Dynamical simulations of quark stars in general relativity

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based on collaboration with
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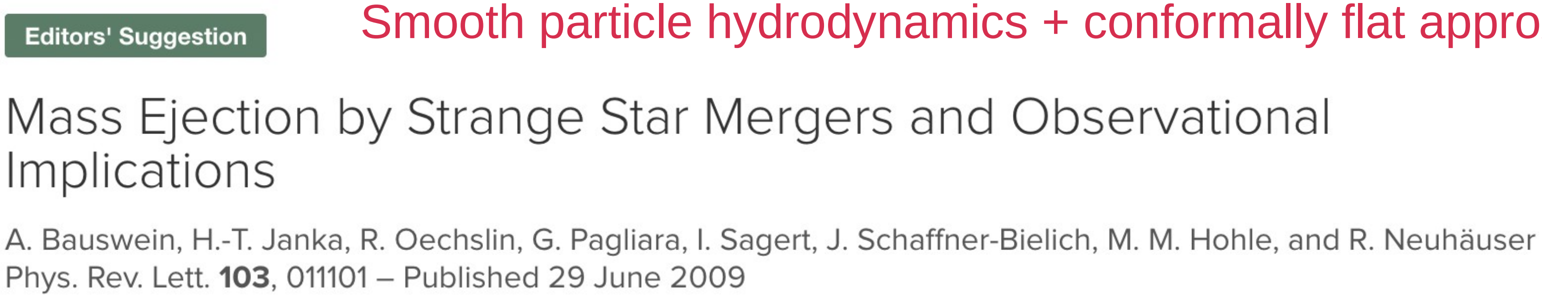
Dialogue at the Dream Field (DDF 2024): Supranuclear Matter
10-15 May 2024, Guizhou

Picture source: Chinese Academy of Sciences

Quark star simulations (a brief history)

Smooth particle hydrodynamics + conformally flat approximation



Editors' Suggestion

Mass Ejection by Strange Star Mergers and Observational Implications

A. Bauswein, H.-T. Janka, R. Oechslin, G. Pagliara, I. Sagert, J. Schaffner-Bielich, M. M. Hohle, and R. Neuhäuser
Phys. Rev. Lett. **103**, 011101 – Published 29 June 2009

Full GR grid-based hydrodynamics approach

Evolution of bare quark stars in full general relativity: Single star case

Enping Zhou, Kenta Kiuchi, Masaru Shibata, Antonios Tsokaros, and Kōji Uryū
Phys. Rev. D **103**, 123011 – Published 8 June 2021

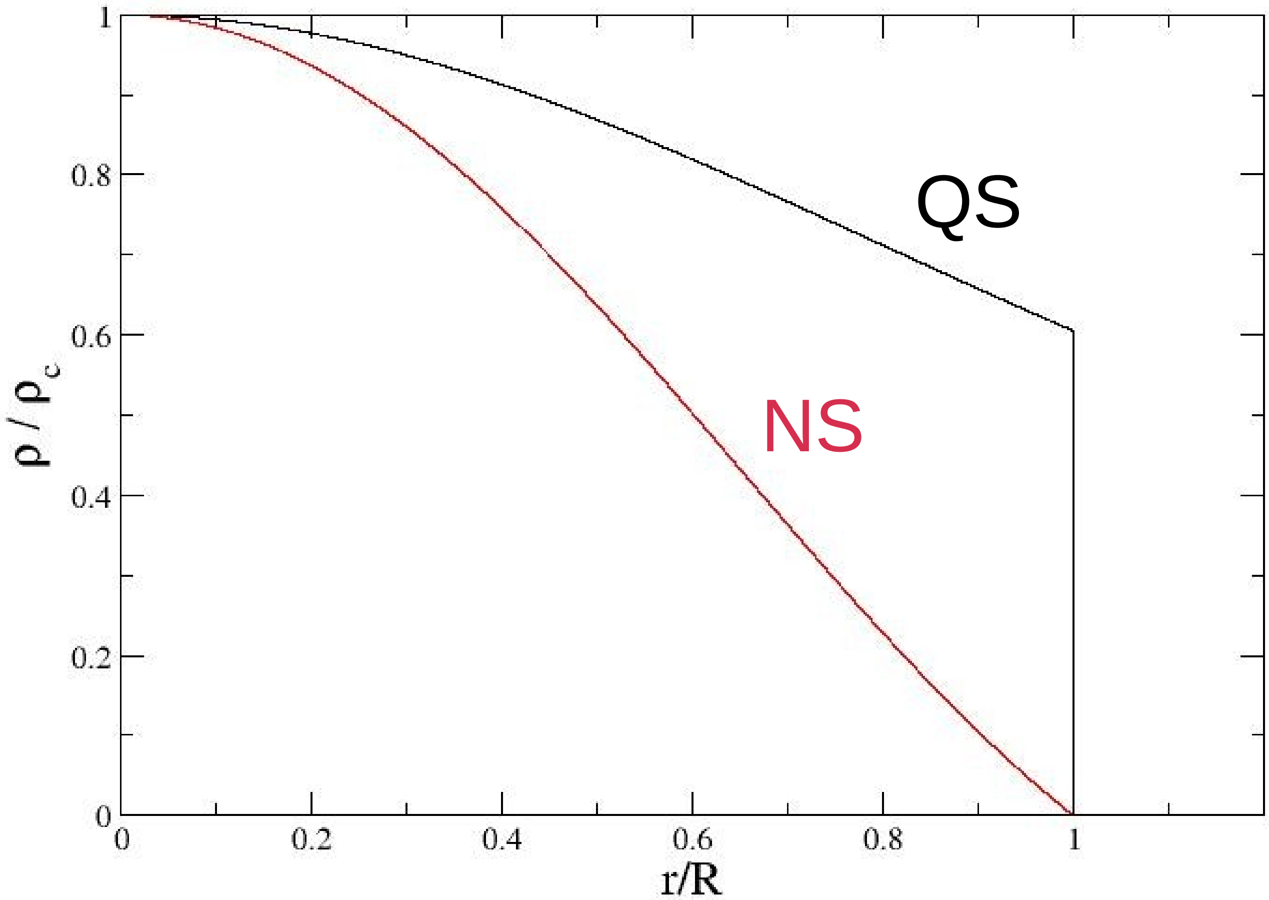
Fully general-relativistic simulations of isolated and binary strange quark stars

Zhenyu Zhu and Luciano Rezzolla
Phys. Rev. D **104**, 083004 – Published 1 October 2021

Evolution of equal mass binary bare quark stars in full general relativity: Could a supramassive merger remnant experience prompt collapse?

Enping Zhou, Kenta Kiuchi, Masaru Shibata, Antonios Tsokaros, and Kōji Uryū
Phys. Rev. D **106**, 103030 – Published 23 November 2022

Quark star vs Neutron Star



MIT bag model

$$P = \frac{1}{3}(\rho - 4B)$$

The surface of QS poses a numerical challenge!

Our recent simulations of quark stars in full GR

-- Oscillations of rapidly rotating QS

Chen & LML, PRD, 108, 064007 (2023)

Kenneth Chen

Ongoing work:

-- Binary QS merger

-- Gravitational collapse of rapidly rotating QS

GR hydrodynamics in one page

3+1 metric: $ds^2 = -\alpha^2 dt^2 + \gamma_{ij}\left(dx^i + \beta^i dt\right)\left(dx^j + \beta^j dt\right)$

Einstein equation: $G_{\mu\nu} = 8\pi T_{\mu\nu}$ → Constraint equations → Evolution equations

Fluid equations:

$$\boxed{\begin{aligned} \nabla_\mu(\rho u^\mu) &= 0, \\ \nabla_\mu T^{\mu\nu} &= 0, \end{aligned}} \longrightarrow \frac{\partial \mathbf{U}}{\partial t} + \underline{\frac{\partial \mathbf{F}^i}{\partial x^i}} = \mathbf{S},$$

Conservative form

Flux term

Many numerical methods have been developed for this type of fluid equation in computational fluid dynamics
(…. essentially how to treat the flux term)

Our quark star simulations

- Use open-source *LORENE* to generate initial data for rotating QS and binary QS

