Puzzles in density regions from nuclear to quark matter

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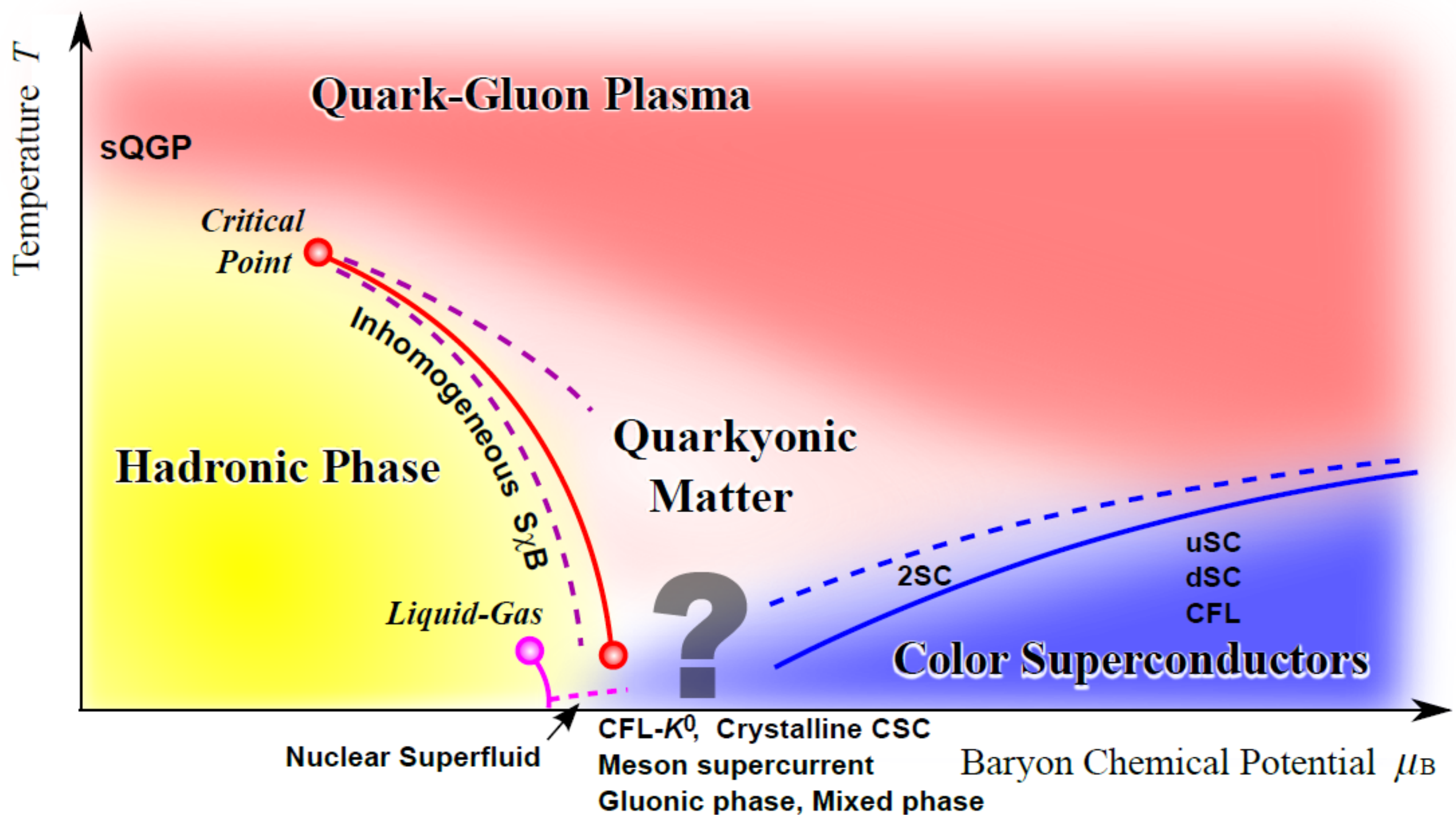
The University of Tokyo

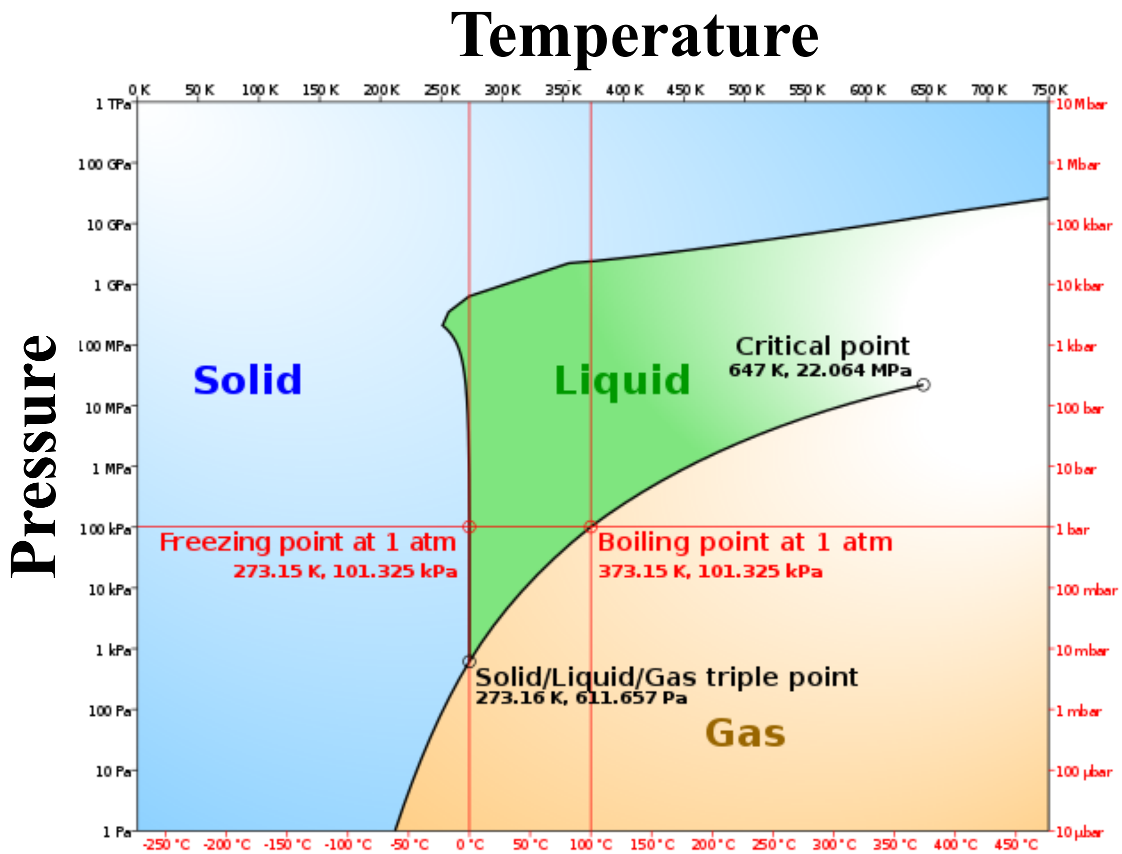
— Dialoque at the Dream Field 2024 —

QCD Phase Diagram



Fukushima-Hatsuda (2010); see also 50 Years of QCD Chap.7 (2023)

