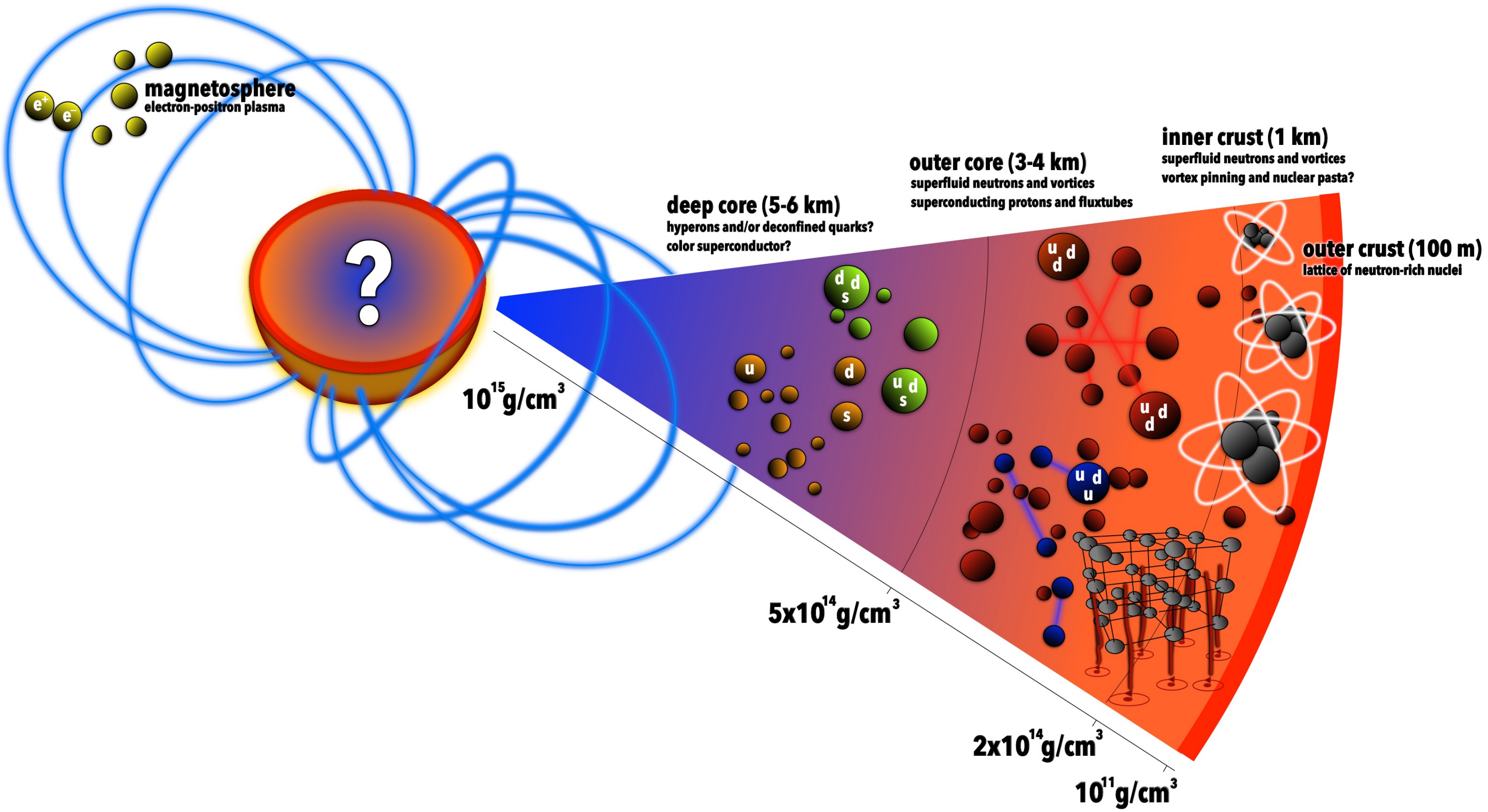
Non-equilibrium aspects of neutron star mergers

