
# Oscillation of Neutron Stars

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贵州 · FAST

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# Contents

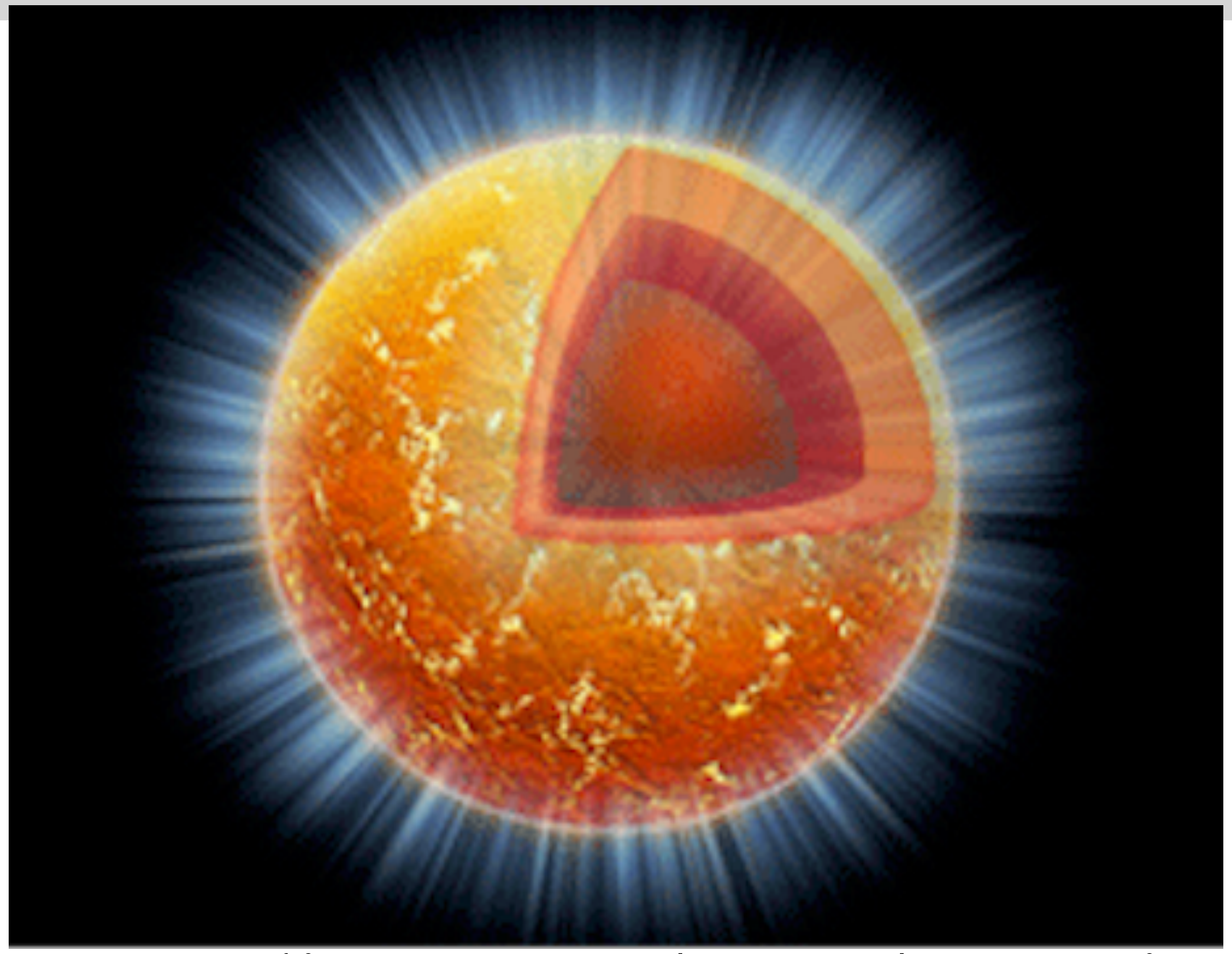
# Equation of state (EOS) and structure of neutron star

Neutron Stars - Physics in extremes

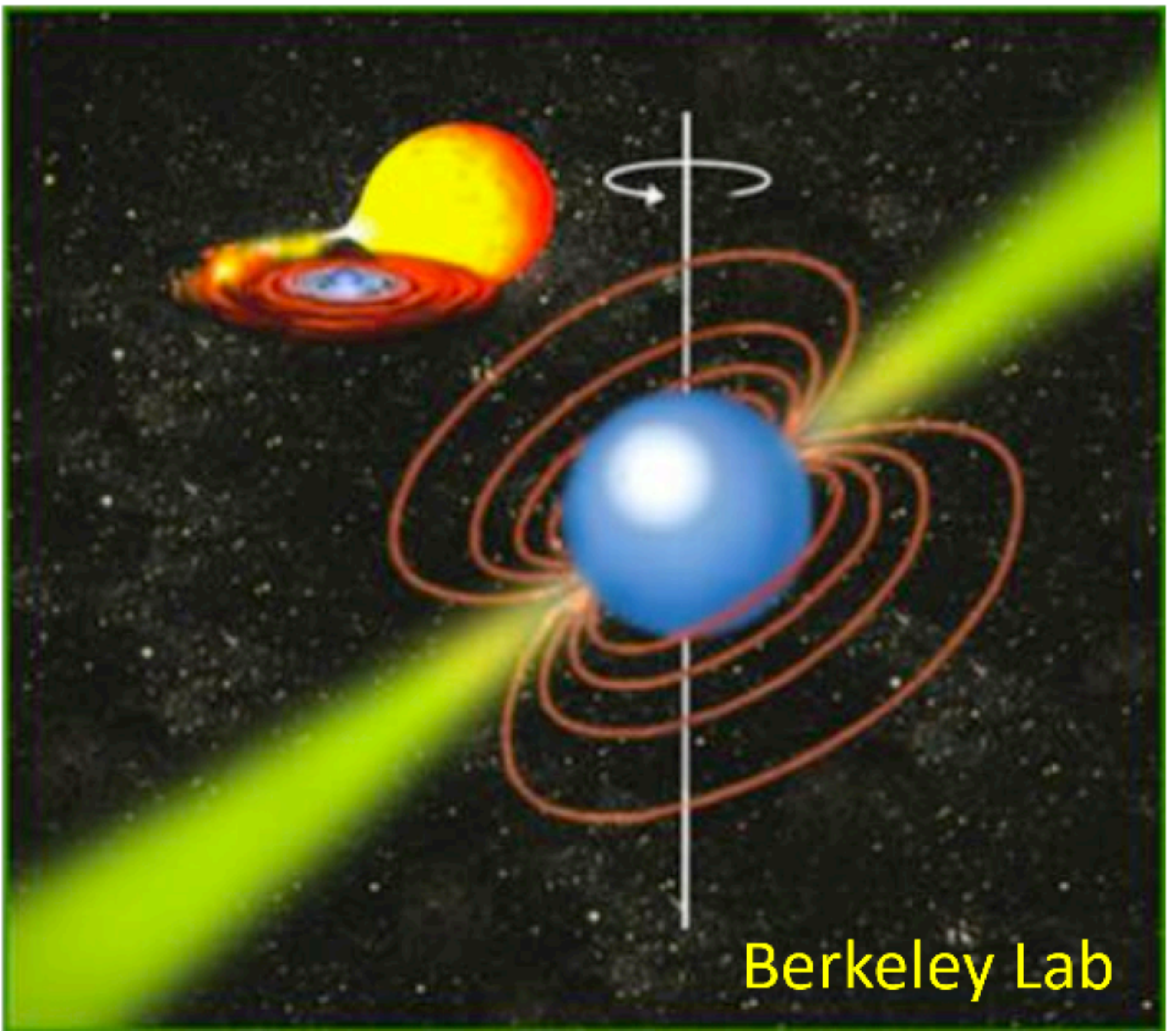
- Compactness $GM/Rc^2 \sim 0.2$, need GR
- Strong interactions are more important

