

# Two-flavor color superconducting quark stars may not exist

**Wen-Li Yuan (苑文莉)**  
Peking University

*Dialogue at the Dream Field (DDF 2024): Supranuclear Matter  
Huaxi Guest Hotel & FAST-Light Years Away  
Guizhou, China  
10 - 15 May, 2024*

# Outline

1

The states of dense quark matter and normal quark stars

2

Color superconducting quark matter and 2SC quark stars ?

3

Summary and outlook

# Outline

1

The states of dense quark matter and normal quark stars

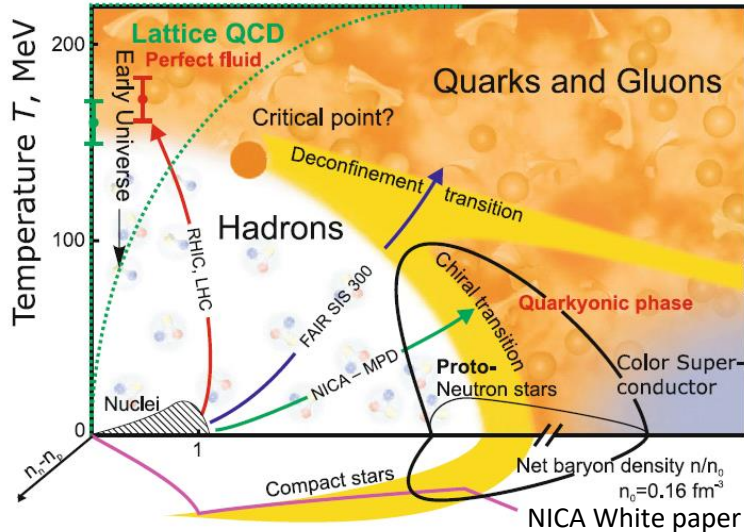
2

Color superconducting quark matter and 2SC quark stars?

3

Summary and outlook

## What is the state of supra-nuclear matter?



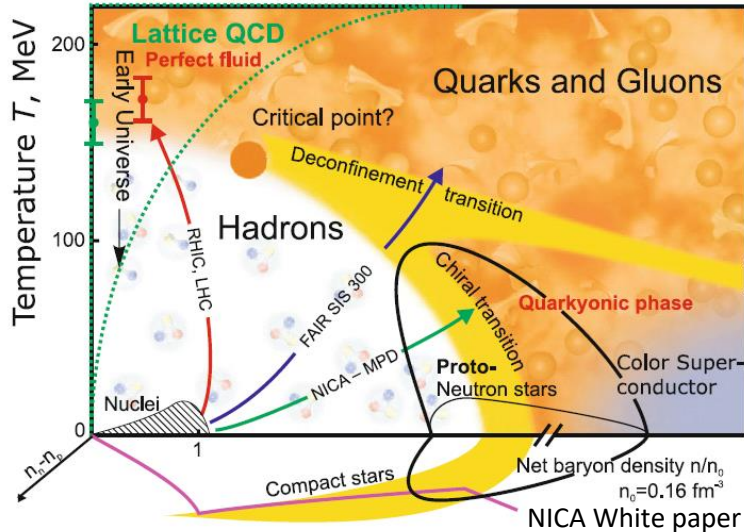
- Hadronic phase:  
 $\langle \bar{\psi}\psi \rangle \neq 0, \langle \psi\psi \rangle = 0$
- Quark-gluon plasma:  
 $\langle \bar{\psi}\psi \rangle \approx 0, \langle \psi\psi \rangle = 0$
- Two-flavor color superconductor (2SC):  
 $\langle \bar{\psi}\psi \rangle \approx 0, \langle ud \rangle \neq 0$
- Color-flavor locking (CFL):  
 $\langle ud \rangle \approx \langle us \rangle = \langle ds \rangle \neq 0$

- Terrestrial experiment: hard to reach

- **Compact stars**: natural laboratory

Maybe multi-messenger observations can tell us more about dense matter.

# What is the state of supra-nuclear matter?

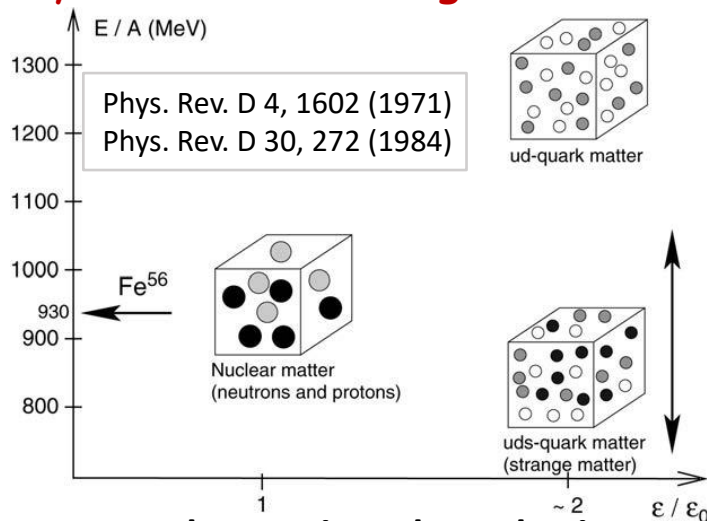


- Hadronic phase:  
 $\langle \bar{\psi}\psi \rangle \neq 0, \langle \psi\psi \rangle = 0$
- Quark-gluon plasma:  
 $\langle \bar{\psi}\psi \rangle \approx 0, \langle \psi\psi \rangle = 0$
- Two-flavor color superconductor (2SC):  
 $\langle \bar{\psi}\psi \rangle \approx 0, \langle ud \rangle \neq 0$
- Color-flavor locking (CFL):  
 $\langle ud \rangle \approx \langle us \rangle = \langle ds \rangle \neq 0$

