

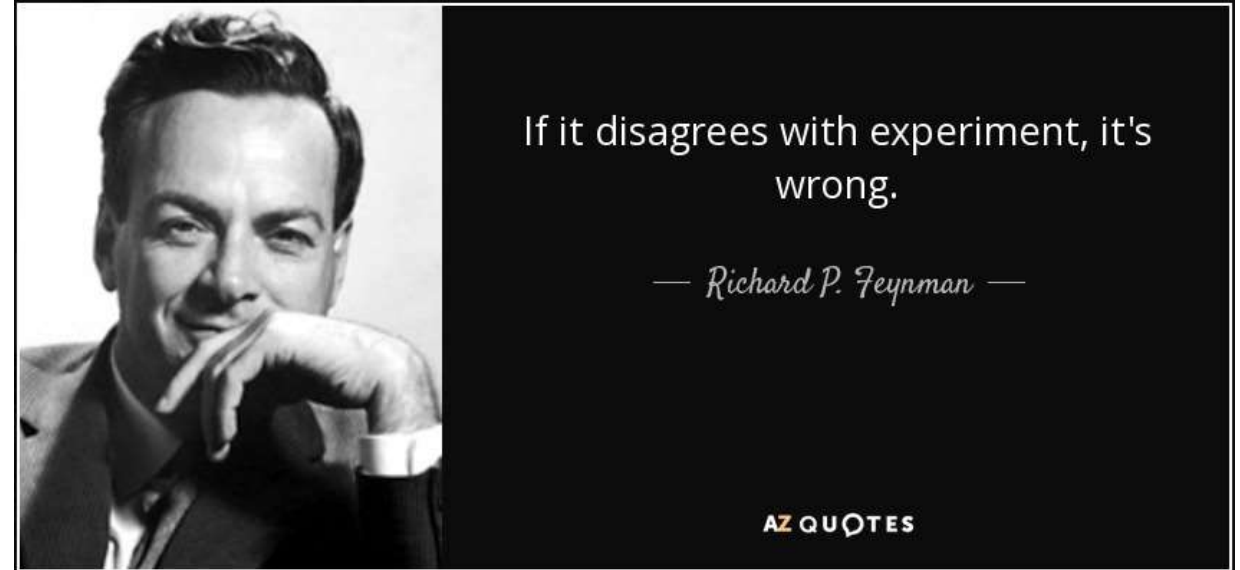
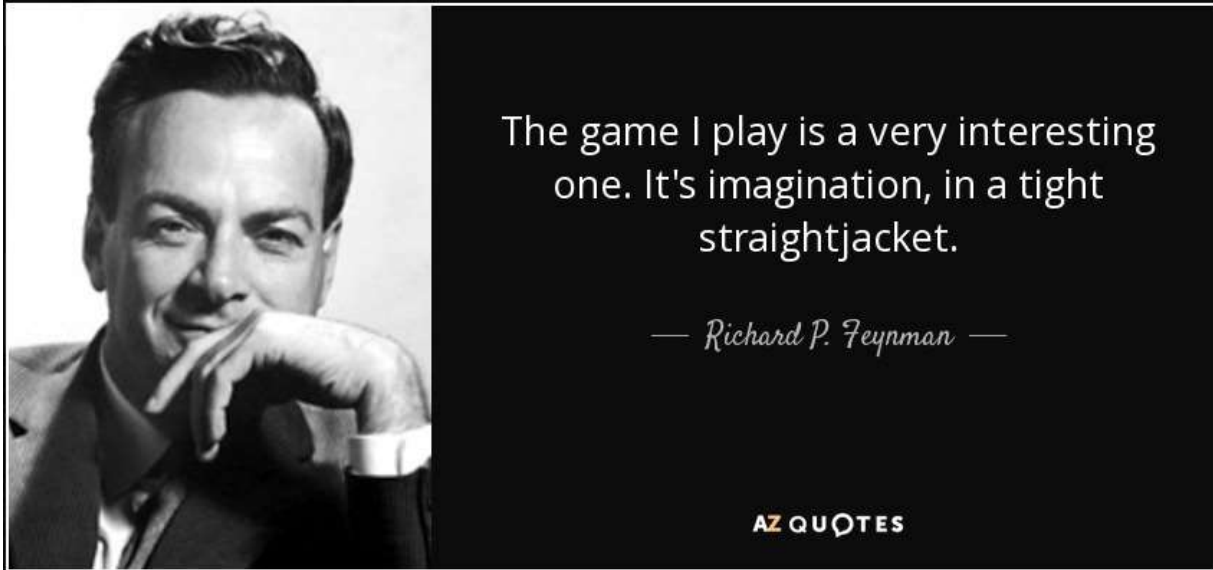
DDF 2024

# Alternatives to conventional Neutron Stars and Hybrid Stars

Chen Zhang (张晨)  
IAS, HKUST

based on

- B. Holdom, J. Ren, **C. Z**, Phys.Rev.Lett. 120 (2018) , 222001
- **C. Z**, Phys.Rev.D 101 (2020) 4, 043003
- J.Ren, **C. Z**, Phys.Rev.D 102 (2020) 8, 083003
- **C.Z**, Jing Ren, Phys.Rev.D 108 (2023) 6, 063012
- **C. Z**, Yudong Luo, Hongbo Li, Lijing Shao, Renxin Xu, Phys.Rev.D 109 (2024), 063020
- **C. Z**, Yong Gao , Cheng-jun Xia, Renxin Xu, Phys.Rev.D 108 (2023) , 12



### Tight straightjacket:

- Theory: Einstein gravity, Standard Model physics (QCD phase diagram)
- Experiment:  $M_{TOV} > 2 M_{\odot}$ , GW170817, NICER e.t.c..

### Imagination with the tight straightjacket:

Alternative possibility of

- **Matter: Up-down Quark matter (and up-down Quark Stars)** [Phys.Rev.Lett. 120 (2018), 222001, Phys.Rev.D 101 (2020) , 043003]
- **Structure: Inverted hybrid stars** [Phys.Rev.D 108 (2023), 063012, Phys.Rev.D 109 (2024), 063020 ]
- **Matter and Structure: Hybrid Strangeon Stars** [Phys.Rev.D 108 (2023) , 12]

# Part 1

## up-down Quark Matter (udQM) and up-down Quark Stars (udQSs)

*Based on*

- *B. Holdom, J. Ren, **C. Z.** Phys.Rev.Lett. 120 (2018) , 222001*
- ***C. Z.** Phys.Rev.D 101 (2020) 4, 043003*
- *J.Ren, **C. Z.** Phys.Rev.D 102 (2020) 8, 083003*

# Strange Quark Matter (SQM) hypothesis

[1984 E. Witten]

QM with nearly equal number of  $u, d, s$  quarks, **may be the most stable form of matter** even at zero temperature and zero pressure

However, assumed that

1. QCD vacuum is flavor-independent (a universal bag const)
2. stable  $u, d$  quark matter ( $udQM$ ) can not exist, considering we don't see proton&neutron being converted to  $udQM$ .

PHYSICAL REVIEW D

VOLUME 30, NUMBER 2

15 JULY 1984

## Cosmic separation of phases

Edward Witten\*

*Institute for Advanced Study, Princeton, New Jersey 08540*

(Received 9 April 1984)

A first-order QCD phase transition that occurred reversibly in the early universe would lead to a surprisingly rich cosmological scenario. Although observable consequences would not necessarily survive, it is at least conceivable that the phase transition would concentrate most of the quark excess in dense, invisible quark nuggets, providing an explanation for the dark matter in terms of QCD effects only. This possibility is viable only if quark matter has energy per baryon less than 938 MeV. Two related issues are considered in appendices: the possibility that neutron stars generate a quark-matter component of cosmic rays, and the possibility that the QCD phase transition may have produced a detectable gravitational signal.

# Up-down Quark Matter ( $udQM$ ) hypothesis

[B. Holdom, J. Ren, C. Z, Phys.Rev.Lett. 120 (2018) , 222001]

$u, d$  quark matter could **be more stable than** SQM and ordinary nuclear matter at a sufficiently large baryon number  $A_{min} > 300$

Derived from an extended SU(3) chiral quark-meson model with the **flavor dependence of the QCD vacuum naturally accounted** and **fits well with all the mass spectrum and decay widths of light scalar and pseudoscalar mesons.**

PHYSICAL REVIEW LETTERS 120, 222001 (2018)

## Quark Matter May Not Be Strange

Bob Holdom,\* Jing Ren,† and Chen Zhang‡

*Department of Physics, University of Toronto, Toronto, Ontario M5S1A7, Canada*

Ⓜ (Received 30 July 2017; revised manuscript received 23 October 2017; published 31 May 2018)

If quark matter is energetically favored over nuclear matter at zero temperature and pressure, then it has long been expected to take the form of strange quark matter (SQM), with comparable amounts of  $u$ ,  $d$ , and  $s$  quarks. The possibility of quark matter with only  $u$  and  $d$  quarks ( $udQM$ ) is usually dismissed because of the observed stability of ordinary nuclei. However, we find that  $udQM$  generally has lower bulk energy per baryon than normal nuclei and SQM. This emerges in a phenomenological model that describes the spectra of the lightest pseudoscalar and scalar meson nonets. Taking into account the finite size effects,  $udQM$  can be the ground state of baryonic matter only for baryon number  $A > A_{min}$  with  $A_{min} \gtrsim 300$ . This ensures the stability of ordinary nuclei and points to a new form of stable matter just beyond the periodic table.

DOI: 10.1103/PhysRevLett.120.222001



## New form of matter may lie just beyond the periodic table

Long Room, 15 Jun 2018

Currently, the heaviest element on the periodic table is oganesson, which has a

madriod

## Hay "materia extraña" más allá de la Tabla Periódica

Madrid, 10 jul 2018

Un equipo de investigadores cree que existen elementos ultrapesados que no 'funcionan' como la materia normal.

ZAP

## As estrelas de neutrões podem ter uma "estranha matéria" que não cabe na tabela periódica

ZAP, 25 Jun 2018

NASA Duas estrelas de neutrões colidem num enorme big bang Um grupo de físicos está a questionar a nossa compreensão sob

science alert

## A Strange Type of Matter Might Lie Inside Neutron Stars, And It Breaks The Periodic Table

Science Alert, 20 Jun 2018

A group of physicists are questioning our understanding of how quarks - a type of elementary particle - arrange themselves...

europa press

## Una nueva forma de materia se vislumbra más allá de la tabla periódica

Europa Press, 18 Jun 2018

(EUROPA PRESS) - Científicos predicen que los elementos con masas atómicas superiores a aproximadamente 300 abre la posibilidad

NEWS CAREERS COMMENTARY JOURNALS

Science

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Hong Kong University...

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# Weird new form of nuclear matter might lie just beyond experimenters' grasp

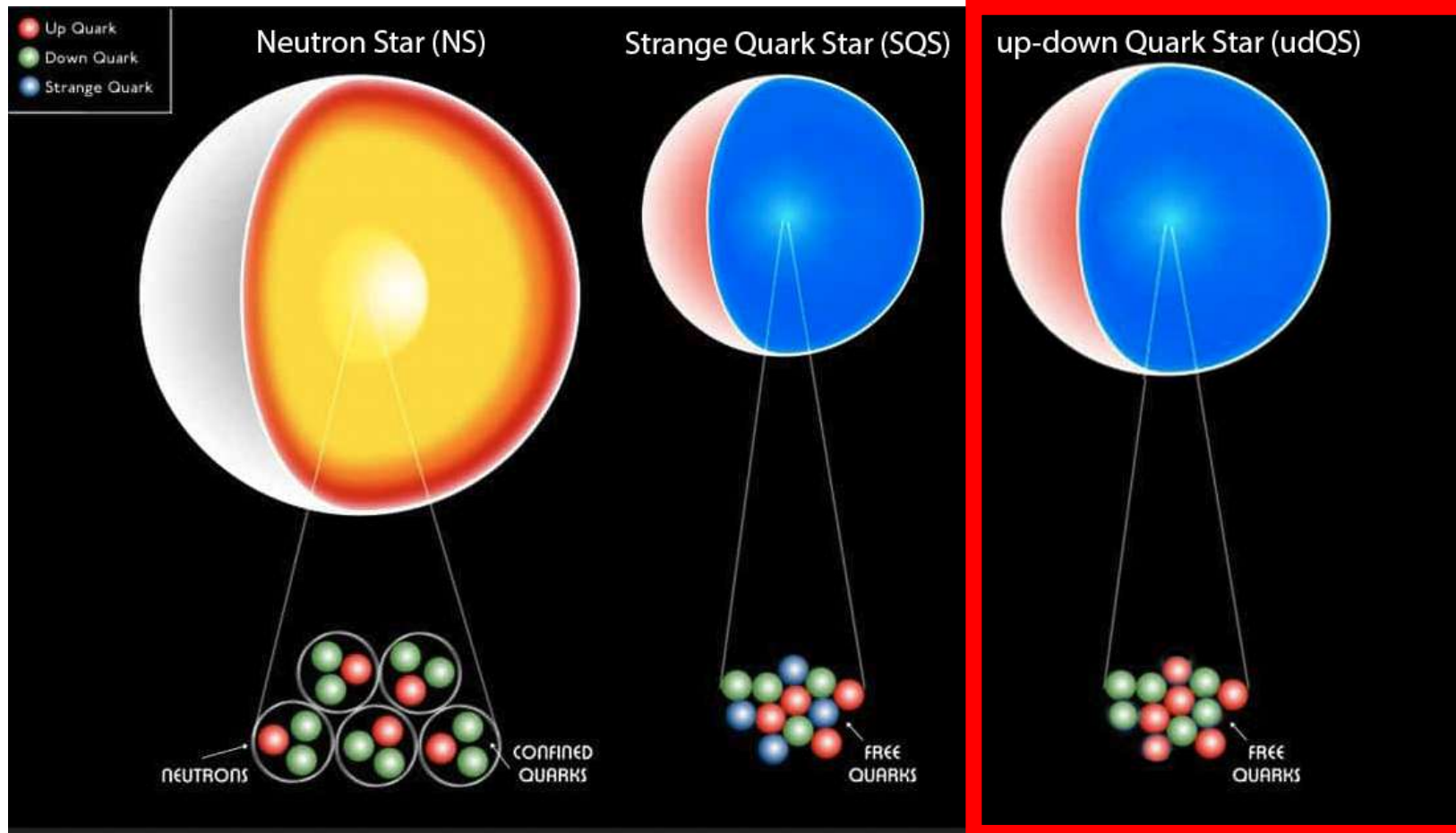
Rethink of "quark matter" also torpedoes notion of Earth-eating particles called

15 MAY 2018 • BY [ADRIAN CHO](#)

Science

penalty than previously thought, so high that cold quark matter should consist of just up and down quarks, the researchers report in a paper in press at *Physical Review Letters*.

Atomic nuclei clearly don't readily convert into up-down quark matter either.



# Up-Down Quark Stars (udQSs)

[C. Z, Phys.Rev.D 101 (2020) , 043003]

[J.Ren, C. Z, Phys.Rev.D 102 (2020) , 083003]

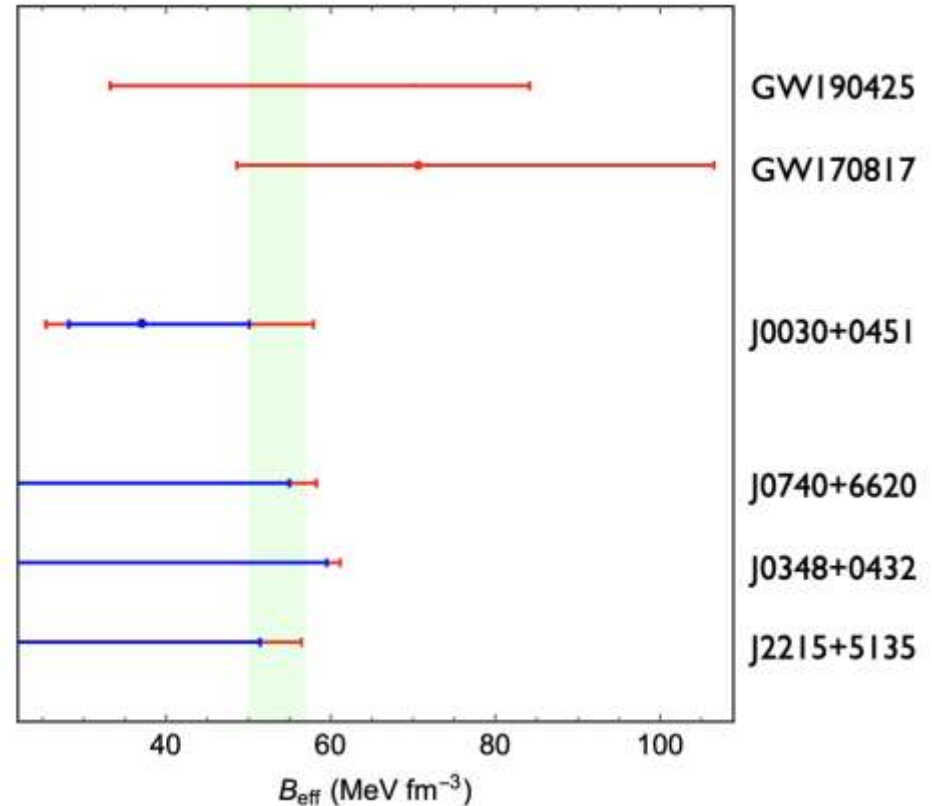
$$P = \frac{1}{3}(\rho - \rho_0) = \frac{1}{3}(\rho - 4B_{\text{eff}})$$

$$\frac{E}{A} = 3\sqrt{2\pi}(\chi^3 B_{\text{eff}})^{1/4} \in (900, 930) \text{ MeV}$$

**➔**  $50 \text{ MeV fm}^{-3} \lesssim B_{\text{eff}} \lesssim 57 \text{ MeV fm}^{-3}$

**Blue:** 68% confidence level

**Red:** for 90% confidence level



# Part 2

## Inverted Hybrid Stars and their radial & non-radial oscillations

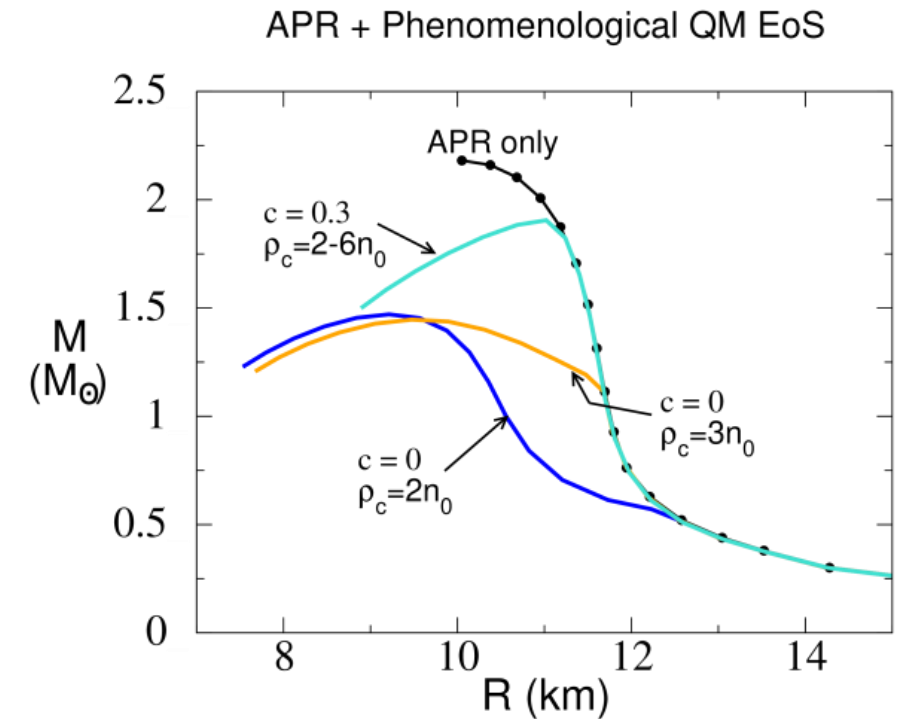
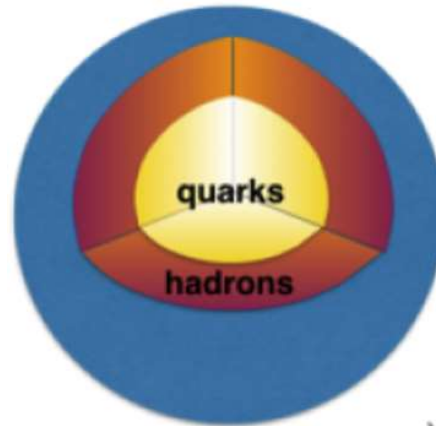
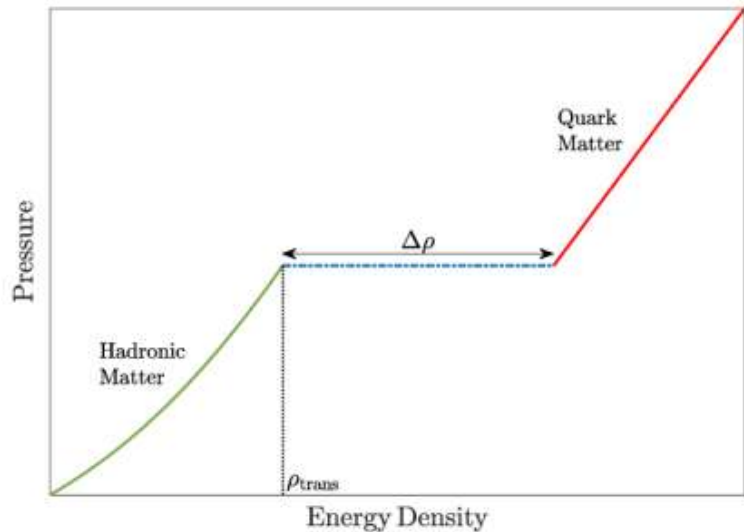
*Based on*

- **C. Z.**, Jing Ren *Phys.Rev.D 108 (2023) 6, 063012*
- **C. Z.**, Yudong Luo, Hongbo Li, Lijing Shao, Renxin Xu, *Phys.Rev.D 109 (2024) 6, 063020*



# (Conventional) Hybrid Star

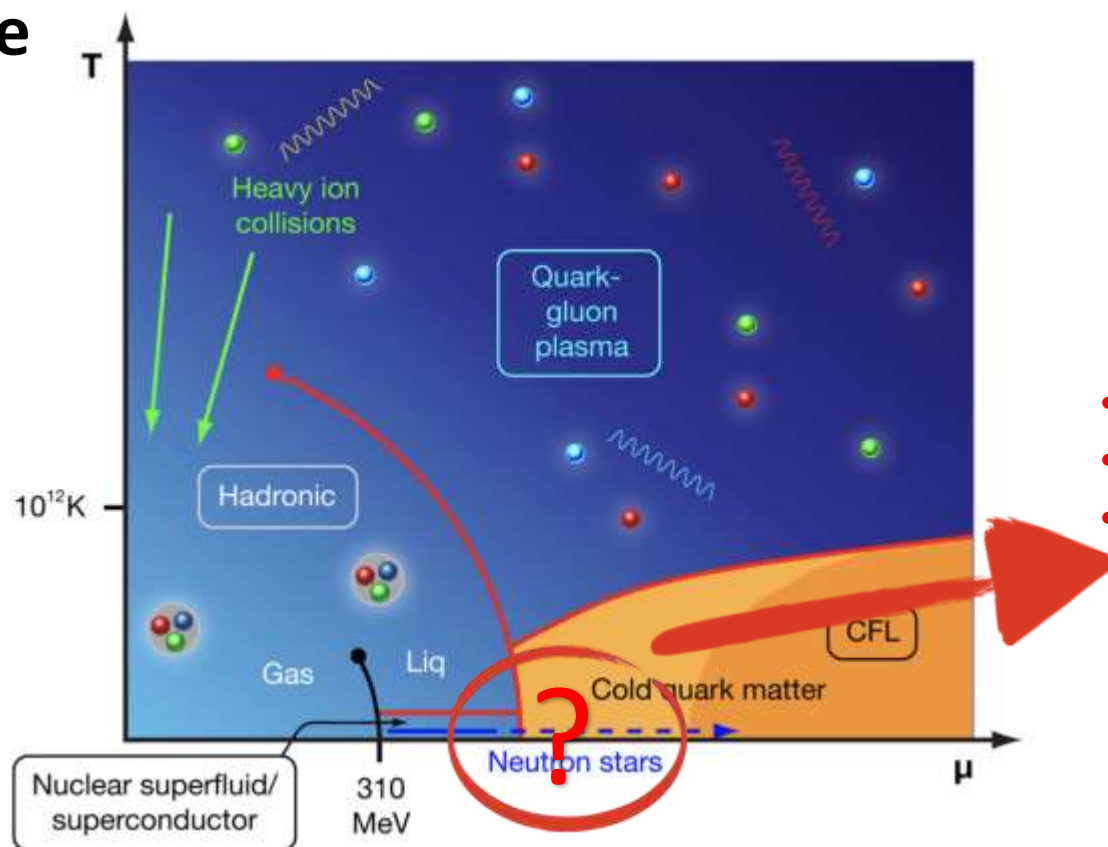
Basic picture: at high pressure, hadrons deconfine into quark matter



Graph adapted from [arXiv:1612.09485 \[nucl-th\]](https://arxiv.org/abs/1612.09485), [arXiv:nucl-th/0411016](https://arxiv.org/abs/nucl-th/0411016)

