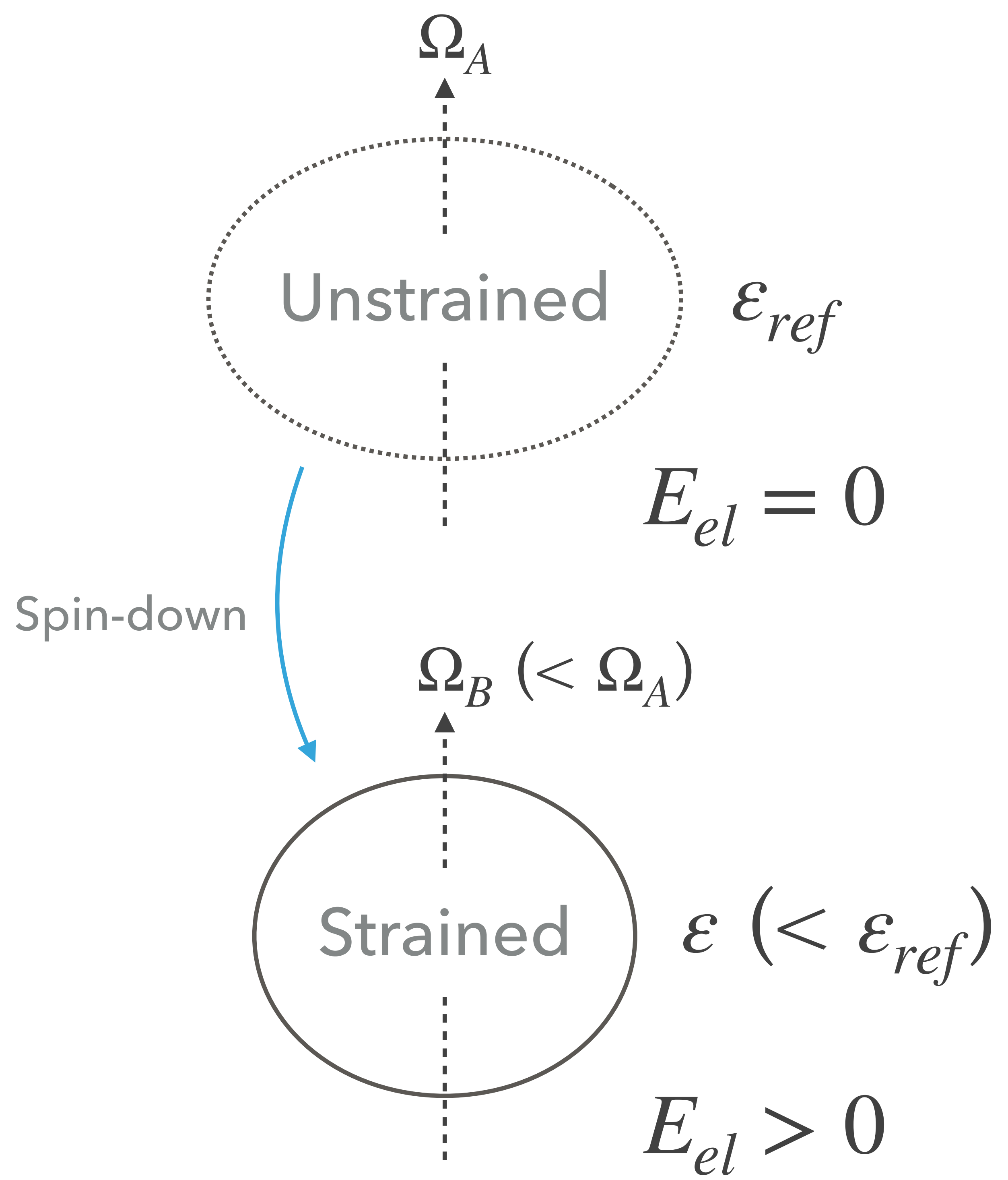
$$\varepsilon = \frac{I_{sph}\Omega^2}{4(A+B)} + \frac{B}{A+B}\varepsilon_{ref}$$

Time derivative →

$$\dot{E}_{el} = -2B\dot{\varepsilon}(\varepsilon_{ref} - \varepsilon)$$

$$\dot{\varepsilon} = \frac{I_{sph}\Omega\dot{\Omega}}{2(A+B)} = \frac{\dot{E}_{spin-down}}{2(A+B)}$$

where E_{el} is the elastic energy, ε is the oblateness, and A and B are constants due to gravitational and elastic energy corrections.



