



Dialogue at the Dream Field 2024
**May 10-15, 2024, Huaxi Guest Hotel &FAST-
Light Years Away, Guizhou, China**

The equation of state for the massive neutron stars

Jinniu Hu (胡金牛)

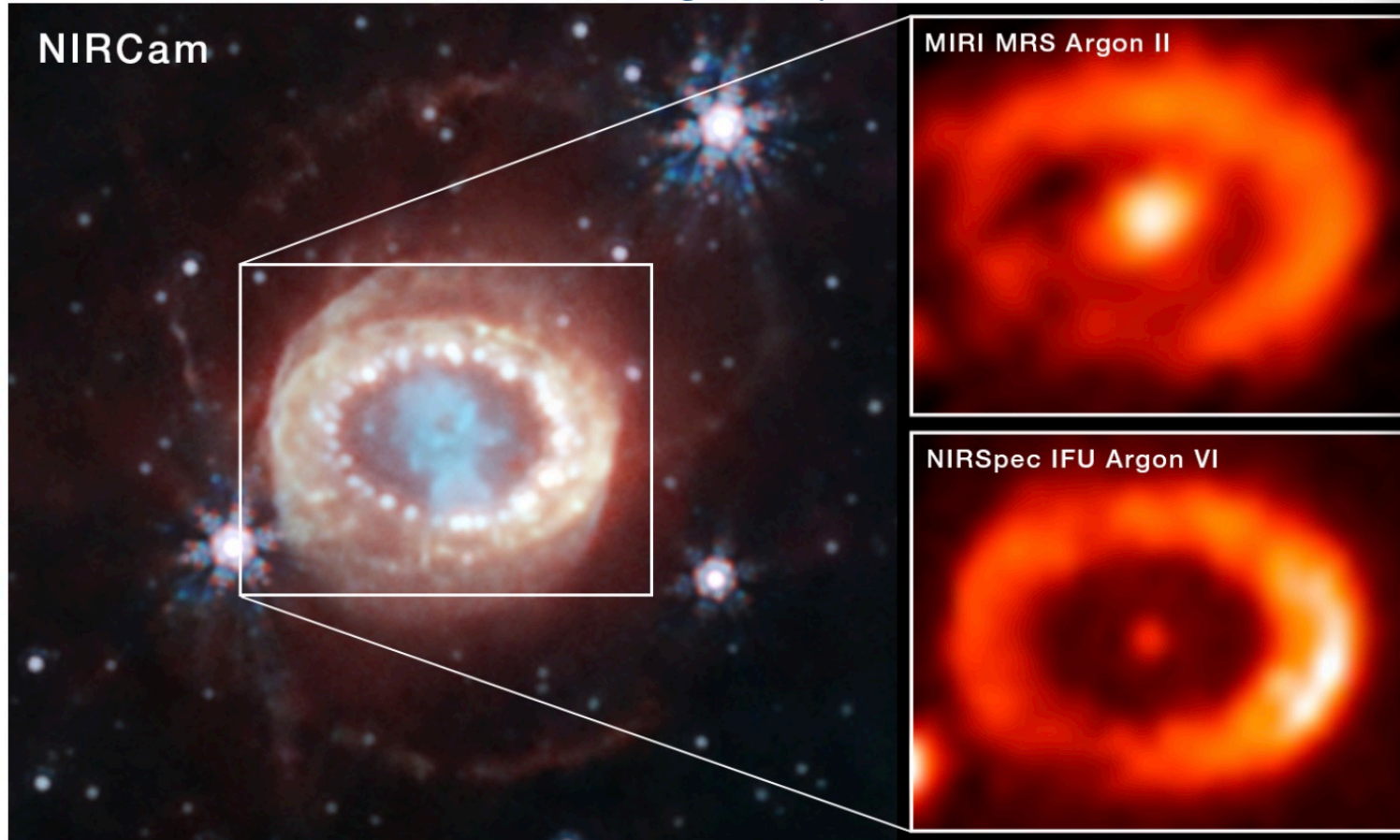
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- Introduction
- The massive neutron star from DDRMF
- The equation of state from machine learning
- Summary

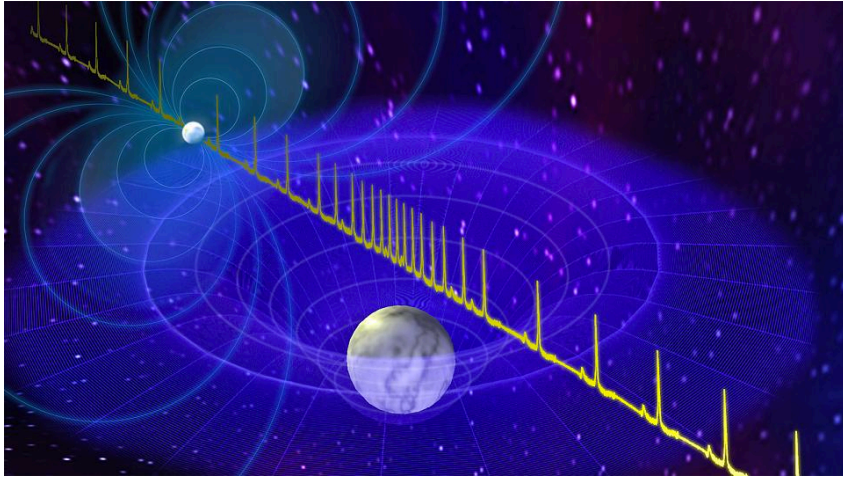
Webb Finds Evidence for Neutron Star at Heart of Young Supernova Remnant



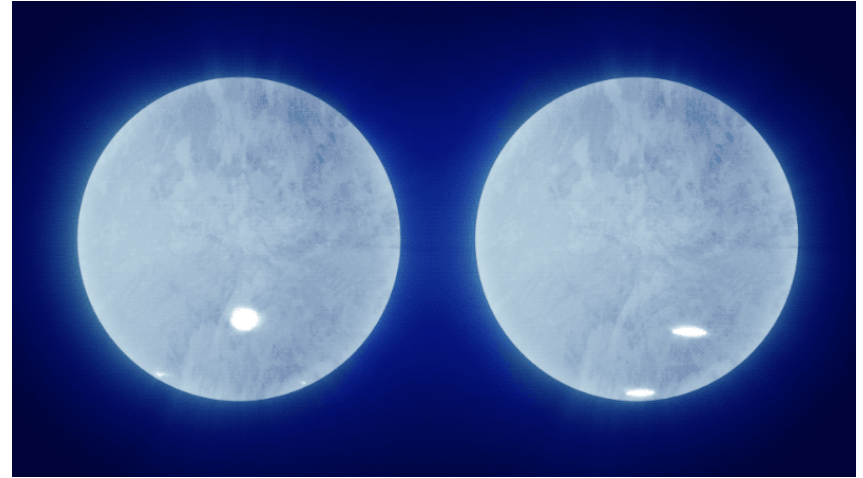
Credits from NASA

The radii and masses

Shapiro delay measurement



Neutron Star Interior Composition Explorer



The massive neutron star

PSR J1614-2230 ($1.928 \pm 0.017 M_{\odot}$),

P. B. Demorest, et al., *Nature*. 467(2010)108

E. Fonseca et al., *Astrophys. J.* 832, 167 (2016).

PSR J0348+0432 ($2.01 \pm 0.04 M_{\odot}$),

P. J. Antoniadis et al., *Science* 340, 1233232 (2013).

PSR J0740+6620 ($2.08 \pm 0.07 M_{\odot}$)

H. T. Cromartie et al., *Nat. Astron.* 4, 72 (2020)

M. C. Miller et al. *Astrophys. J. Lett.* 918(2021)L28

The NICER Measurement

PSR J0740+6620 ($2.08 \pm 0.07 M_{\odot}$,

12.35 ± 0.75 km)

H. T. Cromartie et al., *Nat. Astron.* 4, 72 (2020)

M. C. Miller et al. *Astrophys. J. Lett.* 918(2021)L28

PSR J0030+0451 ($1.44 \pm 0.15 M_{\odot}$,

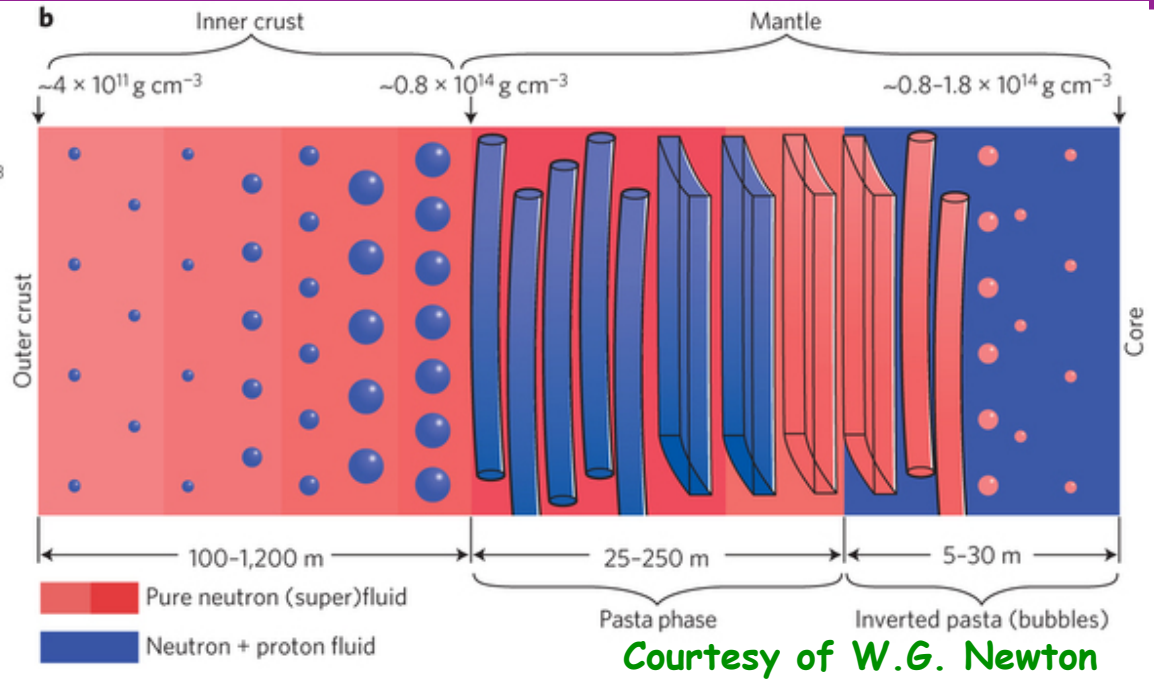
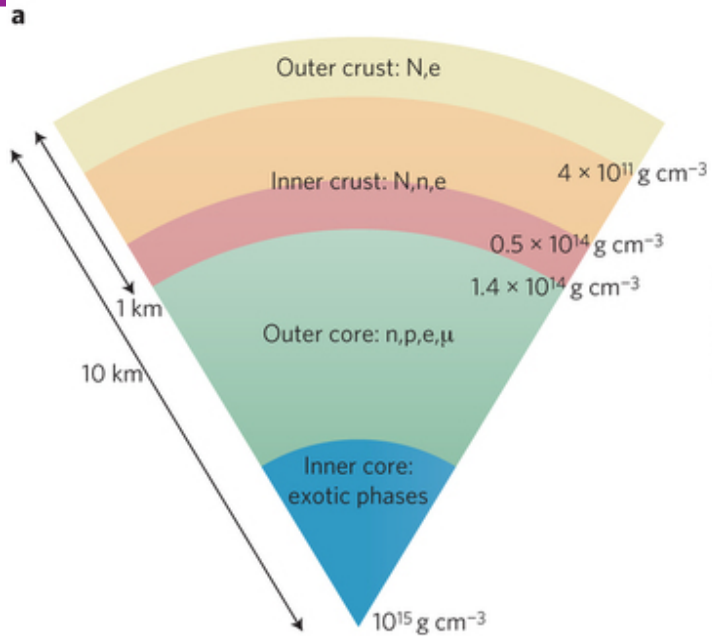
13.02 ± 1.24 km)

M. C. Miller et al. *Astrophys. J. Lett.* 887(2019)L42

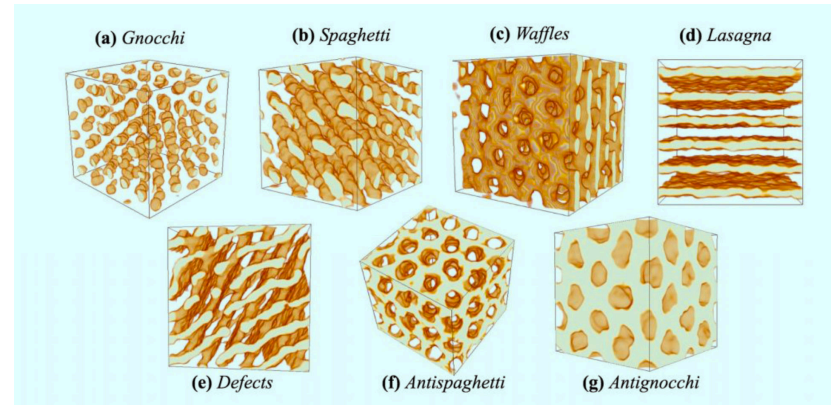
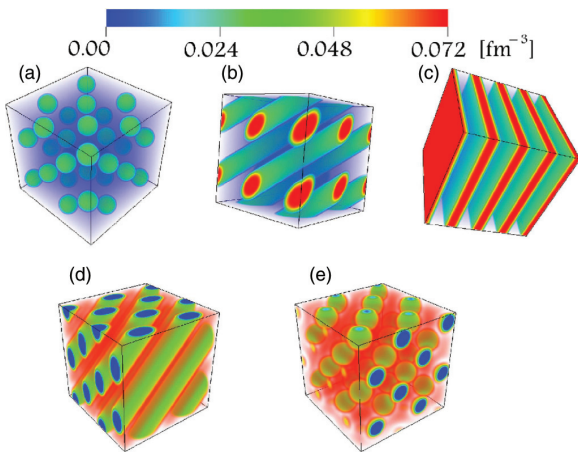
Neutron star structure