# However

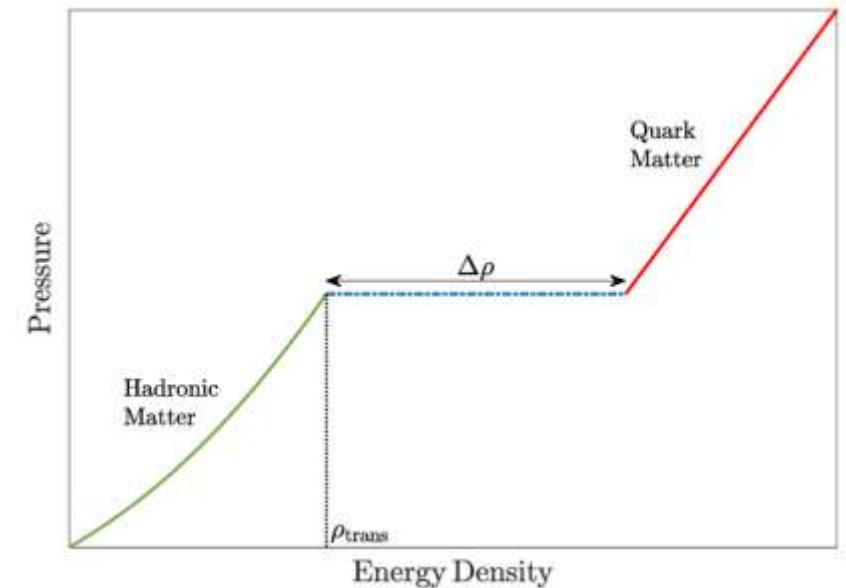
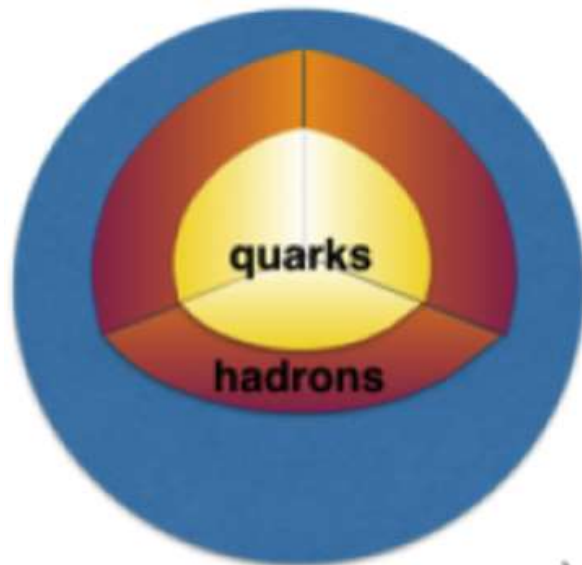
- The picture of hybrid star aforementioned **assumed that quark matter only become absolutely stable over hadronic matter at high pressure**
- **But in context of stable QM hypothesis, QM is more stable than HM at low pressure**



- Non-perturbative QCD
- Inaccessible by Lattice and earth experiment
- With stable QM hypothesis, **cannot rule out the possibility of QM to HM transition**

# Inverted Hybrid Stars

C. Z, J. Ren *Phys.Rev.D* 108 (2023) 6, 063012



## Motivation:

- At low pressure (crust), SQM/udQM hypothesis allows quark matter to be absolutely stable
- At high pressure (core), it's possible to have **<quark matter to hadronic matter transition>** associated **with the chemical potential crossing.**
- For brevity, we named it **Cross stars (CrSs).**

# Model inverted hybrid stars

## QM sector

- Interacting Quark matter (IQM)

*C. Z., R.B. Mann, Phys.Rev.D 103 (2021) 6, 063018*

$$\Omega = -\frac{\xi_4}{4\pi^2}\mu^4 + \frac{\xi_4(1-a_4)}{4\pi^2}\mu^4 - \frac{\xi_{2a}\Delta^2 - \xi_{2b}m_s^2}{\pi^2}\mu^2 - \frac{\mu_e^4}{12\pi^2} + B_{\text{eff}}$$

$$(\xi_4, \xi_{2a}, \xi_{2b}) = \begin{cases} ((\frac{1}{3})^{\frac{4}{3}} + (\frac{2}{3})^{\frac{4}{3}})^{-3}, 1, 0) & \text{2SC phase} \\ (3, 1, 3/4) & \text{2SC+s phase} \\ (3, 3, 3/4) & \text{CFL phase} \end{cases}$$

$$\lambda = \frac{\xi_{2a}\Delta^2 - \xi_{2b}m_s^2}{\sqrt{\xi_4 a_4}}$$



$$p = \frac{1}{3}(\rho - 4B_{\text{eff}}) + \frac{4\lambda^2}{9\pi^2} \left( -1 + \sqrt{1 + 3\pi^2 \frac{(\rho - B_{\text{eff}})}{\lambda^2}} \right)$$

$$\bar{\lambda} = \frac{\lambda^2}{4B_{\text{eff}}}$$

$$\bar{p} = \frac{1}{3}(\bar{\rho} - 1) + \frac{4}{9\pi^2} \bar{\lambda} \left( -1 + \text{sgn}(\lambda) \sqrt{1 + \frac{3\pi^2}{\bar{\lambda}} (\bar{\rho} - \frac{1}{4})} \right)$$

$$\mu_{\text{QM}} = \frac{3\sqrt{2}}{(a_4 \xi_4)^{1/4}} \sqrt{[(P + B)\pi^2 + \lambda^2]^{1/2} - \lambda}$$

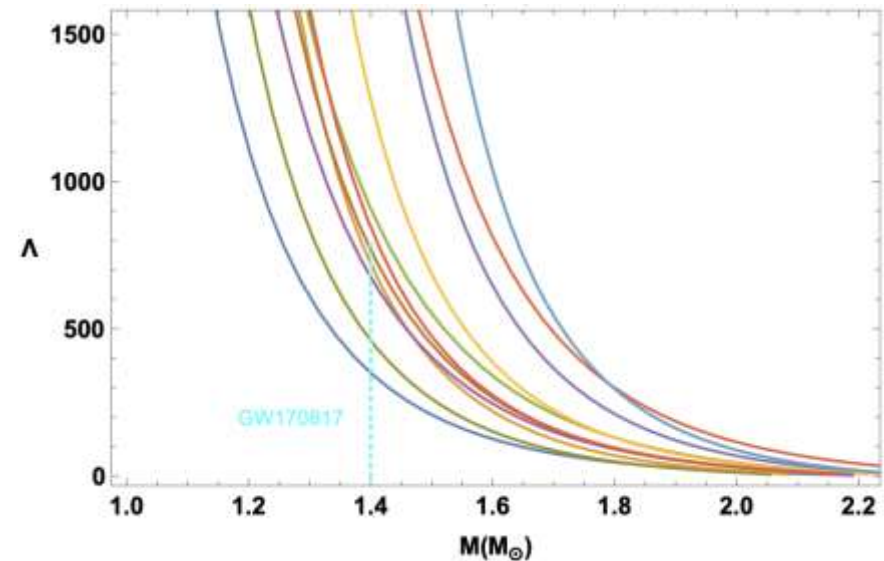
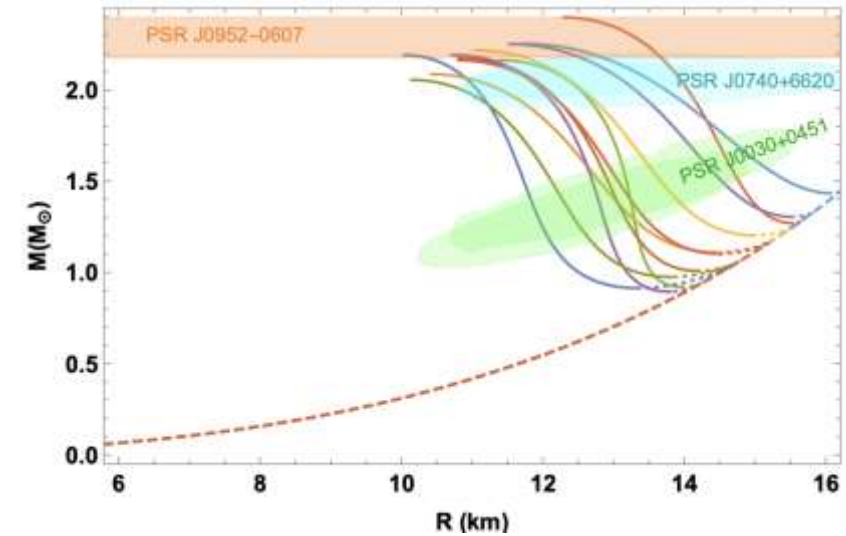
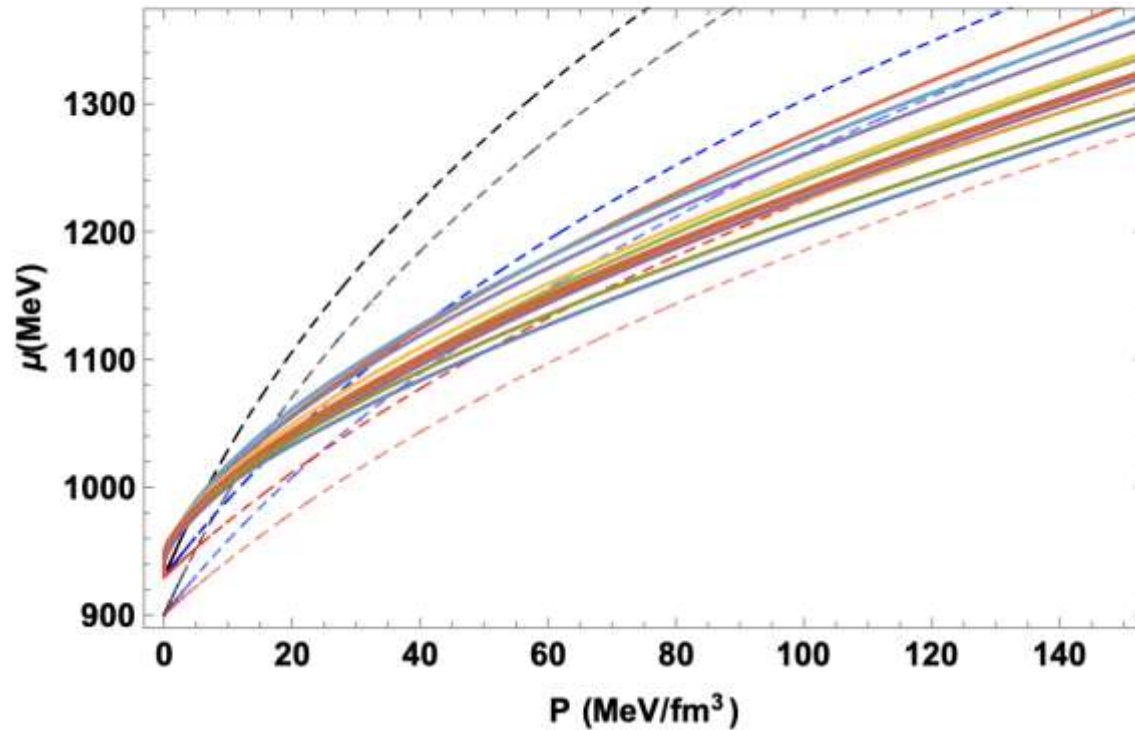
$$\left(\frac{E}{A}\right)_{\text{QM}} = \frac{3\sqrt{2}\pi}{(\xi_4 a_4)^{1/4}} \frac{B^{1/4}}{\sqrt{(\lambda^2/B + \pi^2)^{1/2} + \lambda/\sqrt{B}}}$$

For simplicity, in this work we ignore color-superconductivity, thus  $\lambda = 0$  for *udQM*  $\lambda = -\sqrt{3}m_s^2/(4\sqrt{a_4})$  for *SQM*

# Variations of HM EOSs

- Fix QM EOS, but vary HM EOSs

— APR — BL — DDH $\delta$  — GM1 — Sk13 — Sk14  
— Sk15 — SKa — SKb — SLy4 — SLy9

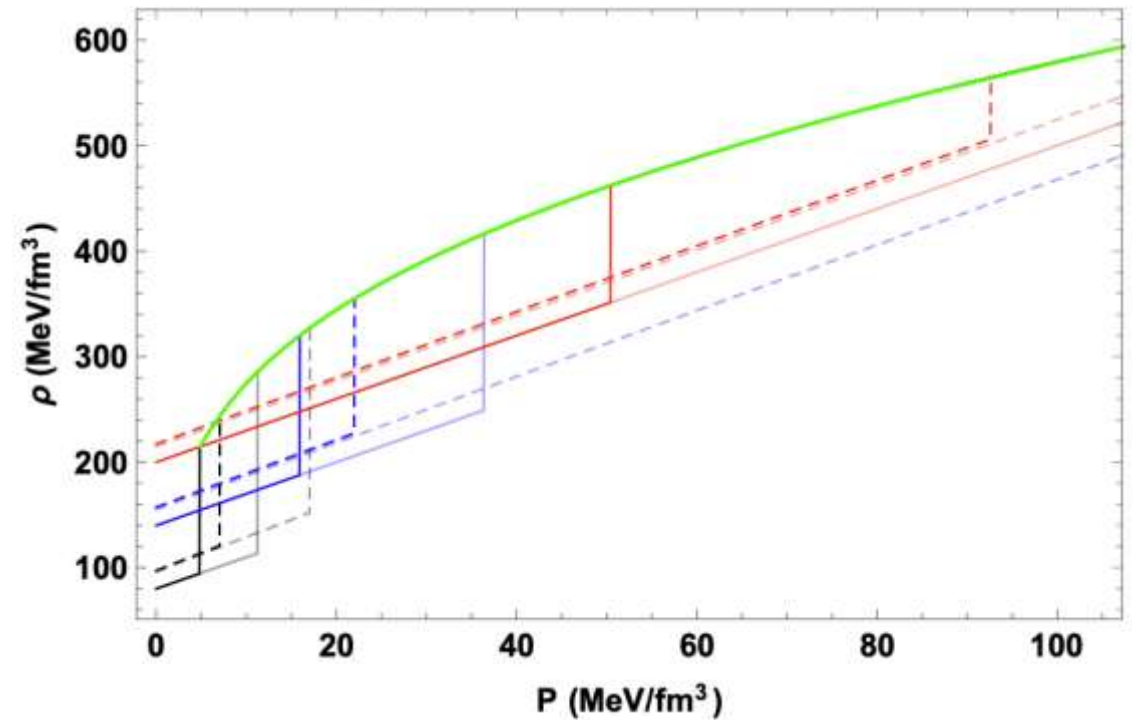
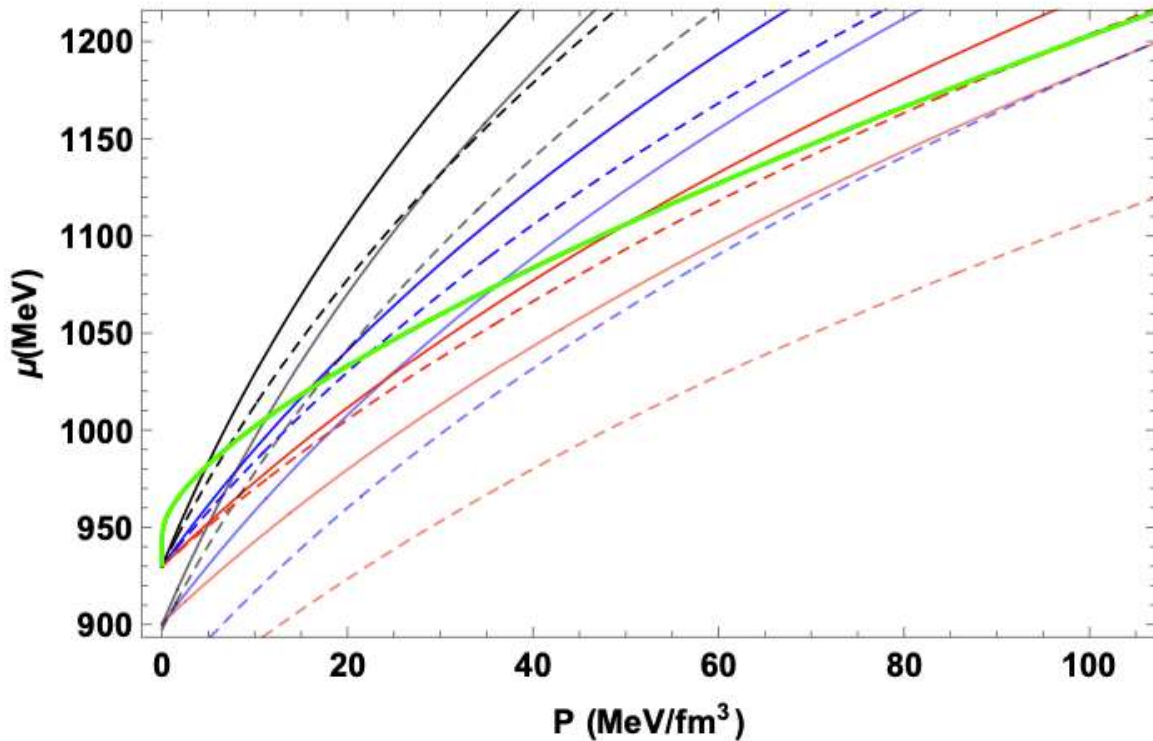




# Variations of QM EOSs

Next, we fix HM EOS (APR), and vary the QM EOSs

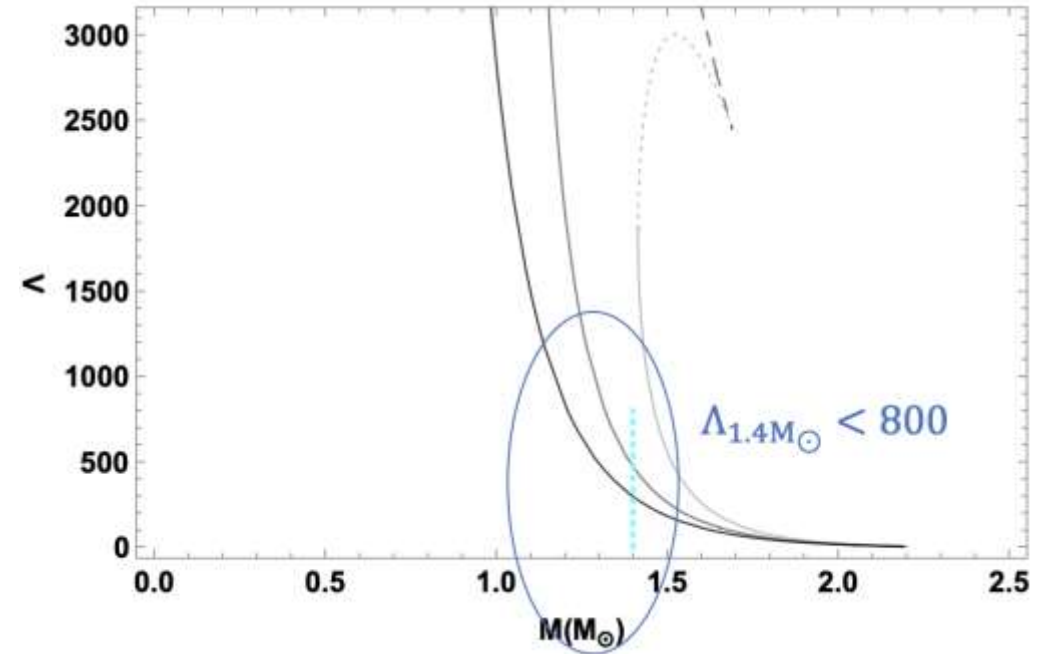
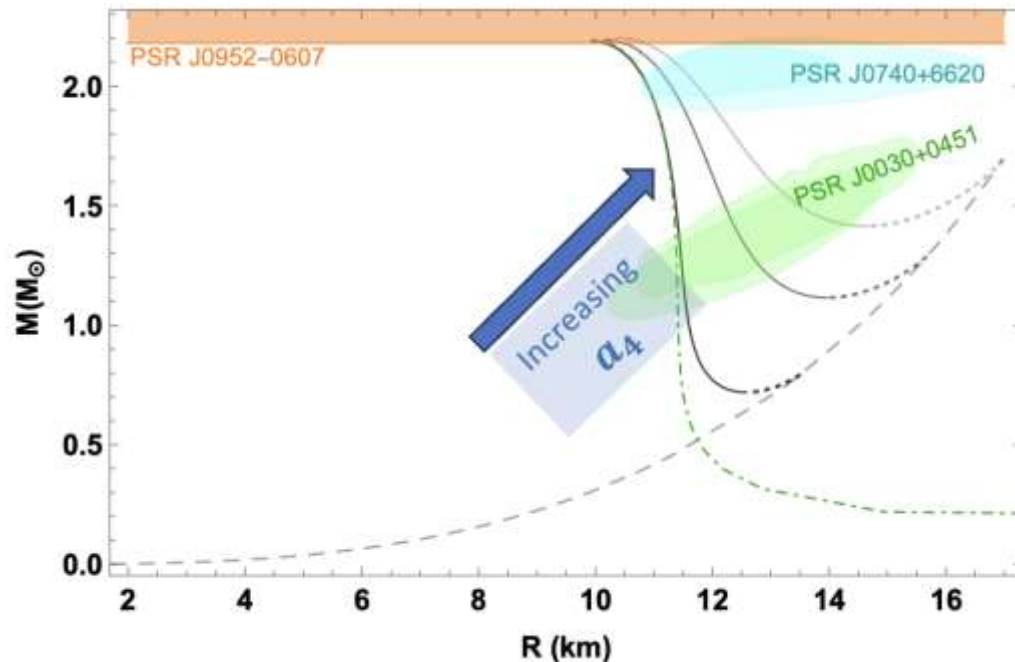
- Solid lines are for udQM
- Dashed line are for SQM
- darker= $a_{4,min}$ , lighter= $a_{4,max}$



# Complementarity of CrSs an illustrative example

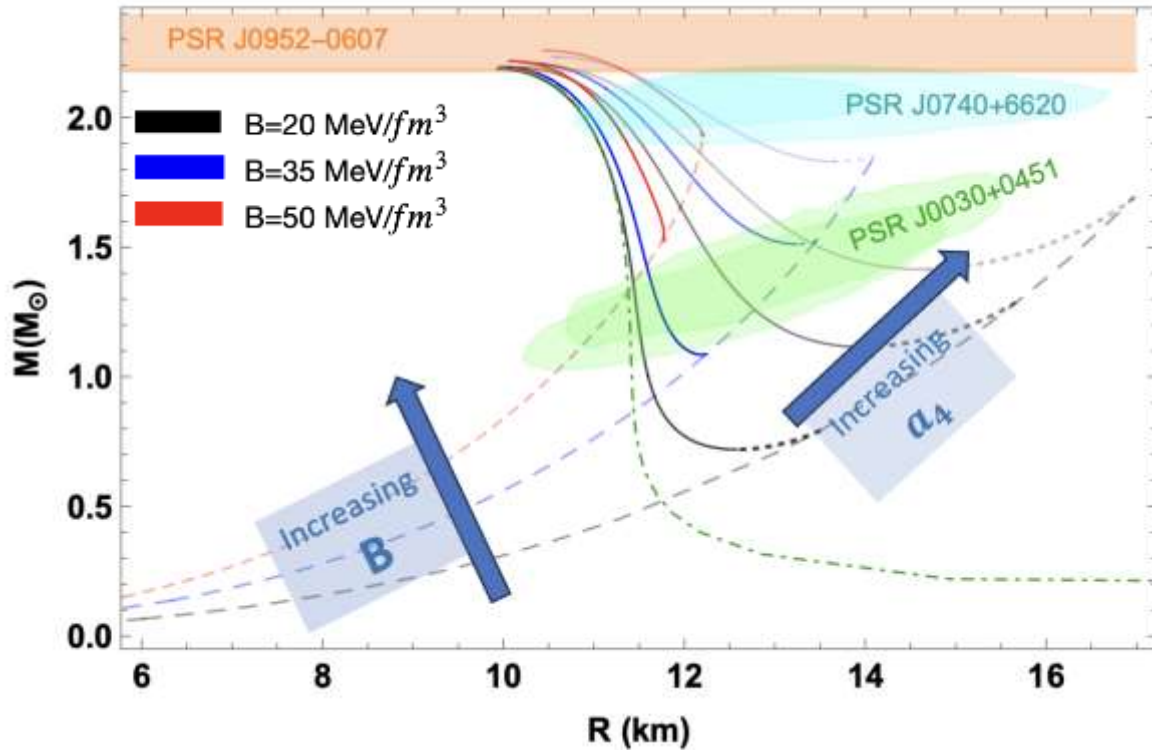


- NS with APR cannot well meet NICER PSR J0740+6620; udQS with  $B=20 \text{ MeV/fm}^3$  cannot meet GW170817. But inverted hybrid stars with (APR & udQM  $B=20 \text{ MeV/fm}^3$ ) can well meet both constraints