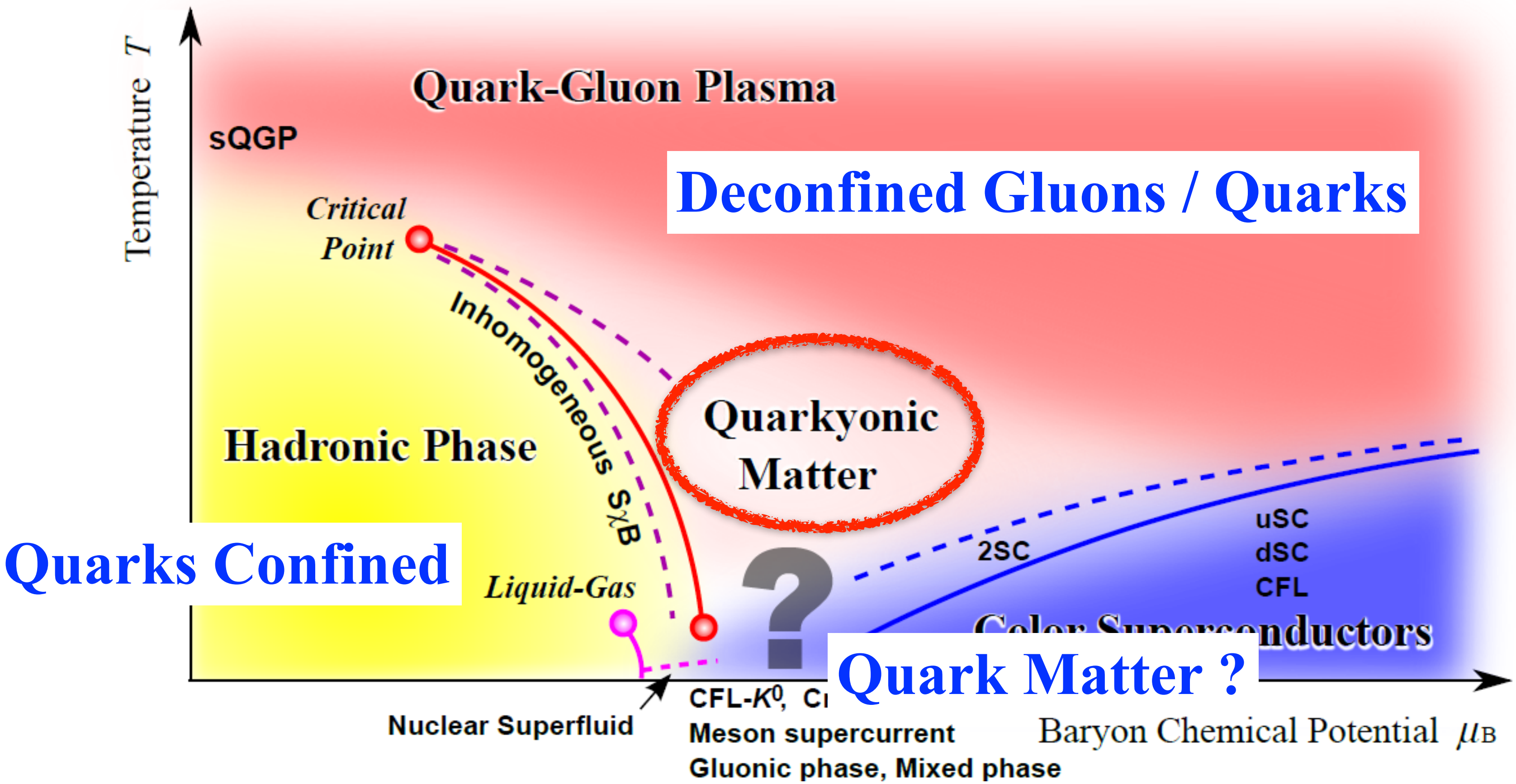
QCD Phase Diagram



Wikipedia

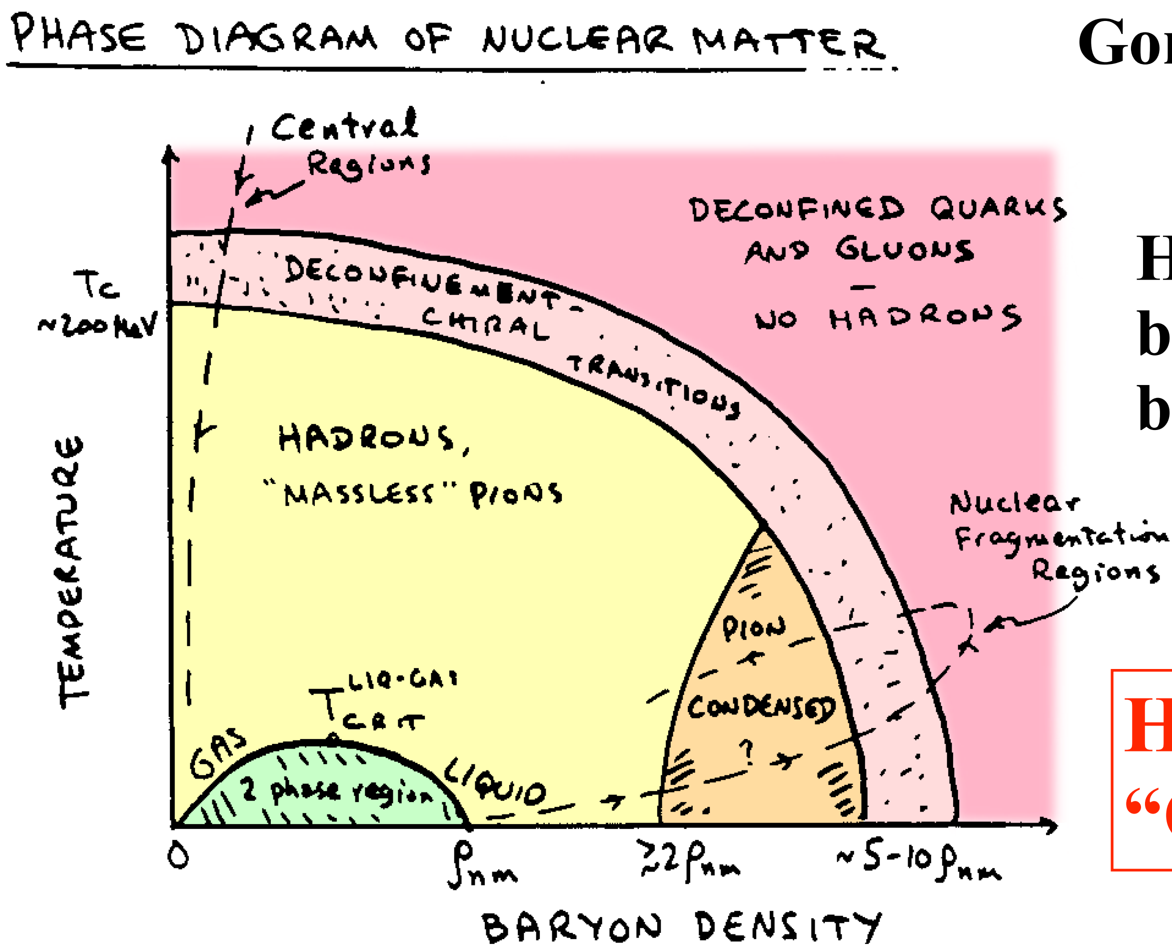
Phyics simiar to "water phase diagram" but more complicated…

QCD Phase Diagram



These are all gaseous states and physical d.o.f are different!

QCD Phase Diagram: Prototype ’86



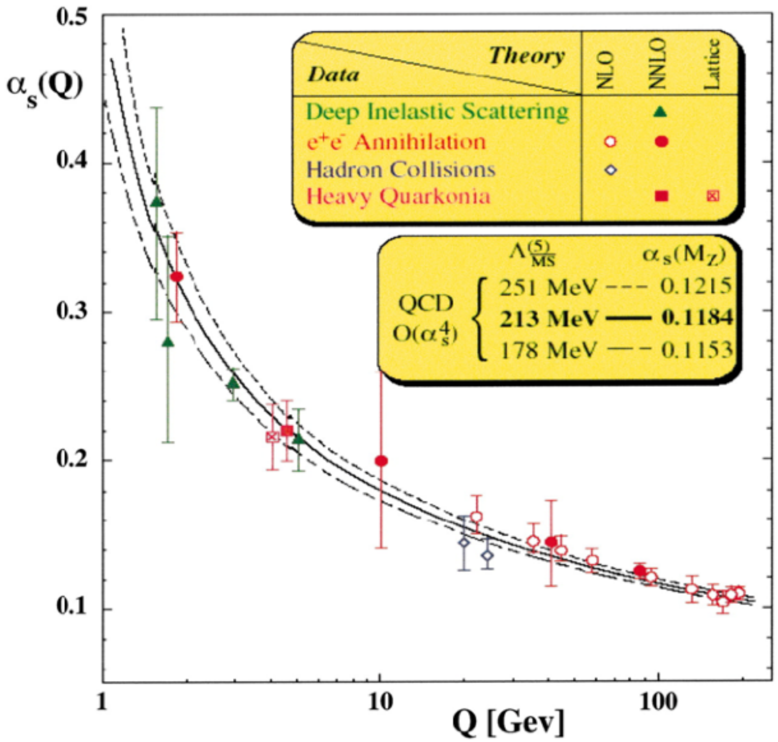
Gordon Baym (1986)

High density regions better understood, but puzzles arise!

How to find “Quark Matter” ?

Phase Transition at High Energies

Asymptotic Freedom in QCD



$$\alpha_s(Q^2) = \frac{1}{\beta_0 \ln(Q^2/\Lambda^2_{QCD})}$$

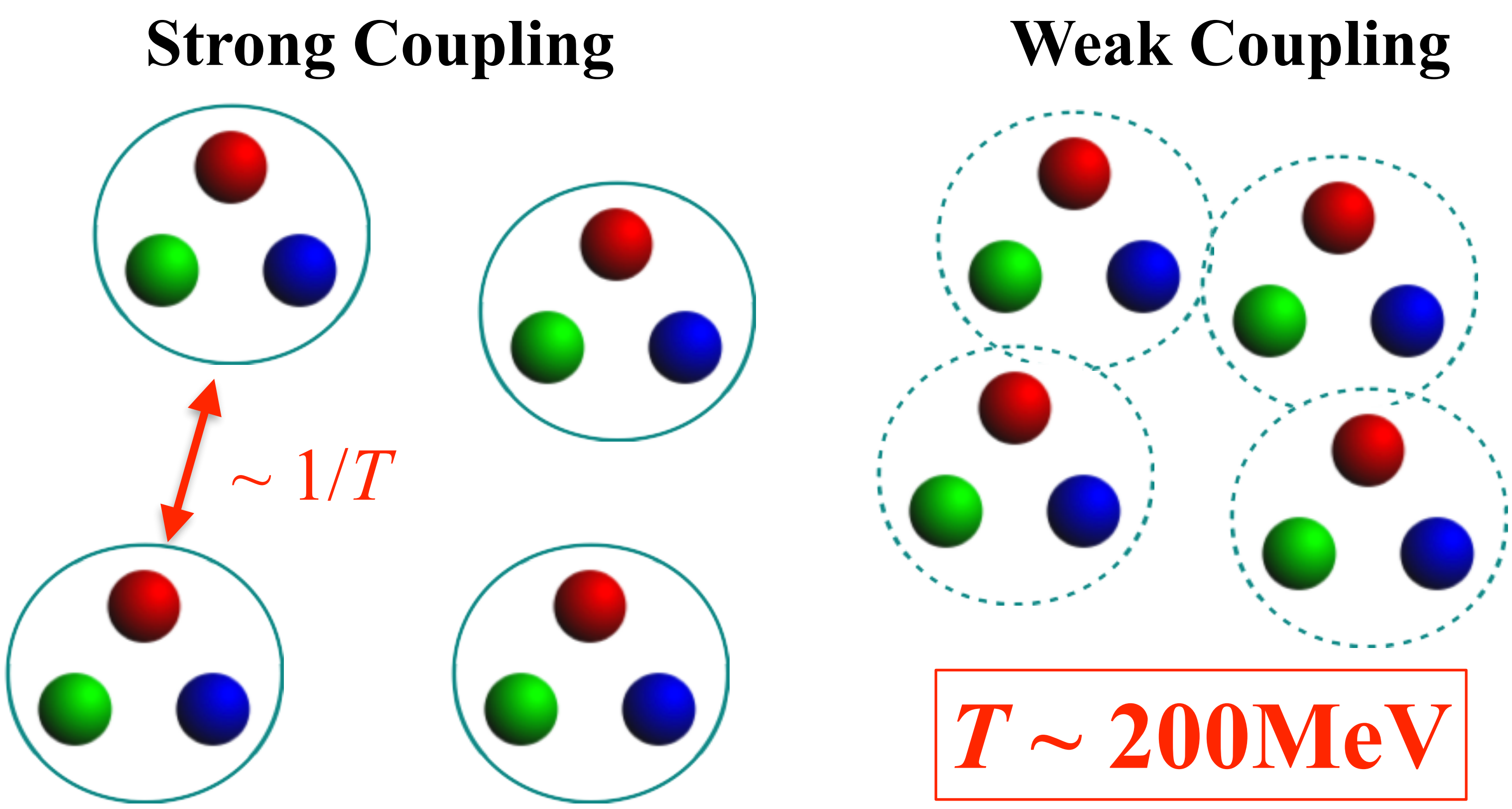
The theory is perturbative if the exchanged momenta are larger than Λ_{QCD} .

Empirically Λ_{QCD} ~ 200MeV.

Nuclear physics scale is characterized by Λ_{QCD} only.

Phase Transition at High Energies

Deconfinement Transition



Phase Transition at High Energies

Chiral Transition

$T \sim 200\text{MeV}$

Strong Coupling **Weak Coupling**

Quarks get mass from Strong Interaction non-perturbatively:

$$M_q \sim M_N/3 \sim 350\text{MeV}$$

Perturbative mass of quarks are much smaller than Λ_{QCD}:

$$m_u \sim 3\text{MeV},\ m_d \sim 5\text{MeV}$$

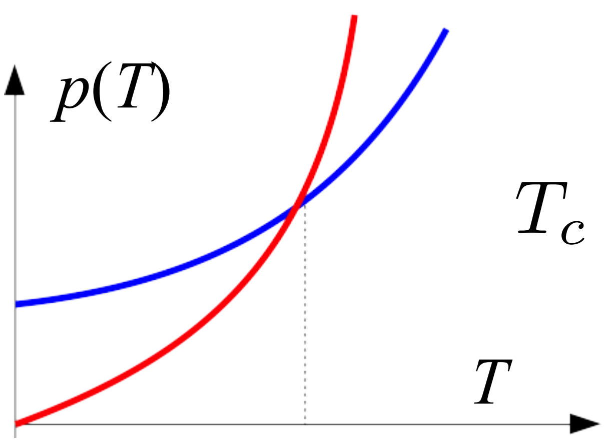
Dynamical mass generated by Nambu-BCS mechanism.

