

# QUARK MODEL WITH HIDDEN LOCAL SYMMETRY AND ITS APPLICATION TO THE HADRON SPECTRUM

## 具有隐藏定域对称性的夸克模型及其在强子谱 中的应用

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2306.03526  
2307.16280

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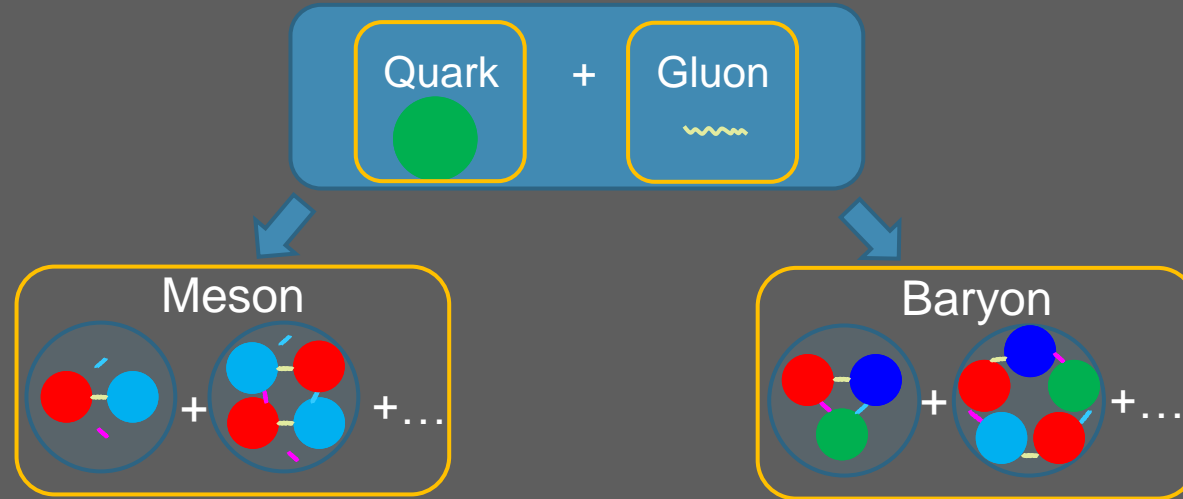
# Outline

## ➤ Introduction

- ⊙ Chiral quark model with HLS
- ⊙  $SU2$  ground states + excited states
- ⊙  $SU3$  ground states
- ⊙ Summary

# Introduction

Hadron are made by quarks and gluons



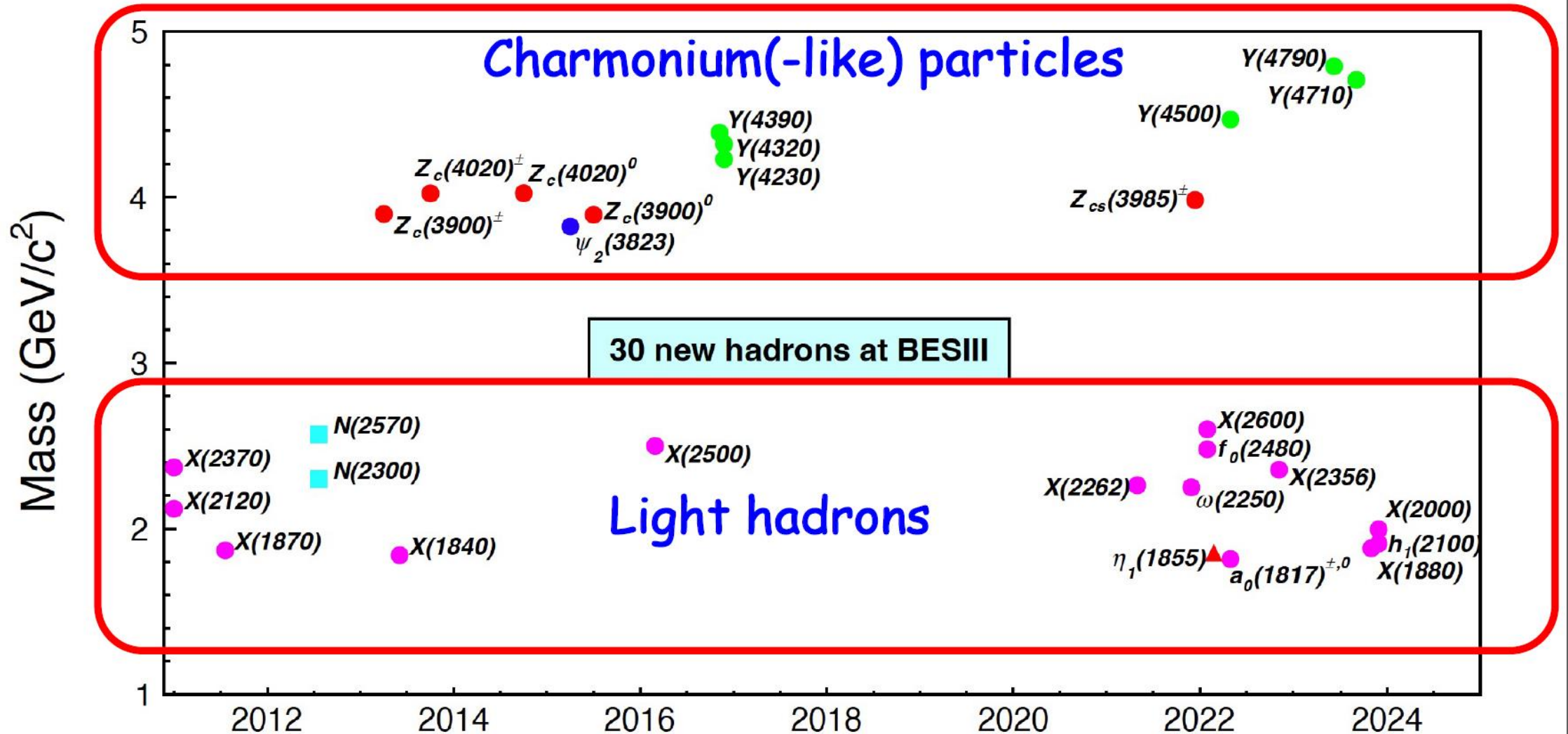
The dynamics of quarks and gluons are described by Quantum chromodynamics (QCD)

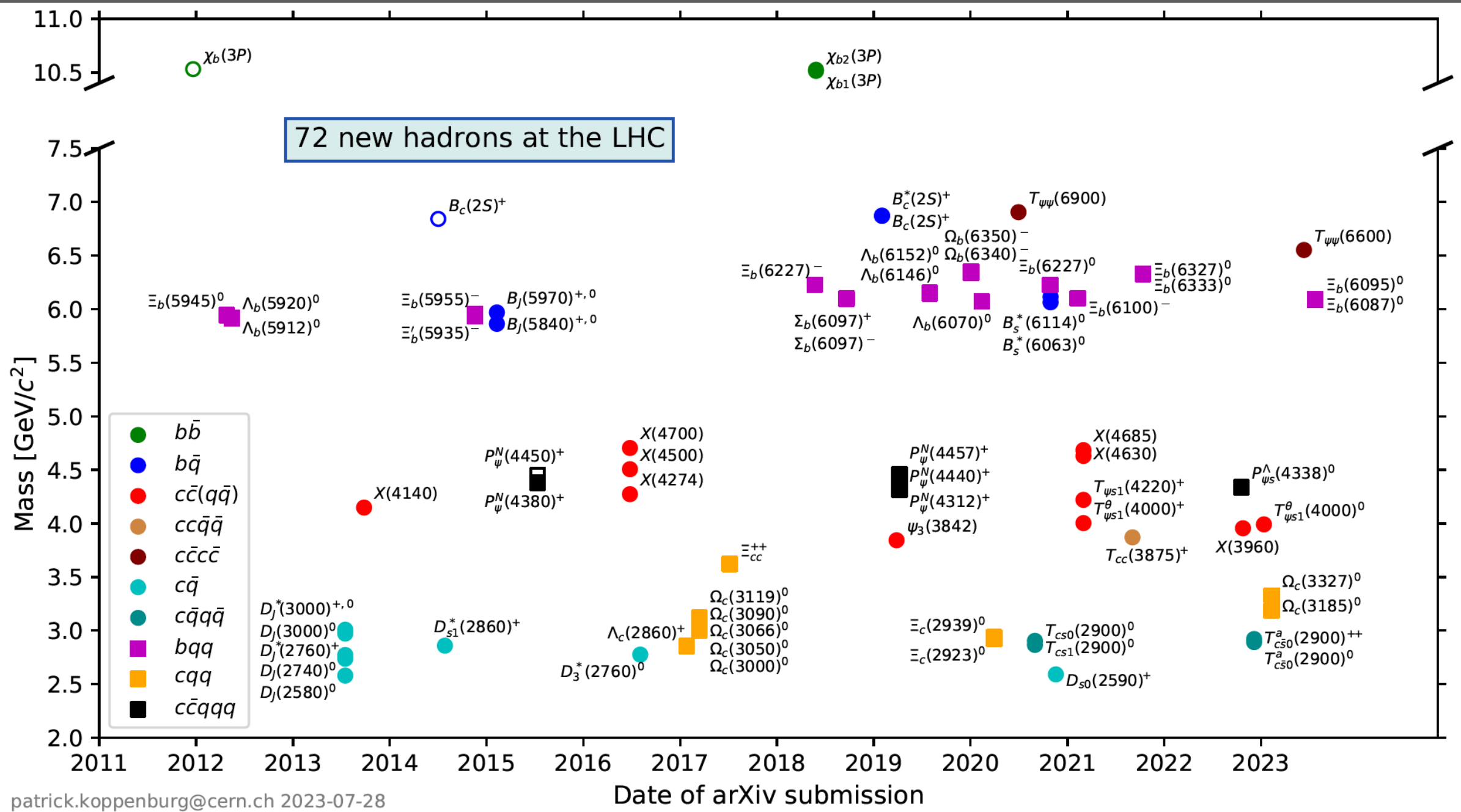
- QCD have two important features:
  - ◆ Color confinement
  - ◆ Asymptotic freedom
- In low energy region the perturbative calculation for QCD is impossible, alternatively:
  - ◆ Lattice QCD (non-perturbative calculation)
  - ◆ Effective models (chiral perturbation theory, quark model, etc...)



