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Tidal-excited g-mode from inspiralling neutron stars as probes of the high-density equation of state

Zhiqiang Miao (缪志强)
zq_miao@sjtu.edu.cn
Tsung-Dao Lee Institute, SJTU

In collaboration with：
Enping Zhou（周恩平） @HUST and Ang Li（李昂）@XMU

Based on MZQ+2024 ApJ 964, 31 (2305.08501)

Dialogue at the Dream Field
2024.5.13 @FAST · Light Year Away

Outline

- Background
- Neutron star seismology
- Tidal seismology and GW
- Summary

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❖Neutron star and equation of state (EOS)

- The densest observable object in the universe. For M = 1.4 $M_{\odot}$, R≈10km, average density ~few times nuclear density ($n_0 \approx 2.7 \times 10^{14}\,\mathrm{g/cm^3}$)

- At density $\gtrsim 2n_0$
  what is composition?
  what is phase state?

Fujimoto+ PRL 2022

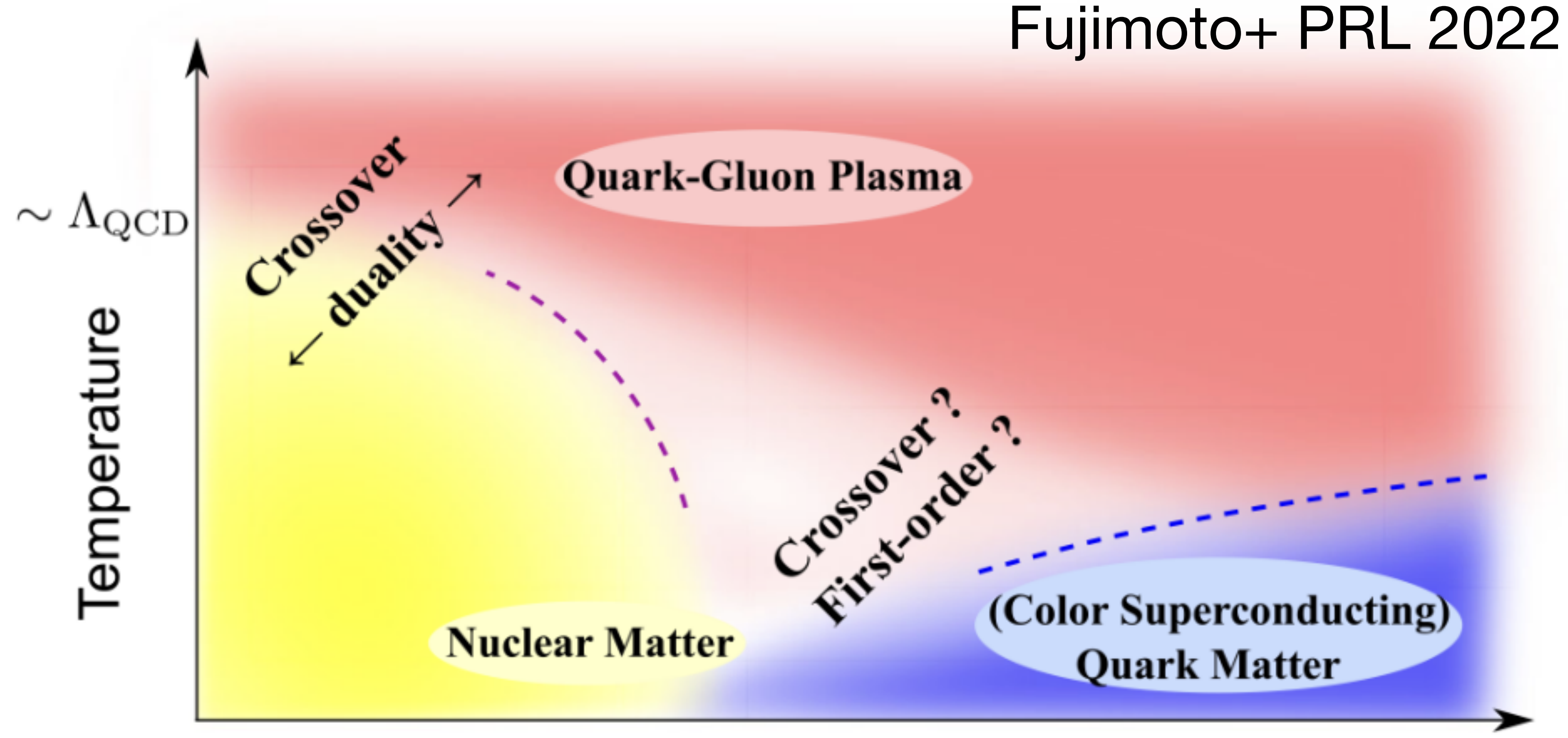
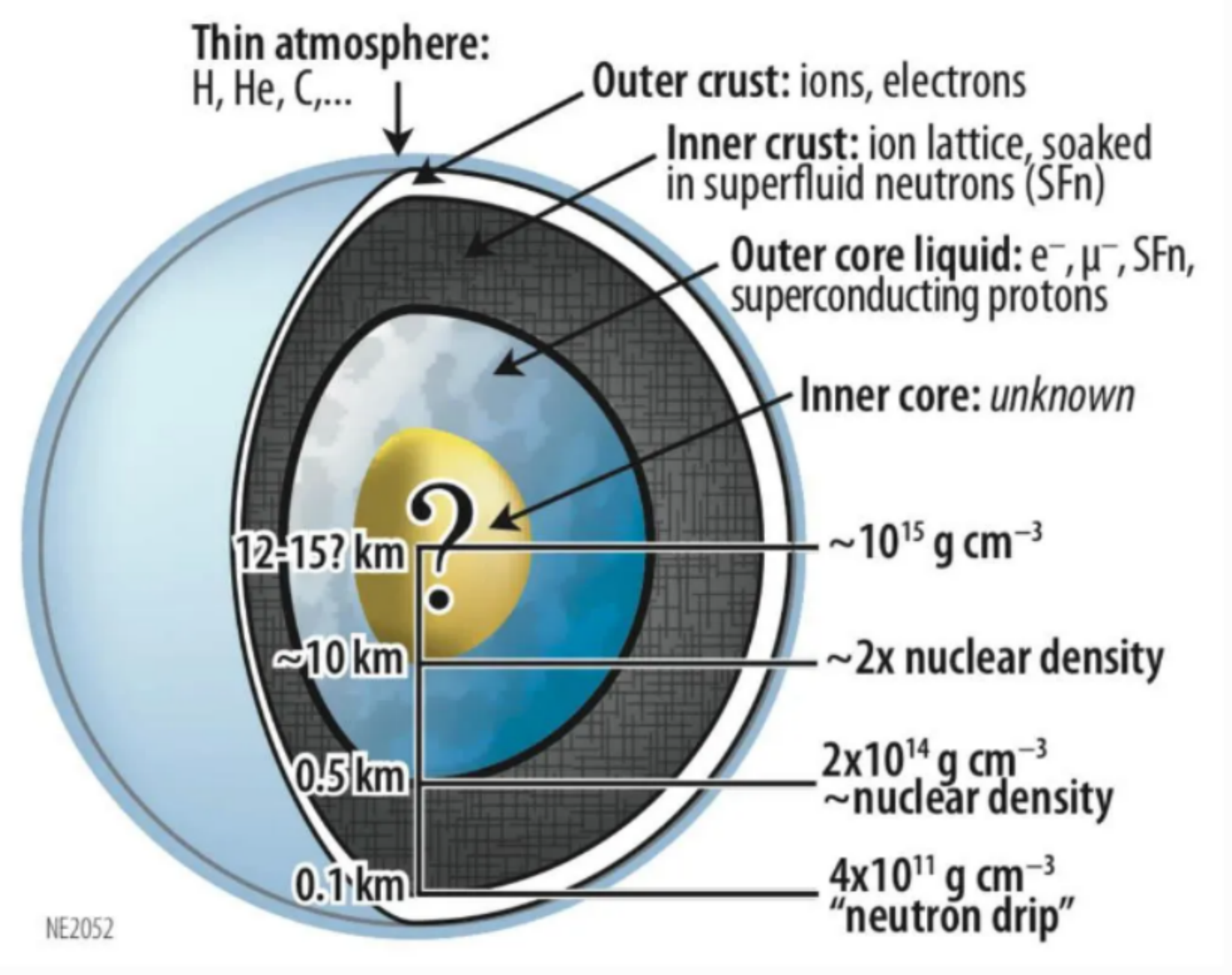


FIG. 1. Schematic QCD phase diagram. Deconfinement at high temperature and low density has been established to be a smooth crossover. A change to QM at low temperature is yet unresolved.



The core of a neutron star is expected to be made of neutrons and neutral quark-gluon plasmas, with the outermost layers containing free, charged particles. The rotating star was thought to lead to a dipole magnetic field, but the true field may be even more complicated. [-] NASA / GSFC / NICER

❖Connection between neutron star properties and the EOS

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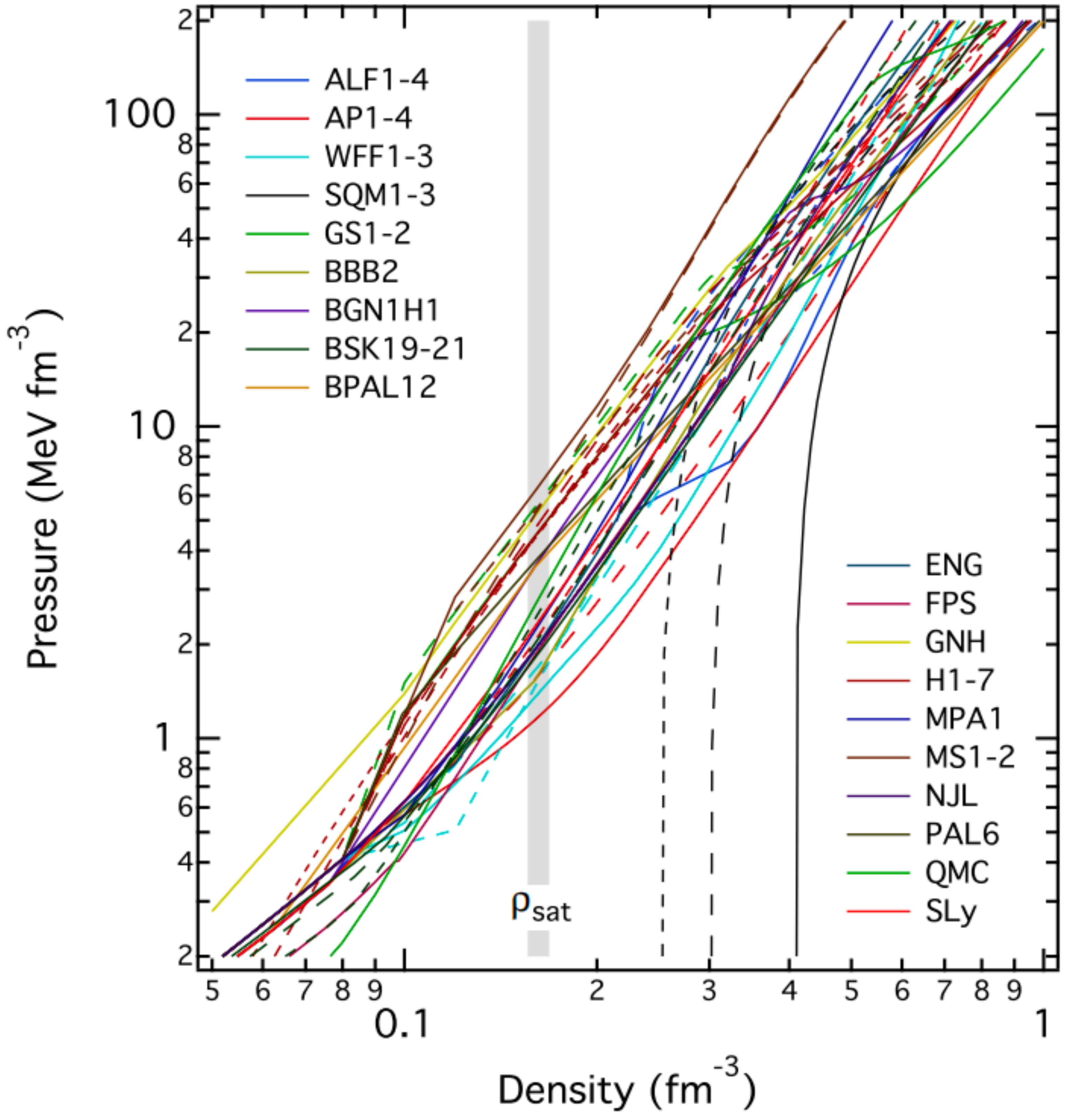
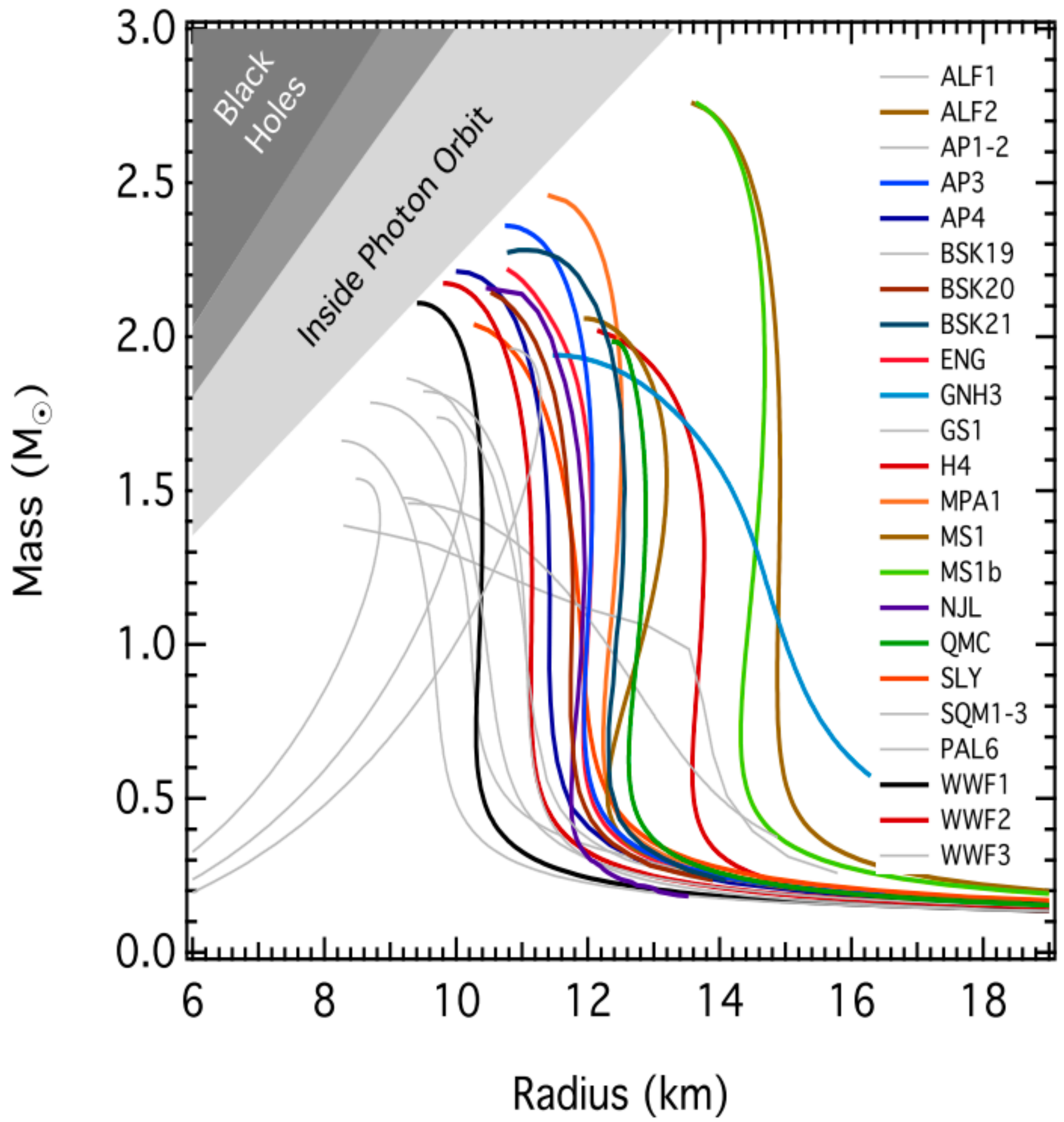
E.g., the one-to-one mapping from EOS to the mass-radius relation, the tidal deformability …