南开大学



Courtesy of W.G. Newton



M.Okamoto, T.Maruyama, K.Yabana,
T.Tatsumi, Phys. Rev. C 88 (2013) 025801

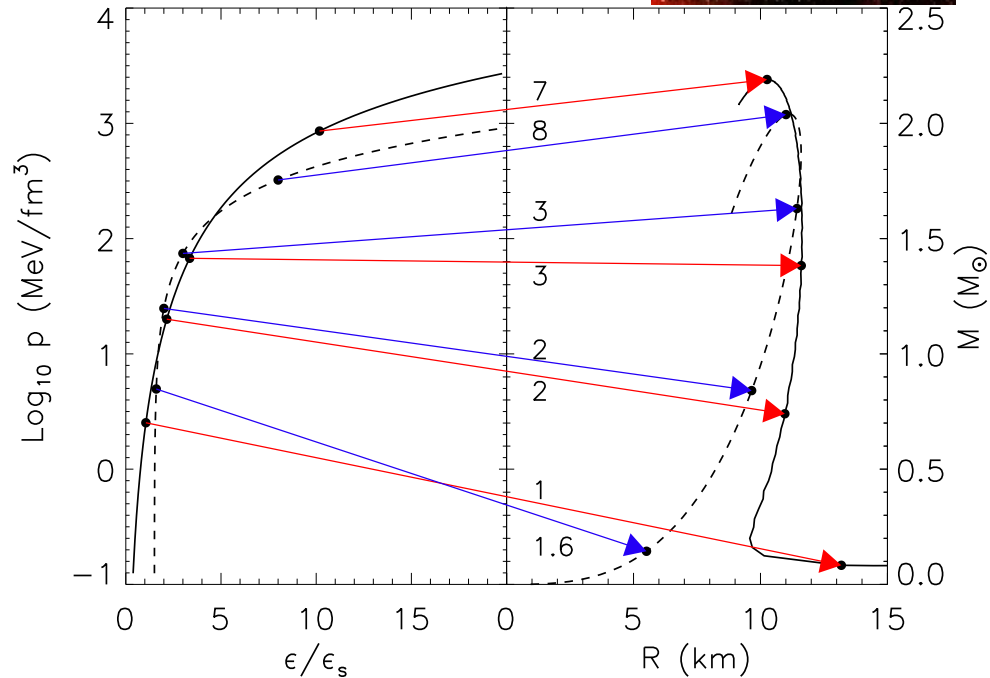
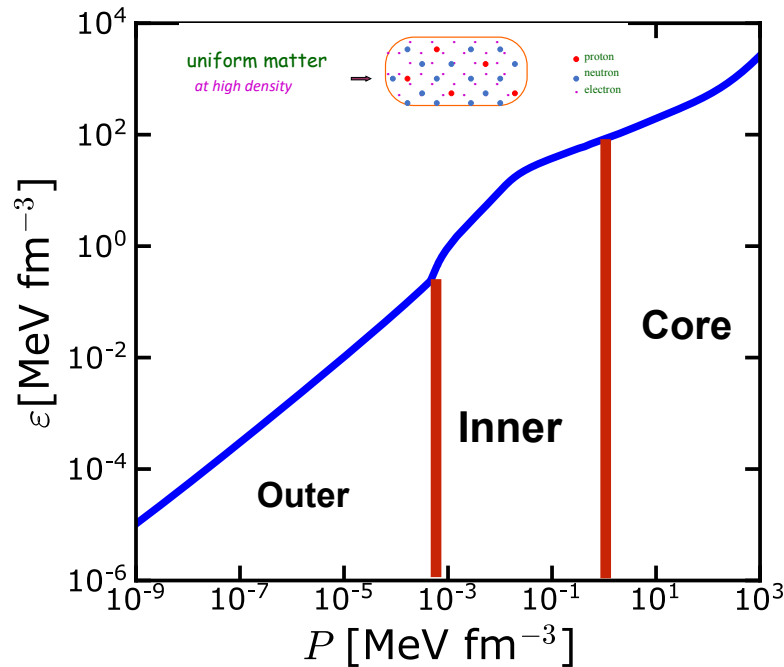
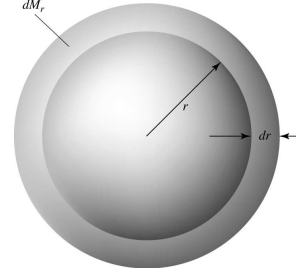
M. E. Caplan and C. J. Horowitz,
Rev. Mod. Phys. 89(2017)041002

Tolman–Oppenheimer–Volkoff equation

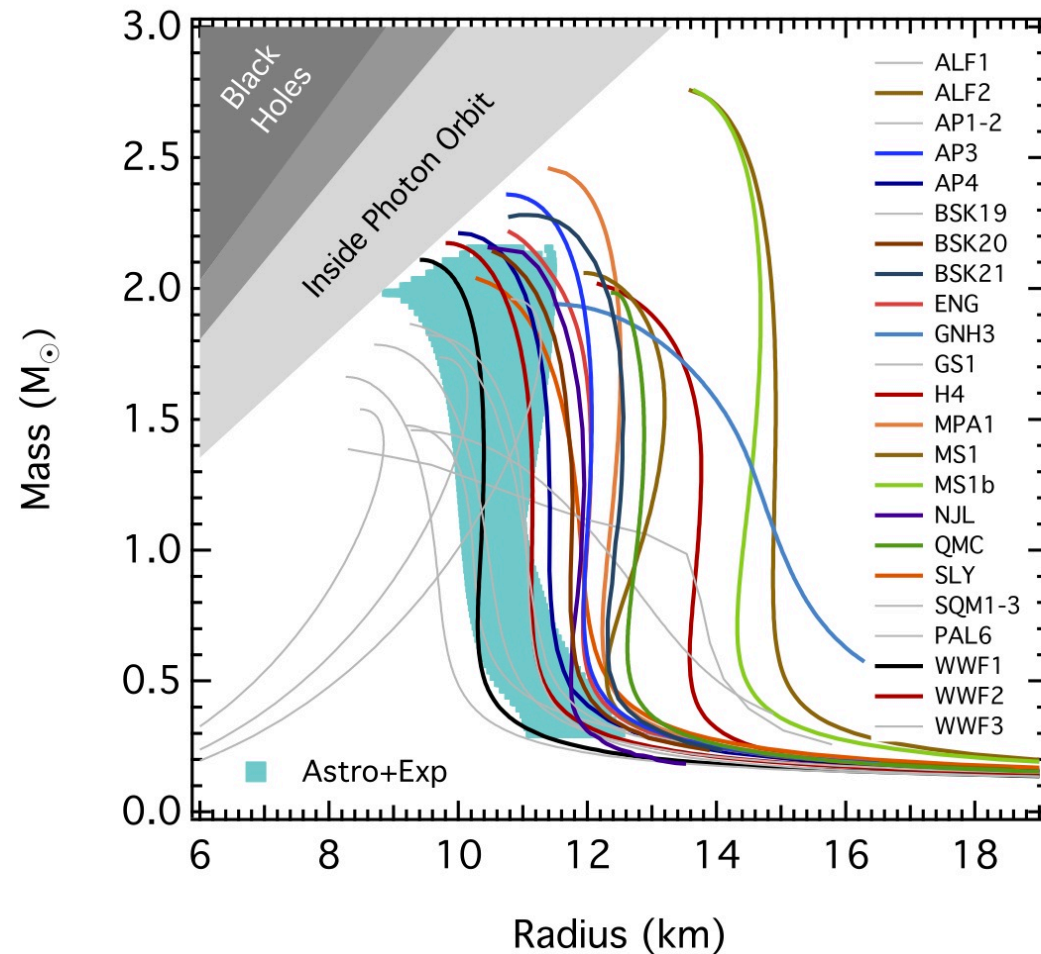
$$\frac{dP}{dr} = -\frac{G\rho M(r)}{r^2} \left(1 + \frac{P}{\rho c^2}\right) \left(1 + \frac{4\pi Pr^3}{M(r)c^2}\right) \left(1 - \frac{2GM(r)}{c^2 r}\right)^{-1} dM_r$$

$$M(r) = 4\pi \int_0^r \xi^2 \rho(\xi) d\xi$$

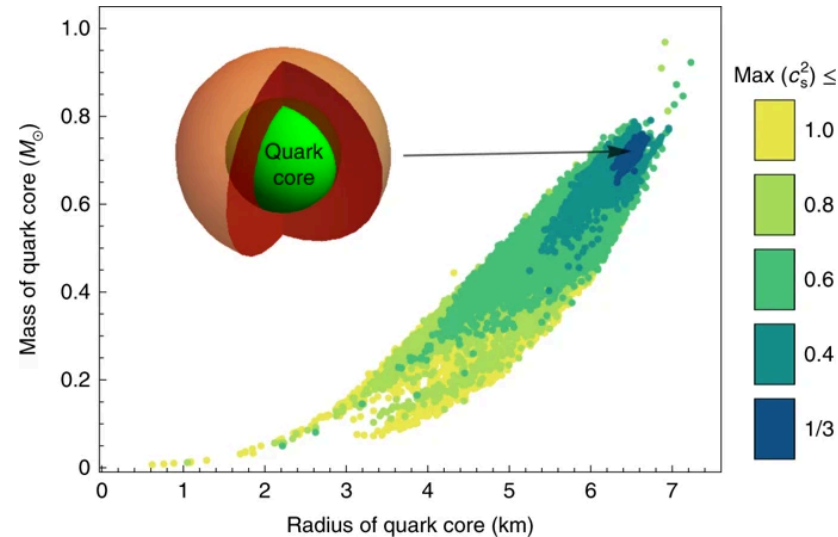
$$\rho(r) = \varepsilon(r)/c^2$$



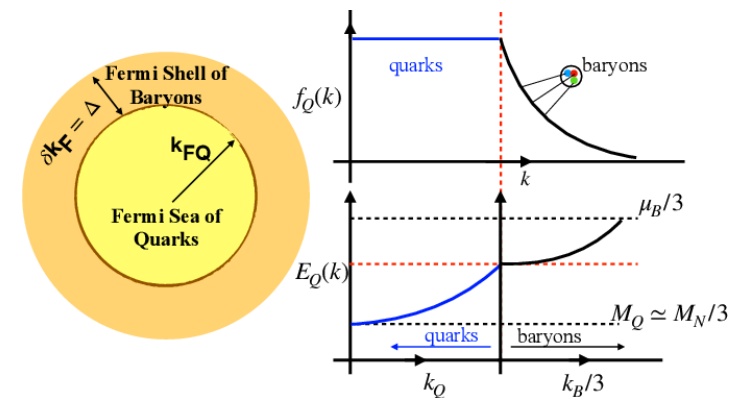
The equations of state



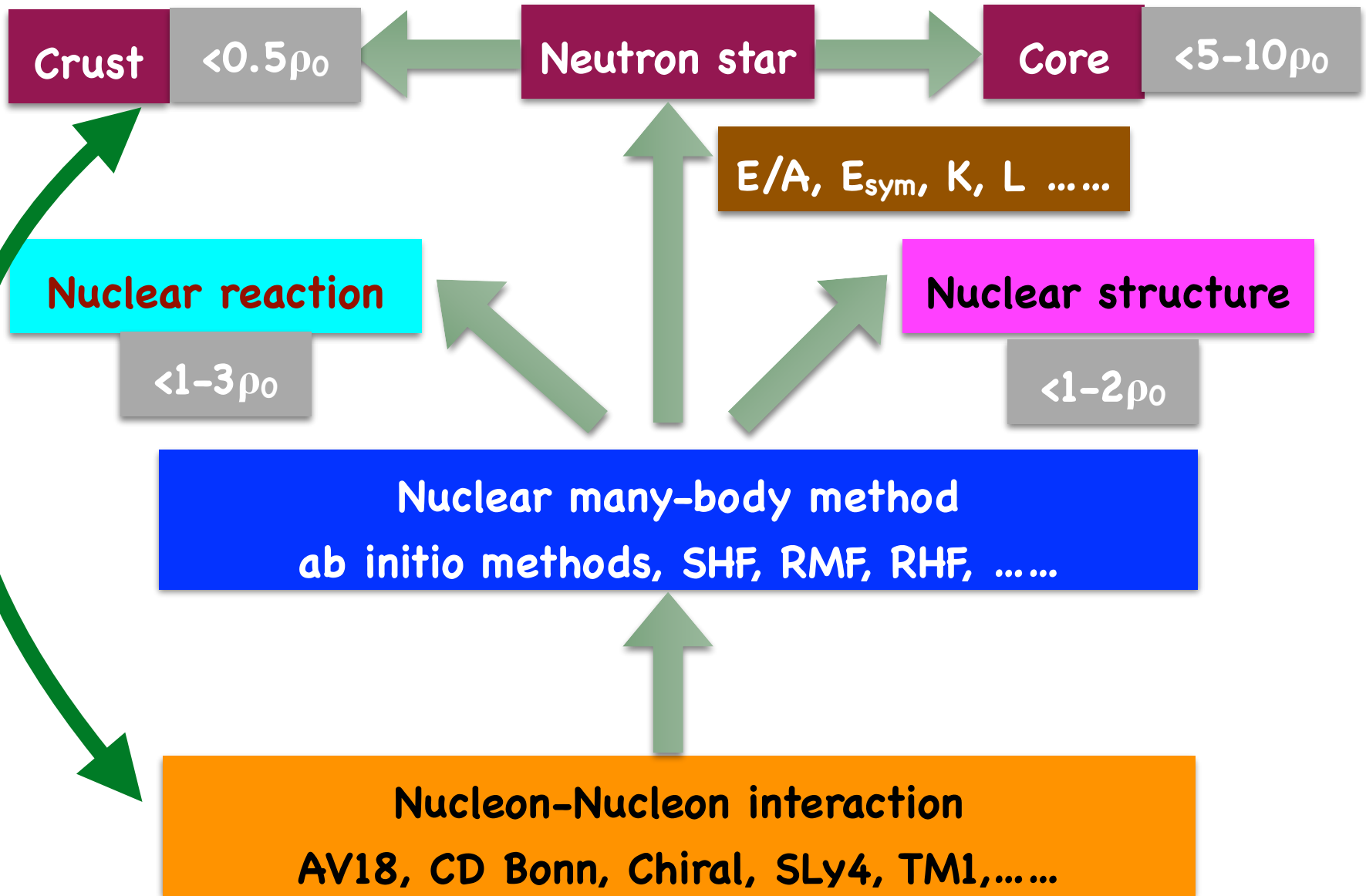
F. Oezel and P. Freire *Annu. Rev. Astron. Astrophys.* 54 (2016)401



E. Annala et al. *Nat. Phys.* (2020)



L. McLerran and S. Reddy *Phys. Rev. Lett.* 122 (2019)122701





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The GW190814-2.6 M_{\odot} object