Non-perturbative QCD + many-body interactions

- Electroweak interactions drive the astronomical emission



Credit: NASA/CXC/M.Weiss

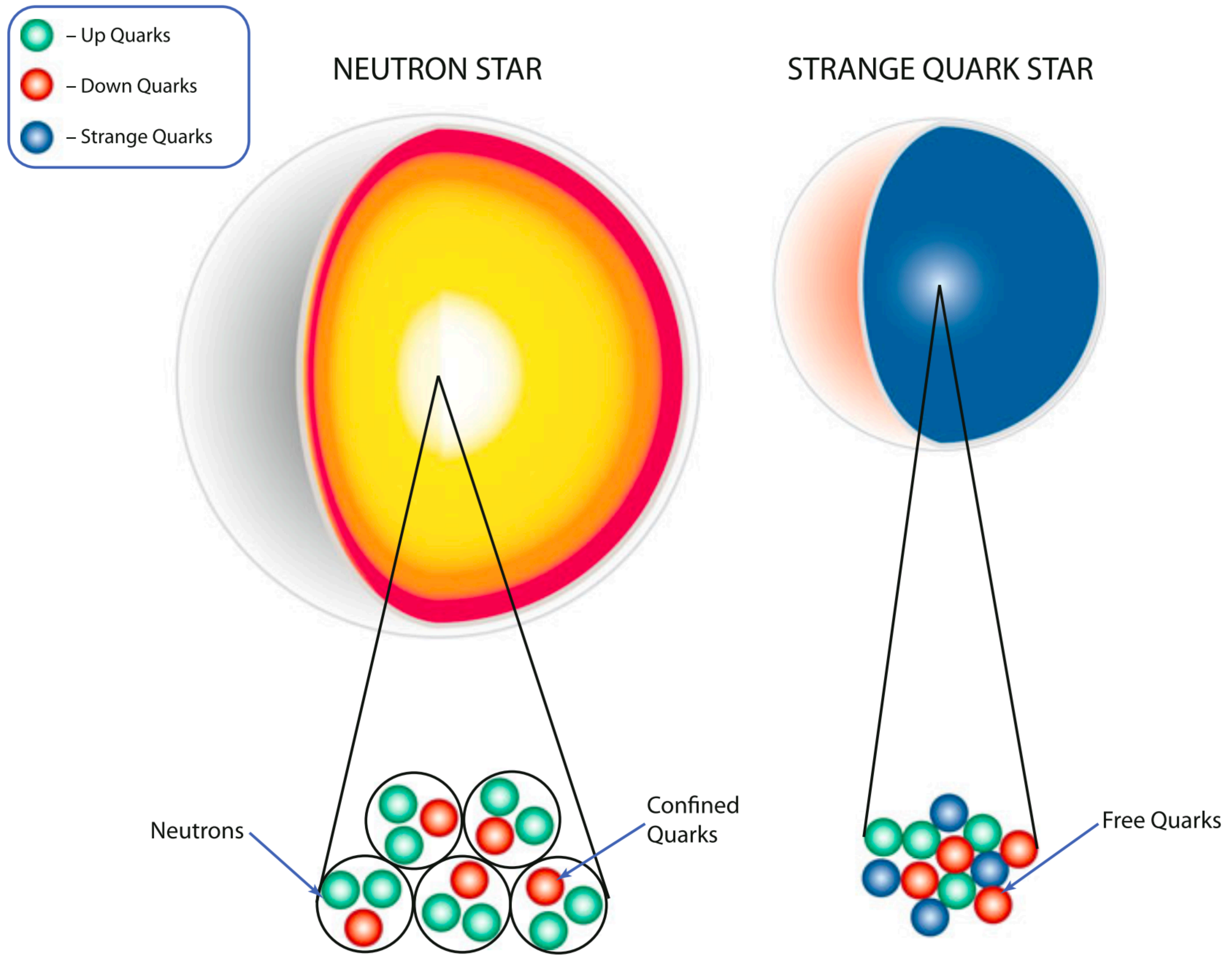



Neutron Stars - a unique interplay among

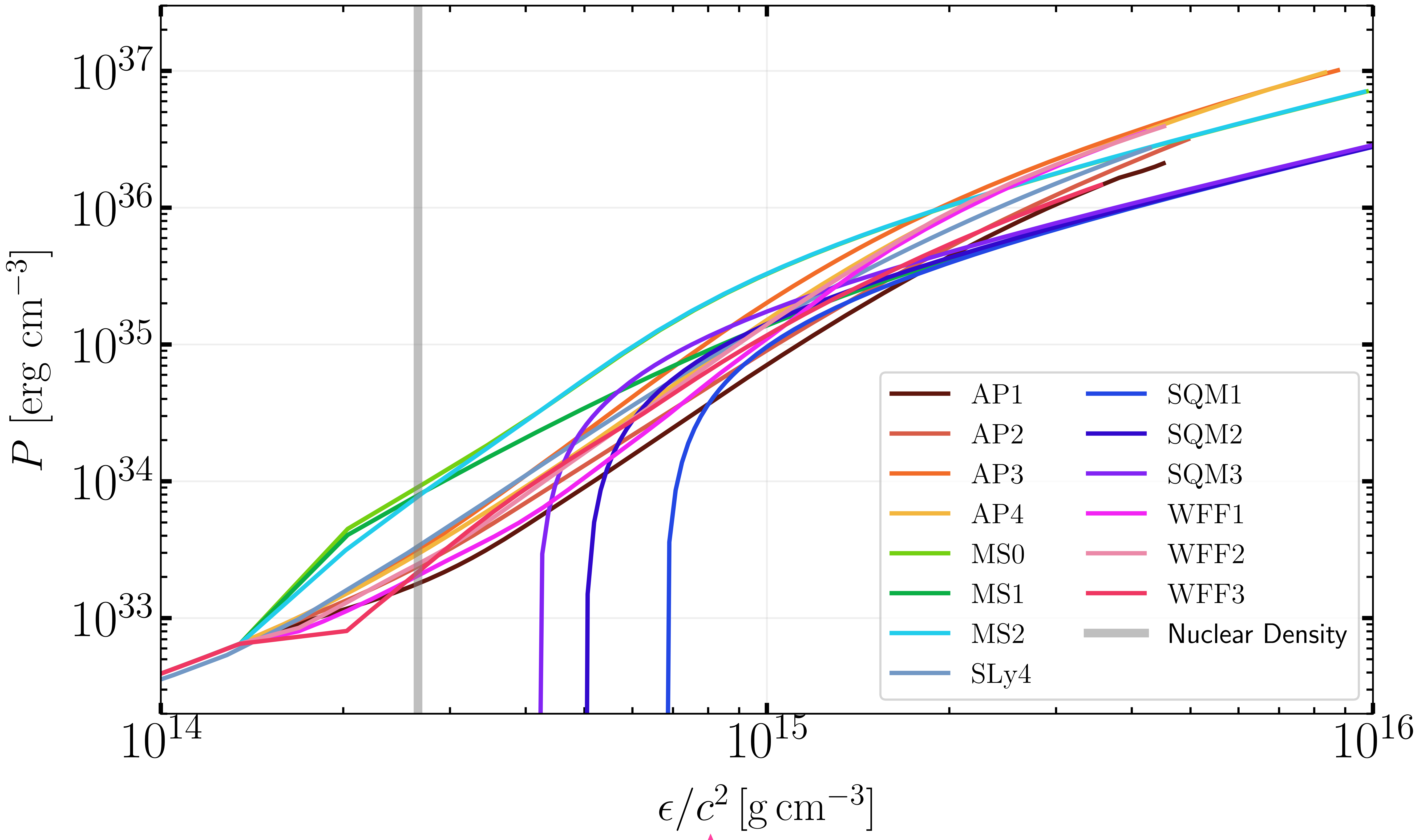
- astrophysics
- gravitational physics
- nuclear physics

After half a century since the discovery, we are still far from understanding the composition of matter in their cores!