Ozel+2016

❖Astrophysical observation for EOS constraining

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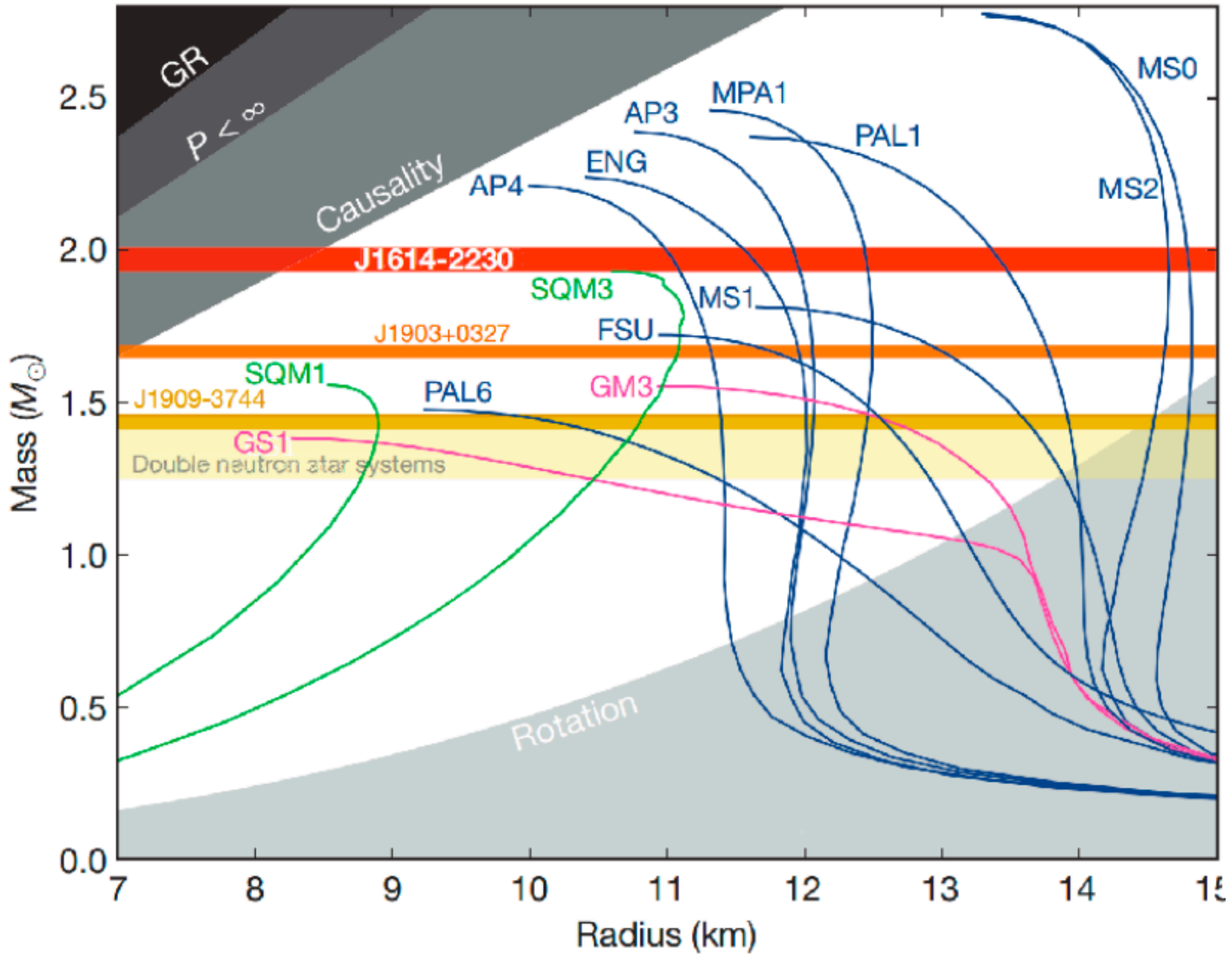
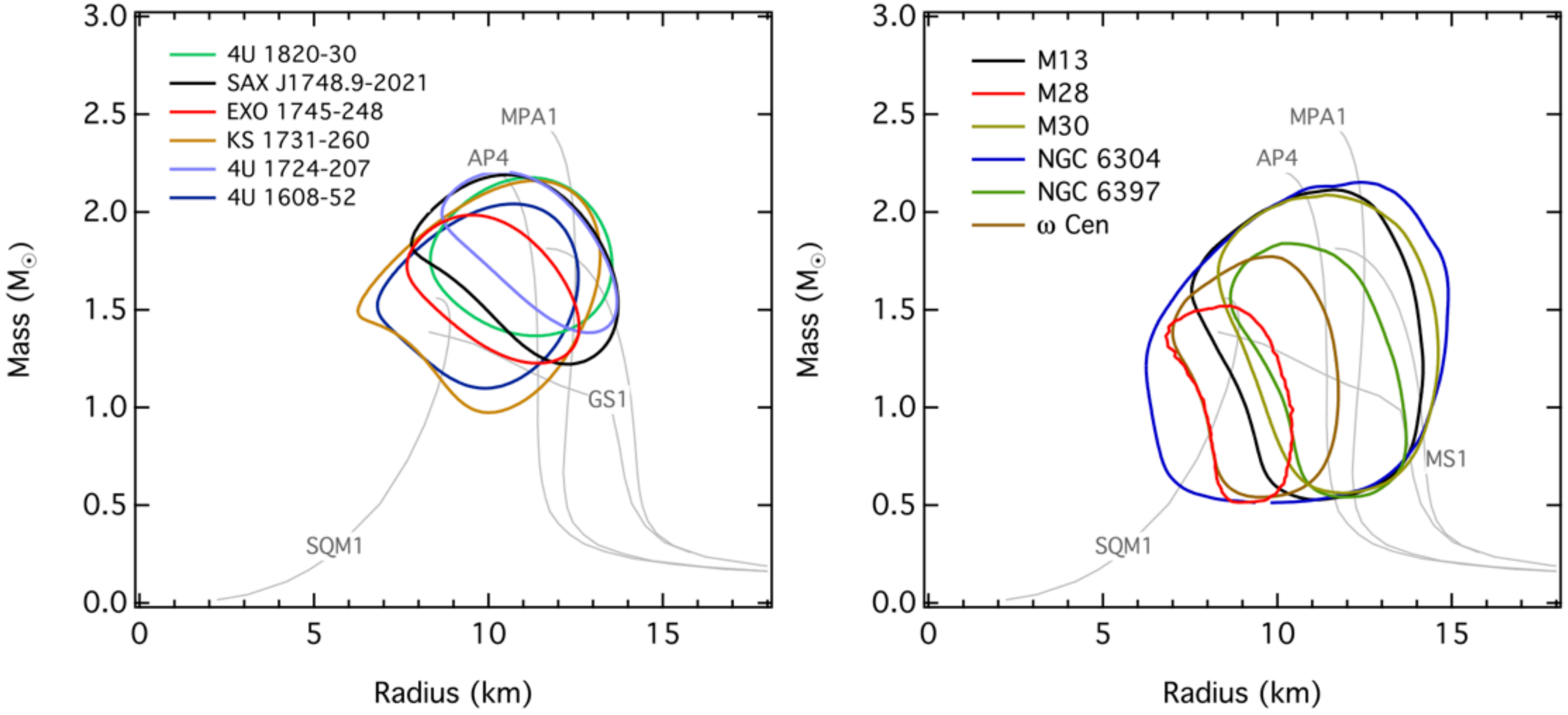
Maximum mass