CAN ELASTICITY POWER THESE MODES?

- Back-of-the-envelope calculation using Baym & Pines (1971) → axisymmetric spin-down

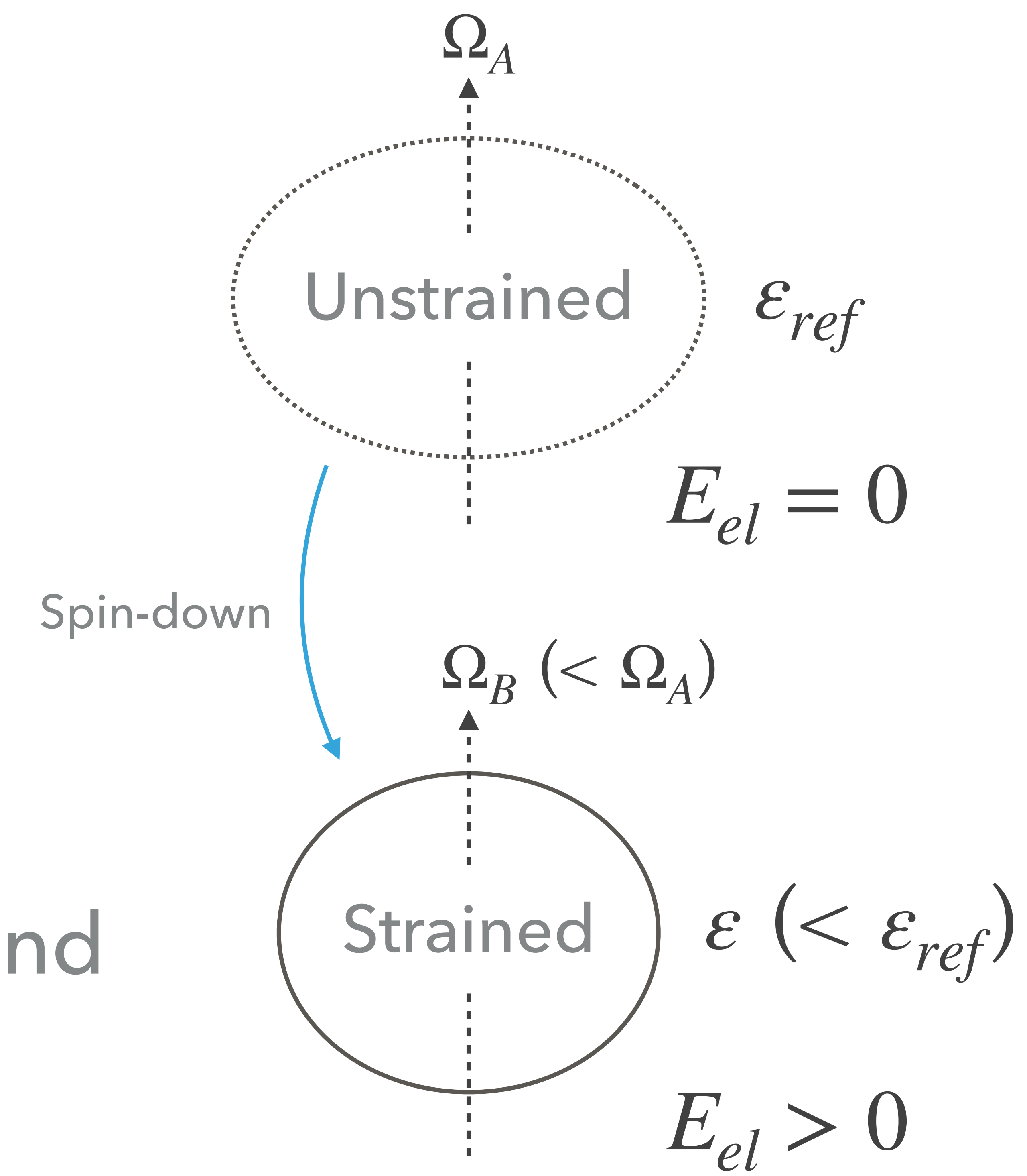
$$E_{el} = B(\varepsilon_{ref} - \varepsilon)^2$$

$$\varepsilon = \frac{I_{sph}\Omega^2}{4(A+B)} + \frac{B}{A+B}\varepsilon_{ref}$$