• Terrestrial experiment: hard to reach

• **Compact stars:** natural laboratory

**Maybe multi-messenger observations can tell us more about dense matter.**



**Bodmer-Witten hypothesis**

A recent study: [Phys. Rev. Lett. 120,222001 \(2018\)](#)

Bob Holdom, Jing Ren, and Chen Zhang

*Strange/Nonstrange quark stars could also exist.*



*The Strangeon matter is absolutely stable.*

Renxin Xu [Astrophys. J. 596:L59–L62 \(2003\)](#)

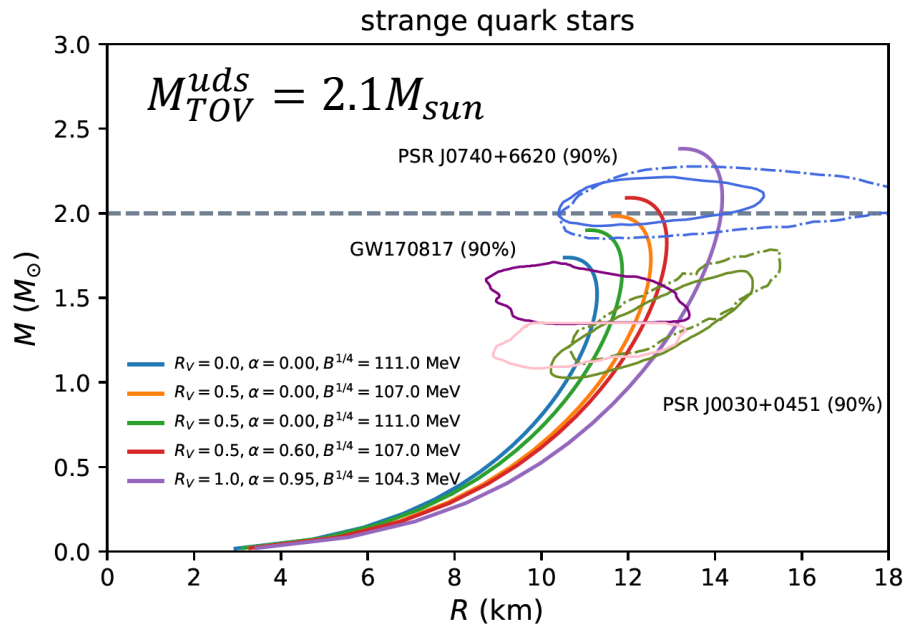
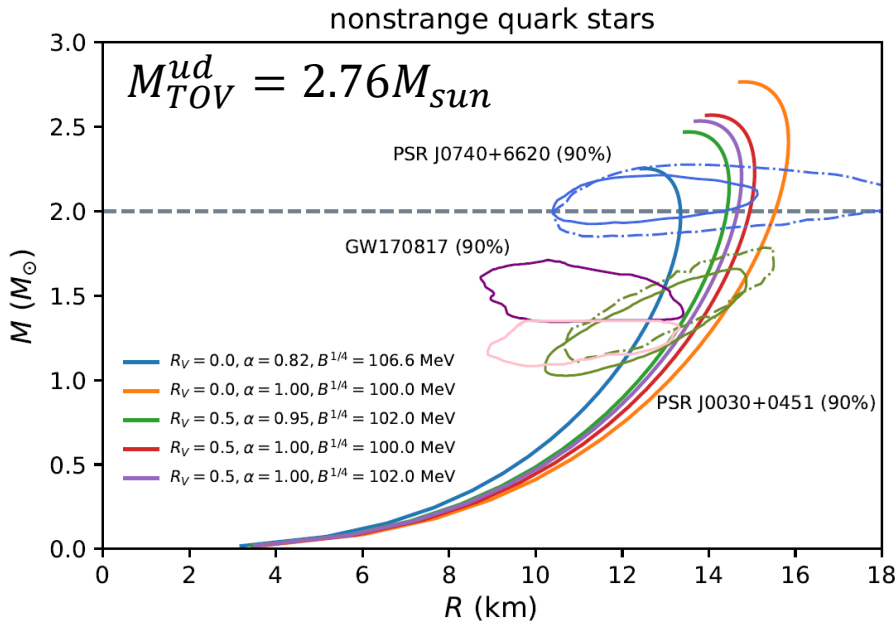
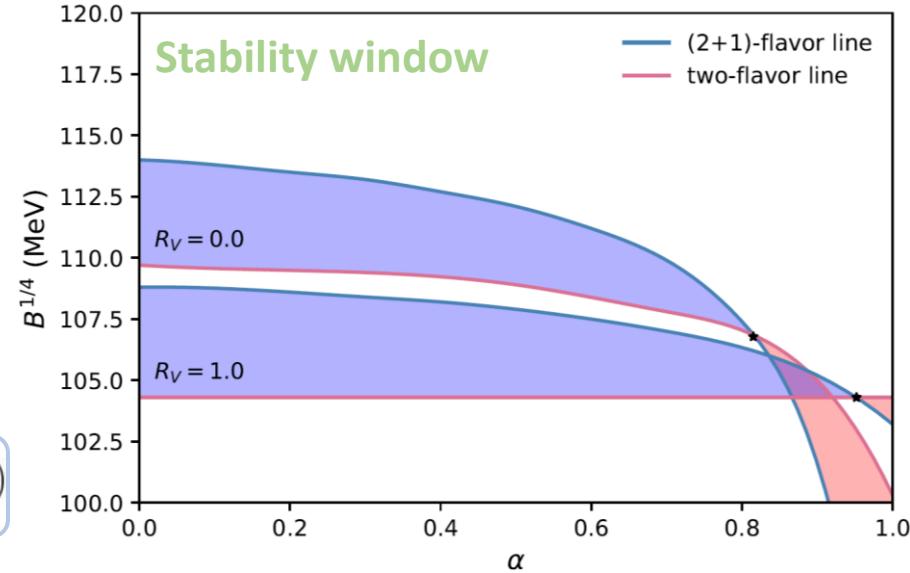
# Interacting $u$ s and $uds$ quark matter and quark stars