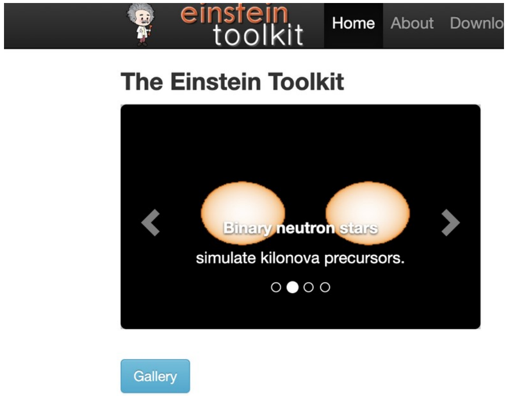
https://lorene.obspm.fr/

LORENE

Langage Objet pour la RElativité NumériquE

- Use *Einstein Toolkit* to perform GR hydro evolution

https://einsteintoolkit.org/



The Einstein Toolkit

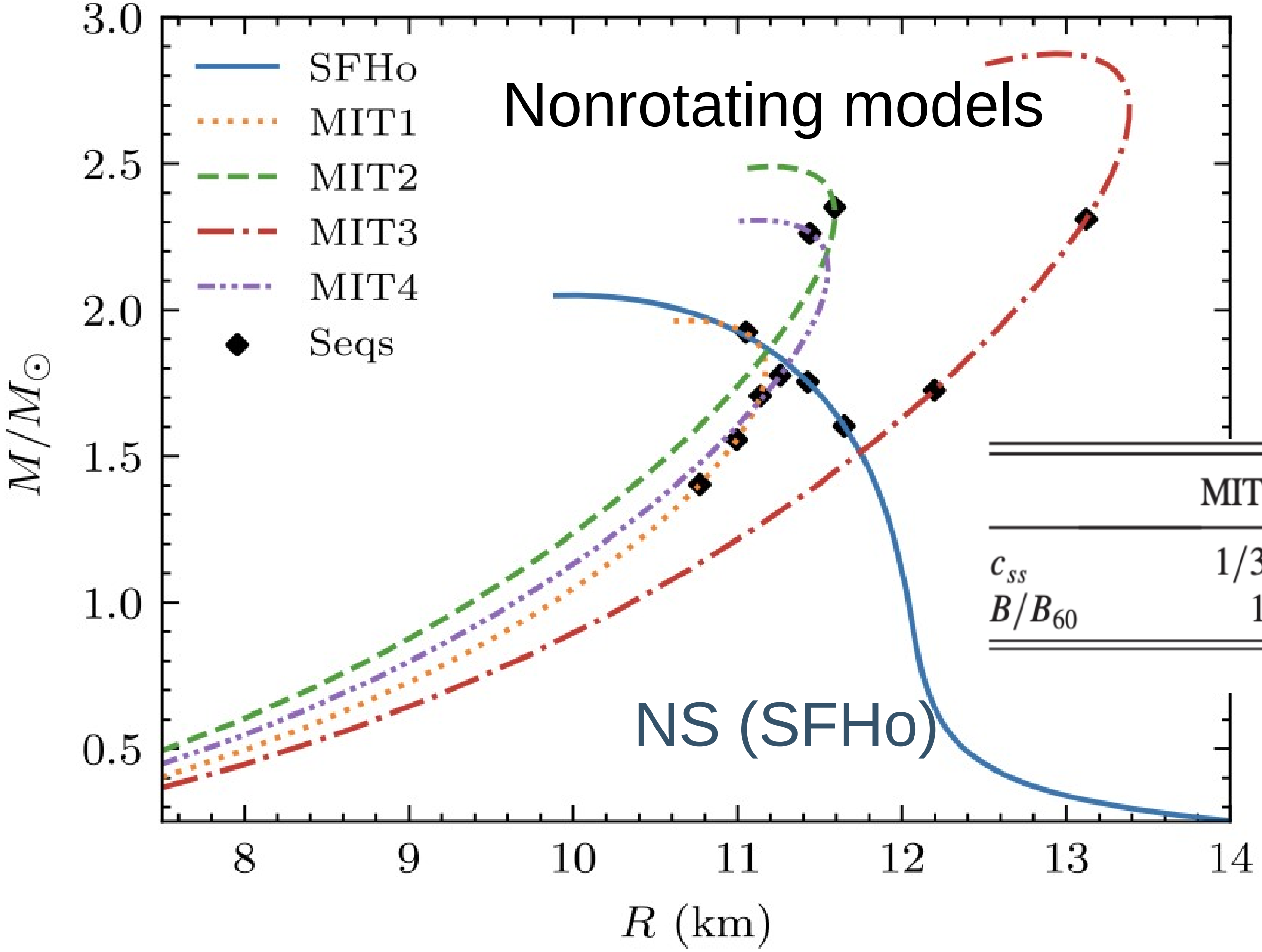
Binary neutron stars

simulate kilonova precursors.

- We need to implement our own hydro scheme to handle the surface density discontinuity of quark stars

- Special treatment of a “dust” atmosphere outside the star

Our quark star models



Nonrotating models

NS (SFHo)

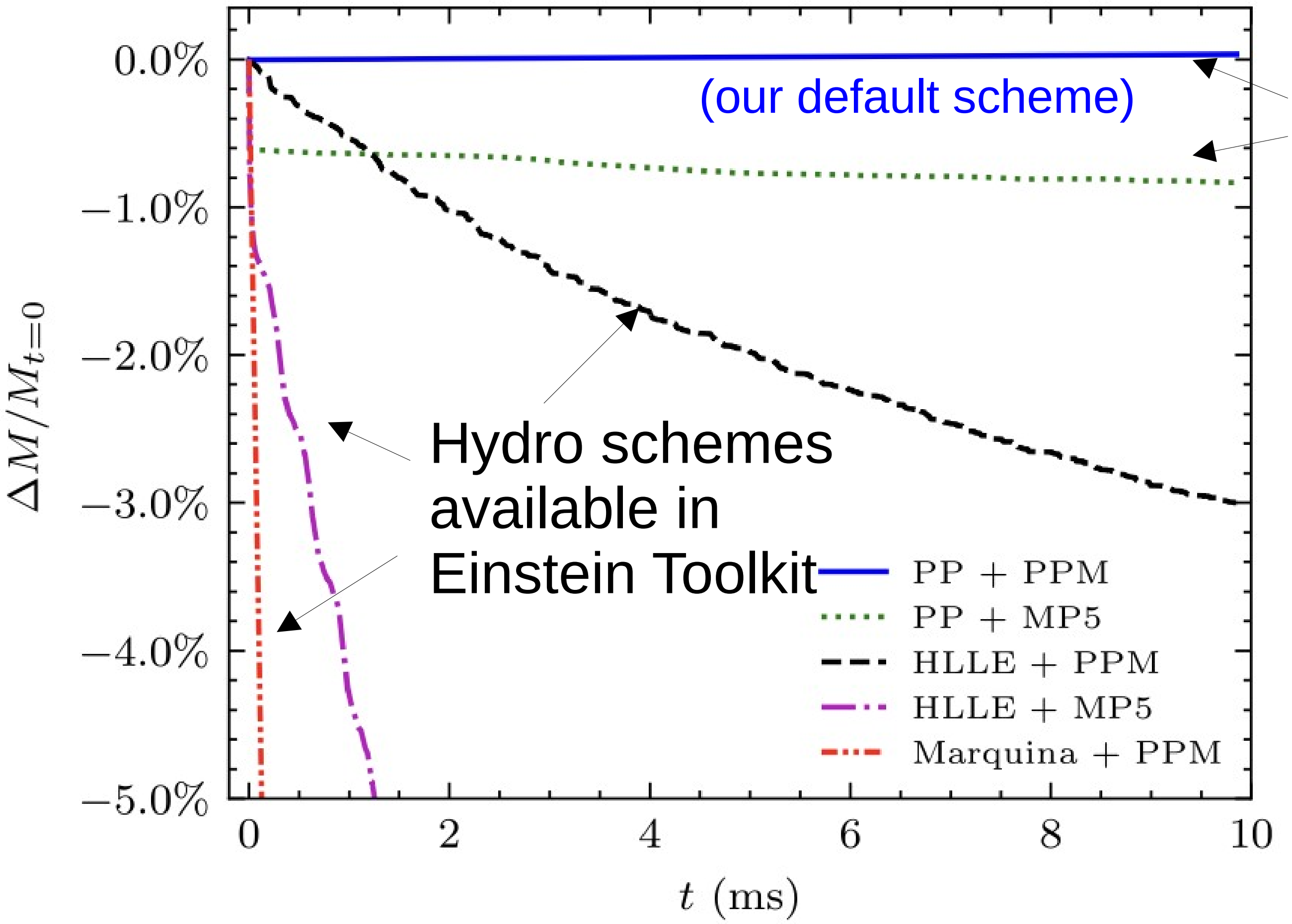
Parametrized model:

$$P = c_{ss}\,\epsilon - (1 + c_{ss})\,B$$

	MIT1	MIT2	MIT3	MIT4
c_{ss}	1/3	1	2/3	1/2
B/B_{60}	1	3	3/2	3/2

B_{60} = 60 MeV/fm^3

Mass conservation



(our default scheme)

Hydro schemes available in Einstein Toolkit

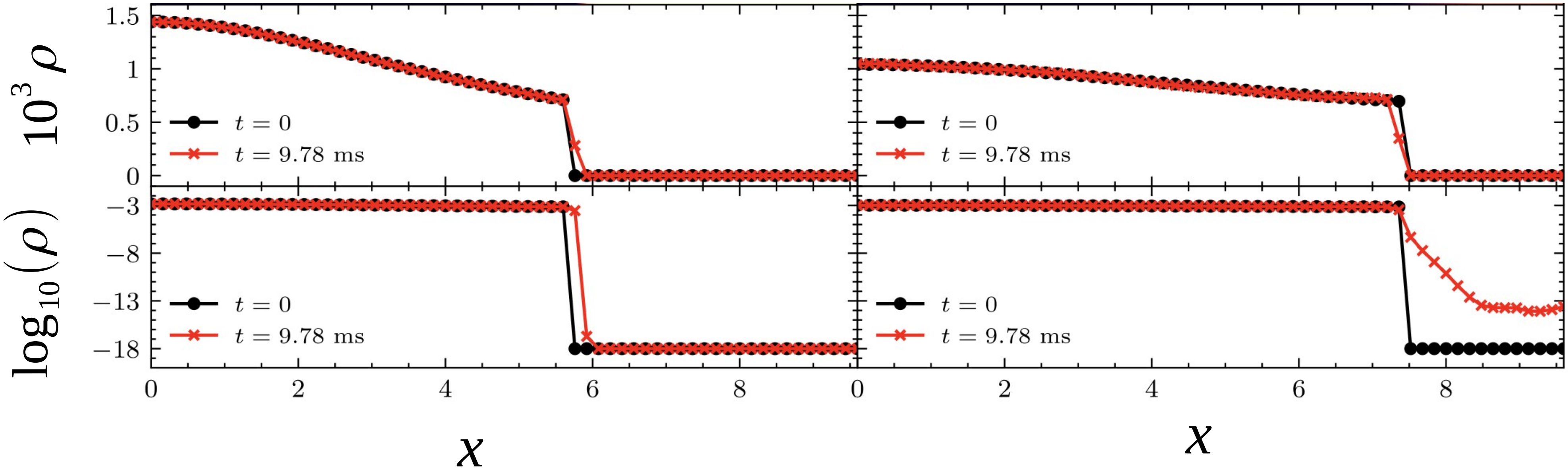
~0.03% deviation

Our hydro method (Positivity preserving Riemann solver)

Basically, the scheme preserves the positivity of density and pressure near the surface

Einstein Toolkit does not work out of the box for quark stars!

Stability of rest-mass density profiles

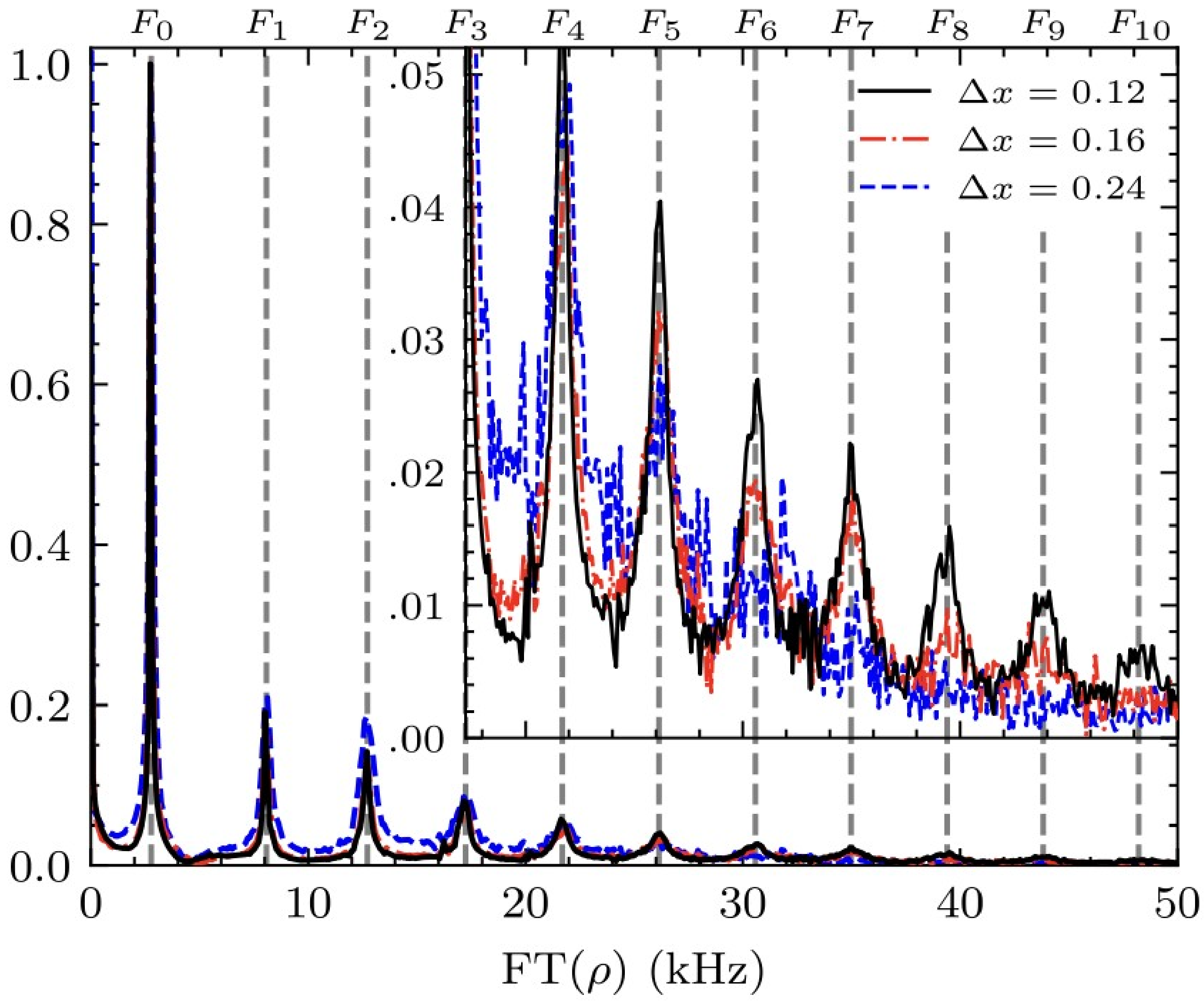


Rotation frequency = 300 Hz

Rotation frequency = 1200 Hz (~max frequency)