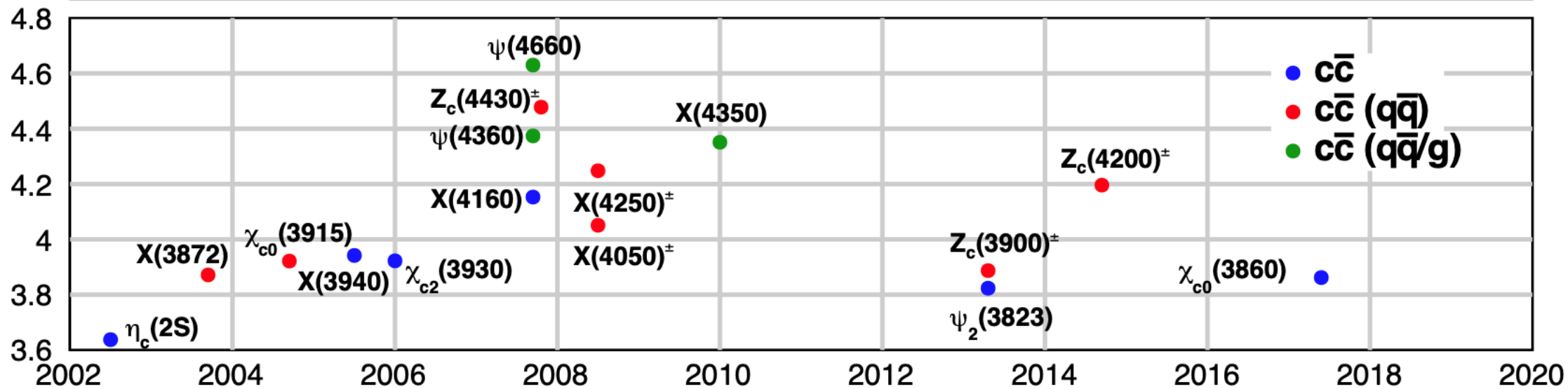
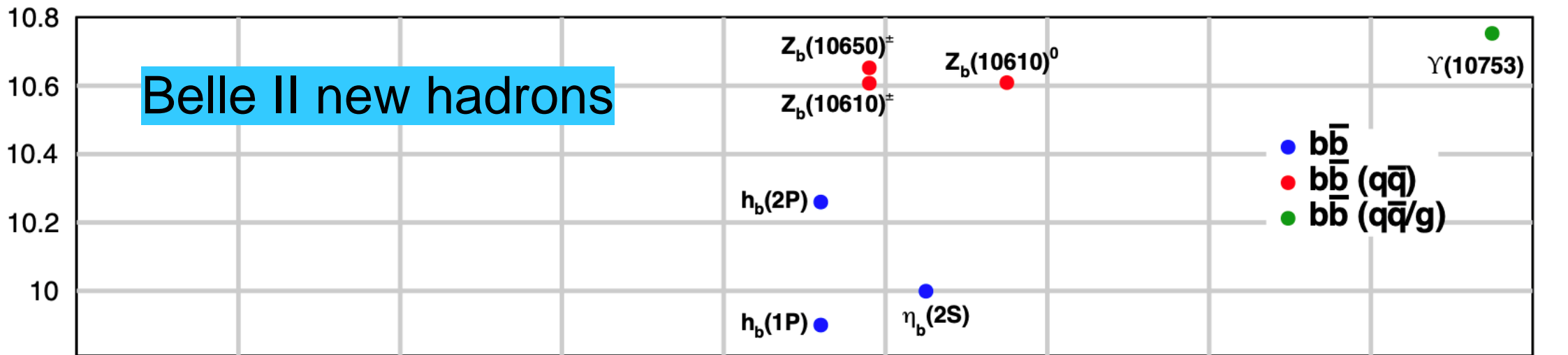
# New resonant structures at BESIII

From Prof. Shuang-Shi Fang



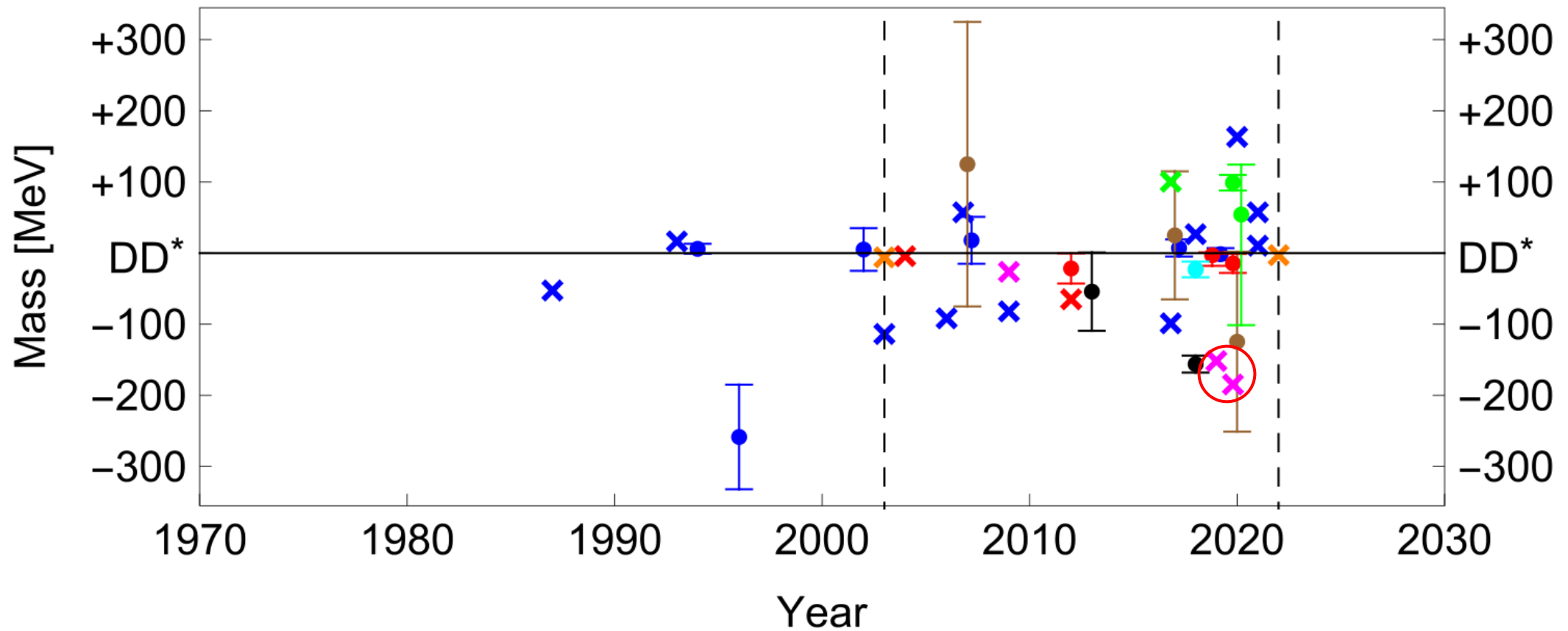


Mass [GeV]



Data of arXiv submission





**Figure 42.** Theoretical predictions on the mass of the doubly charmed tetraquark state  $cc\bar{u}\bar{d}$  with  $(I)J^P = (0)1^+$ , with uncertainties (error bars) and without uncertainties (crosses), calculated based on the compact tetraquark picture through various quark models [454, 500–515] (blue), QCD sum rules [462, 516, 517] (brown), heavy quark symmetry [467–469] (green), and others [461, 466] (black), as well as those calculated through the hadronic molecular picture [477, 479, 518–520] (red), the quark model considering the mixture of the meson-meson and diquark–antidiquark structures [481–483] (magenta), and lattice QCD [499] (cyan). The two dashed lines with orange crosses denote the  $\chi_{c1}(3872)$  ( $X(3872)$ ) first observed by Belle in 2003 [30] and the  $T_{cc}^+$  recently observed by LHCb in 2021 [42, 43].