THE ASTROPHYSICAL JOURNAL LETTERS, 896:L44 (20pp), 2020 June 20

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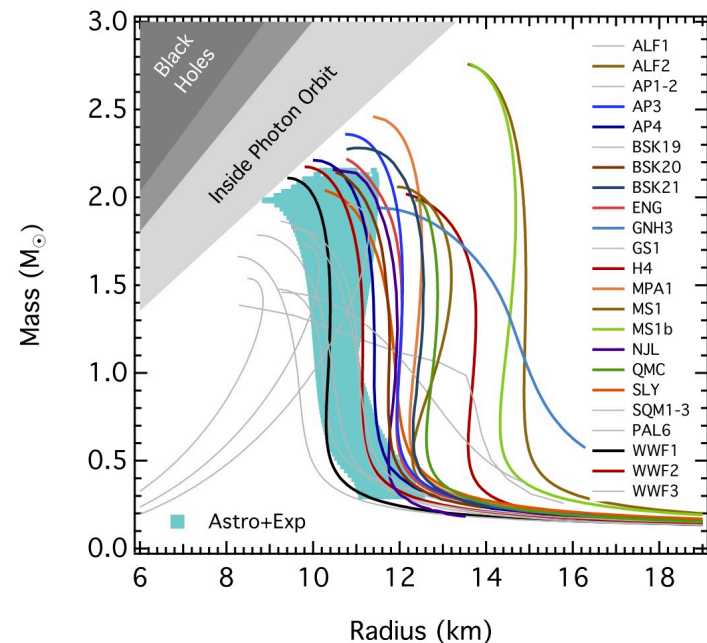
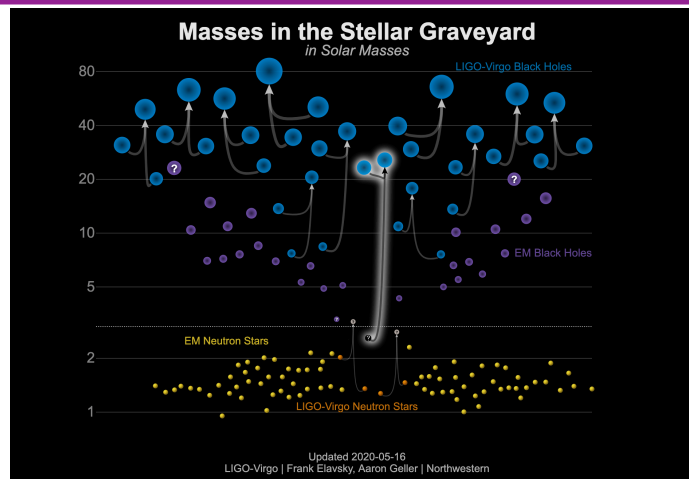
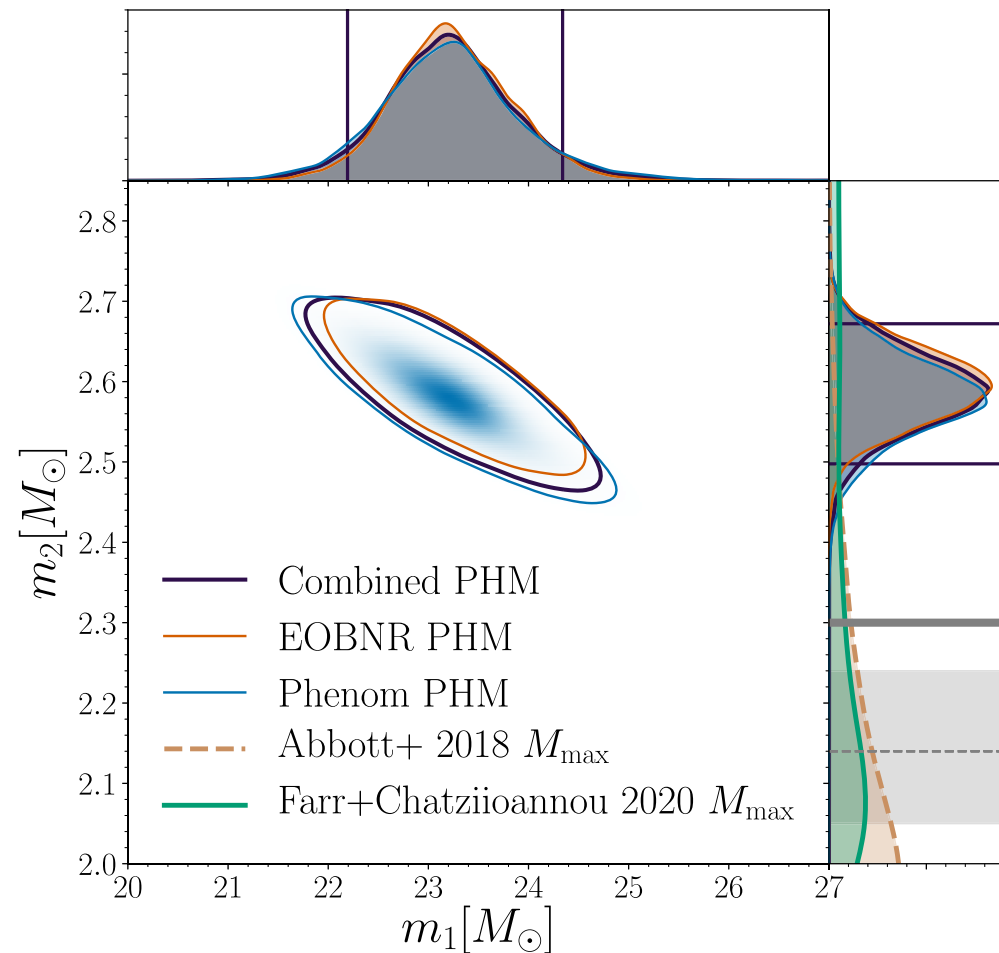
OPEN ACCESS

<https://doi.org/10.3847/2041-8213/ab960f>



CrossMark

GW190814: Gravitational Waves from the Coalescence of a 23 Solar Mass Black Hole with a 2.6 Solar Mass Compact Object



F. Oezel and P. Freire *Annu. Rev. Astron. Astrophys.* **54** (2016)401

The Lagrangian of DDRMF model

$$\begin{aligned}\mathcal{L}_{DD} = & \sum_{i=p, n} \bar{\psi}_i \left[\gamma^\mu \left(i\partial_\mu - \Gamma_\omega(\rho_B)\omega_\mu - \frac{\Gamma_\rho(\rho_B)}{2}\gamma^\mu \vec{\rho}_\mu \vec{\tau} \right) - \left(M - \Gamma_\sigma(\rho_B)\sigma - \Gamma_\delta(\rho_B)\vec{\delta} \vec{\tau} \right) \right] \psi_i \\ & + \frac{1}{2} (\partial^\mu \sigma \partial_\mu \sigma - m_\sigma^2 \sigma^2) + \frac{1}{2} (\partial^\mu \vec{\delta} \partial_\mu \vec{\delta} - m_\delta^2 \vec{\delta}^2) \\ & - \frac{1}{4} W^{\mu\nu} W_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} \vec{R}^{\mu\nu} \vec{R}_{\mu\nu} + \frac{1}{2} m_\rho^2 \vec{\rho}_\mu \vec{\rho}^\mu,\end{aligned}$$

The density dependent coupling constants

for σ and ω mesons

$$\Gamma_i(\rho_B) = \Gamma_i(\rho_{B0}) f_i(x), \quad \text{with} \quad f_i(x) = a_i \frac{1 + b_i(x + d_i)^2}{1 + c_i(x + d_i)^2}, \quad x = \rho_B / \rho_{B0},$$

for ρ and δ mesons

$$\Gamma_i(\rho_B) = \Gamma_i(\rho_{B0}) \exp[-a_i(x - 1)].$$

K. Huang, J. N. Hu, Y. Zhang, and H. Shen, *Astrophys. J.* 904(2020)39

The Lagrangian of DDRMF model

$$\begin{aligned} \mathcal{L}_{DD} = & \sum_{i=p, n} \bar{\psi}_i \left[\gamma^\mu \left(i\partial_\mu - \Gamma_\omega(\rho_B)\omega_\mu - \frac{\Gamma_\rho(\rho_B)}{2}\gamma^\mu \vec{\rho}_\mu \vec{\tau} \right) - \left(M - \Gamma_\sigma(\rho_B)\sigma - \Gamma_\delta(\rho_B)\vec{\delta} \vec{\tau} \right) \right] \psi_i \\ & + \frac{1}{2} (\partial^\mu \sigma \partial_\mu \sigma - m_\sigma^2 \sigma^2) + \frac{1}{2} (\partial^\mu \vec{\delta} \partial_\mu \vec{\delta} - m_\delta^2 \vec{\delta}^2) \\ & - \frac{1}{4} W^{\mu\nu} W_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} \vec{R}^{\mu\nu} \vec{R}_{\mu\nu} + \frac{1}{2} m_\rho^2 \vec{\rho}_\mu \vec{\rho}^\mu, \end{aligned}$$

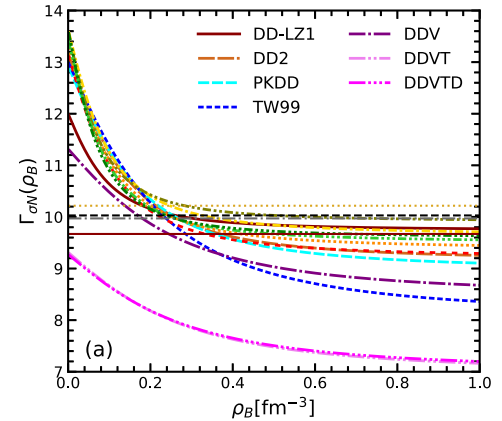
The density dependent coupling constants

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$$\Gamma_i(\rho_B) = \Gamma_i(\rho_{B0}) f_i(x), \quad \text{with} \quad f_i(x) = a_i \frac{1 + b_i(x + d_i)^2}{1 + c_i(x + d_i)^2}, \quad x = \rho_B / \rho_{B0},$$

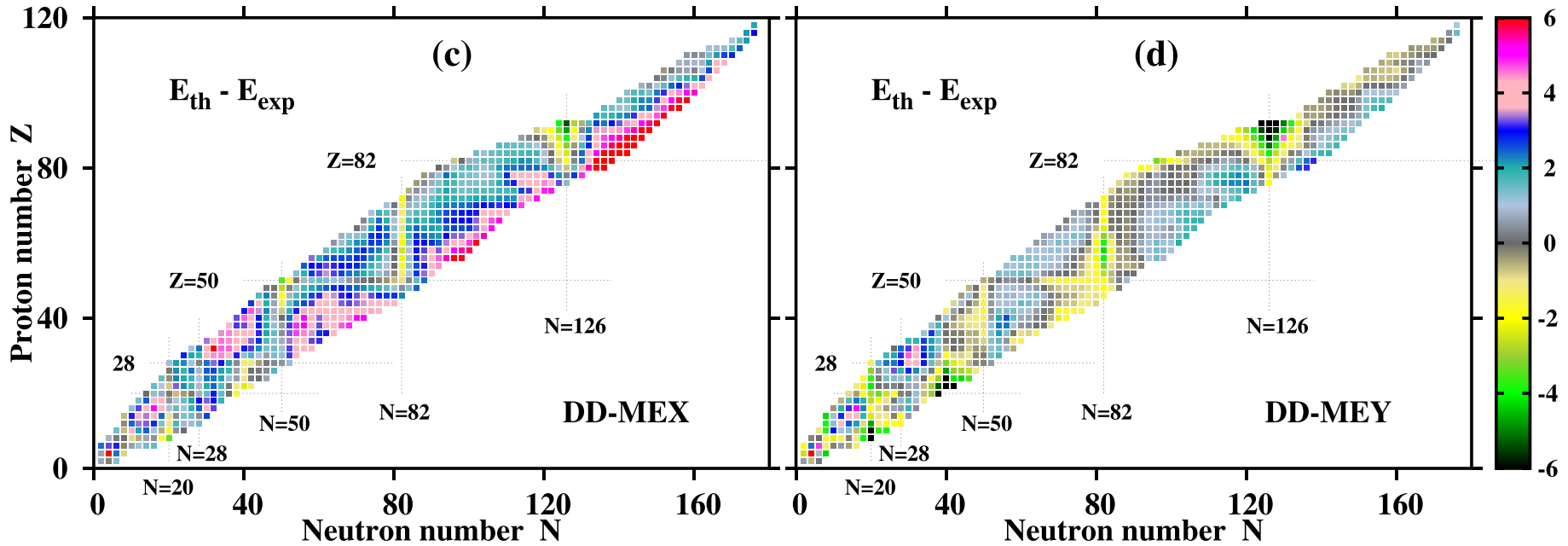
for ρ and δ mesons

$$\Gamma_i(\rho_B) = \Gamma_i(\rho_{B0}) \exp[-a_i(x - 1)].$$



K. Huang, J. N. Hu, Y. Zhang, and H. Shen, *Astrophys. J.* 904(2020)39

1	ΔE_{rms} (MeV)	$\Delta(S_{2n})_{\text{rms}}$ (MeV)	$\Delta(S_{2p})_{\text{rms}}$ (MeV)	$\Delta(r_{\text{ch}})_{\text{rms}}$ (fm)	K_0 (MeV)	J (MeV)	L_0 (MeV)
2	3	4	5	6	7	8	
DD-ME2 [22]	2.436 (2.300)	1.056 (0.854)	0.949 (0.750)	0.0266 (0.0262)	250.9	32.9	49.4
DD-MEX [18]	2.849 (2.963)	1.095 (0.972)	0.978 (0.847)	0.0247 (0.0249)	267.0	32.9	47.8
DD-MEX1	1.637 (1.539)	1.045 (0.873)	0.896 (0.704)	0.0261 (0.0263)	291.8	32.5	51.8
DD-MEX2	2.236 (1.791)	1.228 (0.913)	1.271 (0.928)	0.0466 (0.0488)	255.8	35.9	85.3
DD-MEY	1.734 (1.414)	1.259 (0.876)	1.026 (0.755)	0.0264 (0.0244)	265.8	32.8	51.8