Time derivative

$$\dot{E}_{el} = -2B\dot{\varepsilon}(\varepsilon_{ref} - \varepsilon)$$

$$\dot{\varepsilon} = \frac{I_{sph}\Omega\dot{\Omega}}{2(A+B)} = \frac{\dot{E}_{spin-down}}{2(A+B)}$$



where E_{el} is the elastic energy, ε is the oblateness, and A and B are constants due to gravitational and elastic energy corrections.

- For a NS crust, $B/A \sim 10^{-5}$ with $B \sim \mu V$, where μ is the shear modulus and V is the volume of stressed elastic material

$$\frac{|\dot{E}_{el}|_{max}}{\dot{E}_{spin-down}} = \frac{B}{A+B}(\varepsilon_{ref} - \varepsilon)_{max}$$

CAN ELASTICITY POWER THESE MODES?

- Back-of-the-envelope calculation using Baym & Pines (1971) → axisymmetric spin-down

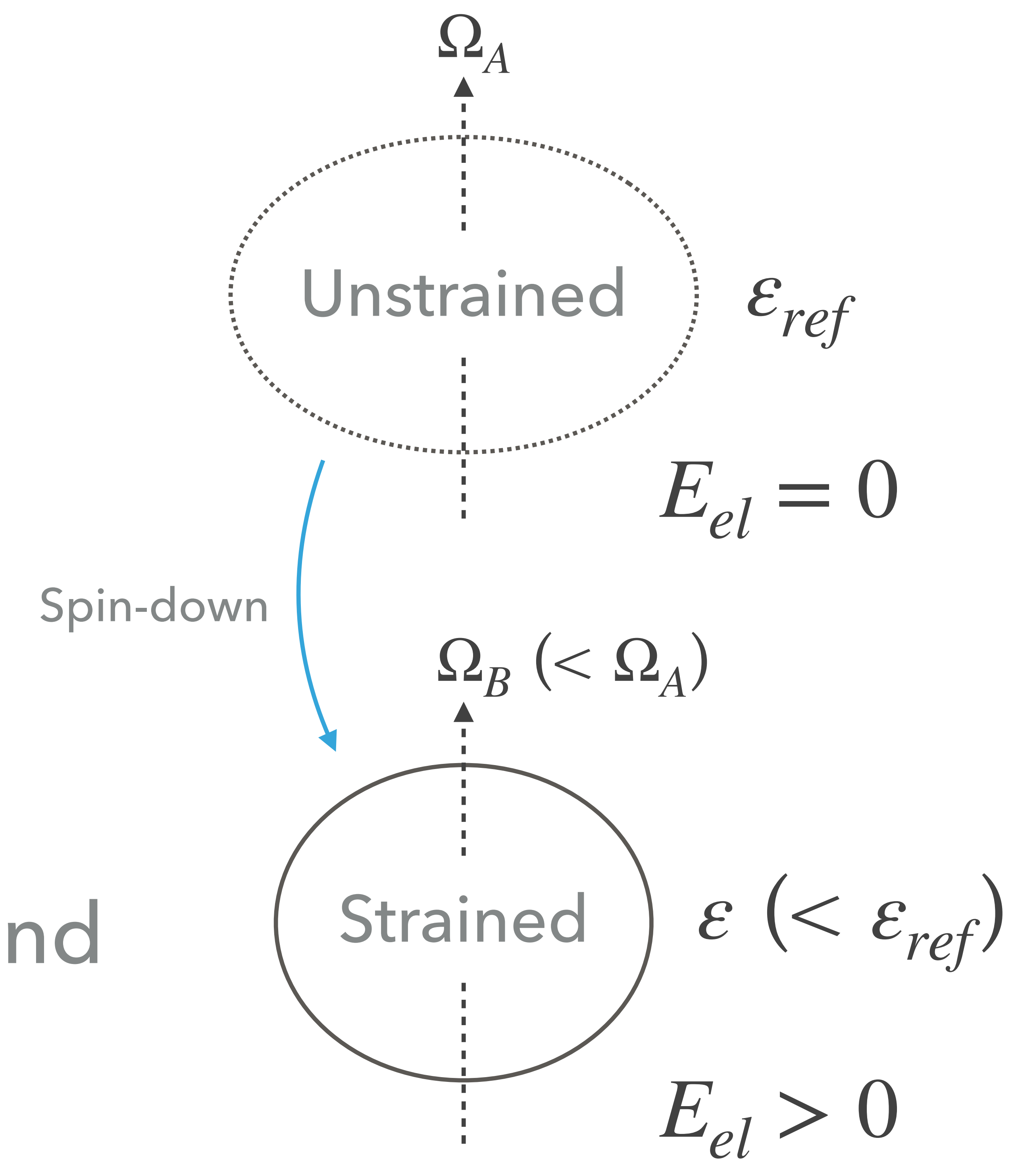
$$E_{el} = B(\varepsilon_{ref} - \varepsilon)^2$$