# Meson exchange in nuclear force:

- $\pi(138)$  **Long-ranged** tensor force
- $\sigma(500)$  **intermediate-ranged**, attractive central force plus LS force
- $\omega(782)$  **short-ranged**, repulsive central force plus strong LS force
- $\rho(770)$  **short-ranged** tensor force, opposite to pion

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- ◎  $SU2$  ground states + excited states
- ◎  $SU3$  ground states
- ◎ Summary

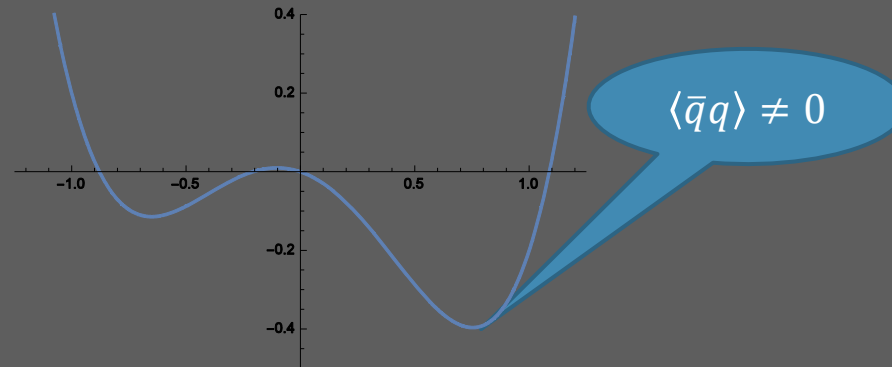
# The chiral symmetry

The chiral symmetry:



Spontaneously breaking of chiral symmetry:

$$\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$$



The effective theory based on chiral symmetry:

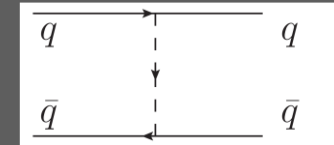
- Nonlinear sigma model
- Chiral perturbation theory

# Chiral quark model

## Naïve quark model:

- Quark mass term
- Kinetic term
- Color confinement potential (CON)
- One gluon exchange (OGE)

gluon exchange

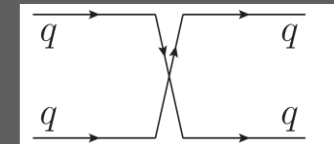
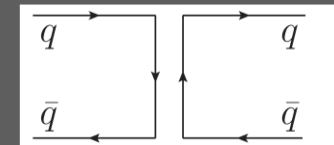


- Gell-Mann, M., 1964, Phys. Lett. 8, 214.
- Zweig, G., 1964, CERN Reports No. 8182/TH. 401 and No. 8419/TH. 412).
- N. Isgur, G. Karl, Phys.Lett.B 72 (1977) 109.

## The Nambu–Goldstone boson exchange:

- Chiral symmetry is spontaneously broken
- Pseudoscalars ( $\pi$ ,  $K$ ,  $\eta$ ) are the Nambu–Goldstone (NG) bosons of chiral symmetry breaking
- Scalar meson  $\sigma$  as the chiral partner of NG bosons

meson exchange



- K. Shimizu, Phys. Lett. B 148, 418-422 (1984)
- Z.Y. Zhang, Y.W. Yu, P.N. Shen, L.R. Dai, A. Faessler, U. Straub, Nucl. Phys. A 625 (1997) 59.
- J. Vijande, F. Fernandez, A. Valcarce, J. Phys. G 31, 481(2005)

# Chiral quark model

pseudoscalar + vector meson exchange + CON

- Quark mass term
- Kinetic term
- Color confinement potential (CON)
- Pseudo-scalar + vector meson

- L. Y. Glozman and D. O. Riska, Phys. Rept. 268, 263-303 (1996).
- L. Y. Glozman, Nucl. Phys. A 663, 103-112 (2000).

Scalar + pseudoscalar + vector meson exchange + CON + OGE

- Quark mass term
- Kinetic term
- Color confinement potential (CON)
- One gluon exchange (OGE)
- Scalar + pseudoscalar + vector meson

- L. R. Dai, Z. Y. Zhang, Y. W. Yu and P. Wang, Nucl. Phys. A 727, 321-332 (2003).
- Bing-Ran He, Masayasu Harada, Bing-Song Zou 2306.03526, 2307.16280

# Incorporate the vector meson contribution

## The hidden local symmetry:

M. Bando, T. Kugo, K. Yamawaki.  
Phys.Rept. 164 (1988) 217-314  
M. Harada, K. Yamawaki.  
Phys.Rept. 381 (2003) 1-233

$$U = \xi_L^\dagger \xi_R = e^{2i \frac{\pi(x)}{f_\pi}}$$
$$\xi_{L,R} \rightarrow h(x) \xi_{L,R} \cdot g_{L,R}^\dagger$$
$$\xi_{L,R} = e^{i \frac{V(x)}{f_V}} e^{\mp i \frac{\pi(x)}{f_\pi}}$$
$$h(x)^\dagger h(x) = 1$$

$$h(x) \in H_{\text{local}}, \quad g_{L,R} \in G_{\text{global}}$$

- The transformation for U do not changes, which seems that the freedom of vector meson is “hidden”

$$[SU(N_f)_L \times SU(N_f)_R]_{\text{global}} \times [SU(N_f)_V]_{\text{local}} \rightarrow [SU(N_f)_V]_{\text{global}}$$



- Hidden local symmetry is an extension of chiral perturbation theory
- Hidden local symmetry is a systematic way to include pseudo-scalar mesons ( $\pi, K, \eta, \eta'$ ) and vector mesons ( $\rho, K^*, \omega, \phi$ )

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- ***SU2 ground states + excited states***
- ◎ *SU3* ground states
- ◎ Summary

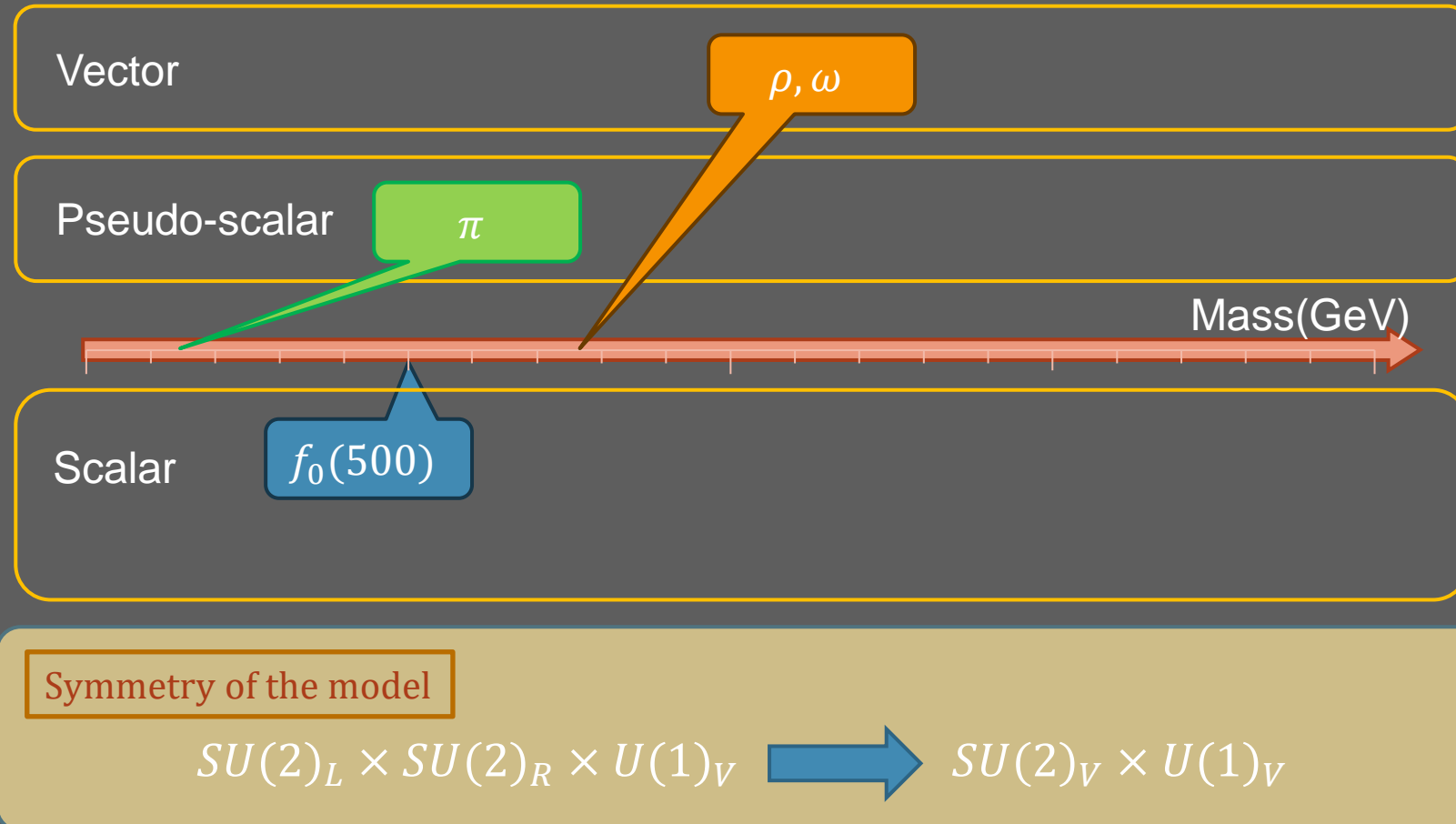
# The Hamiltonian

$$H = \sum_{i=1} \left( m_i + \frac{p_i^2}{2m_i} \right) - T_{CM} + \sum_{j>i=1} (V_{ij}^{\text{CON}} + V_{ij}^{\text{OGE}} + V_{ij}^{\sigma} + V_{ij}^{\pi} + V_{ij}^{\omega} + V_{ij}^{\rho})$$