High-T: Old Handwaving View

Definition of "deconfinement" in old days

Pion Gas + Bag Pressure $$p_{\rm hadron}(T) = \frac{3\pi^2}{90}T^4 + B$$

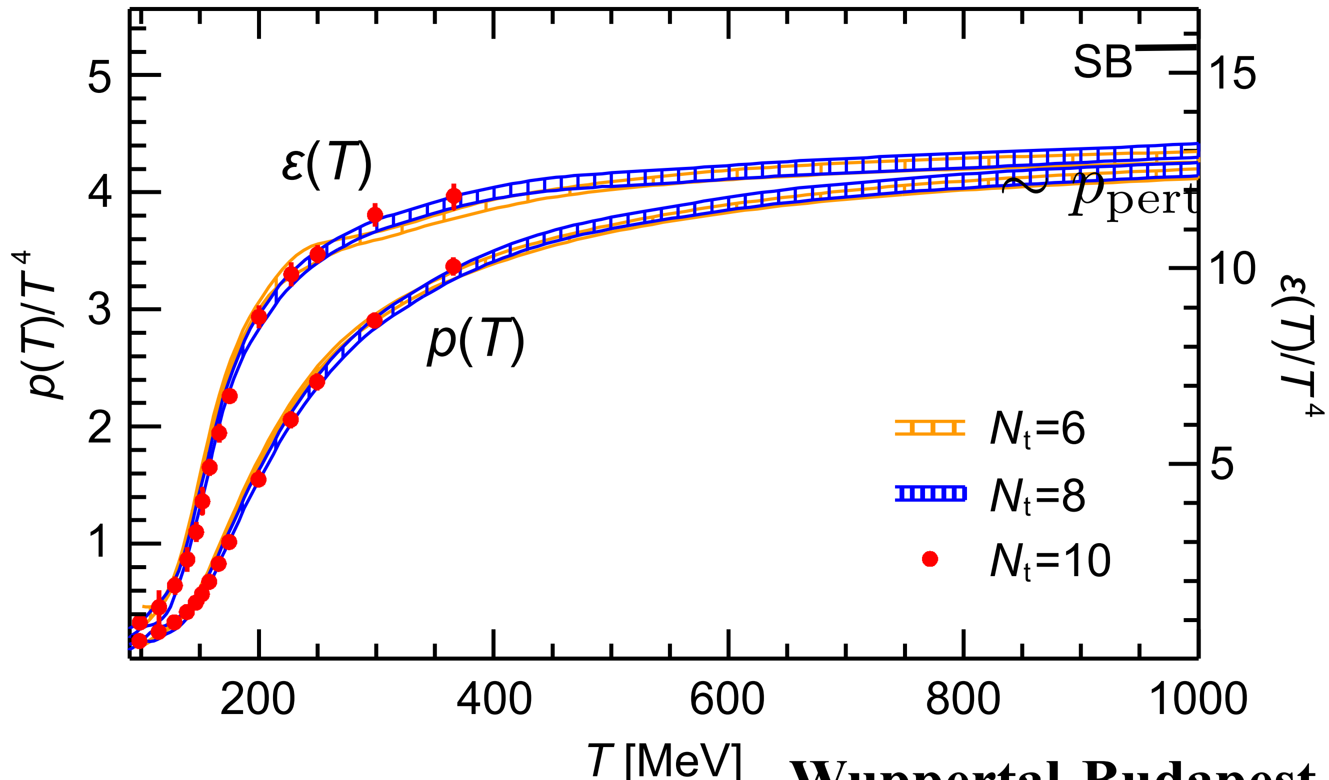
Quark-Gluon Plasma $$p_{\rm pert}(T) = \frac{(16+21)\pi^2}{90}T^4$$



$$T_c = \left[\frac{90}{(37-3)\pi^2}B\right]^{1/4} \sim 160\ \text{MeV}$$

High-T: QCD First-Principles

No clear phase transition in reality…?

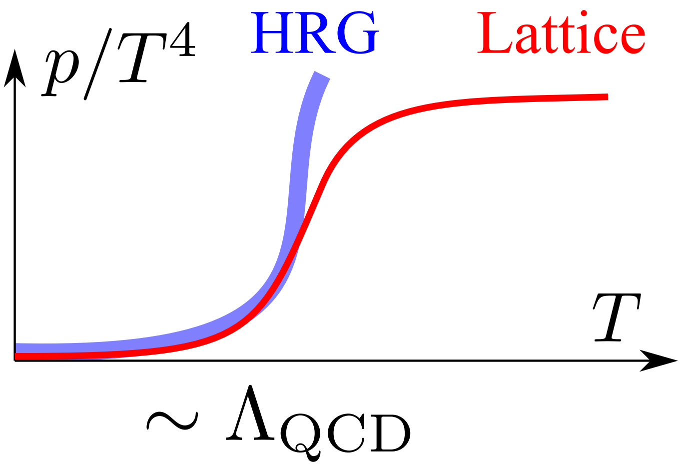


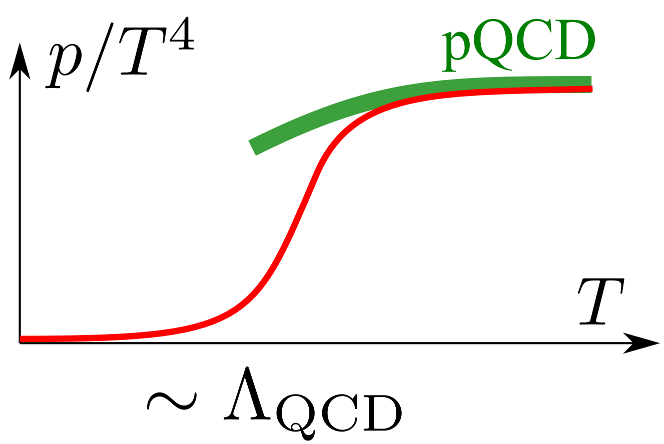
Wuppertal-Budapest (2010)

High-T: Interpretation

Crossover = Duality Point

Rising *p* from small *T* is understood by a free gas of (thousands of) mesons (Hadron Resonance Gas).





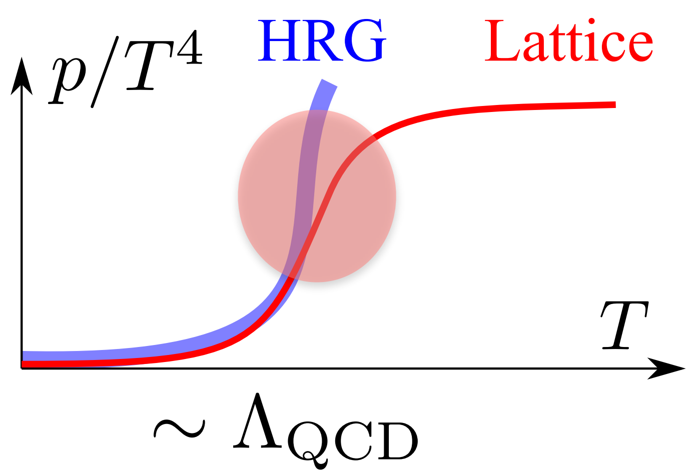
Experimentally confirmed picture

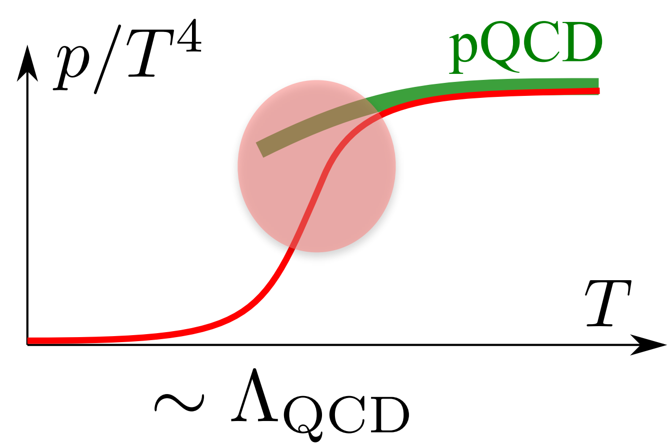
Quark-gluon plasma is matter well approximated by perturbative QCD.

High-T: Interpretation

Called “Crossover” = Duality Point

Interacting hadronic gas
~ Non-perturbative gas of quarks / gluons





High-T: Lessons

* Relativistic Heavy-Ion Collisions

Quark-gluon plasma confirmed experimentally
Inconsistency / consistency with hadronic gas / pQCD

* Lattice QCD First-Principles Simulations

Energy / entropy density approaching the SB limit
Approximate order parameters show phase transitions
pQCD calculations reproducing LQCD around $\sim 2T_c$

* Effective Models

Chiral quark models with gluons working well
Even AdS/CFT models good with phenomenology

High-μ_B: Difficulties

* Relativistic Heavy-Ion Collisions

Temperature too high
Isospin / strangeness contents very different

* Lattice QCD First-Principles Simulations

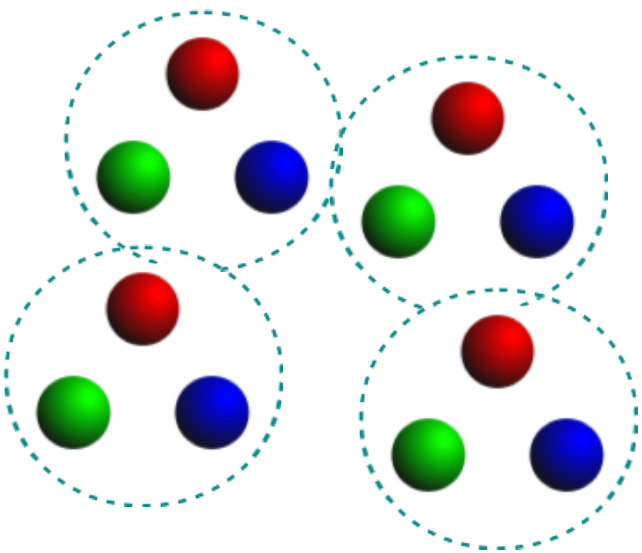
Sign problem extremely serious for $T < (2\text{-}3)\,\mu_B$
pQCD convergence slow (resummation necessary)

* Effective Models

Order parameter to distinguish nuclear matter and quark matter DOES NOT EXIT !

Quark Matter ?

Remember the high-T arguments:



This works because both gluons and quarks carry typical momenta of order $p \sim T$.

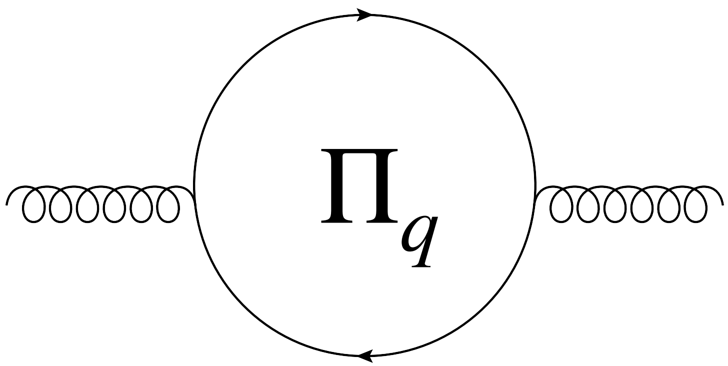
However, high-density implies quarks carrying $p \sim T$ but gluons can carry $p \ll T$.