Demorest+2010 Nature

Mass-radius measurement



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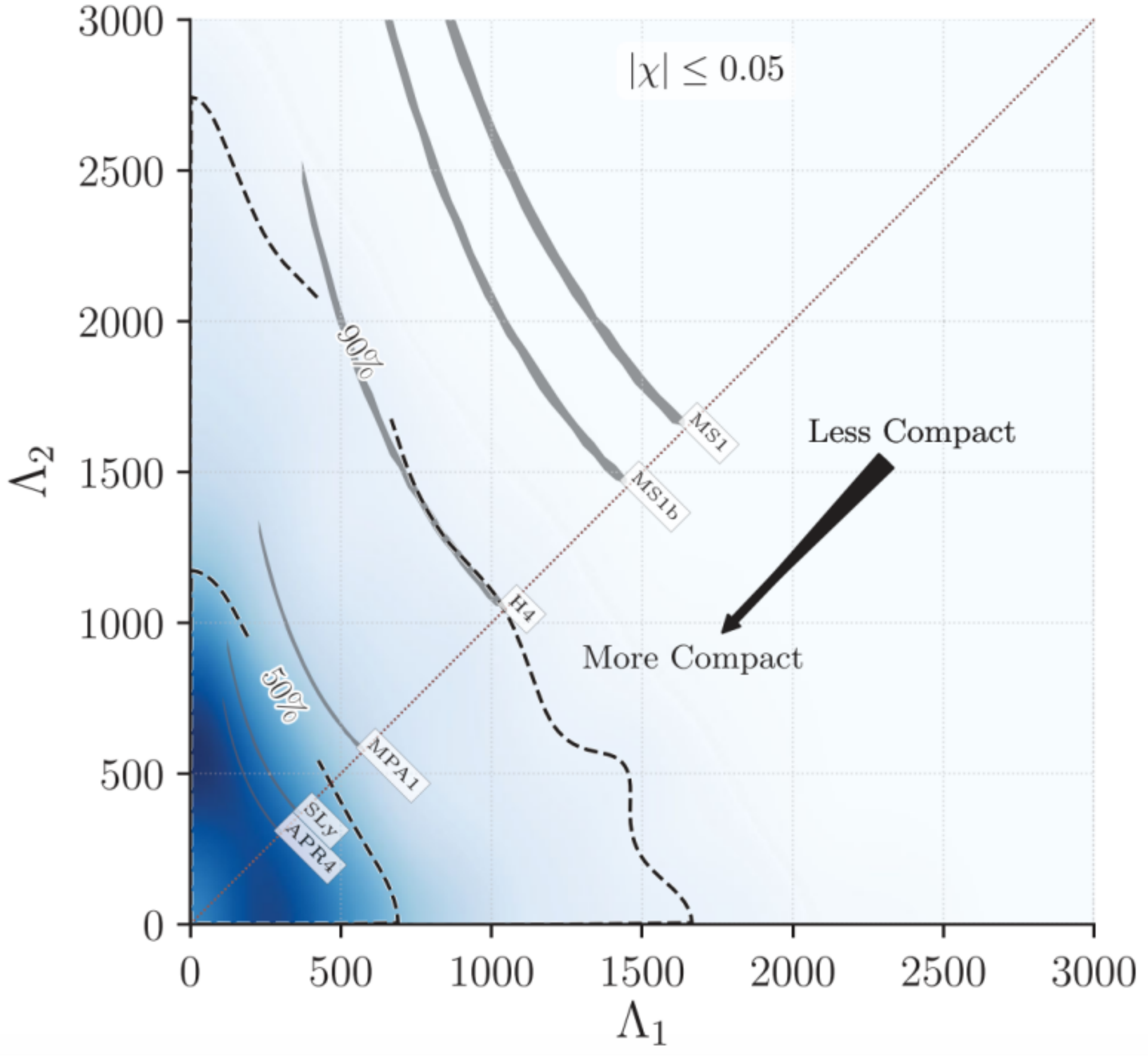
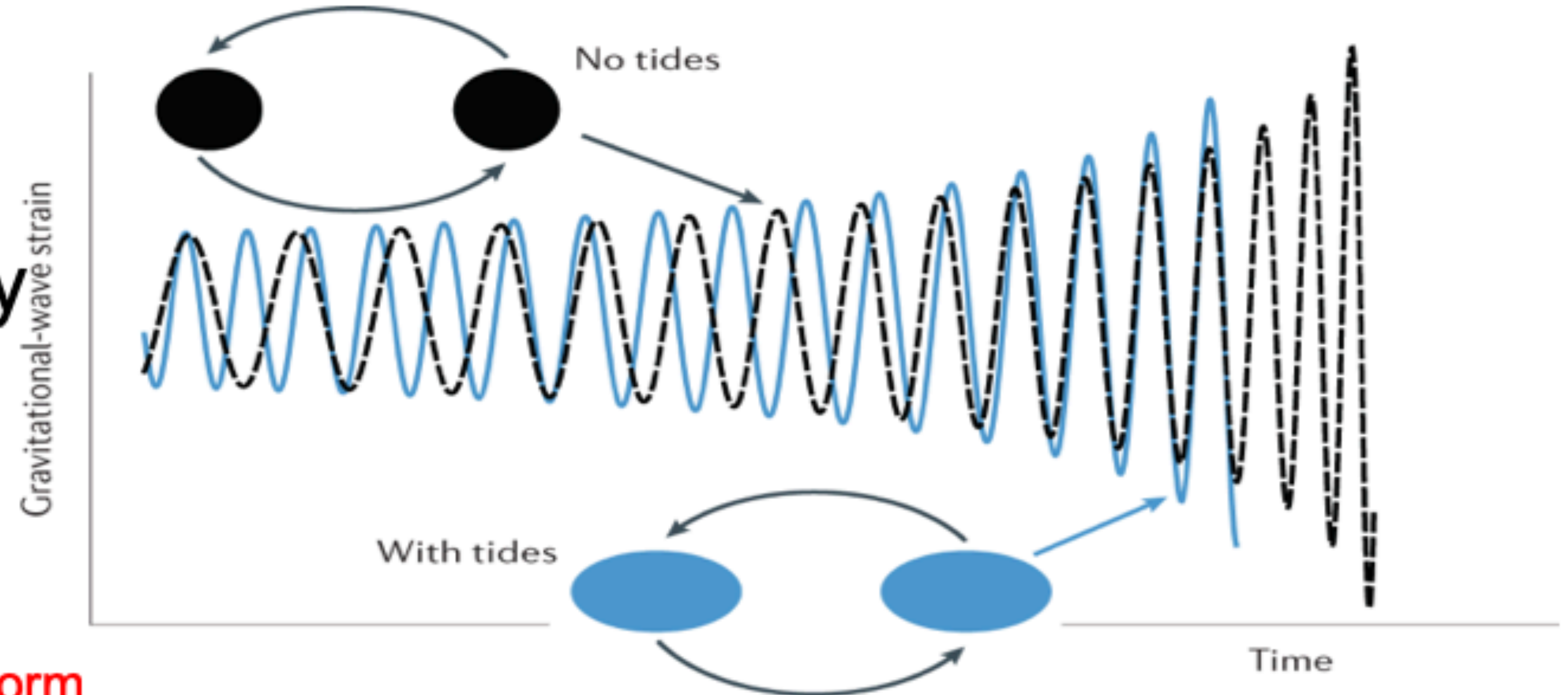


GW (Inspiralling phase)

Mass and tidal deformability

GW170817: $\Lambda_{1.4} < 800$

Abbott et al. PRL, 2017



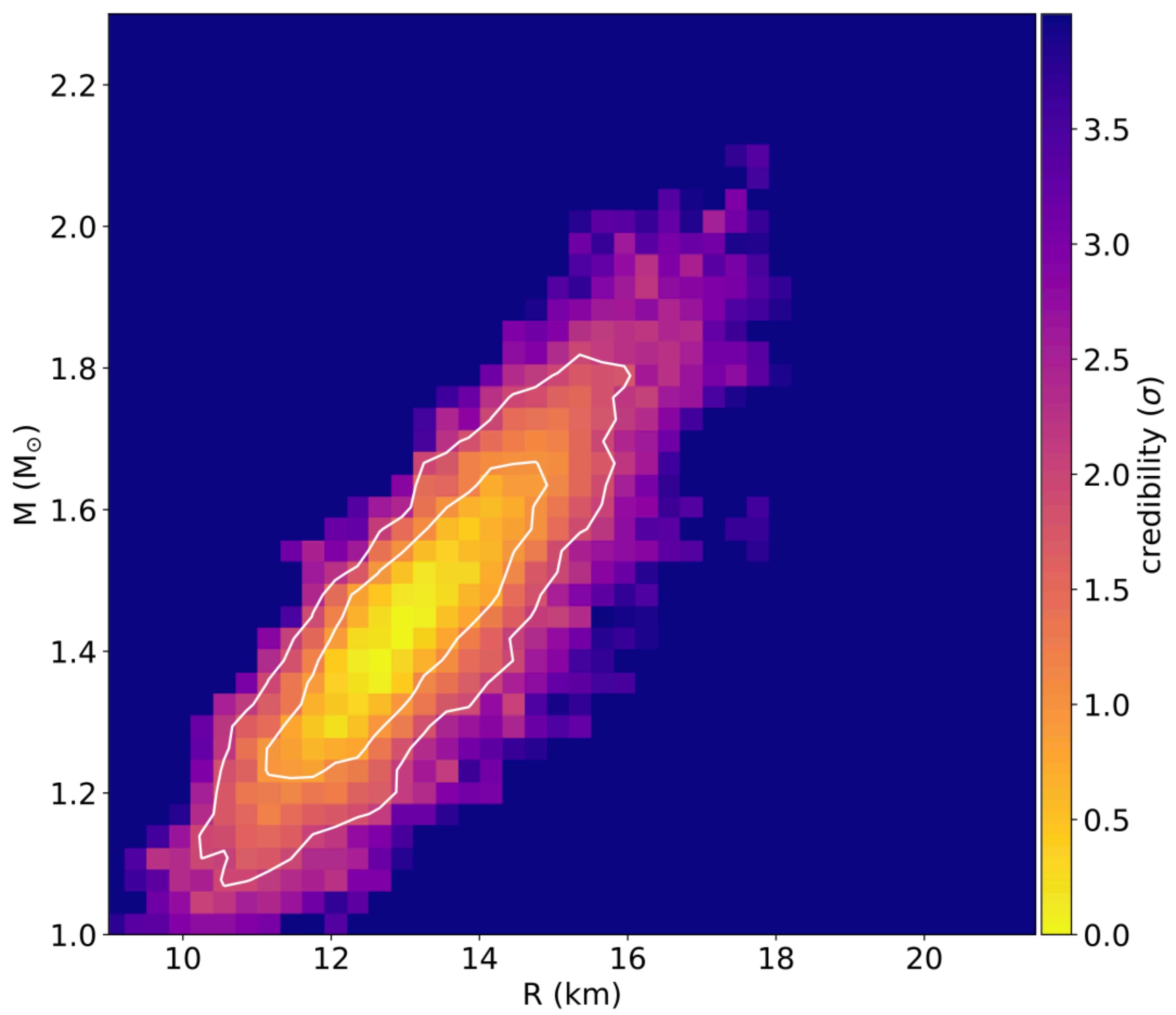
Yunes et al. Nature Review Physics, 2022

$$\mathcal{L}_{\rm GW}(d_{\rm GW}|\boldsymbol{\theta}_{\rm GW}, \mathbb{M}) \propto \exp\left[-2\int_0^\infty \frac{|\tilde{d}(f) - \tilde{h}(f, \boldsymbol{\theta}_{\rm GW})|^2}{S_n(f)} df\right]$$

strain, waveform, PSD

$$\boldsymbol{\theta}_{\rm GW} = \{M_1, M_2, \Lambda_1, \Lambda_2, \chi_{1z}, \chi_{2z}, \varphi, \Psi, \theta_{\rm jn}, t_c, d_L, \mathrm{R.A.}, \mathrm{Decl.}\}$$

$\Lambda(\boldsymbol{\theta}_{\rm EOS}; M)$



X-ray (Pulse profile)