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Bing-Song Zou, 2306.03526

$$V_{ij}^v = \frac{\Lambda_v^2}{\Lambda_v^2 - m_v^2} \left\{ \frac{g_v^2}{4\pi} m_v \left[ Y(m_v r) - \left( \frac{\Lambda_v}{m_v} \right) Y(\Lambda_v r) \right] + \frac{m_v^3}{m_i m_j} \left( \frac{g_v(2f_v + g_v)}{16\pi} + \frac{\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j (f_v + g_v)^2}{6 \cdot 4\pi} \right) \times \left[ Y(m_v r) - \left( \frac{\Lambda_v}{m_v} \right)^3 Y(\Lambda_v r) \right] - \boldsymbol{S}_+ \cdot \boldsymbol{L} \frac{g_v(4f_v + 3g_v)}{8\pi} \frac{m_v^3}{m_i m_j} \times \left[ G(m_v r) - \left( \frac{\Lambda_v}{m_v} \right)^3 G(\Lambda_v r) \right] - \boldsymbol{S}_{ij} \frac{(f_v + g_v)^2}{4\pi} \frac{m_v^3}{12m_i m_j} \times \left[ H(m_v r) - \left( \frac{\Lambda_v}{m_v} \right)^3 H(\Lambda_v r) \right] \right\}$$

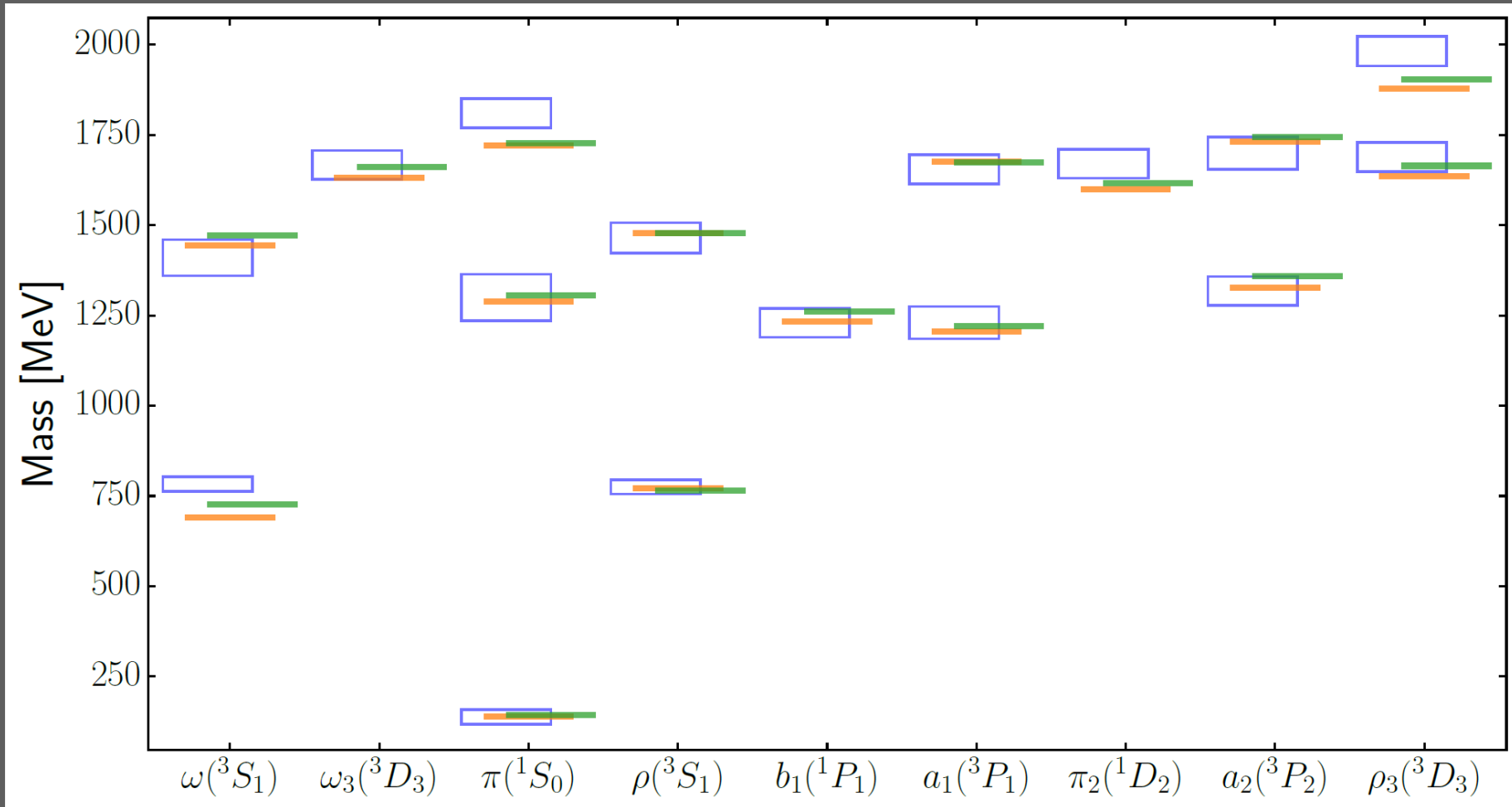
# The exchanged mesons in $SU2$ model



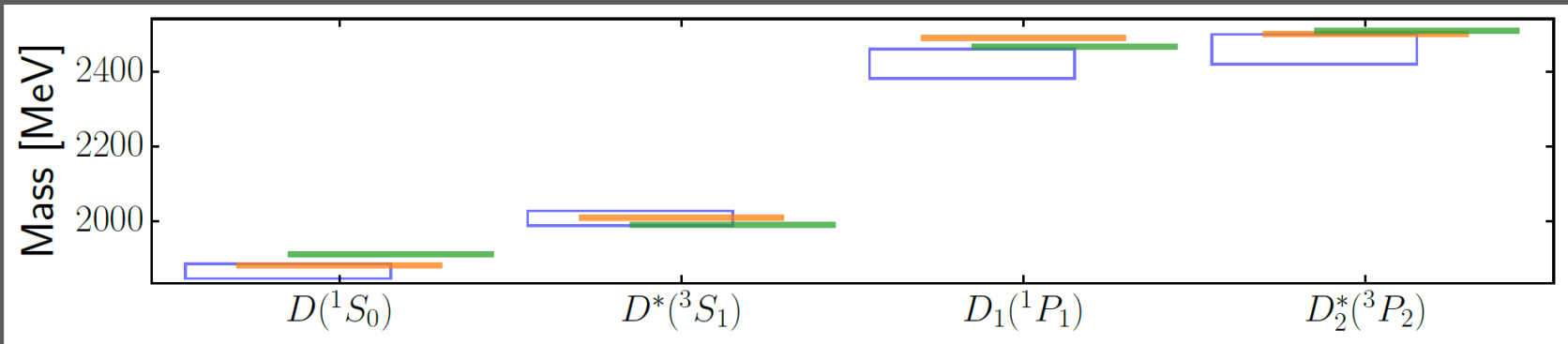
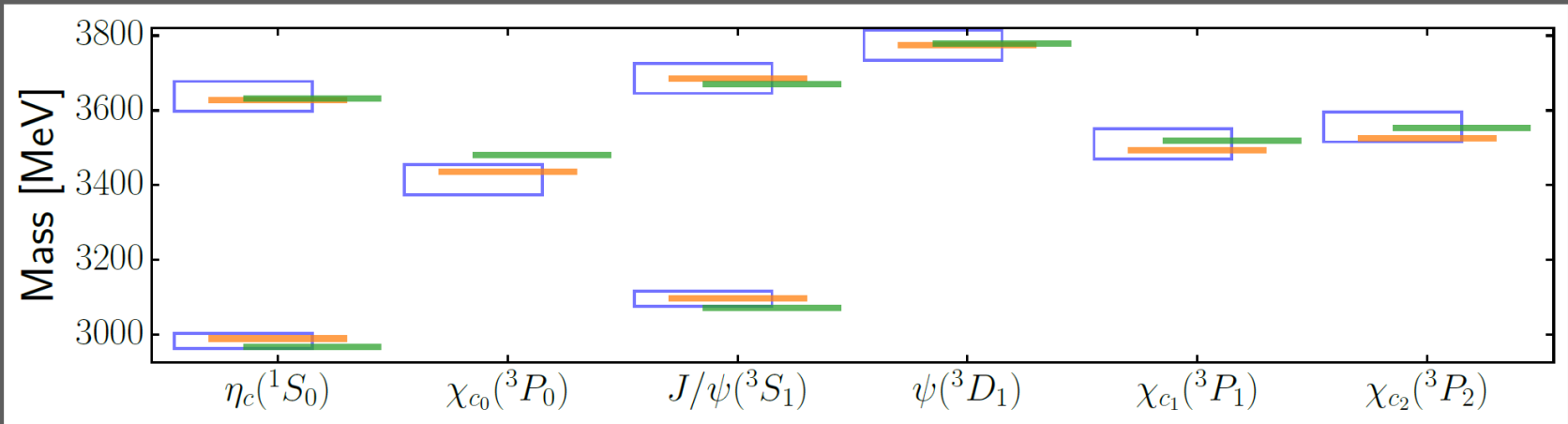
# Wave functions