Modified NJL model:

Consider the effect of a rearrangement of fermion field operators.

$$\mathcal{L}_{\text{eff}}^{2f} = \bar{\psi}(i\gamma^\mu \partial_\mu - m + \mu\gamma^0)\psi + (1 - \alpha)\mathcal{L}_{\text{int}}^{2f} + \alpha\mathcal{F}(\mathcal{L}_{\text{int}}^{2f})$$

$$\mathcal{L}_{\text{eff}}^{3f} = \bar{\psi}(i\gamma^\mu \partial_\mu - m + \mu\gamma^0)\psi + (1 - \alpha)\mathcal{L}_{\text{int}}^{3f} + \alpha\mathcal{F}(\mathcal{L}_{\text{int}}^{3f})$$



# Outline

1

The states of dense quark matter and normal quark stars

2

Color superconducting quark matter and 2SC quark stars?

3

Summary and outlook

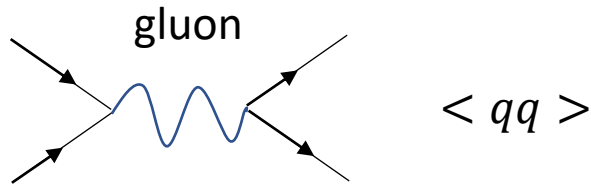
# Color superconducting phases

## The birth of CSC

1977-1984

$\Delta \sim 1 \text{ MeV}$

The interaction between quarks at high density is dominated by *one-gluon-exchange interaction*, which is *attractive* in the *color-antitriplet channel*.

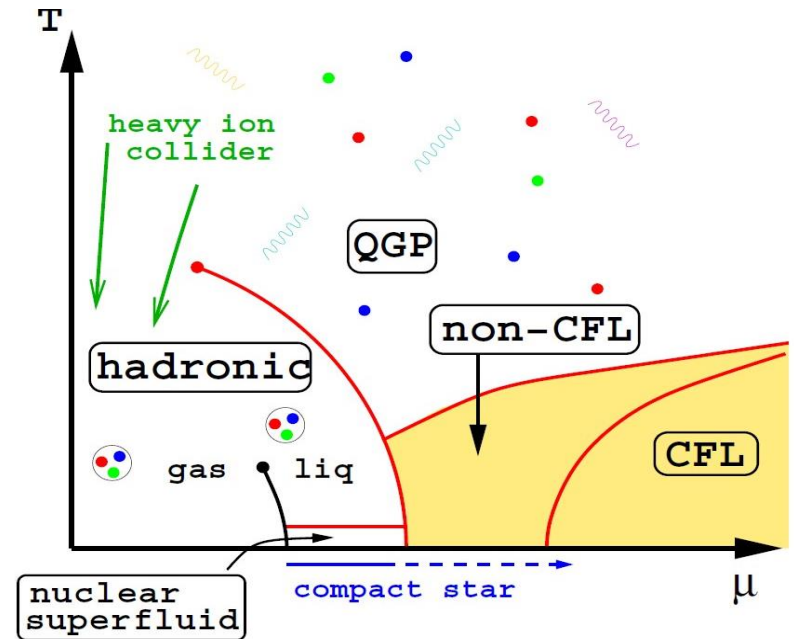


Barrois, Nucl. Phys. B 129, 390 (1977);  
Bailin, Love, Phys. Rep. 107, 325 (1984).

1988

$\Delta \sim 100 \text{ MeV}$

Rapp, Schaefer, Shuryak, Velkovsky,  
Phys. Rev. Lett. 81, 53 (1998);  
Alford, Rajagopal, Wilczek, Phys. Lett. B 422, 247 (1998)



Alford 1999



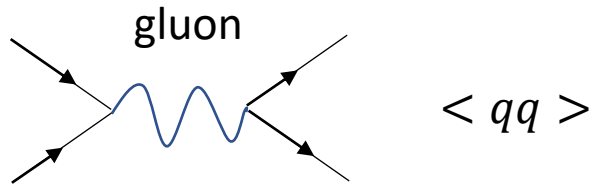
# Color superconducting phases

## The birth of CSC

1977-1984

$\Delta \sim 1 \text{ MeV}$

The interaction between quarks at high density is dominated by *one-gluon-exchange interaction*, which is *attractive* in the *color-antitriplet channel*.

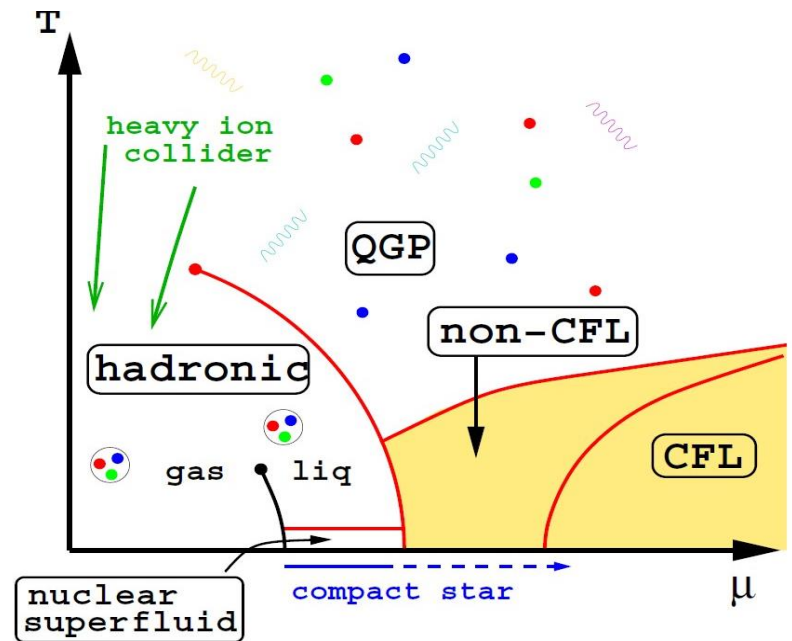


Barrois, Nucl. Phys. B 129, 390 (1977);  
Bailin, Love, Phys. Rep. 107, 325 (1984).