(Code’s length unit: 1 ≈ 1.47 km)

Radial oscillation modes of nonrotating quark star



Dashed lines = perturbative normal-mode results

Important test for our treatments of the QS surface

Computation of oscillation modes

- Nonrotating stars:

Perturbed scalar variables: $\delta\rho = f(r) Y_{lm}(\theta,\phi) e^{i\omega t}$

$$\delta G_{\mu\nu} = 8\pi\,\delta T_{\mu\nu} \quad ; \quad \delta(\nabla_\mu T^{\mu\nu}) = 0$$

$$\Big\downarrow$$

eigenvalue problem

For spherical stars, we only need to consider *m* = 0.

- Rotating stars

$$\delta\rho = f(r,\theta)\, e^{i(m\phi+\omega t)}$$

Degeneracy in *m* is lifted due to rotation (similar to Zeeman effect)

We shall focus on the $l = |m| = 2$ f-modes.

How to obtain oscillation modes of rapidly rotating stars?

Oscillation modes of rapidly rotating stars

Add a “suitable” perturbation to an initial rotating star

Hydrodynamics code

Options:

Exact GR / approximated GR / fixed spacetime (no gravity perturbation)

+

Nonlinear hydro / Linearized hydro

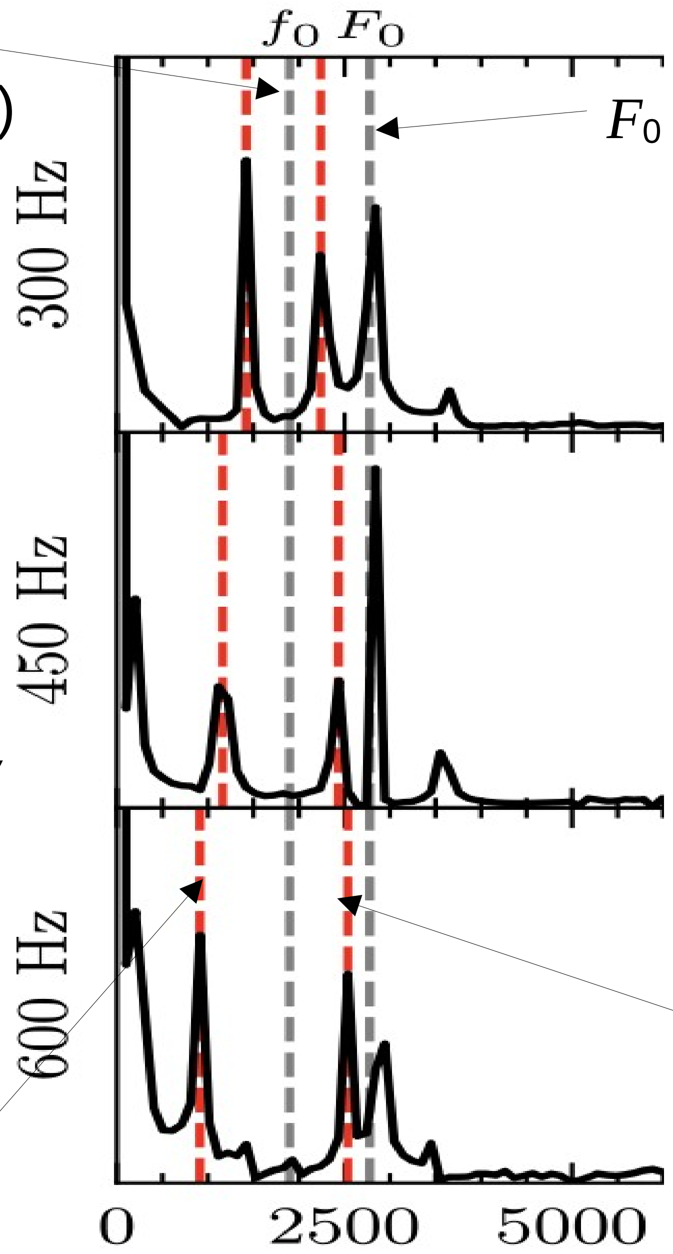
Time evolution

Perform Fourier transform for fluid variables to extract oscillation modes

f_0 = f-mode
(nonrotating star)

F_0 = Fundamental radial mode
(nonrotating star)

Rotation frequency



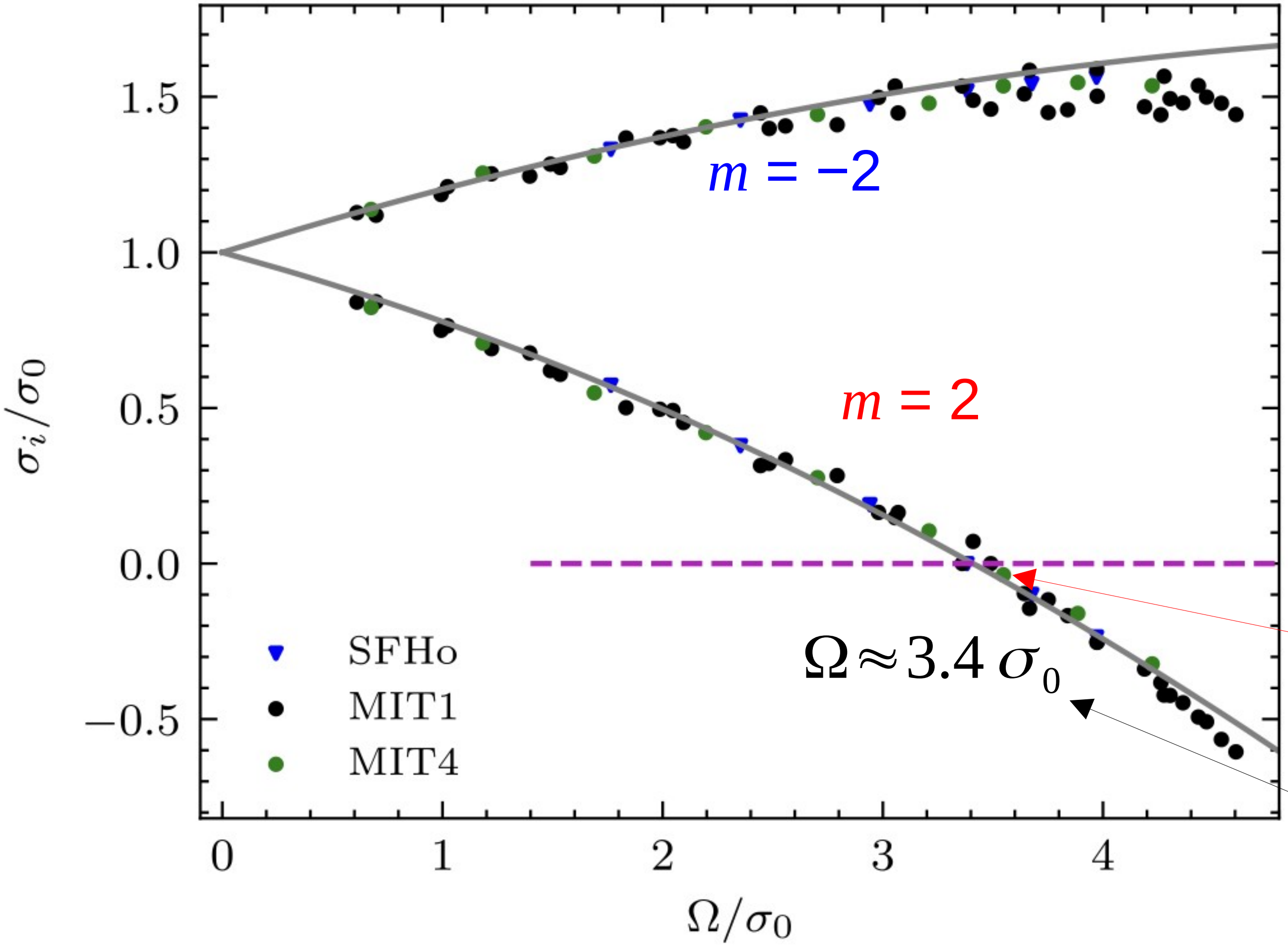
(Hz)

Fourier spectra of density

Right red lines:
$m = -2$ mode
(co-rotating)

Left red lines:
$m = 2$ mode
(counterrotating)

f-mode frequencies (observed in the inertial frame) for sequences of constant central energy density



$m = -2$

$m = 2$

$\Omega \approx 3.4\,\sigma_0$

Solid lines:
“Universal” relations for NS

Krüger & Kokkotas,
PRL, 125, 111106
(2020)

CFS instability onset
(GW driven)