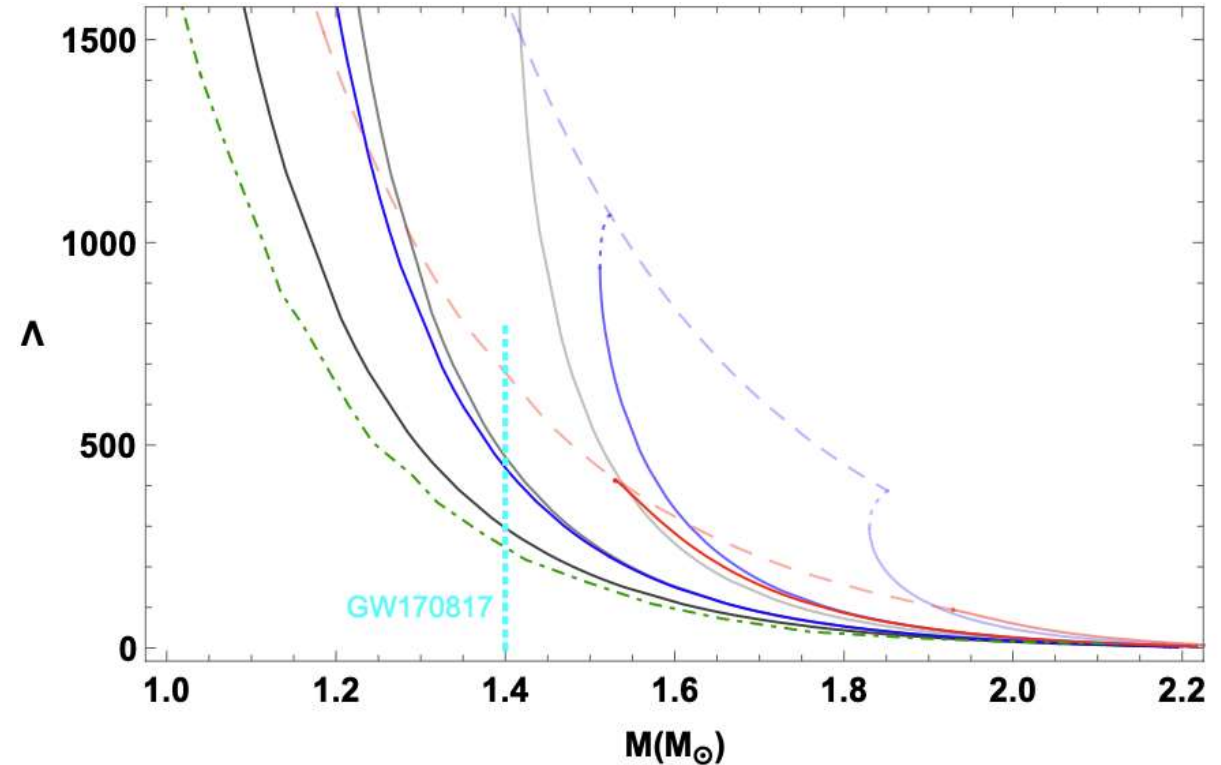


the interplay between the HM and QM compositions helps to reconcile astrophysical constraints at low and high masses

# CrSs with $ud$ QM, more examples



$B = 20$  (black),  $35$  (blue),  $50$  (red)  $\text{MeV}/\text{fm}^3$



$a_4 = a_{4,\text{min}}, (a_{4,\text{min}} + a_{4,\text{max}})/2, a_{4,\text{max}}$

CrSs with SQM are similar

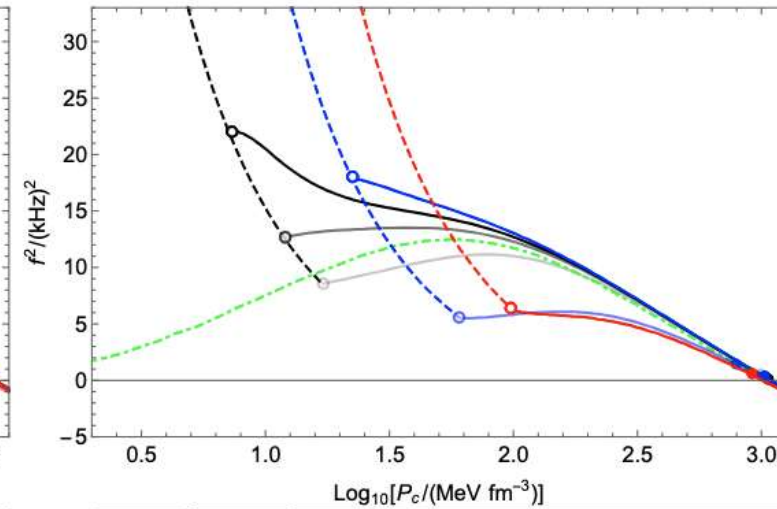
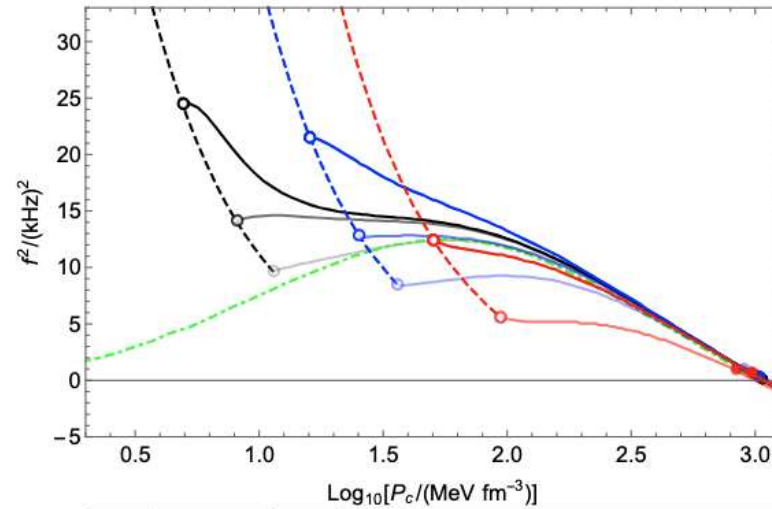
# Radial and Non-Radial Oscillations of of Inverted Hybrid Stars

**C. Z.**, *Yudong Luo, Hongbo Li, Lijing Shao, Renxin Xu,*  
*Phys.Rev.D 109 (2024) 6, 063020*

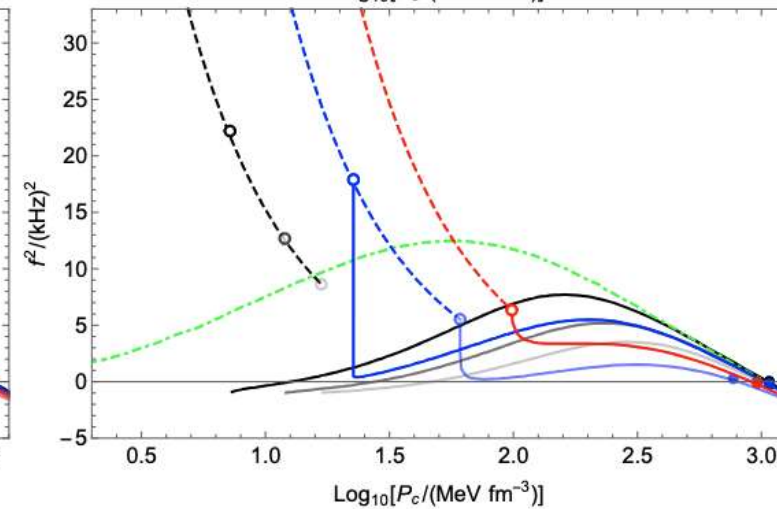
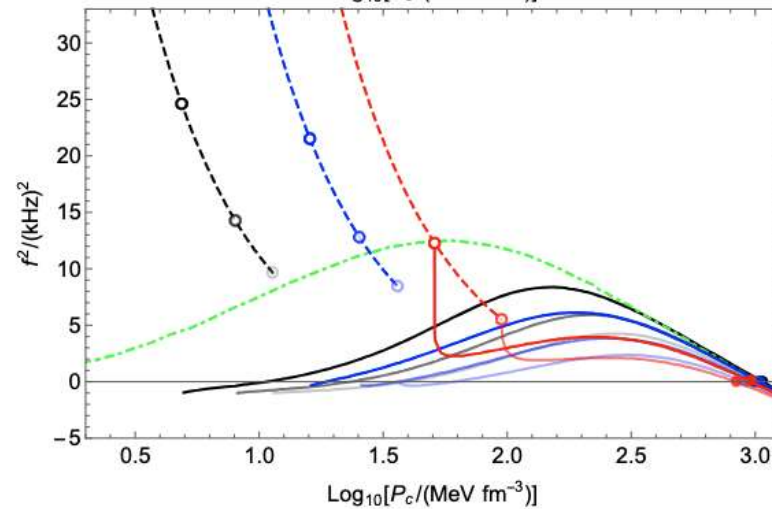
# Fundamental mode of Radial Oscillations



Slow conversion:



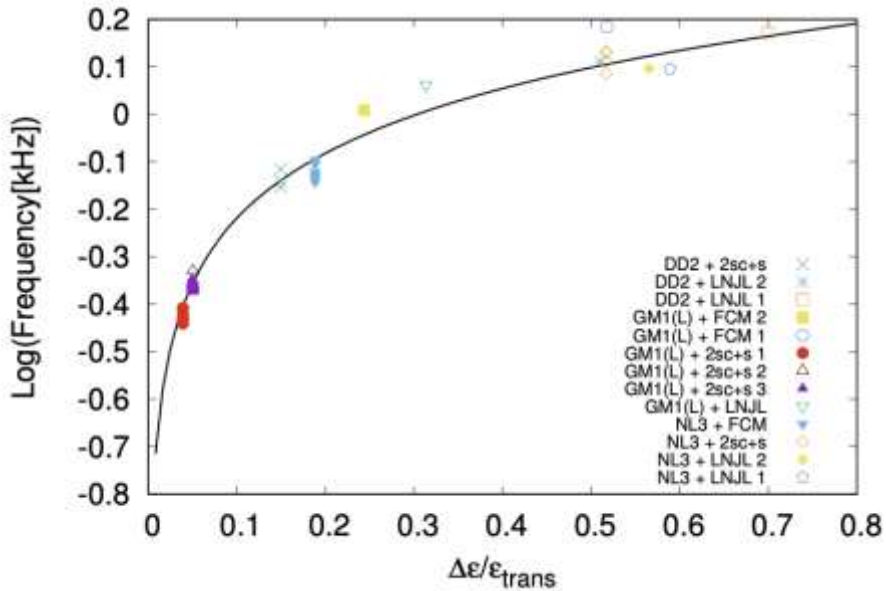
Rapid conversion:



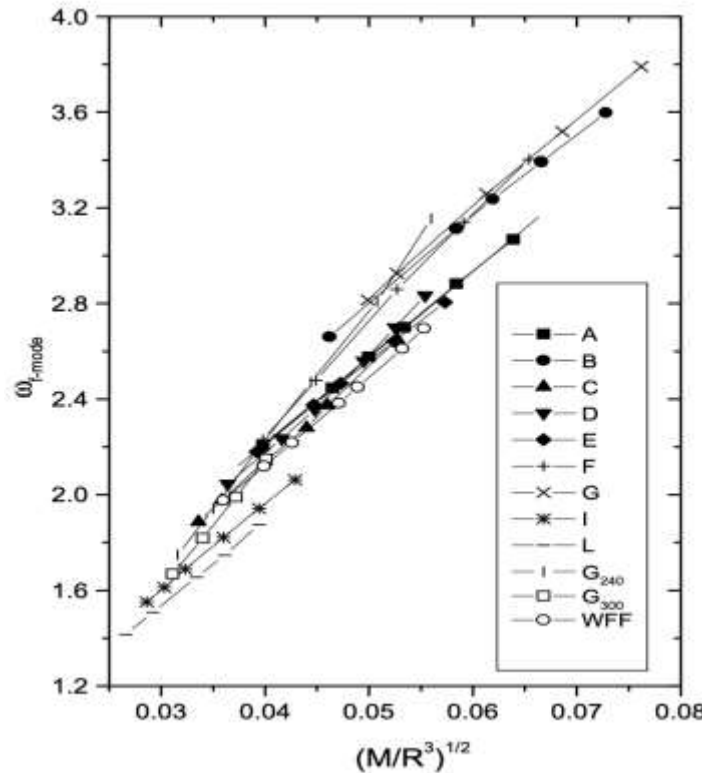


# Non-Radial Oscillations: Some General Correlations(NSs and Hybrid stars)

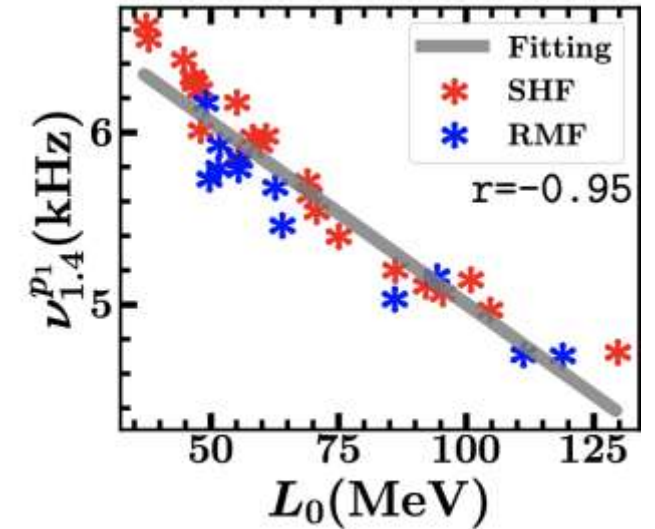
For physics origin of different non-radial modes, see Dr. Miao's yesterday talk



Ranea-Sandoval, I.F., Guilera, O.M., Mariani, M. and Orsaria, M.G., 2018  
*Journal of Cosmology and Astroparticle Physics*, 2018(12), p.031.

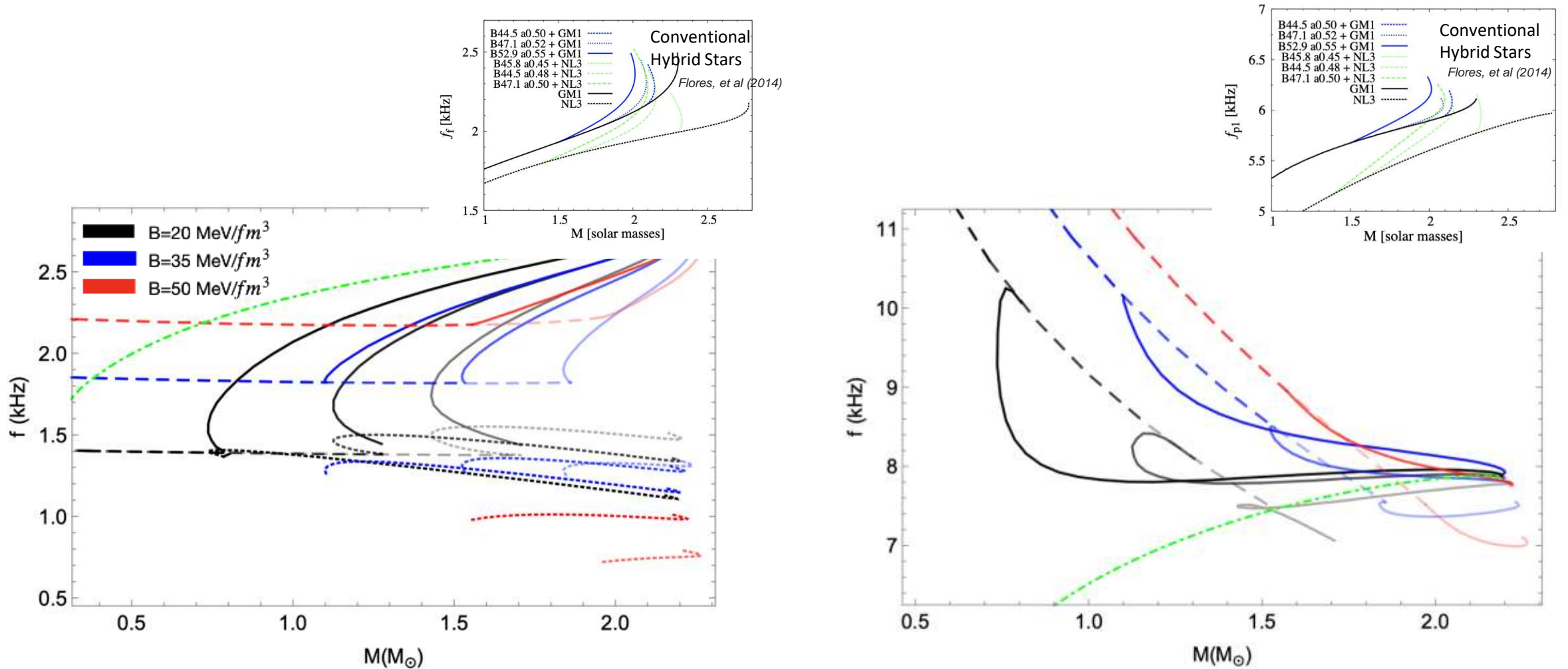


Andersson, N. and Kokkotas, K.D., 1998.

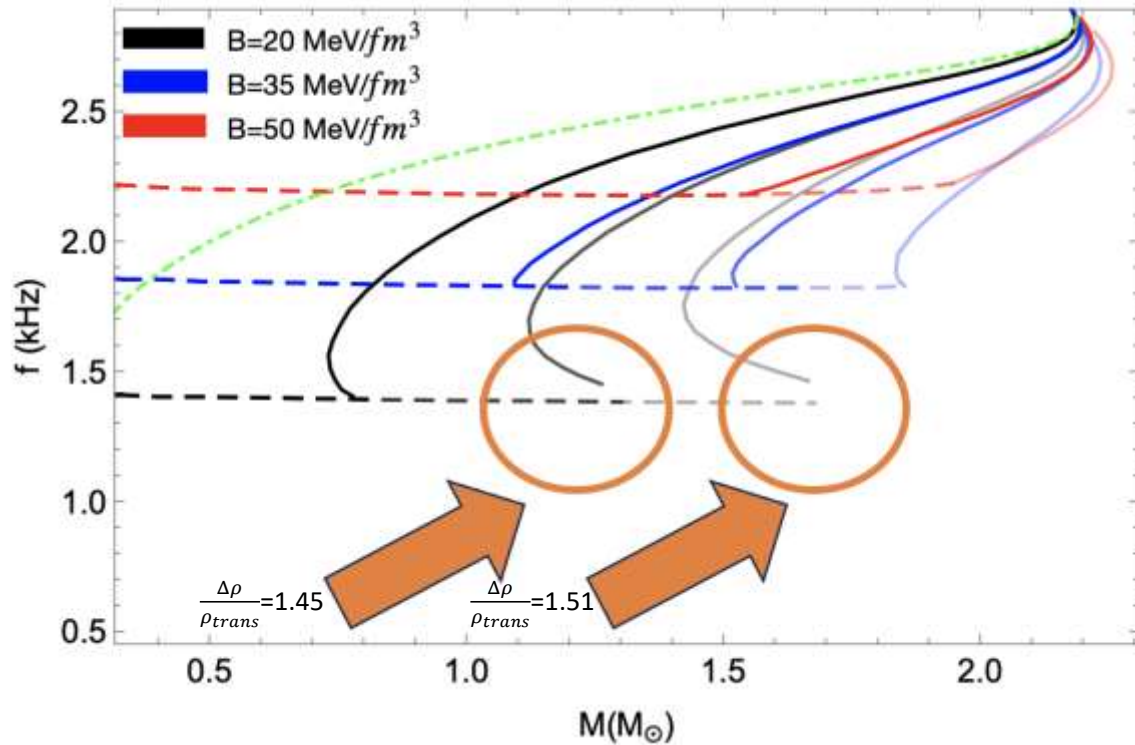


Kunjipurayil, A., Zhao, T., Kumar, B., Agrawal, B.K. and Prakash, M., 2022.. *Physical Review D*, 106(6), p.063005.

# $g, f, p_1$ modes of Non-Radial Oscillations (Cowling Approximation)



# bizarre jump in $f$ mode



- This bizarre jump appear **even for conventional hybrid stars** With large  $\Delta\rho/\rho_{trans}$  (e.g.  $>1.3$ ), but been overlooked by others due to their choice of  $\Delta\rho/\rho_{trans}<1$  for  $M_{TOV} > 2M_\odot$  consideration.
- Confirmed in Full GR by Dr. Tianqi Zhao

# Part 2

## Hybrid **Strangeon** Stars

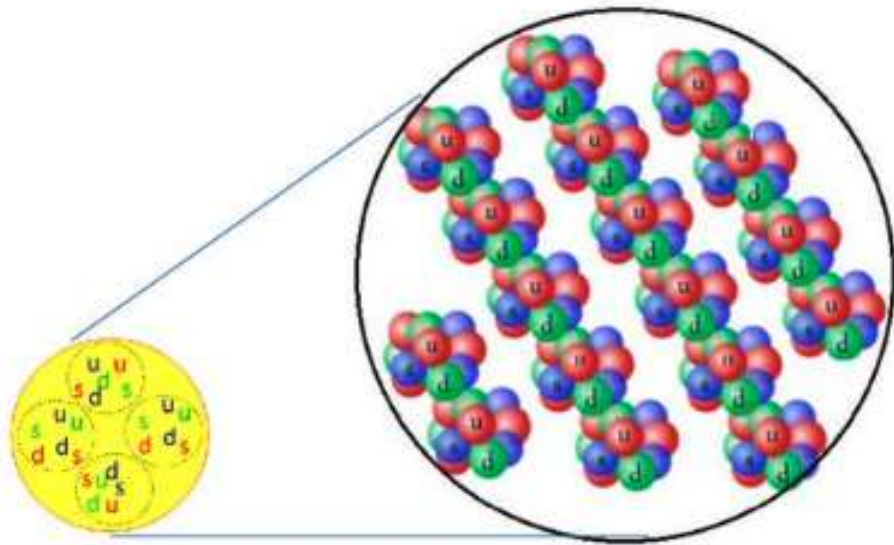
*Based on*

- **C. Z.**, Yong Gao , Cheng-jun Xia, Renxin Xu, Phys.Rev.D 108 (2023) , 12

# Strangeon (Strange-Cluster) matter

Renxin Xu 2003 ApJ 596 L59

See Prof. X.Y Lai's talk



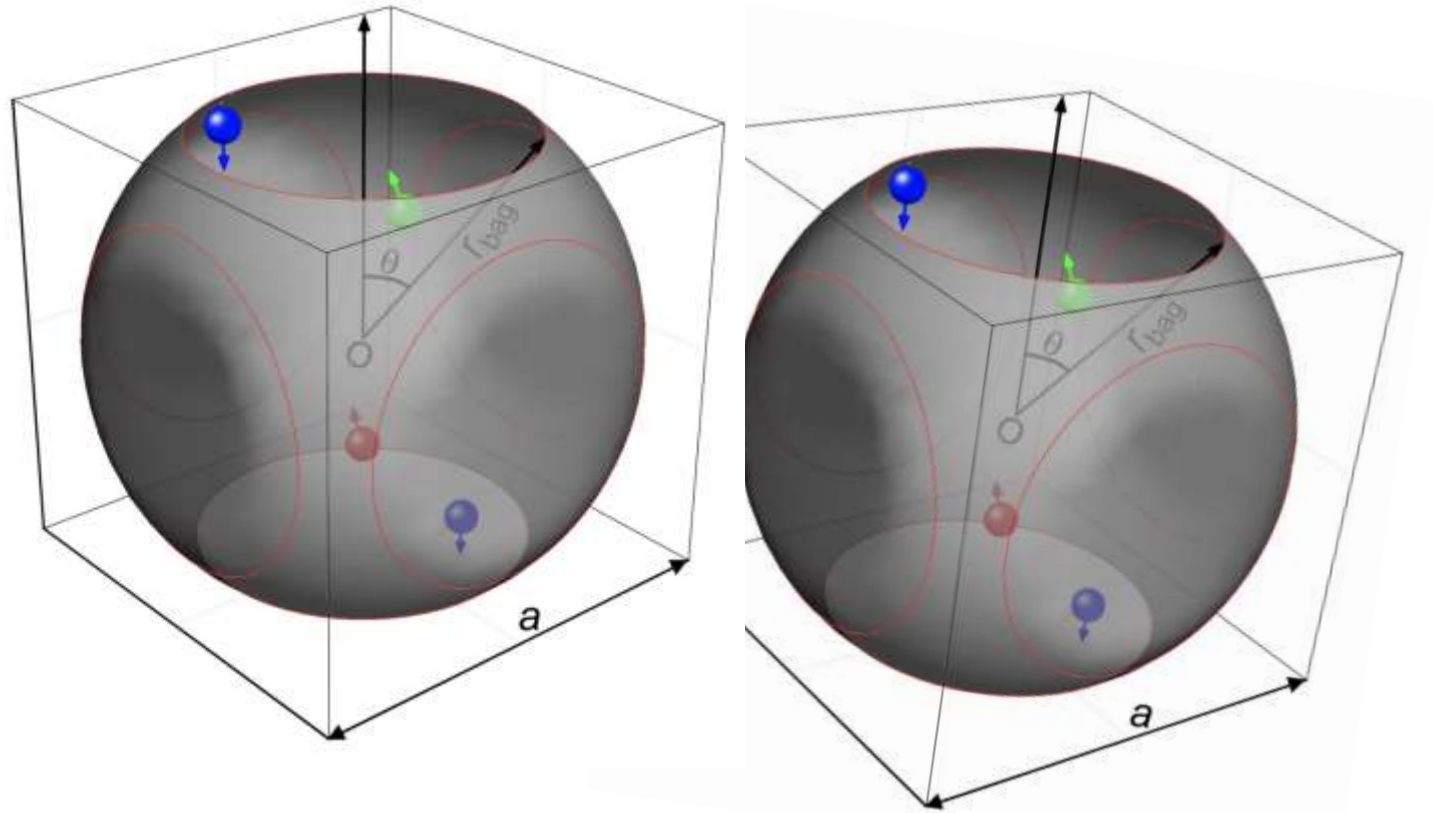
**Strangeon**  
(strange+nucleon)

Strangeon matter could be in a classical solid state— because of (1) a small quantum wave packet  $\lambda_c \simeq \hbar/(mc)$  since the mass of a strangeon is much higher than that of a nucleon, and (2) a significant interaction energy between strangeons ( $\sim$  a few tens of MeVs) which is much higher than their temperature which is typically  $\lesssim 1$  keV.