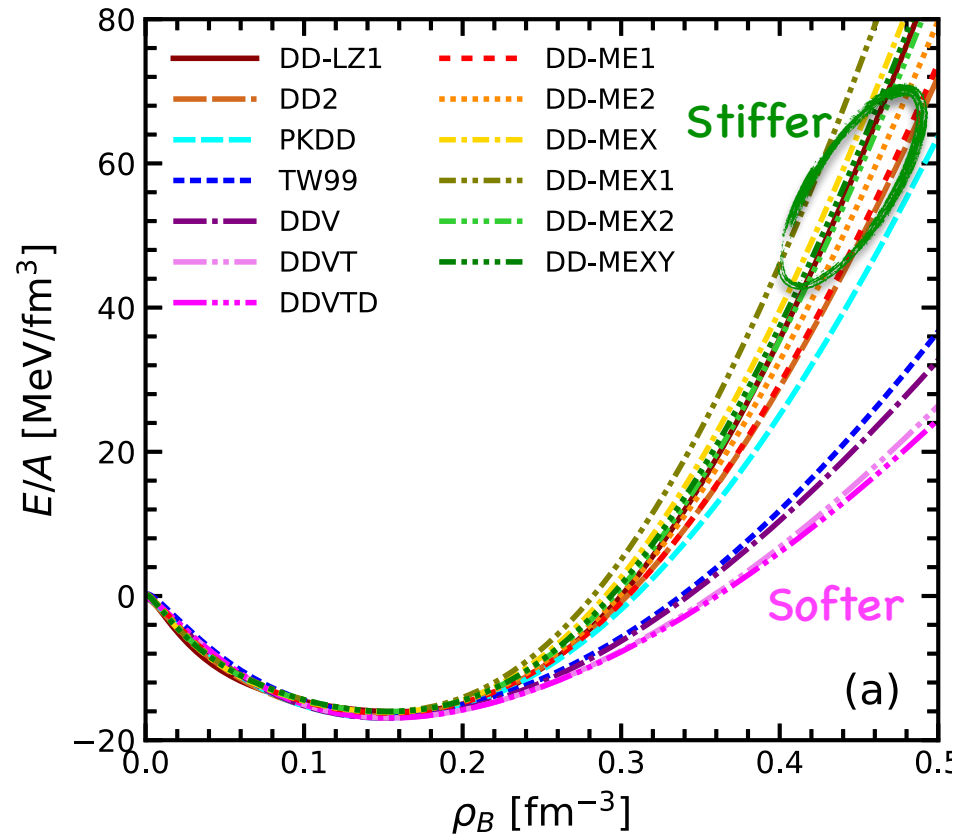


A. Taninah and A. V. Afanasjev, Phys. Rev. C107(2023)L041301

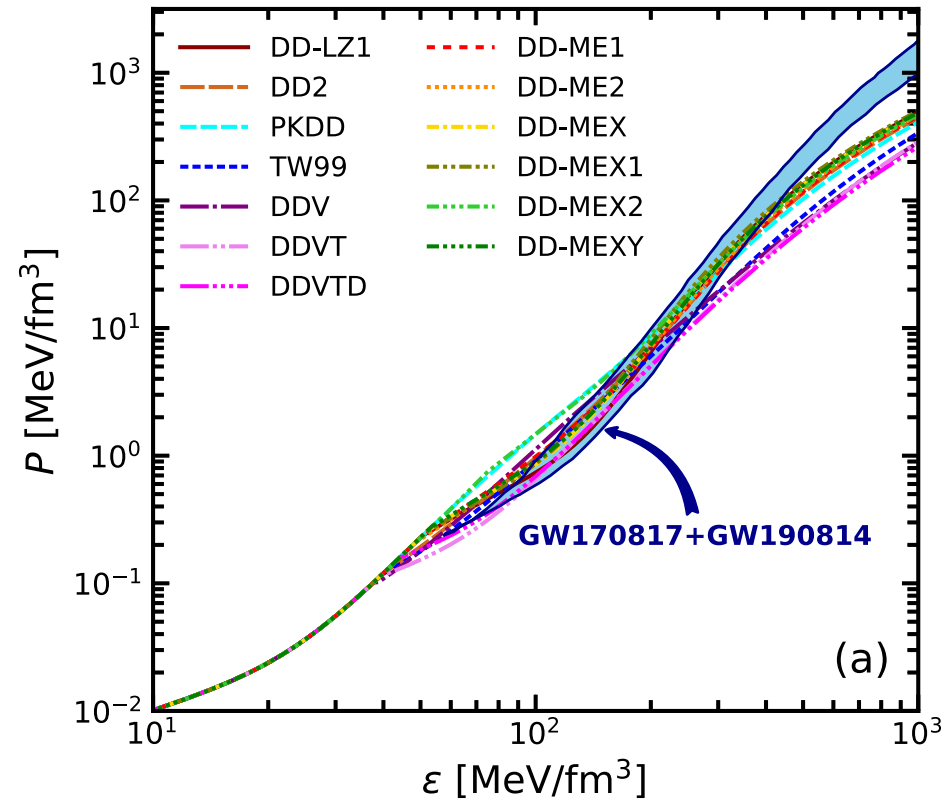
	Saturation density	Binding energy per nucleon		Symmetry energy	Slope of Symmetry energy	Effective Mass	
	$\rho_{B0}[\text{fm}^{-3}]$	$E/A[\text{MeV}]$	$K_0[\text{MeV}]$	$E_{\text{sym}}[\text{MeV}]$	$L_0[\text{MeV}]$	M_n^*/M	M_p^*/M
DD-LZ1	0.1581	-16.0598	231.1030	31.3806	42.4660	0.5581	0.5581
DD-MEX	0.1519	-16.0973	267.3819	32.2238	46.6998	0.5554	0.5554
DD-MEX1	0.1505	-16.0368	291.1968	31.8312	53.4254	0.5709	0.5709
DD-MEX2	0.1520	-16.0376	255.0925	35.2921	86.8244	0.5780	0.5780
DD-MEXY	0.1535	16.0243	367.9365	32.0355	53.2101	0.5811	0.5811
DD-ME2	0.1520	-16.1418	251.3062	32.3094	51.2653	0.5718	0.5718
DD-ME1	0.1522	-16.2328	245.6657	33.0899	55.4634	0.5776	0.5776
DD2	0.1491	-16.6679	242.8509	31.6504	54.9529	0.5627	0.5614
PKDD	0.1495	-16.9145	261.7912	36.7605	90.1204	0.5713	0.5699
TW99	0.1530	-16.2472	240.2022	32.7651	55.3095	0.5549	0.5549
DDV	0.1511	-16.9279	239.9522	33.5969	69.6813	0.5869	0.5852
DDVT	0.1536	-16.9155	239.9989	31.5585	42.3414	0.6670	0.6657
DDVTD	0.1536	-16.9165	239.9137	31.8168	42.5829	0.6673	0.6660

K. Huang, J. N. Hu, Y. Zhang, and H. Shen, *Astrophys. J.* 904(2020)39
 K. Huang, J. N. Hu, Y. Zhang, and H. Shen, *Nucl. Phys. Rev.* 39(2022)35

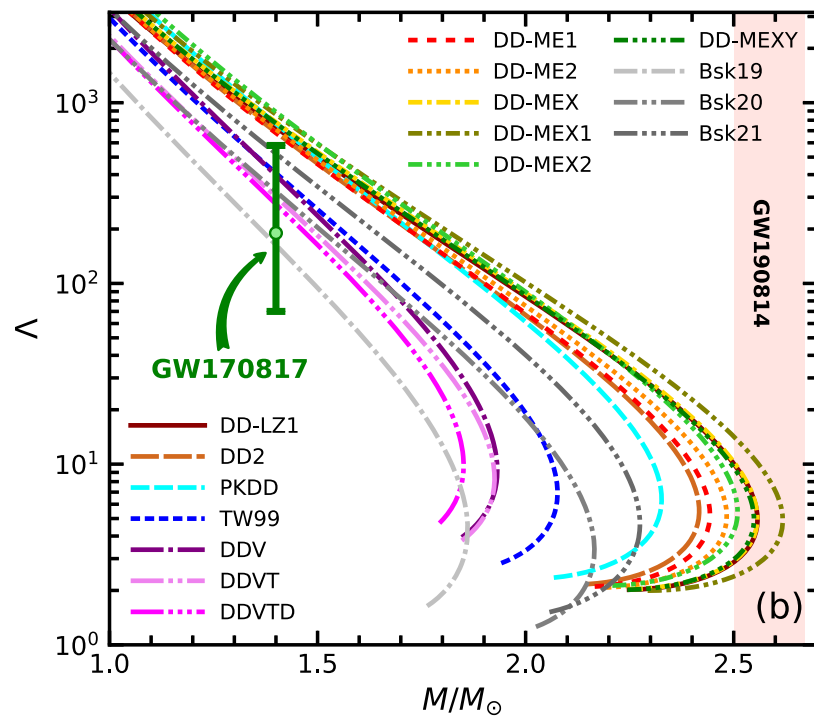
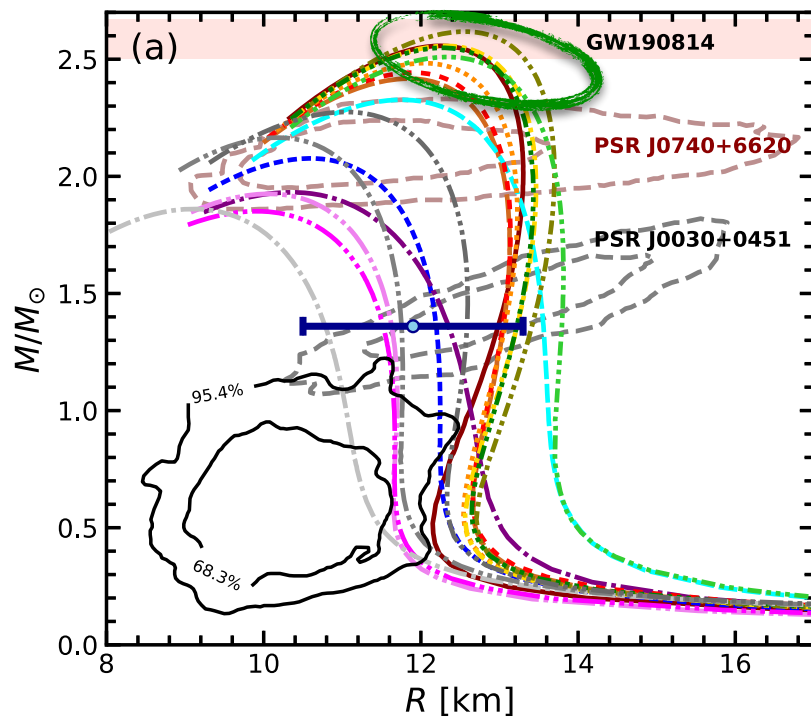
Symmetric nuclear matter



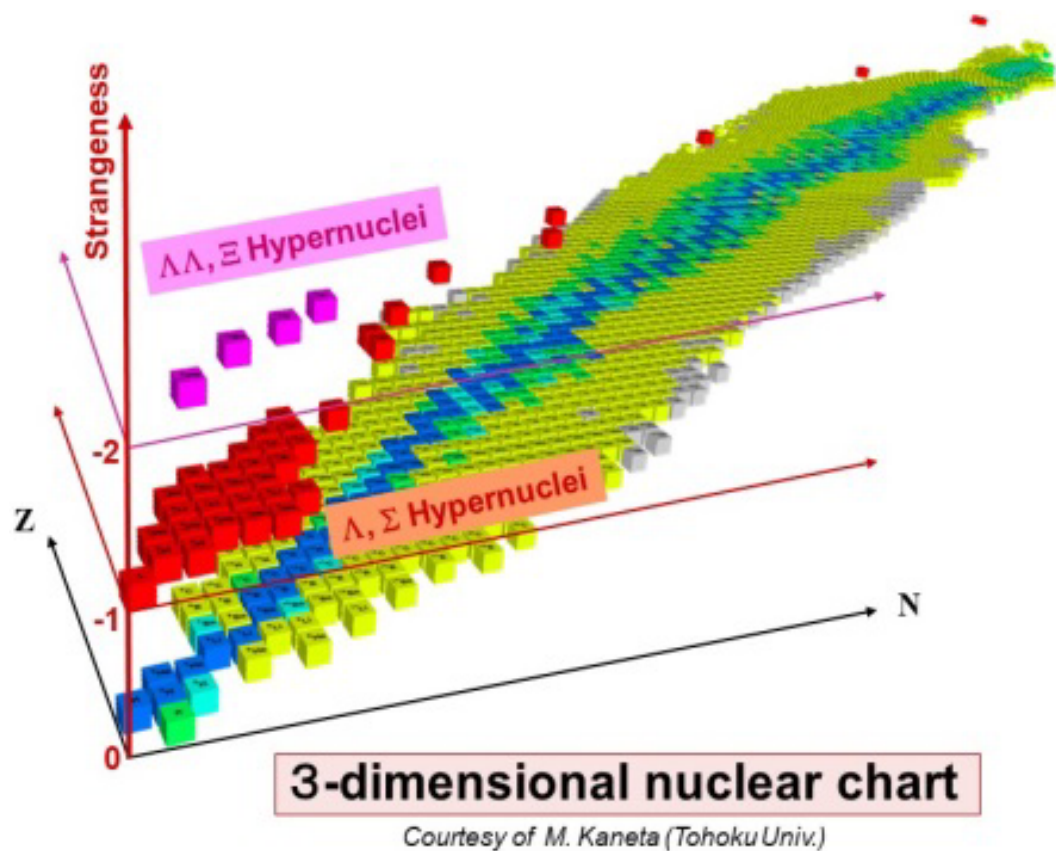
Neutron star matter



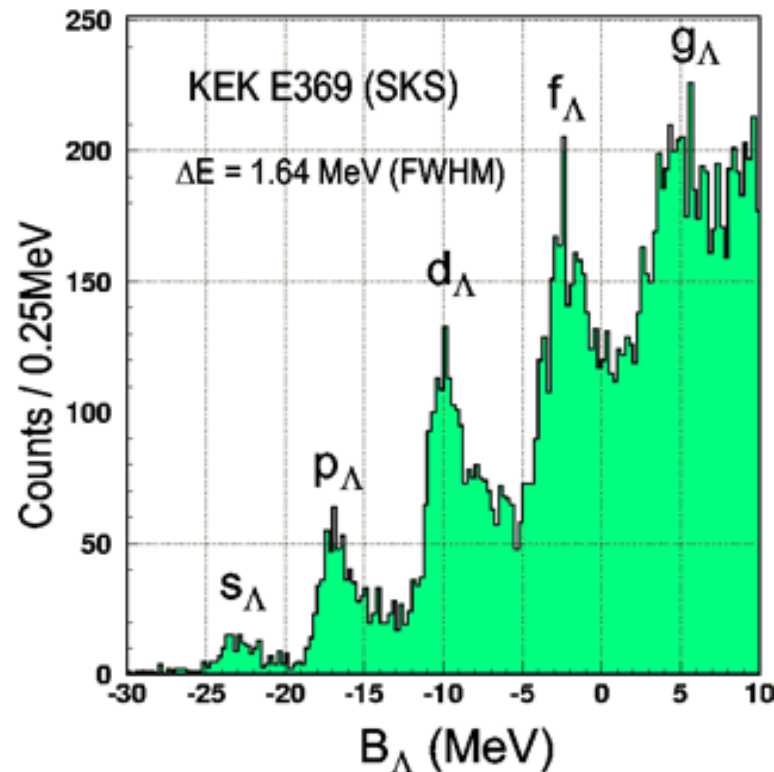
K. Huang, J. N. Hu, Y. Zhang, and H. Shen, *Astrophys. J.* 904(2020)39



	DD-LZ1	DD2	DD-ME1	DD-ME2	DD-MEX	DDV	DDVT	DDVTD
M_{\max}/M_{\odot}	2.5545	2.4168	2.4426	2.4829	2.5566	1.9317	1.9251	1.8507
R_{\max} [km]	12.178	11.826	11.885	12.012	12.274	10.336	10.023	9.850
ρ_{\max} [fm $^{-3}$]	0.786	0.845	0.832	0.813	0.777	1.188	1.237	1.306
$R_{1.4}$ [km]	12.864	12.938	12.931	12.961	13.118	12.195	11.511	11.396
$\Lambda_{1.4}$	727.071	639.032	686.786	730.737	790.051	390.005	301.388	274.908



$^{89}\text{Y} (\pi^+, K^+) ^{89}_{\Lambda}\text{Y}$



The interaction between vector mesons and baryons

$$\Gamma_{\omega\Lambda} = \Gamma_{\omega\Sigma} = 2\Gamma_{\omega\Xi} = \frac{2}{3}\Gamma_{\omega N},$$

$$2\Gamma_{\phi\Sigma} = \Gamma_{\phi\Xi} = -\frac{2\sqrt{2}}{3}\Gamma_{\omega N}, \quad \Gamma_{\phi N} = 0,$$

$$\Gamma_{\rho\Lambda} = 0, \quad \Gamma_{\rho\Sigma} = 2\Gamma_{\rho\Xi} = 2\Gamma_{\rho N},$$

$$\Gamma_{\delta\Lambda} = 0, \quad \Gamma_{\delta\Sigma} = 2\Gamma_{\delta\Xi} = 2\Gamma_{\delta N}.$$

The hyperon-nucleon potentials

$$U_Y^N(\rho_{B0}) = -R_{\sigma Y}\Gamma_{\sigma N}(\rho_{B0})\sigma_0 + R_{\omega Y}\Gamma_{\omega N}(\rho_{B0})\omega_0,$$

Empirical potential values

$$U_{\Lambda}^N = -30 \text{ MeV}, \quad U_{\Sigma}^N = +30 \text{ MeV} \quad U_{\Xi}^N = -14 \text{ MeV}$$