1988

$\Delta \sim 100 \text{ MeV}$

Rapp, Schaefer, Shuryak, Velkovsky,  
Phys. Rev. Lett. 81, 53 (1998);  
Alford, Rajagopal, Wilczek, Phys. Lett. B 422, 247 (1998)



Alford 1999

$\langle q_{ia}^\alpha q_{jb}^\beta \rangle$  Quark Cooper pair

color  $\alpha, \beta = r, g, b$

flavor  $i, j = u, d, s$

spin  $a, b = \uparrow \downarrow$

# Color superconducting phases

Two flavor CSC (2SC) : spin-0

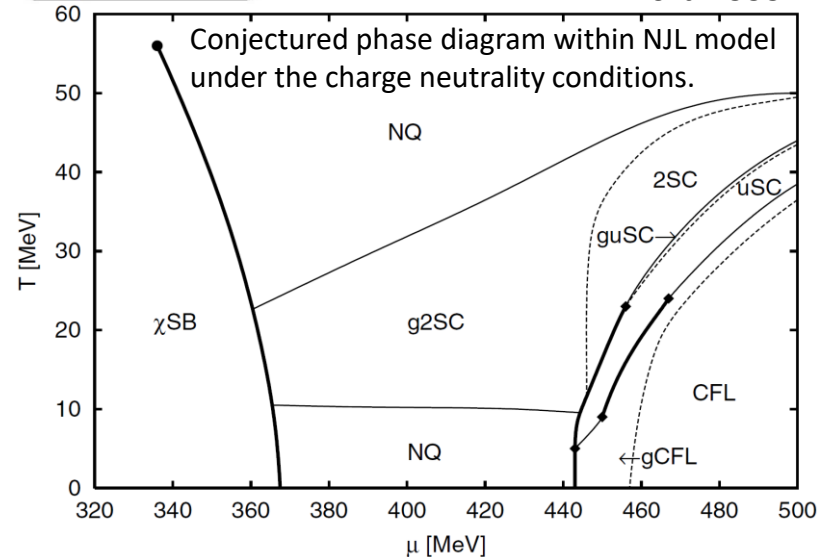
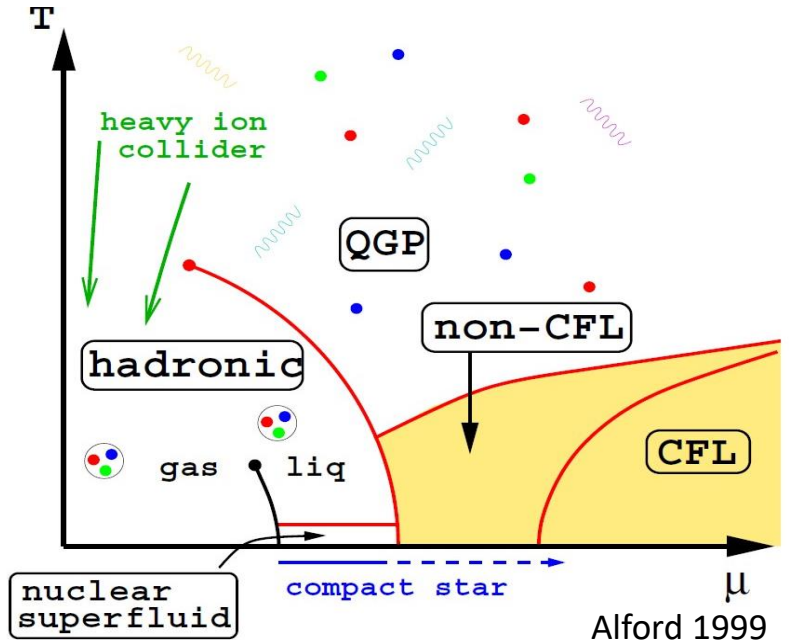
s quarks do not participate in pairing.



$\langle ud \rangle \neq 0$

$\langle du \rangle \neq 0$

Color-flavor-locked (CFL) phase: spin-0



Ruster et al., Phys. Rev. D 72, 034004 (2005)

# Color superconducting phases

Two flavor CSC (2SC) : spin-0

s quarks do not participate in pairing.



$$\langle ud \rangle \neq 0$$

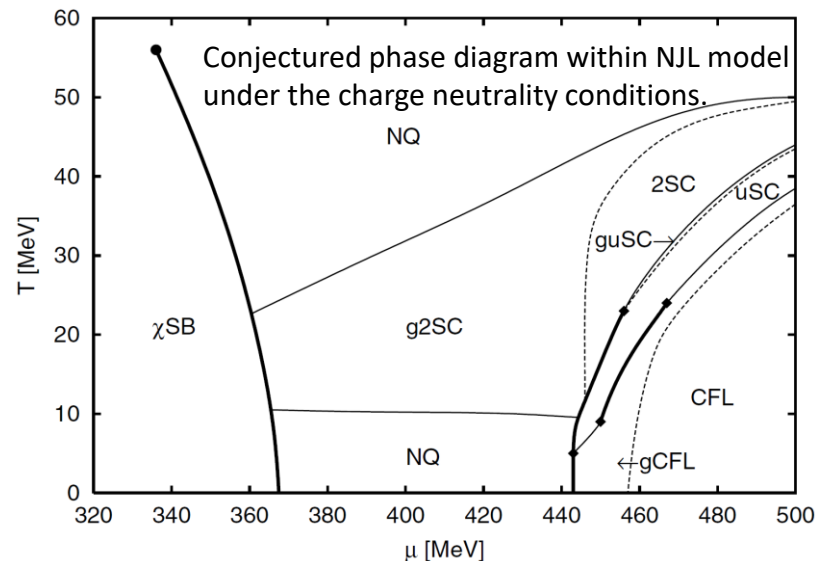
$$\langle du \rangle \neq 0$$

Color-flavor-locked (CFL) phase: spin-0



## Where can we find CSC?

CSC phase may exist inside compact stars, which has an effect on *cooling*, *r-mode instability*, *pulsar glitch*...