Nils Andersson

STAG RESEARCH CENTRE

University of Southampton

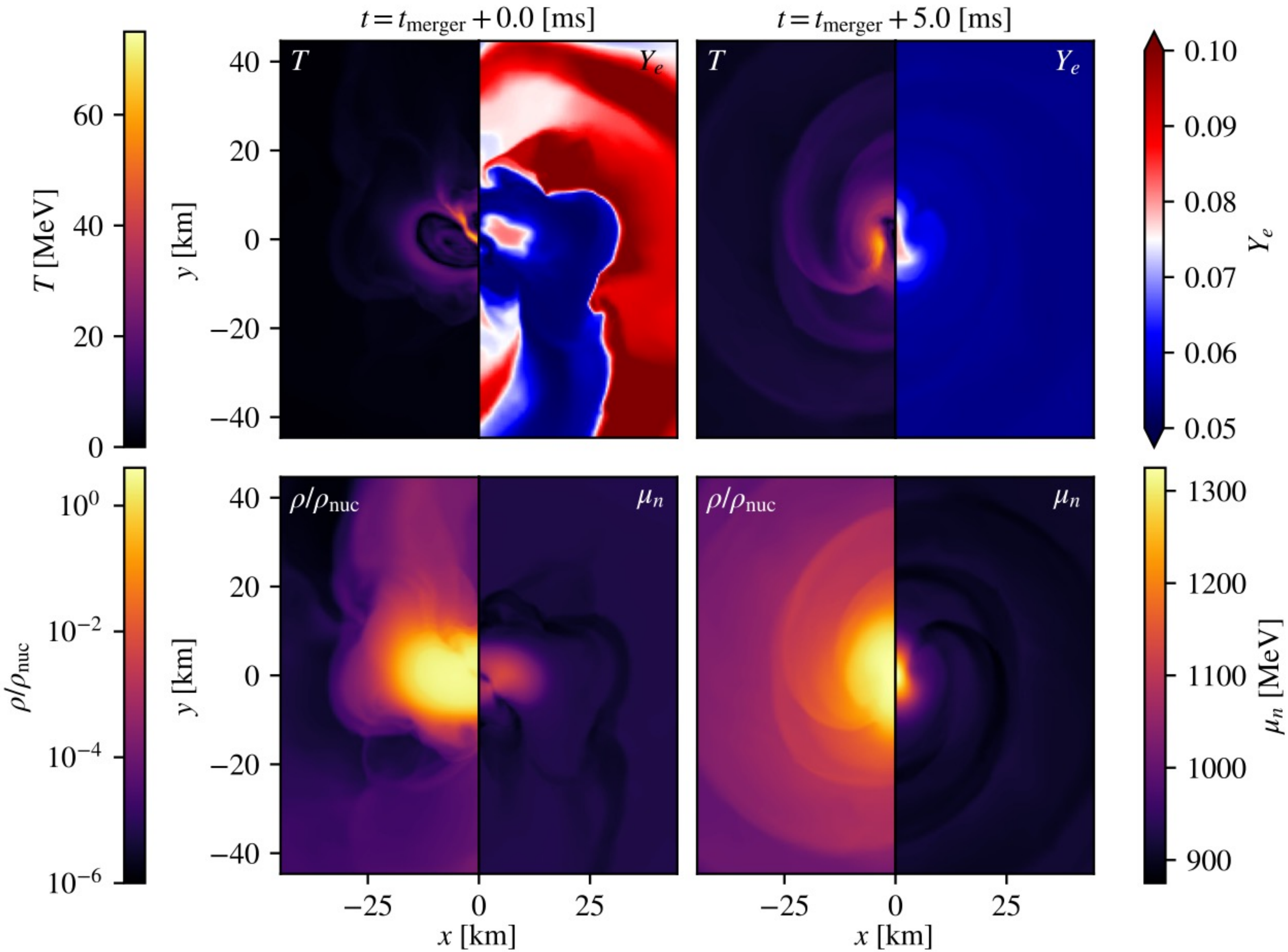


Gravity, holds the star together (gravitational waves!)

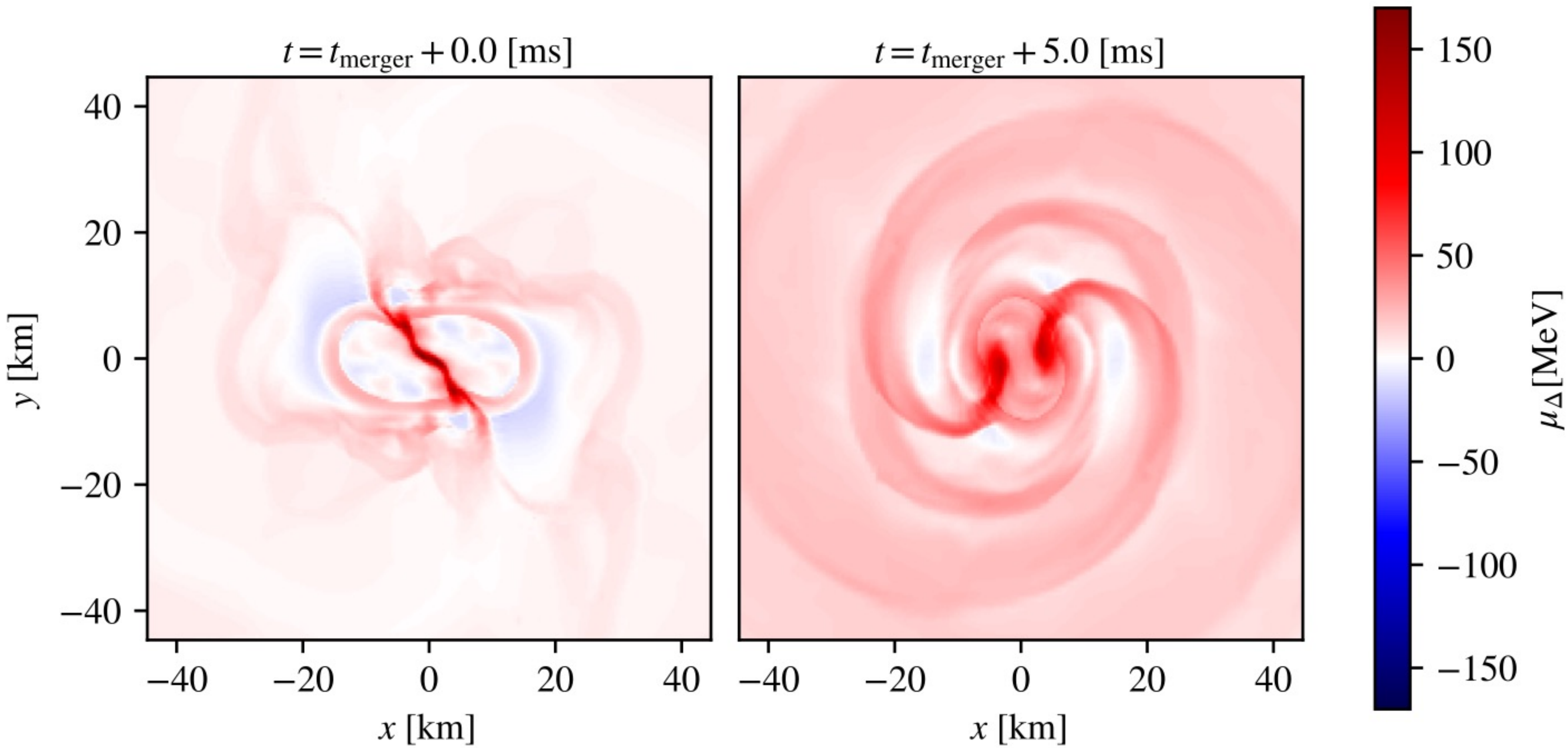
Electromagnetism, makes pulsars pulse and magnetars flare