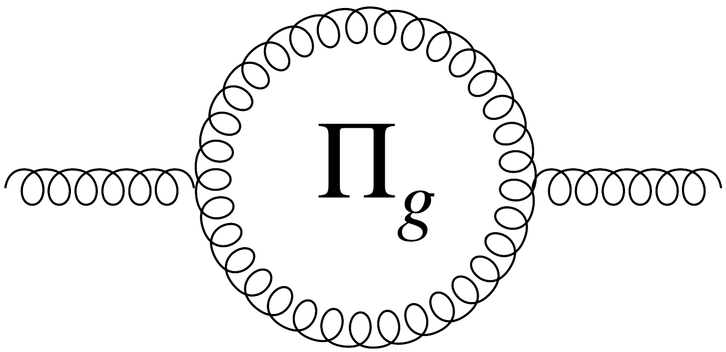
Asymptotic freedom not necessarily realized?

Quark Matter ?

Screening vs. Confinement

Collins-Perry (1975) **McLerran-Pisarski (2008)**





Debye screened by $m_D^2 \sim g^2\mu^2$

Enhanced by the number of gluons >> quarks

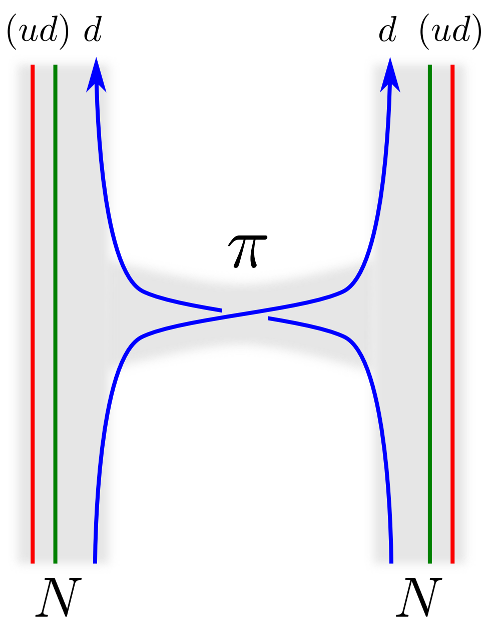
Confinement should persist as long as $\Pi_g > \Pi_q$

Quark Matter ?

Quarkyonic Matter

Quark matter keeps confinement (see the previous page).

Baryonic matter has the nature of deconfinement.



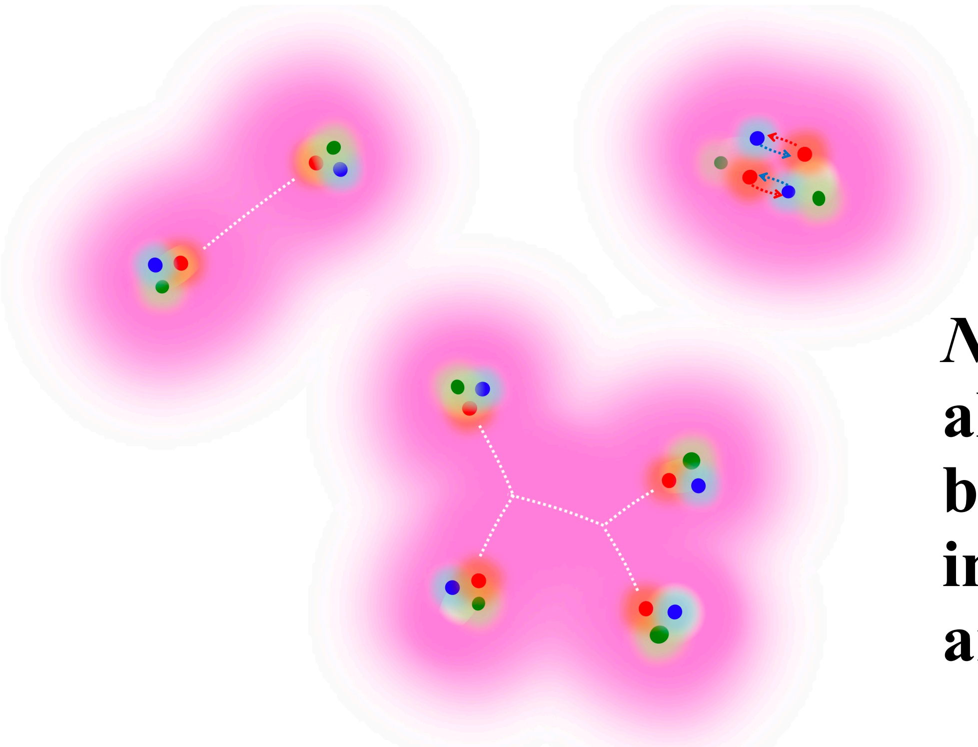
Baryon interaction scales with the number of quarks — quark matter?

Nuclear Matter

≃ Quark-like Baryonic Matter

≃ Quark Matter

Quark Matter ?

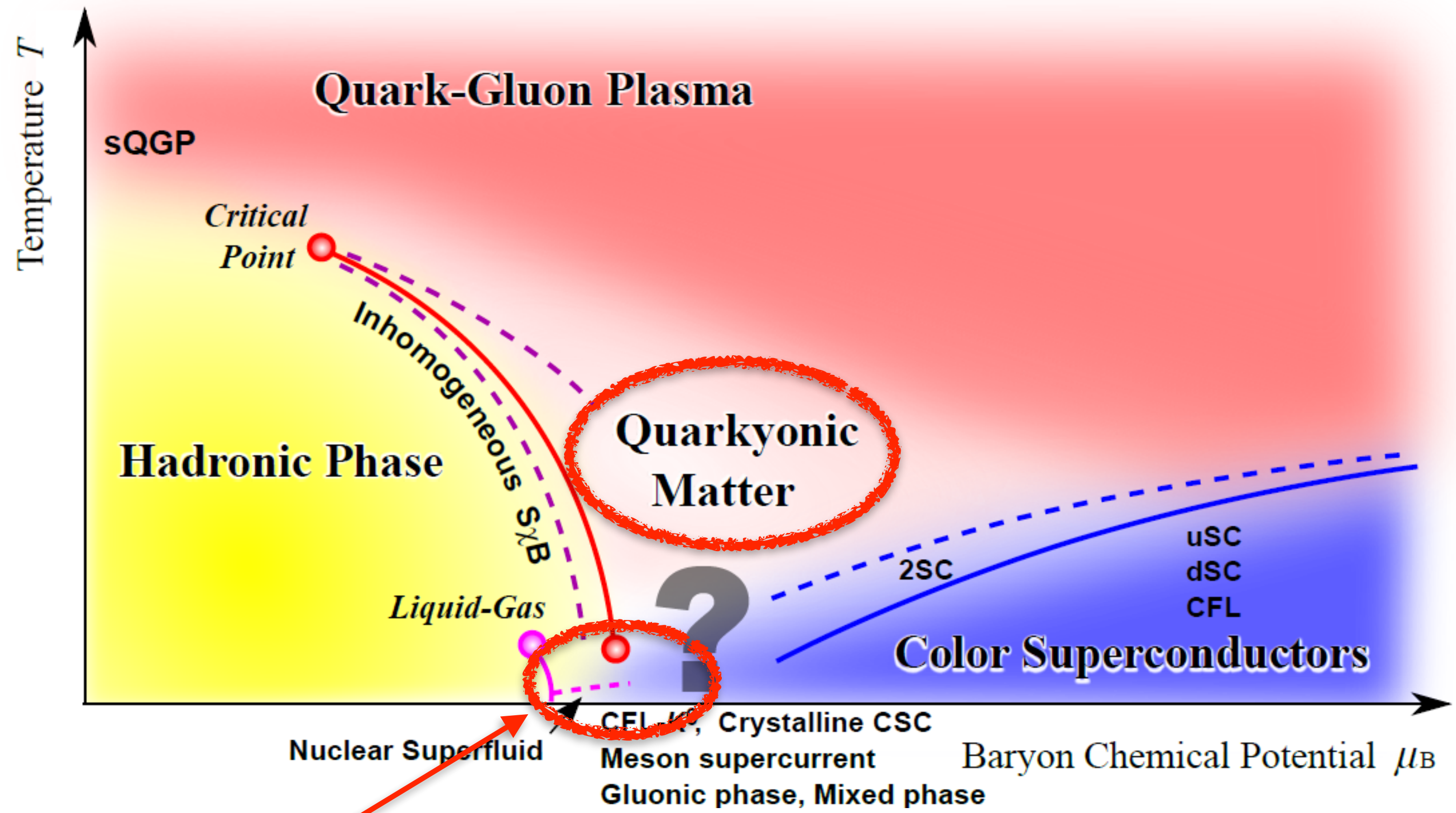


Transition to quark matter ~ Quantum percolation

***NN*, *NNN*, *NNNN*, ….**
all many-body interactions become the same order in the color-number scaling around ~ 2n_0

Fukushima-Kojo-Weise (2020)

Cold and Dense Matter



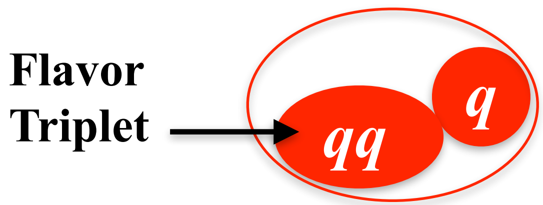
No phase transition? What happens there?

Cold and Dense Matter

Baryons ~ (Diquark) + Quarks

Baryons: 8+1 (low-lying)

Quarks: 3color × 3flavor = 9



Nuclear Matter

$$\langle nn \rangle$$

Superfluid

Quark Matter

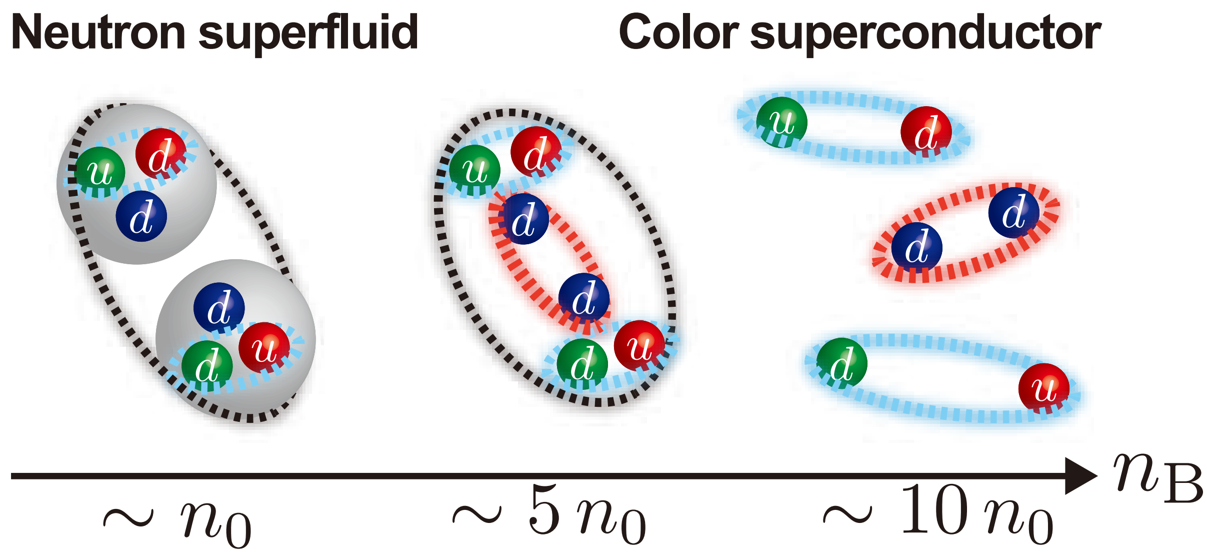
$$\langle ud \rangle$$

Superconductor

They *can* be smoothly connected: **Schaefer-Wilczek (1999)**

Cold and Dense Matter

Schematic picture of quark-hadron continuity



Fujimoto
-Fukushima
-Weise (2020)

These two states cannot be distinguished.
(Color is not a physical observable.)