Rüster et al., Phys. Rev. D **72**, 034004 (2005)

The existing research: gap size and free energy

Our work: first study to examine the

absolute stability of the 2SC quark matter (NJL)

# Nambu-Jona-Lasinio (NJL) model



“for the discovery of the origin of the broken symmetry which predicts the existence of at least three families of quarks in nature”

—— the Nobel Prize (2008)

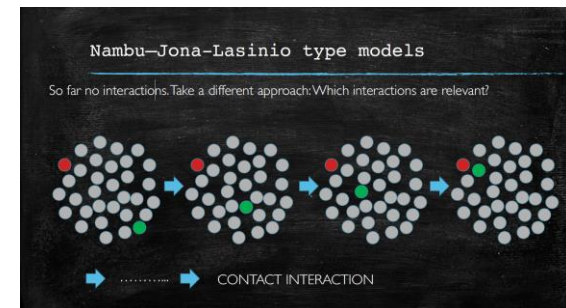


Yoichiro Nambu



Giovanni Jona-Lasinio

- An important and valid effective quark theory ( a suitable approximation to QCD in the low-energy and long-wavelength limit)
- Reproduce the basic symmetries of QCD
- Dynamical chiral symmetry breaking (DCSB)
- Confinement



# Modified Nambu-Jona-Lasinio (NJL) model

The original two-flavor NJL model:

$$\mathcal{L} = \bar{q}(i\gamma^\mu\partial_\mu - m_0 + \mu\gamma^0)q + G_S \left[ (\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2 \right]$$

✓ The **vector interaction** is important for the study of compact stars.

✓ We aim to explore the **stability of two-flavor color superconducting quark matter**.

$$M = m_0 - 2G_S\sigma$$

The modified two-flavor NJL model:

$$\sigma = \langle \bar{q}q \rangle$$

$$\mathcal{L} = \bar{q}(i\gamma^\mu\partial_\mu - m_0 + \mu\gamma^0)q + G_S \left[ (\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2 \right]$$

$$\boxed{-G_V (\bar{\psi}\gamma^\mu\psi)^2} + \boxed{G_D \left[ (\bar{q}i\gamma_5\tau_2\lambda_A q_c)(\bar{q}_c i\gamma_5\tau_2\lambda_A q) \right]}$$

Parameter fixing in two-flavor NJL model:

$$\Delta = -2G_D \langle \bar{q}_c i\gamma_5\tau_2\lambda_2 q \rangle$$

$\Lambda, G_S$ : determined by fitting experimental data on the pion decay constant and pion mass

$G_V$  and  $G_D$ : treated as free parameter for our purpose of exploring whether a parameter space exists for absolutely stable 2SC quark matter.

# 2SC quark matter under charge neutrality

Beta-equilibrium and charge neutrality :

$$\mu_{ur} = \mu_{ug} = \frac{1}{3}\mu_B - \frac{2}{3}\mu_e + \frac{1}{3}\mu_{8c}$$

$$\mu_{dr} = \mu_{dg} = \frac{1}{3}\mu_B + \frac{1}{3}\mu_e + \frac{1}{3}\mu_{8c}$$

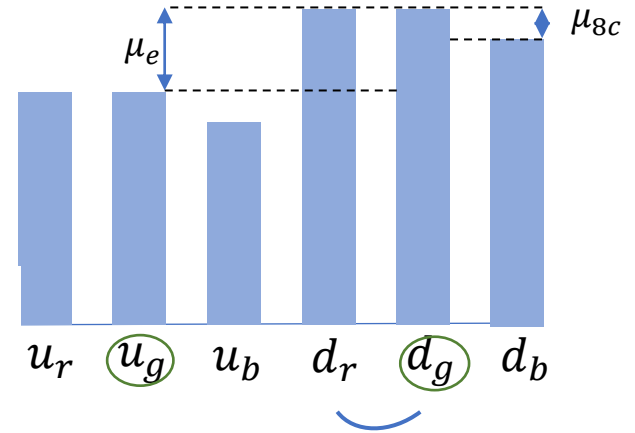
$$\mu_{ub} = \frac{1}{3}\mu_B - \frac{2}{3}\mu_e - \frac{2}{3}\mu_{8c}$$

$$\mu_{db} = \frac{1}{3}\mu_B + \frac{1}{3}\mu_e - \frac{2}{3}\mu_{8c}$$

The thermodynamical potential:

$$\begin{aligned} \Omega_q = & \frac{(m_0 - M)^2}{4G_S} - \frac{(\mu - \tilde{\mu})^2}{4G_V} + \frac{\Delta^2}{4G_D} \\ & - 2 \int \frac{d^3p}{(2\pi)^3} \{ 2E_p + 2E_{\Delta}^+ + 2E_{\Delta}^- \\ & + T \ln [1 + \exp(-\beta E_{ub}^+)] + T \ln [1 + \exp(-\beta E_{ub}^-)] \\ & + T \ln [1 + \exp(-\beta E_{db}^+)] + T \ln [1 + \exp(-\beta E_{db}^-)] \\ & + 2T \ln [1 + \exp(-\beta E_{\Delta+}^+)] + 2T \ln [1 + \exp(-\beta E_{\Delta-}^+)] \\ & + 2T \ln [1 + \exp(-\beta E_{\Delta+}^-)] + 2T \ln [1 + \exp(-\beta E_{\Delta-}^-)] \} \end{aligned}$$