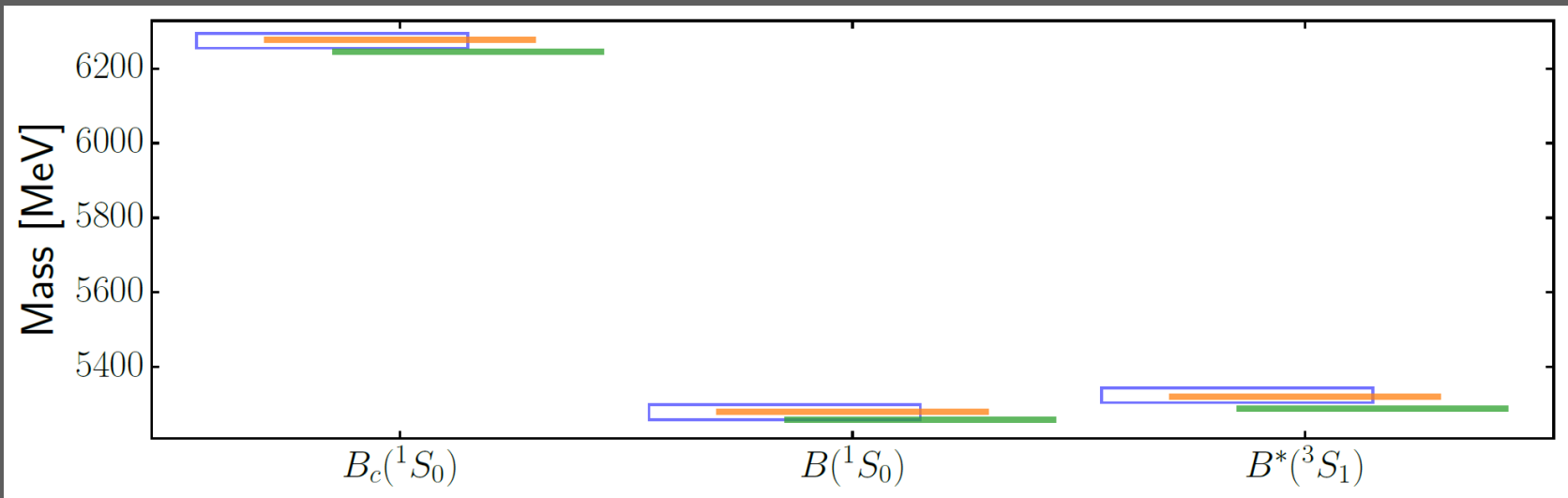
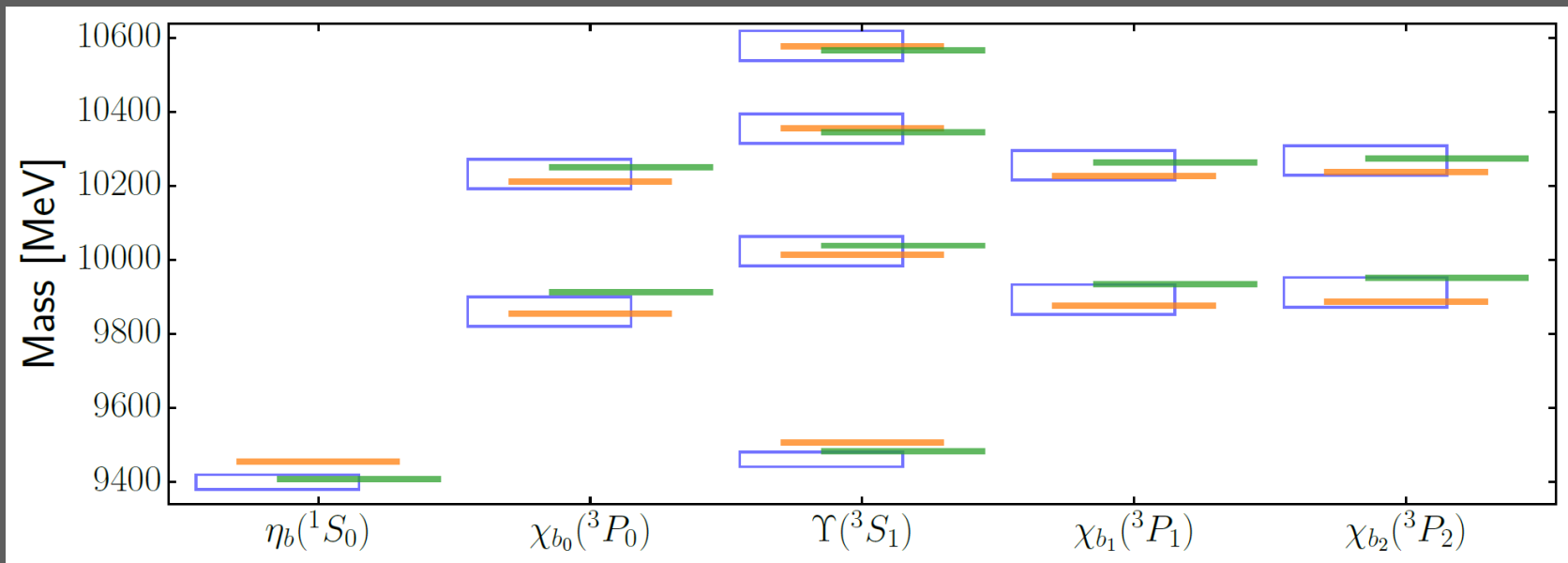
- Orbital (SO(3)):  $(\psi_L)$
- Spin (SU(2)):  $(\chi_S^\sigma)$
- Flavor (SU(2)):  $(\chi_I^f)$
- Color (SU(3)):  $(\chi^c)$

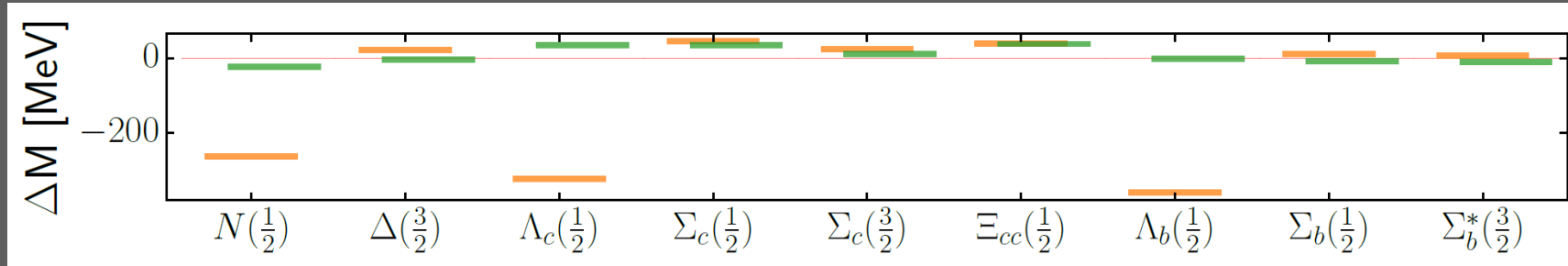
$$\Psi_{JM_J IM_I}^{ijk} = \mathcal{A} \left[ [\psi_L \chi_S^{\sigma i}]_{JM_J} \chi_I^{fj} \chi_k^c \right]$$











|                      | 1   | $\tau_i \tau_j$ | $\sigma_i \sigma_j$ | $\sigma_i \sigma_j \cdot \tau_i \tau_j$ |
|----------------------|-----|-----------------|---------------------|---|
| $\sigma$             | -/- |                 |                     |   |
| $\pi$                |     |                 |                     | +/-                                     |
| $a_0$                |     | -/+             |                     |   |
| OGE                  | -/- |                 | +/+                 |   |
| CON                  | +/+ |                 |                     |   |
| $\omega$ (This work) | +/- |                 | +/-                 |   |
| $\rho$ (This work)   |     | +/+             |                     | +/+                                     |

$qq/q\bar{q}$

| Channel                                       | $E_B$ |     | Channel                                       | $E_B$ |     |
|---|-------|-----|---|-------|-----|
| $[DD^*]_{1\otimes 1}$                         | 6.2   | 39% | $[BB^*]_{1\otimes 1}$                         | 0.5   | 12% |
| $[D^*D]_{1\otimes 1}$                         | 6.2   | 39% | $[B^*B]_{1\otimes 1}$                         | 0.5   | 12% |
| $[D^*D^*]_{1\otimes 1}$                       | 83.1  | 5%  | $[B^*B^*]_{1\otimes 1}$                       | 31    | 32% |
| $[DD^*]_{8\otimes 8}$                         | 383.1 | 0%  | $[BB^*]_{8\otimes 8}$                         | 253.6 | 0%  |
| $[D^*D]_{8\otimes 8}$                         | 383.1 | 0%  | $[B^*B]_{8\otimes 8}$                         | 253.6 | 0%  |
| $[D^*D^*]_{8\otimes 8}$                       | 337.3 | 0%  | $[B^*B^*]_{8\otimes 8}$                       | 233.3 | 0%  |
| $[(cc)(\bar{q}\bar{q})^*]_{6\otimes \bar{6}}$ | 337.5 | 0%  | $[(bb)(\bar{q}\bar{q})^*]_{6\otimes \bar{6}}$ | 233.7 | 0%  |
| $[(cc)^*(\bar{q}\bar{q})]_{\bar{3}\otimes 3}$ | 120.3 | 17% | $[(bb)^*(\bar{q}\bar{q})]_{\bar{3}\otimes 3}$ | -37.8 | 44% |
| Mixed   | -4.9  |     | Mixed   | -88.2 |     |

|          | $r_{cc}$ | $r_{\bar{q}c}$ | $r_{\bar{q}\bar{q}}$ | $r_{\bar{q}b}$ | $r_{bb}$ |
|----------|----------|----------------|----------------------|----------------|----------|
| $T_{cc}$ | 1.56     | 1.24           | 1.70                 |                |          |
| $T_{bb}$ |          |                | 0.75                 | 0.65           | 0.37     |