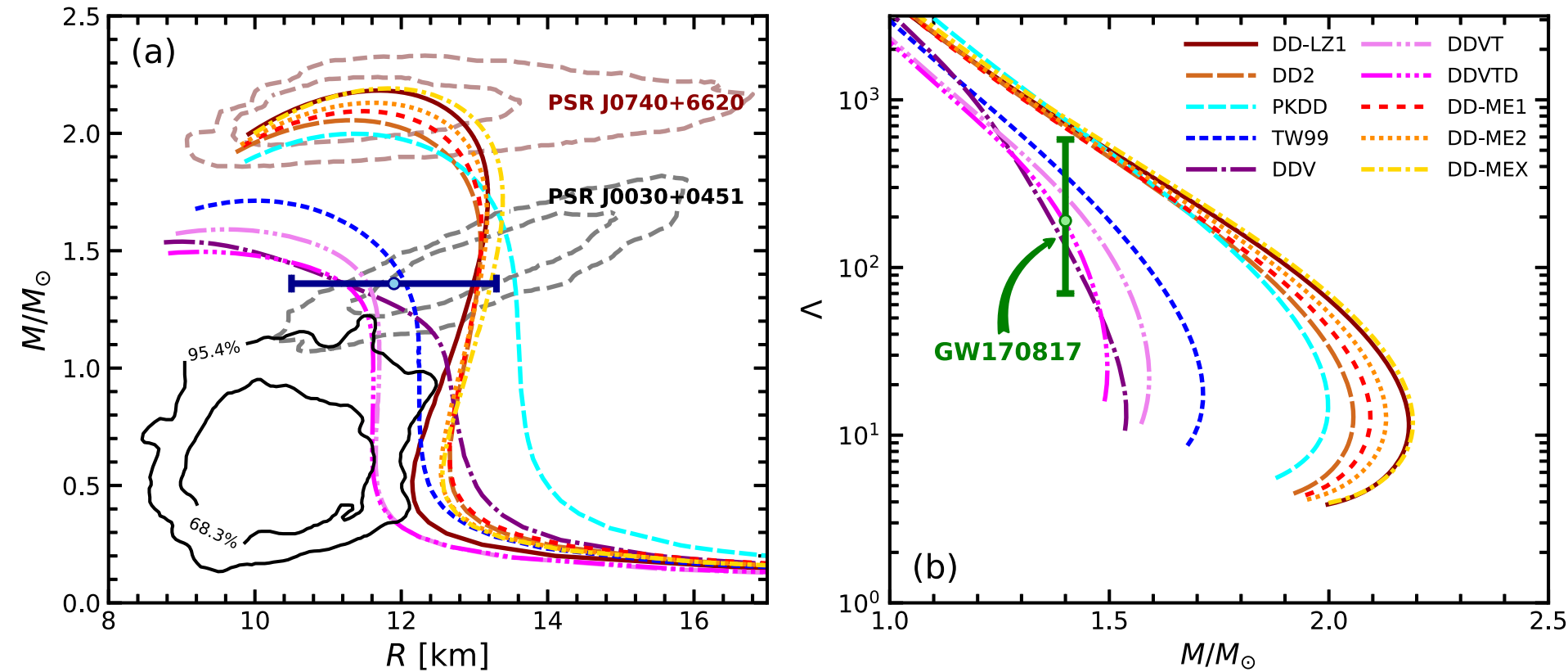
The hyperon-hyperon potentials

$$U_{\Lambda}^{\Lambda}(\rho_{B0}) = -R_{\sigma\Lambda}\Gamma_{\sigma N}(\rho_{B0})\sigma_0 - R_{\sigma^*\Lambda}\Gamma_{\sigma N}(\rho_{B0})\sigma_0^* \\ + R_{\omega Y}\Gamma_{\omega N}(\rho_{B0})\omega_0 + R_{\phi\Lambda}\Gamma_{\omega N}(\rho_{B0})\phi_0,$$

$$U_{\Lambda}^{\Lambda}(\rho_{B0}) = -10 \text{ MeV},$$

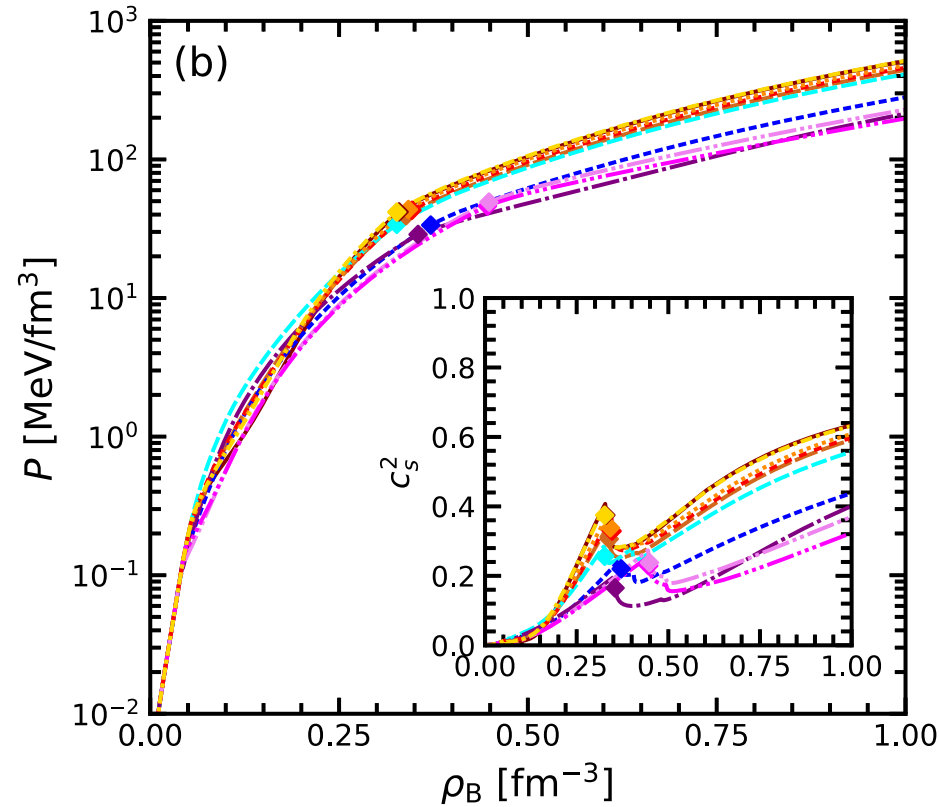
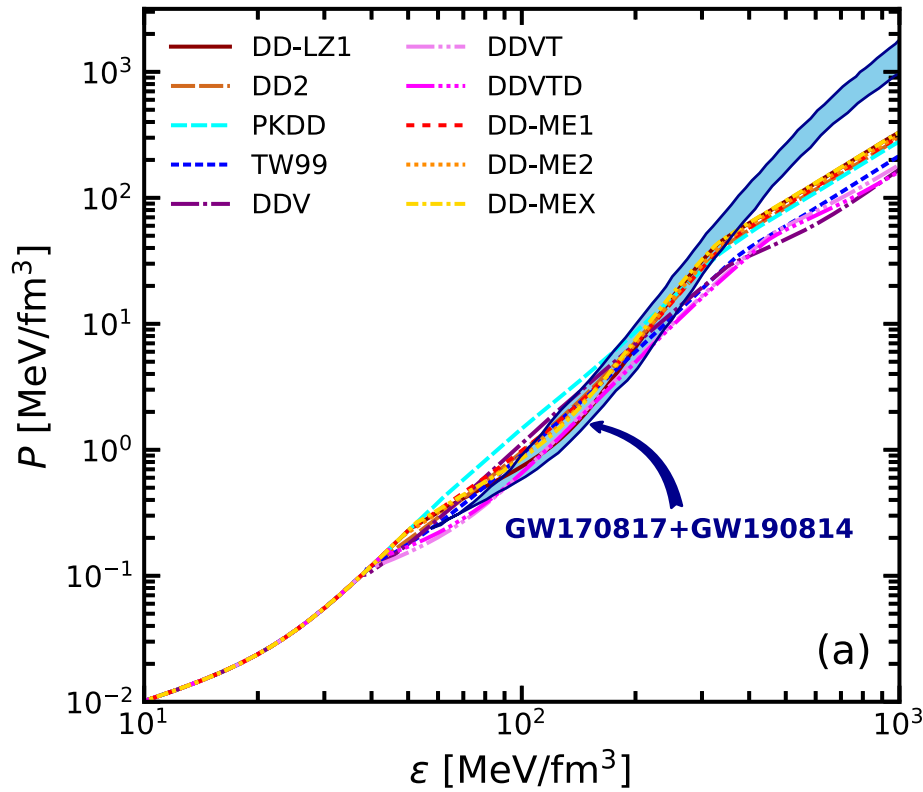
The radius-mass relation of neutron star and hyperonic star

K. Huang, J. N. Hu, Y. Zhang, and H. Shen, Nucl. Phys. Rev. 39(2022)35



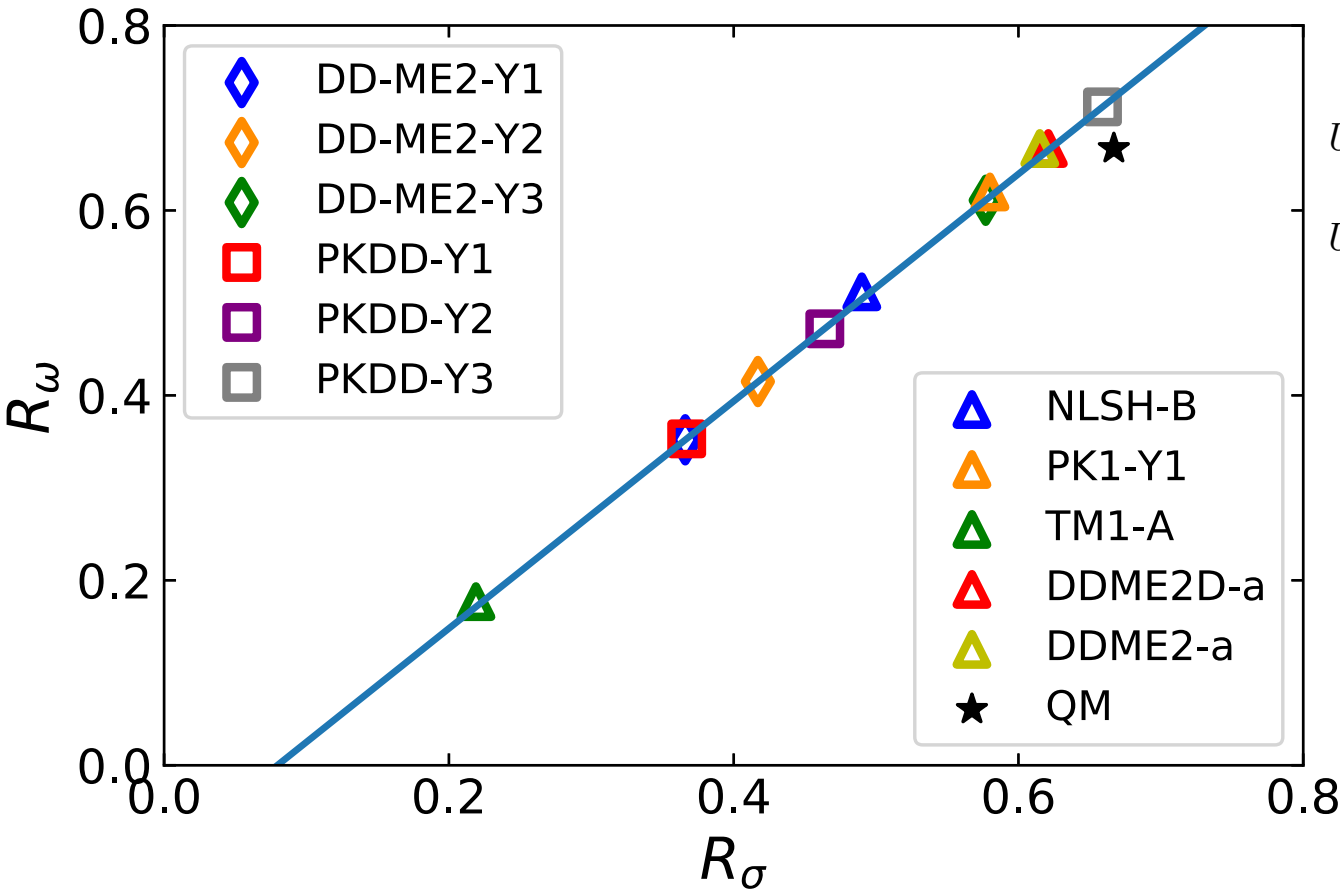
The equations of state for hyperonic star

K. Huang, J. N. Hu, Y. Zhang, and H. Shen, Nucl. Phys. Rev. 39(2022)35





The correlations between the coupling strengths



$$U_\Lambda^N = -30 \text{ MeV}, U_\Sigma^N = +30 \text{ MeV}$$

$$U_\Xi^N = -14 \text{ MeV},$$

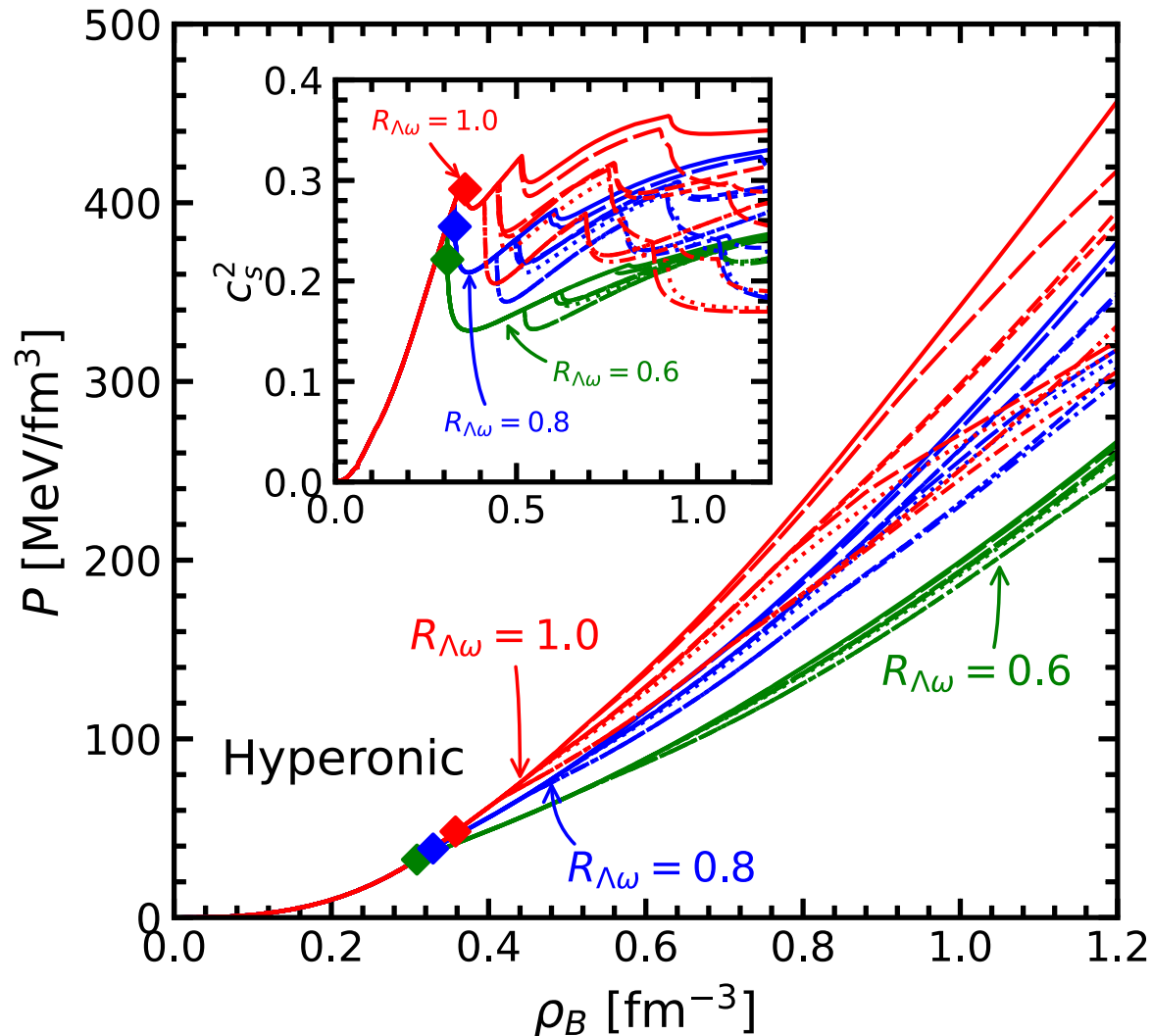
$$R_{\omega\Lambda} = 1.24969R_{\sigma\Lambda} - 0.10946,$$