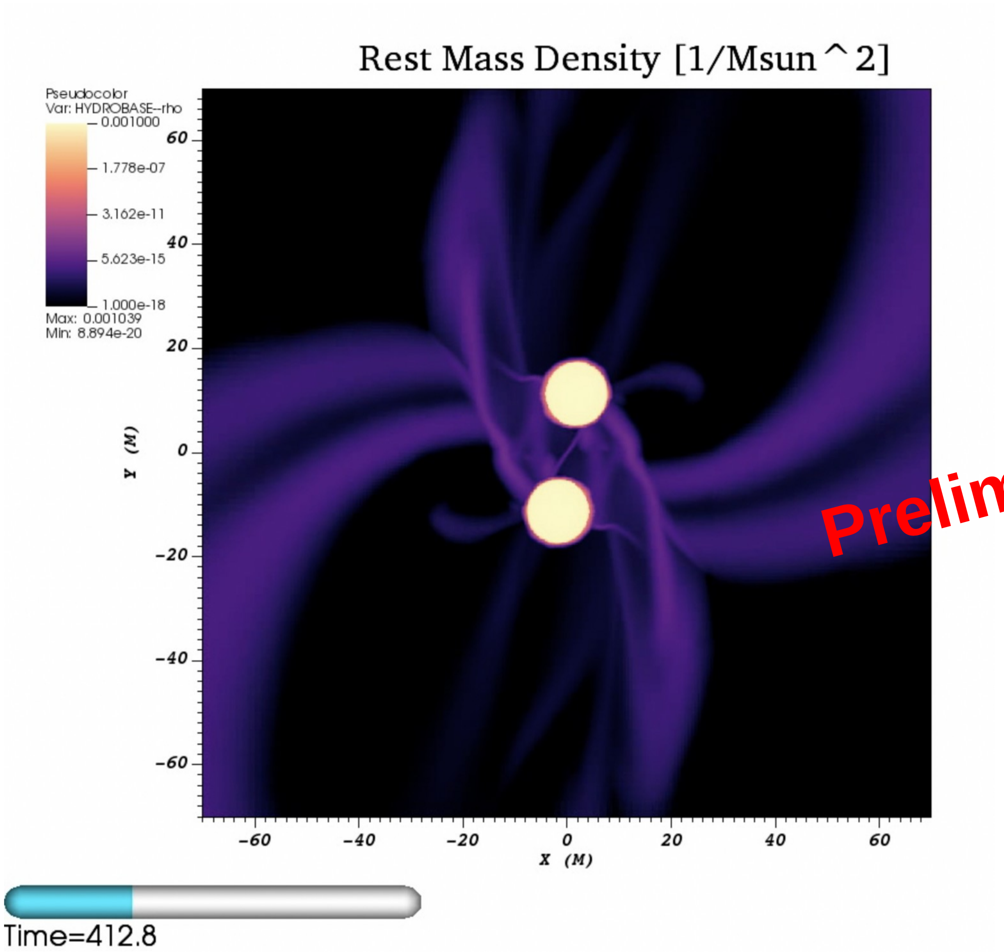
f-mode frequency
of nonrotating model

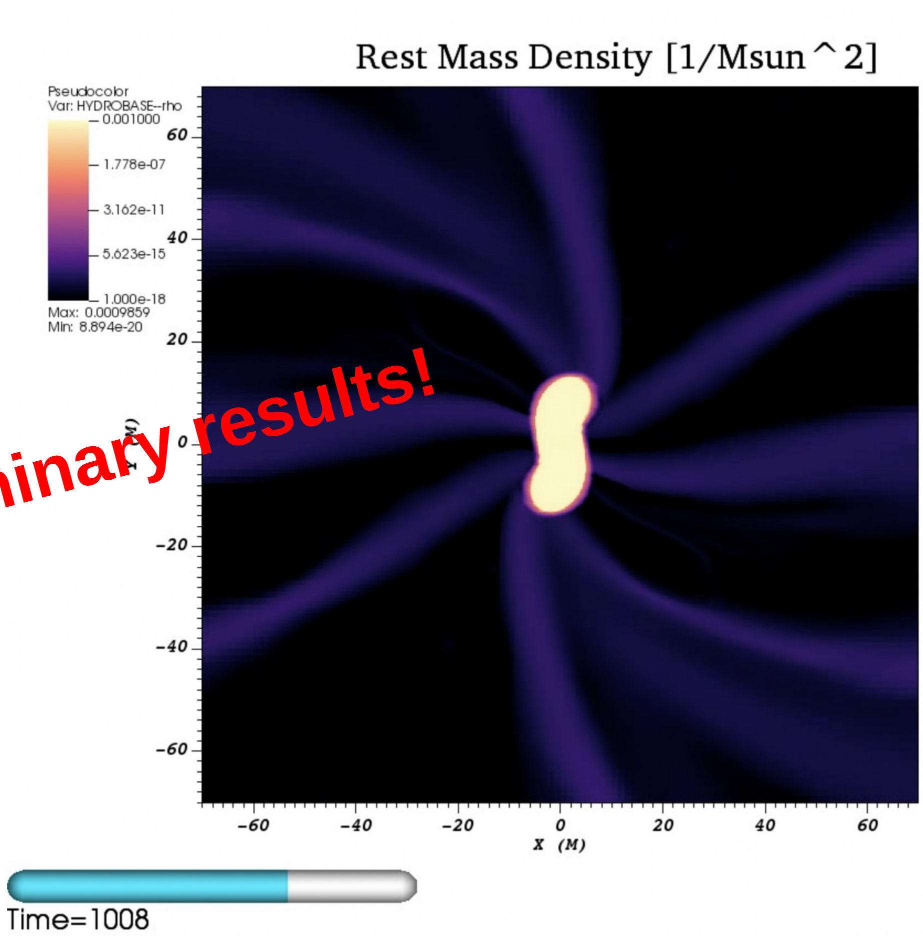
Rapidly rotating quark stars still satisfy the same universal relations and the onset condition for CFS instability as neutron stars

Chen & LML, PRD, 108, 064007 (2023)

Some preliminary results:
Binary QS mergers

Binary QS merger





Preliminary results!

- GW frequency “at amplitude maximum” $f_{\max}$ in BNS simulations

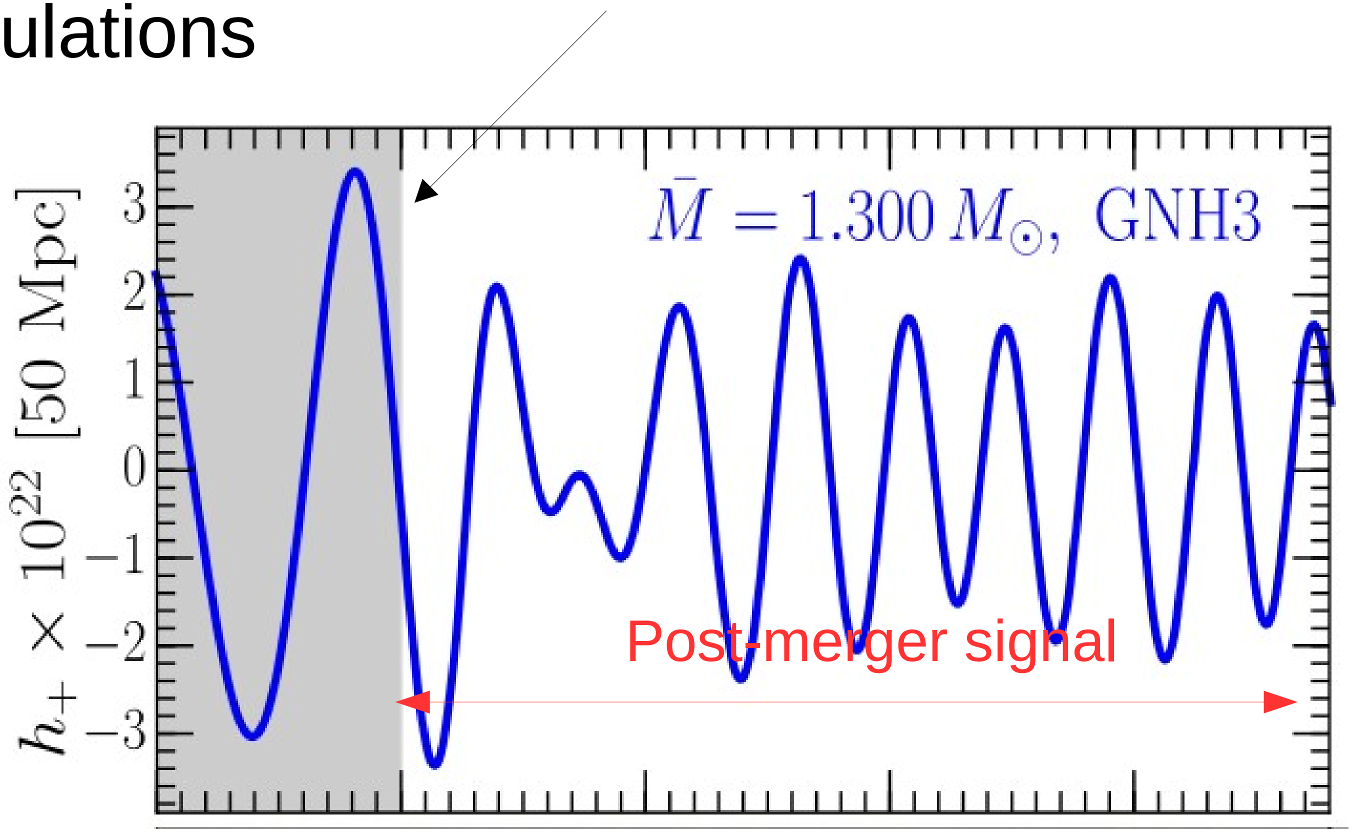
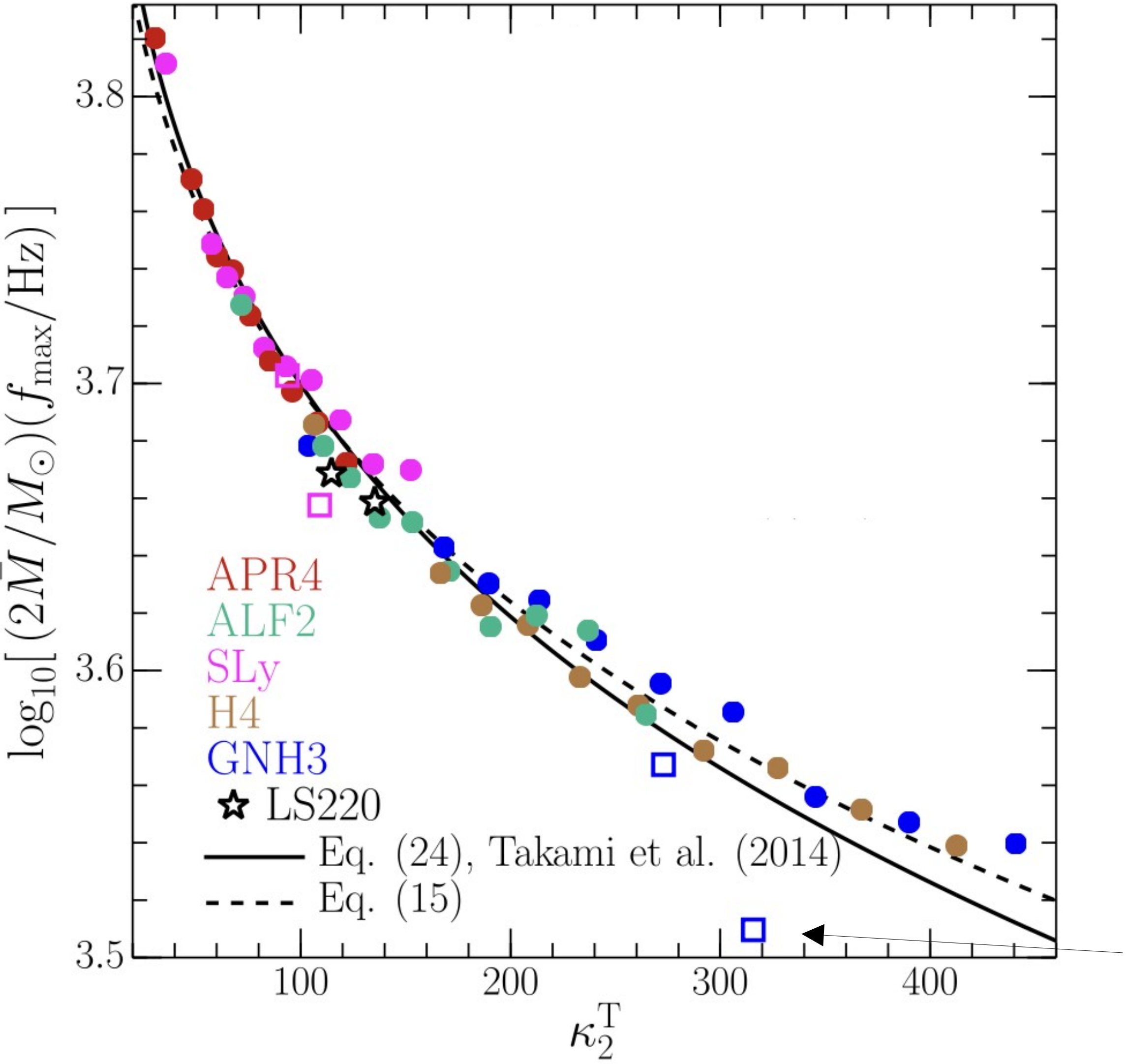