Mass and radius

PSR J0030+0451

Miller et al, ApJL, 2019

$M = 1.44^{+0.15}_{-0.14} M_\odot \quad R = 13.02^{+1.24}_{-1.06}$ km

Riley et al, ApJL, 2019

$M = 1.34^{+0.15}_{-0.16} M_\odot \quad R = 12.71^{+1.14}_{-1.19}$ km

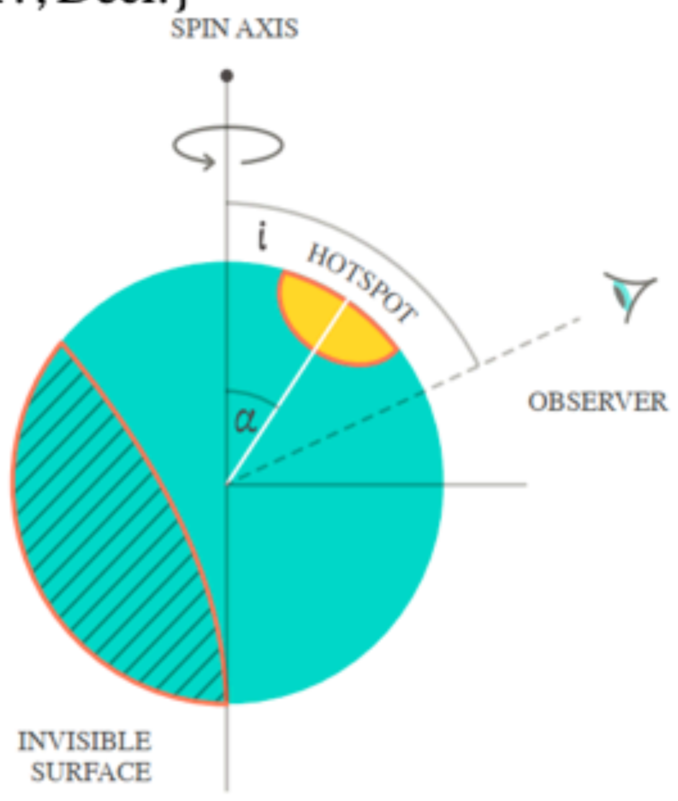
PSR J0740+6620

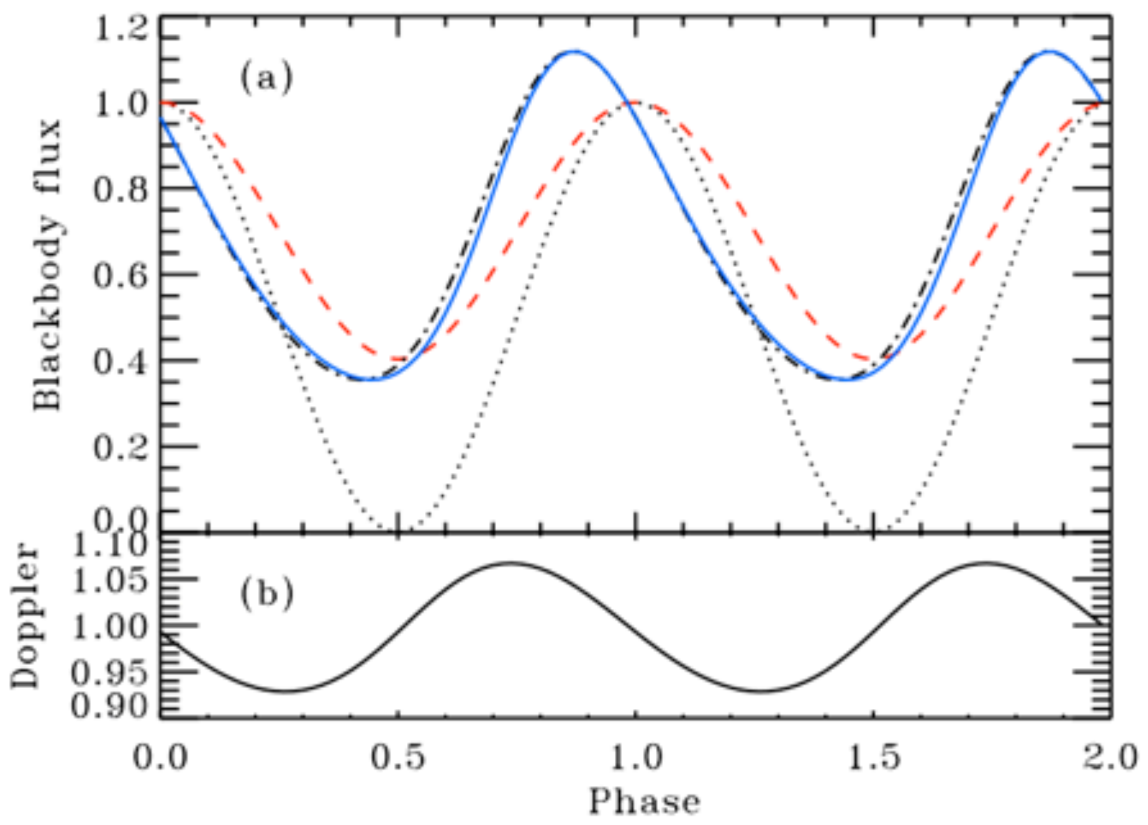
Miller et al, ApJL, 2021

$M = 2.062^{+0.090}_{-0.091} M_\odot \quad R = 13.71^{+2.61}_{-1.50}$ km

Riley et al, ApJL, 2021

$M = 2.072^{+0.067}_{-0.066} M_\odot \quad R = 12.39^{+1.30}_{-0.98}$ km





Poutanen et al. AIP Conference Proceedings, 2008

$$\mathcal{L}_{\rm NICER}(M, R|\boldsymbol{\theta}_{\rm EOS} \cup \{\varepsilon_c\}, \mathbb{M}) = P_{\rm KDE}(M(\boldsymbol{\theta}_{\rm EOS}; \varepsilon_c), R(\boldsymbol{\theta}_{\rm EOS}; \varepsilon_c))$$

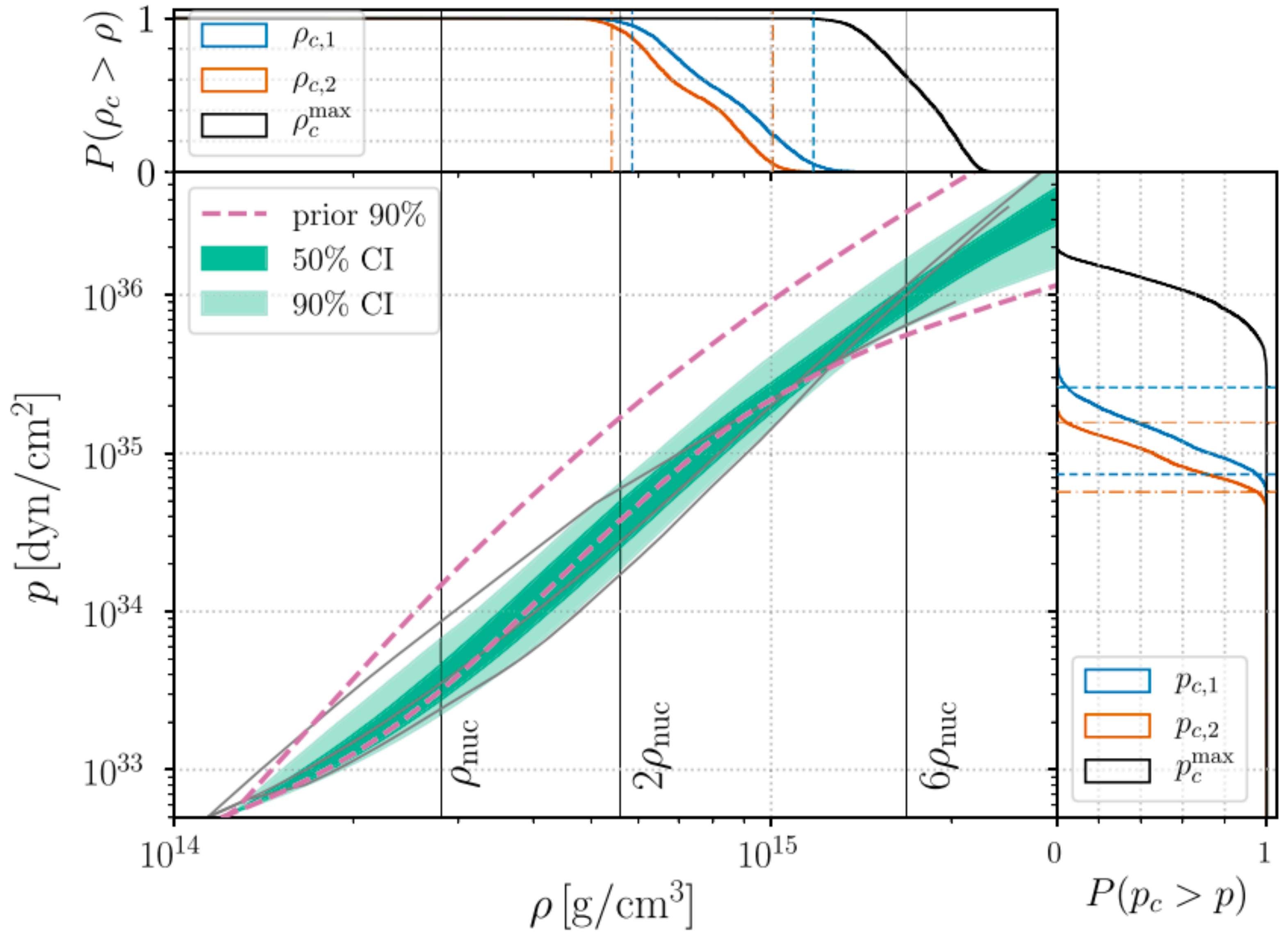
❖From equation of state to phase state?

- Nucleons?

- Hyperons?

- Quarks?

❖From equation of state to phase state?



Abbott+2018

"Nevertheless, even if we understand the (effective) stiffness of the EoS, a further challenge is the particle degree of freedom in cold, dense matter".

universe MDPI

Review

A Gravitational-Wave Perspective on Neutron-Star Seismology

Nils Andersson

…"What is less clear is to what extent this progress will allow us to probe aspects associated with the neutron star interior, e.g. the state and composition of matter… it is natural to (try to) formulate a seismology strategy".

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❖Normal modes and Quasi-normal modes

- Basic equations (Newtonian, normal modes)
  - Non-rotating star in equilibrium (the background star)

$$\frac{dp}{dr} = -\frac{\rho M}{r^2}$$

  - perturbation equations

$$\xi(r,t) = (\xi^r \hat{\boldsymbol{r}} + \xi^h r \nabla) Y_{lm}(\theta, \phi) e^{i\omega t}$$

$$\partial_t^2 \boldsymbol{\xi} = \frac{\delta\rho}{\rho^2}\nabla p - \frac{1}{\rho}\nabla\delta p - \nabla\delta\Phi$$ (Euler equation)

$$\delta\rho = -\nabla\cdot(\rho\boldsymbol{\xi})$$ (Continuity)

⬇

$$\mathscr{L}\boldsymbol{\xi} - \rho\omega^2\xi = 0$$

$$\mathscr{L}\boldsymbol{\xi} = \rho[-\nabla(\frac{\Gamma p}{\rho}\nabla\cdot\boldsymbol{\xi}) - \nabla(\frac{1}{\rho}\boldsymbol{\xi}\cdot\nabla p) + \nabla\delta\Phi]$$

In GR, the frequency is a complex, the imaginary part represent the damping due to gravitational wave radiation.

❖Zoo of modes

▸f-modes

no nodes, depends on the mean density $\sim \sqrt{G\bar{\rho}} \approx 2000\mathrm{Hz}$

▸p-modes

restored by pressure, few kHz, depends on the sound speed $c_s = \sqrt{\partial p/\partial \rho}$

▸g-modes

restored by buoyancy, arise from stable stratification:

- Density jump at some location, depends on $\Delta\rho$
- Composition gradient, $\sim N = g\sqrt{(\frac{\partial \rho}{\partial Y_e})_p(\frac{dY_e}{dp})}$ Brunt-Vaisala frequency

▸s-modes

restored by shear force, depends on the shear velocity $\sqrt{\mu/\rho}$

▸r-modes

restored by Coriolis force, depends on rotation velocity Ω_s

❖The g-modes reflect the ingredients of the matter

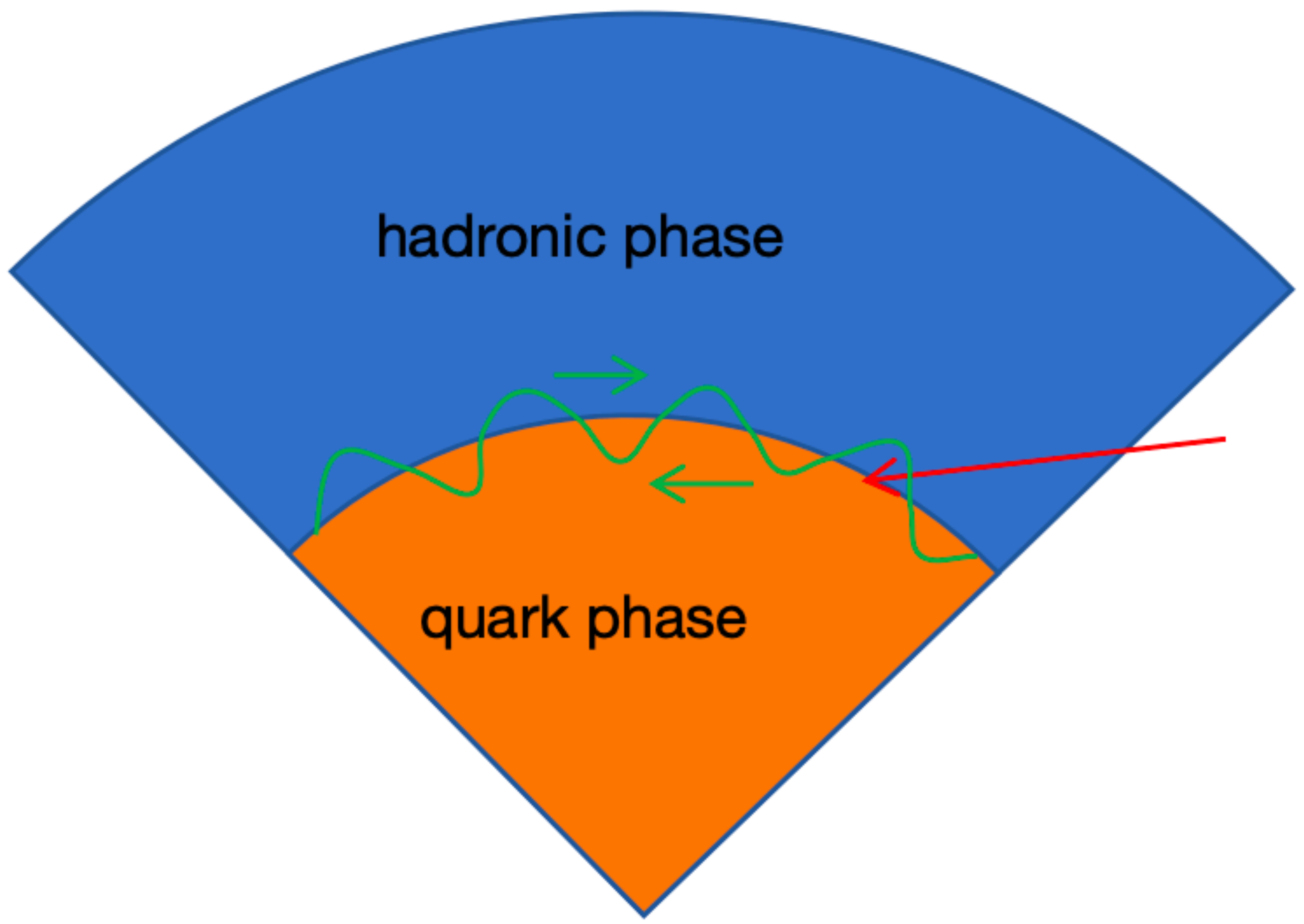
- Neutron star vs quarks star? (Flores+2017, Fu+2008 PRL)