Strong interaction, determines internal composition

Weak interaction, affects reaction rates - cooling and internal viscosity

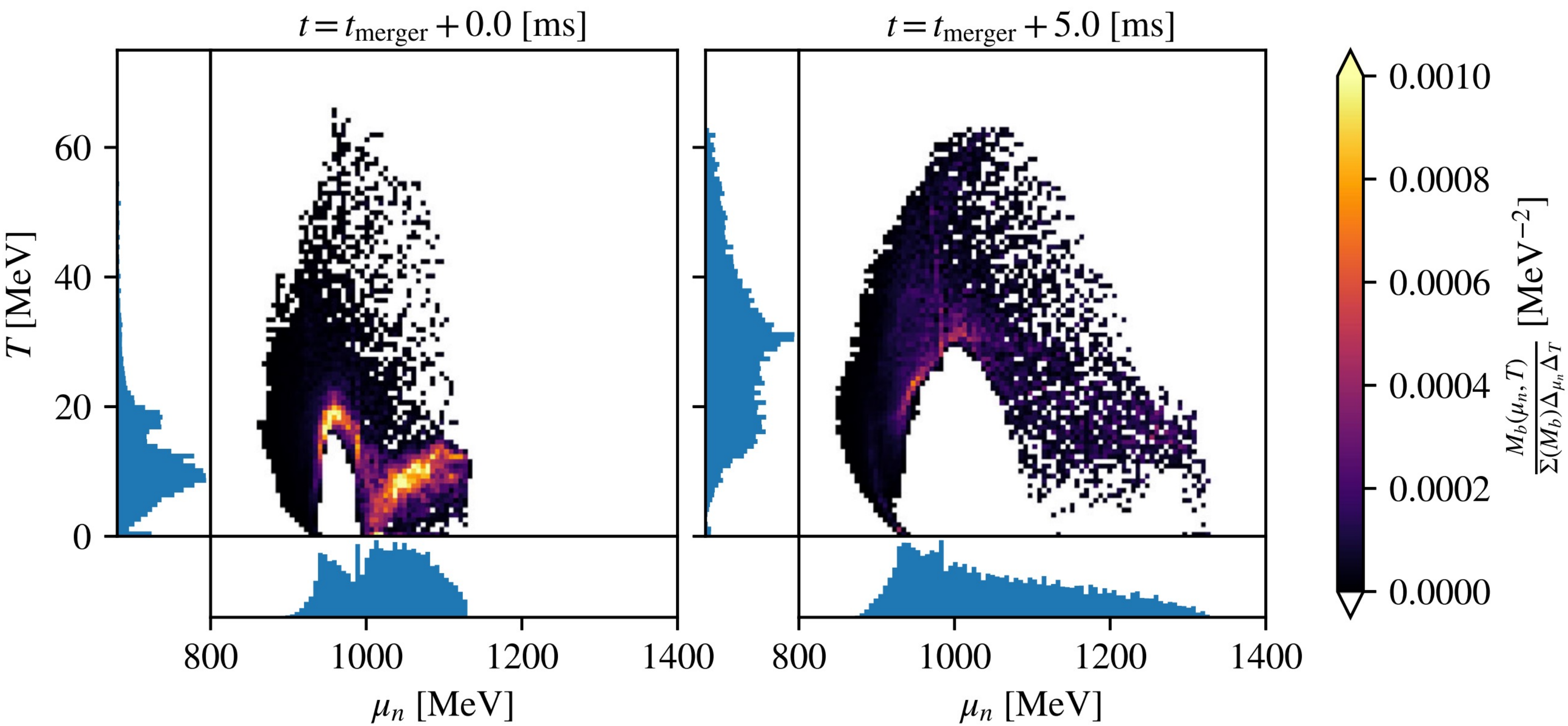

$t = t_{\rm merger} + 0.0$ [ms]
$t = t_{\rm merger} + 5.0$ [ms]
T
Y_e
$\rho/\rho_{\rm nuc}$
μ_n
T [MeV]
Y_e
$\rho/\rho_{\rm nuc}$
μ_n [MeV]
y [km]
x [km]
0.10
0.09
0.08
0.07
0.06
0.05
60
40
20
0
10^0
10^{-2}
10^{-4}
10^{-6}
1300
1200
1100
1000
900
−25
0
25
40
20
0
−20
−40