# The EOS model of NSs




Credit: Fortov, Extreme States of Matter, 2016



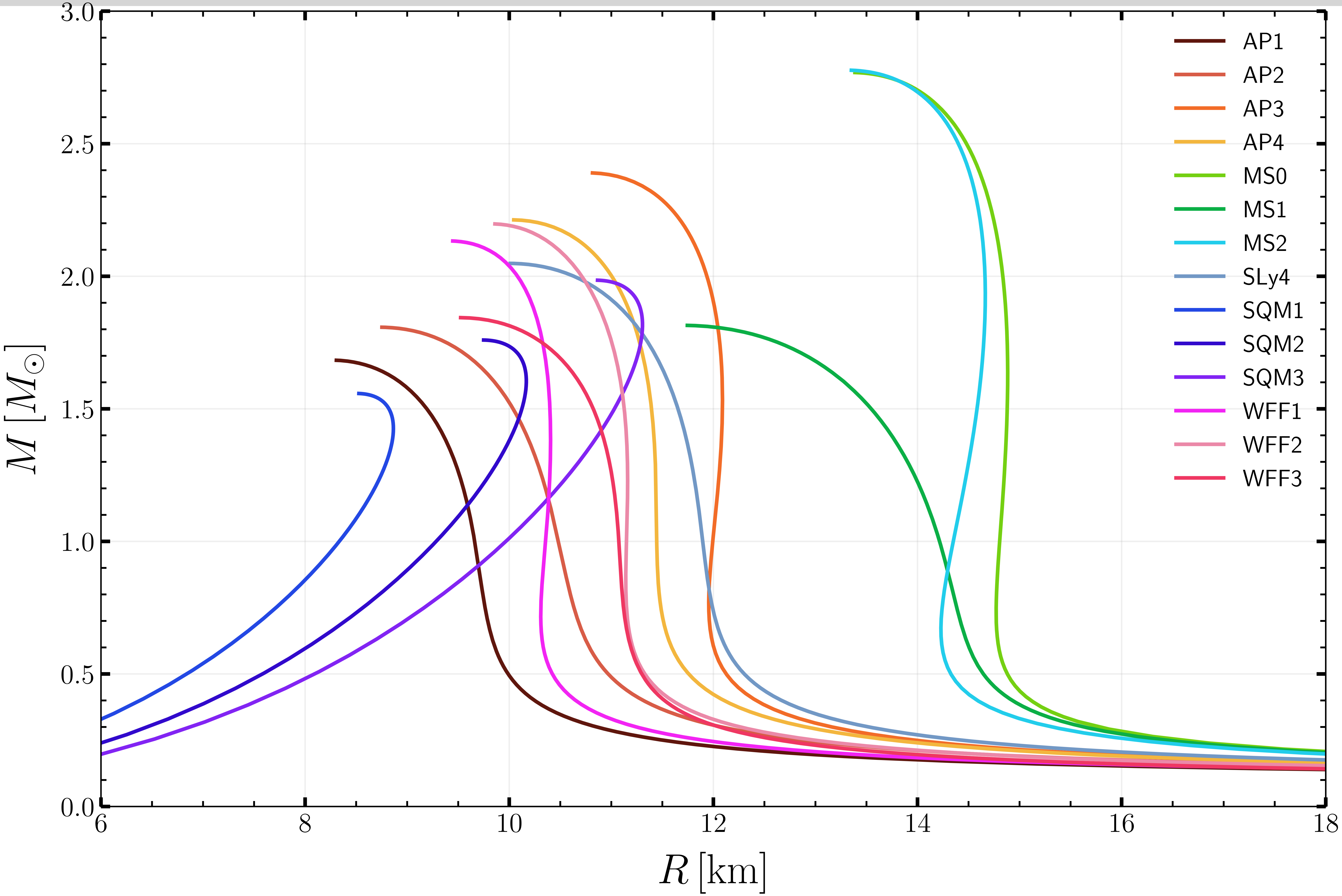

$$\frac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2 \epsilon / c^2$$

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{Gm\epsilon/c^2}{r^2}\left(1+\frac{P}{\epsilon}\right)\left(1+\frac{4\pi r^3 P}{mc^2}\right)\left(1-\frac{2Gm}{rc^2}\right)^{-1}$$

→ **mass and radius**

↑ Inverse problem

# Mass and radius relation



# Perturbation analysis in Newtonian gravity

## Oscillation modes

### Fluid equations:

Mass conservation $$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$$

Euler equations $$\rho \frac{\mathrm{d}\vec{\mathrm{v}}}{\mathrm{dt}} = -\rho \nabla \Phi - \nabla P$$

Poisson's equation $$\nabla^2 \Phi = 4\pi G \rho$$

The system is completed by providing an equation of state (EOS):

$$P = P(\rho, \ldots)$$

# Oscillation modes

Consider the small adiabatic oscillations of a self-gravitating fluid globe

$$\rho = \rho_0 + \delta\rho \quad P = P_0 + \delta P \quad \Phi = \Phi_0 + \delta\Phi$$

Linearly perturbed fluid equations:

$$\delta\rho + \nabla \cdot (\rho\ \vec{\xi}) = 0$$

$$\frac{\partial^2 \vec{\xi}}{\partial t^2} = -\nabla\delta\Phi - \frac{\delta\rho}{\rho}\nabla\Phi - \frac{1}{\rho}\nabla\delta P$$

$$\nabla^2(\delta\Phi) = 4\pi G(\delta\rho)$$

$$\frac{\Delta P}{P} = \Gamma_1 \frac{\Delta\rho}{\rho}$$

Assuming $\vec{\xi}(t, \vec{r}) = \vec{\xi}(\vec{r})e^{i\omega t}$

$$-\omega^2\vec{\xi} + \vec{C}(\vec{\xi}) = 0$$

+

boundary conditions

⇓

$\omega, \vec{\xi}$

Eulerian ($\delta$) and Lagrangian ($\Delta$) perturbations are related via: $\Delta Q = \delta Q + \vec{\xi} \cdot \nabla Q$

# Oscillation modes

$$\frac{\partial^2 \vec{\xi}}{\partial t^2} = -\underbrace{\nabla\left(\frac{\delta P}{\rho}\right)}_{\text{pressure}} + \underbrace{\frac{\Gamma_1 P}{\rho}\vec{A}(\nabla \cdot \vec{\xi})}_{\text{buoyancy}} - \nabla \delta\Phi$$