- Quarks appear in a mixed phase (Gibbs, crossover)? (Jaikumar+2021, Constantinou+2021)

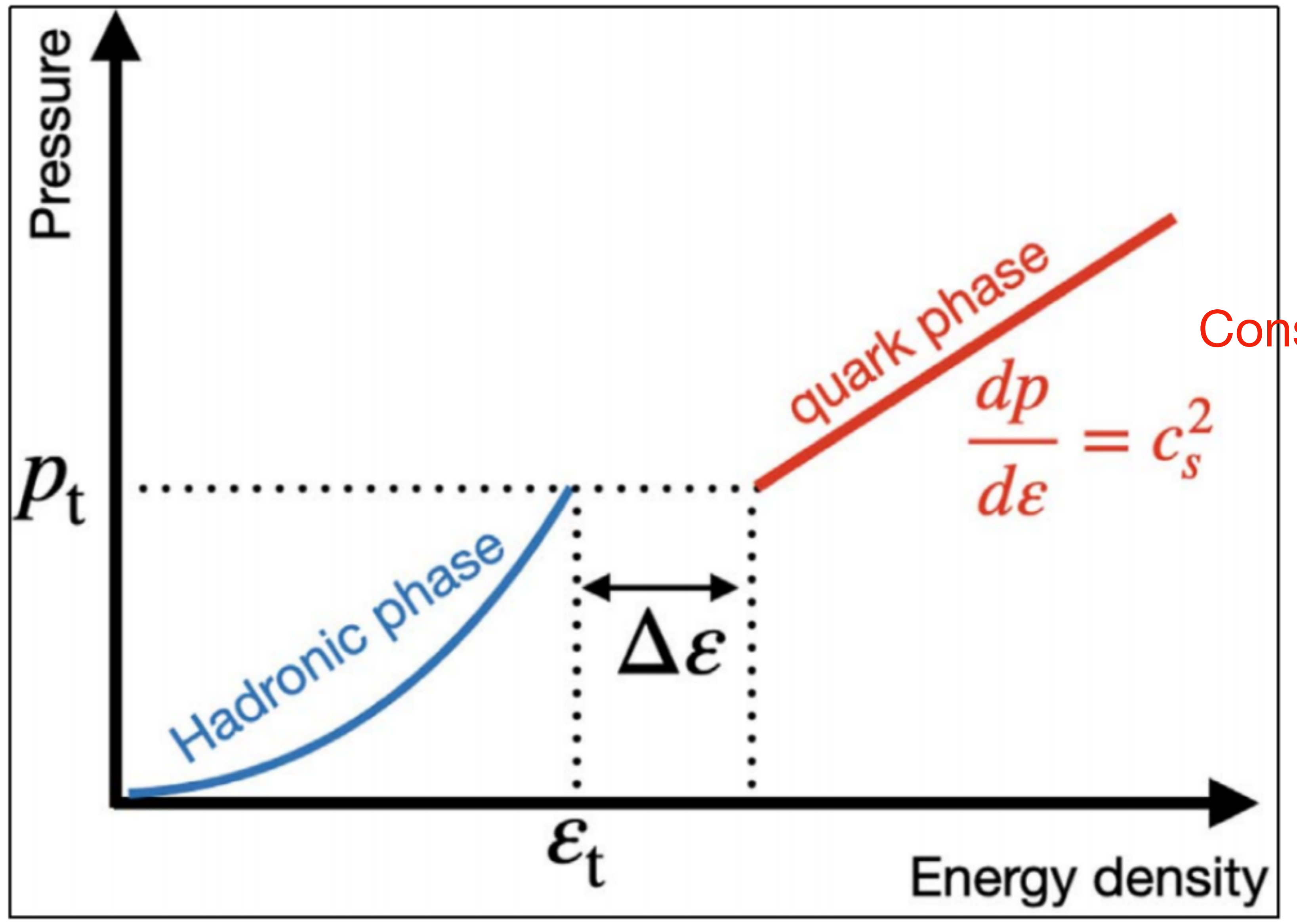
- Quarks appear with a sharp 1st phase transition (Maxwell)? (Zhao+2022, **MZQ**+2024)

- Superfluid neutrons? (Andersson+2001, Prix+2002) hyperons, muons? (Kantor+2014, Yu+2017,Dommes+2016)

❖The (discontinuity) g-mode



interface



Constant-speed-of-sound (CSS) model

$$\varepsilon_t, \Delta\varepsilon, c_s^2$$

Hadronic EOS:
NL3ωρ (stiff) and QMF(soft)

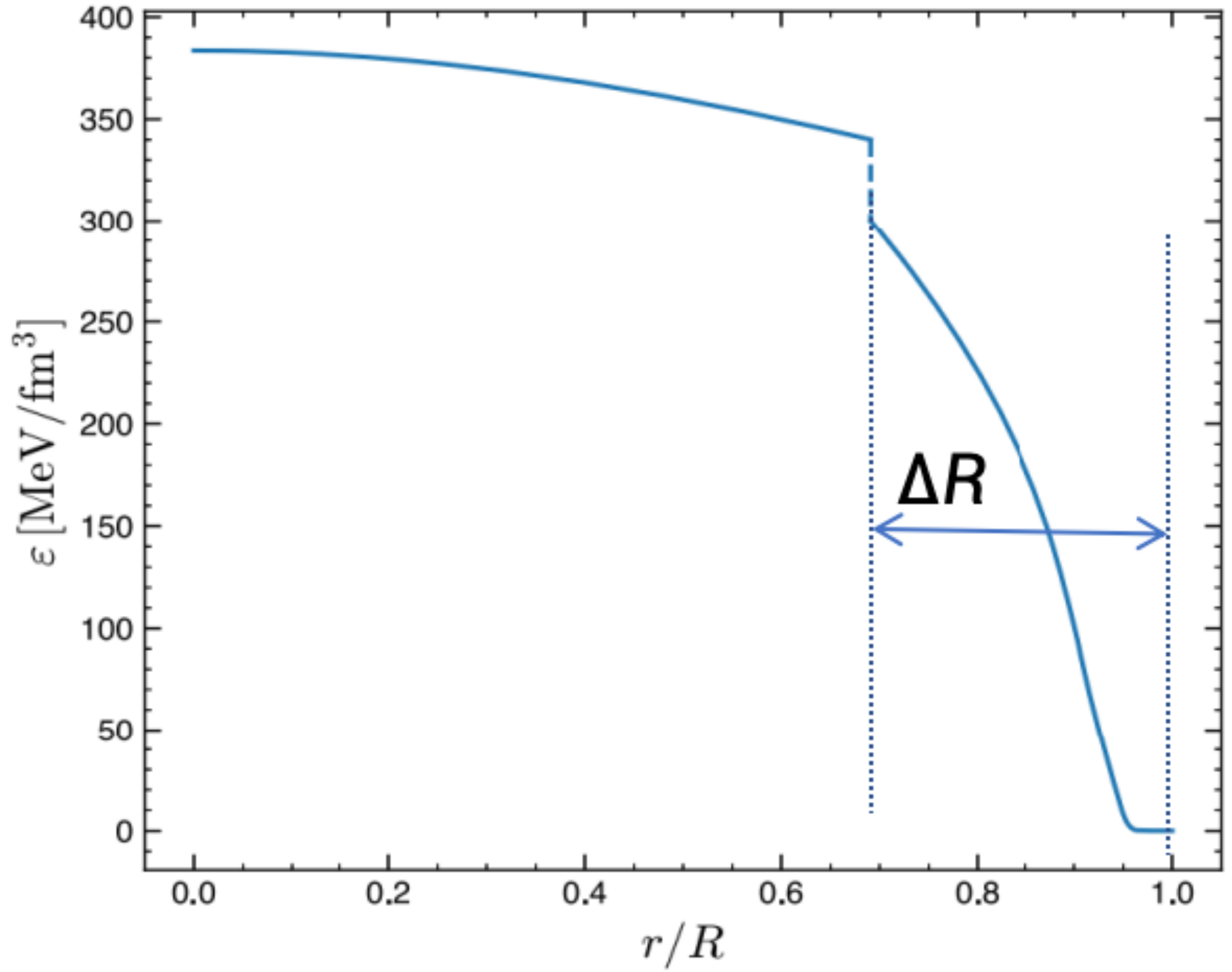
$$\omega^2 \propto \frac{\Delta\varepsilon}{\varepsilon_t}\frac{\Delta R}{R}\frac{GM}{R^3}$$

$$\omega^2 \approx K\left[\frac{6}{5}\frac{\Delta\varepsilon}{\varepsilon_{\rm trans}}(1-x^5) + 1 + \eta - \sqrt{(1+\eta)^2 - \frac{12}{5}\frac{\Delta\varepsilon}{\varepsilon_{\rm trans}}(\kappa-1)(1-x^5)}\right]$$
$$\times\left[\frac{\varepsilon_{\rm trans}+\Delta\varepsilon}{\varepsilon_{\rm trans}} - \frac{2}{5}\frac{\Delta\varepsilon}{\varepsilon_{\rm trans}}(1-x^5)\right]^{-1}\frac{M}{R^3},$$

$\eta = (1/5)(\Delta\varepsilon/\varepsilon_{\rm trans})[3(\kappa-1) + (3+2\kappa)x^5]$

$\kappa = (M_c/M)(R/R_{\rm trans})^3$

$x = R_{\rm trans}/R$



MZQ+2024

❖How can we detect them?

- Different from geoseismology on Earth and helioseismology on Sun, we can't directly detect the seismic wave of NS oscillation and can hardly to resolve the surface emission of NSs.

- GW signals are very faint, only possible for galactic events (like supernova or pulsar glitch) and f-mode.

$$h \approx 4 \times 10^{-23} \left(\frac{E}{10^{-9} M_\odot c^2} \right)^{1/2} \left(\frac{\tau}{0.1\ \mathrm{s}} \right)^{-1/2} \left(\frac{f}{2\ \mathrm{kHz}} \right)^{-1} \left(\frac{d}{10\ \mathrm{kpc}} \right)^{-1}$$

- We may detect the orbital phase change induced by mode excitation in BNS inspiral.

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❖Stellar response to tidal field

- Oscillation under tidal force

$$\left(\rho\frac{\partial^2}{\partial t^2}+\mathcal{L}\right)\vec{\xi}=-\rho\nabla U$$

$$U=-GM'\sum_{lm}\frac{4\pi}{2l+1}\frac{r^l}{D^{l+1}}Y^*_{lm}\left(\frac{\pi}{2},\Phi\right)Y_{lm}(\theta,\phi)$$

$$=-GM'\sum_{lm}W_{lm}\frac{r^l}{D(t)^{l+1}}\mathrm{e}^{-im\Phi(t)}Y_{lm}(\theta,\phi)$$