$$R_{\omega\Sigma} = 1.24969R_{\sigma\Sigma} + 0.10946,$$

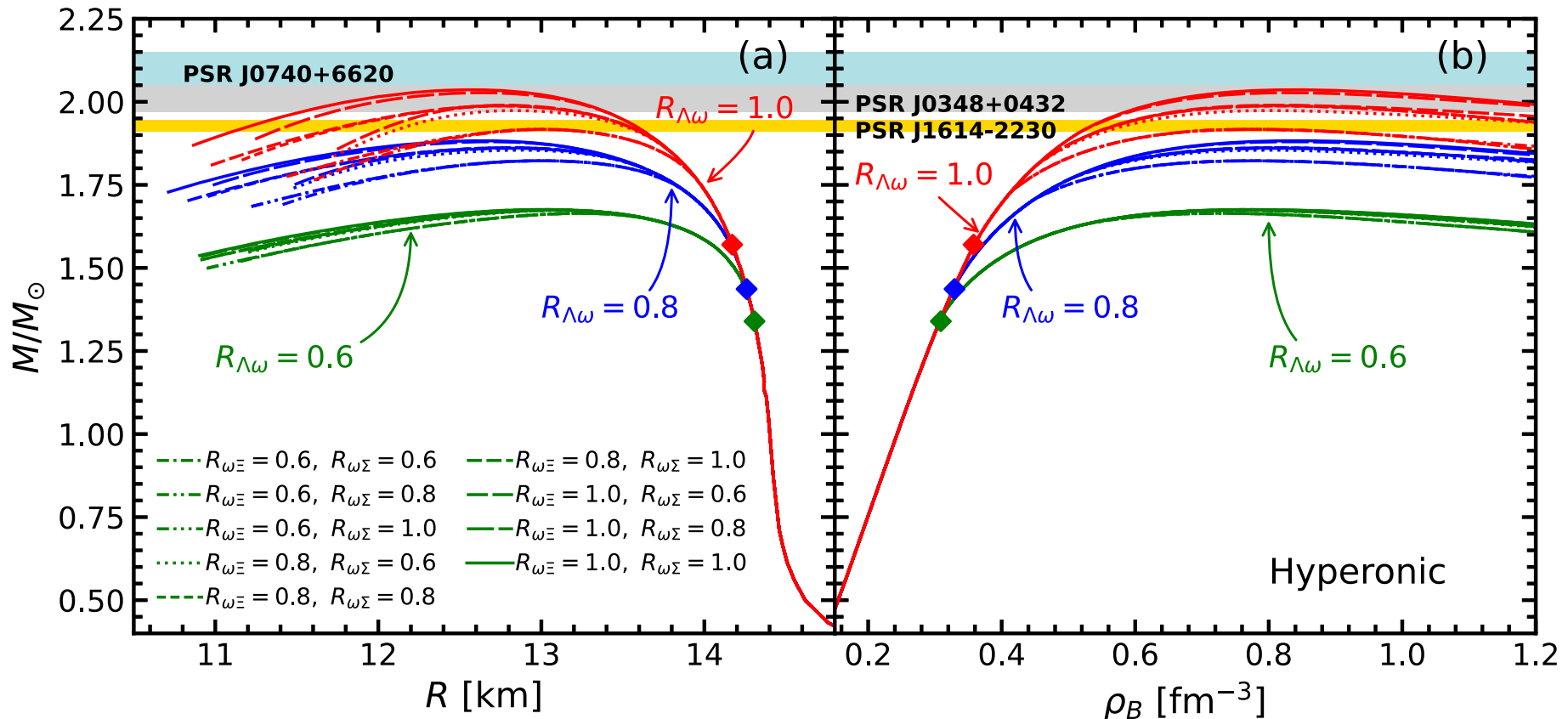
$$R_{\omega\Xi} = 1.24969R_{\sigma\Xi} - 0.05108.$$

Y. T. Rong, Z. H. Tu, S. G. Zhou, Phys. Rev. C 104(2021)054321

The EoSs with different the coupling strengths



The Mass-radius relation with different the coupling strengths





- Introduction
- The massive neutron star from DDRMF
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- Summary

➤ Parametric Bayesian inference

F. Özel, G. Baym, and T. Güver, *Phys. Rev. D* 82 (2010) 101301(R)

A. W. Steiner, J. M. Lattimer, and E. Brown, *Astrophys. J* 722(2010)33

D. Alvarez-Castillo, et al. *Eur. Phys. J. A* 52 (2016) 69

Z. Miao, J. L. Jiang, A. Li, and L. W. Chen, *Astrophys. J. Lett.* 917 (2021) L22

➤ Nonparametric Bayesian inference

P. Landry and R. Essick, *Phys. Rev. D* 99 (2019) 084049

P. Landry, R. Essick, and K. Chatziioannou, *Phys. Rev. D* 101 (2020) 123007

Support Vector Machine

P. Magierski and P. H. Heenen, *Phys. Rev. C* 65(2002)045804

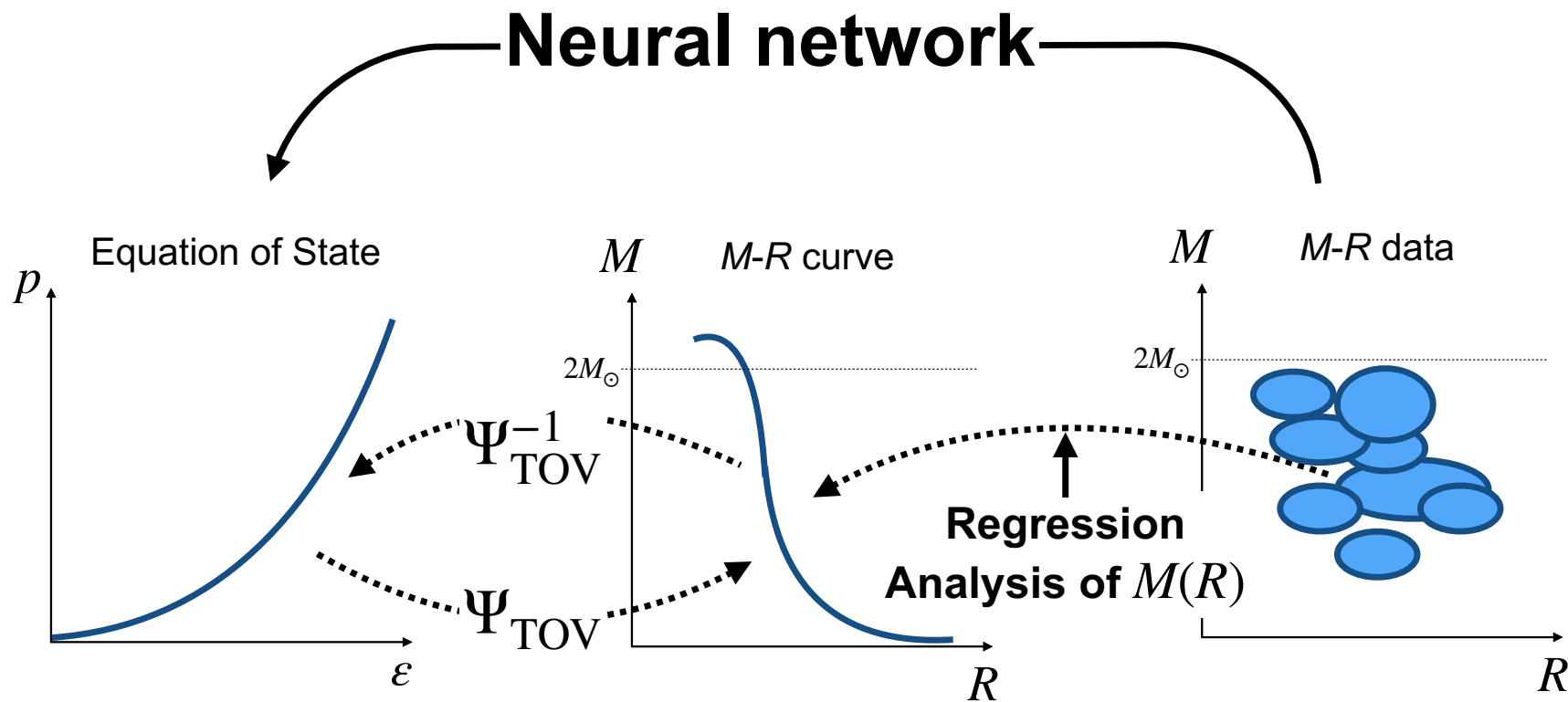
➤ Deep neural network (Parametric EOS)

Y. Fujimoto, K. Fukushima, K. Murase, *Phys. Rev. D*, 98 (2018) 023019

Y. Fujimoto, K. Fukushima, K. Murase, *JHEP*, 2021 (2021) 1

D. Farrell, et al. arXiv: 2209.02817

.....

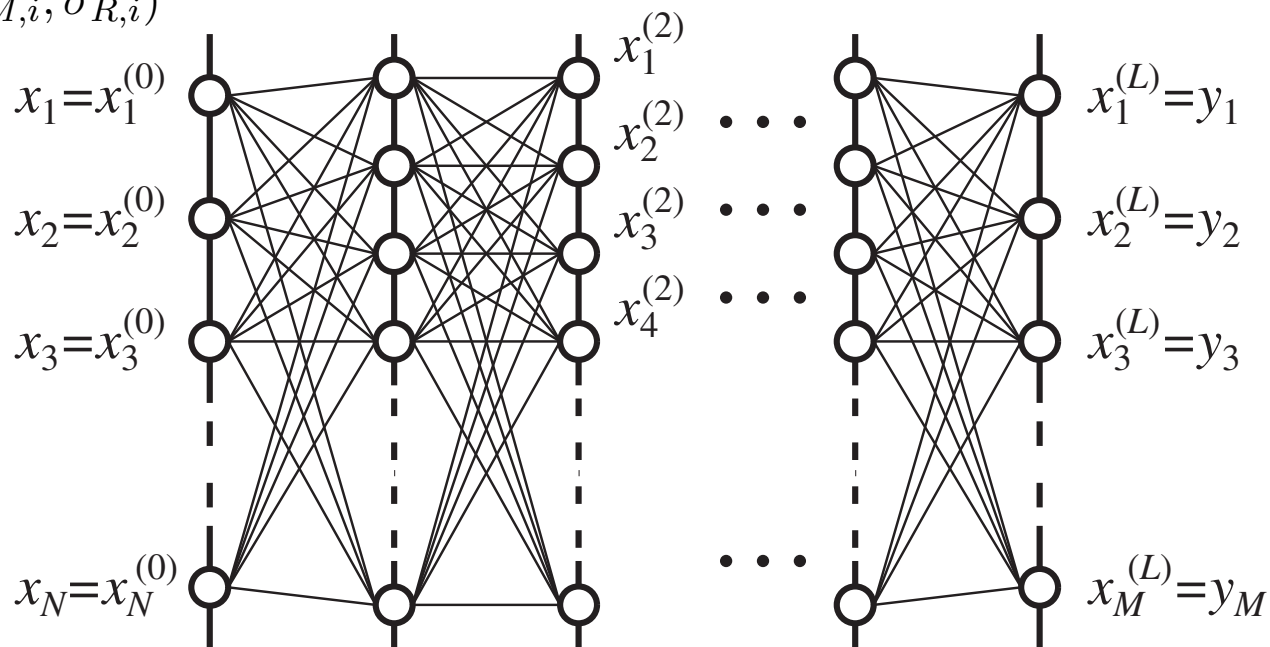


Y. Fujimoto, K. Fukushima, K. Murase, *Phys. Rev. D*, 98 (2018) 023019

Input

Output

$(M_i, R_i; \sigma_{M,i}, \sigma_{R,i})$



$$x_i^{(k+1)} = \sigma^{(k+1)} \left(\sum_{j=1}^{N_k} W_{ij}^{(k+1)} x_j^{(k)} + a_i^{(k+1)} \right),$$

Assume the function K. Huang, J. N. Hu, Y. Zhang, and H. Shen, Astrophys. J. 935(2022)88

to satisfy

$$\begin{bmatrix} f(x_1) \\ f(x_1) \\ \vdots \\ f(x_n) \end{bmatrix} \sim \mathcal{N} \left(\begin{bmatrix} \mu(x_1) \\ \mu(x_2) \\ \vdots \\ \mu(x_n) \end{bmatrix}, \begin{bmatrix} \kappa(x_1, x_1) & \kappa(x_1, x_2) & \dots & \kappa(x_1, x_n) \\ \kappa(x_2, x_1) & \kappa(x_2, x_2) & \dots & \kappa(x_2, x_n) \\ \vdots & \vdots & \ddots & \vdots \\ \kappa(x_n, x_1) & \kappa(x_n, x_2) & \dots & \kappa(x_n, x_n) \end{bmatrix} \right)$$

The observation data is

$$(\mathbf{x}_1, y_1), (\mathbf{x}_2, y_2), \dots, (\mathbf{x}_n, y_n)$$

The prediction value of \mathbf{f} is

$$\begin{bmatrix} f(x_1) \\ f(x_2) \\ \vdots \\ f(x_n) \\ f(x_*) \end{bmatrix} \sim \mathcal{N} \left(\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \kappa(x_1, x_1) & \kappa(x_1, x_2) & \dots & \kappa(x_1, x_n) & \kappa(x_1, x_*) \\ \kappa(x_2, x_1) & \kappa(x_2, x_2) & \dots & \kappa(x_2, x_n) & \kappa(x_2, x_*) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \kappa(x_n, x_1) & \kappa(x_n, x_2) & \dots & \kappa(x_n, x_n) & \kappa(x_n, x_*) \\ \kappa(x_*, x_1) & \kappa(x_*, x_2) & \dots & \kappa(x_*, x_n) & \kappa(x_*, x_*) \end{bmatrix} \right)$$

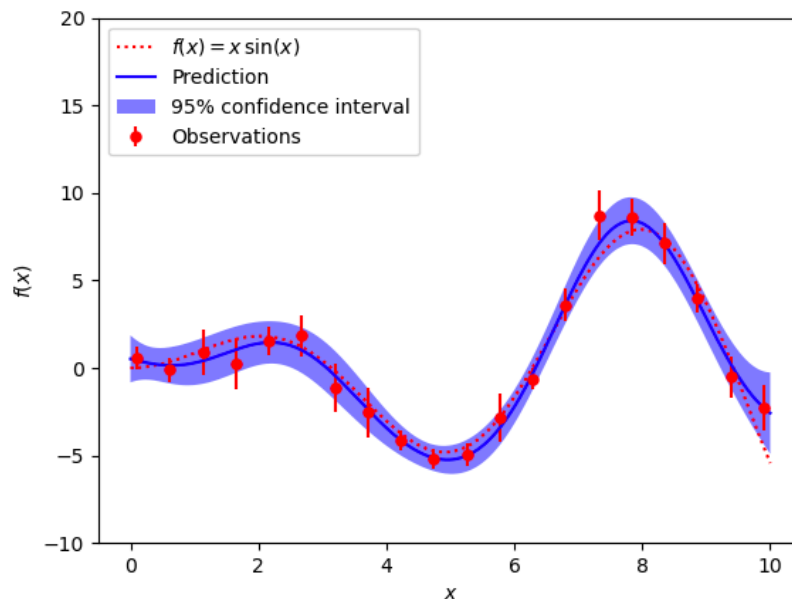
It can be use the matrix notation

$$\begin{bmatrix} \mathbf{y} \\ f(x_*) \end{bmatrix} \sim \mathcal{N} \left(\begin{bmatrix} \mathbf{0} \\ 0 \end{bmatrix}, \begin{bmatrix} K(\mathbf{X}, \mathbf{X}) & K(x_*, \mathbf{X}) \\ K(\mathbf{X}, x_*) & K(x_*, x_*) \end{bmatrix} \right)$$

where the mean function is zero for notational simplicity.