[Renxin Xu, Special Issue of QCS2023]



# Strangeons to SQM transition?



[Z. Q. Miao, C. J. Xia, X. Y. Lai, T. Maruyama, R. X. Xu and E. P. Zhou, Int. J. Mod. Phys. E 31, no.04, 2250037 (2022)]

A increasing pressure reduces strangeon lattice spacing

→ the lattice constant becomes smaller than the radius of individual quark bags

→ Strangeons to SQM transition

# EOS For Strangeons

[X. Y. Lai , R. X. Xu, MNRAS, 398, 1, 2009]

See Prof. X.Y Lai's talk

- Leonard-Jones Model

$$\rho = 2\epsilon (A_{12}\sigma^{12}n^5 - A_6\sigma^6n^3) + nN_qm_q,$$

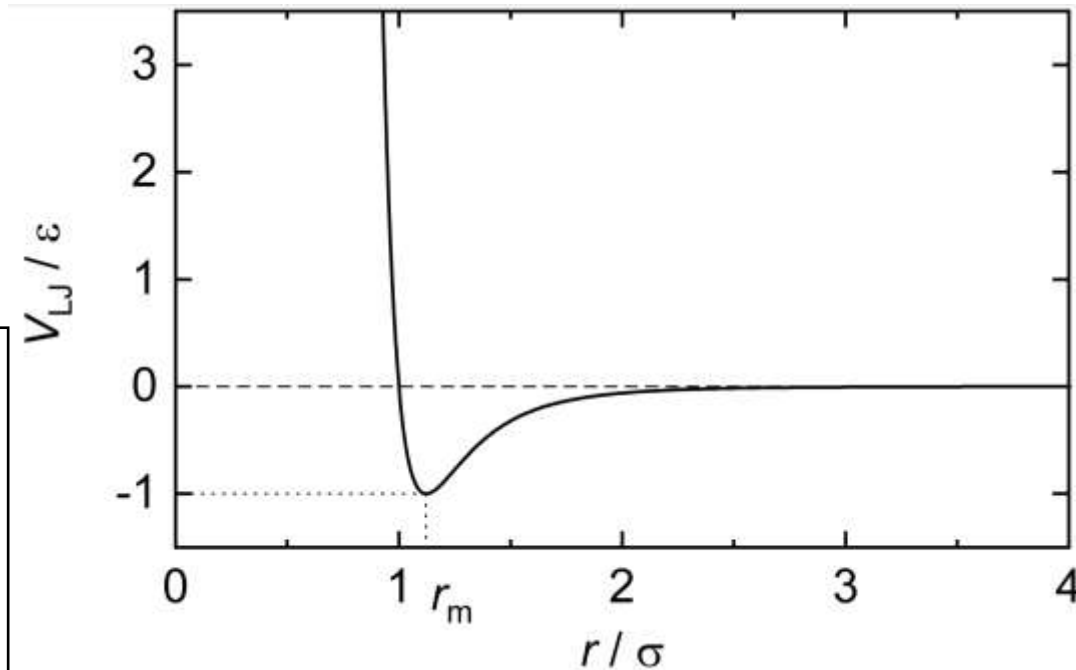
$$p = n^2 \frac{d(\rho/n)}{dn} = 4\epsilon (2A_{12}\sigma^{12}n^5 - A_6\sigma^6n^3)$$

$$n_s = \left(\frac{A_6}{2A_{12}}\right)^{1/2} \frac{N_q}{3\sigma^3} \quad a = A_6^2/A_{12} \approx 11.38$$

$$\rho = \frac{a}{9} \epsilon \left( \frac{N_q^4}{18n_s^4} n^5 - \frac{N_q^2}{n_s^2} n^3 \right) + m_q N_q n$$

$$p = \frac{2a}{9} \epsilon \left( \frac{N_q^4}{9n_s^4} n^5 - \frac{N_q^2}{n_s^2} n^3 \right).$$

$$V_{LJ} = 4\epsilon \left[ \left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^6 \right]$$



# Model hybrid strangeon stars: Strangeon sector

- EOS with  $N_q$  absorbed:

$$\begin{aligned}\frac{\rho}{n_s} &= \frac{a}{9}\tilde{\epsilon}\left(\frac{1}{18}\bar{n}^5 - \bar{n}^3\right) + m_q\bar{n}, \\ \frac{P}{n_s} &= \frac{2a}{9}\tilde{\epsilon}\left(\frac{1}{9}\bar{n}^5 - \bar{n}^3\right),\end{aligned}\tag{10}$$

where  $a = A_6^2/A_{12} = 8.4^2/6.2 \approx 11.38$ ,  $\tilde{\epsilon} = \epsilon/N_q$  and  $\bar{n} = N_q n/n_s$ . Note that  $\bar{n} = 3$  at star surface where  $P = 0$ .

$$\Rightarrow \mu_{\text{strangeon}} = \frac{3\mu}{N_q} = 3\frac{\rho/n_s + P/n_s}{\bar{n}} = 3m_q + a\tilde{\epsilon}\left(\frac{5}{54}\bar{n}^4 - \bar{n}^2\right).$$

$$\Rightarrow \left(\frac{E}{A}\right)_{\text{strangeon}} = 3m_q - \frac{3a}{2}\tilde{\epsilon}$$

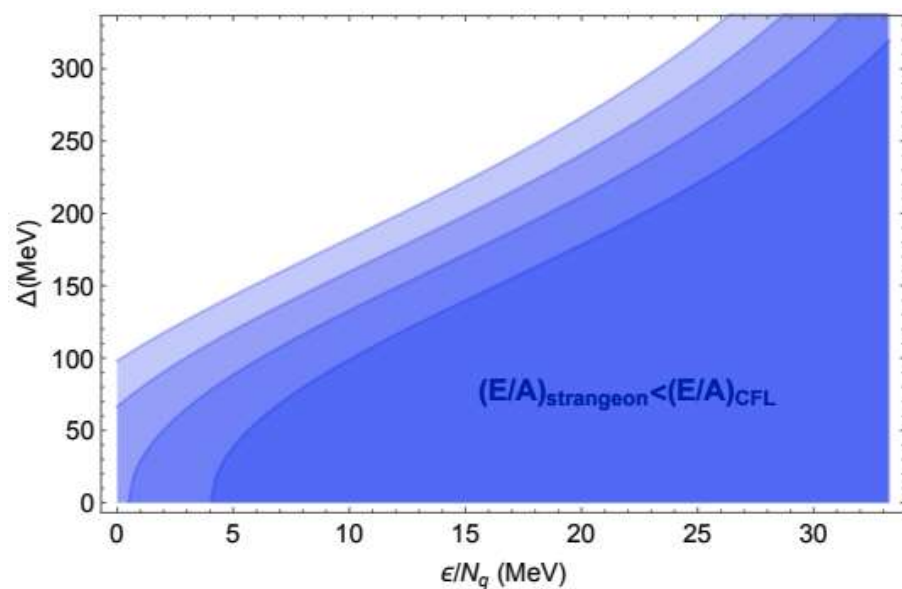
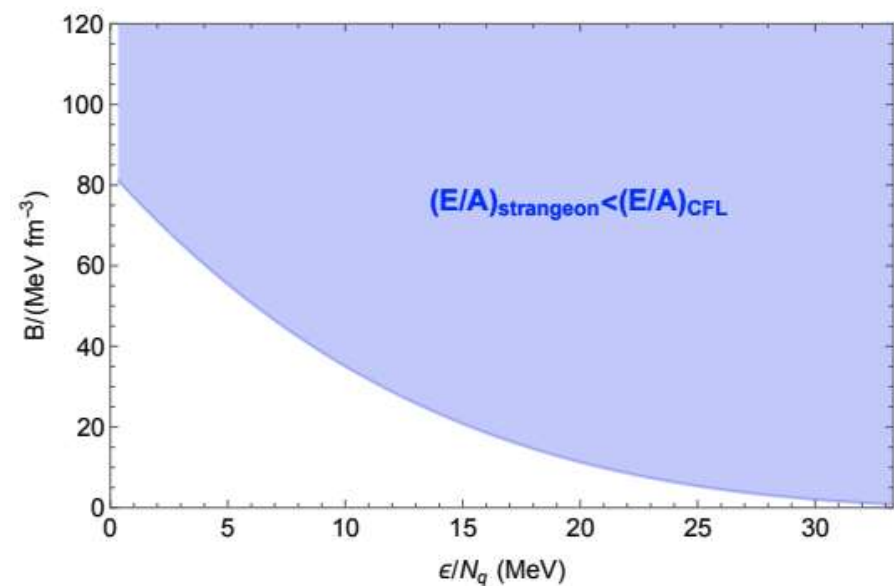
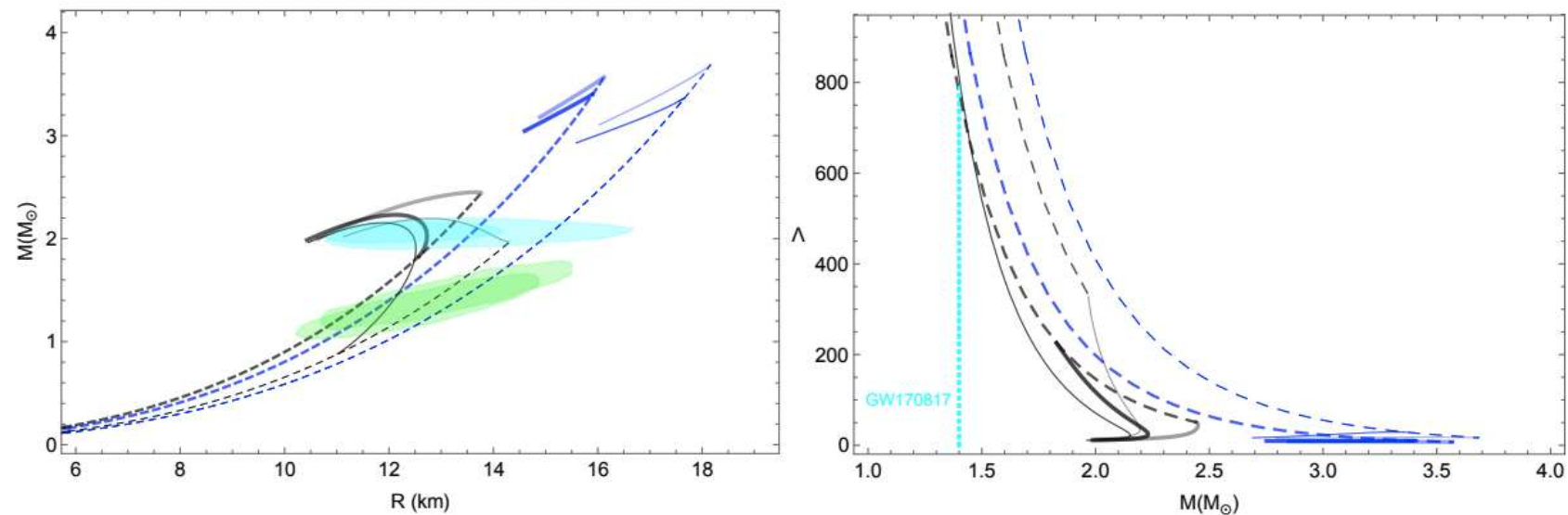


Figure 1: Allowed parameter space (blue-shaded) for the existence of hybrid strangeon stars from stability consideration  $(E/A)_{\text{strangeon}} < (E/A)_{\text{CFL}}$ . Top: CFL bag constant  $B$  and bottom: CFL superconductivity gap  $\Delta$  versus parameter  $\epsilon/N_q$  of strangeon matter. For the bottom sub-figure, the shaded region with lighter-colored contour lines represents larger bag constant, sampling  $B = 60, 80, 100, 120 \text{ MeV}/\text{fm}^3$  (bottom to top).

$B=60 \text{ MeV/fm}^3$



$B=80 \text{ MeV/fm}^3$

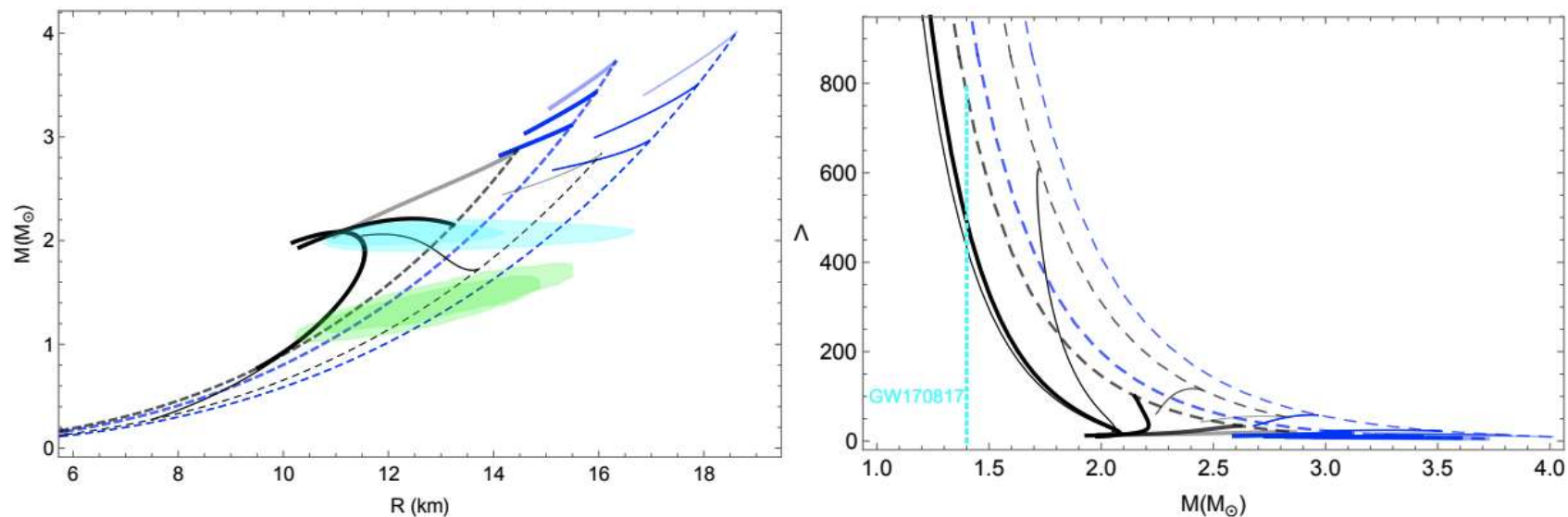
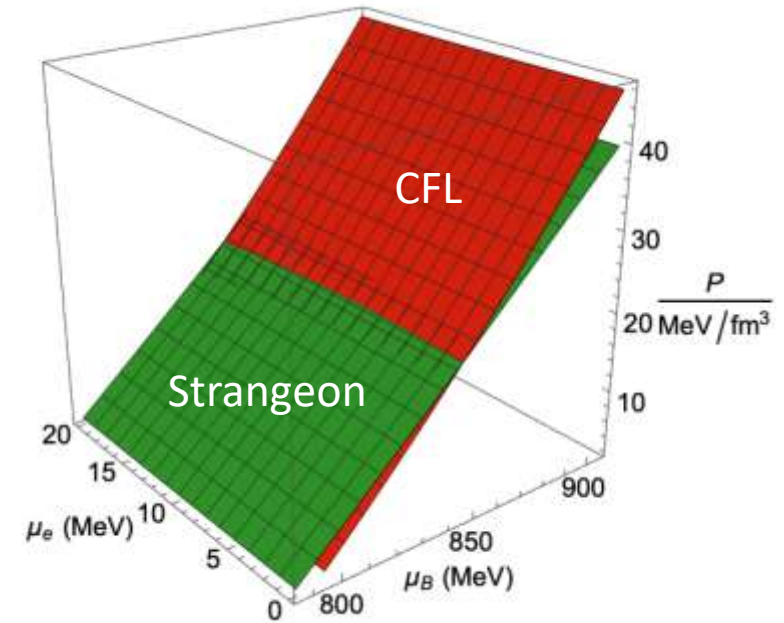
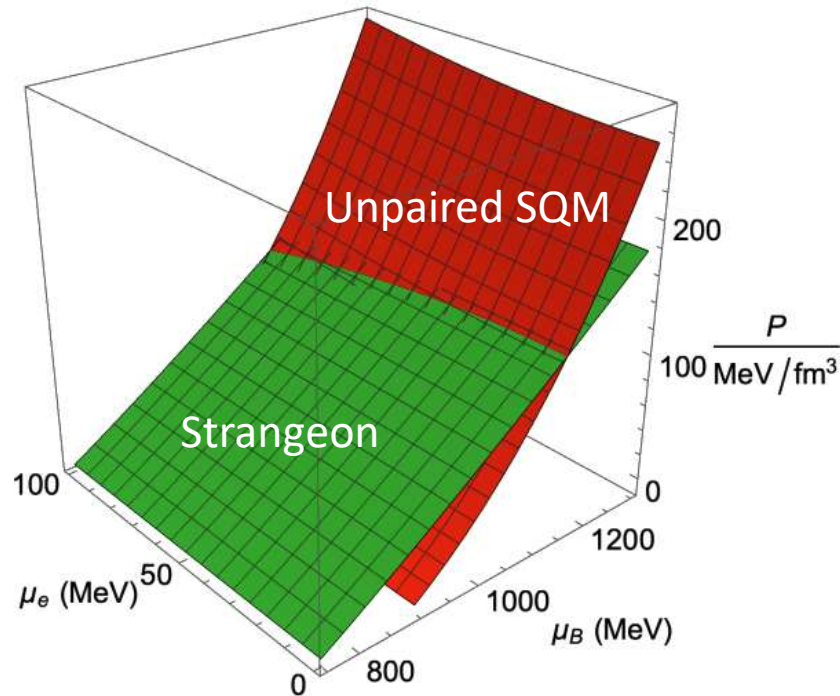


Figure 2: The curves  $M-R$  (left) and  $\Lambda-M$  (right) of hybrid strangeon stars (solid lines) with  $\epsilon/N_q = 80/9 \approx 8.9$  (black),  $120/9 \approx 13.3$  (blue) MeV,  $n_s = 0.22$  (thin),  $0.30$  (thick)  $\text{fm}^{-3}$  for the strangeon composition, and  $B = 60$  (top),  $80$  (bottom)  $\text{MeV/fm}^3$  for the CFL composition. Lines with darker colors denote larger  $\Delta$ , sampling 60, 80 MeV for top panels and 60, 100, 120 MeV for bottom panels, respectively.



# Mixed Phase?



$\Delta\mu_B \sim$  only 1 MeV  
 $\Rightarrow$  Mixed phase is not preferred

# Summary

- Up-down quark matter can be the ground state of matter. They can form up-down quark stars that can naturally have large  $M_{\text{TOV}}$ .
- Both theory and experiment can not exclude the possibility of **inverted hybrid stars**. We have shown their consistency with recent observations. They have distinct p-mode behaviour compared to conventional hybrid stars.
- Strangeon matter can transit to SQM at high pressure, form **Hybrid strangeon stars**. They are consistent with recent observations.
- Open questions: search for udQM? microscopic picture of inverse phase transition? radiation signals? Seismology of hybrid strangeon stars?....

**Thank you**

Back Up Slides

# (2018) up-down Quark Matter Hypothesis

*B. Holdom, J. Ren, C. Z, Phys.Rev.Lett. 120 (2018) 22, 222001*

- With the **flavor dependence of the QCD vacuum** accounted, we found that  $u, d$  quark matter ( $udQM$ ) could be more stable than SQM and ordinary nuclear matter at a sufficiently large baryon number  $A_{\min}$
- We find  $A_{\min} > 300$ , which guarantees the stability of ordinary nuclei on earth
- Derived from an extended quark-meson model I devised that fits well with all the mass spectrum and decay widths of light scalar and pseudoscalar mesons.

PHYSICAL REVIEW LETTERS 120, 222001 (2018)

## Quark Matter May Not Be Strange

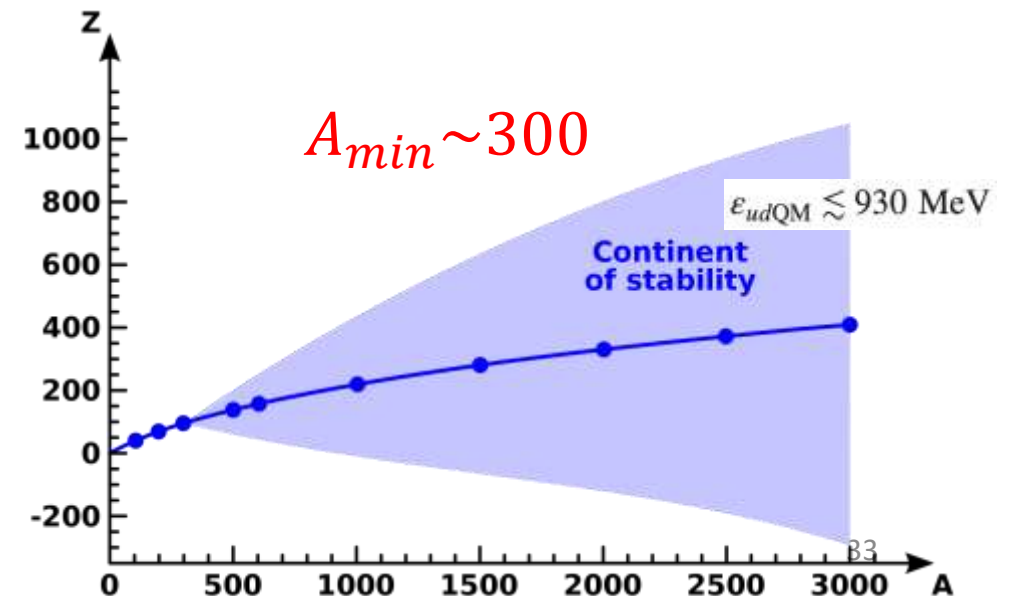
Bob Holdom,<sup>\*</sup> Jing Ren,<sup>†</sup> and Chen Zhang<sup>‡</sup>

*Department of Physics, University of Toronto, Toronto, Ontario M5S1A7, Canada*

(Received 30 July 2017; revised manuscript received 23 October 2017; published 31 May 2018)

If quark matter is energetically favored over nuclear matter at zero temperature and pressure, then it has long been expected to take the form of strange quark matter (SQM), with comparable amounts of  $u$ ,  $d$ , and  $s$  quarks. The possibility of quark matter with only  $u$  and  $d$  quarks ( $udQM$ ) is usually dismissed because of the observed stability of ordinary nuclei. However, we find that  $udQM$  generally has lower bulk energy per baryon than normal nuclei and SQM. This emerges in a phenomenological model that describes the spectra of the lightest pseudoscalar and scalar meson nonets. Taking into account the finite size effects,  $udQM$  can be the ground state of baryonic matter only for baryon number  $A > A_{\min}$  with  $A_{\min} \gtrsim 300$ . This ensures the stability of ordinary nuclei and points to a new form of stable matter just beyond the periodic table.