- Decompose into normal modes

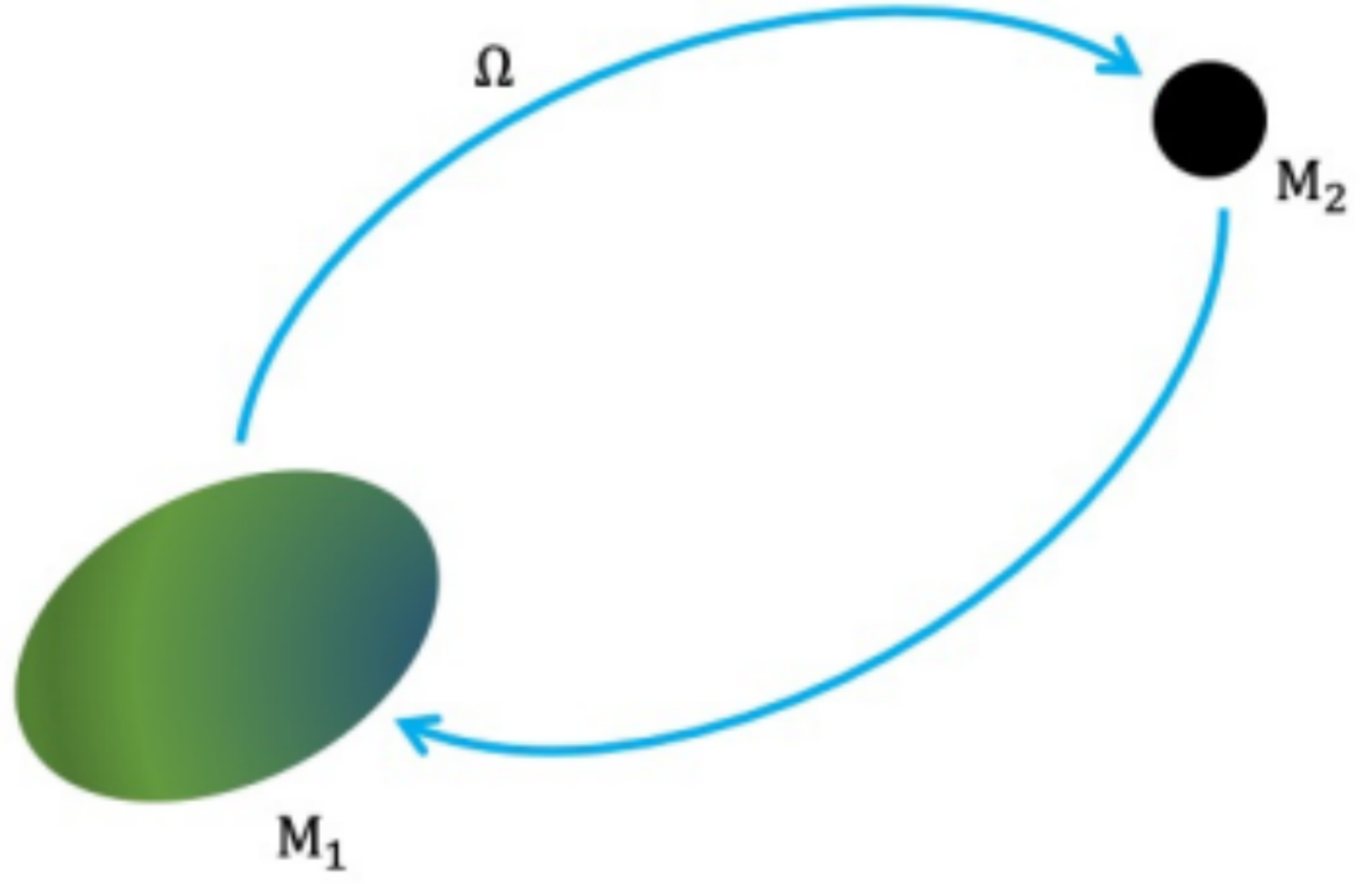
$$\vec{\xi}(\mathbf{r},t)=\sum_\alpha a_\alpha(t)\vec{\xi}_\alpha(\mathbf{r}), \qquad \left(\mathcal{L}-\rho\omega_\alpha^2\right)\vec{\xi}_\alpha(\mathbf{r})=0,$$

$$\ddot{a}_\alpha+\omega_\alpha^2 a_\alpha=\frac{GM_2W_{lm}Q_\alpha}{D^{l+1}}e^{-im\Omega_{\rm orb}t}$$

- Tidal overlap integral

$$Q_{nl}=\int d^3x\,\rho\,\boldsymbol{\xi}^*_{nlm}\cdot\nabla[r^lY_{lm}(\theta,\phi)]$$

$$=\int_0^R \rho l r^{l+1}\,dr[\xi^r_{nl}(r)+(l+1)\xi^\perp_{nl}(r)]$$



- Quasi-equilibrium (static) tide

$$\omega_\alpha \gg m\Omega_{\rm orb} \qquad a_\alpha\sim\frac{e^{i\Omega_{\rm orb}t}}{\omega_\alpha^2}$$

- Resonant tide

$$\omega_\alpha\simeq m\Omega_{\rm orb} \qquad a_\alpha\sim\frac{e^{i\Omega_{\rm orb}t}}{\omega_\alpha^2-m^2\Omega_{\rm orb}^2}$$

❖Stellar response to tidal field

- Oscillation under tidal force

$$\left(\rho\frac{\partial^2}{\partial t^2}+\mathcal{L}\right)\vec{\xi}=-\rho\nabla U$$

$$U=-GM'\sum_{lm}\frac{4\pi}{2l+1}\frac{r^l}{D^{l+1}}Y$$

$$=-GM'\sum_{lm}W_{lm}\frac{r^l}{D(t)^{l+1}}e$$

- Decompose into normal modes

$$\vec{\xi}(\mathbf{r},t)=\sum_\alpha a_\alpha(t)\vec{\xi}_\alpha(\mathbf{r}), \qquad \left(\mathcal{L}-\rho\omega_\alpha^2\right)\vec{\xi}_\alpha(\mathbf{r})=0,$$

$$\ddot{a}_\alpha+\omega_\alpha^2 a_\alpha=\frac{GM_2W_{lm}Q_\alpha}{D^{l+1}}e^{-im\Omega_{\rm orb}t} \qquad Q_{nl}=$$

$$=$$

- Quasi-equilibrium (static) tide

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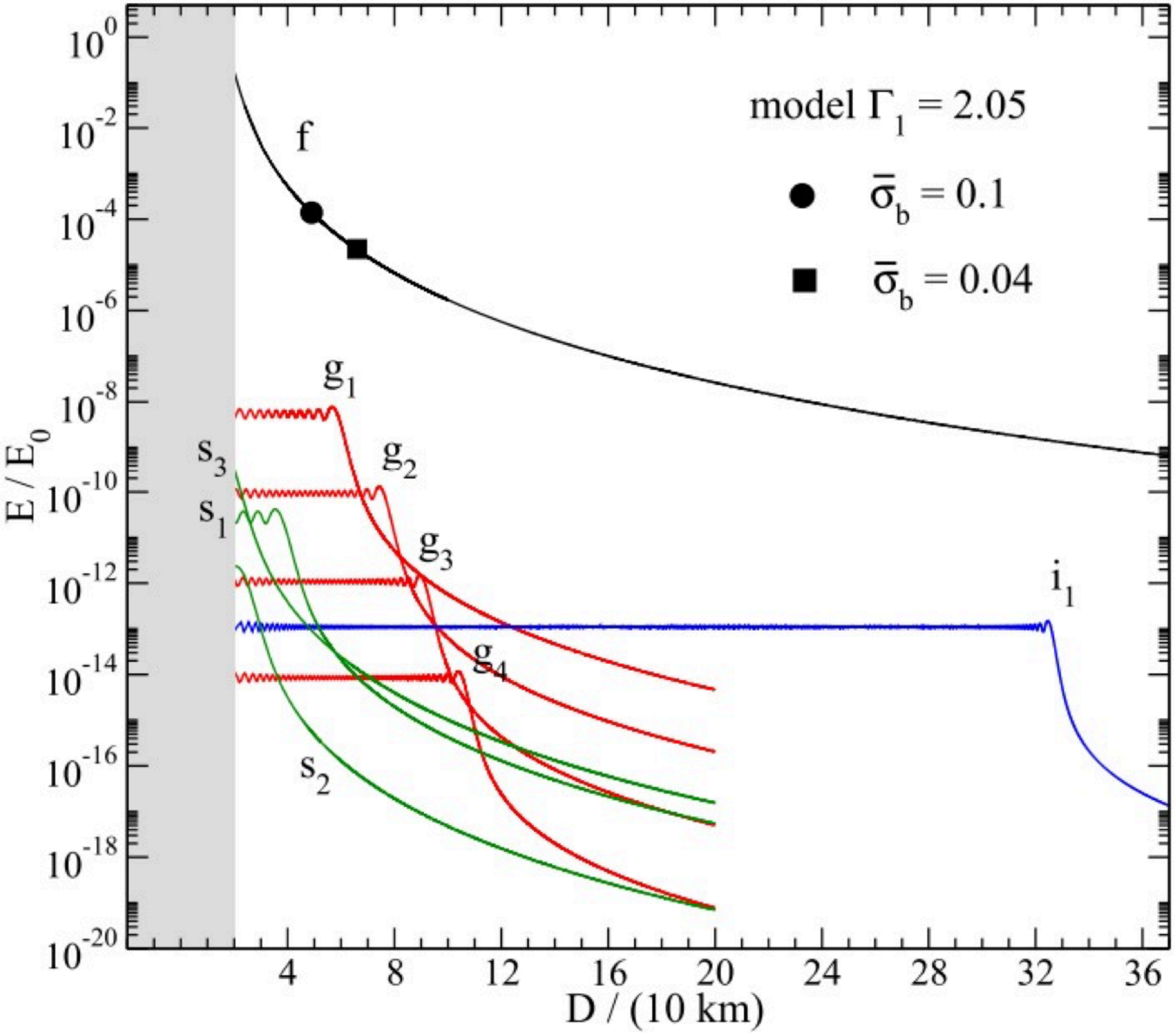
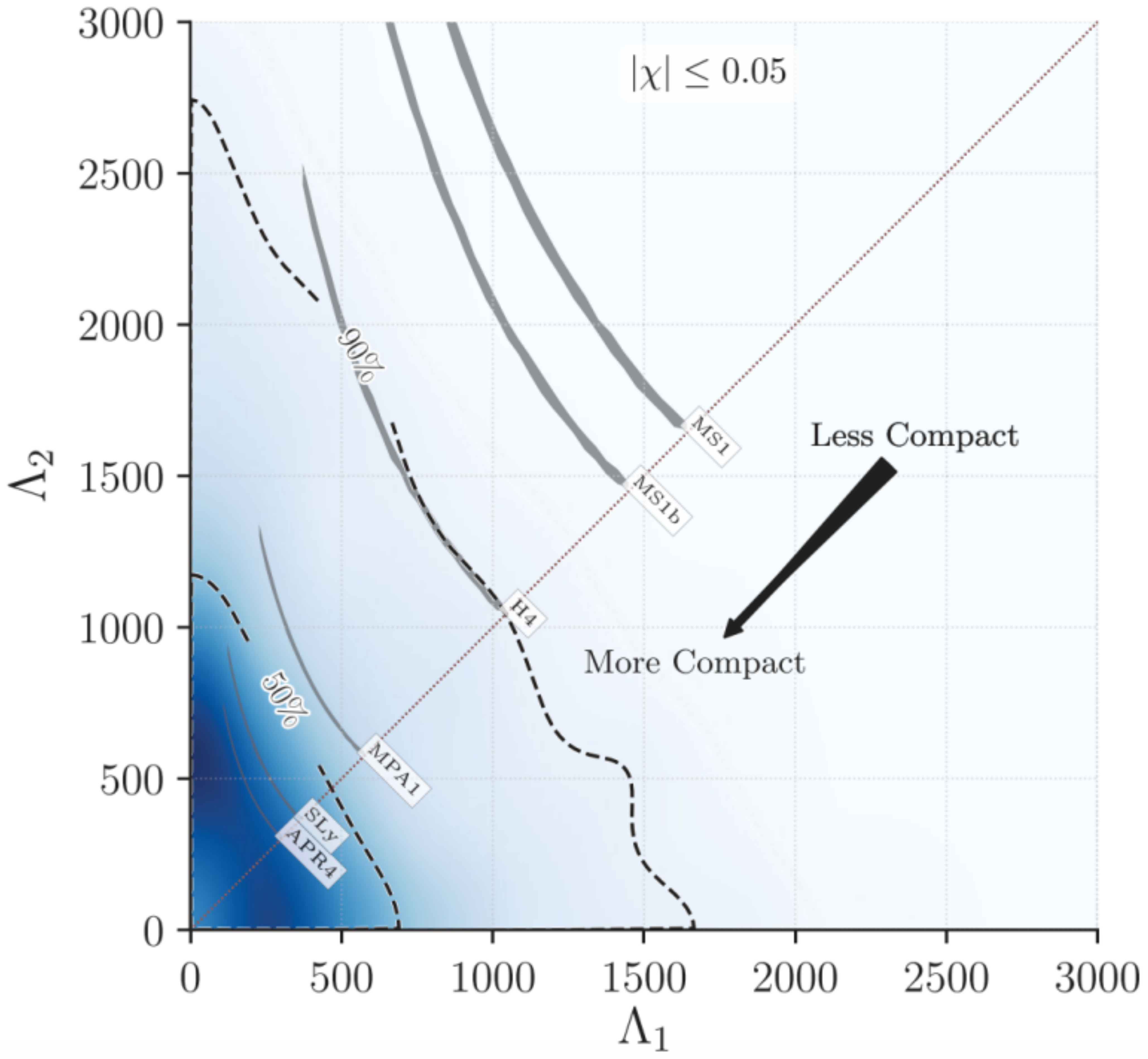


Passamonti+21

❖Quasi-equilibrium (static) tide: observed in GW170817



$$\Lambda_{1.4} < 800$$

Abbottet al. PRL 2017

❖Resonant tides

- The resonance is almost instaneous at lower frequency

$$t_{res} \simeq 0.01s\mathcal{M}_{1.2}^{-5/6}f_{600}^{-11/6} \ll t_D \simeq 0.1s\mathcal{M}_{1.2}^{-5/3}f_{600}^{-8/3}$$