[Hammond+]



[Hammond+]

For npe matter use $\beta = \mu_{\rm n} - \mu_{\rm p} - \mu_{\rm e}$ to encode the deviation from (cold) beta-equilibrium (but "warm" equilibrium is different…)



$t = t_{\mathrm{merger}} + 0.0$ [ms]
$t = t_{\mathrm{merger}} + 5.0$ [ms]
T [MeV]
0
20
40
60
800
1000
1200
1400
μ_n [MeV]
0.0010
0.0008
0.0006
0.0004
0.0002
0.0000
$\frac{M_b(\mu_n, T)}{\Sigma(M_b)\Delta_{\mu_n}\Delta_T}$ [MeV^{-2}]

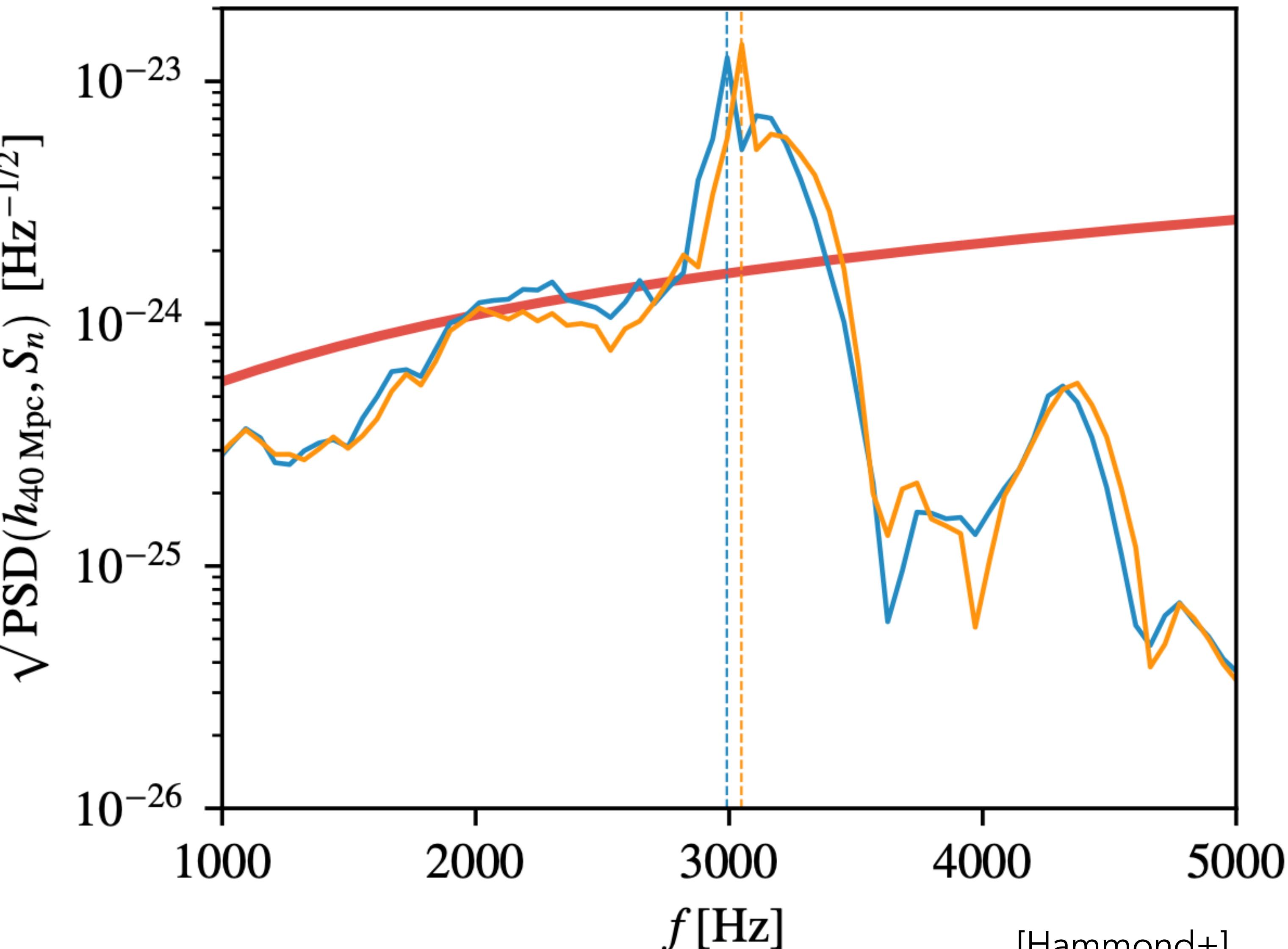
[Hammond+]

The merger dynamics should be within reach of the next-generation gravitational-wave detectors and we (naturally!) want to extract as much physics from these signals as possible.

Requires **robust simulations with a reliable physics implementation** (beyond the equilibrium equation of state).