DOI: 10.1103/PhysRevLett.120.222001





# The Quark-Meson Model

- QCD has approximate chiral symmetry  $SU(3)_L \times SU(3)_R$

$$\mathcal{L}_Q = \bar{\Psi} (i\not{\partial} - g_\Phi \Phi) \Psi + \text{Tr} (\partial_\mu \Phi^\dagger \partial^\mu \Phi) - V(\Phi)$$

$$V(\Phi) = V_{\text{inv}}(\Phi) + V_b(\Phi),$$

$$\Phi = T_a \Phi_a = T_a (\sigma_a + i\pi_a), \quad \langle \Phi \rangle \equiv T_a \bar{\sigma}_a \equiv T_0 \bar{\sigma}_0 + T_8 \bar{\sigma}_8,$$

$$\begin{pmatrix} \sigma_n \\ \sigma_s \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} \sqrt{2} & 1 \\ 1 & -\sqrt{2} \end{pmatrix} \begin{pmatrix} \sigma_0 \\ \sigma_8 \end{pmatrix}$$

$$\longrightarrow V(\sigma_n, \sigma_s)$$

Meson Potential Plays the role of the effective bag constant

# Generalized Meson Potential

$$V(\Phi) = V_{inv}(\Phi) + V_b(\Phi),$$

$$V_{inv} = \lambda_1 (\text{tr } \Phi^\dagger \Phi)^2 + \lambda_2 \text{tr} (\Phi^\dagger \Phi \Phi^\dagger \Phi) + m^2 \text{tr} (\Phi^\dagger \Phi) + c (\det \Phi + \det \Phi^\dagger).$$

$$V_b = \sum_{i=1}^8 V_{bi},$$

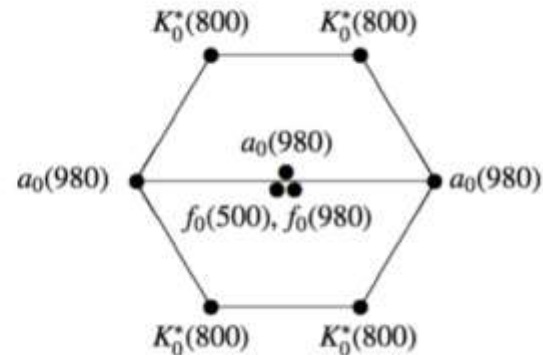
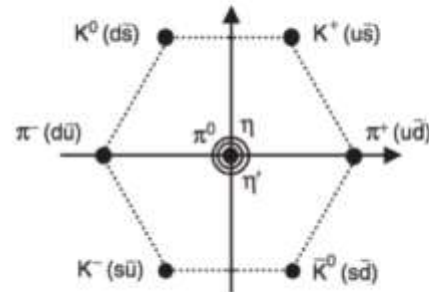
$$V_{b1} = b_1 \text{Tr} (\Phi^\dagger \mathcal{M} + h.c.), \quad V_{b2} = b_2 \epsilon_{ijk} \epsilon_{mnl} \mathcal{M}_{im} \Phi_{jn} \Phi_{kl} + h.c.,$$

$$V_{b3} = b_3 \text{Tr} (\Phi^\dagger \Phi \Phi^\dagger \mathcal{M}) + h.c., \quad V_{b4} = b_4 \text{Tr} (\Phi^\dagger \Phi) \text{Tr} (\Phi^\dagger \mathcal{M}) + h.c.,$$

$$V_{b5} = b_5 \text{Tr} (\Phi^\dagger \mathcal{M} \Phi^\dagger \mathcal{M}) + h.c., \quad V_{b6} = b_6 \text{Tr} (\Phi \Phi^\dagger \mathcal{M} \mathcal{M}^\dagger + \Phi^\dagger \Phi \mathcal{M}^\dagger \mathcal{M}),$$

$$V_{b7} = b_7 (\text{Tr } \Phi^\dagger \mathcal{M} + h.c.)^2, \quad V_{b8} = b_8 (\text{Tr } \Phi^\dagger \mathcal{M} - h.c.)^2,$$

$$\mathcal{M} = \text{diag}(m_{u0}, m_{d0}, m_{s0})$$



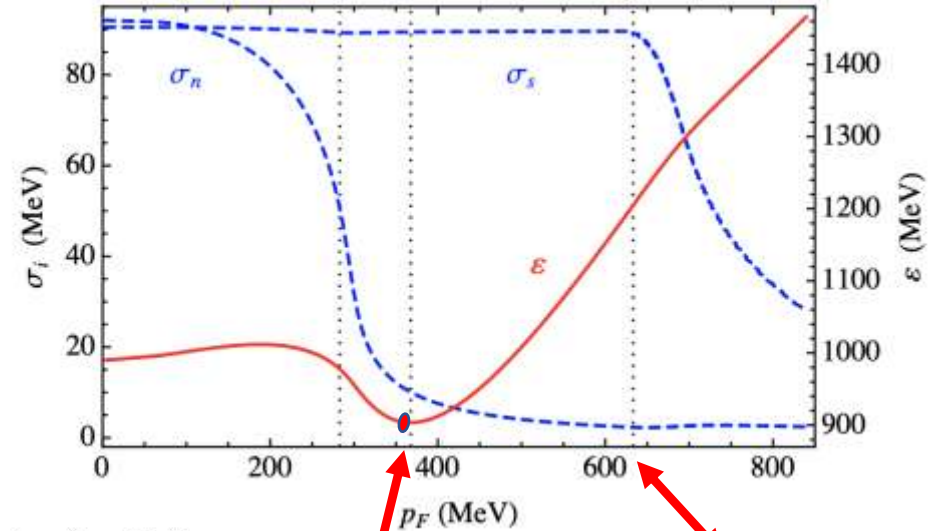
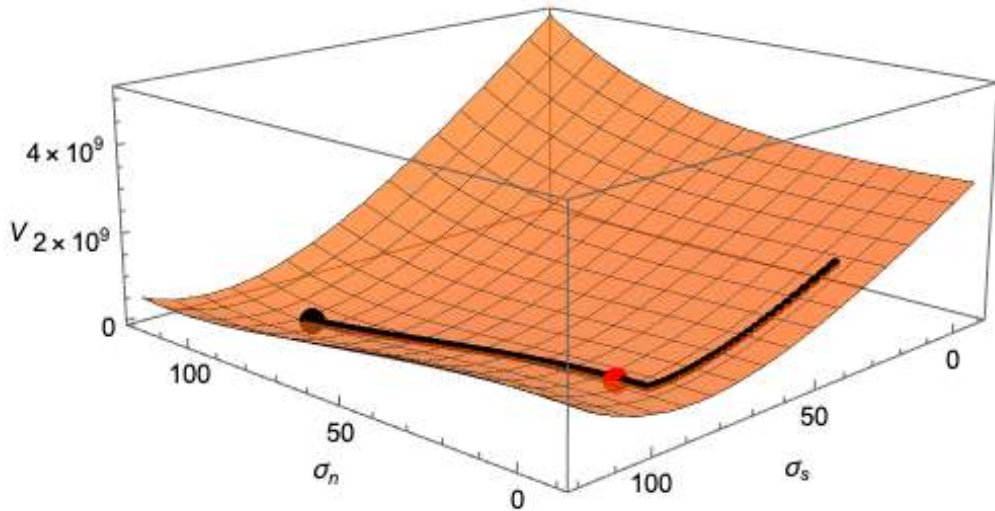
# Generalized Meson Potential

	$\bar{\lambda}_1$	$\bar{\lambda}_2$	$\bar{m}^2$	$\bar{c}$	$\bar{b}_1$	$\bar{b}_2$
Set 1	-0.06	0.33	-0.13	0.33	-4.4	0.19
Set 2	0.04	0.16	0.05	0.27	-1.6	-0.14
	$\bar{b}_3$	$\bar{b}_4$	$\bar{b}_5$	$\bar{b}_6$	$\bar{b}_7$	$\bar{b}_8$
Set 1	-4.2	2.5	-3.0	50	1.4	4.7
Set 2	-0.18	0.09	4.0	5.2	-3.9	-5.5

	$m_\pi$	$m_K$	$m_\eta$	$m'_\eta$	$\theta_p$
Exp	138	496	548	958	NA
Set 1	138	496	548	958	-15.0°
Set 2	148	454	569	922	-10.8°
	$m_{a_0}$	$m_\kappa$	$m_\sigma$	$m_{f_0}$	$\theta_s$
Exp	980 ± 20	700-900	400-550	990 ± 20	NA
Set 1	980	900	555	990	31.5°
Set 2	887	916	555	955	21.7°
	$\Gamma_{\eta \rightarrow \gamma\gamma}$	$\Gamma_{\eta' \rightarrow \gamma\gamma}$	$\Gamma_{\sigma \rightarrow \pi\pi}$	$\Gamma_{\kappa \rightarrow K\pi}$	
Exp	0.52-0.54	4.2-4.5	400-700	~ 500	
Set 1	0.59	4.90	442	451	
Set 2	0.54	4.87	422	537	
	$\Gamma_{f_0 \rightarrow \pi\pi}$	$R_{f_0}$	$\Gamma_{a_0 \rightarrow \eta\pi}$	$R_{a_0}$	
Exp	10-100	3.8-4.7	50-100	1.2-1.6	
Set 1	11	4.3	37.4	2.4	
Set 2	20	4.0	52.0	1.2	

# up down Quark Matter ( $ud$ QM)

$\langle \sigma_n \rangle$  give masses to u,d quarks  
 $\langle \sigma_s \rangle$  give masses to s quarks



$$p_F = (3\pi^2 n_A)^{1/3}$$

Strangeness starts to appear

$$\left(\frac{E}{A}\right)_{udQM} \approx 903 \text{ MeV} < \left(\frac{E}{A}\right)_{^{56}\text{Fe}} \approx 930 \text{ MeV}$$

$$\text{Equation of States} \Rightarrow P \approx \frac{1}{3}(\rho - \rho_0)$$

# Summary and Outlook

We have shown the possibility of **up-down quark stars, inverted hybrid stars, hybrid strangeon stars** and their consistency with recent observations.

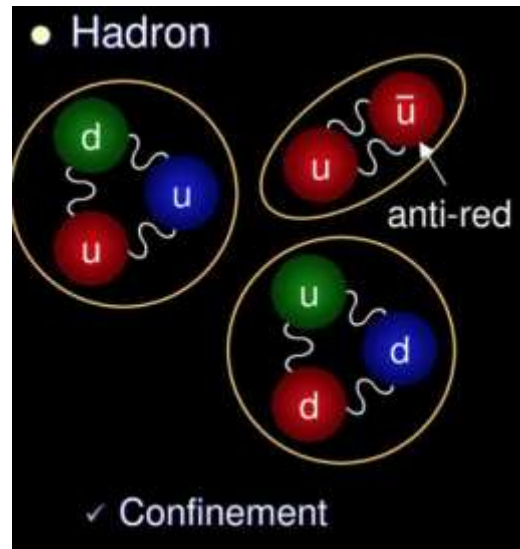
- A microphysics picture of quark-to-hadron phase transition?
- What if the phase transition is cross-over, instead of 1<sup>st</sup> order?

## Hybrid Strangeon Stars

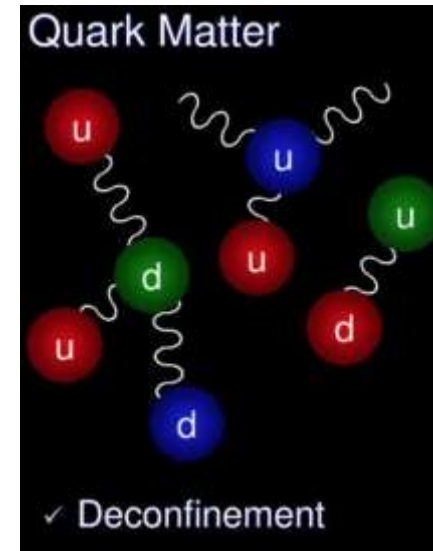
- Nonradial oscillations of hybrid strangeon stars?
- Other distinct astrophysical signatures? (cooling, merger remnant, e.t.c )



# Hadronic Matter vs Quark Matter



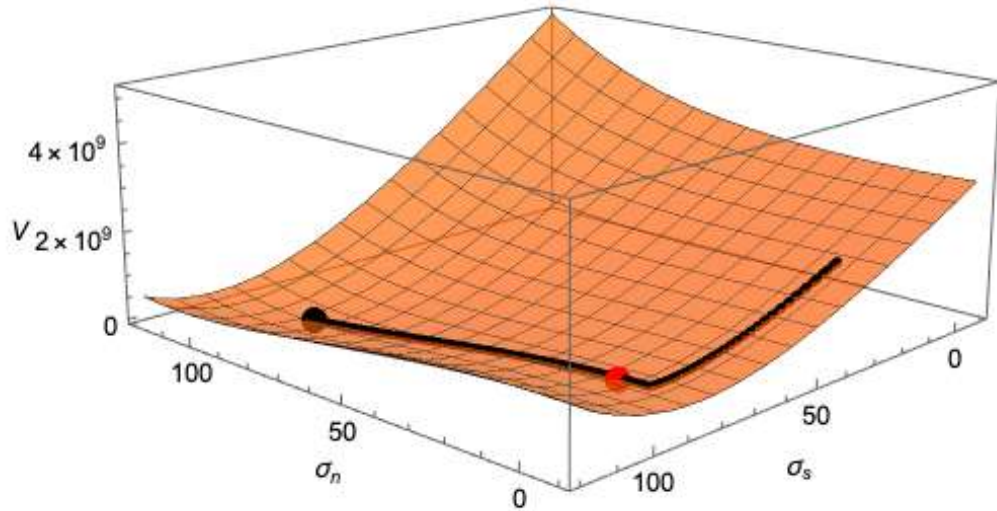
High Temperature  
or  
High Pressure (densities)



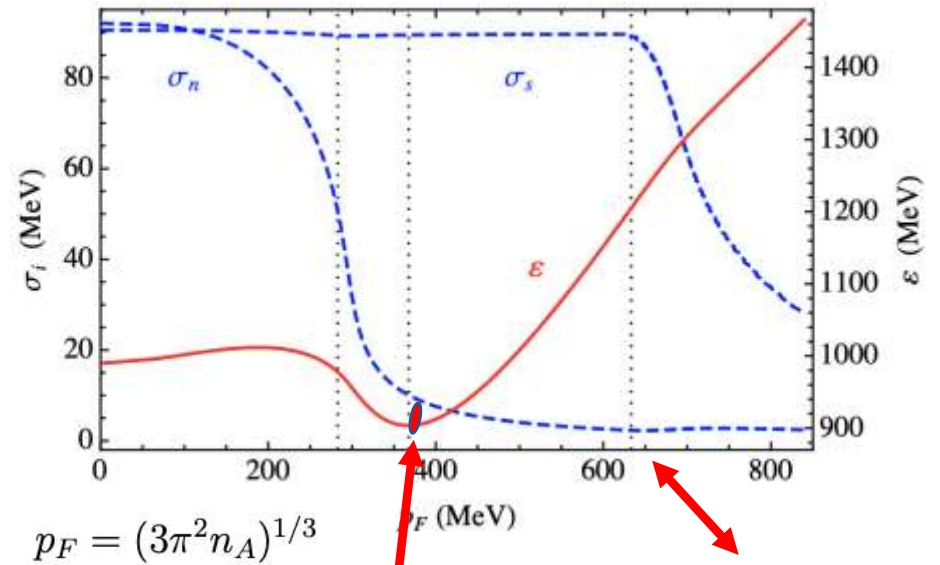
Tomohiro Inagaki [k2.sci.u-toyama.ac.jp](http://k2.sci.u-toyama.ac.jp) › Inagaki\_Tomohiro › files › Inagaki\_Tomohiro1

# up down Quark Matter ( $udQM$ )

- At large particle number (bulk) limit



$\langle \sigma_n \rangle$  give masses to u,d quarks  
 $\langle \sigma_s \rangle$  give masses to s quarks



Strangeness starts to appear

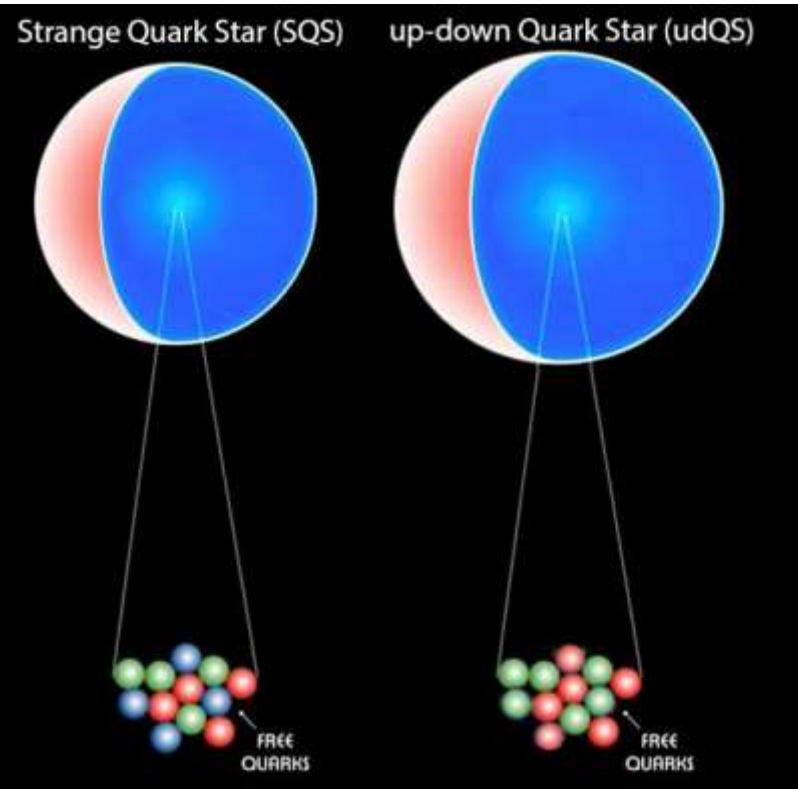
$$\left(\frac{E}{A}\right)_{udQM} \approx 903 \text{ MeV} < \left(\frac{E}{A}\right)_{^{56}\text{Fe}} \approx 930 \text{ MeV}$$

$$\text{Equation of States} \Rightarrow P \approx \frac{1}{3}(\rho - \rho_0)$$

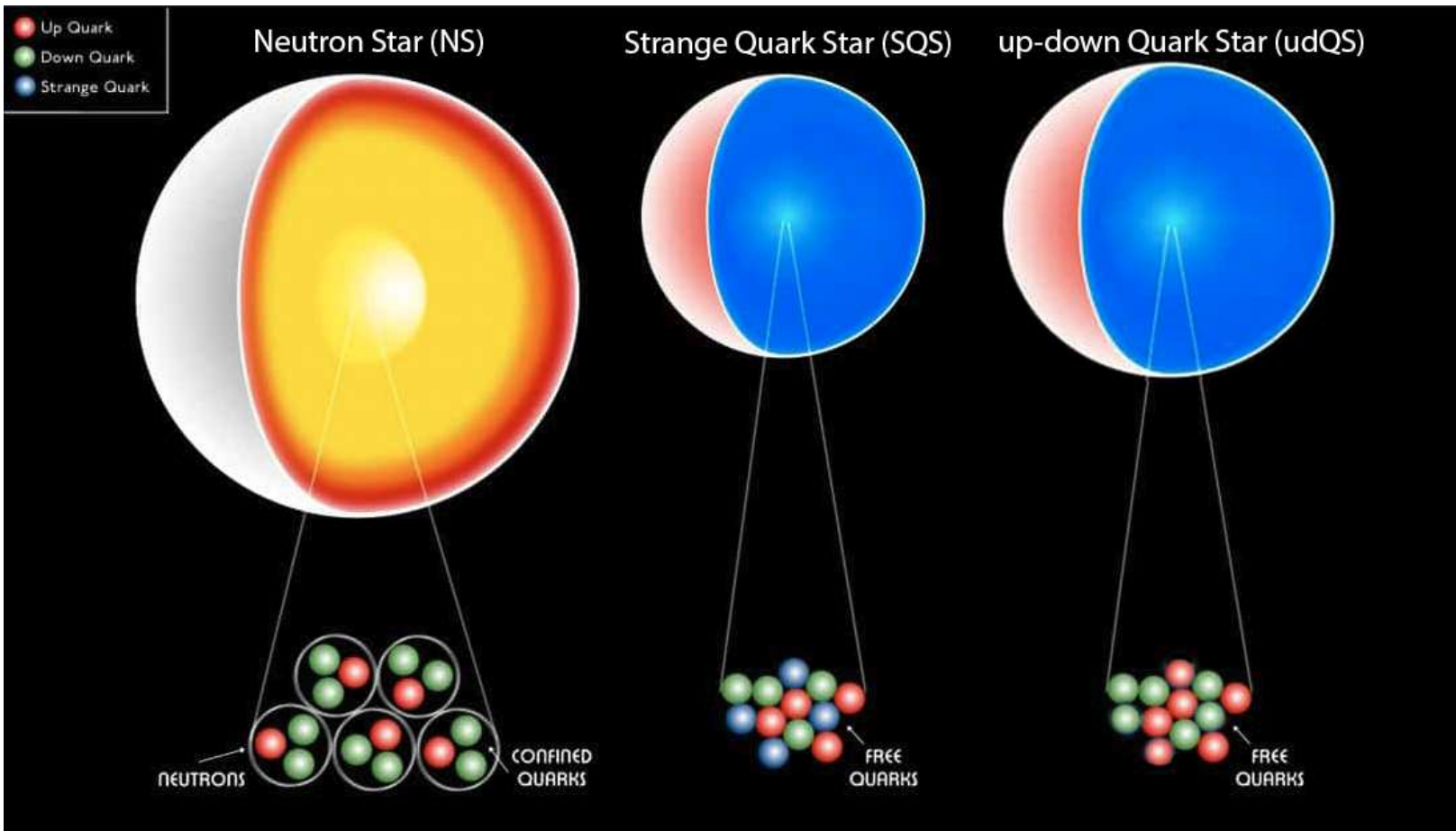
# udQM vs SQM

- *udQM* is more positively charged
- $A_{min}^{udQM} \ll A_{min}^{SQM}$  [Cheng-Jun Xia, et al. Phys. Rev. D 101, 103031 (2020)]
- *udQM* could be possibly formed during a supernove core collapse from conversion of superheavy nuclei. [Kei Iida, et al., JPS Conf. Proc. , 011057 (2020)]

# udQS vs SQS



- SQSs generally have smaller masses and radii than udQSs due to the large strange quark mass and the larger bag constant (unless introduce interquark effects like color superconductivity). So **udQSs are general easier to satisfy the large mass constraint.**  
[C. Z, *Phys.Rev.D* 101 (2020) 4, 043003]
- Compared to NS-> SQS case, **NS can transit to udQS at much faster rate** without the need of converting in strange quarks. We found that **it is possible all NSs observed are actually udQSs.**  
[J.Ren, C. Z, *Phys.Rev.D* 102 (2020) 8, 083003]

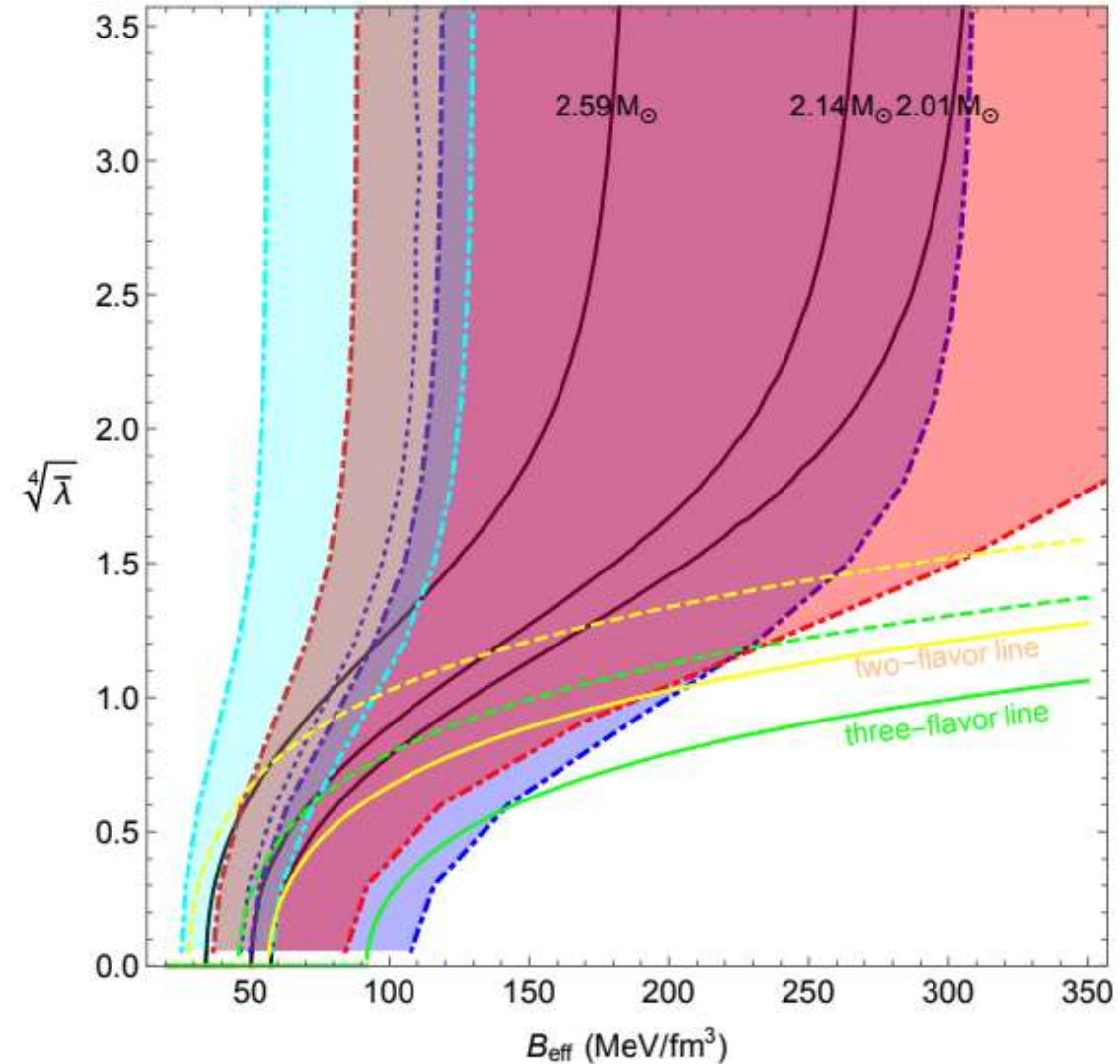




# Intro: Interacting Quark Stars (IQS)

C.Zhang and R.B. Mann *Phys.Rev.D* 103 (2021) 6, 063018; e-Print: [2009.07182](https://arxiv.org/abs/2009.07182) [astro-ph.HE]

blue band for the GW170817,  
Red band for GW190425,  
cyan band for the recent NICER analysis of PSR J0030+0451  
The 2.59 solar mass line is from GW190814



# Model Cross stars: QM sector

- Stability constraints:

Energy per Baryon number

$$\varepsilon_Q = \frac{3\sqrt{2}\pi}{(\xi_4 a_4)^{1/4}} \frac{B^{1/4}}{\sqrt{(\lambda^2/B + \pi^2)^{1/2} + \lambda/\sqrt{B}}}$$

- For udQM hypothesis:

- $\varepsilon_Q < \varepsilon_{Fe} = 930 \text{ MeV} \rightarrow a_{4,min}^{udQM} \approx 174\pi^2 B / \varepsilon_{Fe}^4$

- $A_{min} > 300 \rightarrow \varepsilon_Q > 900 \text{ MeV} \rightarrow a_{4,max}^{udQM} = a_{4,min}^{udQM}(\varepsilon_{Fe} \rightarrow \varepsilon_{900})$ . caveat

- For SQM hypothesis:

- $\varepsilon_Q < \varepsilon_{Fe} = 930 \text{ MeV} \rightarrow a_{4,min}^{SQM} = 3(36\pi^2 B + 3m_s^2 \varepsilon_{Fe}^2) / \varepsilon_{Fe}^4$

- udQM is unstable by definition  $\rightarrow a_{4,max}^{SQM} = a_{4,min}^{udQM}$

- Benchmark choices:

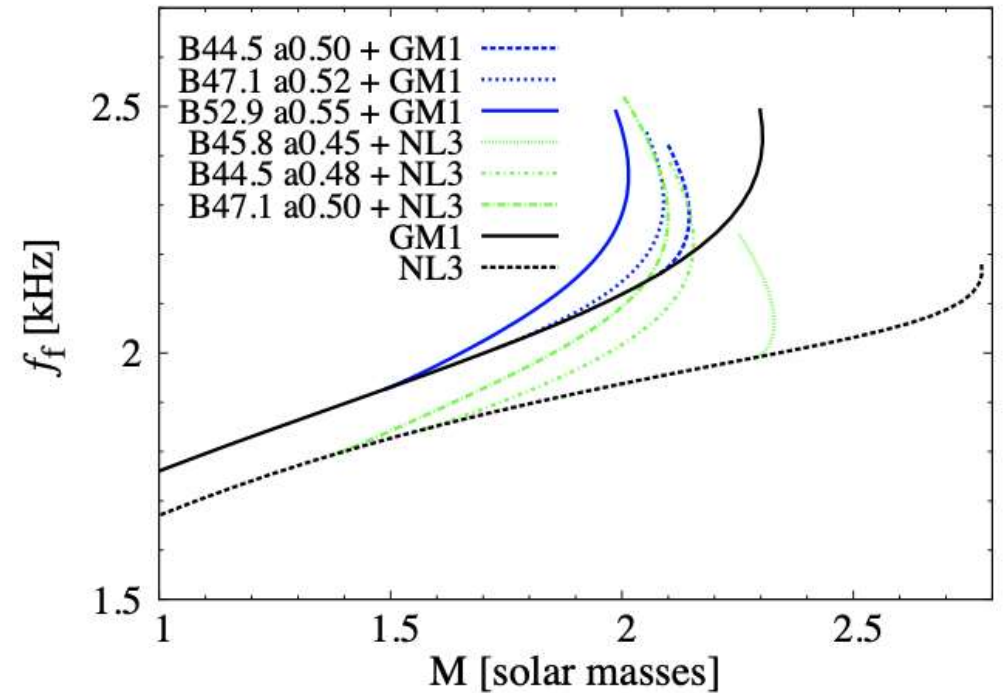
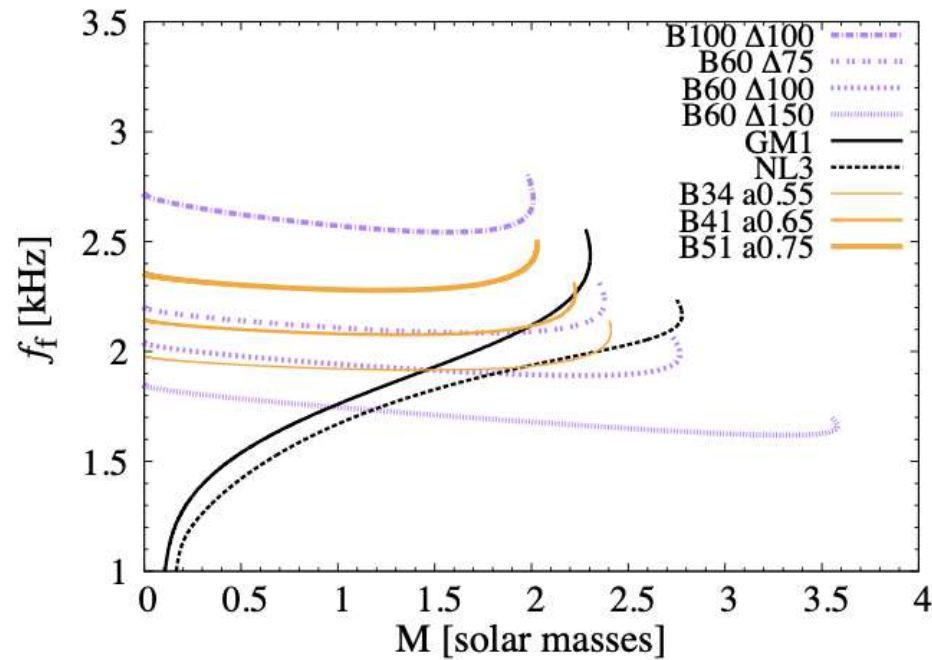
$$B = (20, 35, 50) \text{ MeV/fm}^3 \quad a_{4,min}^{udQM} \approx (0.35, 0.62, 0.88) \quad \text{and} \quad a_{4,max}^{udQM} \approx (0.40, 0.70, 1.0)$$

$$\approx (111^4, 128^4, 140^4) \text{ MeV}^4 \quad a_{4,min}^{SQM} \approx (0.32, 0.49, 0.64) \quad \text{and} \quad a_{4,max}^{SQM} \approx (0.35, 0.62)$$

# Typical results for conventional stars

Flores, C.V. and Lugones, G., 2014 *Classical and Quantum Gravity*, 31(15), p.155002

- $f$  mode



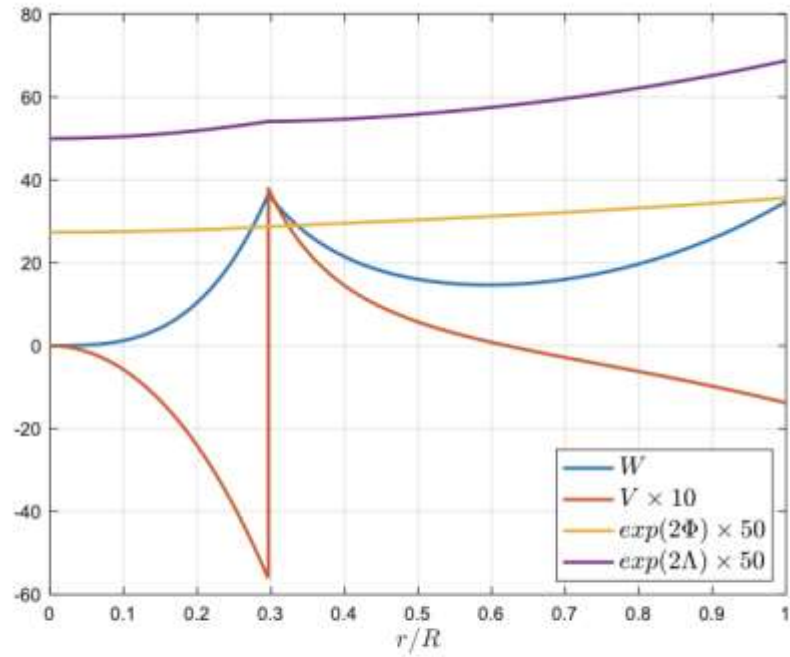
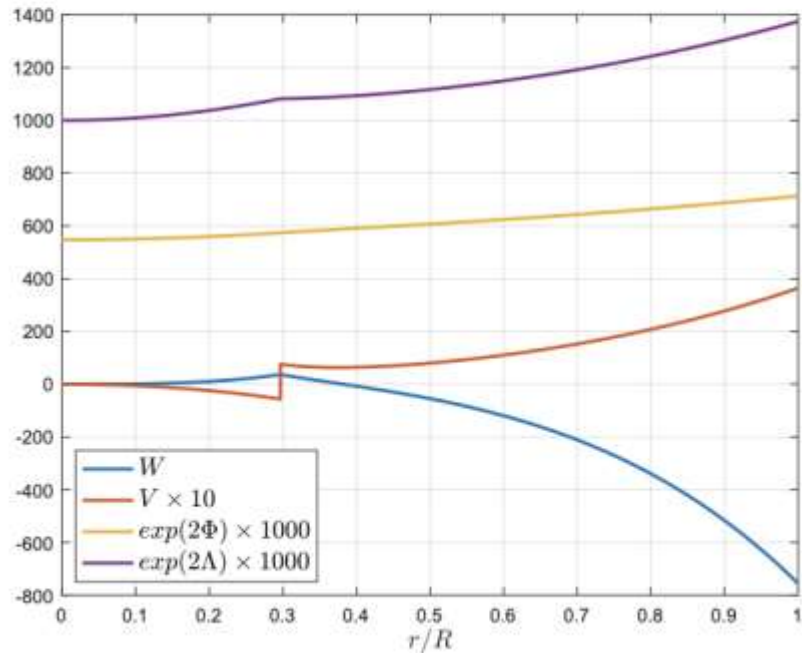
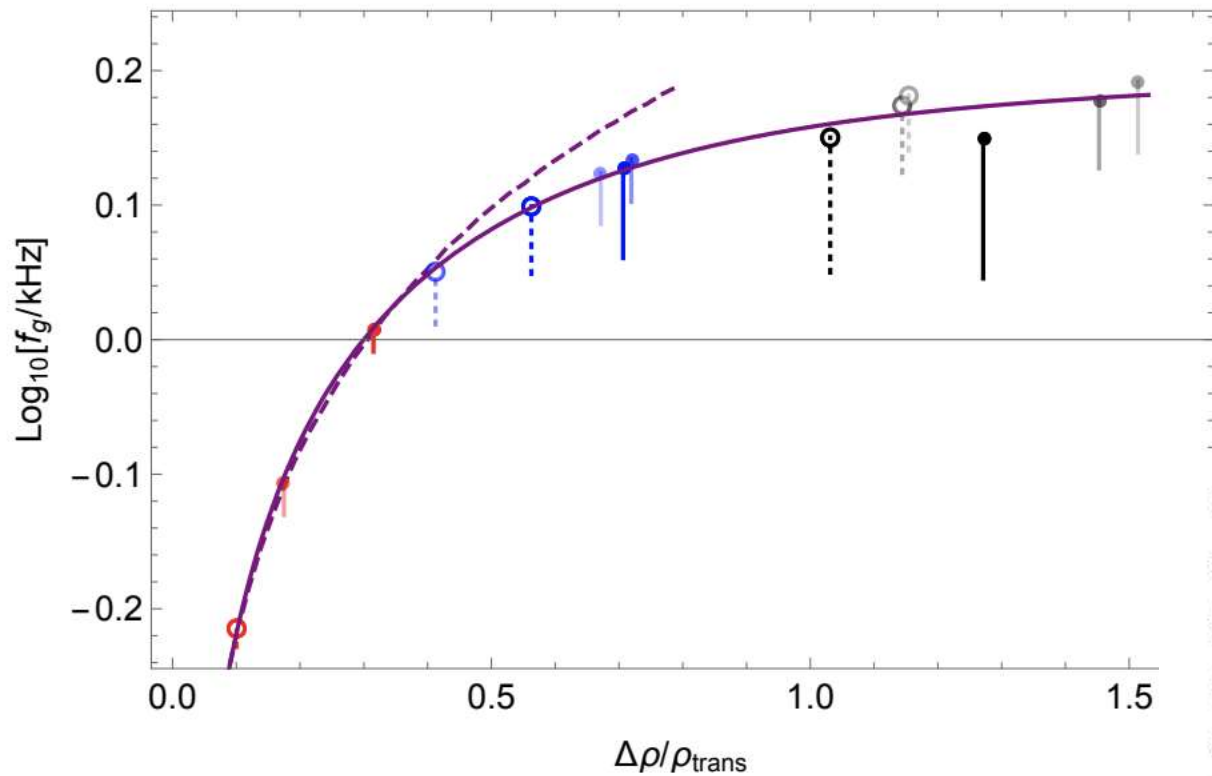


Figure 5: Eigenfunctions of  $f$ -mode (top) and  $g$ -mode (bottom) for the twin branch with a frequency jump at the transition point. The CrS has center pressure  $15.68 \text{ MeV fm}^{-3}$  (star mass  $1.60 M_{\odot}$ ), above and close to the transition pressure, consisting of a small hadronic core. The set of parameters is chosen as the lightest black lines in the left top panel of Fig. 3 in the manuscript ( $B = 20 \text{ MeV/fm}^3$ ,  $a_4 = a_{4,\text{max}}$ ).



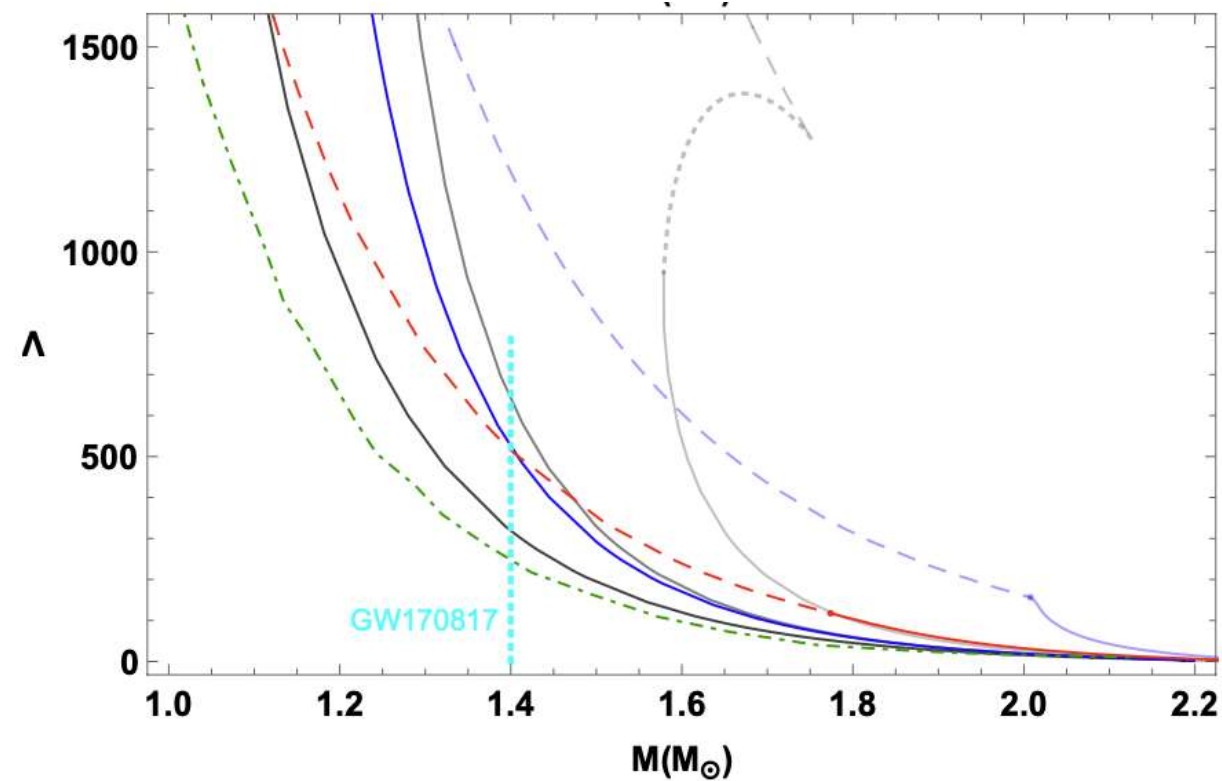
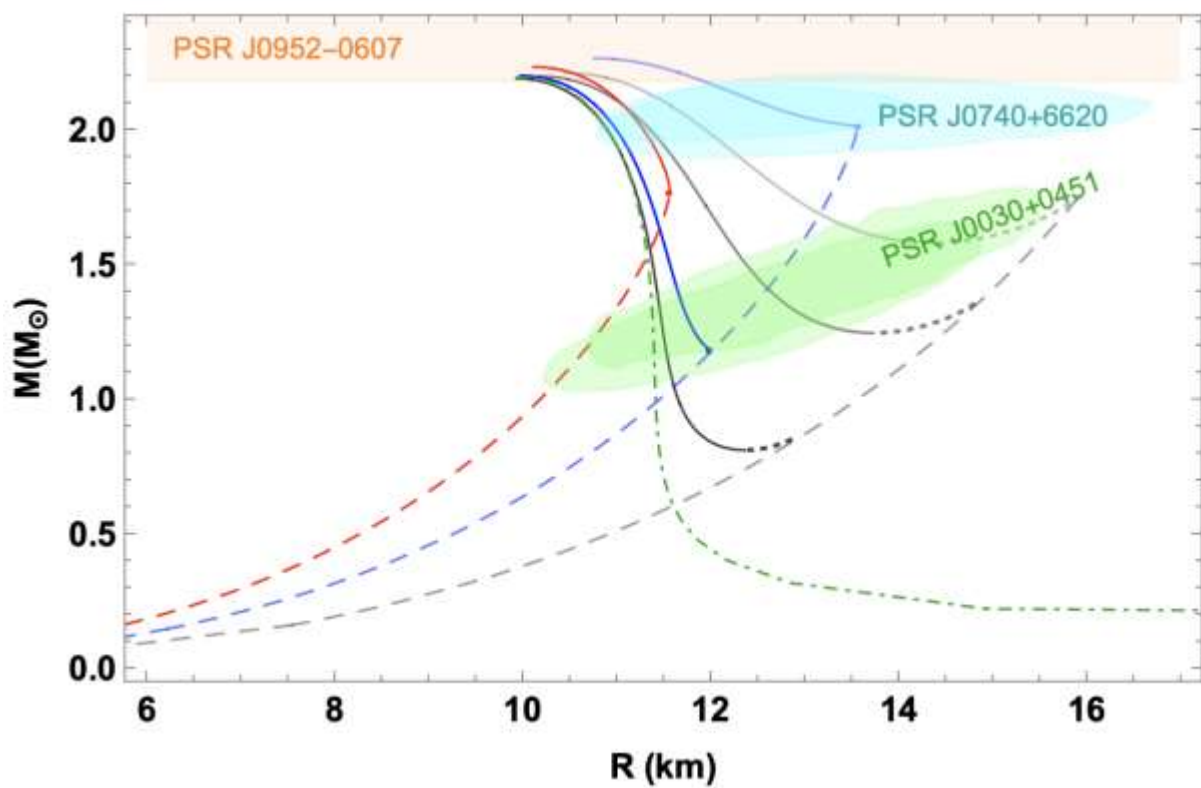


$\frac{\Delta\rho}{\rho_{\text{trans}}}$	$a_{4,\text{min}}$		$\frac{a_{4,\text{min}}+a_{4,\text{min}}}{2}$		$a_{4,\text{max}}$	
	<i>udQM</i>	<i>SQM</i>	<i>udQM</i>	<i>SQM</i>	<i>udQM</i>	<i>SQM</i>
$B_{20}$	1.27	1.03	1.45	1.14	1.51	1.15
$B_{35}$	0.71	0.56	0.72	0.41	0.67	NA
$B_{50}$	0.32	0.10	0.17	NA	NA	NA

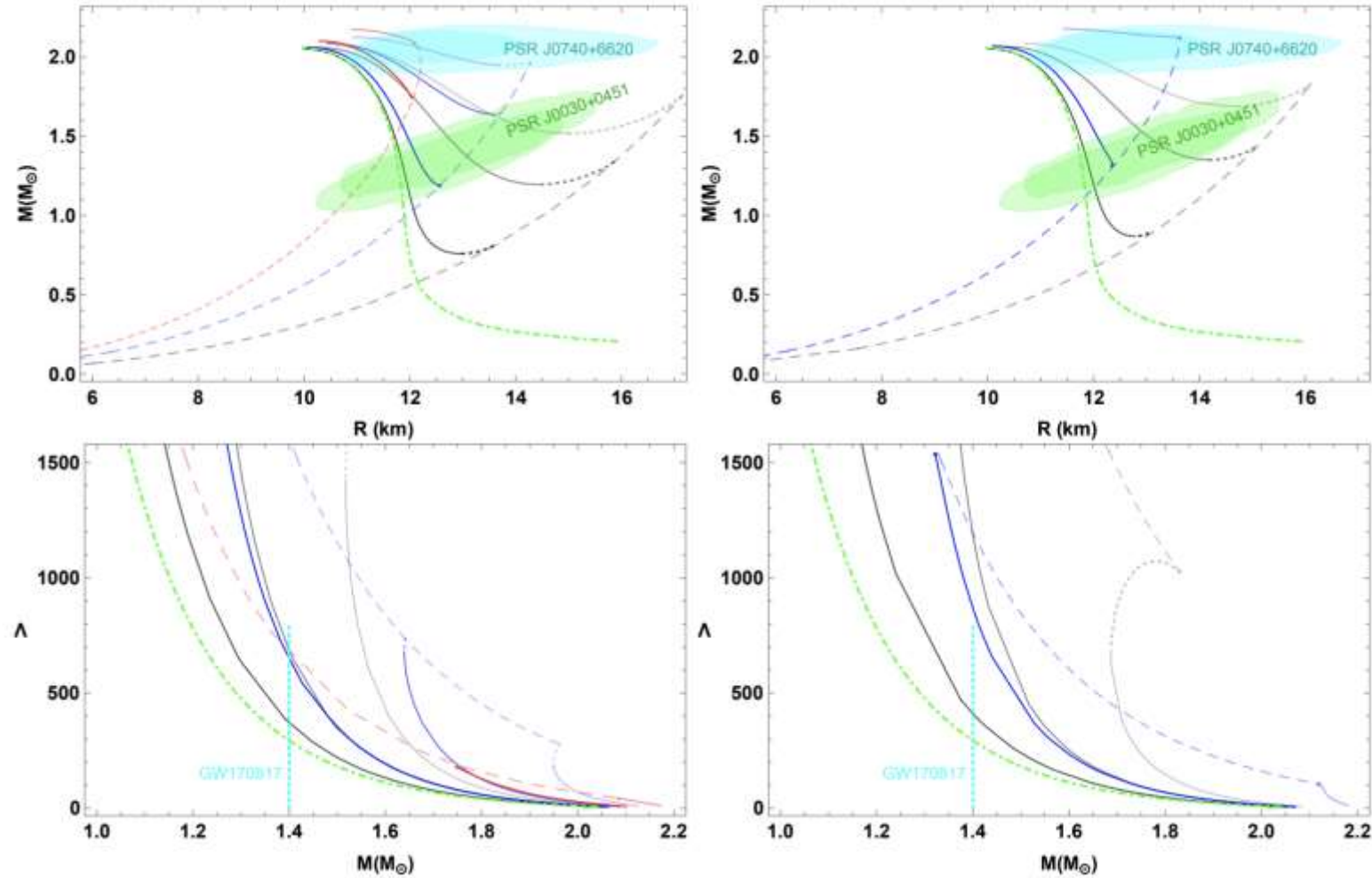
Figure 4:  $\Delta\rho/\rho_{\text{trans}}$  vs  $g$ -mode frequencies for different  $(B, a_4)$  sets. Filled and empty circles denote the maximum  $g$ -mode frequencies of CrSs with *udQM* and *SQM*, respectively, with the color convention being same as Fig. 1, and the vertical bars denote the frequencies ranges. The dashed purple line represents the fit for conventional hybrid stars adapted from Ref. [109], while the solid purple line denotes our fit for CrSs.