Cold and Dense Matter

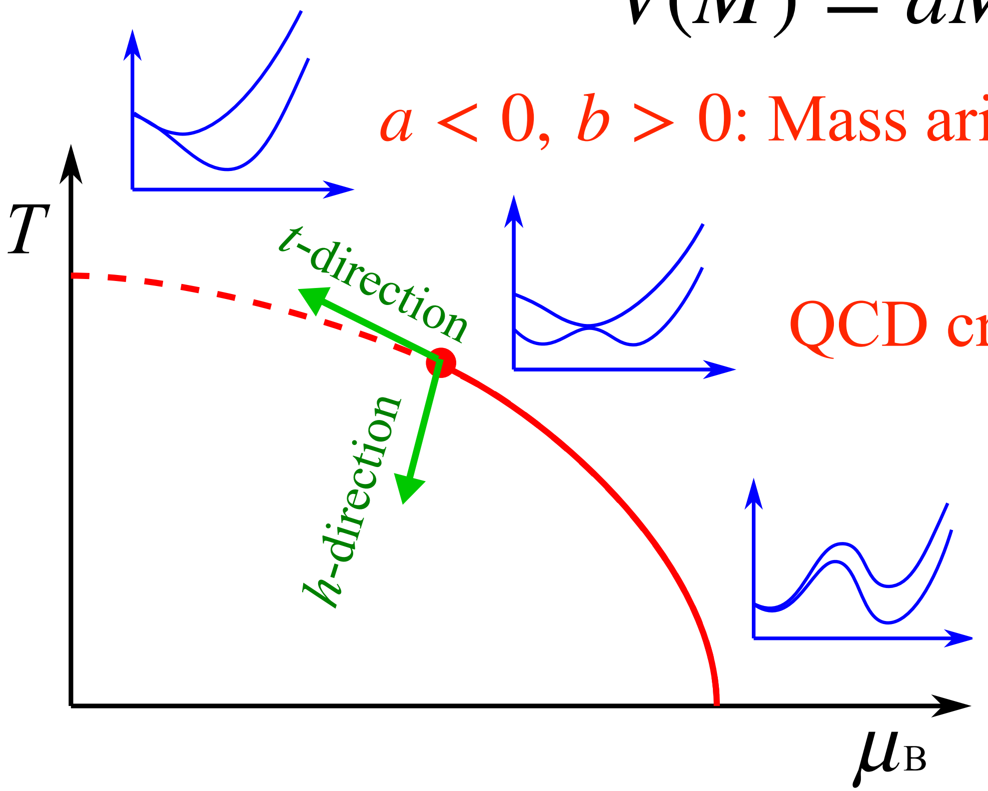
1st-order disfavored by diquarks

$$V(M) = aM^2 + bM^4 + cM^6 + \cdots$$

$a < 0$, $b > 0$: Mass arises by Nambu mechanism

QCD critical point

$a > 0$, $b < 0$: 1st-order



Cold and Dense Matter

$$V(M) = aM^2 + bM^4 + cM^6 + \cdots$$

Chiral Anomaly induces:

$\bar{q}q\bar{q}q\bar{q}q$ **Kobayashi-Maskawa-'t Hooft int.**

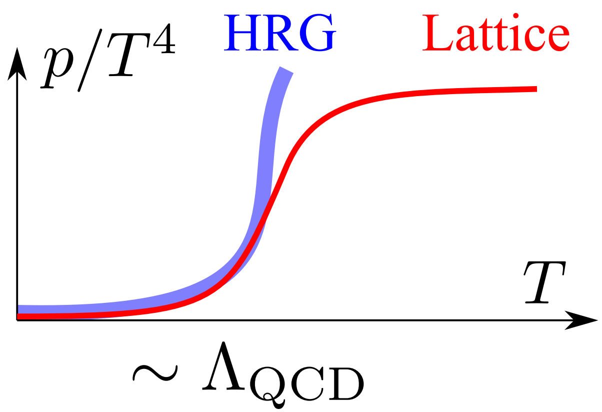
$$\langle qq \rangle \langle \bar{q}\bar{q} \rangle \langle \bar{q}q \rangle \sim |\Delta|^2 M$$

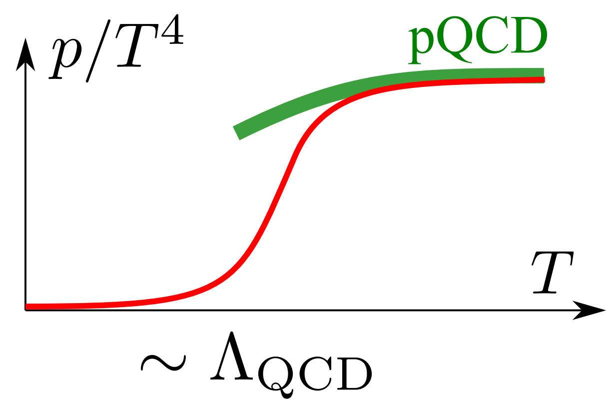
Linear term washes out the 1st-order PT.

Hatsuda-Tachibana-Yamamoto-Baym (2006)

Physics of Crossover

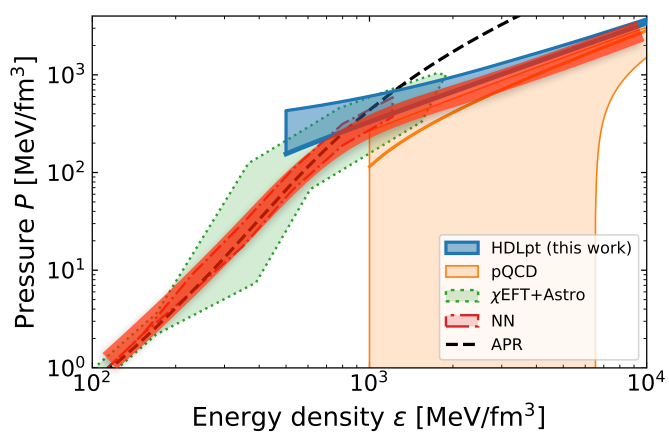
High-*T* has crossover verified by experiments and QCD.





High-Density

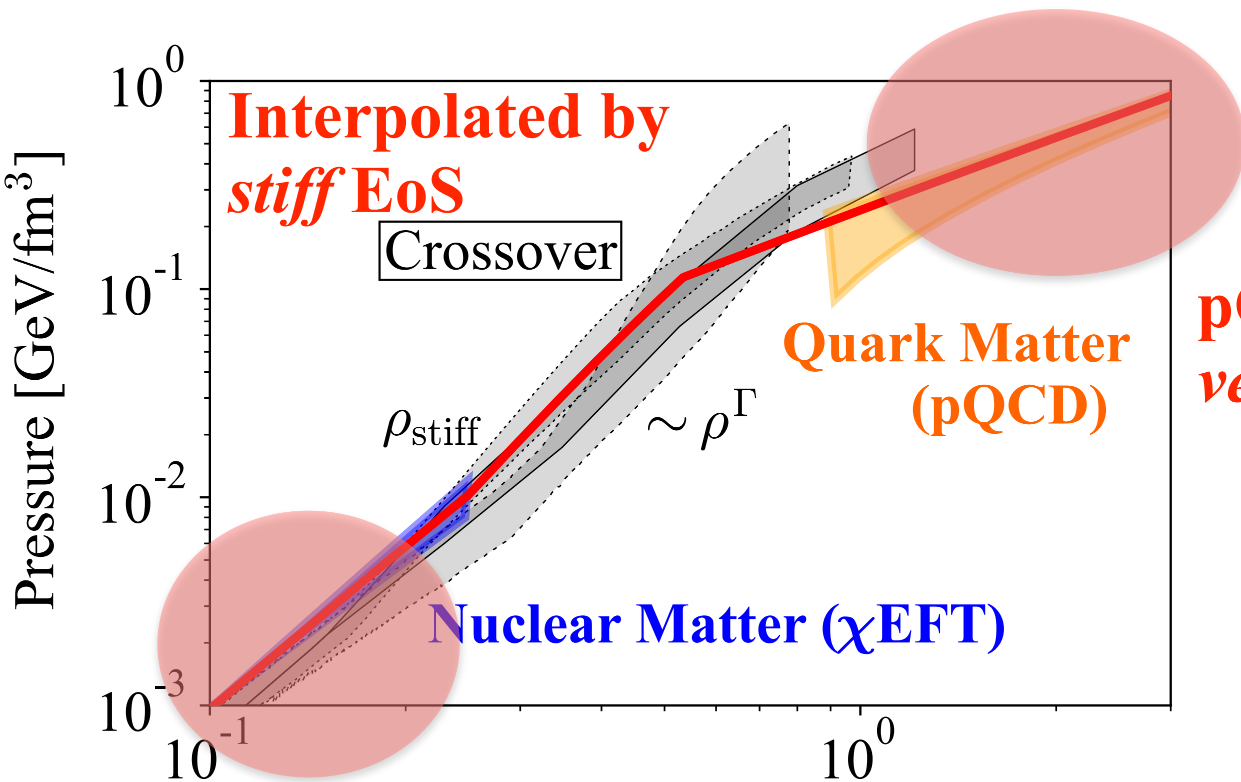
A duality region where the hadrons and quarks may coexist.



pQCD

Physics of Crossover

Fujimoto-Fukushima-Hotokezaka-Kyutoku (2022)