Assuming a 3-parameter model $p = p(n, \varepsilon, Y_\mathrm{e})$ and stepping up the complexity, we may

- assume that reactions are fast enough that the matter remains in equilibrium, or
- slow enough that the composition is frozen,
- try to add the relevant reactions+neutrino aspects,
- add whatever other physics we may be interested in…



10^{-23}
10^{-24}
10^{-25}
10^{-26}
$\sqrt{\mathrm{PSD}(h_{40\,\mathrm{Mpc}}, S_n)}$ [Hz$^{-1/2}$]
1000
2000
3000
4000
5000
f [Hz]

[Hammond+]

For a **reactive** system we need to evolve

$$u^a \nabla_a Y_e = \Gamma_e / n$$

which may look fairly innocent… but is problematic.

Focussing on the numerical implementation (leaving reaction rates etc to the nuclear physics experts), the main issue relates to what can be resolved and what can not.

- When reactions are slow enough that they can be resolved, they should be resolved - we have to evolve the reactive system.
- When reactions are fast enough that they cannot be resolved, they may still leave an imprint on the dynamics. We need to figure out how to approximate this.

In reality, the parameter space of a neutron star merger will have regions where each assumption holds (and there will be “grey” areas in between…)

The **key timescales** to worry about are:

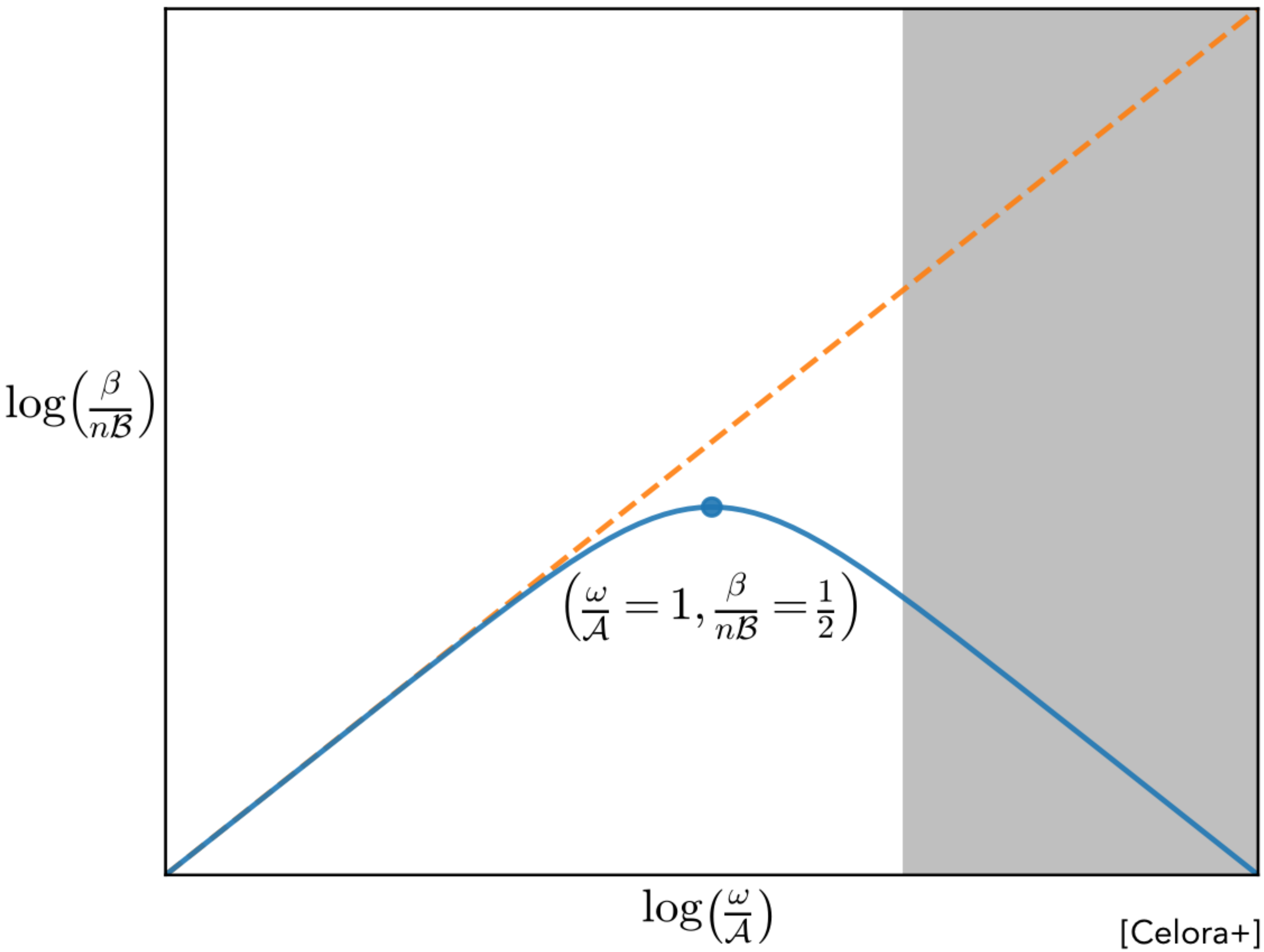
- The full merger event $\sim$ seconds
- Post-merger dynamics $\sim\ 10^{-3}$s
- Numerical resolution$\sim\ 10^{-7}-10^{-8}$s
- Nuclear reactions (Urca) $t_R = 1/\mathcal{A} \sim 10^{-8}-10^{-10}$s

Might consider classic Israel-Stewart approach, but… there are issues with this.