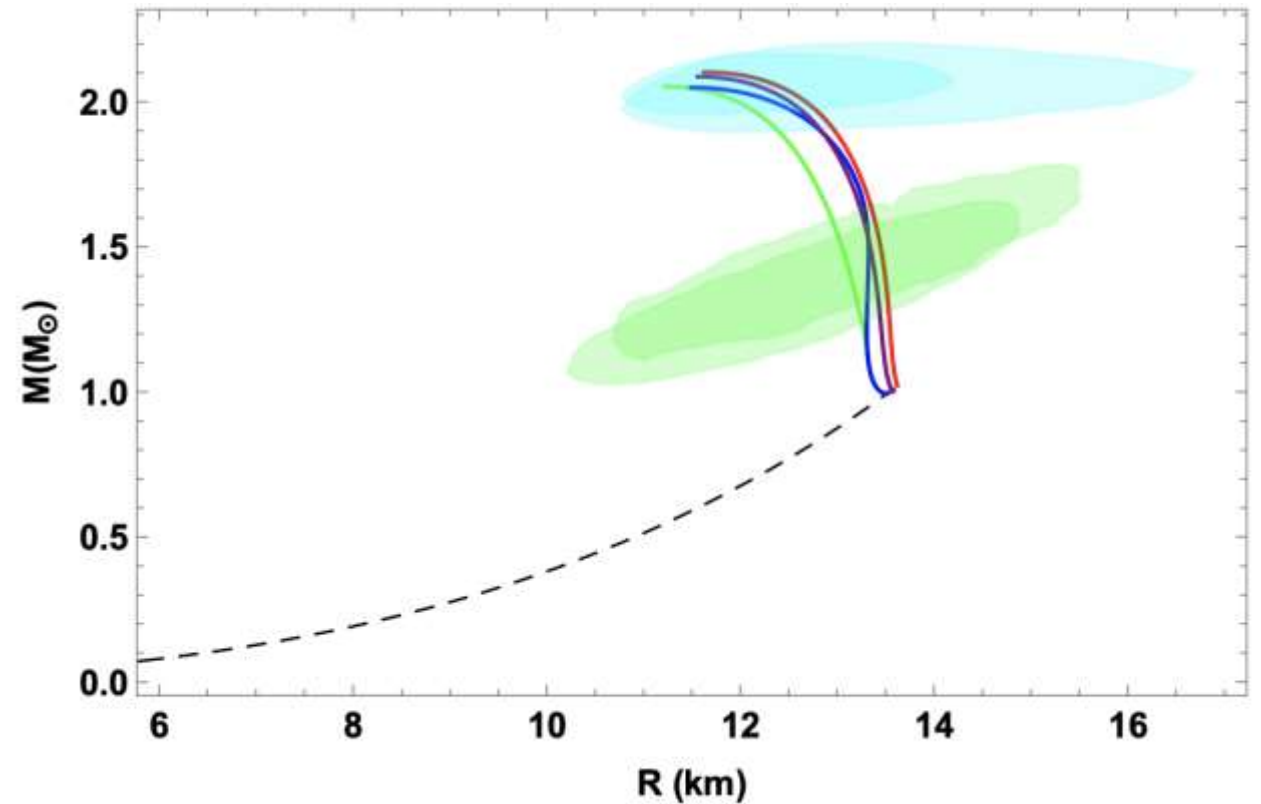
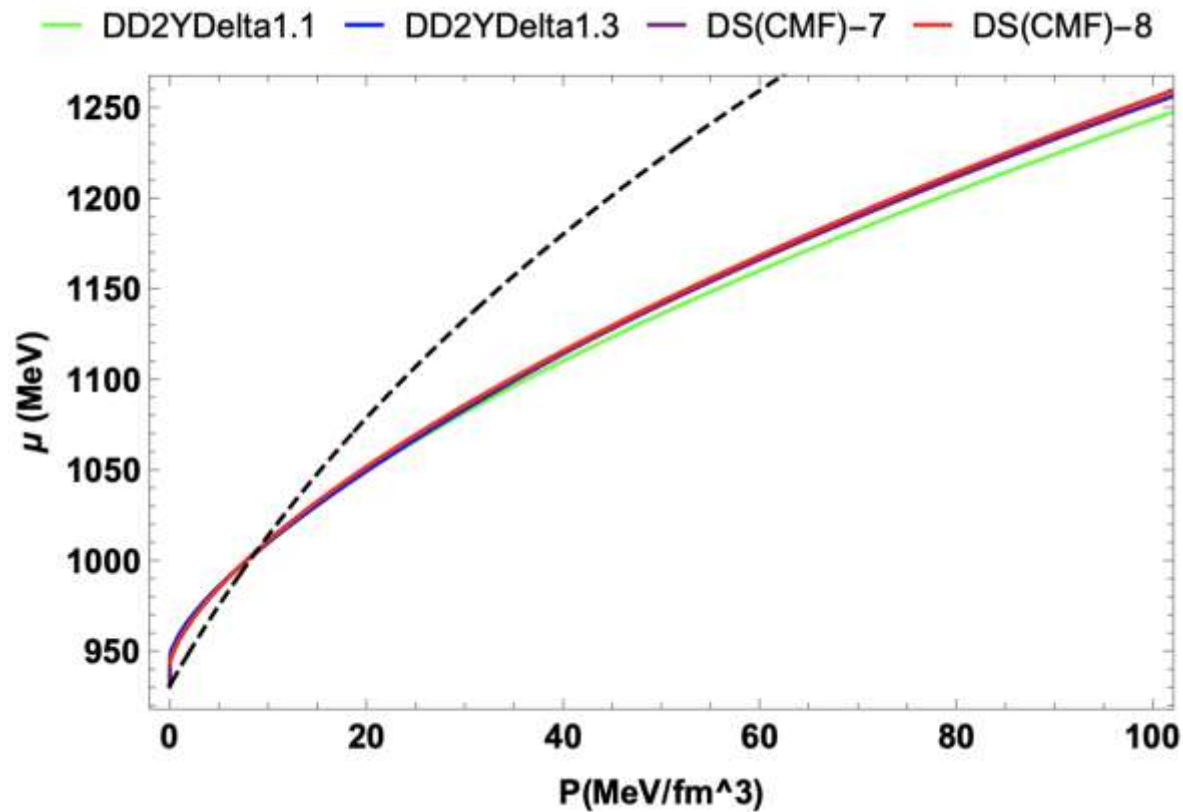
# CrSs with SQM



# CrSs for SLy4 hadronic EOS

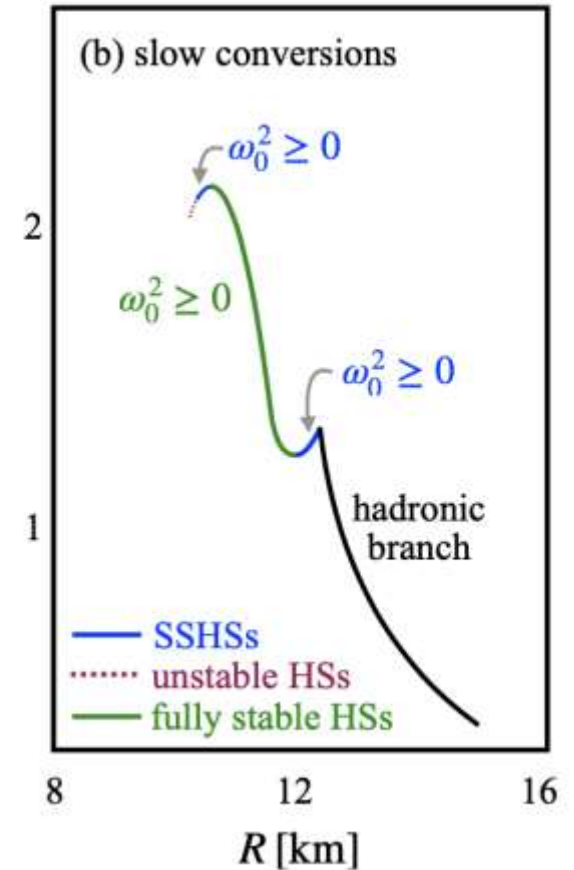
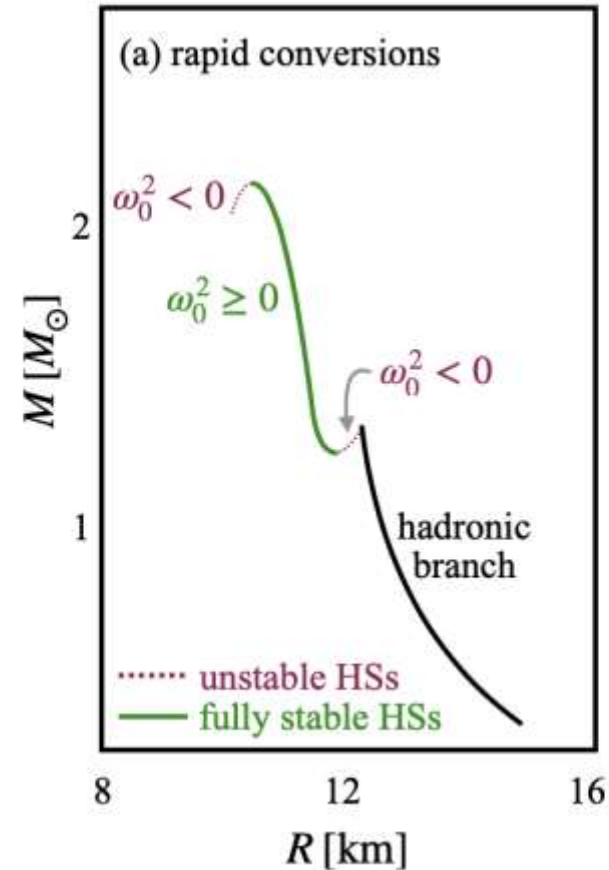
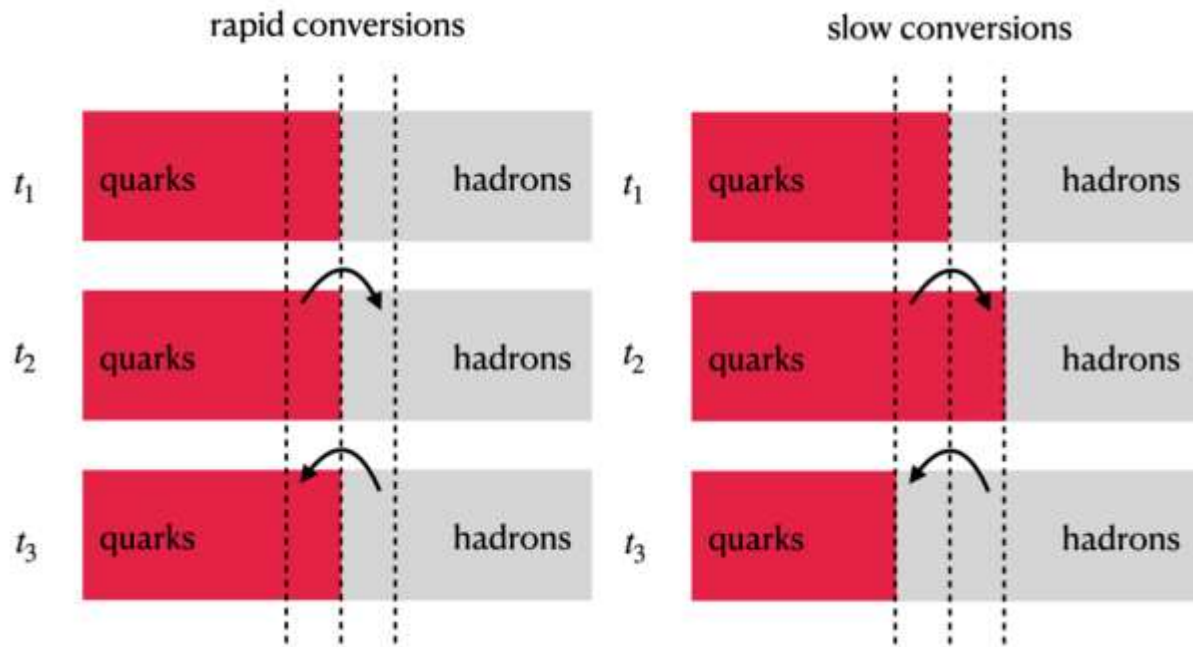


# Variations of HM EOSs: Hyperonic (with Delta resonances, as expected for core region)



# Radial Oscillations

- Conventional Hybrid stars



# Radial Oscillations

- Solving Scheme:

$$\xi = \Delta r / r$$

$$\begin{aligned} \frac{d\xi}{dr} &= \mathcal{V}(r)\xi + \mathcal{W}(r)\Delta P, \\ \frac{d\Delta P}{dr} &= X(r)\xi + Y(r)\Delta P, \end{aligned}$$

Initial Conditions:

$$(\Delta P)_{r=0} = -3(\xi\Gamma P)_{r=0} \text{ and } \xi(0) = 1$$

Solve the eigenfrequencies via **shooting method**,  
Until match the boundary Condition:  $(\Delta P)_{r=R} = 0$

Junction conditions: **Slow:**  $[\xi]_{-}^{+} = 0, [\Delta P]_{-}^{+} = 0$ ; **Rapid:**  $[\Delta P]_{-}^{+} = 0, \left[\xi - \frac{\Delta P}{rP'}\right]_{-}^{+} = 0$

$$\begin{aligned} \mathcal{V}(r) &= -\frac{3}{r} - \frac{dP}{dr} \frac{1}{(P + \rho)}, \\ \mathcal{W}(r) &= -\frac{1}{r} \frac{1}{\Gamma P}, \quad \Gamma = \frac{n}{P} \frac{dP}{dn} = \frac{\rho + P}{P} \frac{dP}{d\rho} \\ X(r) &= \omega^2 e^{2\Lambda - 2\Phi} (P + \rho) r - 4 \frac{dP}{dr} \\ &\quad + \left(\frac{dP}{dr}\right)^2 \frac{r}{(P + \rho)} - 8\pi e^{2\Lambda} (P + \rho) P r, \\ Y(r) &= \frac{dP}{dr} \frac{1}{(P + \rho)} - 4\pi (P + \rho) r e^{2\Lambda}, \end{aligned}$$

For small discontinuity, slow  $\approx$  rapid

# Non-Radial treatment via Cowling Approximation

$$\xi^i = (e^{-\Lambda}W, -V\partial_\theta, -V\sin^{-2}\theta\partial_\phi)$$

$$\frac{dW}{dr} = \frac{d\rho}{dP} \left[ \omega^2 r^2 e^{\Lambda-2\Phi} V + \frac{d\Phi}{dr} W \right] - \ell(\ell+1)e^\Lambda V$$

$$\frac{dV}{dr} = 2\frac{d\Phi}{dr} V - e^\Lambda \frac{W}{r^2}$$

## Initial Conditions:

$$W(r)|_{r \rightarrow 0} = Cr^{\ell+1} \text{ and } V(r)|_{r \rightarrow 0} = -Cr^\ell/\ell$$

Solve the eigenfrequencies via **shooting method**,  
Until match **the boundary Condition**:

$$\omega^2 r^2 e^{\Lambda-2\Phi} V + \Phi' W = 0$$

$$\omega_g < \omega_f < \omega_{p1}$$

## Junction conditions

$$W_+ = W_-$$

$$V_+ = \frac{e^{2\Phi}}{\omega^2 r_d^2} \left[ \frac{\rho_- + P}{\rho_+ + P} (\omega^2 r_d^2 e^{-2\Phi} V_- + e^{-\Lambda} \Phi' W_-) - e^{-\Lambda} \Phi' W_+ \right],$$



# A New Rescaling Scheme

C. Z, Yong Gao , Cheng-jun Xia, Renxin Xu Phys.Rev.D 108 (2023) 6, 063002

$$\rho = \frac{a}{9} \epsilon \left( \frac{N_q^4}{18 n_s^4} n^5 - \frac{N_q^2}{n_s^2} n^3 \right) + m_q N_q n$$

$$p = \frac{2a}{9} \epsilon \left( \frac{N_q^4}{9 n_s^4} n^5 - \frac{N_q^2}{n_s^2} n^3 \right).$$

$$\bar{\rho} = \frac{\rho}{m_q n_s}, \quad \bar{p} = \frac{p}{m_q n_s}, \quad \bar{n} = \frac{N_q n}{n_s}, \quad \bar{\epsilon} = \frac{\epsilon}{N_q m_q}$$

$$\bar{\rho} = \frac{a}{9} \bar{\epsilon} \left( \frac{1}{18} \bar{n}^5 - \bar{n}^3 \right) + \bar{n},$$

$$\bar{p} = \frac{2a}{9} \bar{\epsilon} \left( \frac{1}{9} \bar{n}^5 - \bar{n}^3 \right)$$

At zero pressure,  $\bar{n} = 3$ .

Requiring  $\bar{\rho} \geq 0$  at zero pressure thus yield  $\bar{\epsilon} \leq \frac{2}{a} \approx 0.1757$  ( $\bar{\epsilon}_{max}^{theo}$ )

We take  $\bar{\epsilon}_{max}^{em} = \frac{120 \text{ MeV}}{930 \text{ MeV}} = 0.13$

# TOV Equations

$$ds^2 = -e^{2\Phi(r)} dt^2 + e^{2\Lambda(r)} dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)$$

where  $e^{-2\Lambda(r)} = 1 - \frac{2m(r)}{r}$

$$\frac{dP(r)}{dr} = -\frac{[m(r) + 4\pi r^3 P(r)] [\rho(r) + P(r)]}{r(r - 2m(r))}$$

$$\frac{dm(r)}{dr} = 4\pi\rho(r)r^2,$$

$$\frac{d\Phi}{dr} = -\frac{1}{\epsilon + p} \frac{dP}{dr},$$

with the boundary conditions

$$\rho(0) = \rho_c, \quad \Phi(R) = -\Lambda(R),$$

# Non-Radial Oscillations

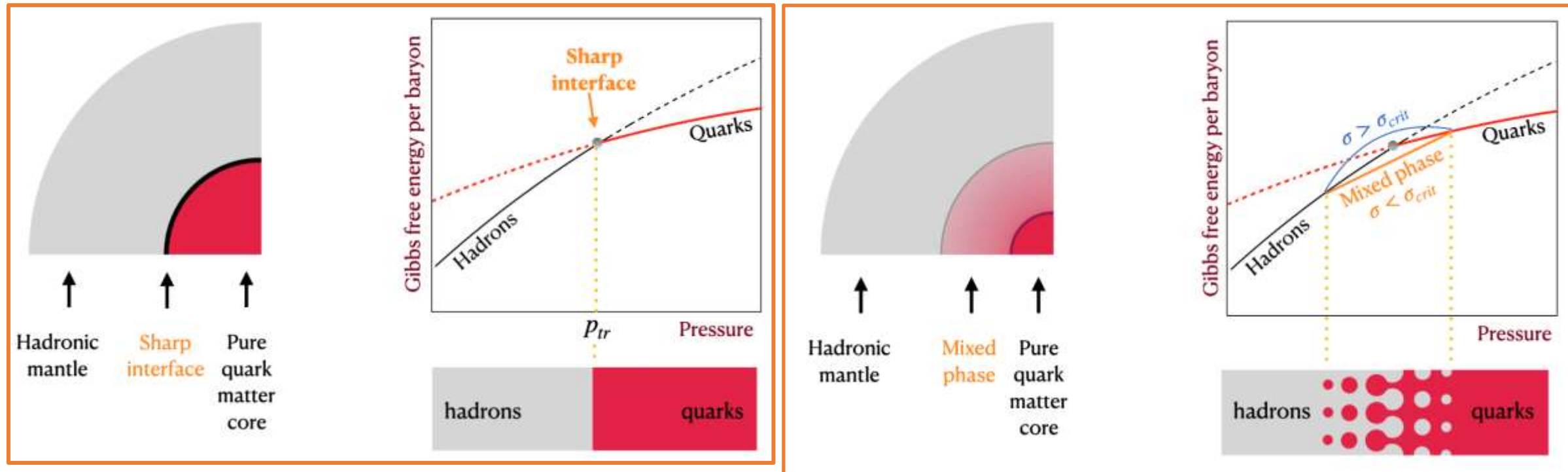
- When compact stars oscillate non-radially, various modes of GWs are radiated
- The gravity ( $g$ ), fundamental ( $f$ ) and pressure ( $p$ ) modes are related to the fluid oscillations while  $\omega$  modes correspond to the oscillations of spacetime itself
  - $f$  mode and  $p$  mode result from pressure forces, with  $f$  mode a particular zero-radial-node mode of  $p$  mode in its oscillation eigenfunction
  - $f$  mode is positively correlated to average mass density,  $p$  mode is mostly sensitive to the outer core/crust.
  - $g$  mode is originated from buoyancy forces and thus is excited if the star has temperature/composition gradients or **density discontinuities (like hybrid stars and cross stars)**
  - In chemically homogeneous, zero temperature stars, all  $g$ -modes are zero frequency
  - $g$  mode vanishes for rapid conversions

# Non-Radial Oscillations

## Cowling Approximation

- Assumes that the fluid would oscillate on a fixed background metric with the metric perturbation ( $\omega$  mode) being neglected
- there is no damping (the eigenfrequencies of oscillation modes only have the real parts)
- compared to full treatments:
  - differences by less than 20% for f modes, around 10% for p-modes [66], and 5%–10% for g-modes

# Sharp vs Mixed



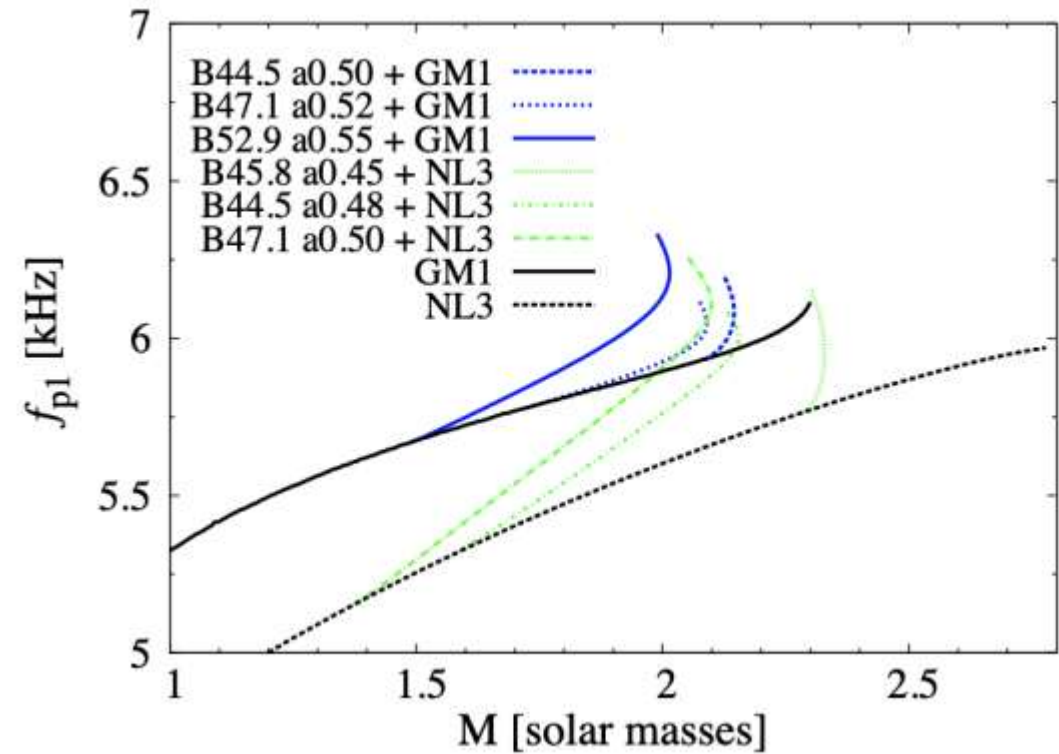
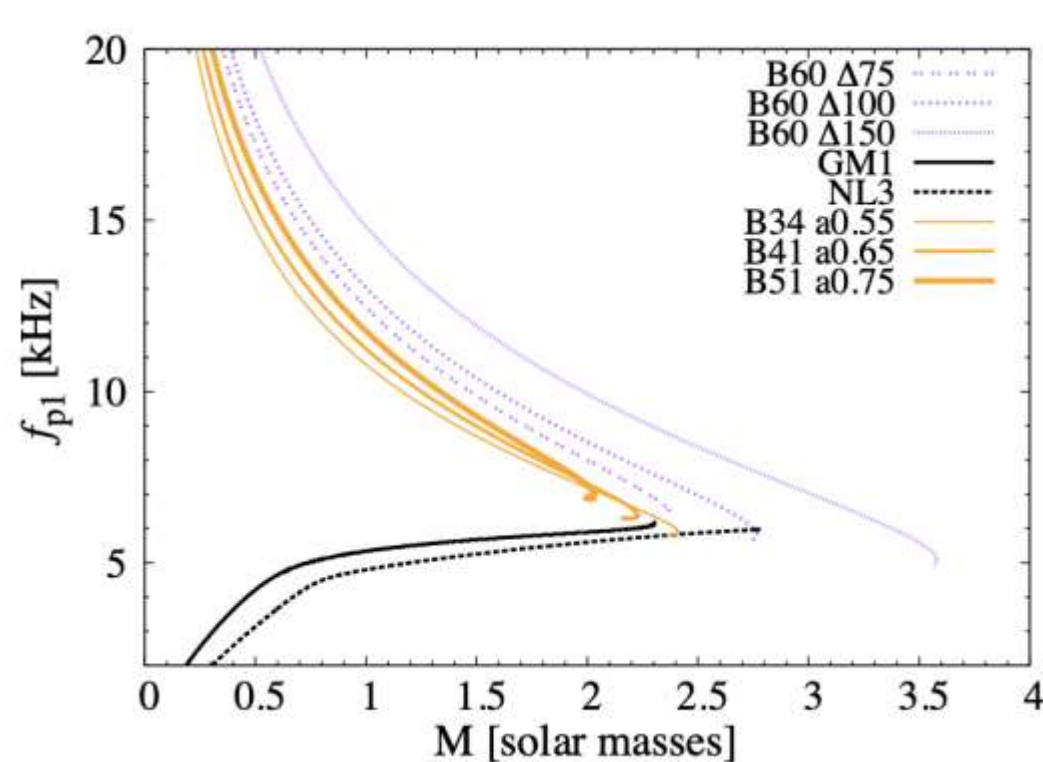
Lugones, G., & Grunfeld, A. G. (2021). *Universe*, 7(12), 493.