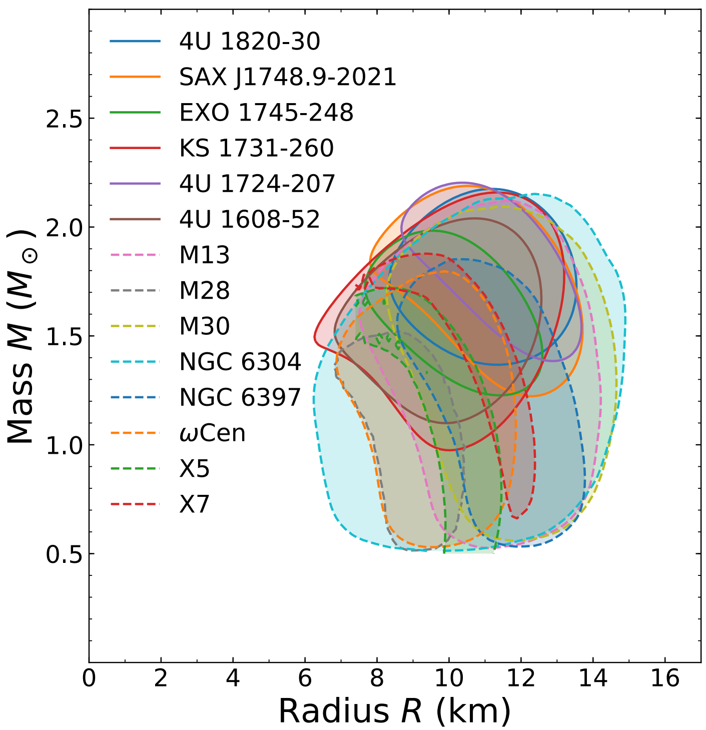


pQCD gives *very soft* EoS

χEFT gives *very soft* EoS

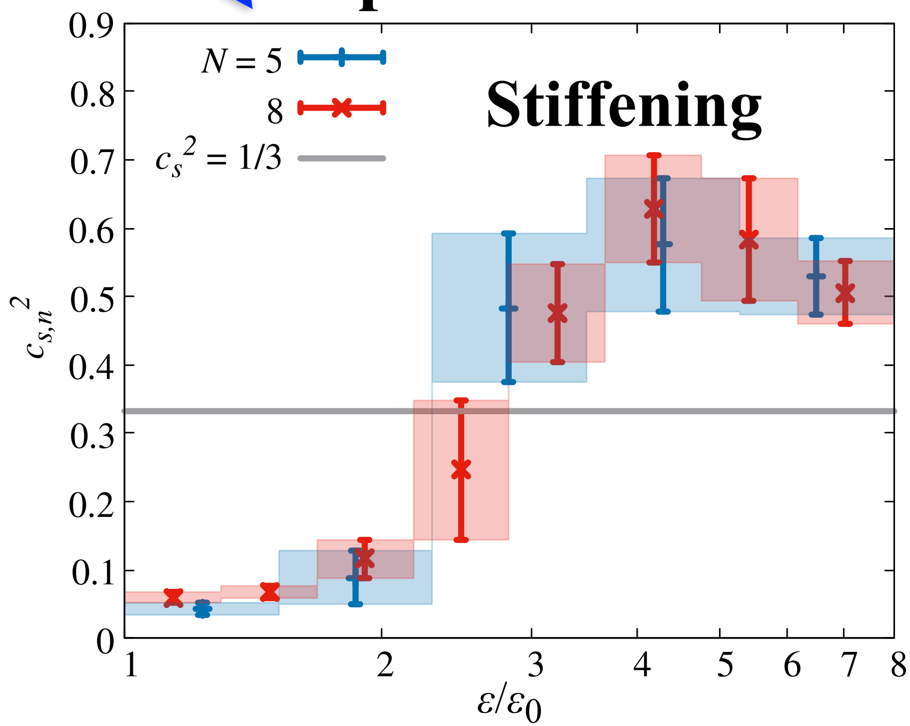
Physics of Crossover

Fujimoto-Fukushima-Murase (2019-2021)



Deep Neural Network

Speed of Sound



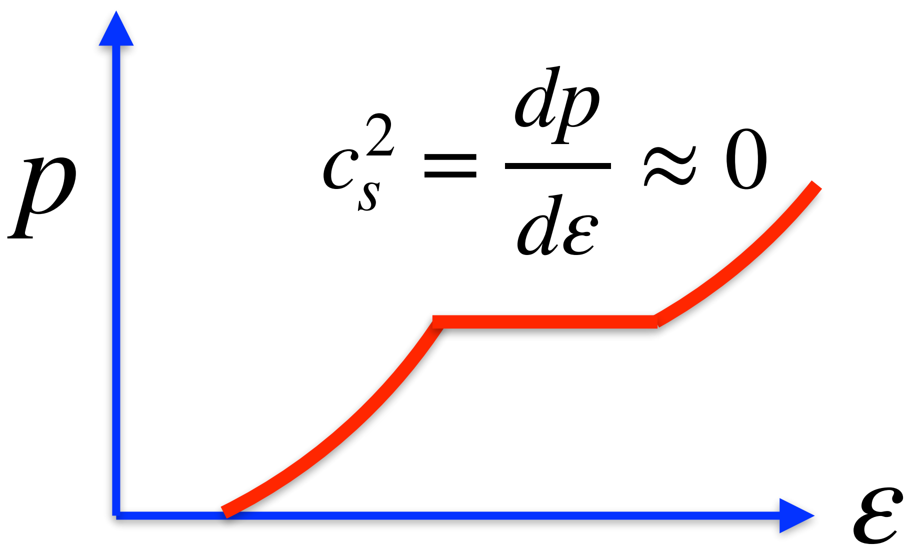
cf. Talk by A. Li

Physics of Crossover

A peak (or enhancement) in the speed of sound ???

What is the physics underlying this behavior ???

[1st-order]

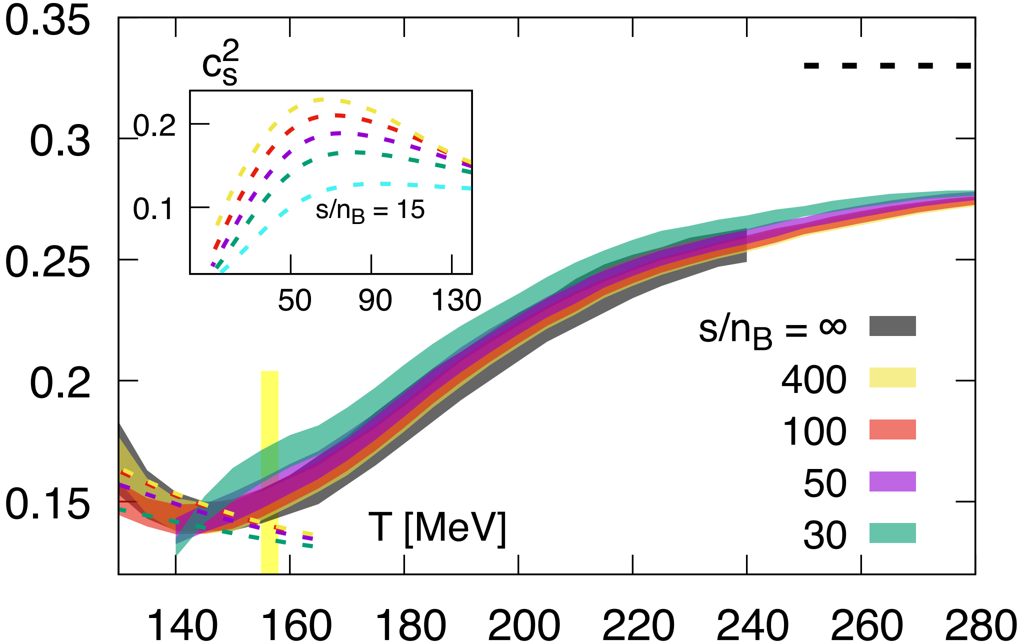


$$c_s^2 = \frac{dp}{d\varepsilon} \approx 0$$

Phase transition pushes down the speed of sound!?

Physics of Crossover

What do we know about high-*T* and small-*μ* matter?



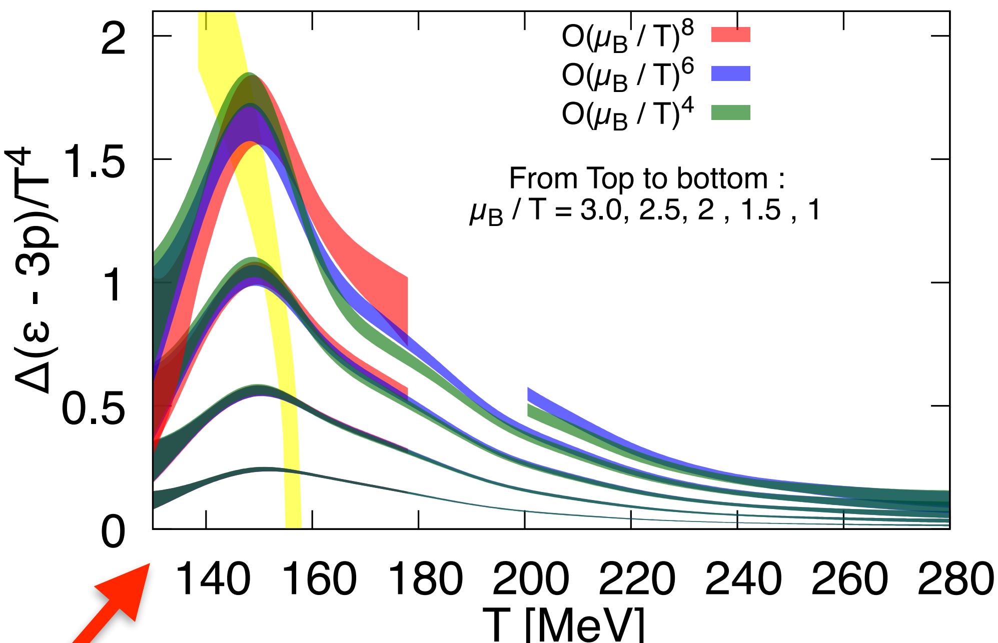
HotQCD Collab. (2212.09043)

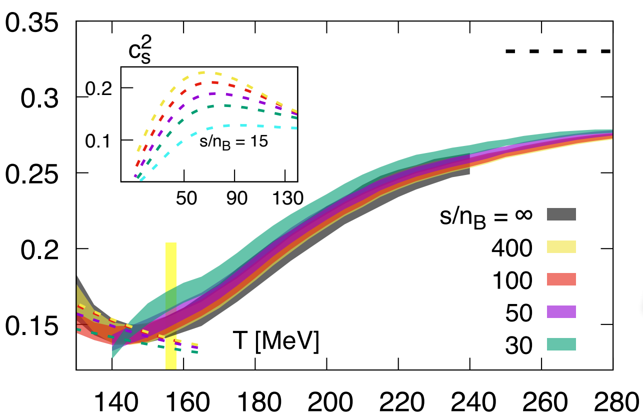
Suppressed around the phase transition (~1st-order PT) and approaching the conformal limit at high *T*…

Physics of Crossover

Energy-momentum tensor in hydro variables:

$$\Theta^{\mu}{}_{\nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix}$$





Any relation? Yes!!