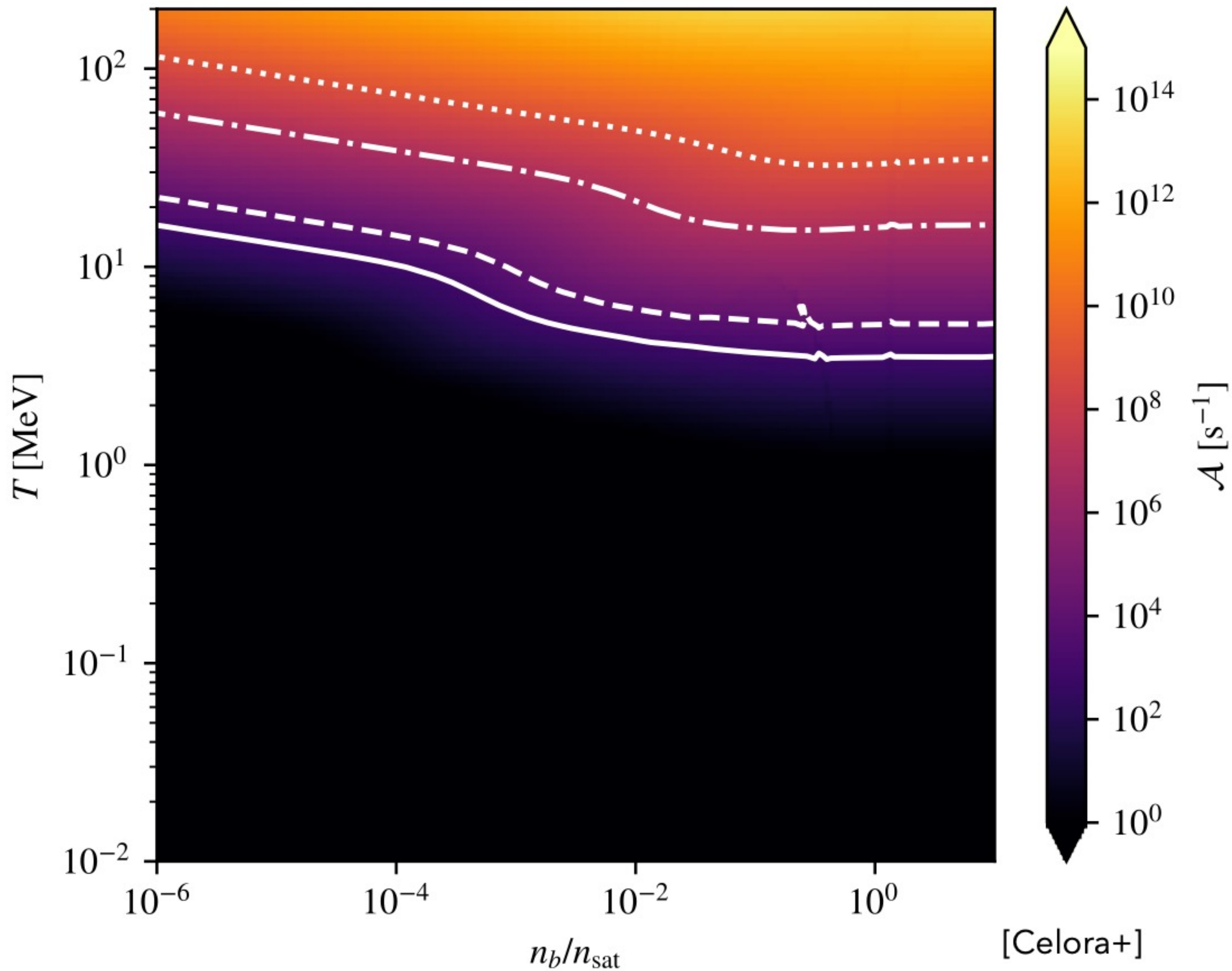
- Stiffness (could be dealt with through an implicit method, but…)
- Expansion around equilibrium, which may not "make sense" in the nonlinear regime (suprathermal perturbations).

Multi-scale methods lead to a Navier-Stokes type **bulk-viscous pressure**:

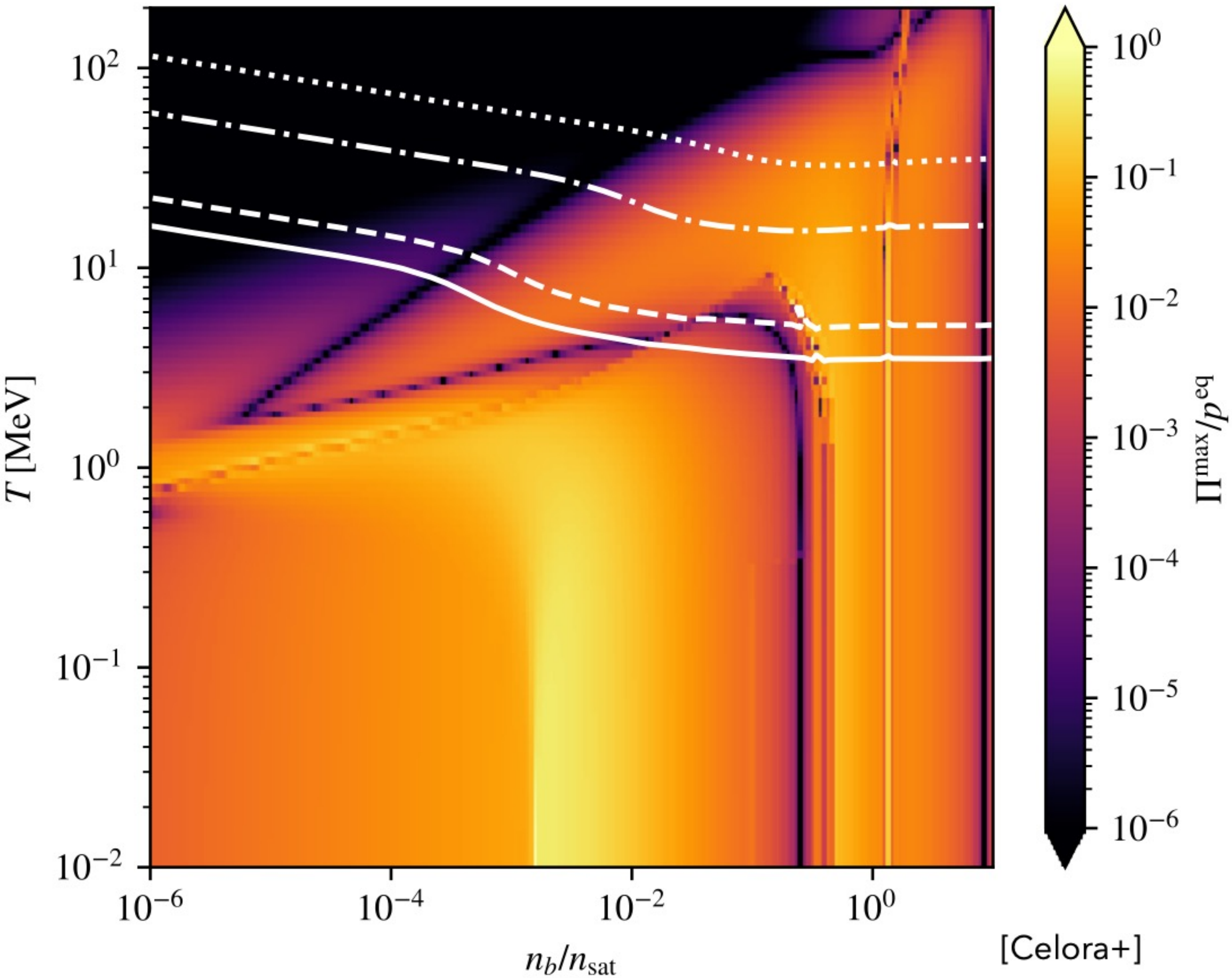
$$p = p^{\mathrm{eq}} + \left(\frac{\partial p}{\partial \beta}\right)_{\beta=0} \beta = p^{\mathrm{eq}} + \Pi$$



Bulk viscosity vs relaxation and numerical resolution.



Reaction timescale with contours representative of numerical resolution.



Maximum relative importance of bulk viscosity, same contours.

Also need to worry about **small-scale dynamics** (turbulence).

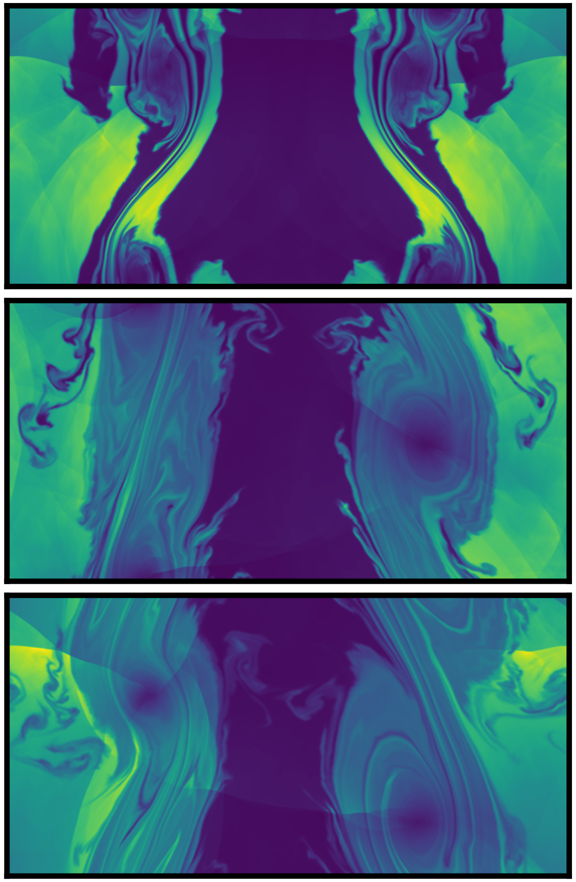
Severe scale discrepancy:

- neutron star core: m.f.p. ∼0.1mm
- best resolution in simulations ∼10m

Idea: account for unresolved features through Large Eddy Filtering scheme.