We focus on Sharp interface (i.e., Maxwell construction) in our current study

# Typical results for conventional stars

Flores, C.V. and Lugones, G., 2014 *Classical and Quantum Gravity*, 31(15), p.155002

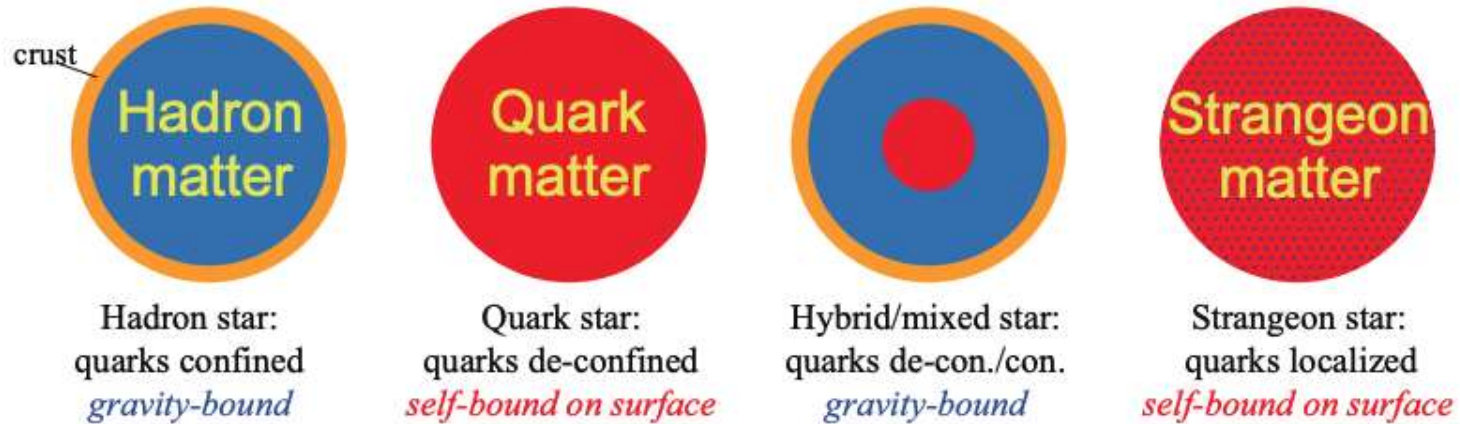
- $p_1$  mode



$p$  modes of conventional hybrid stars ( $\sim 4\text{--}6$  kHz for stars with  $M_{TOV} \gtrsim 2M_{\odot}$ )



# Strangeon Stars



Graph adapted from X.~Lai and R.~Xu, *J. Phys. Conf. Ser.* 861, no.1, 012027 (2017)

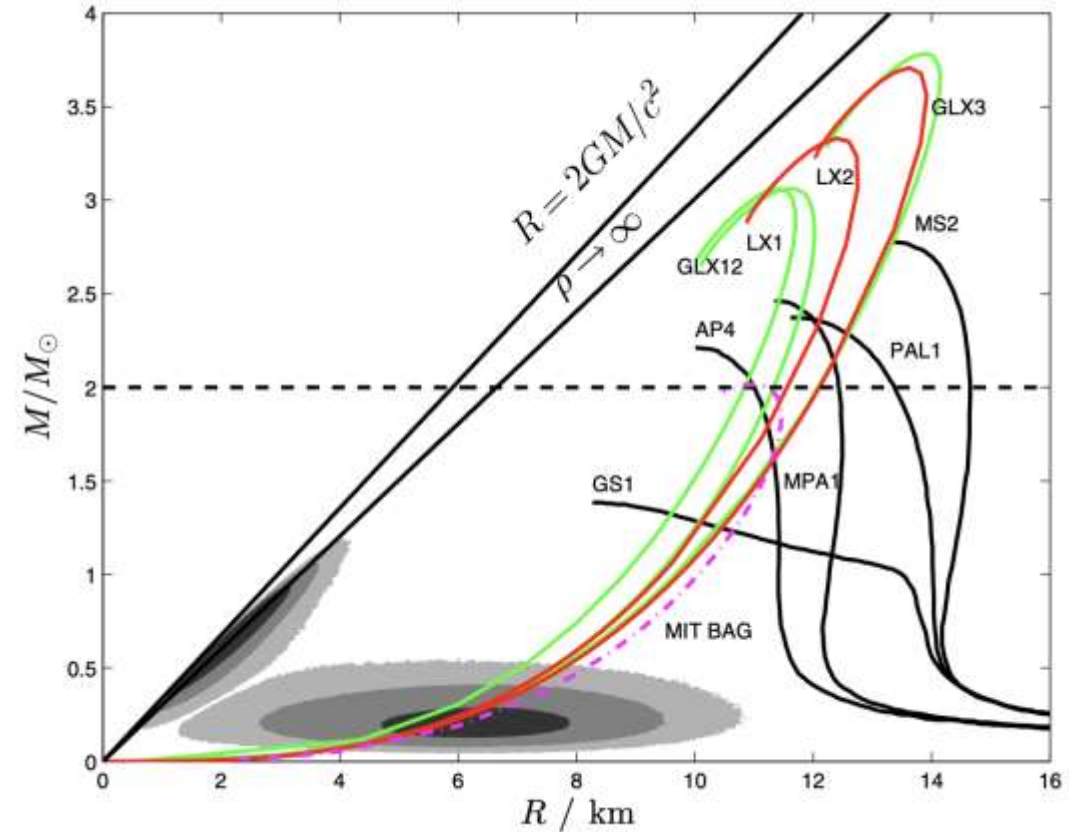
# Strangeon Stars

$$\rho = \frac{a}{9} \epsilon \left( \frac{N_q^4}{18n_s^4} n^5 - \frac{N_q^2}{n_s^2} n^3 \right) + m_q N_q n$$

$$p = \frac{2a}{9} \epsilon \left( \frac{N_q^4}{9n_s^4} n^5 - \frac{N_q^2}{n_s^2} n^3 \right).$$

$$\frac{dm}{dr} = 4\pi \rho r^2,$$

$$\frac{dp}{dr} = (\rho + p) \frac{m + 4\pi p r^3}{2mr - r^2}$$



Graph adapted from  
LI, Z, QU, Z, CHEN, L, et al. *Astrophys J*, 2015, 798: 56

# Model Hybrid strangeon stars: QM sector

- Interacting Quark matter (IQM)

*C. Z, R.B. Mann, Phys.Rev.D 103 (2021) 6, 063018*

$$\Omega = -\frac{\xi_4}{4\pi^2}\mu^4 + \frac{\xi_4(1-a_4)}{4\pi^2}\mu^4 - \frac{\xi_{2a}\Delta^2 - \xi_{2b}m_s^2}{\pi^2}\mu^2 - \frac{\mu_e^4}{12\pi^2} + B_{\text{eff}}$$

$$(\xi_4, \xi_{2a}, \xi_{2b}) = \begin{cases} ((\frac{1}{3})^{\frac{4}{3}} + (\frac{2}{3})^{\frac{4}{3}})^{-3}, 1, 0) & \text{2SC phase} \\ (3, 1, 3/4) & \text{2SC+s phase} \\ (3, 3, 3/4) & \text{CFL phase} \end{cases}$$

$$\lambda = \frac{\xi_{2a}\Delta^2 - \xi_{2b}m_s^2}{\sqrt{\xi_4 a_4}}$$



$$p = \frac{1}{3}(\rho - 4B_{\text{eff}}) + \frac{4\lambda^2}{9\pi^2} \left( -1 + \sqrt{1 + 3\pi^2 \frac{(\rho - B_{\text{eff}})}{\lambda^2}} \right)$$

$$\bar{\lambda} = \frac{\lambda^2}{4B_{\text{eff}}}$$

$$\bar{p} = \frac{1}{3}(\bar{\rho} - 1) + \frac{4}{9\pi^2} \bar{\lambda} \left( -1 + \text{sgn}(\lambda) \sqrt{1 + \frac{3\pi^2}{\bar{\lambda}} (\bar{\rho} - \frac{1}{4})} \right)$$

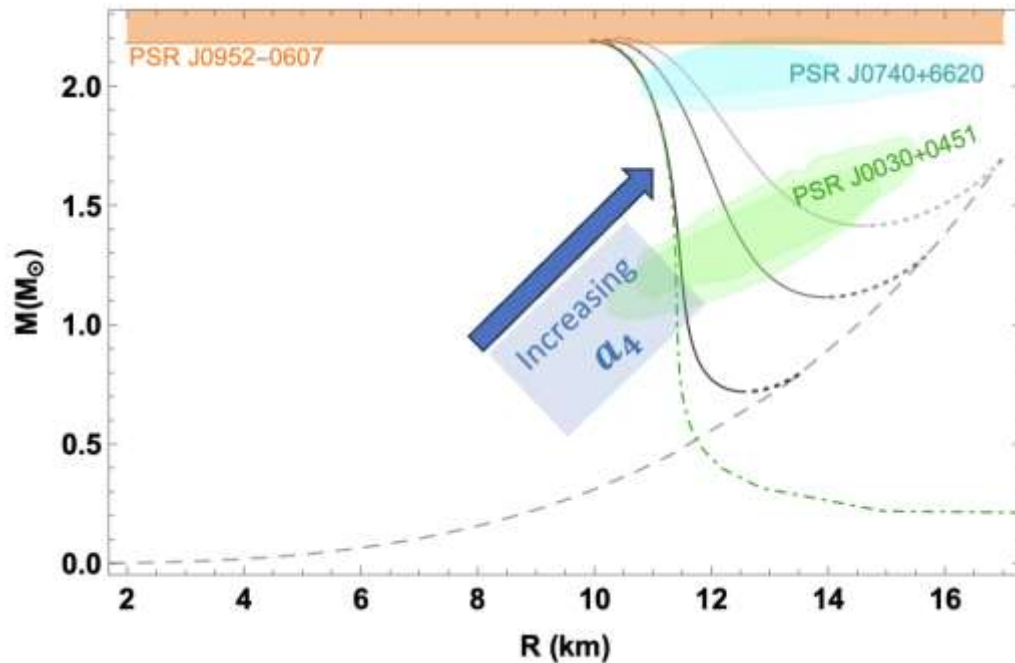
$$\mu_{\text{QM}} = \frac{3\sqrt{2}}{(a_4 \xi_4)^{1/4}} \sqrt{[(P + B)\pi^2 + \lambda^2]^{1/2} - \lambda}$$

$$\left( \frac{E}{A} \right)_{\text{QM}} = \frac{3\sqrt{2}\pi}{(\xi_4 a_4)^{1/4}} \frac{B^{1/4}}{\sqrt{(\lambda^2/B + \pi^2)^{1/2} + \lambda/\sqrt{B}}}$$

**We focus on CFL phase ( $\xi_4 = 3$ )**

# M-R and Tidal deformability

- Branch of “Twin stars”: stars with identical masses but very different radii



$\Delta\rho$	$a_{4,min}$		$\frac{a_{4,min} + a_{4,min}}{2}$		$a_{4,max}$	
	$udQM$	$SQM$	$udQM$	$SQM$	$udQM$	$SQM$
$B_{20}$	1.27	1.03	1.45	1.14	1.51	1.15

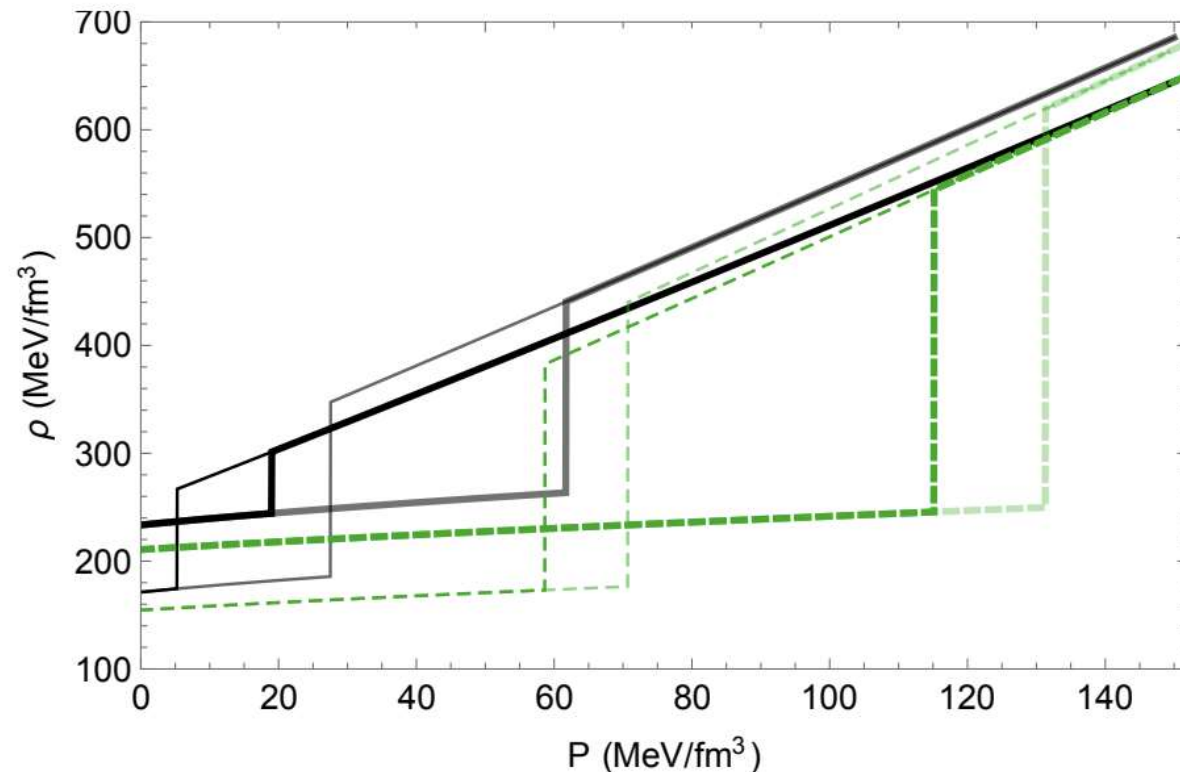
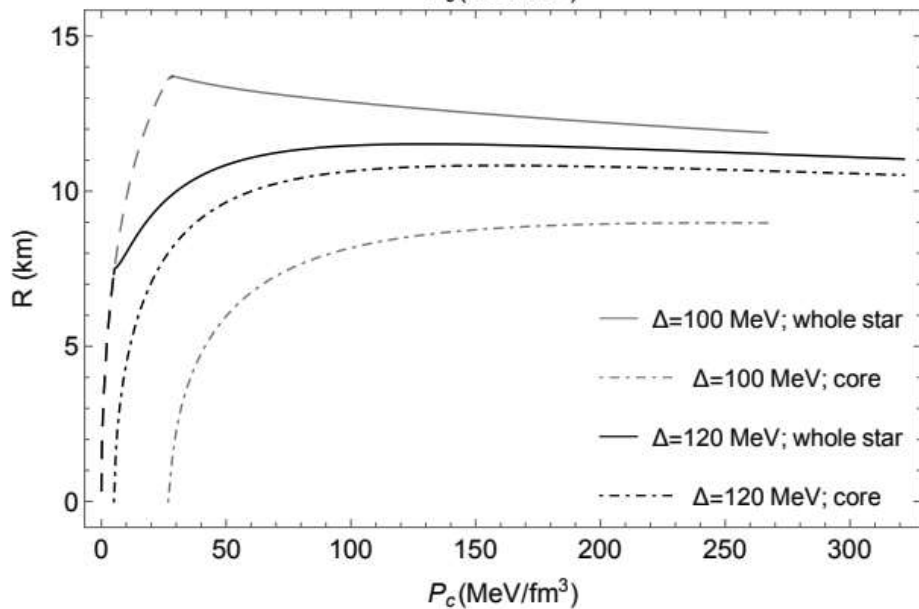
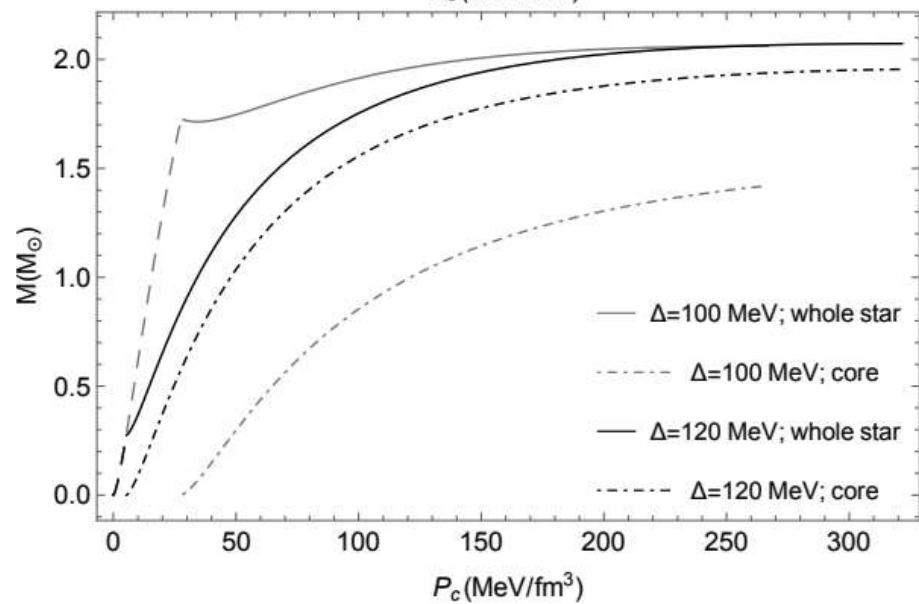
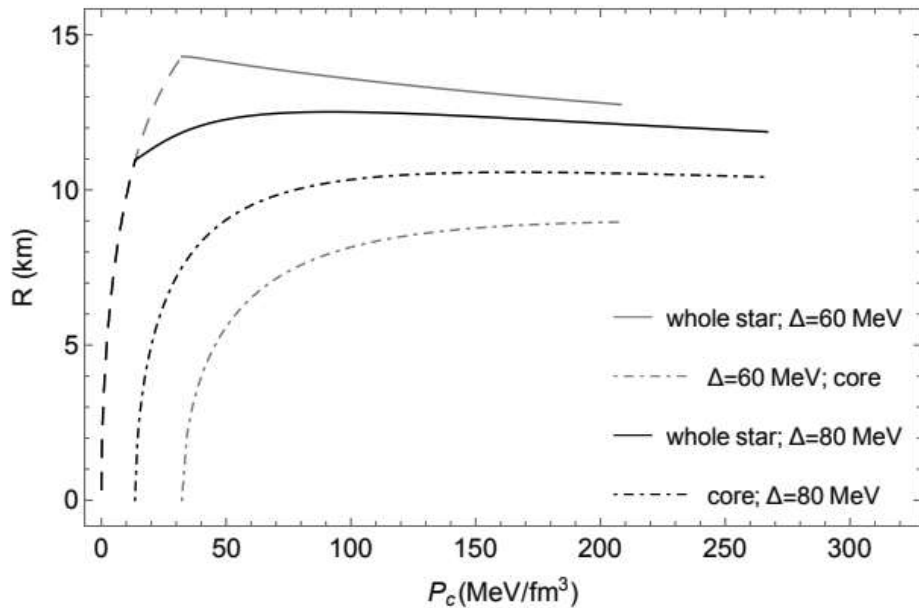
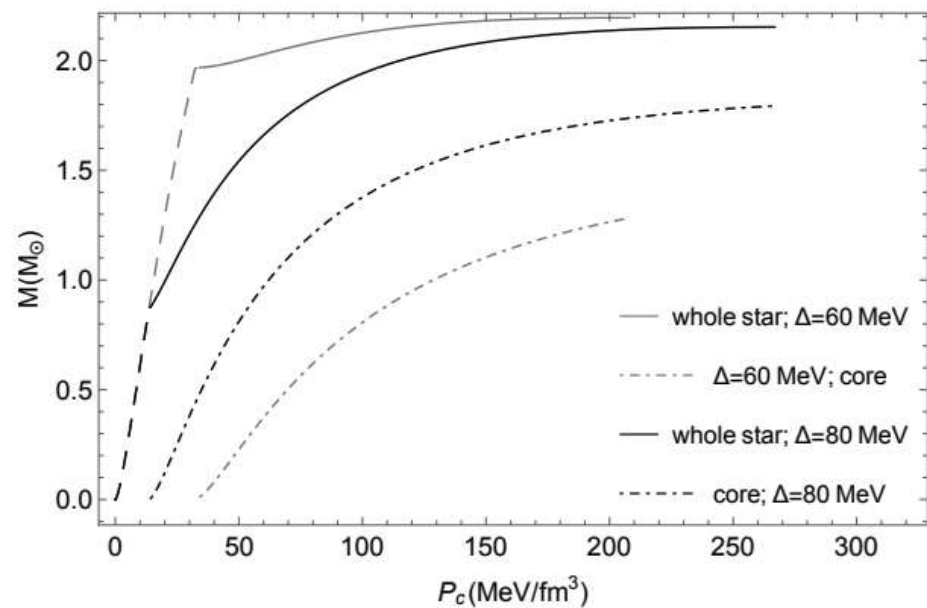


Figure 3: The relations of  $\rho(P)$  for stable hybrid strangeon stars, with  $\epsilon/N_q = 80/9$  MeV,  $n_s = 0.22$  (thin),  $0.30$  (thick)  $\text{fm}^{-3}$  for the strangeon composition, and  $B = 60$  (green dashed),  $80$  (black solid)  $\text{MeV}/\text{fm}^3$  for the CFL composition. Lines with darker colors denote larger  $\Delta$ , sampling  $60, 80$  MeV for green lines and  $100, 120$  MeV for black lines, respectively.





# Mixed Phase?

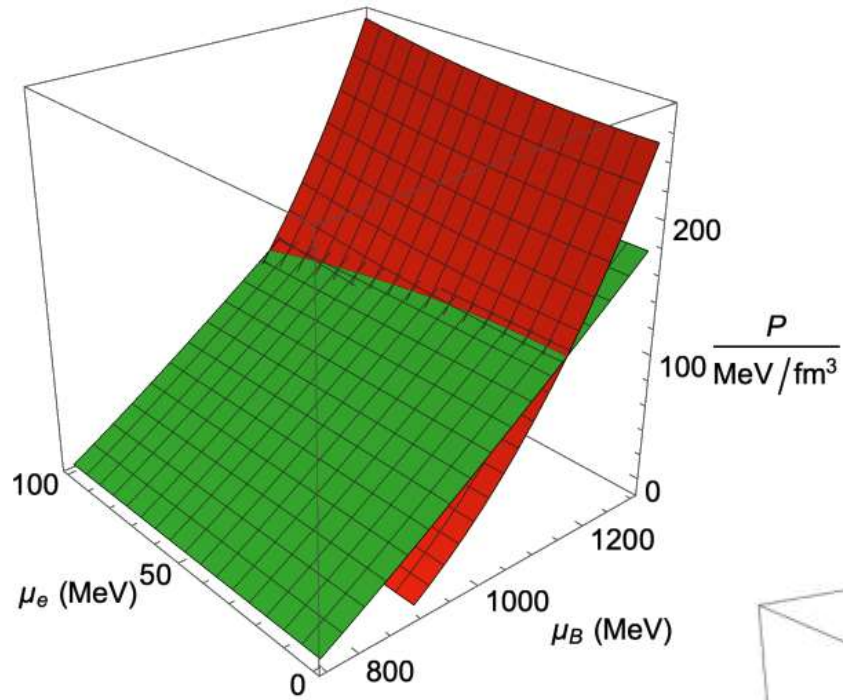
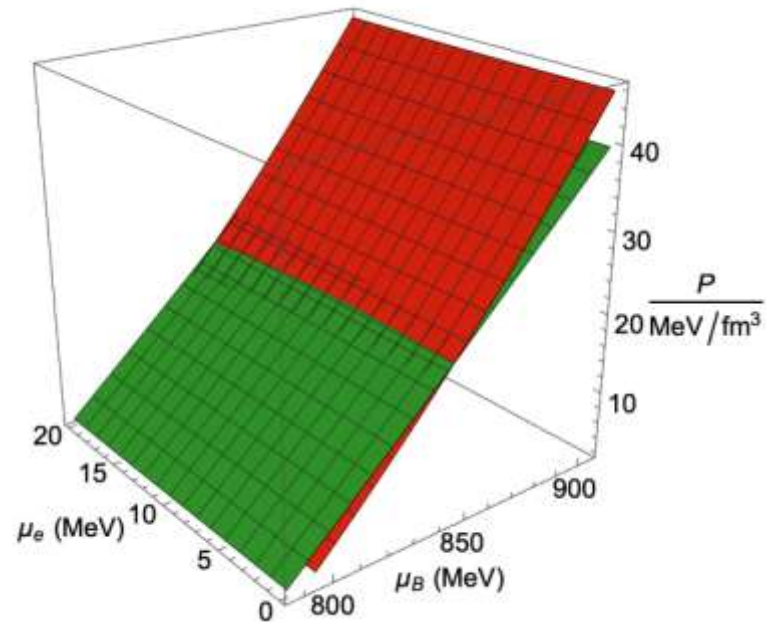


Figure 5: Pressure is plotted as a function of  $\mu_B$  and  $\mu_e$  for strangeon phase (green) and strange quark matter (red) of normal unpaired (top panel) and charged CFL phase (bottom panel) of  $\Delta = 100$  MeV. The mixed phase sits in the intersection of the two surfaces. For illustration, here  $B = 80 \text{ MeV}/\text{fm}^3$ ,  $m_s = 95 \text{ MeV}$  for the SQM phase and  $\epsilon/N_q = 80/9$ ,  $n_s = 0.3/\text{fm}^3$  for strangeon phase.



## New form of matter may lie just beyond the periodic table

Long Room, 15 Jun 2018

Currently, the heaviest element on the periodic table is oganesson, which has an atomic mass of 294 and was officially named in...

## bla Periódica

is ultrapesados que no 'funcionan' como la materia normal.

## "estranha matéria" que não cabe na tabela periódica

big bang Um grupo de físicos está a questionar a nossa compreensão sob

## e Neutron Stars, And It Breaks The Periodic Table

g of how quarks - a type of elementary particle - arrange themselves...

## New form of matter may lie just beyond the periodic table

Phys.org, 15 Jun 2018

Home Physics General Physics June 15, 2018 June 15, 2018 by Lisa Zyga, Phys.org feature Currently, the heaviest element on the...

europa press

## Una nueva forma de materia se vislumbra más allá de la tabla periódica

Europa Press, 18 Jun 2018

(EUROPA PRESS) - Científicos predicen que los elementos con masas atómicas superiores a aproximadamente 300 abre la posibil

# Weird new form of nuclear matter might lie just beyond experimenters' grasp

Rethink of "quark matter" also torpedoed notion of Earth-eating particle

15 MAY 2018 • BY [ADRIAN CHO](#)

penalty than previously thought, so high that cold quark matter should consist of just up and down quarks, the researchers report in a paper in press at *Physical Review Letters*.

Atomic nuclei clearly don't readily convert into up-down quark matter either. The team calculates that for masses below about 300 times that of the proton, ordinary nuclei are stable because effects akin to surface tension