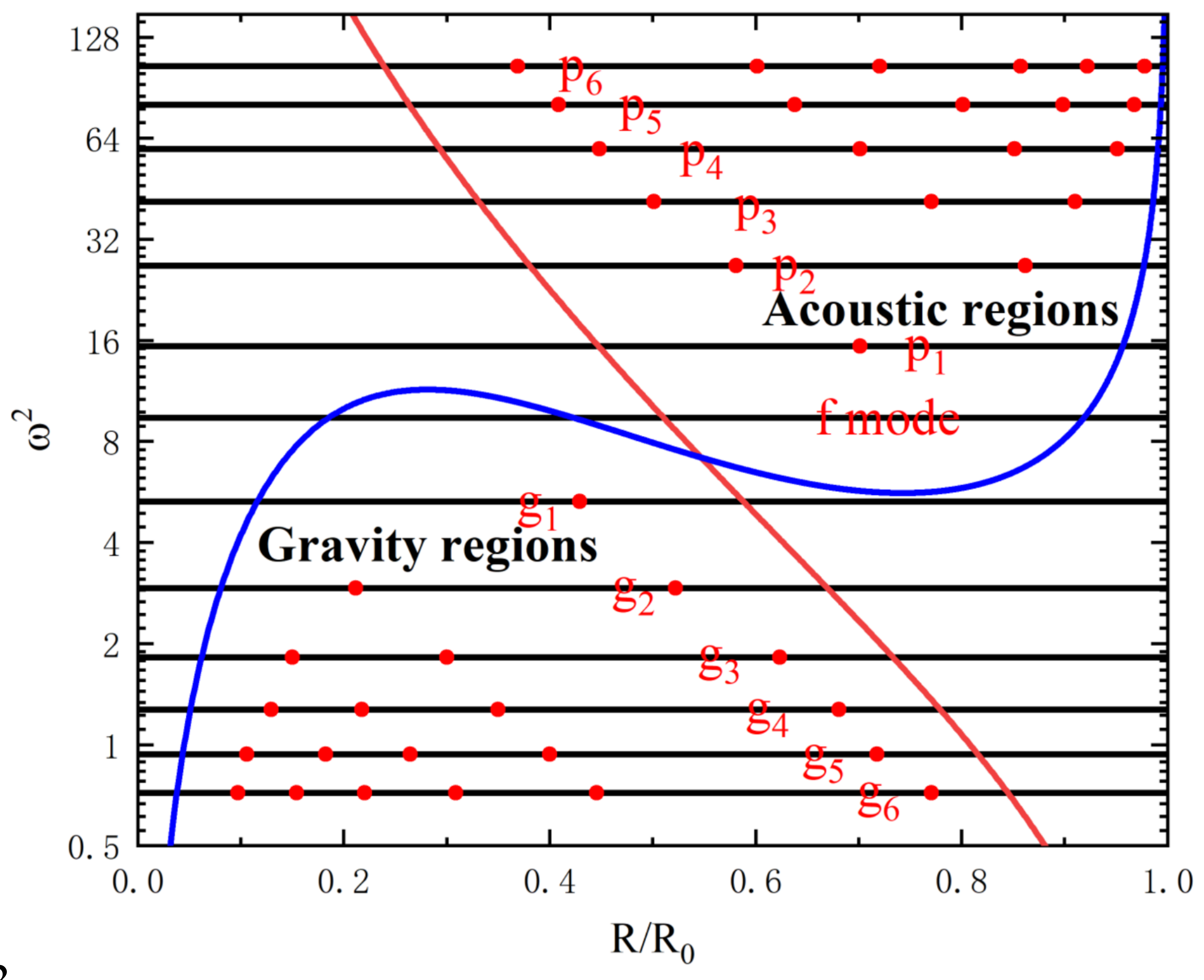
We consider the EOS , $\rho = \rho(P, S)$

Buoyancy frequency (g-mode)

$$N^2 = -Ag = g^2\left(\frac{1}{c_e^2} - \frac{1}{c_s^2}\right) > 0$$

$$A = \frac{1}{\rho}\frac{d\rho}{dr} - \frac{1}{\Gamma_1 P}\frac{dP}{dr}$$

- Composition gradient
- Density discontinuity
- Temperature




Li et al., 2023

# Motivation

## (1) Numerical simulations

I: The pseudo-Newtonian formulations are commonly used in the numerical simulations of core-collapse supernova (CCSN)

Marek et al. (2006), Mueller et al. 2008, Yakunin et al. 2015, Morozova et al. 2018, OConnor et al. 2018

II: Many works have identified features in the GW signals associated with the g-mode of the proto-neutron star (PNS)

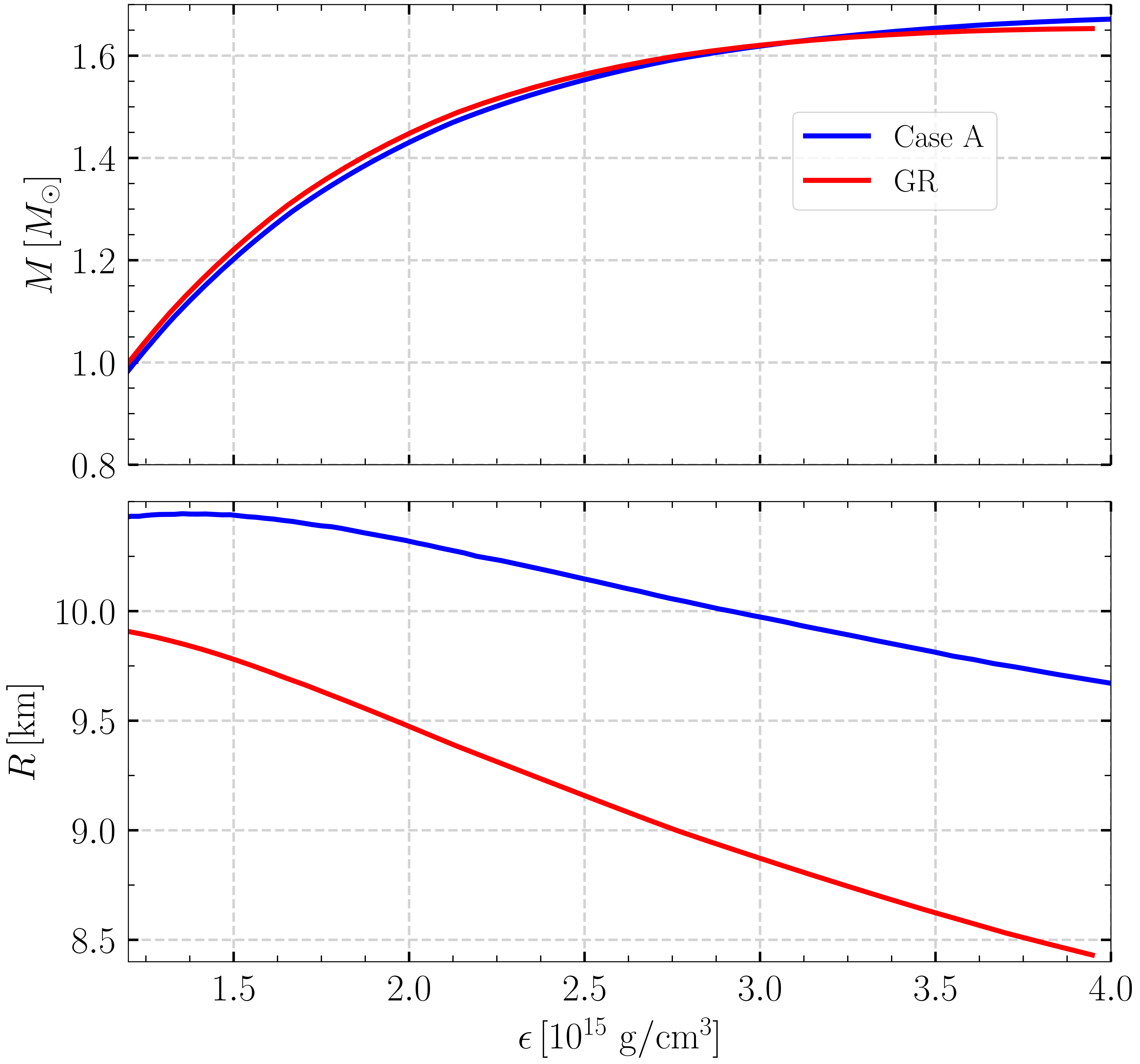
Murphy et al. 2009, Mueller et al. 2012, Cerda-Duran et al. 2013, Kuroda et al. 2016, Andresen et al. 2016