Resonance　　　　Orbit decay

- The energy transfer from orbit to stellar oscillation is

$$\Delta E \simeq 5 \times 10^{49}\mathrm{erg} f_{600}^{1/3}Q_{0.01}^2M_{1.4}^{-2/3}R_{12}^2 q(\frac{2}{1+q})^{5/3}$$

- Which implies a sudden GW phase change at resonance frequency

$$\delta\Phi = \frac{\omega_{mode}\Delta E}{P_{GW}} \simeq -0.12f_{600}^{-2}Q_{0.01}^2M_{1.4}^{-4}R_{12}^2\frac{2q}{1+q}$$

Lai+1994

❖Signature in gravitational waveform

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$$h(f) = \mathcal{A}e^{i\Psi(f)}$$

$$\Psi(f, \phi_c, t_c) = \begin{cases} 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{4}\left(\frac{8\pi G\mathcal{M}f}{c^3}\right)^{-5/3} + \delta\Psi^{\Lambda} \\ 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{4}\left(\frac{8\pi G\mathcal{M}f}{c^3}\right)^{-5/3} + \delta\Psi^{\Lambda} - \left(1 - \frac{f}{f_a}\right)\delta\phi_a \end{cases}$$

- Before resonance, i.e., $f < f_a$
- After the resonance, i.e., $f > f_a$

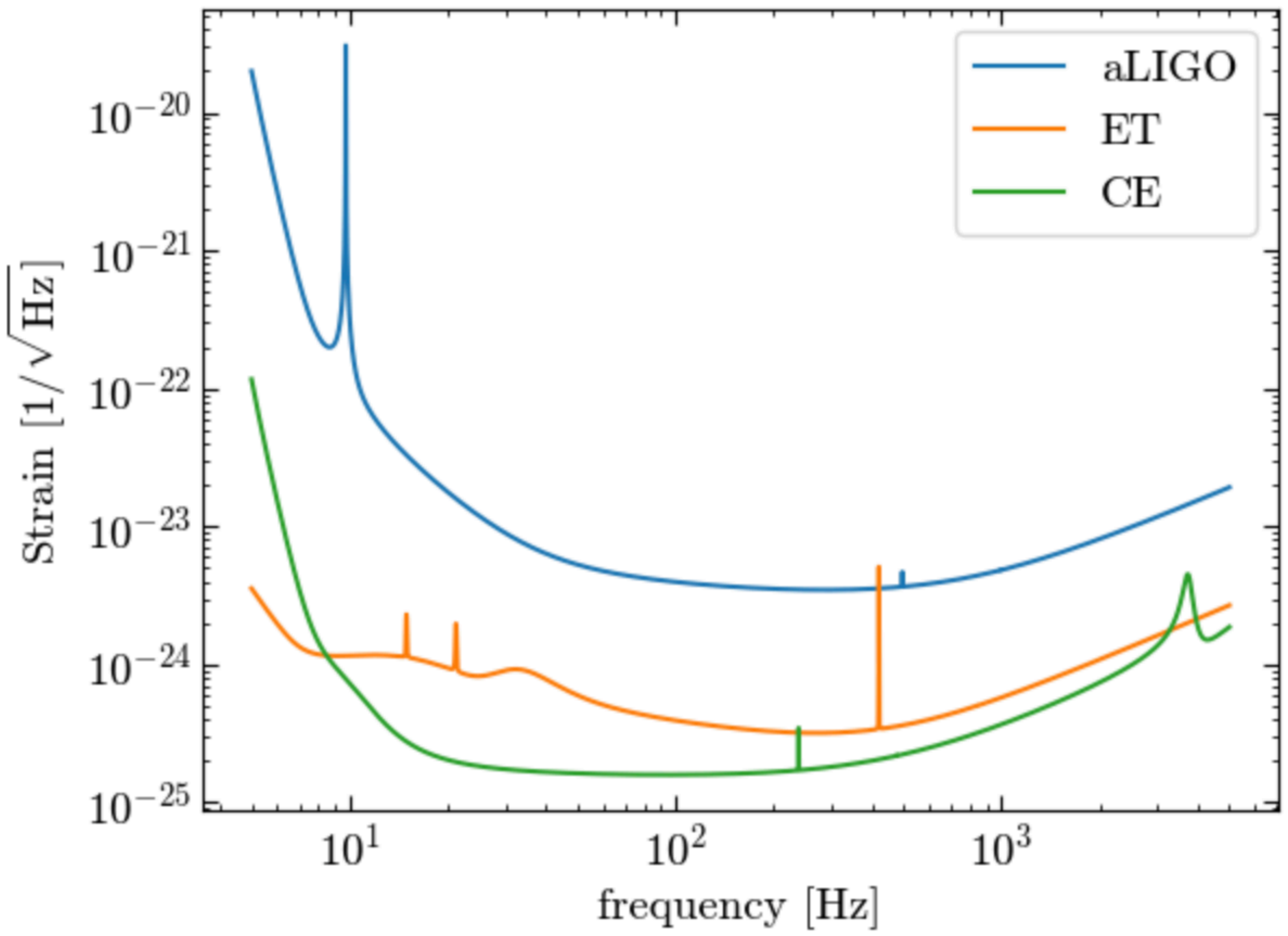
(Flanagan+2007, Yu+2017)

❖Sensitivity curve with future ground-base detectors

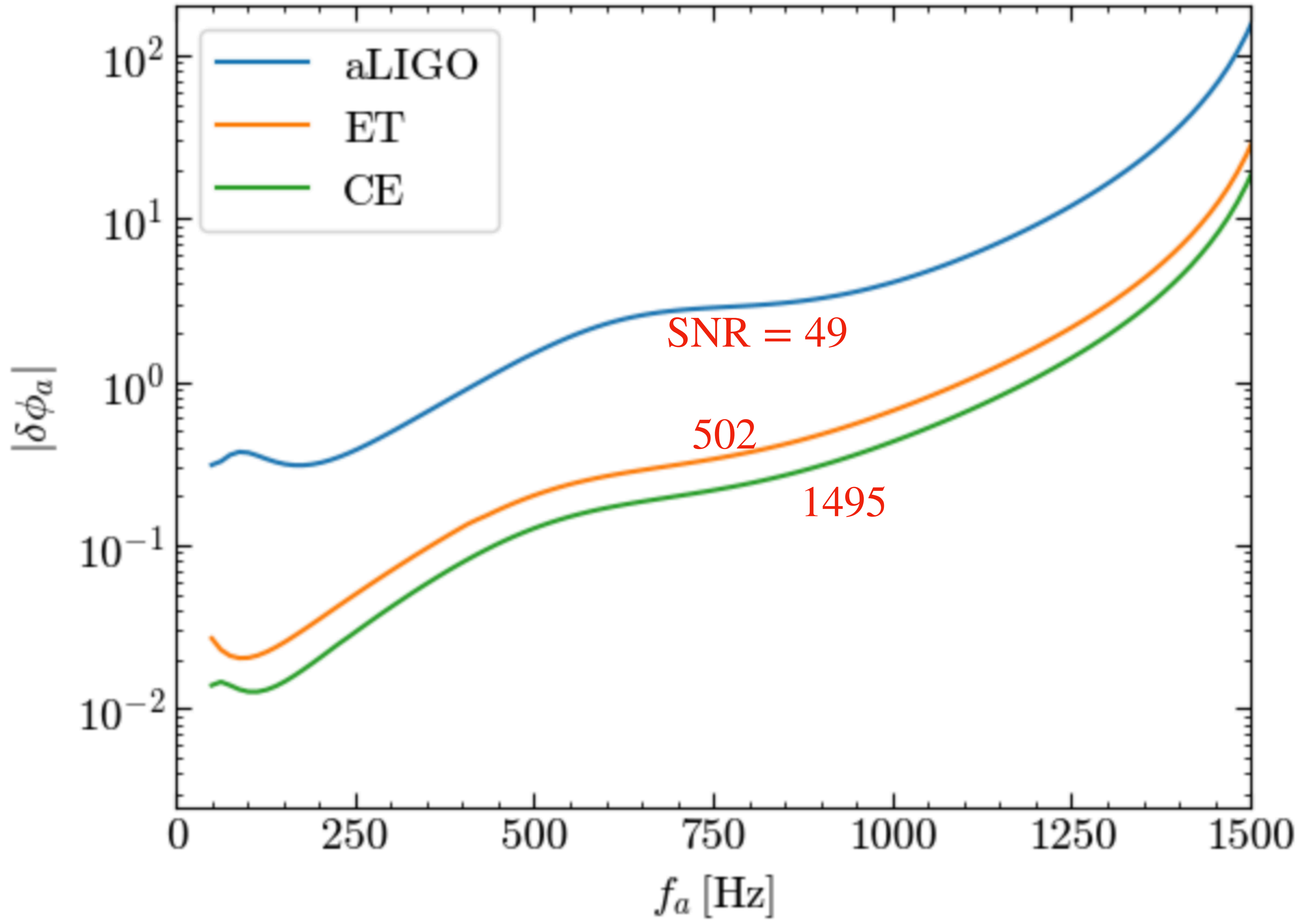
Fisher Information Matrix

$$\Gamma_{ij} = \left(\frac{\partial h}{\partial \theta_i} \Big| \frac{\partial h}{\partial \theta_j}\right), \qquad (h_1|h_2) = 2\int_0^\infty \frac{\tilde{h}_1^*(f)\tilde{h}_2(f) + \tilde{h}_1(f)\tilde{h}_2^*(f)}{S_n(f)} df, \qquad \Delta\theta_i = \sqrt{(\Gamma^{-1})_{ii}},$$

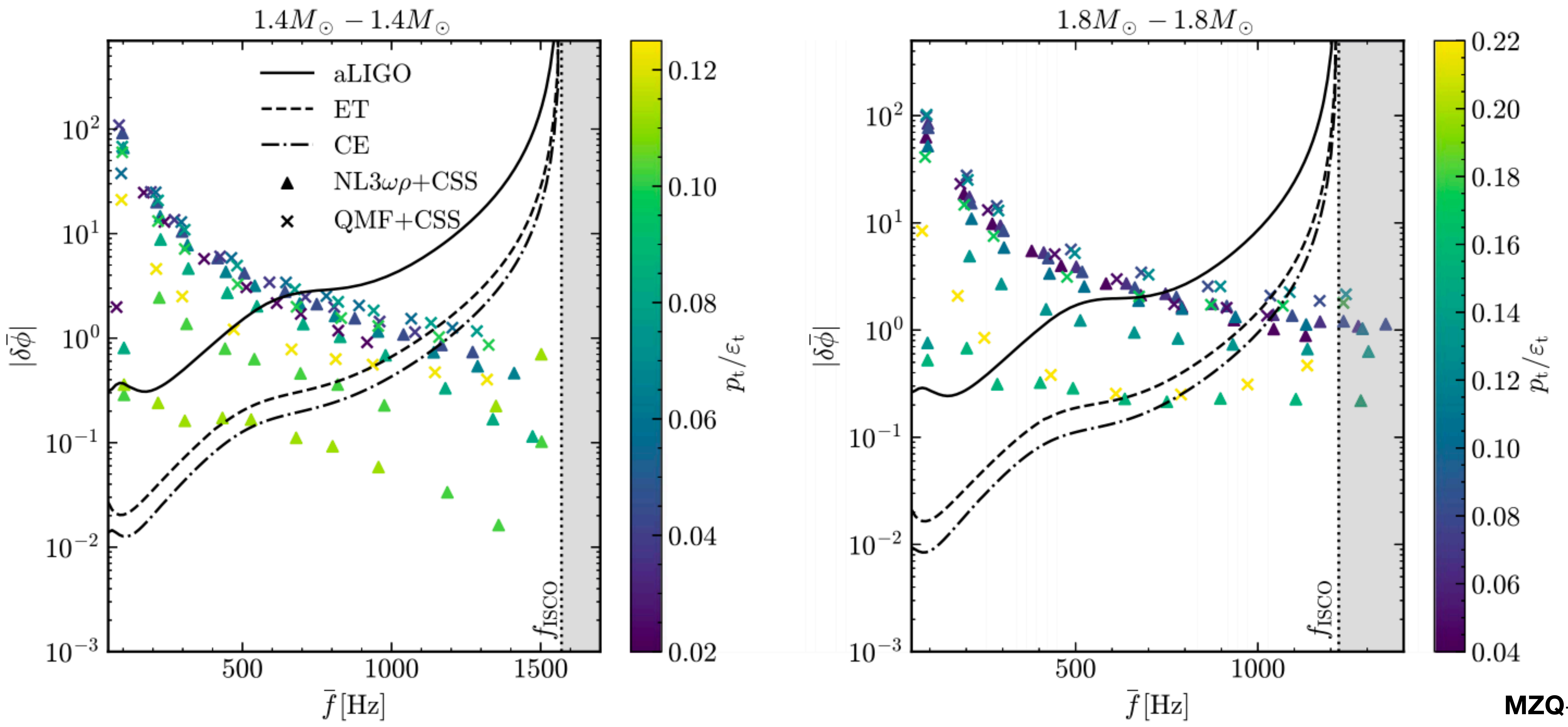
$$\{\theta_i\} = \{\mathscr{M}, q, \Lambda_1, \Lambda_2, d_L, t_c, \phi_c, |\delta\phi_a|, f_a\}$$