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# The Hamiltonian

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$$V_{ij}^{\bar{\sigma}} = V_{ij}^{s=\bar{\sigma}, g_s=g_{\bar{\sigma}q}} \lambda_i^q \lambda_j^q + V_{ij}^{s=\bar{\sigma}, g_s=g_{\bar{\sigma}s}} \lambda_i^s \lambda_j^s,$$

$$V_{ij}^{\eta} = V_{ij}^{p=\eta, g_p=g_{\eta q}} \lambda_i^q \lambda_j^q + V_{ij}^{p=\eta, g_p=g_{\eta s}} \lambda_i^s \lambda_j^s,$$

$$V_{ij}^{\eta'} = V_{ij}^{p=\eta', g_p=g_{\eta'q}} \lambda_i^q \lambda_j^q + V_{ij}^{p=\eta', g_p=g_{\eta's}} \lambda_i^s \lambda_j^s,$$

$$\lambda^q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^s = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$V_{ij}^{\pi} = V_{ij}^{p=\pi, g_p=g_{\pi}} \sum_{a=1}^3 \lambda_i^a \lambda_j^a,$$

$$V_{ij}^K = V_{ij}^{p=K, g_p=g_K} \sum_{a=4}^7 \lambda_i^a \lambda_j^a,$$

$$V_{ij}^{\omega} = V_{ij}^{v=\omega, g_v=g_{\omega q}} \lambda_i^q \lambda_j^q + V_{ij}^{v=\omega, g_v=g_{\omega s}} \lambda_i^s \lambda_j^s,$$

$$V_{ij}^{\phi} = V_{ij}^{v=\phi, g_v=g_{\phi q}} \lambda_i^q \lambda_j^q + V_{ij}^{v=\phi, g_v=g_{\phi s}} \lambda_i^s \lambda_j^s,$$

$$V_{ij}^{\rho} = V_{ij}^{v=\rho, g_v=g_{\rho}} \sum_{a=1}^3 \lambda_i^a \lambda_j^a,$$

$$V_{ij}^{K^*} = V_{ij}^{v=K^*, g_v=g_{K^*}} \sum_{a=4}^7 \lambda_i^a \lambda_j^a$$

The coupling have relations based on  $SU_3$  flavor symmetry:

$$g_{\eta s} = g_{\eta q} - \sqrt{3} \cos \theta_p g_{\pi}$$

$$g_{\eta'q} = -\cot \theta_p g_{\eta q} + \frac{1}{\sqrt{3} \sin \theta_p} g_{\pi}$$

$$g_{\eta's} = -\cot \theta_p g_{\eta q} + \frac{\cos \theta_p \cot \theta_p - 2 \sin \theta_p}{\sqrt{3}} g_{\pi}$$

$$g_{\pi} = g_K$$

$$g_{\omega s} = g_{\omega q} - g_{\rho}$$

$$g_{\phi q} = -\sqrt{\frac{1}{2}} (g_{\omega q} - g_{\rho})$$

$$g_{\phi s} = -\sqrt{\frac{1}{2}} (g_{\omega q} + g_{\rho})$$

$$g_{\rho} = g_{K^*}$$

$$f_{\omega s} = f_{\omega q} - f_{\rho}$$

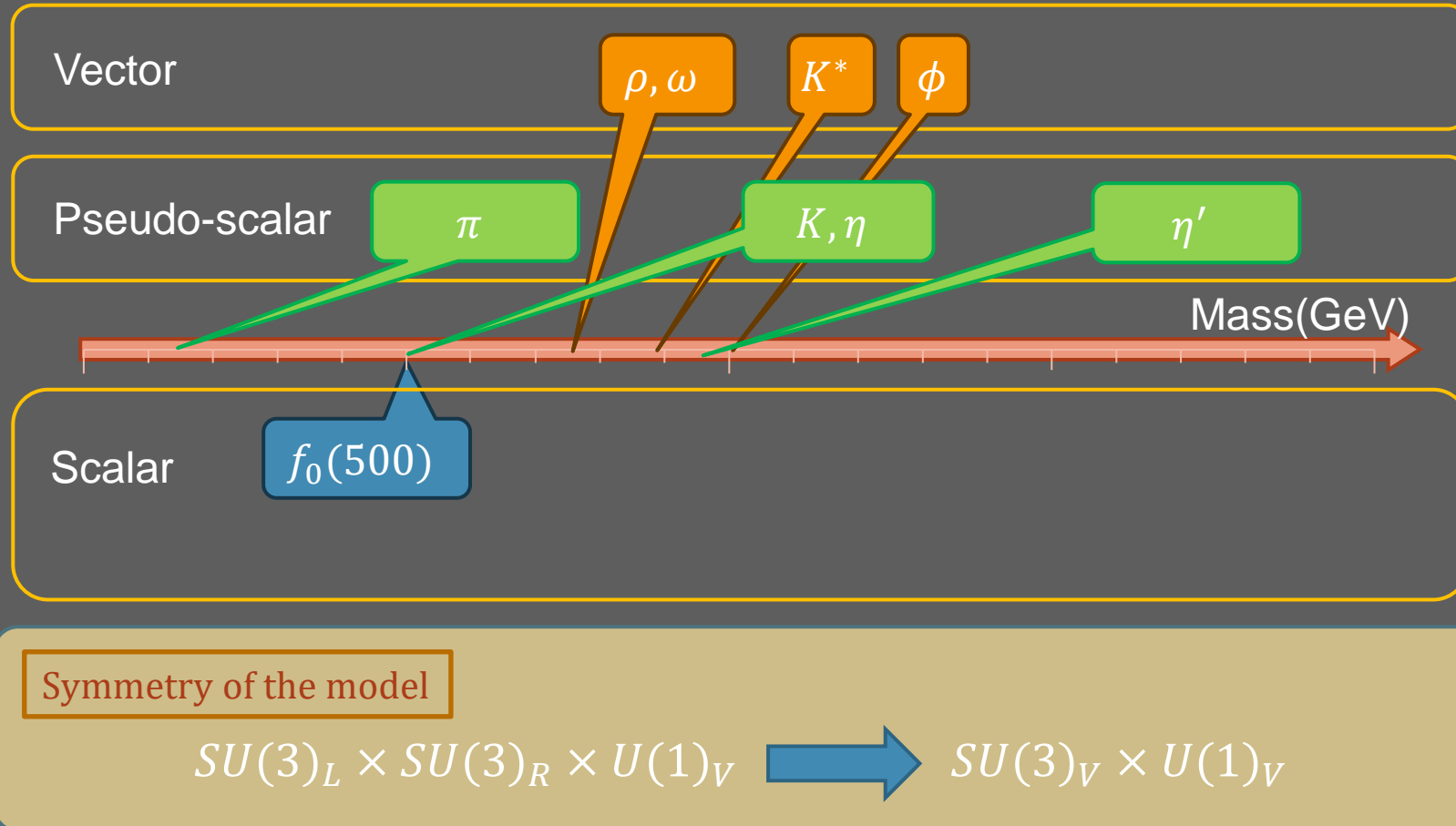
$$f_{\phi q} = -\sqrt{\frac{1}{2}} (f_{\omega q} - f_{\rho})$$