Physics of Crossover

Fujimoto-Fukushima-McLerran-Praszalowicz (2022)

Measure of conformality: $$\Delta := \frac{1}{3} - \frac{p}{\varepsilon}$$

$$c_s^2 = \frac{dp}{d\varepsilon} = c_{s,\,\mathrm{deriv}}^2 + c_{s,\,\mathrm{non-deriv}}^2$$

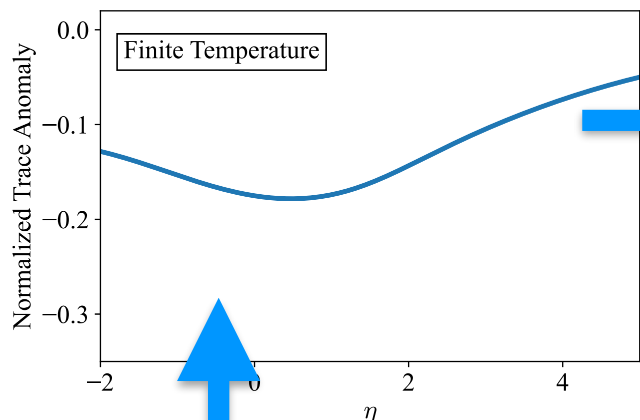
$$c_{s,\,\mathrm{deriv}}^2 = -\varepsilon\frac{d\Delta}{d\varepsilon} \qquad c_{s,\,\mathrm{non-deriv}}^2 = \frac{1}{3} - \Delta$$

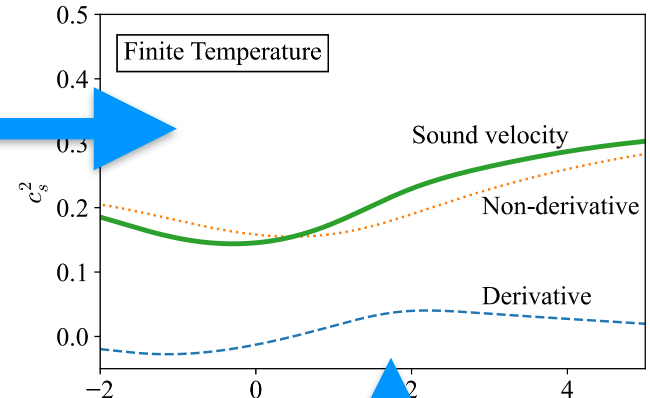
Derivative **Non-Derivative**

Physics of Crossover

High-*T* — Non-Derivative Dominant $c_s^2 \simeq p/\varepsilon$

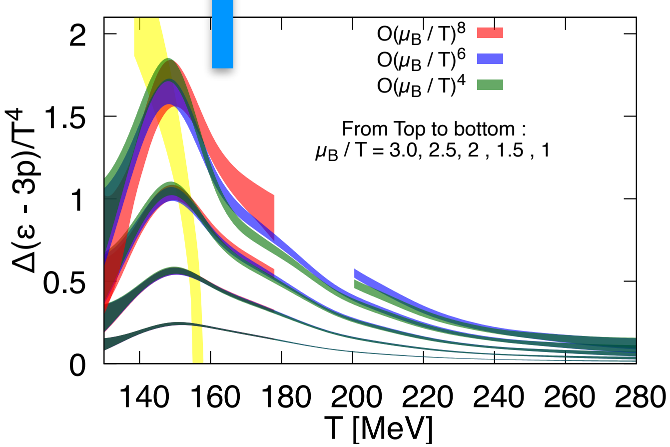
Sign Flipped $-\Delta$

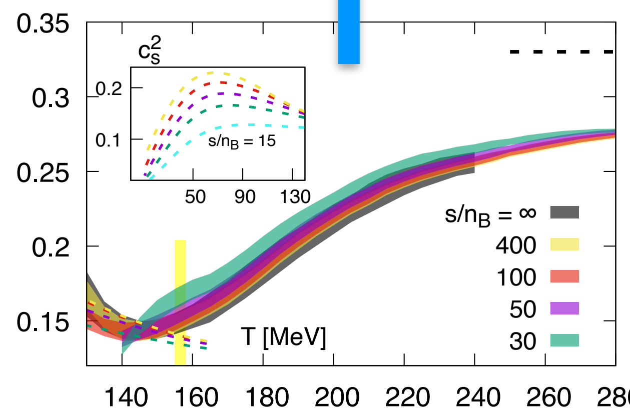




$\eta = \ln(\varepsilon/\varepsilon_0)$

Speed of Sound

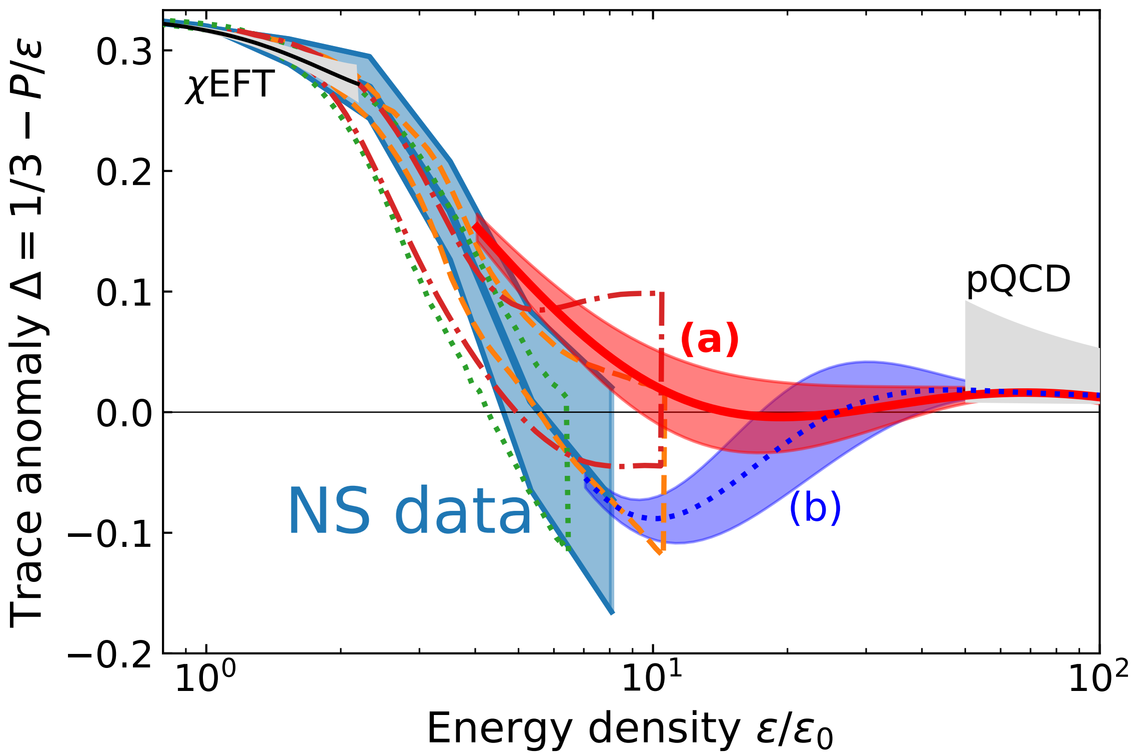




Physics of Crossover

Fujimoto-Fukushima-McLerran-Praszalowicz (2022)

"Trace Anomaly" inferred from the NS data



Quick recovery of conformality ?

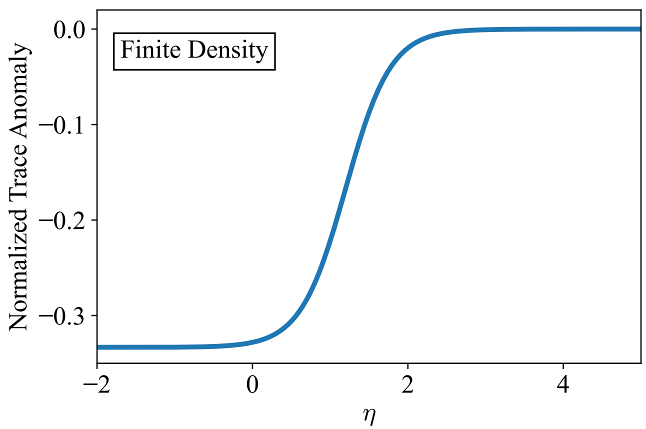
Unexpected from QCD

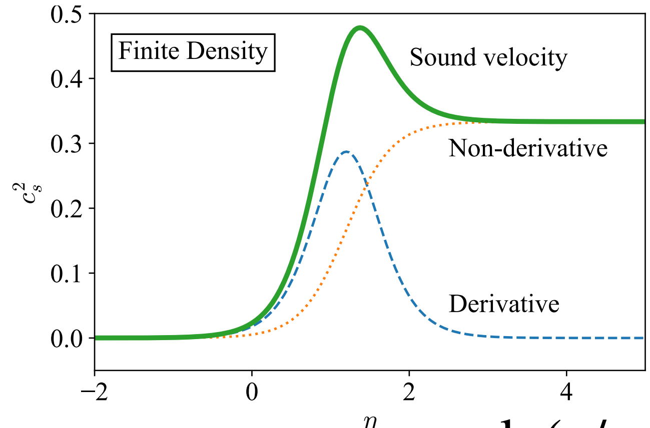
Why? How? Really?

Physics of Crossover

High Density — Derivative Peak

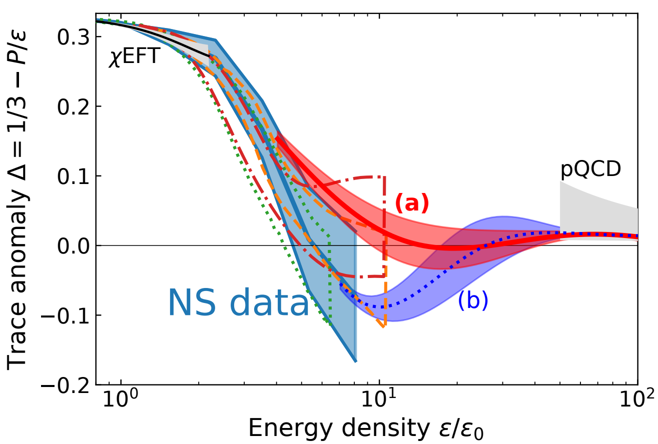
Sign Flipped $-\Delta$

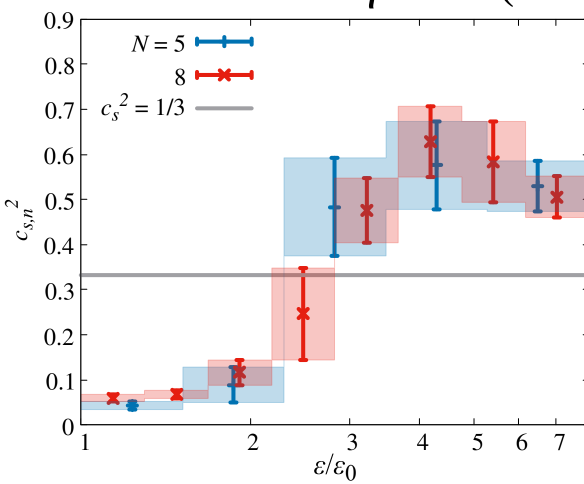




$\eta = \ln(\varepsilon/\varepsilon_0)$

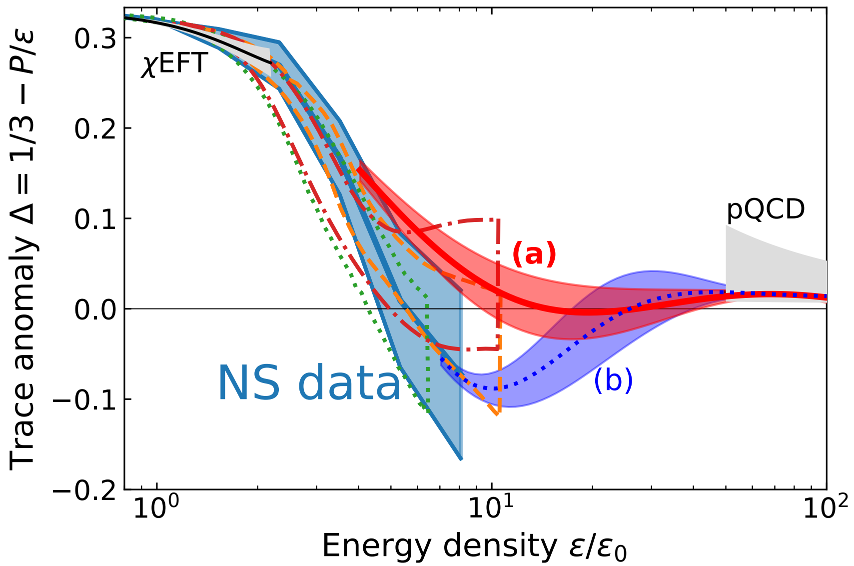
Speed of Sound





Physics of Crossover

Interesting question… $\Delta < 0$???