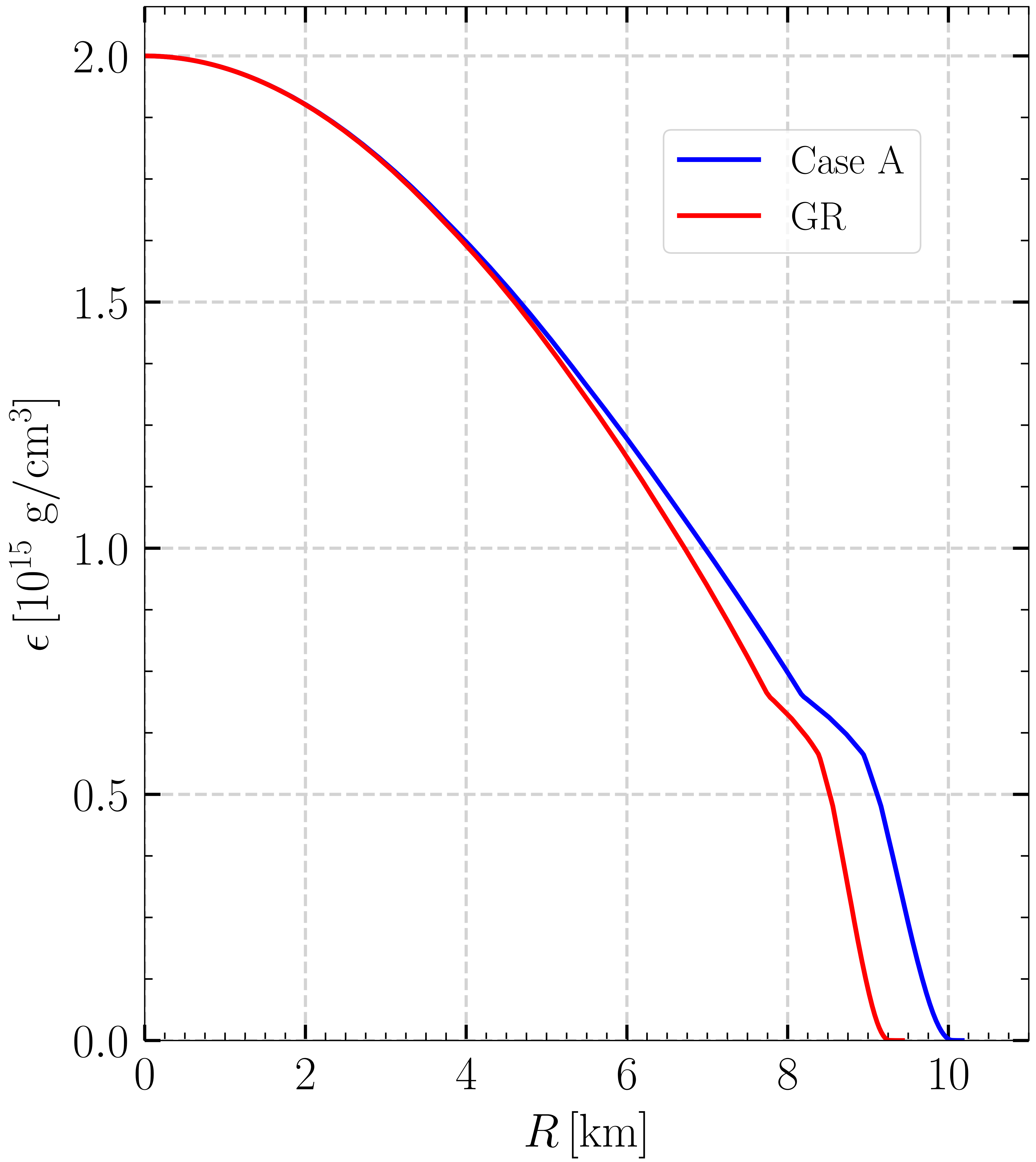
## (2) Perturbative calculations

Tang and Lin (2022) studied the radial and non-radial oscillation modes of NSs in pseudo-Newtonian gravity

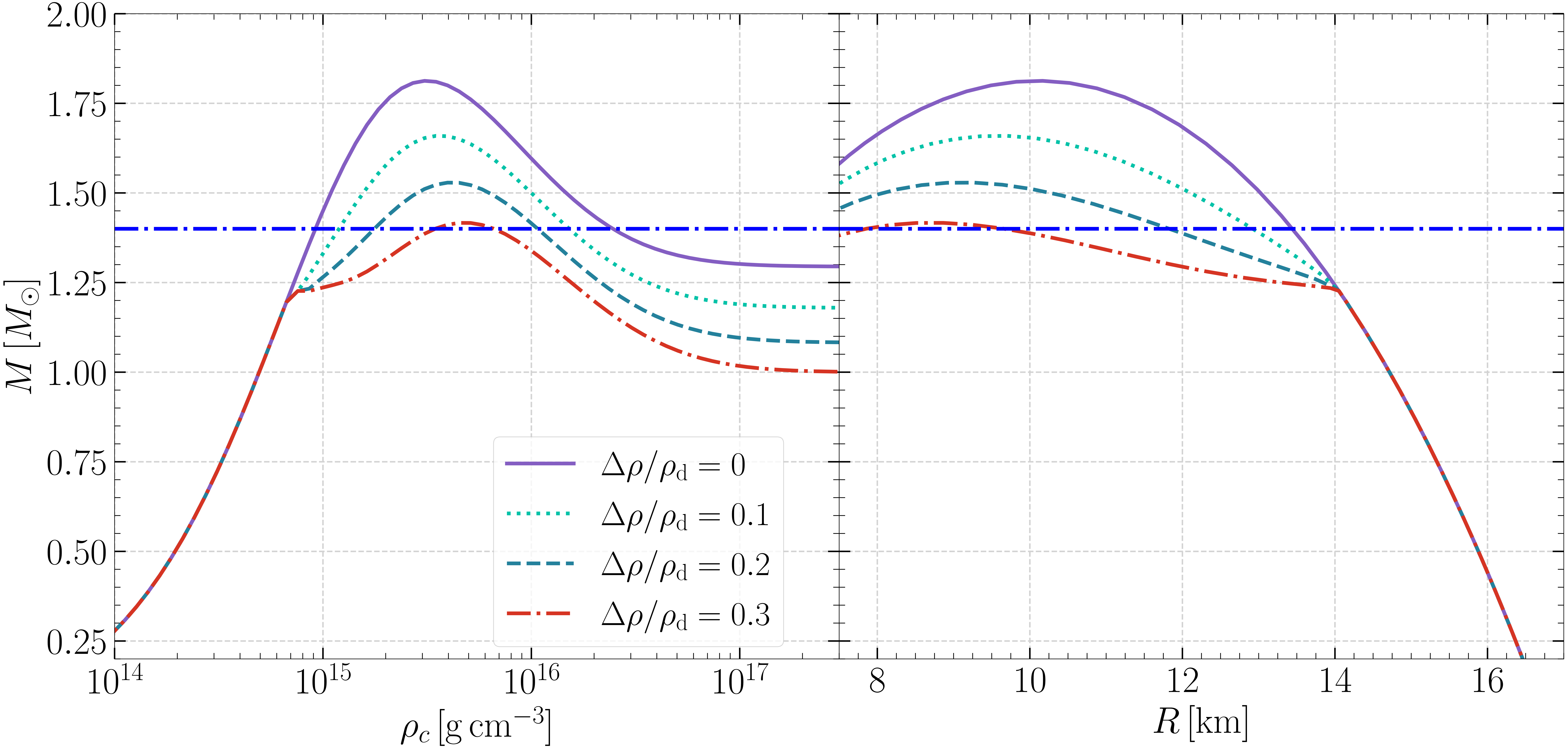
only considered the barotropic oscillations of NSs (f and p modes)