Figure from Rezzolla and Takami, PRD 93, 124051 (2016)

$f_{\max}$ was first introduced in Read et al PRD 88, 044042 (2013)

“Universal” relation between $f_{\max}$ and dimensionless tidal deformability (for equal-mass systems)



For equal-mass systems:

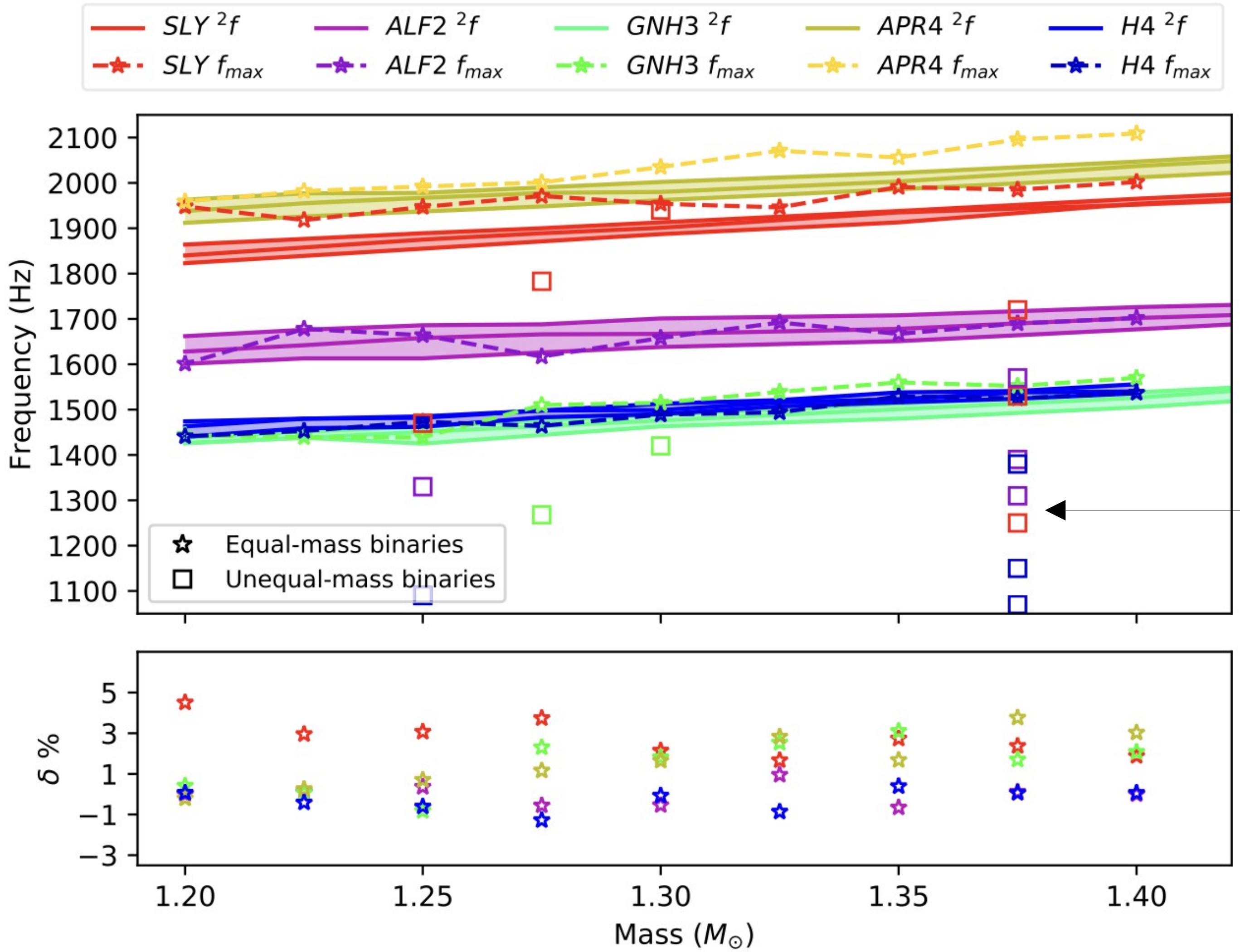
$$\kappa_2^T = \frac{3}{16}\frac{\lambda}{\bar{M}^5}$$

Square data: unequal-mass systems

Figure from Rezzolla and Takami, PRD 93, 124051 (2016)

Equal-mass BNS simulations:

$GW\ f_{\max}$ = f-mode of the initial stars (within a few %)

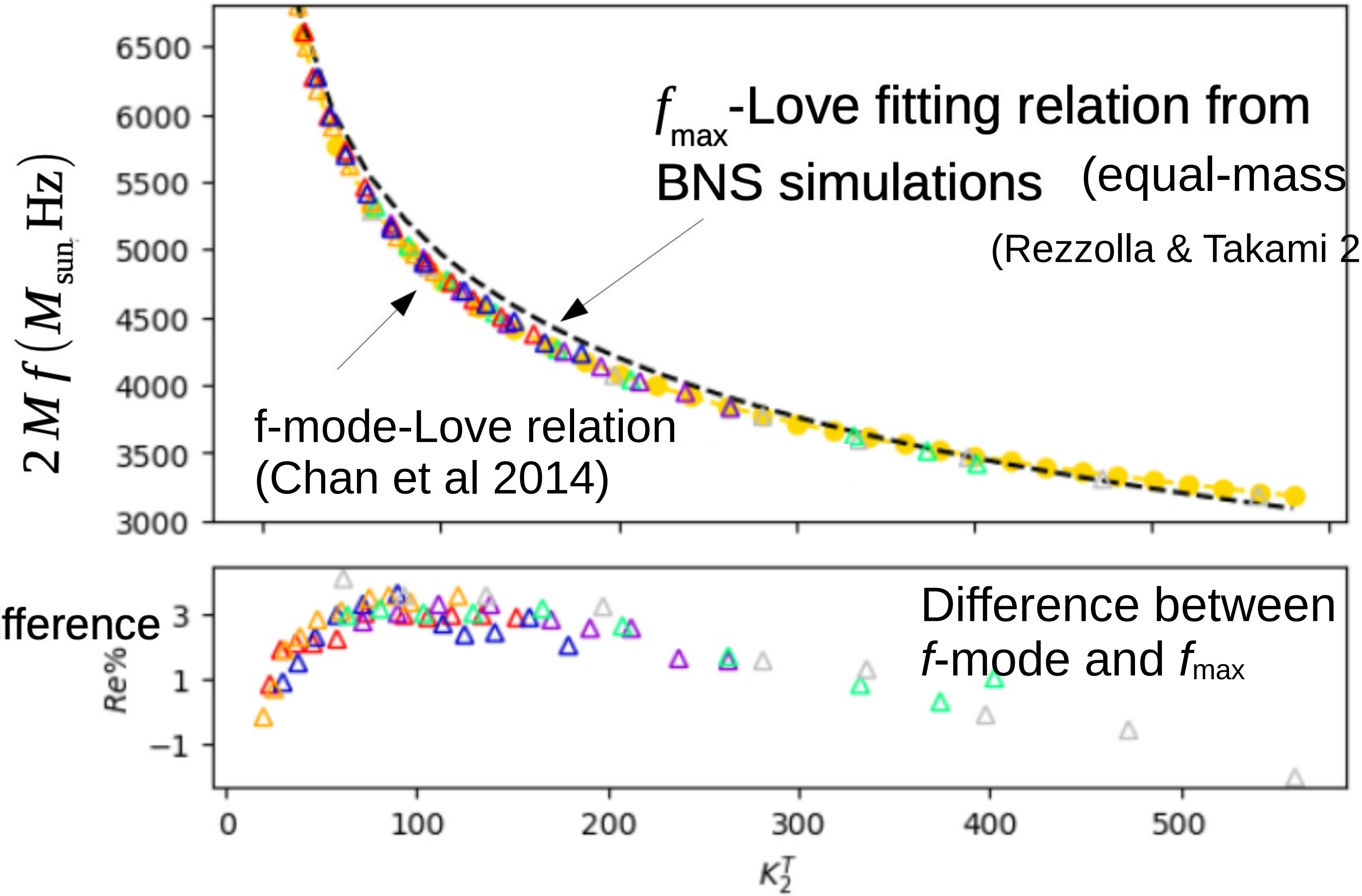


GW $f_{\max}$ of BNS published by other groups (data points)

Square data: unequal-mass systems

Color bands: f-mode of isolated stars with different rotation rates

Ng, Cheong, LML, Li, ApJ, 915, 108 (2021)



$f_{\rm max}$-Love fitting relation from BNS simulations (equal-mass systems)

(Rezzolla & Takami 2016)

f-mode-Love relation
(Chan et al 2014)