$$\varepsilon = \frac{I_{sph}\Omega^2}{4(A+B)} + \frac{B}{A+B}\varepsilon_{ref}$$

Time derivative →

$$\dot{E}_{el} = -2B\dot{\varepsilon}(\varepsilon_{ref} - \varepsilon)$$

$$\dot{\varepsilon} = \frac{I_{sph}\Omega\dot{\Omega}}{2(A+B)} = \frac{\dot{E}_{spin-down}}{2(A+B)}$$



where E_{el} is the elastic energy, ε is the oblateness, and A and B are constants due to gravitational and elastic energy corrections.

- For a NS crust, $B/A \sim 10^{-5}$ with $B \sim \mu V$, where μ is the shear modulus and V is the volume of stressed elastic material

$$\frac{|\dot{E}_{el}|_{max}}{\dot{E}_{spin-down}} = \frac{B}{A+B}(\varepsilon_{ref} - \varepsilon)_{max} \sim 10^{-6}$$

CAN ELASTICITY POWER THESE MODES?

- Back-of-the-envelope calculation using Baym & Pines (1971) → axisymmetric spin-down

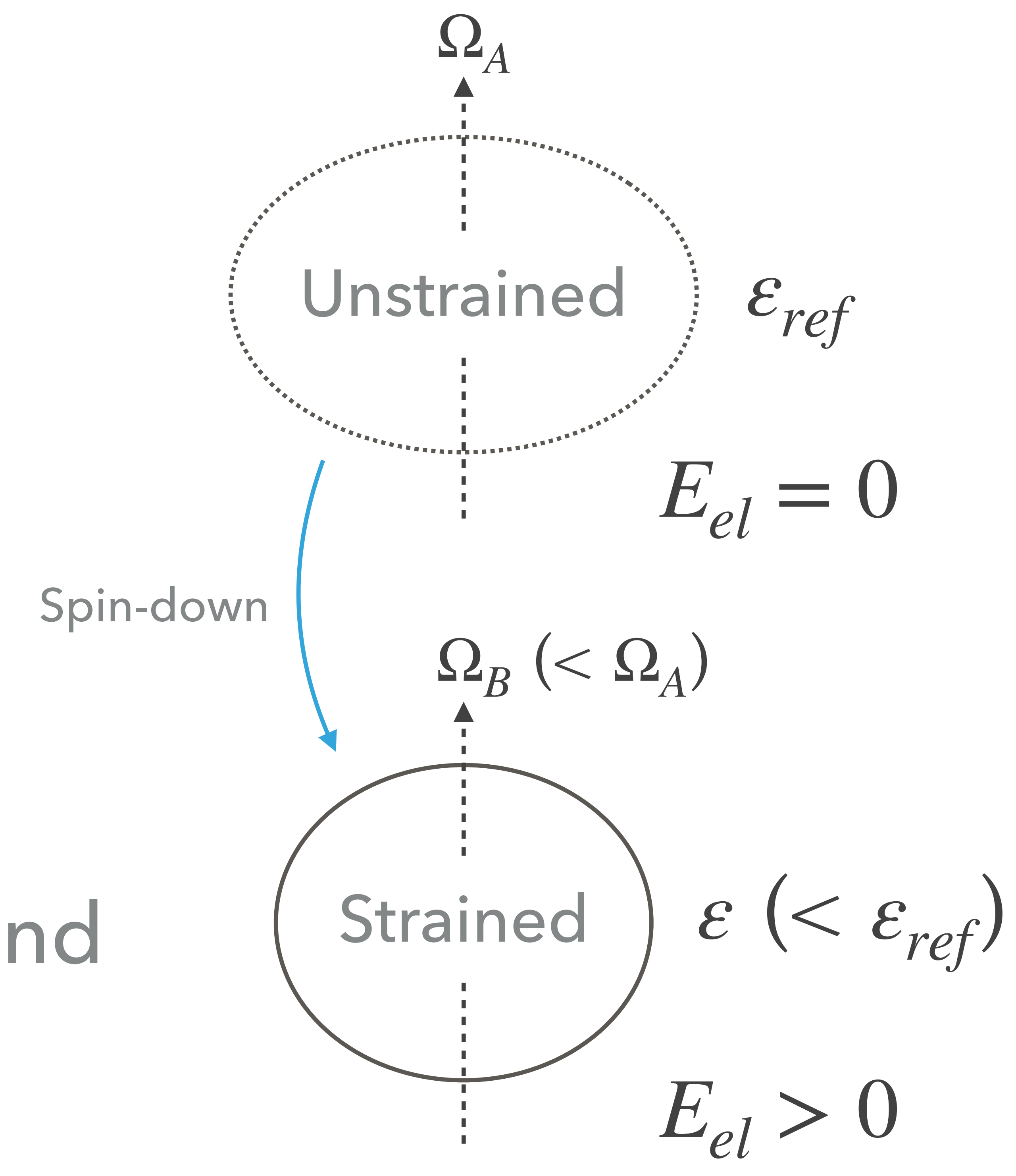
$$E_{el} = B(\varepsilon_{ref} - \varepsilon)^2$$

$$\varepsilon = \frac{I_{sph}\Omega^2}{4(A+B)} + \frac{B}{A+B}\varepsilon_{ref}$$

Time derivative →

$$\dot{E}_{el} = -2B\dot{\varepsilon}(\varepsilon_{ref} - \varepsilon)$$

$$\dot{\varepsilon} = \frac{I_{sph}\Omega\dot{\Omega}}{2(A+B)} = \frac{\dot{E}_{spin-down}}{2(A+B)}$$



where E_{el} is the elastic energy, ε is the oblateness, and A and B are constants due to gravitational and elastic energy corrections.