The distribution of prediction point can be obtained

$$f(x_*) | \mathbf{y} \sim \mathcal{N} \left(K(x_*, \mathbf{X})K(\mathbf{X}, \mathbf{X})^{-1}\mathbf{y}, K(x_*, x_*) - K(x_*, \mathbf{X})K(\mathbf{X}, \mathbf{X})^{-1}K(\mathbf{X}, x_*) \right)$$

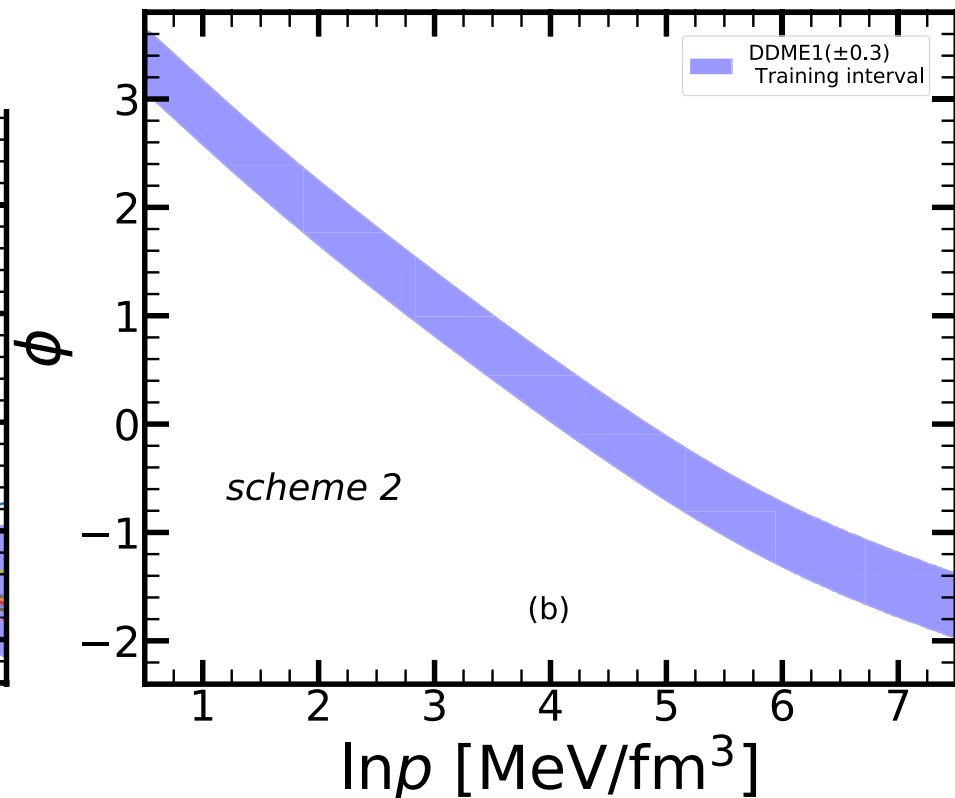
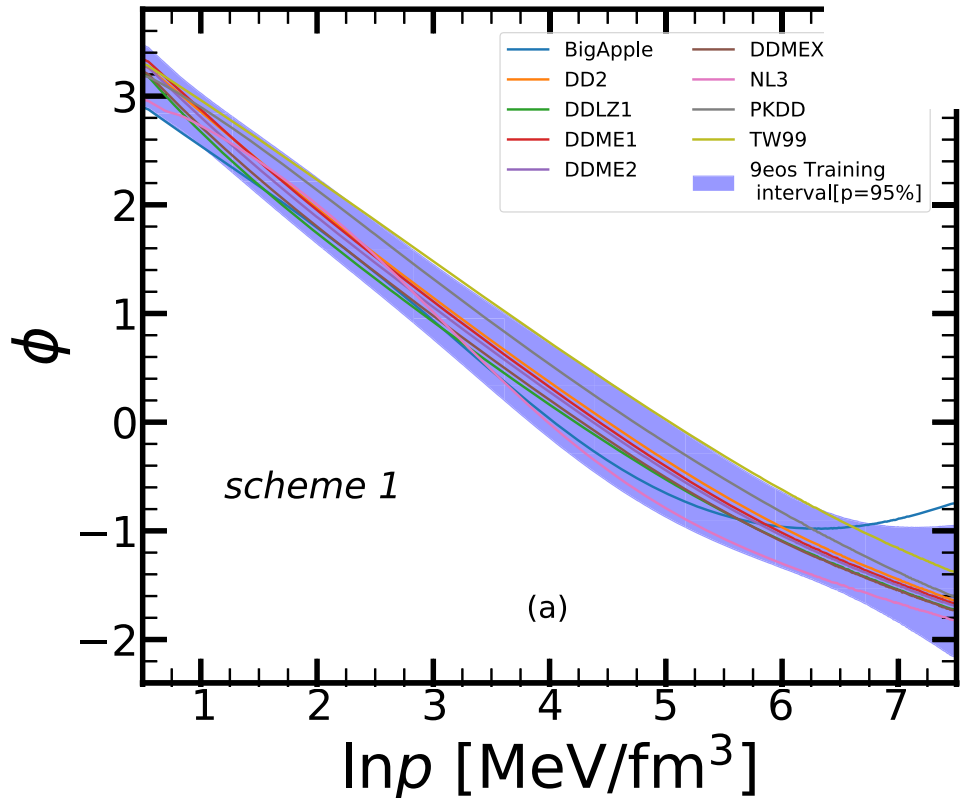


The spectral representation

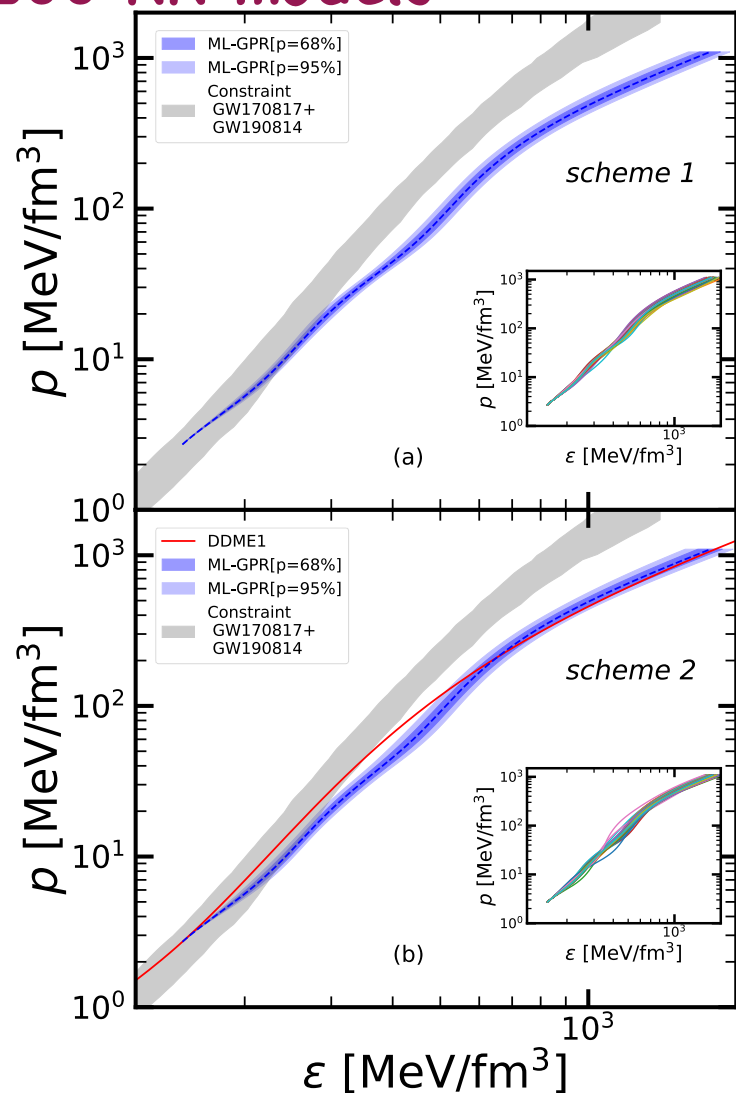
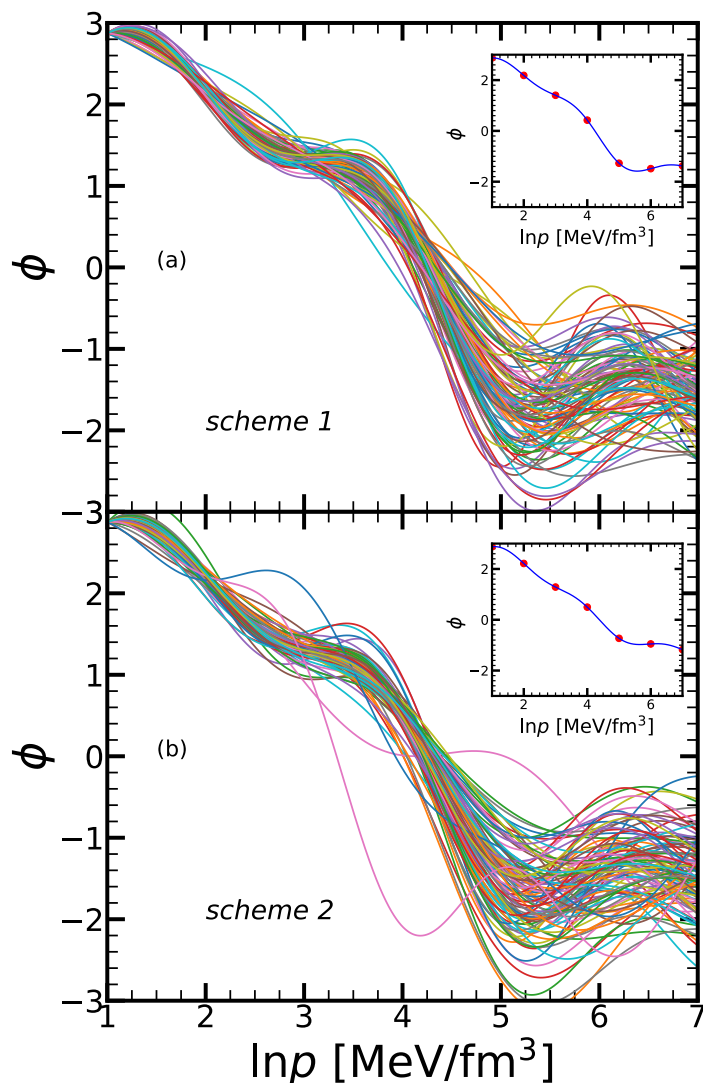
$$\phi = \log \left(c^2 \frac{d\varepsilon}{dp} - 1 \right).$$

$$\phi = \phi(\log p)$$

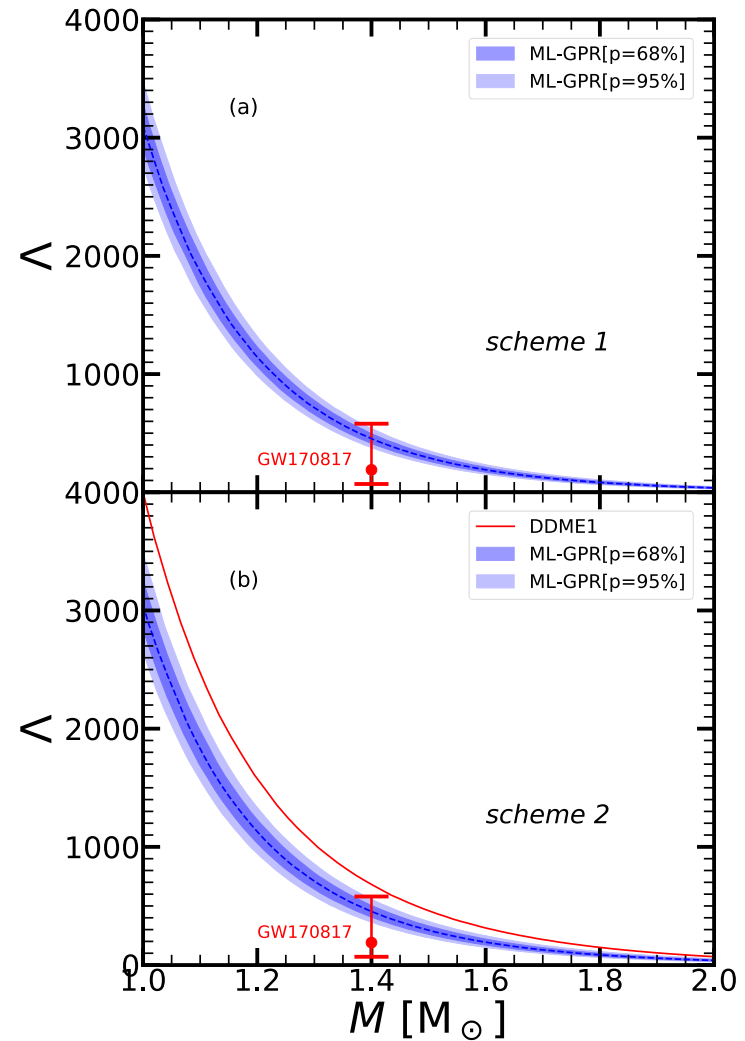
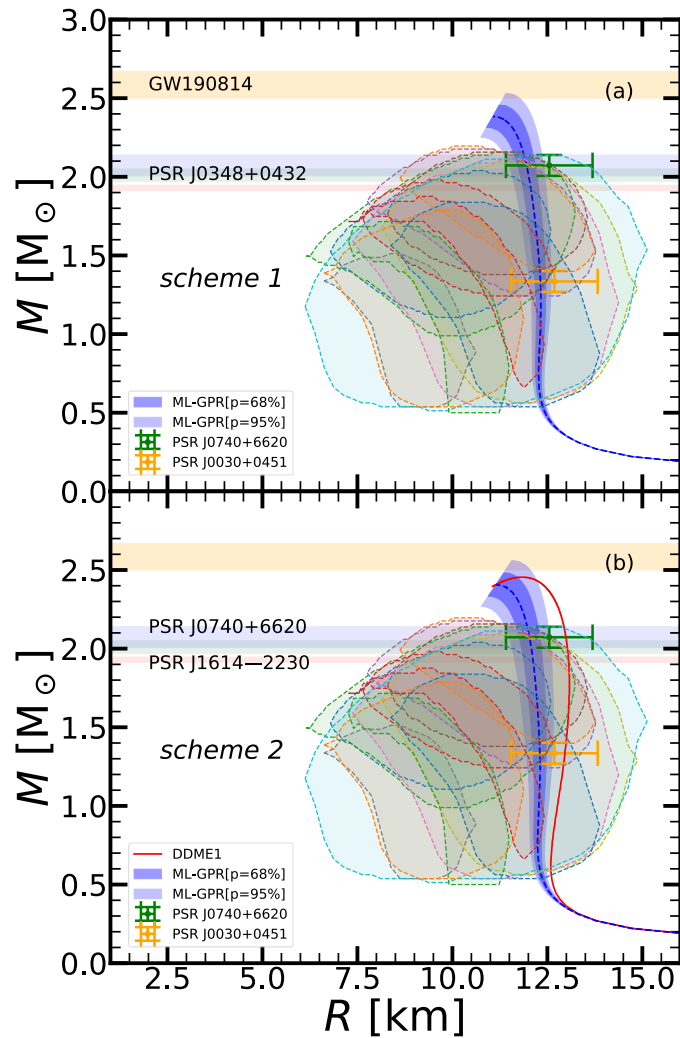
$$\varepsilon = \int \frac{1 + e^\phi}{c^2} dp$$



The EOSs from 200 NN models



W. Zhou, J. N. Hu, Y. Zhang, and H. Shen, *Astrophys. J.* 950(2023)186





- Introduction
- The massive neutron star from DDRMF
- The equation of state from machine learning
- Summary

The neutron star is a natural laboratory to check the nuclear many-body methods

Equations of state of massive neutron star can be described within DDRMF model.

The masses of hyperonic star can approach two times solo mass.