% difference

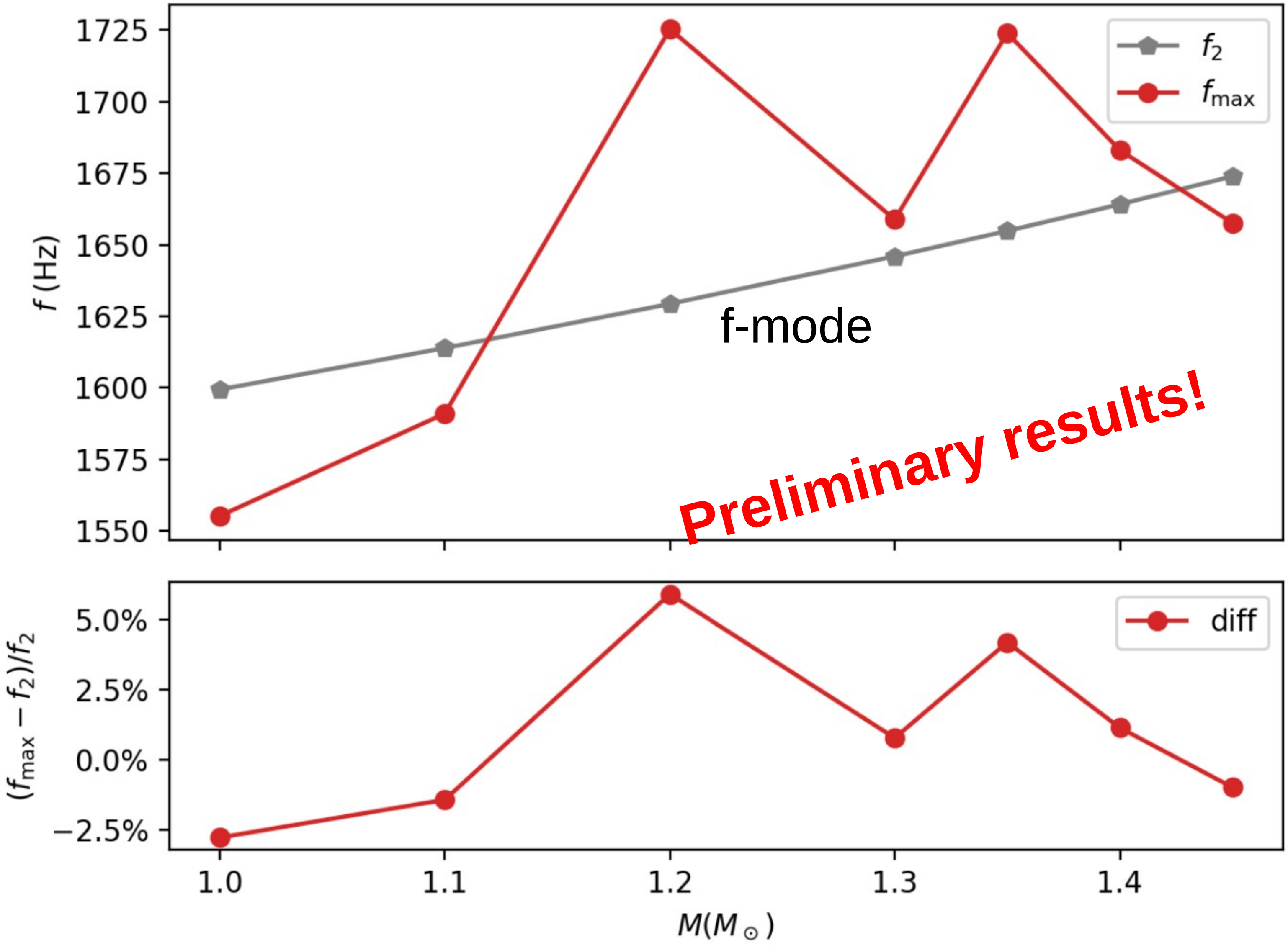
Difference between f-mode and $f_{\rm max}$

- f-mode-Love universal relations (valid for isolated NS and QS)

Chan, Sham, Leung, LML, PRD, 124023 (2014)

Equal-mass BQS simulations:

$GW\ f_{\max}$ = f-mode of the initial stars (within a few %)

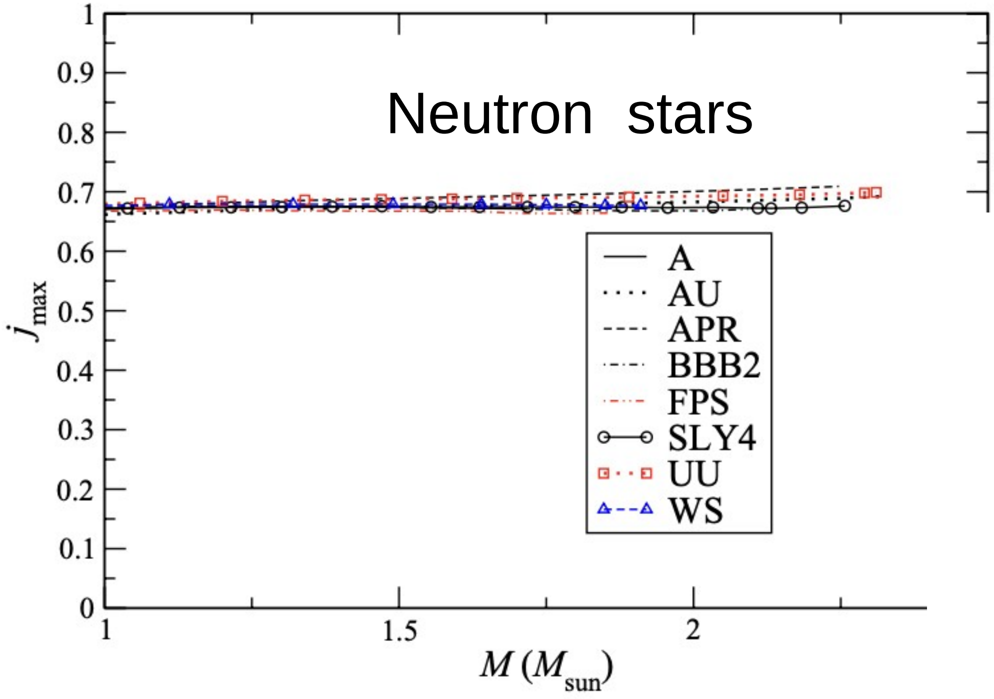


Mass of each star

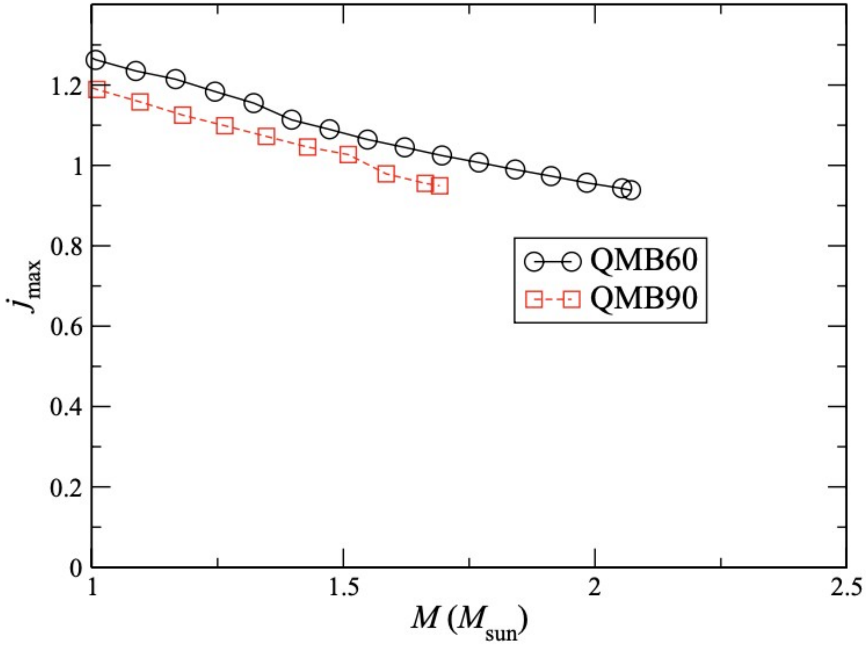
Some preliminary results:
Rotational collapse of unstable QS

(Rotating) neutron stars vs quark stars

Dimensionless spin parameter $j = \frac{cJ}{GM^2}$


Neutron stars

Quark stars
(MIT bag model)

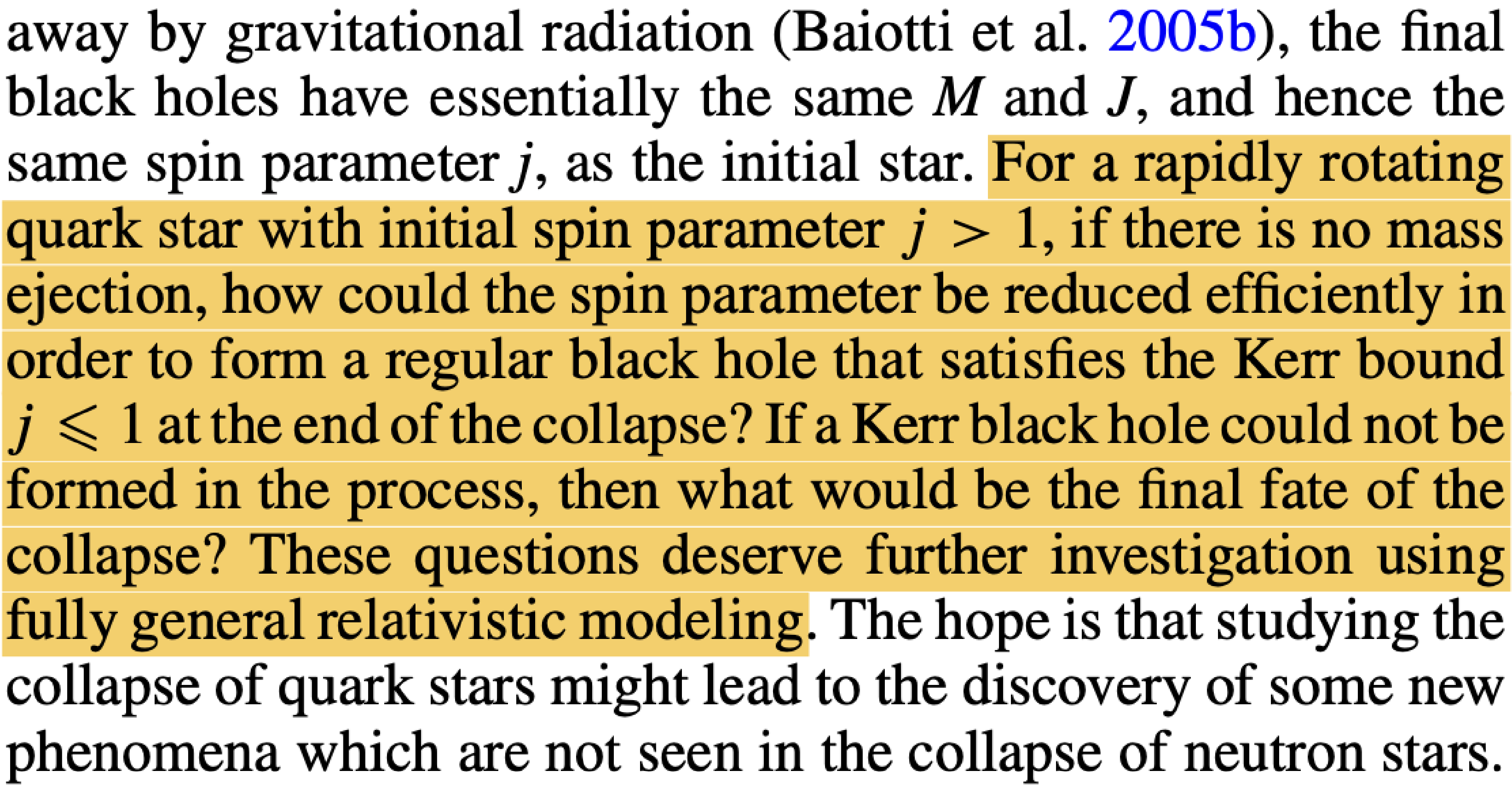


Rotating black hole: $j_{max} = 1$

Lo & LML, ApJ, 728, 12 (2011)

Collapse of rapidly rotating QS?