Recent progress: Covariant framework based on Fermi coordinates.

- local analysis tied to the fluid
- filtering operation explicitly defined
- compatible with the Einstein equations
- explicit link to thermodynamics



Turbulence induced by Kelvin-Helmholtz instability [Celora+]

Filtering leads to an ``effective'' stress-energy tensor:

$$\overline{T^{ab}} = \underbrace{(\overline{\varepsilon} + \overline{p})\,\overline{u}^a\overline{u}^b + \overline{p}g^{ab}}_{\text{ideal terms}} + \overbrace{2\overline{u}^{(a}q^{b)} + s^{ab}}^{\text{“dissipative” terms}}$$

where the dissipative terms depend on ``residuals'' that need to be represented by some **closure scheme**.

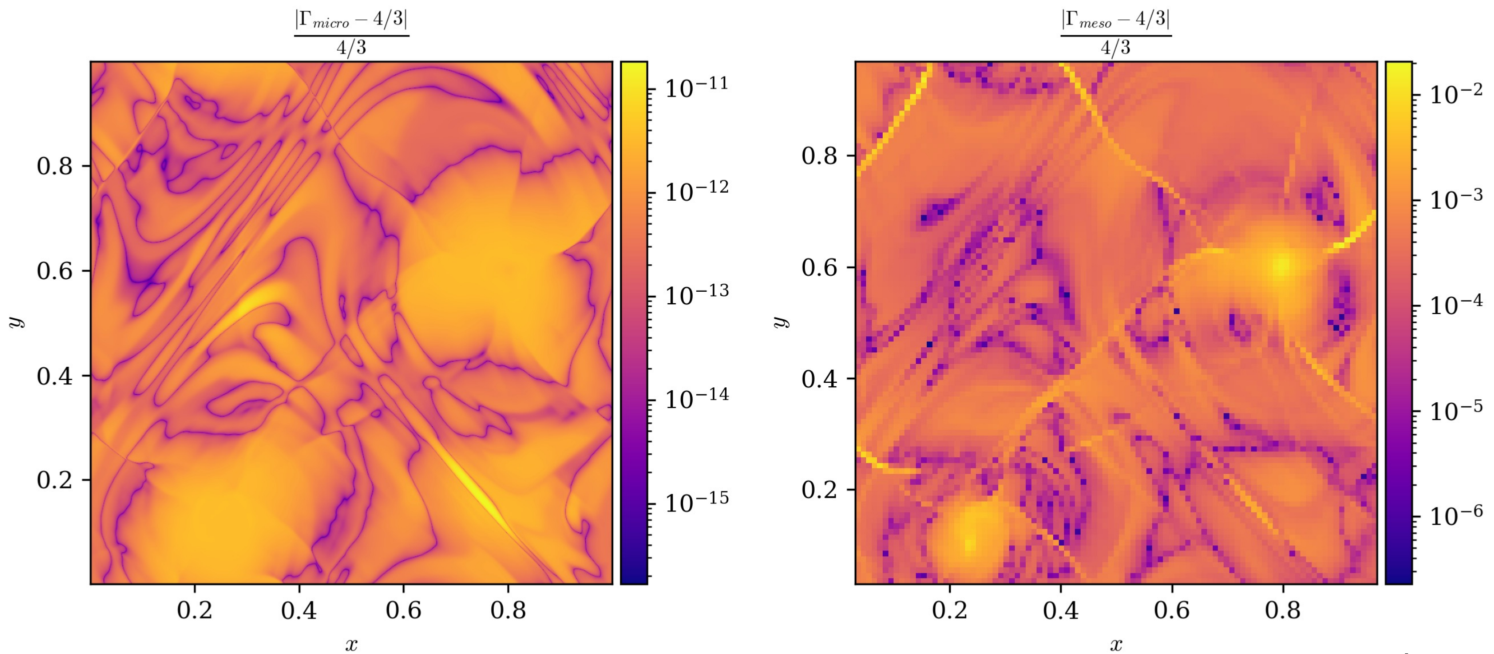
Caution: Filtering (explicit/implicit) affects the thermodynamics:

$$\bar{p} + \bar{\varepsilon} = \bar{n}\bar{\mu} + \mathscr{M}$$

In effect, we may not be simulating the actual equation of state from microphysics. This could be problematic, as the aim is match simulations against observations to constrain the nuclear physics.

Recent **proof-of-principle** results for turbulent Kelvin-Helmholtz instability.

1. Extract Favre-observer.
2. Quantify filtering "residuals".



[Celora+]

Comment: In this example, the equation of state residual is of the same order of magnitude as the bulk-viscous pressure.

take home message

With increasingly sensitive instruments, observations are beginning to constrain neutron-star theory…

In order to match the precision of the next generation of interferometers we need to (continue to) improve numerical simulations.

1. Nuclear reactions are difficult to implement as the timescales vary over many orders of magnitude in the parameter space explored in a merger.

Here I have focussed on the fast-reaction regime, where an effective bulk viscosity may suffice. Still not “easy” to implement, but it would be a natural step forward…

2. Need to represent ``unresolved'' features (turbulence) through a suitable averaging/filtering scheme.

Here I have mentioned our recent covariant large-eddy approach, which is ready to be implemented, but there is a lot of work still to do.

Main message: Beware of the subtleties!