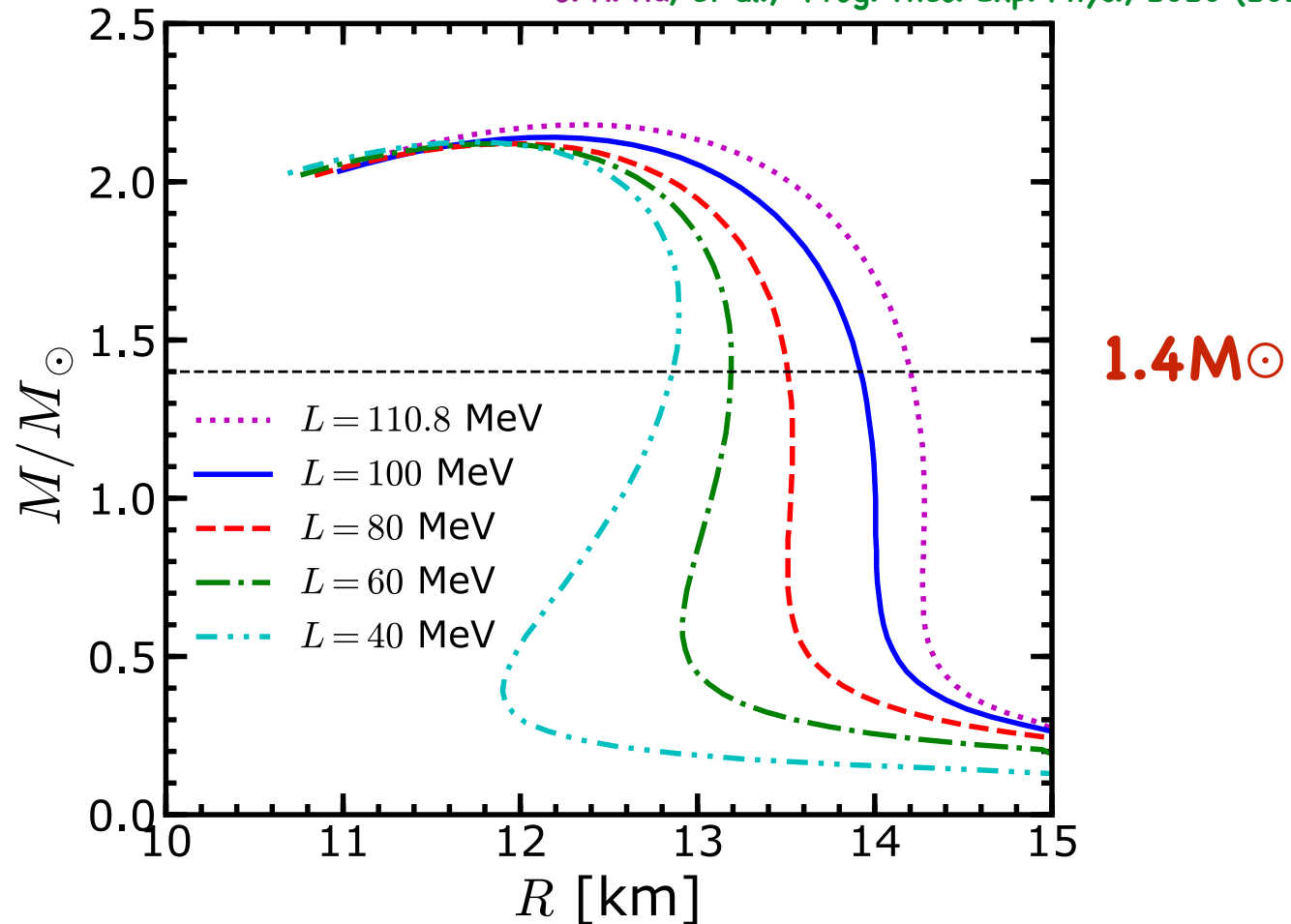
A nonparametric method was proposed to infer the equation of state of compact star with deep neural network.



Thank you very much
for your attention!

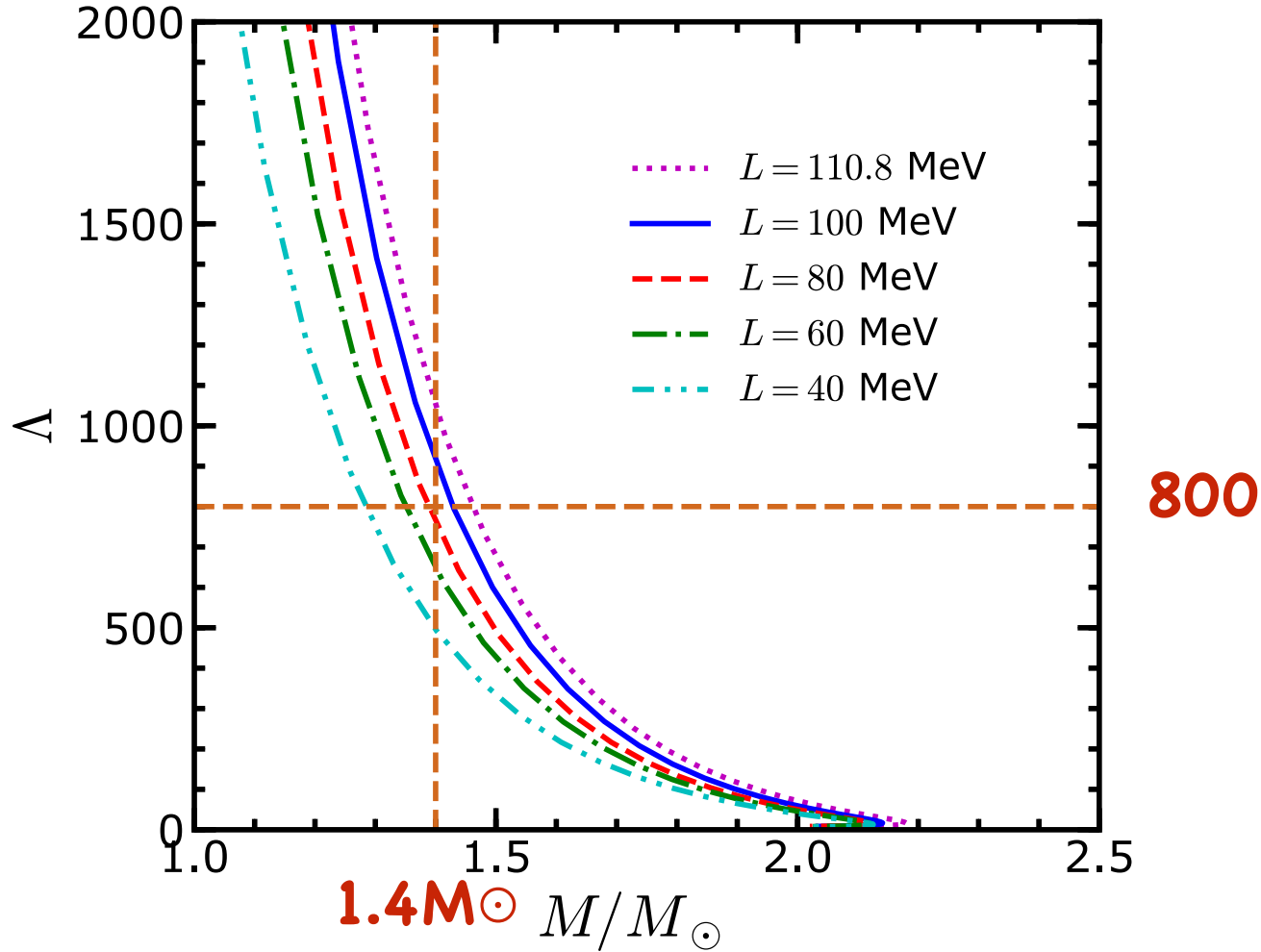
The neutron star mass as function of Radius

J. N. Hu, et al., Prog. Theo. Exp. Phys., 2020 (2020) 043D01



The symmetry energy affects the neutron star at small mass region

The tidal deformability as a function of neutron mass



Fermi-Dirac distribution

$$f_{i\pm}^k = \{1 + \exp[(\sqrt{k^2 + M^{*2}} \mp \nu_i)/T]\}^{-1},$$

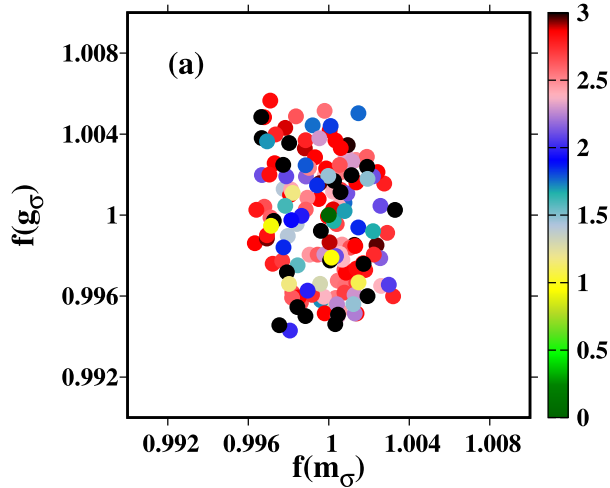
The number density of protons or neutrons

$$n_i = \frac{1}{\pi^2} \int_0^\infty dk k^2 (f_{i+}^k - f_{i-}^k).$$

The energy density

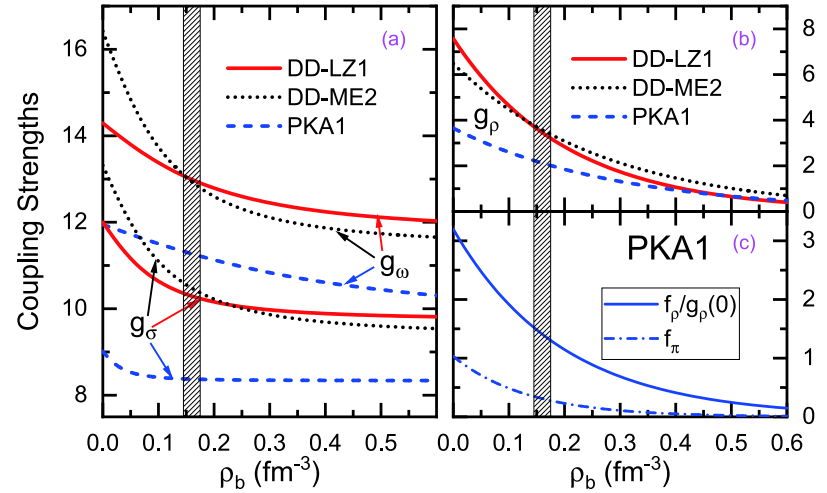
$$\begin{aligned} \epsilon = & \sum_{i=p,n} \frac{1}{\pi^2} \int_0^\infty dk k^2 \sqrt{k^2 + M^{*2}} (f_{i+}^k + f_{i-}^k) \\ & + \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{3} g_2 \sigma^3 + \frac{1}{4} g_3 \sigma^4 \\ & + \frac{1}{2} m_\omega^2 \omega^2 + \frac{3}{4} c_3 \omega^4 + \frac{1}{2} m_\rho^2 \rho^2 + 3 \Lambda_\nu (g_\omega^2 \omega^2) (g_\rho^2 \rho^2), \end{aligned}$$

DD-MEX



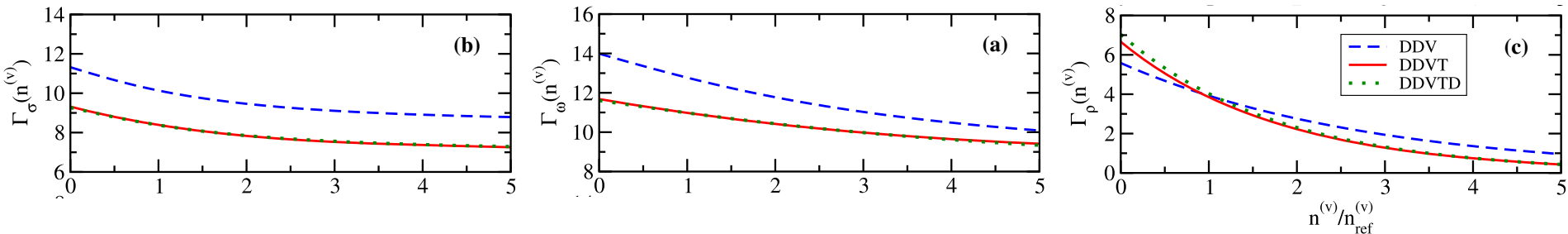
A. Taninah, et al. Phys. Lett. B 800,135065(2020)

DD-LZ1



B. Wei, et al. Chin. Phys. C 44, 074107 (2020)

DDV, DDVT, DDVTD



S. Typel and D. A. Terrero, Eur. Phys. J. A 56, 160 (2020)

DD2

DD-ME1

DD-ME2

DD-MEX

DD-LZ1

DDV

DDVT

DDVTD

	DD-LZ1		DD2	DD-ME1	DD-ME2	DD-MEX	DDV	DDVT	DDVTD
m_n [MeV]	938.900000	m_n	939.56536	939.0000	939.0000	939.0000	939.565413	939.565413	939.565413
m_p [MeV]	938.900000	m_p	938.27203	939.0000	939.0000	939.0000	938.272081	938.272081	938.272081
m_σ [MeV]	538.619216	m_σ	546.212459	549.5255	550.1238	547.3327	537.600098	502.598602	502.619843
m_ω [MeV]	783.0000	m_ω	783.0000	783.0000	783.0000	783.0000	783.0000	783.0000	783.0000
m_ρ [MeV]	769.0000	m_ρ	763.0000	763.0000	763.0000	763.0000	763.0000	763.0000	763.0000
m_δ [MeV]	—	m_δ	—	—	—	—	—	—	980.0000
$\Gamma_\sigma(0)$	12.001429	$\Gamma_\sigma(\rho_{B0})$	10.686681	10.4434	10.5396	10.7067	10.136960	8.382863	8.379269
$\Gamma_\omega(0)$	14.292525	$\Gamma_\omega(\rho_{B0})$	13.342362	12.8939	13.0189	13.3388	12.770450	10.987106	10.980433
$\Gamma_\rho(0)$	15.150934	$\Gamma_\rho(\rho_{B0})$	7.25388	7.6106	7.3672	7.2380	7.84833	7.697112	8.06038
$\Gamma_\delta(0)$	—	$\Gamma_\delta(\rho_{B0})$	—	—	—	—	—	—	0.8487420
ρ_{B0} [fm $^{-3}$]	0.158100	ρ_{B0}	0.149	0.152	0.152	0.153	0.1511	0.1536	0.1536
a_σ	1.062748	a_σ	1.357630	1.3854	1.3881	1.3970	1.20993	1.20397	1.19643
b_σ	1.763627	b_σ	0.634442	0.9781	1.0943	1.3350	0.21286844	0.19210314	0.19171263
c_σ	2.308928	c_σ	1.005358	1.5342	1.7057	2.0671	0.30798197	0.27773566	0.27376859
d_σ	0.379957	d_σ	0.575810	0.4661	0.4421	0.4016	1.04034342	1.09552817	1.10343705
a_ω	1.059181	a_ω	1.369718	1.3879	1.3892	1.3936	1.23746	1.16084	1.16693
b_ω	0.418273	b_ω	0.496475	0.8525	0.9240	1.0191	0.03911422	0.04459850	0.02640016
c_ω	0.538663	c_ω	0.817753	1.3566	1.4620	1.6060	0.07239939	0.06721759	0.04233010
d_ω	0.786649	d_ω	0.638452	0.4957	0.4775	0.4556	2.14571442	2.22688558	2.80617483
a_ρ	0.776095	a_ρ	0.518903	0.5008	0.5647	0.6202	0.35265899	0.54870200	0.55795902
a_δ	—	a_δ	—	—	—	—	—	—	0.55795902