$$\Delta \propto \varepsilon - 3p$$

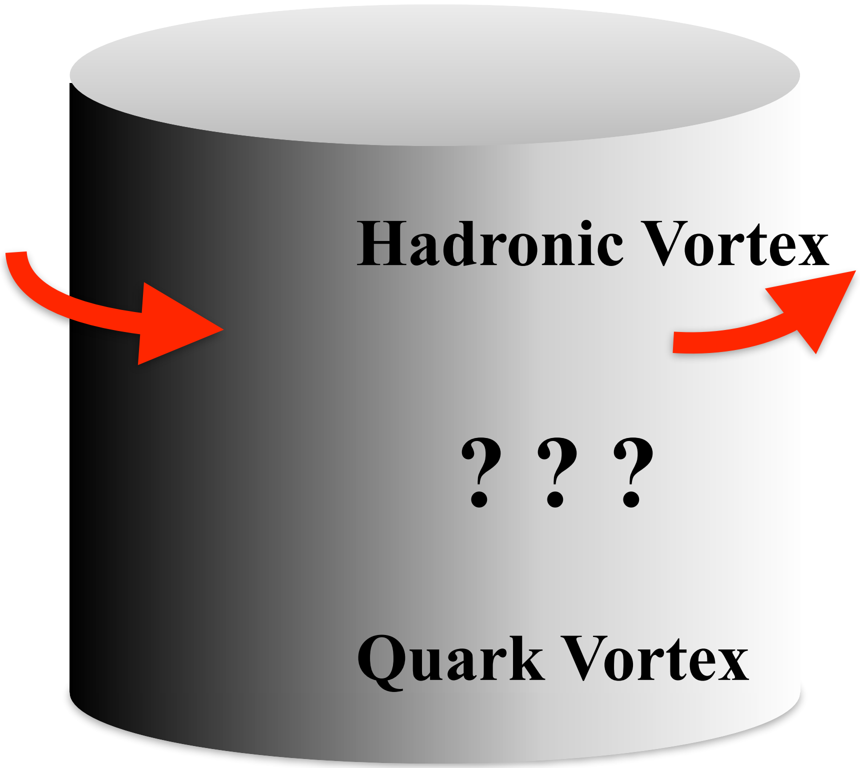
$$\propto \frac{d}{d\mu}\left(\frac{p}{\mu^4}\right)$$

Thermodynamic degrees of freedom

Negative trace anomaly implies the presence of "condensates".

Epilogue: 1st-order ruled out?

Thinking experiment: Rotating superfluid



Rotate the bucket filled with quarks

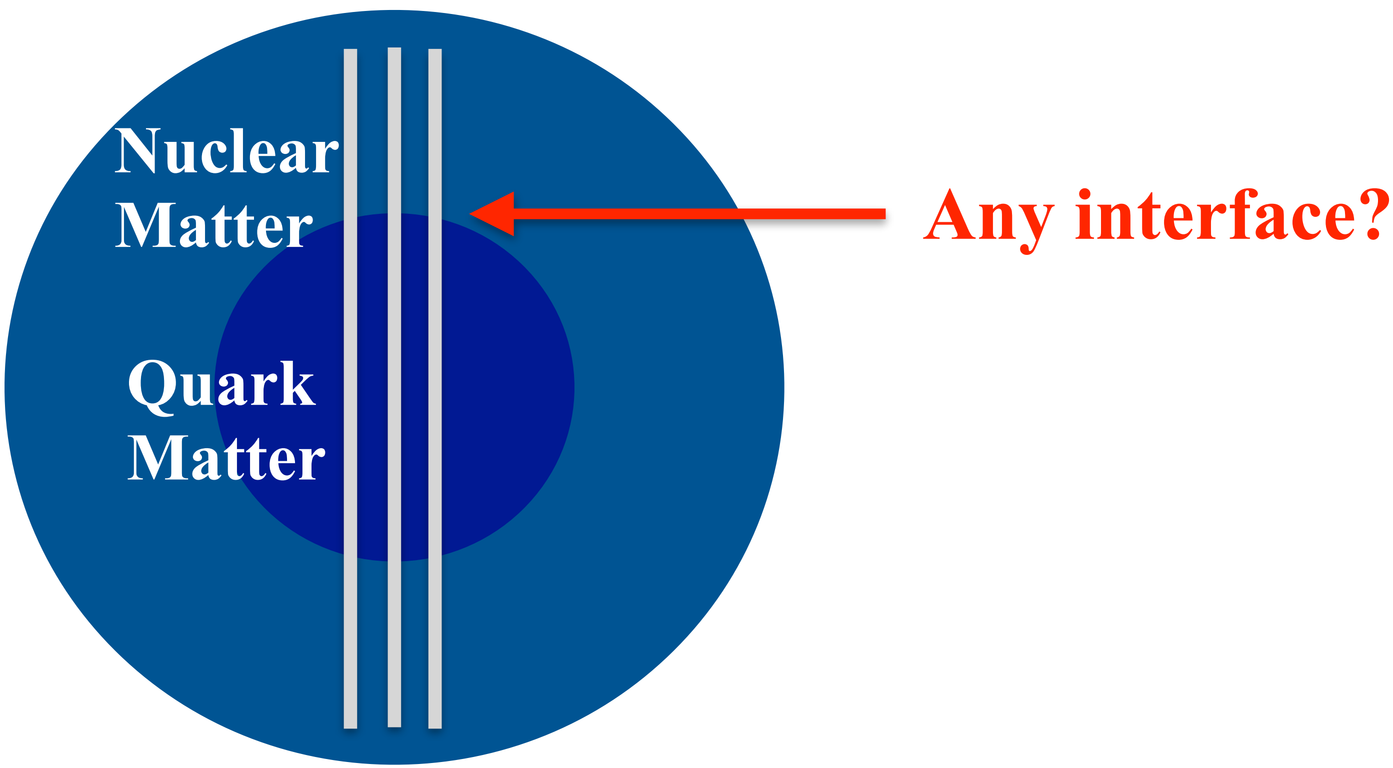
Upper part : Hadronic Vortex

Lower part : Quark Vortex

How can they be connected?

Epilogue: 1st-order ruled out?

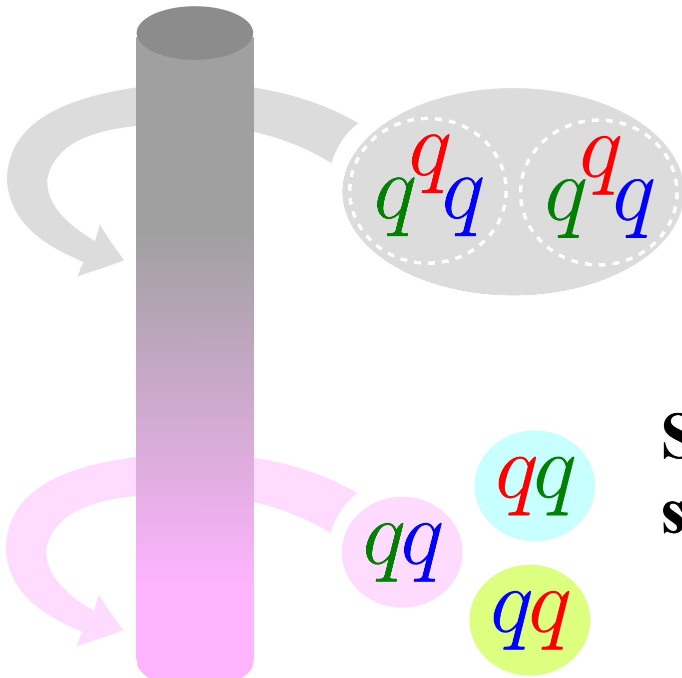
Superfluid vortices pinned in the NS cores



Epilogue: 1st-order ruled out?

Superfluid-vortex continuity

Alford-Baym-Fukushima-Hatsuda-Tachibana (2018)



Supercurrent coupled with surrounding condensates

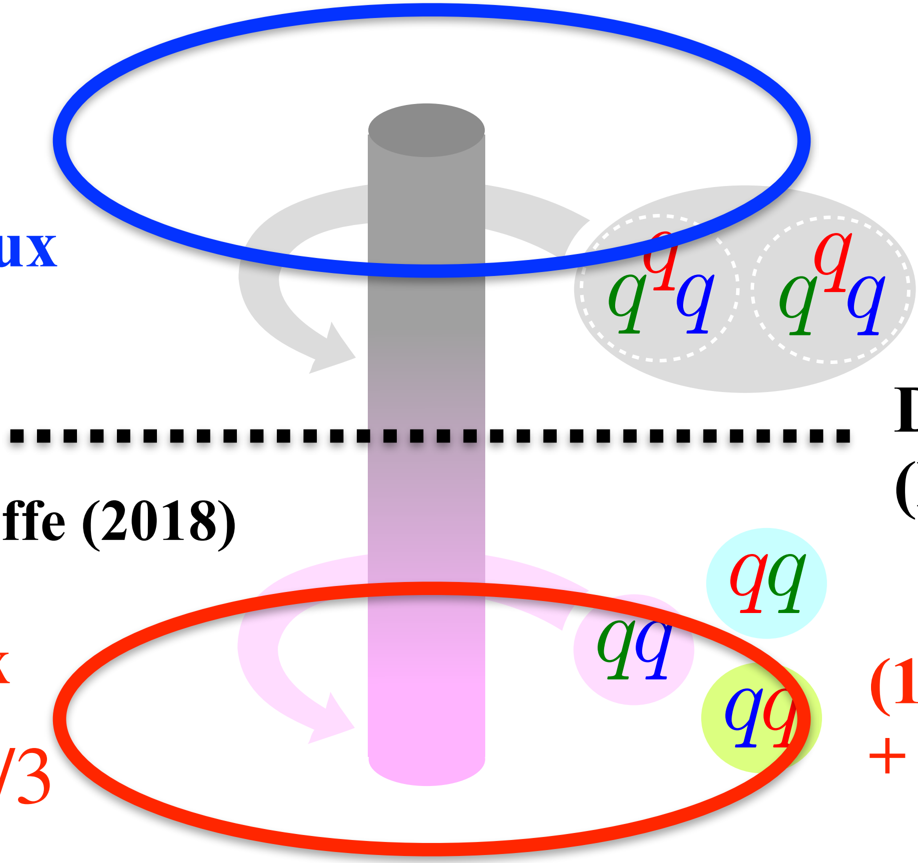
Epilogue: 1st-order ruled out?

Topological 1st-order PT ?

No magnetic flux

$$e^{i\Phi} = 1$$

Cherman-Sen-Yaffe (2018)



Discontinuous (Interface)

Quantized flux

$$e^{i\Phi} = e^{i2\pi/3}$$

(1/3) QED + (2/3) QCD

Summary

- **Our understanding about quark matter completely overriden in recent 15 years.**

- **Nuclear matter and quark matter: no order parameter, no phase transition, probably.**

- **Peak in the speed of sound = Quick recovery of conformal symmetry in dense matter.**

- **Higher-form symmetry may detect 1st-order?**