- For a NS crust, $B/A \sim 10^{-5}$ with $B \sim \mu V$, where μ is the shear modulus and V is the volume of stressed elastic material

$$\frac{|\dot{E}_{el}|_{max}}{\dot{E}_{spin-down}} = \frac{B}{A+B}(\varepsilon_{ref} - \varepsilon)_{max} \sim 10^{-6}$$

(c.f. 10^{-3} required to power modes - elasticity from NS crust not enough!)
Higher shear modulus? Larger volume stressed? → Xiaoyu Lai and Weihua Wang's talks

PART V – CONCLUSION AND OUTLOOK

CONCLUSION AND OUTLOOK

- Showed that small glitches and anti-glitches could be due to the excitation and decay of non-axisymmetric modes, with the model testable with GWs.
- A confident detection requires coherently stacking multiple signals. More work should be done on this front to ensure our detection pipelines are ready.
- Gravitational wave back-reaction from decaying oscillation mode is surprising → extend calculation to higher order in Ω to see if effect is still present.
- We also need good time resolution for these small events. A re-analysis of the radio data should be done but focused on improving accuracy of the event times (e.g. with FAST). It would also be good to know the glitch size distribution for small glitches and anti-glitches. → Weiyang Wang's talk
- If elasticity plays a role in powering these modes, we require something more exotic than just the NS crust. Perhaps superfluidity of interior can also play a role.