$$\mu_{ij,\alpha\beta} = (\mu\delta_{ij} - \mu_e Q_{ij}) + \frac{2}{\sqrt{3}} \mu_8 \delta_{ij} (T_8)_{\alpha\beta}$$



mean-filed approximation

# 2SC quark matter under charge neutrality

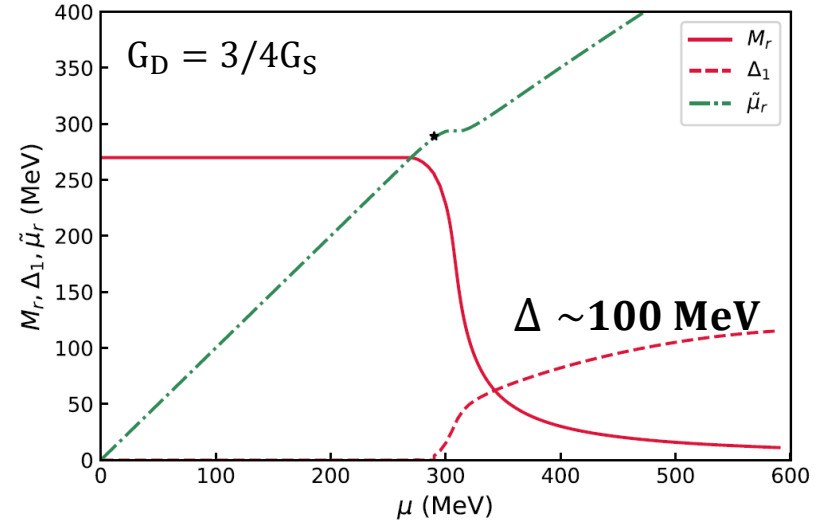
The dynamical quark mass gap equation:

$$M = m_0 - 2G_S\sigma, \quad \sigma = \langle \bar{q}q \rangle$$

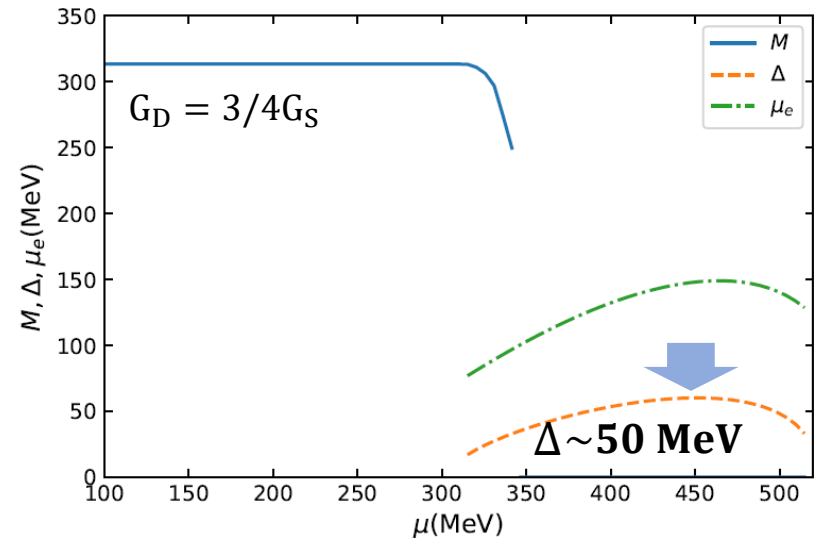
$$M = m_0 + 4G_S M \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{E_p} \left\{ \left[ 1 - f(E_{ub}^+) - f(E_{ub}^-) \right] \right. \\ \left. + \left[ 1 - f(E_{db}^+) - f(E_{db}^-) \right] \right. \\ \left. + 2 \frac{E_p^+}{E_{\Delta}^+} \left[ 1 - f(E_{\Delta^+}^+) - f(E_{\Delta^+}^-) \right] \right. \\ \left. + 2 \frac{E_p^-}{E_{\Delta}^-} \left[ 1 - f(E_{\Delta^-}^+) - f(E_{\Delta^-}^-) \right] \right\}$$

The diquark condensate gap equation:

$$\Delta = 4G_D \Delta \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left\{ \frac{2}{E_{\Delta}^-} \left[ 1 - f(E_{\Delta^+}^-) - f(E_{\Delta^-}^-) \right] \right. \\ \left. + \frac{2}{E_{\Delta}^+} \left[ 1 - f(E_{\Delta^+}^+) - f(E_{\Delta^-}^+) \right] \right\}$$