$$f_{\phi s} = -\sqrt{\frac{1}{2}} (f_{\omega q} + f_{\rho})$$

$$f_{\rho} = f_{K^*}$$



# The exchanged mesons in $SU3$ model



# Wave functions

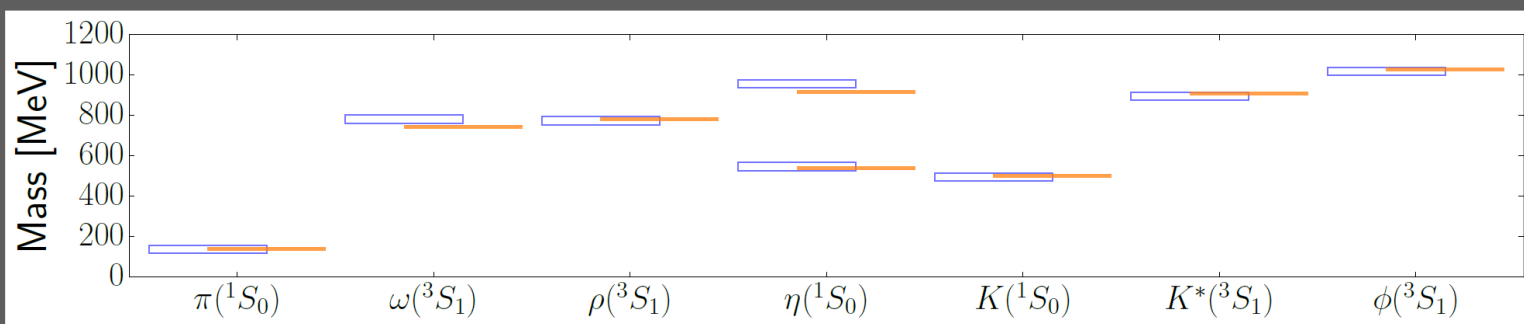
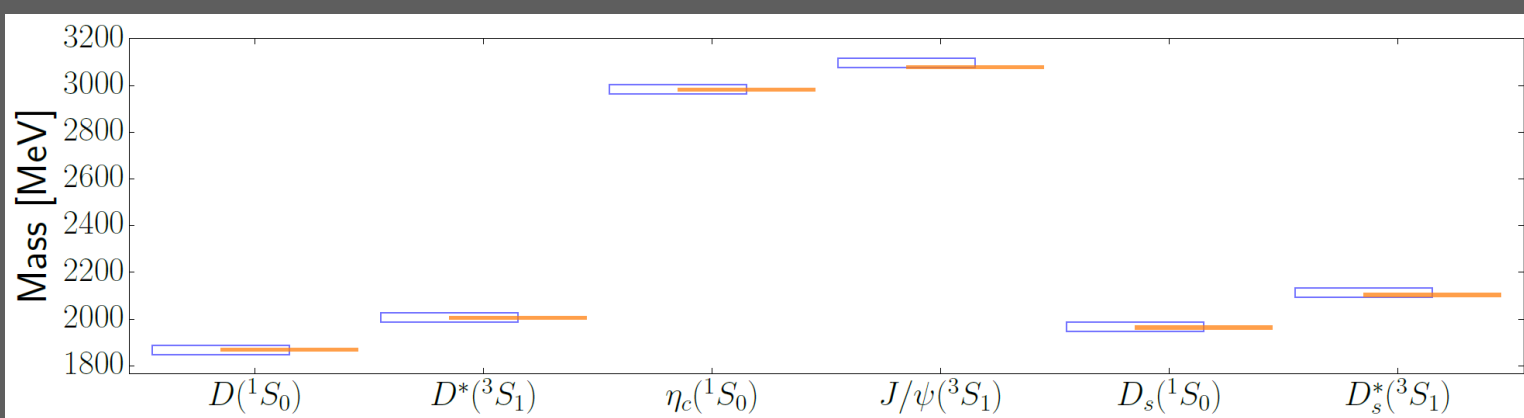
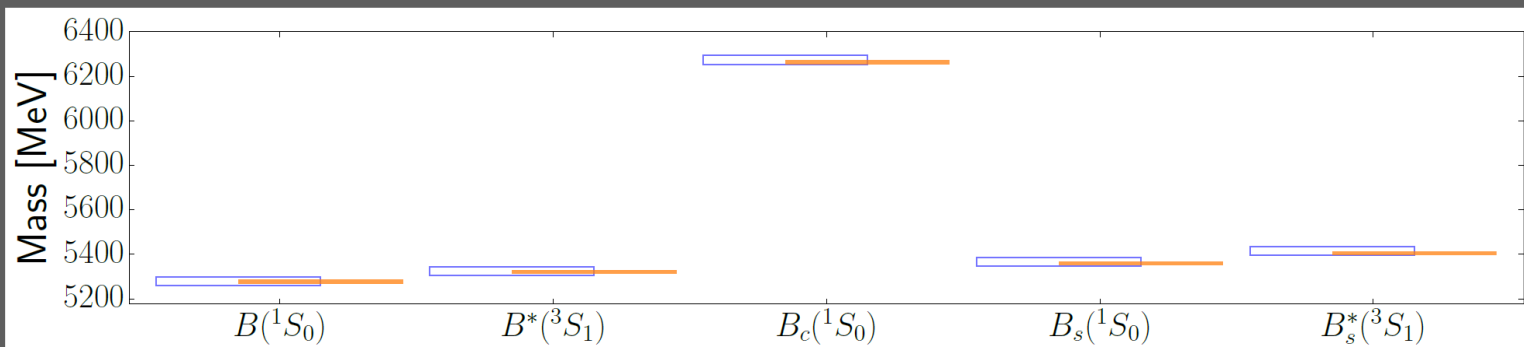
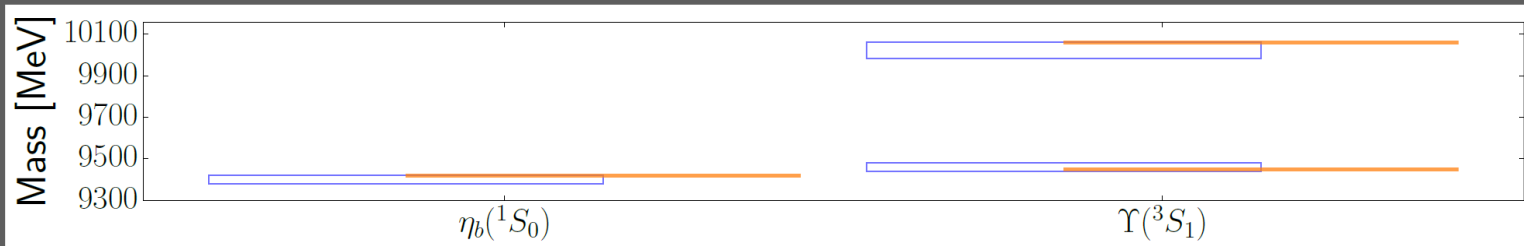
- Orbital (SO(3)):  $(\psi_L)$
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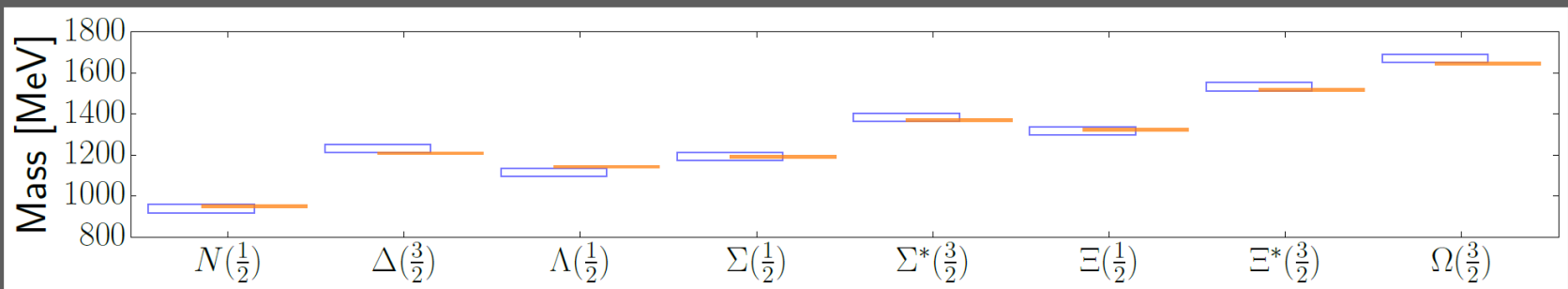
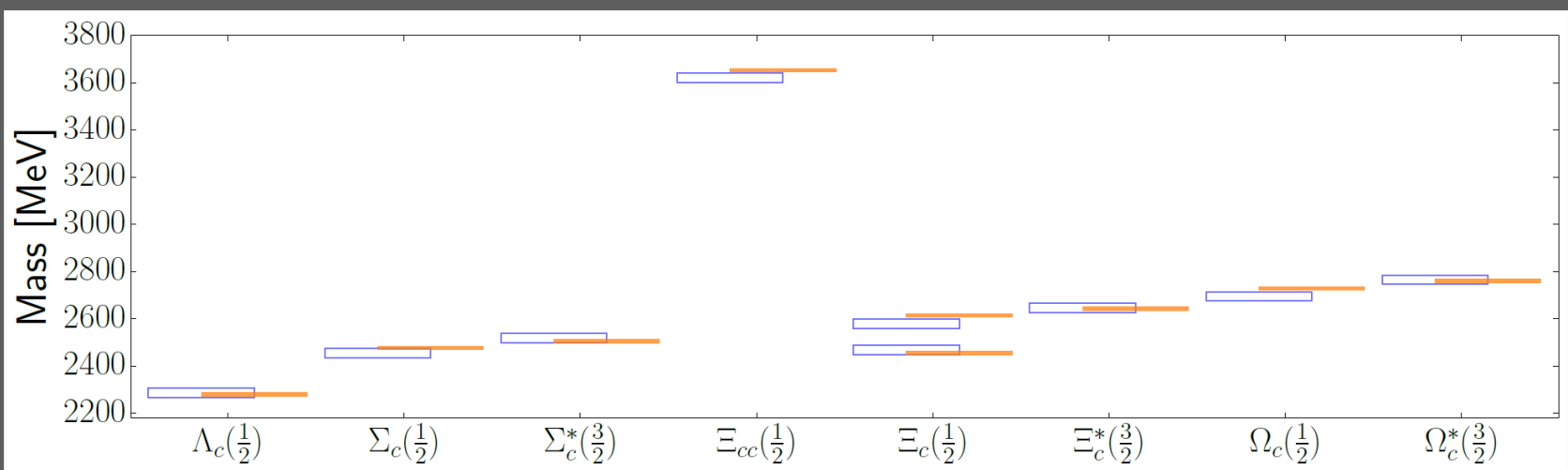
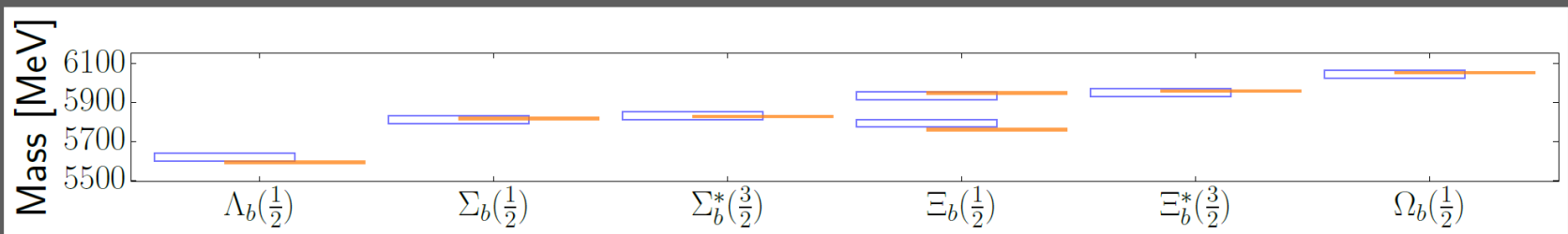
$$\Psi_{JM_J IM_I}^{ijk} = \mathcal{A} \left[ [\psi_L \chi_S^{\sigma i}]_{JM_J} \chi_I^{fj} \chi_k^c \right]$$