# The general relativistic and pseudo-Newtonian gravity



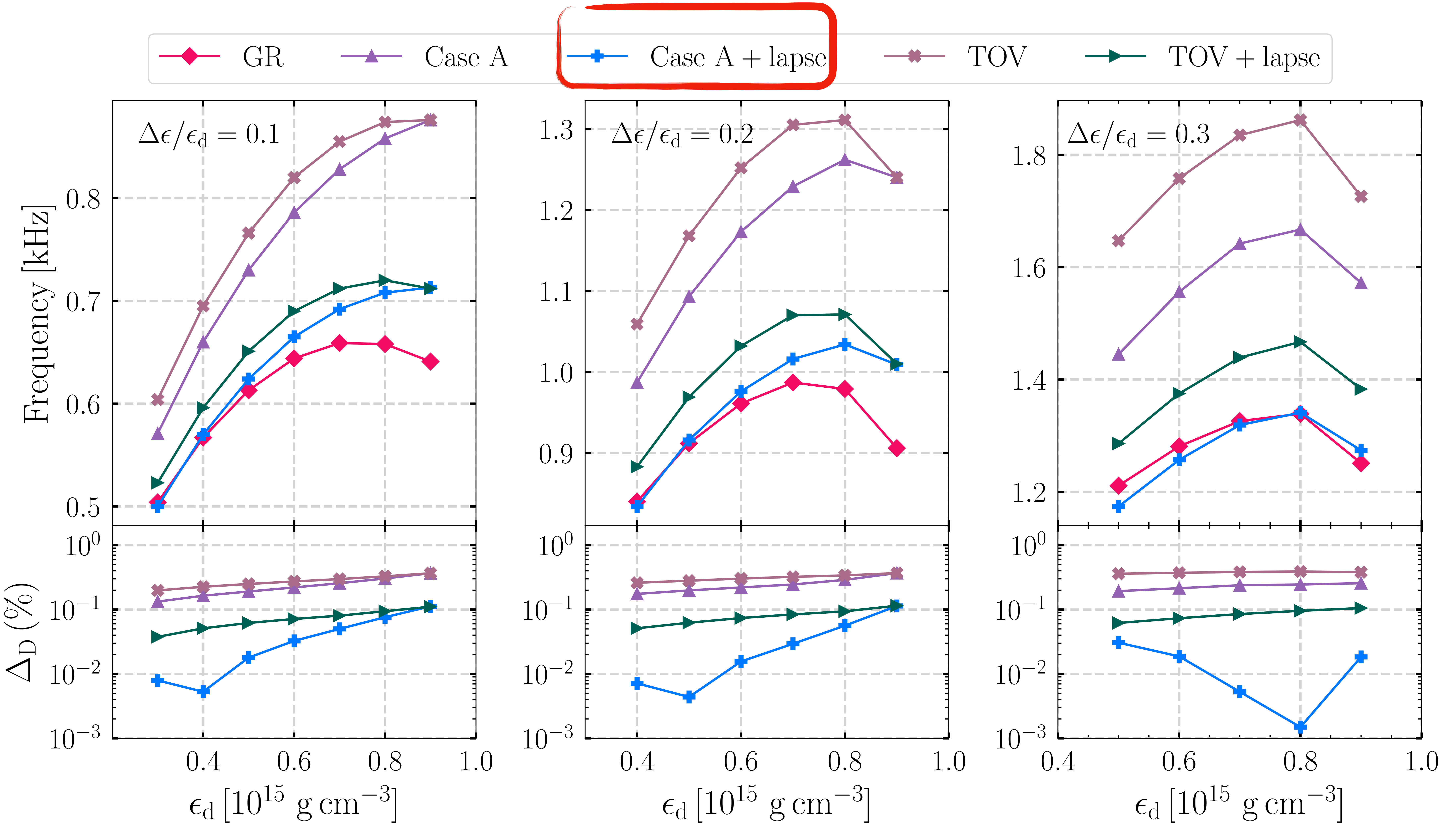
# Density discontinuity



# The g-mode of NSs: density discontinuity

The g-mode frequencies in pseudo-Newtonian treatments can be approximated remarkably well to the GR solutions, with relative errors in the order of 1%

$$\Delta_D = \left| \frac{f - f_{GR}}{f_{GR}} \right| \times 100\ \%$$



# Structure of Neutron Star Crust

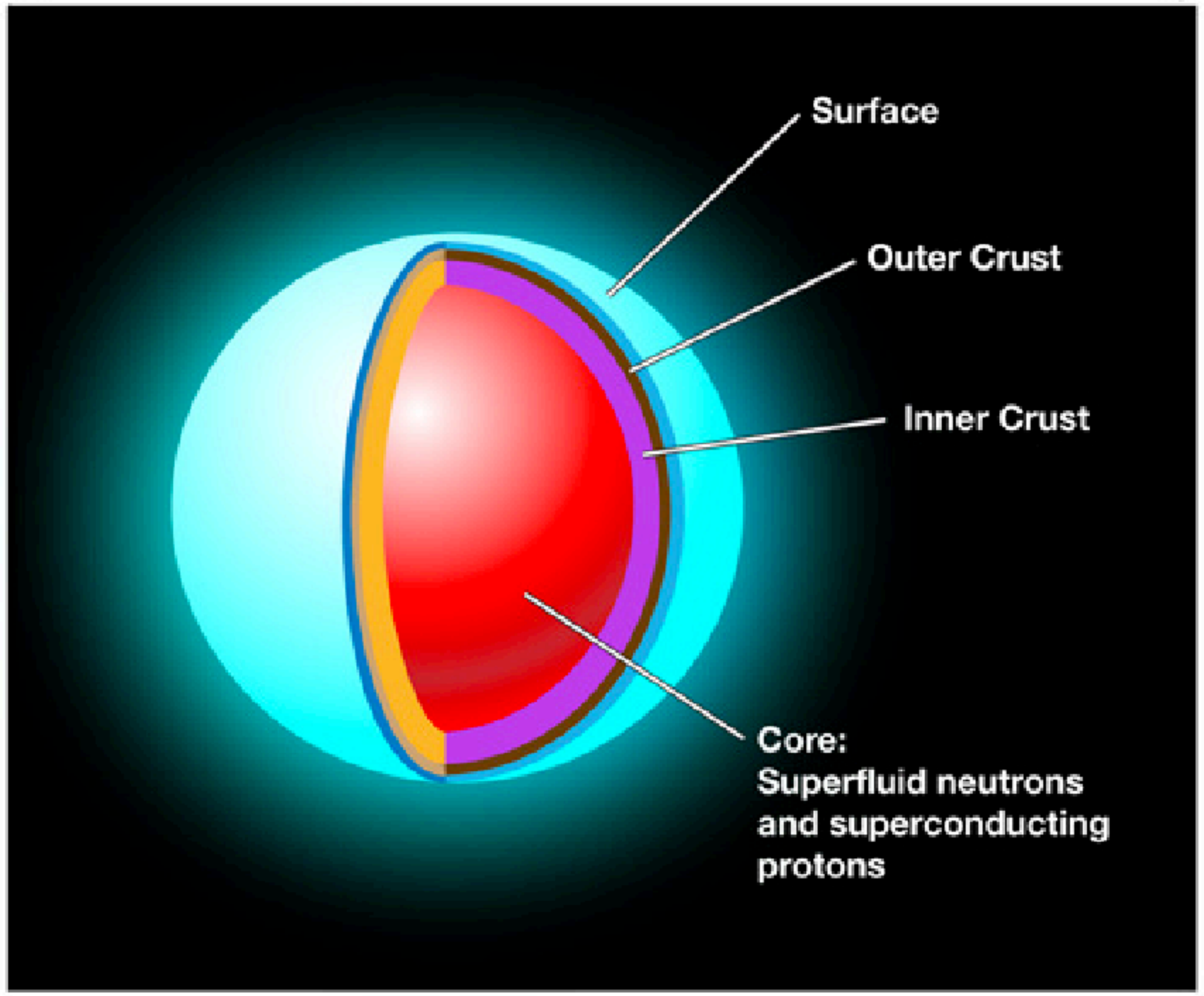
Even though the crust of a neutron star represents about 1% of the stellar mass and 10% of the radius, it is thought to be related to various astrophysical phenomena:

(1) Pulsar glitch

(2) X-ray (super)bursts

(3) Gravitational wave emission (mountains)

(4) Giant flares from soft gamma-ray repeaters and quasiperiodic oscillations (QPOs)




Credit: Chamel, 2016

# Structure of neutron star crust

(1): Structure of the outer crust

The matter at densities below neutron drip is not only relevant for neutron star crusts but also for white dwarfs