Spin parameter: $j = J / M^2$

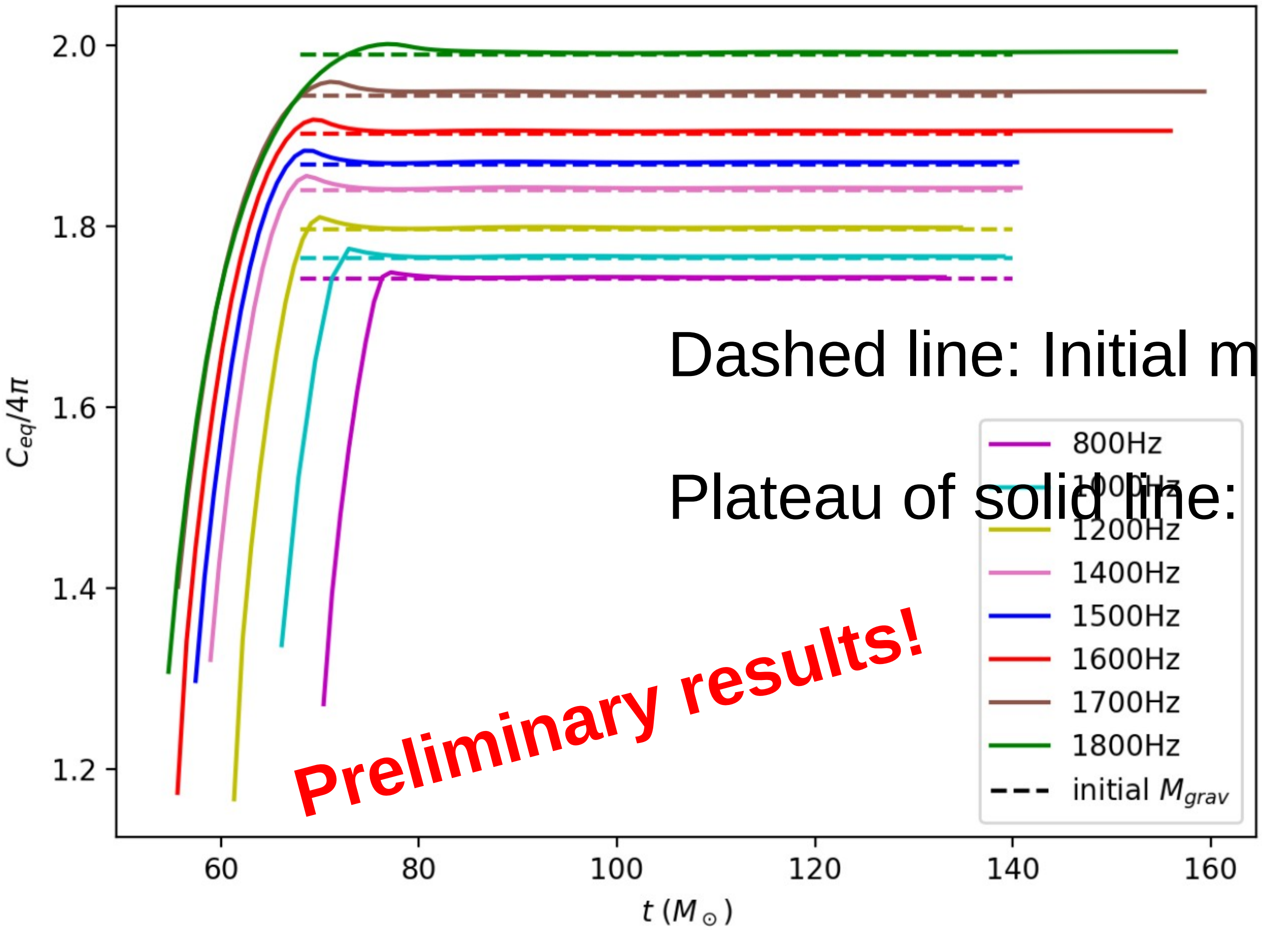


away by gravitational radiation (Baiotti et al. 2005b), the final black holes have essentially the same M and J, and hence the same spin parameter j, as the initial star. For a rapidly rotating quark star with initial spin parameter $j > 1$, if there is no mass ejection, how could the spin parameter be reduced efficiently in order to form a regular black hole that satisfies the Kerr bound $j \leqslant 1$ at the end of the collapse? If a Kerr black hole could not be formed in the process, then what would be the final fate of the collapse? These questions deserve further investigation using fully general relativistic modeling. The hope is that studying the collapse of quark stars might lead to the discovery of some new phenomena which are not seen in the collapse of neutron stars.

Lo & LML, ApJ, 728, 12 (2011)

Properties of final black holes based on the forming apparent horizon (AH)

In some loose sense, AH lies “inside” event horizon



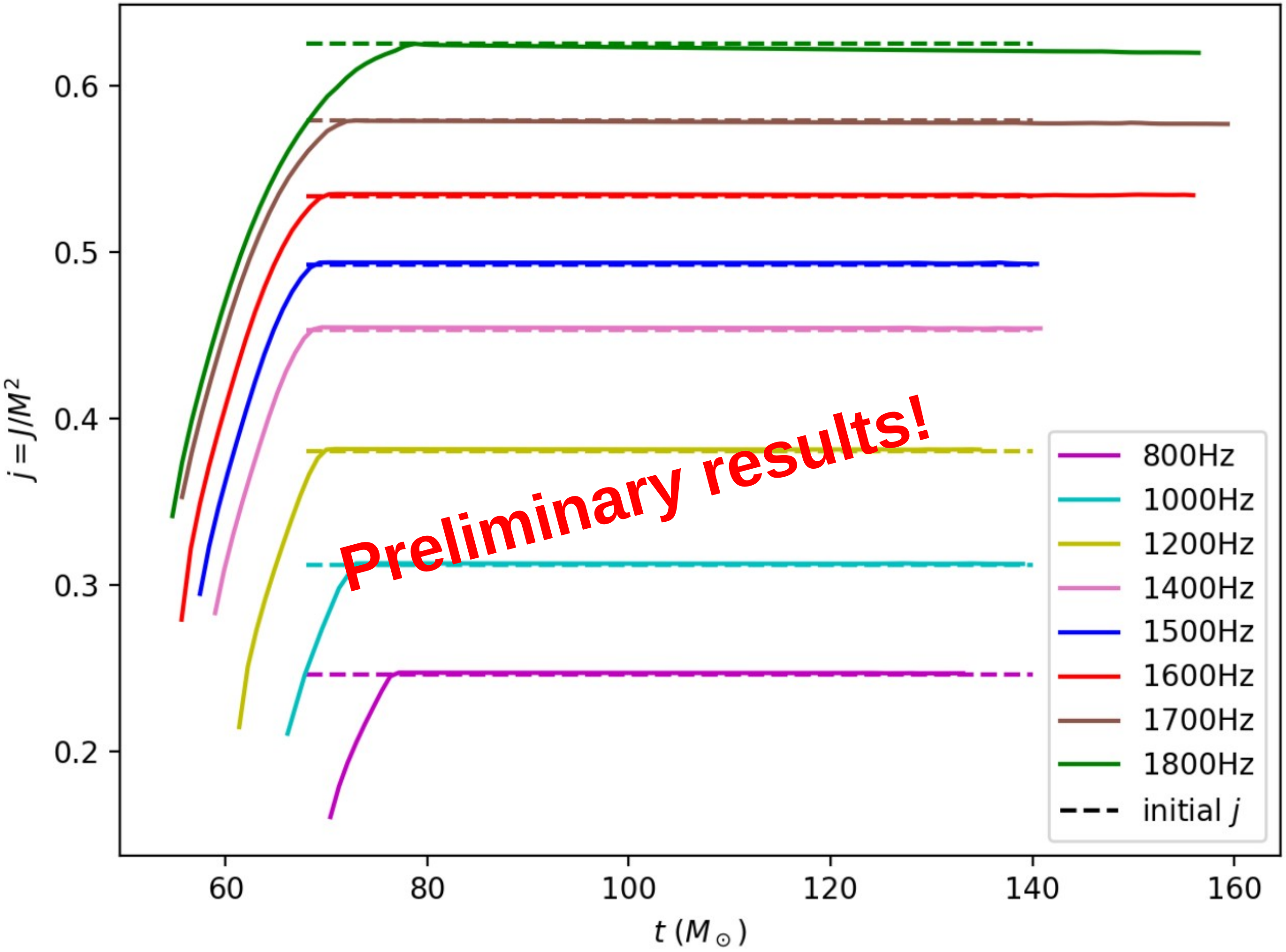
settled down to Kerr BH

Dashed line: Initial mass of QS

Plateau of solid line: Mass of the final black hole

Preliminary results!

Spin parameter



Dashed line: Initial j of QS

Plateau of solid line: final j of black hole

Summary

We have demonstrated our ability to

- simulate stable evolution and oscillations of rapidly rotating QS near the Kepler limit
- study the f-mode and onset on secular instabilities
- study equal-mass binary QS mergers
  (and found that GW $f_{max} \approx$ f-mode, just like BNS)
- study the rotational collapse of QS

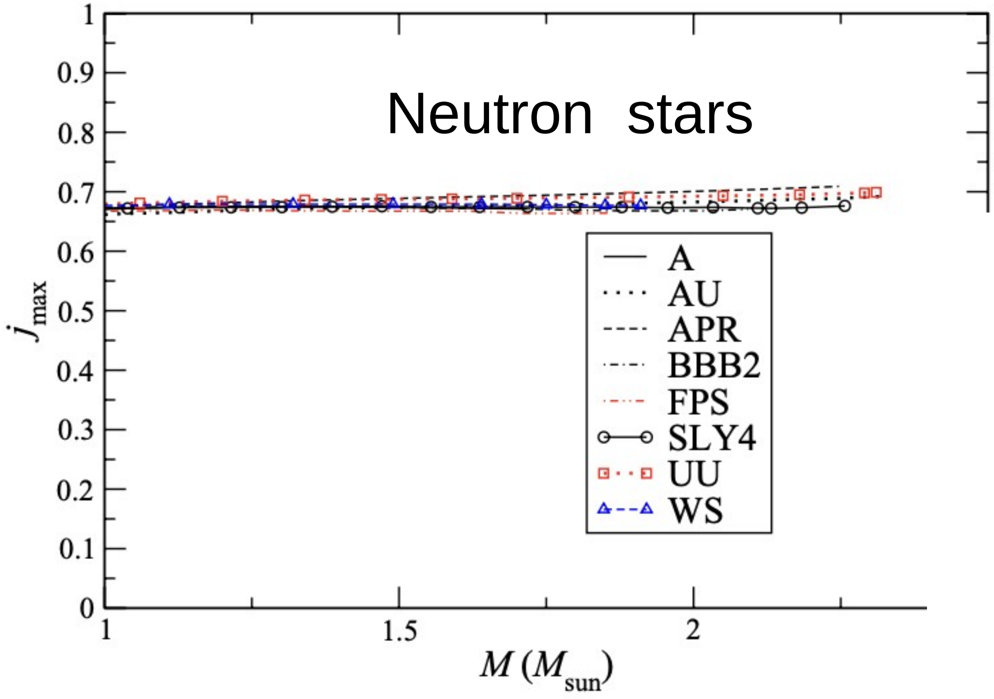
Kenneth Chen

Thank you!