|                |     |                       |                     |   |
|----------------|-----|-----------------------|---------------------|---|
|                | 1   | $\lambda_i \lambda_j$ | $\sigma_i \sigma_j$ | $\sigma_i \sigma_j \cdot \lambda_i \lambda_j$ |
| $\bar{\sigma}$ | -/- |                       |                     |   |
| $\eta/\eta'$   |     |                       | +/+                 |   |
| $\pi/K$        |     |                       |                     | +/-   |
| $\omega/\phi$  | +/- |                       | +/-                 |   |
| $\rho/K^*$     |     | +/+                   |                     | +/+   |
| OGE            | -/- |                       | +/+                 |   |
| CON            | +/+ |                       |                     |   |

|                | No vector<br>J. Phys. G 31, 481(2005) | su2 vector<br>2306.03526 | su3 vector<br>2307.16280 |
|----------------|---------------------------------------|--------------------------|--------------------------|
| $\alpha_s(qq)$ | 0.536                                 | 0.880                    | 0.456                    |
| $\alpha_s(qs)$ | 0.479                                 |                          | 0.426                    |
| $\alpha_s(qc)$ | 0.426                                 | 0.774                    | 0.363                    |
| $\alpha_s(qb)$ | 0.409                                 | 0.749                    | 0.339                    |
| $\alpha_s(ss)$ | 0.419                                 |                          | 0.388                    |
| $\alpha_s(sc)$ | 0.360                                 |                          | 0.308                    |
| $\alpha_s(sb)$ | 0.340                                 |                          | 0.279                    |
| $\alpha_s(cc)$ | 0.288                                 | 0.510                    | 0.205                    |
| $\alpha_s(cb)$ | 0.260                                 | 0.447                    | 0.168                    |
| $\alpha_s(bb)$ | 0.223                                 | 0.366                    | 0.128                    |

$qq/q\bar{q}$



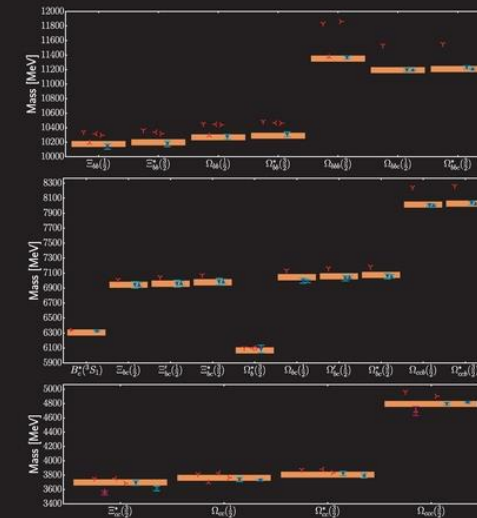
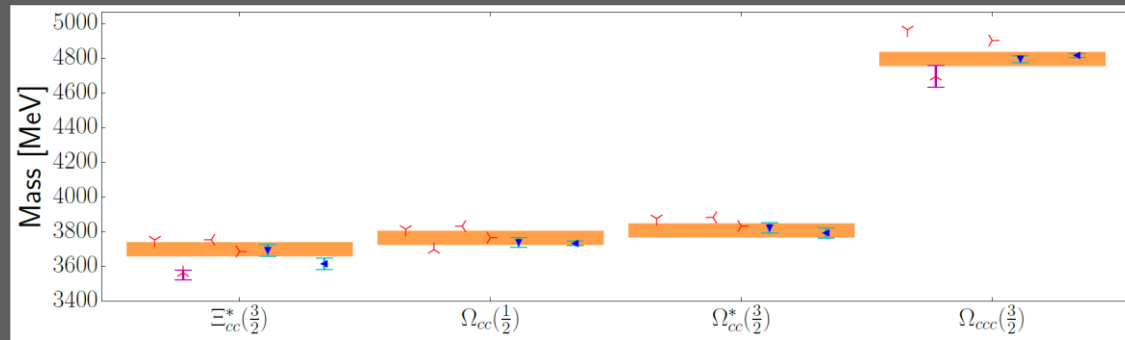
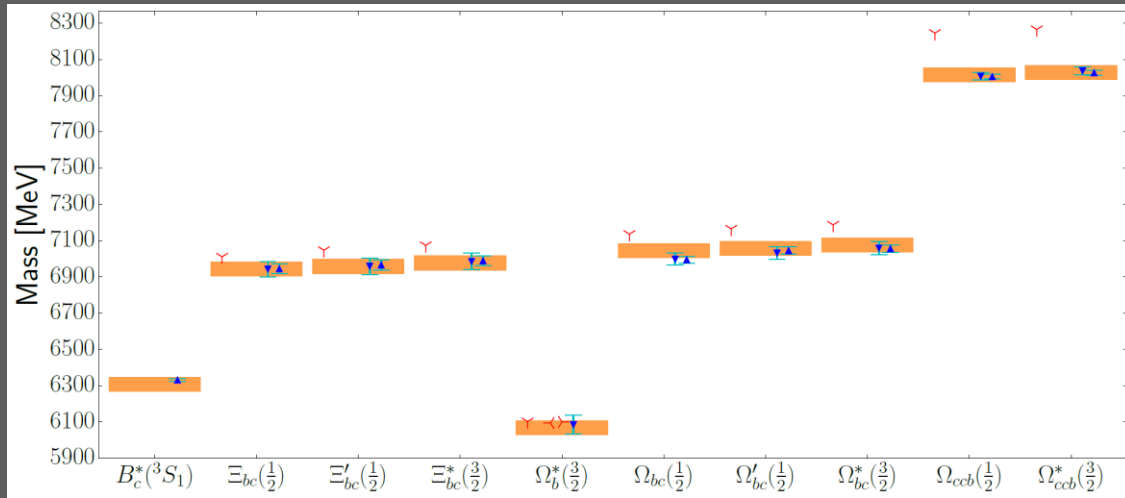
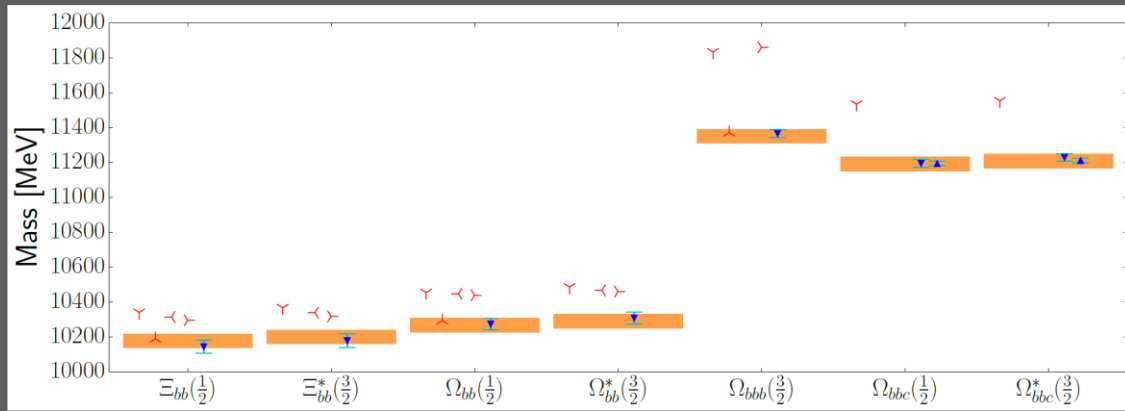


| <b>Models</b>                                   | <b>Meson <math>\chi^2</math><br/>Err(the)=40MeV</b> | <b>Baryon <math>\chi^2</math><br/>Err(the)=40MeV</b> |
|---|---|--|
| Godfrey & Isgur<br>1985                         | 5.3 (21 meson)                                      |  |
| Capstick & Isgur<br>1986                        |   | 3.78 (14 baryon)                                     |
| J. Phys. G 31,<br>481(2005)                     | 9.5 (21 meson)                                      | 305.8 (8 baryon,<br><b>unpublished</b> )             |
| <b>He &amp; Harada &amp; Zou<br/>2307.16280</b> | 2.9 (21 meson)                                      | 5.7 (24 baryon)                                      |

$$\chi^2 = \sum_i \left( \frac{m_i(\text{the}) - m_i(\text{exp})}{\text{Err}_i(\text{sys})} \right)^2$$

$$\text{Err}(\text{sys}) = \sqrt{\text{Err}(\text{exp})^2 + \text{Err}(\text{the})^2}$$





Predicted mass spectrum of missing meson and baryons which have not been experimentally confirmed shown by orange line with 40 MeV error. The values of  $\Omega_{bbb}$  shown here are shifted by  $-3000\text{MeV}$ .

From Bing-Ran He, Masayasu Harada & Bing-Song Zou on: Ground states of all mesons and baryons in a quark model with hidden local symmetry. Eur. Phys. J. C 83, 1159 (2023).



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- Introduction
- Chiral quark model with HLS
- $SU2$  ground states + excited states
- $SU3$  ground states
- **Summary**

- Quark model with hidden local symmetry gives a systematic description from light meson/baryon to heavy meson/baryon
- Quark model with hidden local symmetry gives correct interaction between  $qq$  and  $q\bar{q}$ , thus its easy to extend the model to describe multiquark states, e.g., tetraquark, pentaquark, ...
- Quark model with hidden local symmetry gives size messages and percentage of components, which could help us to identify the particles observed from experiments

Thank you for your attention!