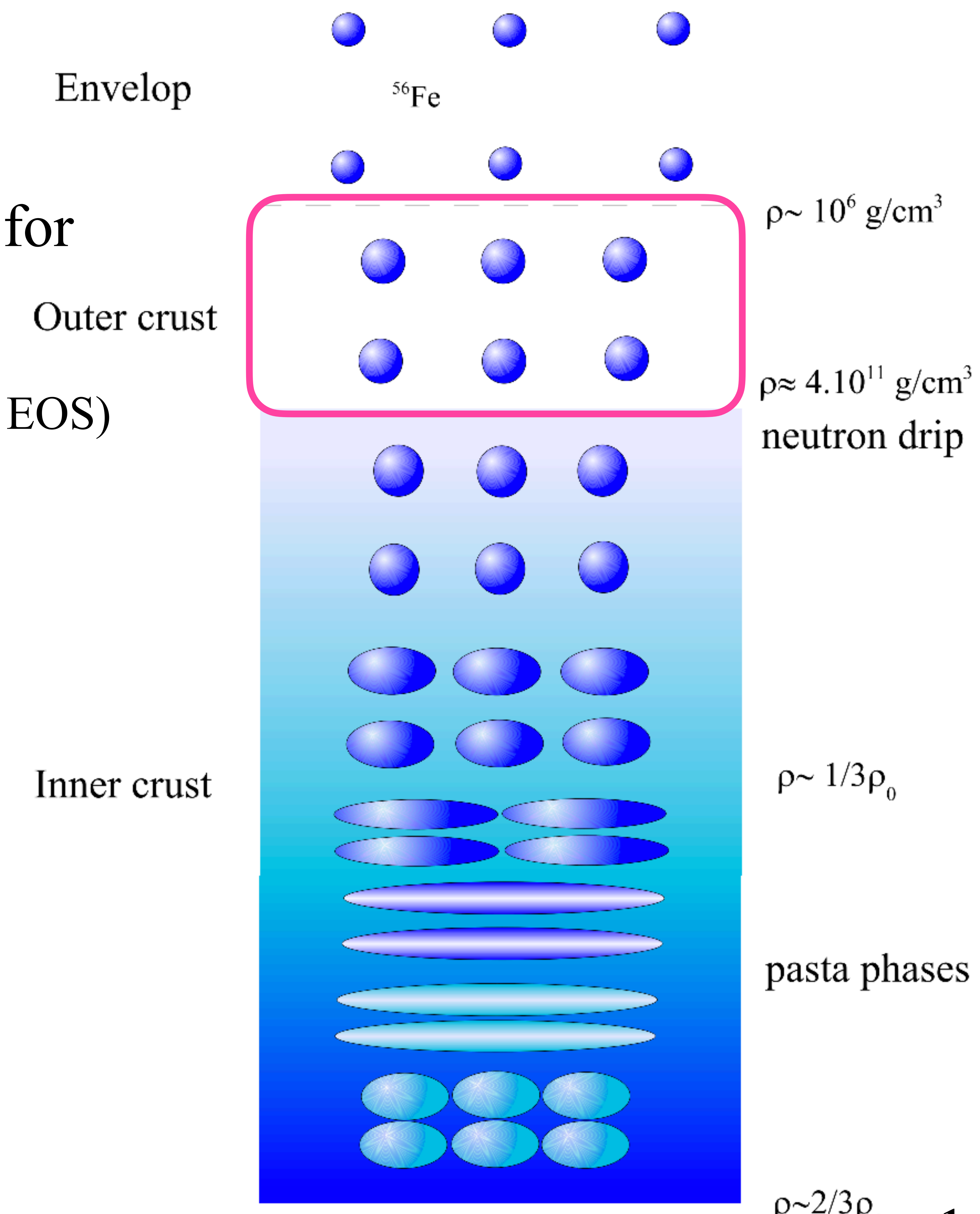
Following the classical paper of Baym, Pethick, and Sutherland, 1971 (BPS EOS)

Total energy density:

$$\epsilon = n_N E\{A,Z\} + \epsilon_e + \epsilon_L$$

$n_N$ is the number density of nuclei

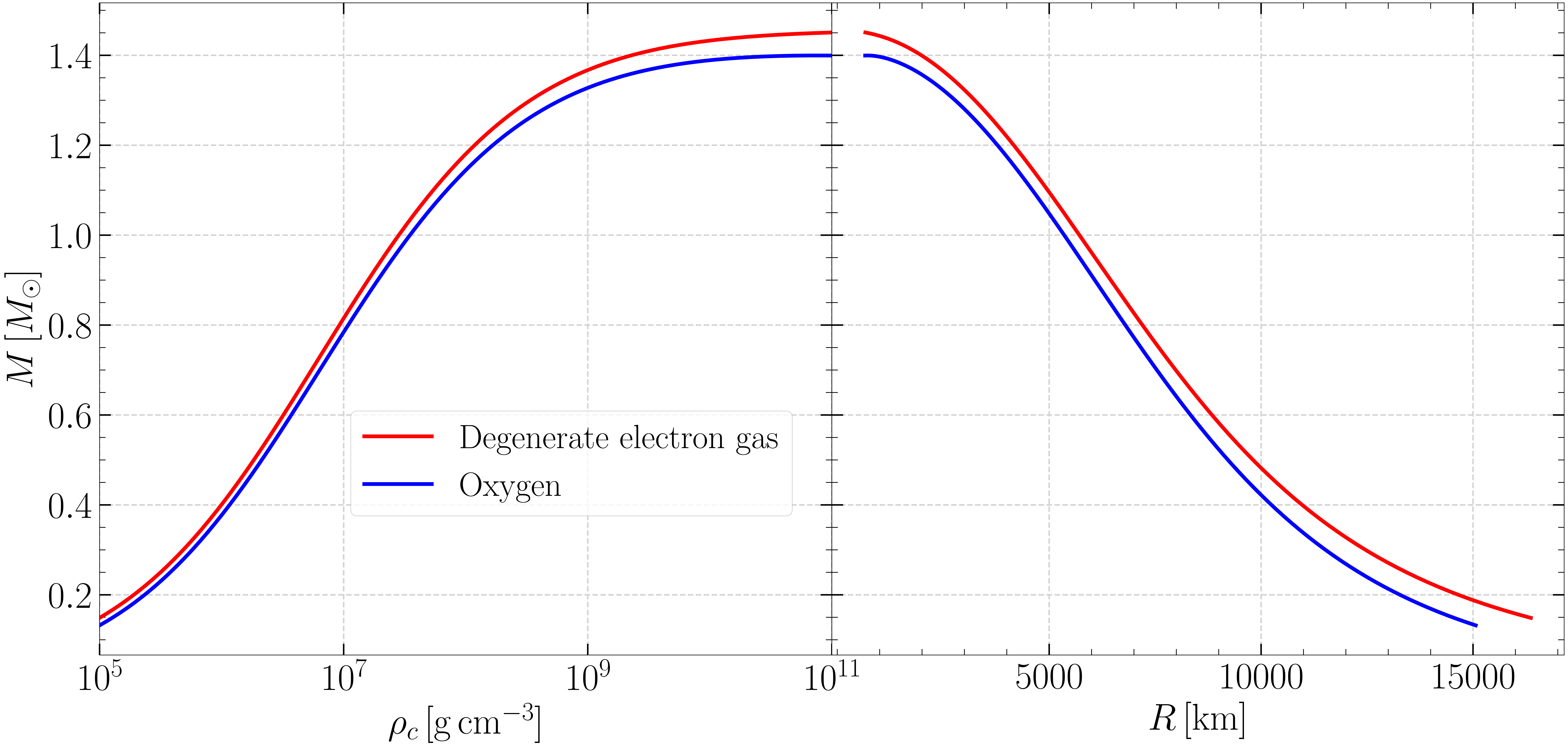
$E\{A,Z\}$ is the energy of a nucleus with Z protons and (A-Z) neutrons