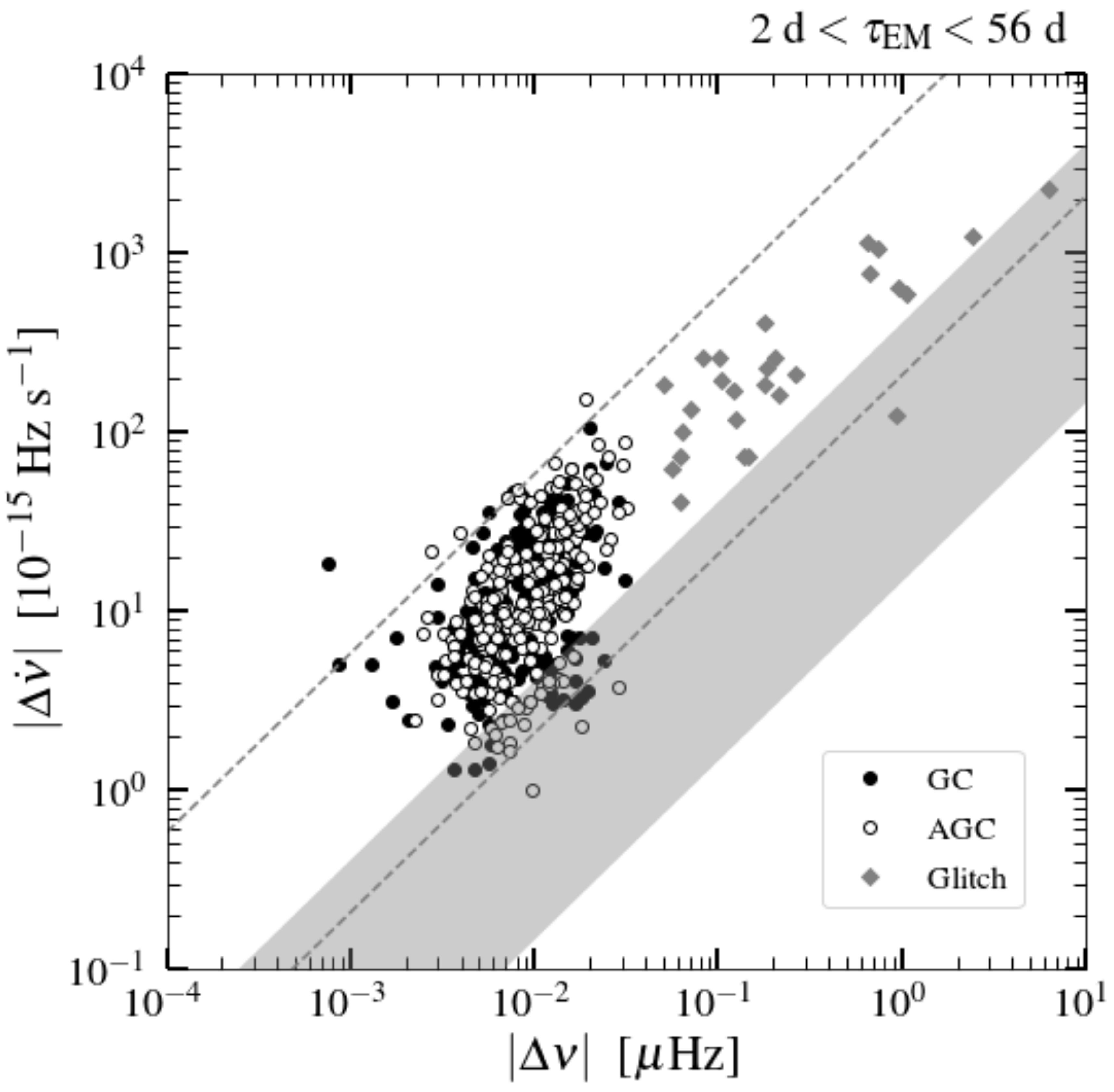
EXTRA SLIDES

EXPLAINING $\Delta\dot{\nu}$ – INCLUDE THE INTERNAL COUPLING TORQUE

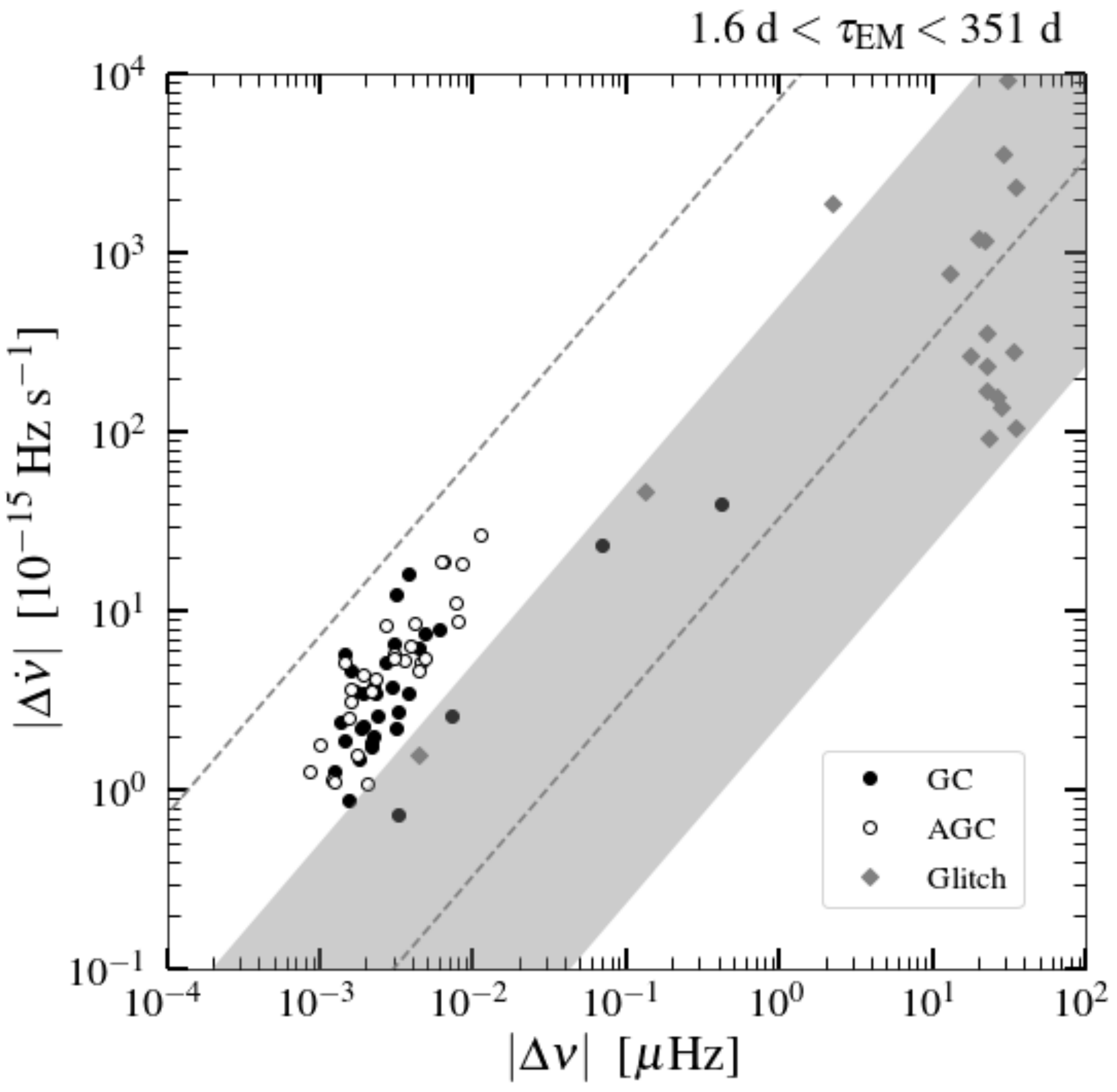
- Two components: pinned superfluid and crust.
- Weakly coupled by coupling torque:

$$\rightarrow N_{coup} \propto \frac{\Omega_s - \Omega_c}{\tau_{coup}}$$

$$\frac{\Delta\dot{\nu}}{\dot{\nu}} = \frac{2\tau_{age}}{\tau_{EM}}\frac{I_n}{I}\left(\frac{\Delta\nu}{\nu}\right)$$



Crab



Vela

Dashed band: $I_n / I = 1$
Grey band: $I_n / I = 0.07$