without charge neutrality



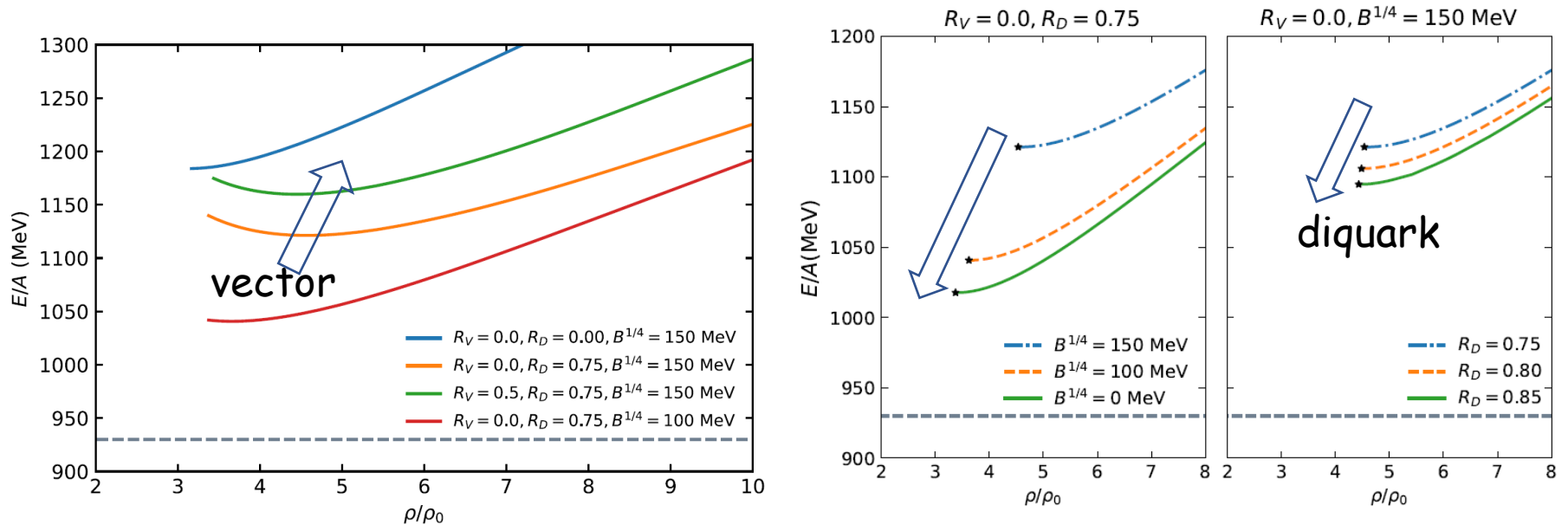
with charge neutrality

# Stability for self-bound 2SC quark matter

## Stability condition for self-bound 2SC quark matter:

- 2SC quark matter is more stable than Fe nuclei.

$$E/A(P = 0) < (56m_N - 56 \times 8.8 \text{ MeV}) / 56 = 930 \text{ MeV}$$

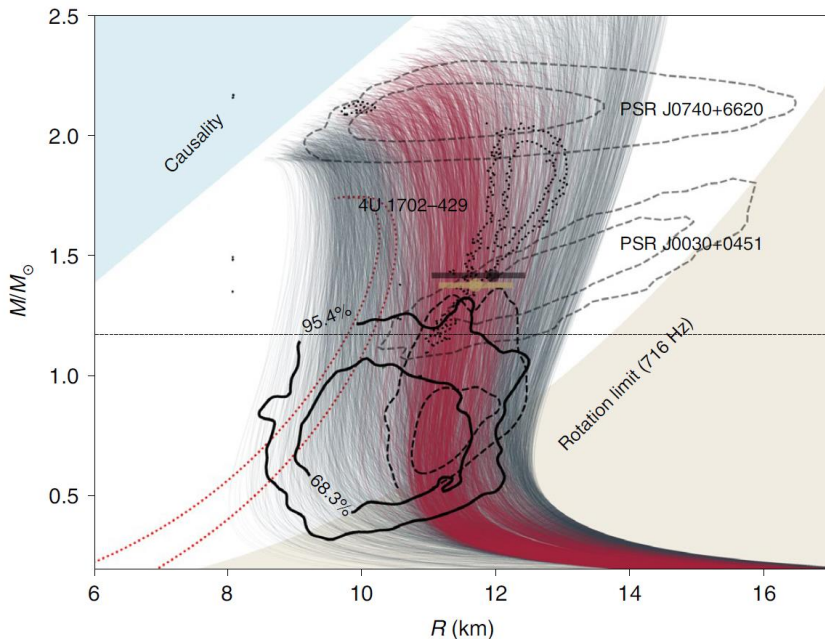


W. L. Yuan, J. Y. Chao, and A. Li, *Phys.Rev.D* 108 (2023); W. L. Yuan, and A. Li, *Astrophys.J.* 966 (2024)



# What is the nature of the HESS J1731-347 compact object?

nature astronomy 2022, HESS J1731-347



$$M = 0.77^{+0.20}_{-0.17} M_{\odot} \quad R = 10.4^{+0.86}_{-0.78} \text{ km}$$

**A self-bound 2SC quark star?**

*A light strange star in the remnant HESS J1731-347: Minimal consistency checks* **A&A 672, L11 (2023)**

*Is the compact object associated with HESS J1731-347 a strange quark star?* **arXiv:2211.07485**

*Nonstrange quark stars within resummed QCD* **Phys. Rev. D 107, 114015 (2023)**

*Color-flavor locked quark stars in light of the compact object in the HESS J1731-347 and the GW190814 event* **Phys. Rev. D 108, 063010 (2023)**

*What is the nature of the HESS J1731-347 compact object?* **arXiv:2306.12326**

.....

*Dark Matter Admixed Neutron Star in the light of HESS J1731-347 and PSR J0952-0607* **arXiv:2307.12748**

*Nuclear and hybrid equations of state in light of the low-mass compact star in HESS J1731-347* **Phys. Rev. C 108, 025806 (2023)**