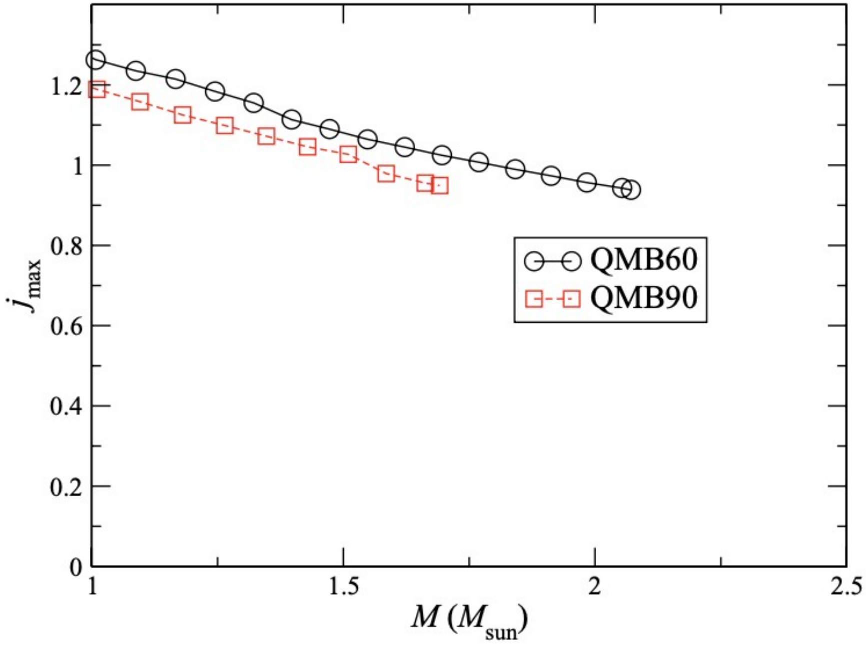
Appendix

(Rotating) neutron stars vs quark stars

Dimensionless spin parameter $j = \frac{cJ}{G M^2}$


Neutron stars

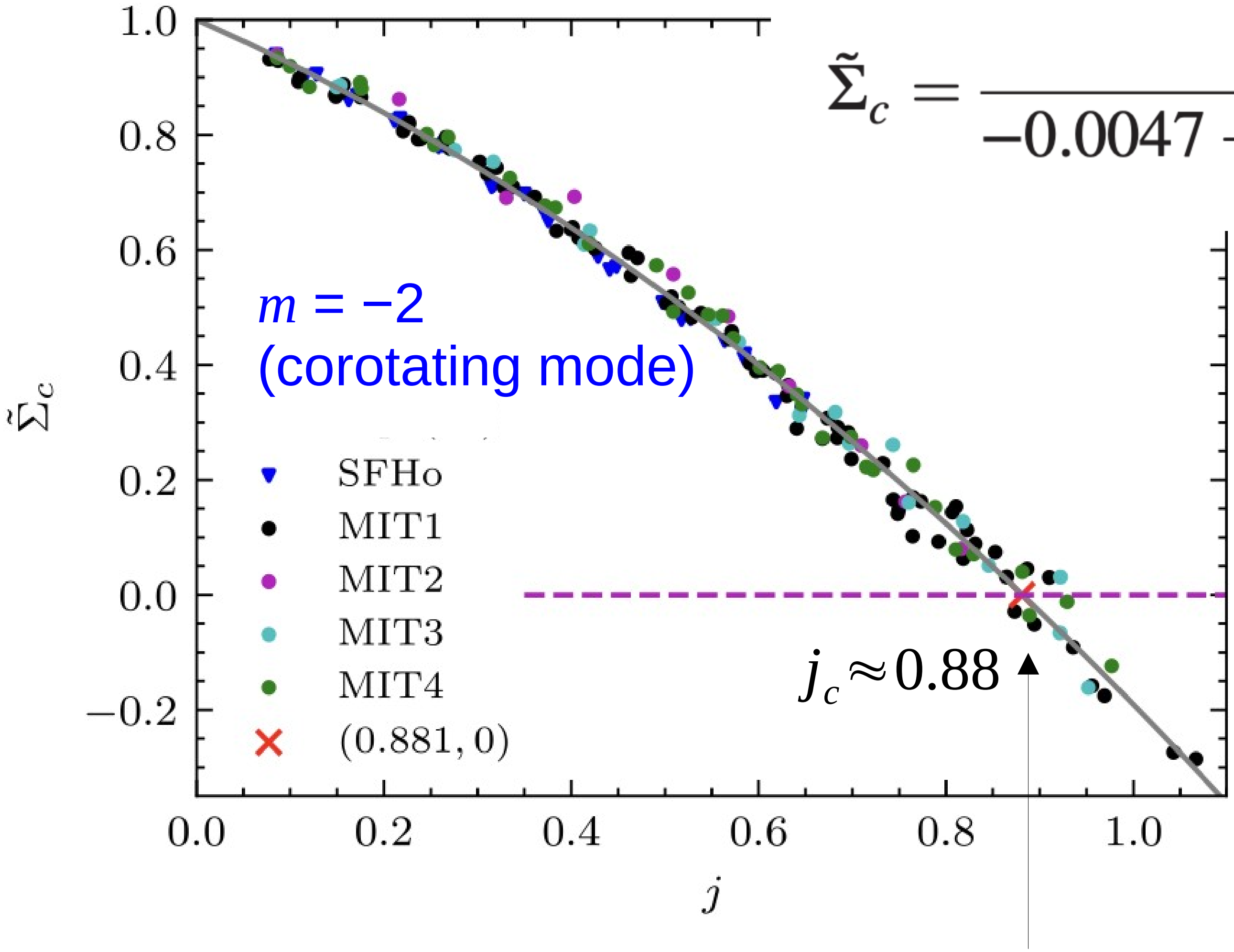
Quark stars
(MIT bag model)



Rotating black hole: $j_{max} = 1$

Lo & LML, ApJ, 728, 12 (2011)

Corotating f-mode frequencies (observed in rotating frame)



m = −2
(corotating mode)

$j_c \approx 0.88$

$$\tilde{\Sigma}_c = \frac{2\pi M \sigma_c}{-0.0047 + 0.133\eta + 0.575\eta^2}$$

$$\eta \equiv \sqrt{\frac{M^3}{I}}$$

Moment of inertia

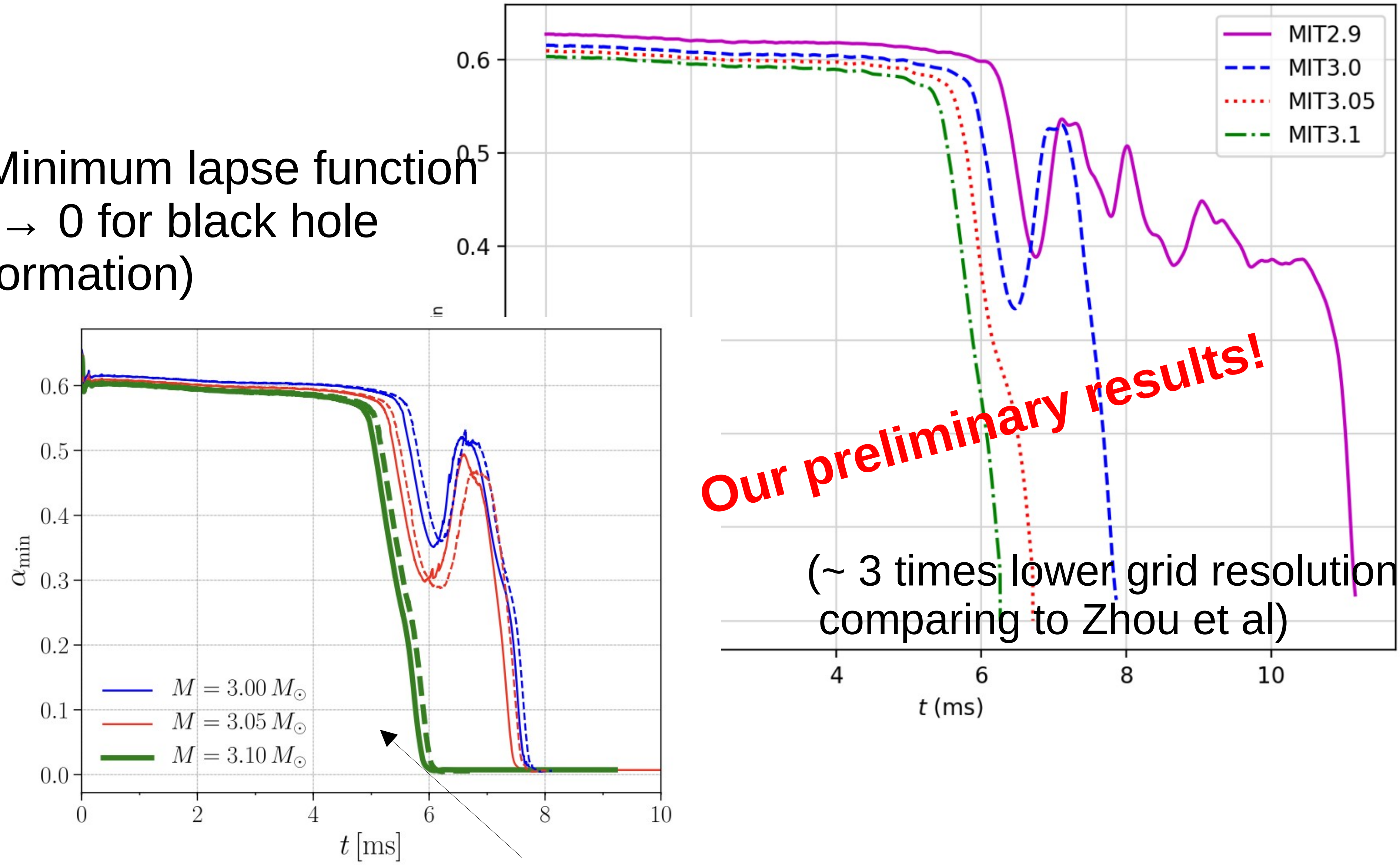
Spin parameter

$$j = \frac{J}{M^2}$$

Onset of viscosity-driven instability for quark stars
(Neutron stars cannot achieve such a high j)

- Study of the critical total mass for prompt collapse

Minimum lapse function (→ 0 for black hole formation)



Our preliminary results!

(~ 3 times lower grid resolution comparing to Zhou et al)

Figure from Zhou etal PRD 106, 103030 (2022)