Chamel & Haensel, LRR, 2008

# Crystallized white dwarfs

## Mass-central density/radius relation for crystallized white dwarfs



# Observations: quasi-periodic oscillations (QPOs)

Credit: Watts and Strohmayer, 2006

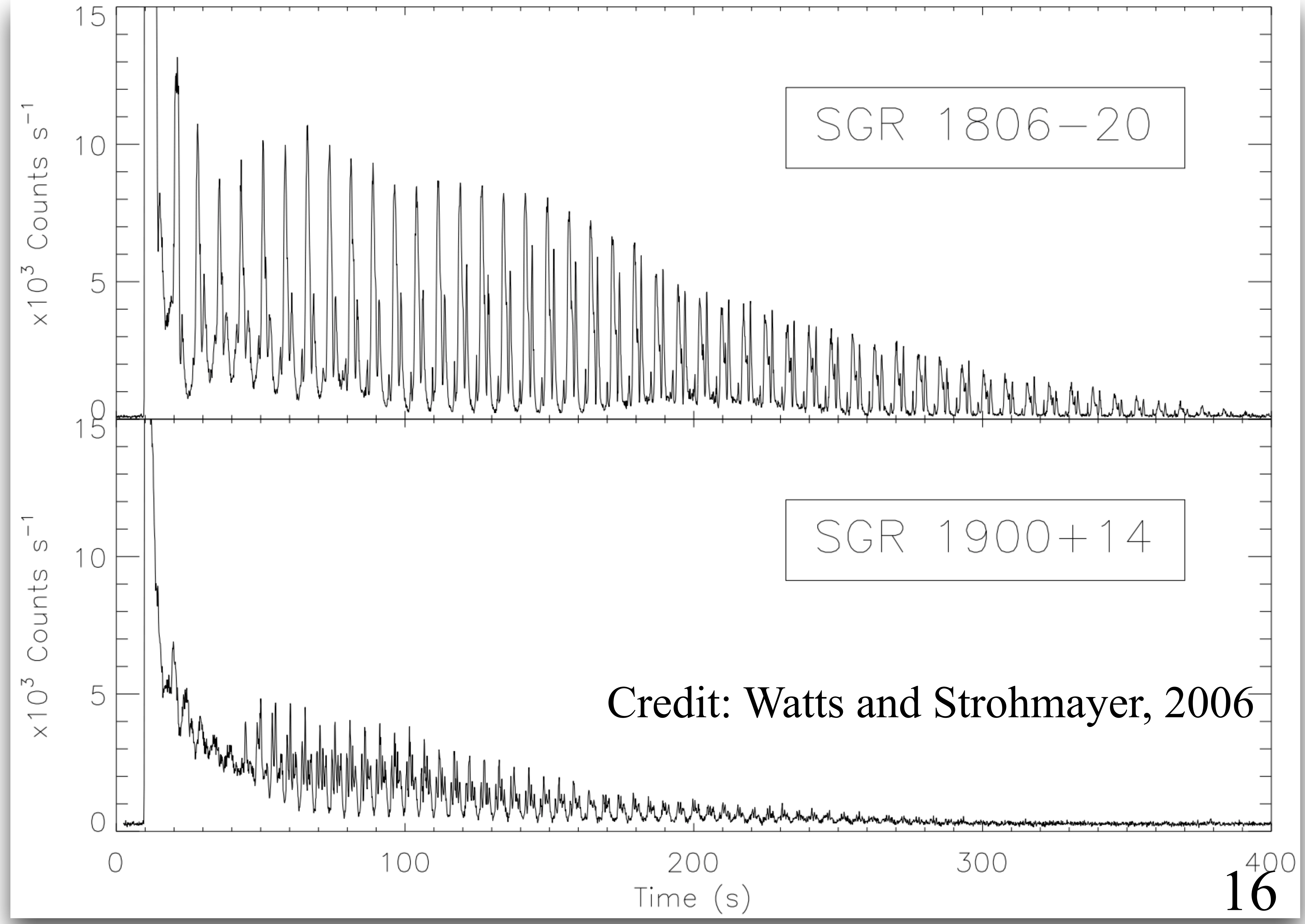
## Giant flares in SGRs

A decaying tail for several hundred seconds follows the flare

## QPOs in decaying tail

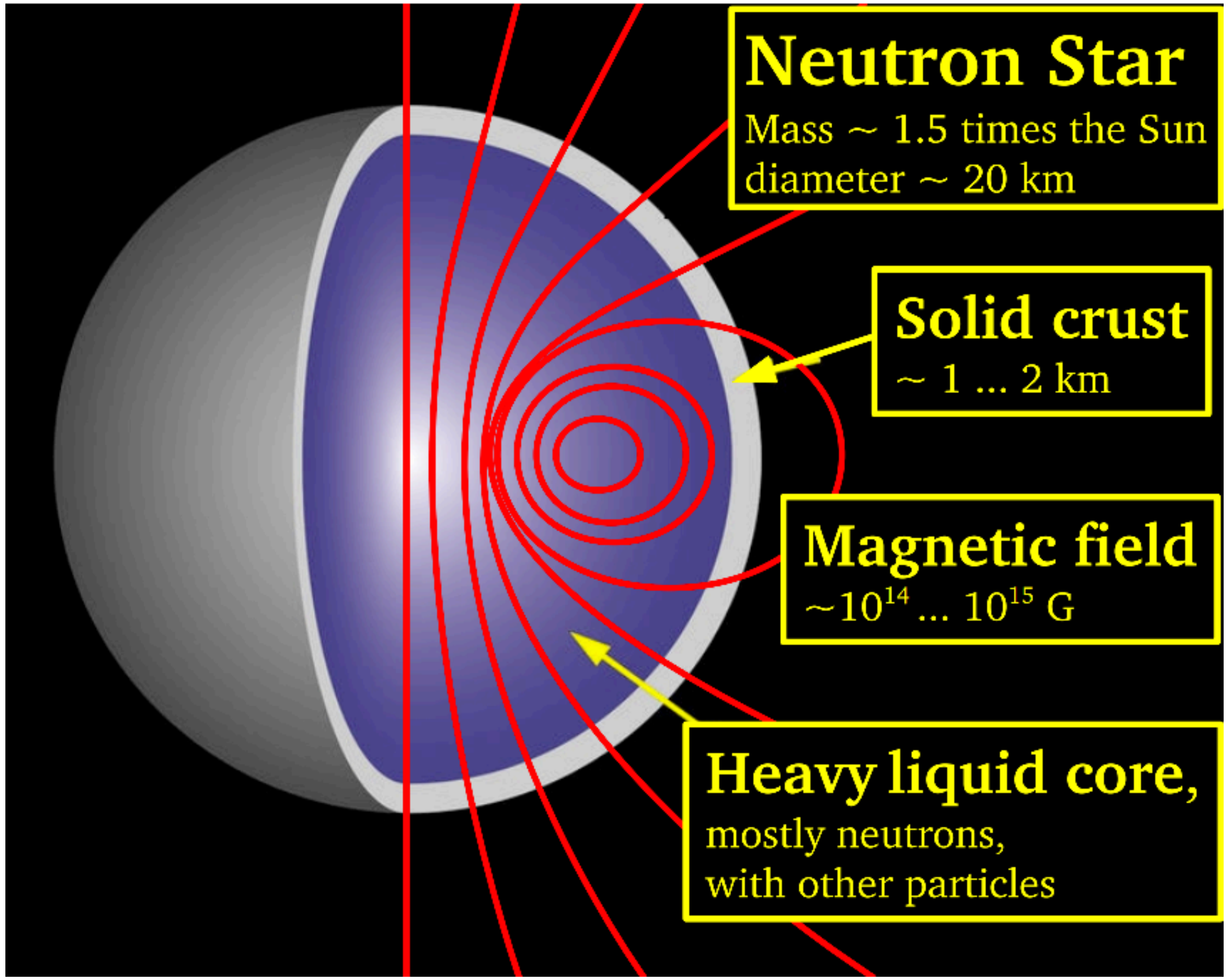
Israel et al., 2005; Strohmayer and Watts, 2005; Watts and Strohmayer, 2006

- SGR 1900+14 (1998): 28, 53, 84, 155 Hz
- SGR 1806-20 (2004): 18, 26, 29, 92.5, 150, 626.5, 720, 976, 1837, 2384 Hz




Credit: Watts and Strohmayer, 2006

# Where do the QPOs come from? Are they Starquakes?




Credit: Michael Gabler, 2014

Possible origin of the observed frequencies

(I) Discrete Shear modes (crust)?

(II) Alfven oscillations:
coupled crust-core oscillations?

with or without superfluid effects, pasta phases, ...

(III) Magnetospheric: oscillations?

(Glampedakis et al. 06; Levin 07; Van Hoven & Levin 11 & 12; Colaiuda et al. 10 & 11 & 12; Gabler et al. 11, 12, 13, 16, &18, Passamonti 12, 13, 14 & 16, Sotani et al. 07, 08, 13, 14, 15…)


# Thank you for your attention!