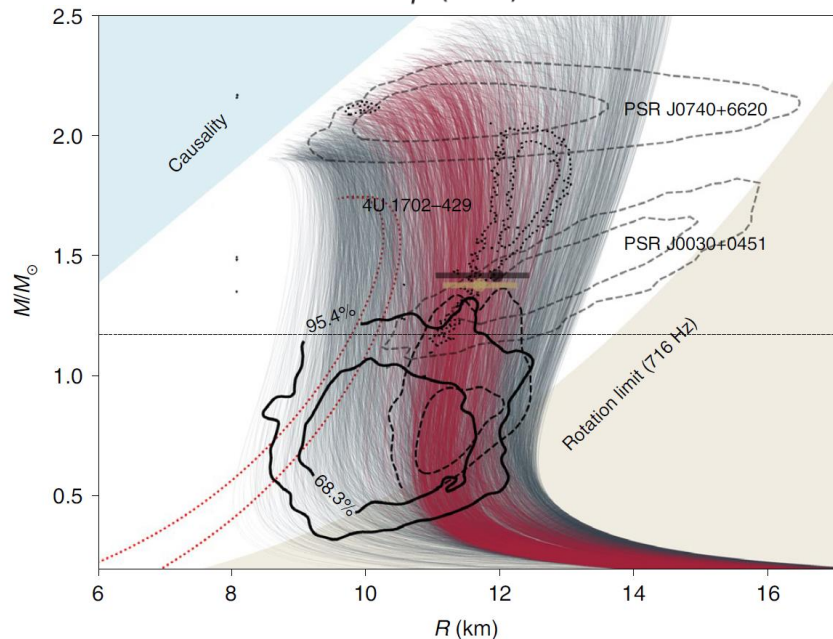
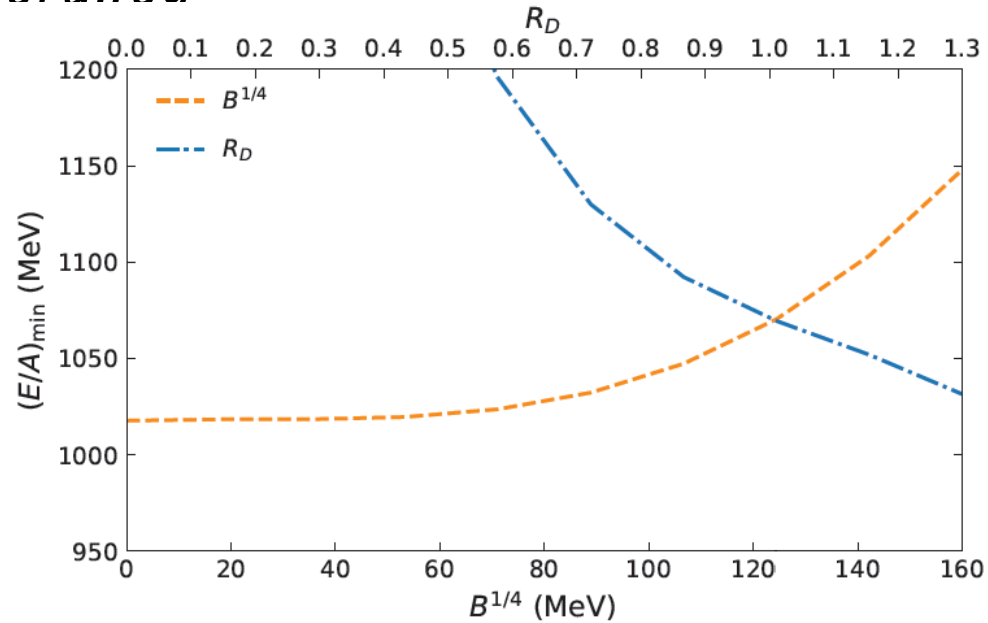
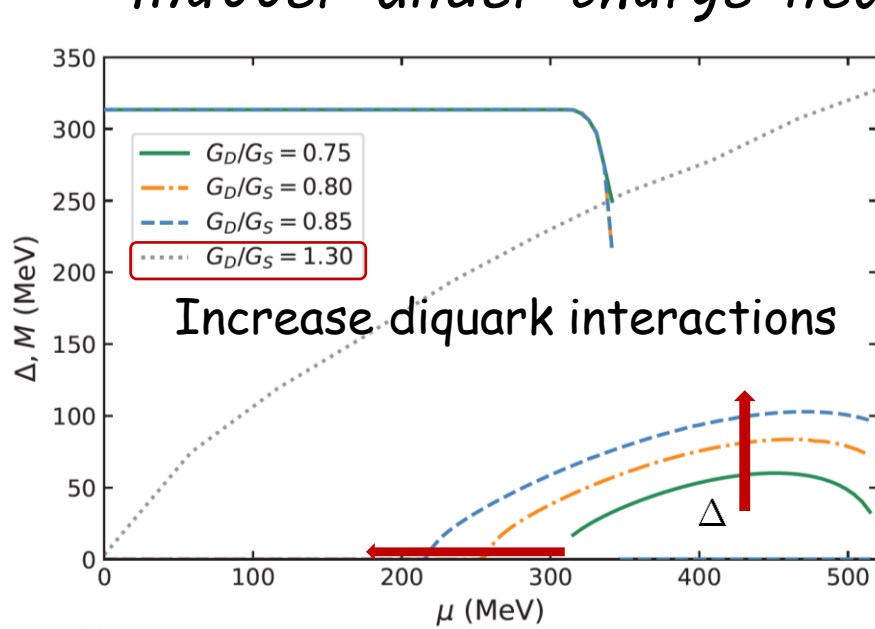
*Relativistic mean-field model for the ultra-compact low-mass neutron star HESS J1731-347* **Phys. Rev. C 108, 045803**

*Baryonic models of ultra-low-mass compact stars for the central compact object in HESS J1731-347* **Phys. Lett. B 844, 138062 (2023)**

*The hadronic equation of state of HESS J1731-347 from the relativistic mean-field model with tensor coupling* **arXiv:2306.04992**

.....

# No parameter space for absolutely stable 2SC quark matter under charge neutrality



Two-flavor color superconducting quark stars may not exist

## *Summary and outlook*

- We find that there is an ample parameter space in the NJL-type model calculations for stable quark matter. **Theoretically supports quark stars as viable alternative physical model for neutron stars.**
- Both **nonstrange and strange quark stars** can, in general, reconcile with the available mass and radius **constraints from observational data.**
- Within the modified NJL model, we investigate the stability of beta-stable two-flavor color superconducting (2SC) phase of quark matter, but find **no physically-allowed parameter space for the existence of 2SC quark stars.**

*Thank you!*