$1.4M_\odot - 1.4M_\odot$ @ 100Mpc



❖Detectability of the g-mode



MZQ+2024

❖Data analysis of GW170817

- Waveform model: IMRPhenomD_NTidal + resonance

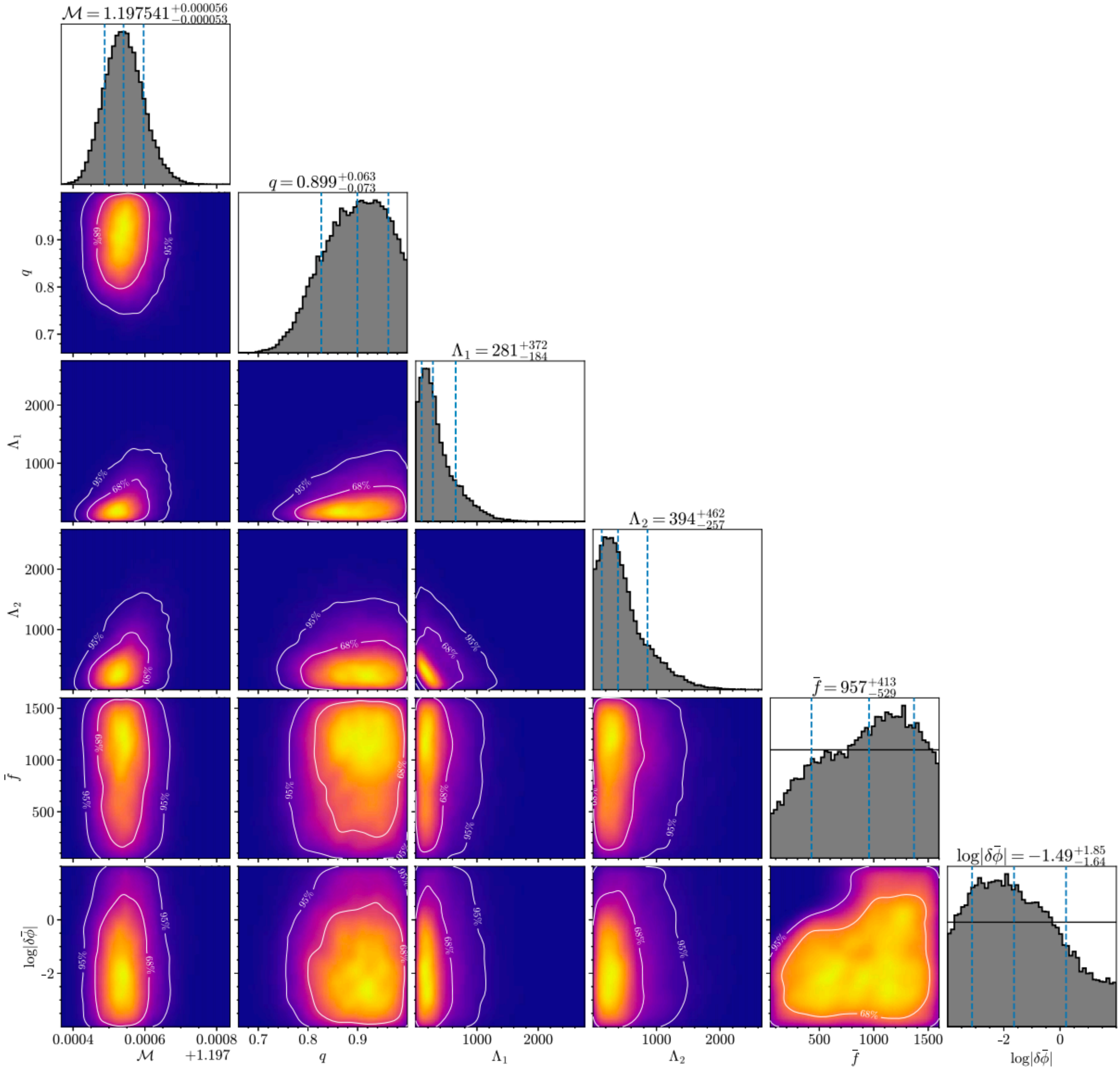
MZQ+2024

$$\{\theta_i\} = \{\mathcal{M},\, q,\, \Lambda_1,\, \Lambda_2,\, \chi_{1z},\, \chi_{2z},\, \theta_{\rm jn},\, t_c,\, \phi_c,\, \Psi,\, |\delta\bar{\phi}|,\, \bar{f}\}. \quad \text{(A3}$$

We fix the location of the source to the position determined electromagnetic observations (Abbott et al. 2017c; Levan et 2017) with $\alpha(J2000) = 197.^\circ45$, $\delta(J2000) = -23.^\circ38$ a $z = 0.0099$. The priors of the parameters are chosen following those used in Abbott et al. (2019), with the excepti of the priors for $|\delta\bar{\phi}|$ and $\bar{f}$. For the mode resonance paramete

- H0: model without mode resonance
- H1: model with mode resonance

No detection of the signal, as $\mathrm{BF}^{H_1}_{H_0} \in [0.72, 1.11]$



❖Data analysis of GW170817

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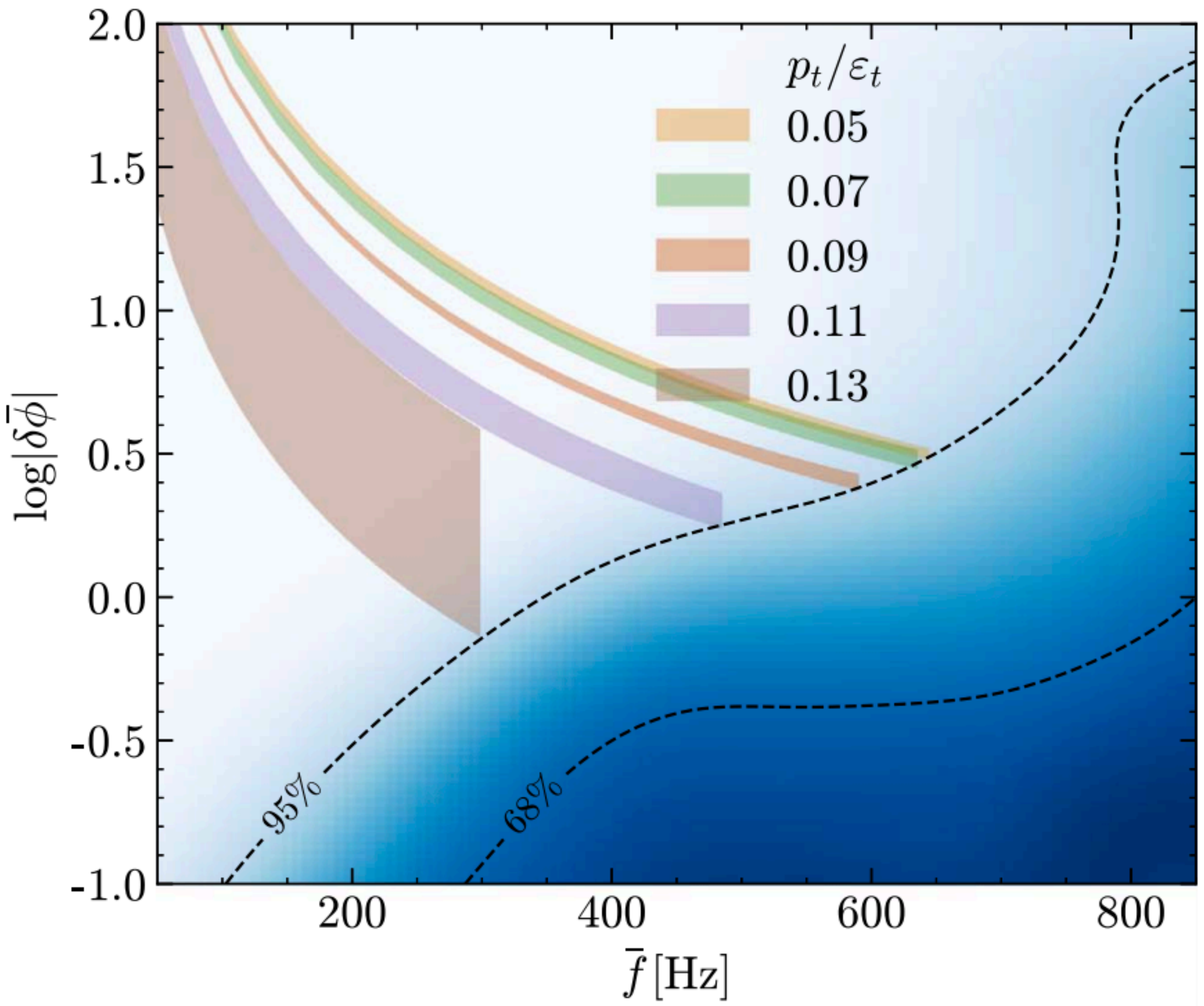
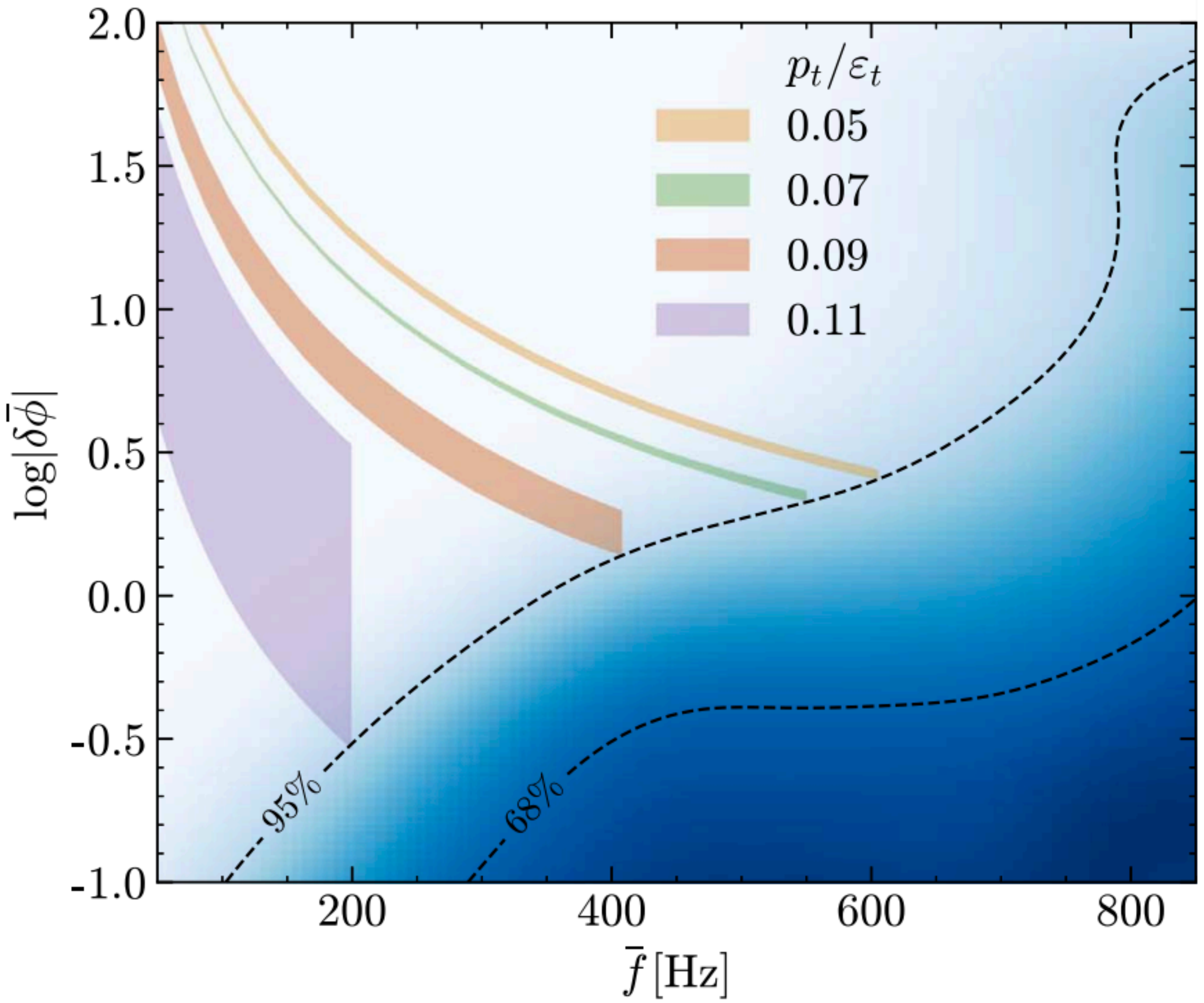
QMF+CSS



NL3ωρ+CSS



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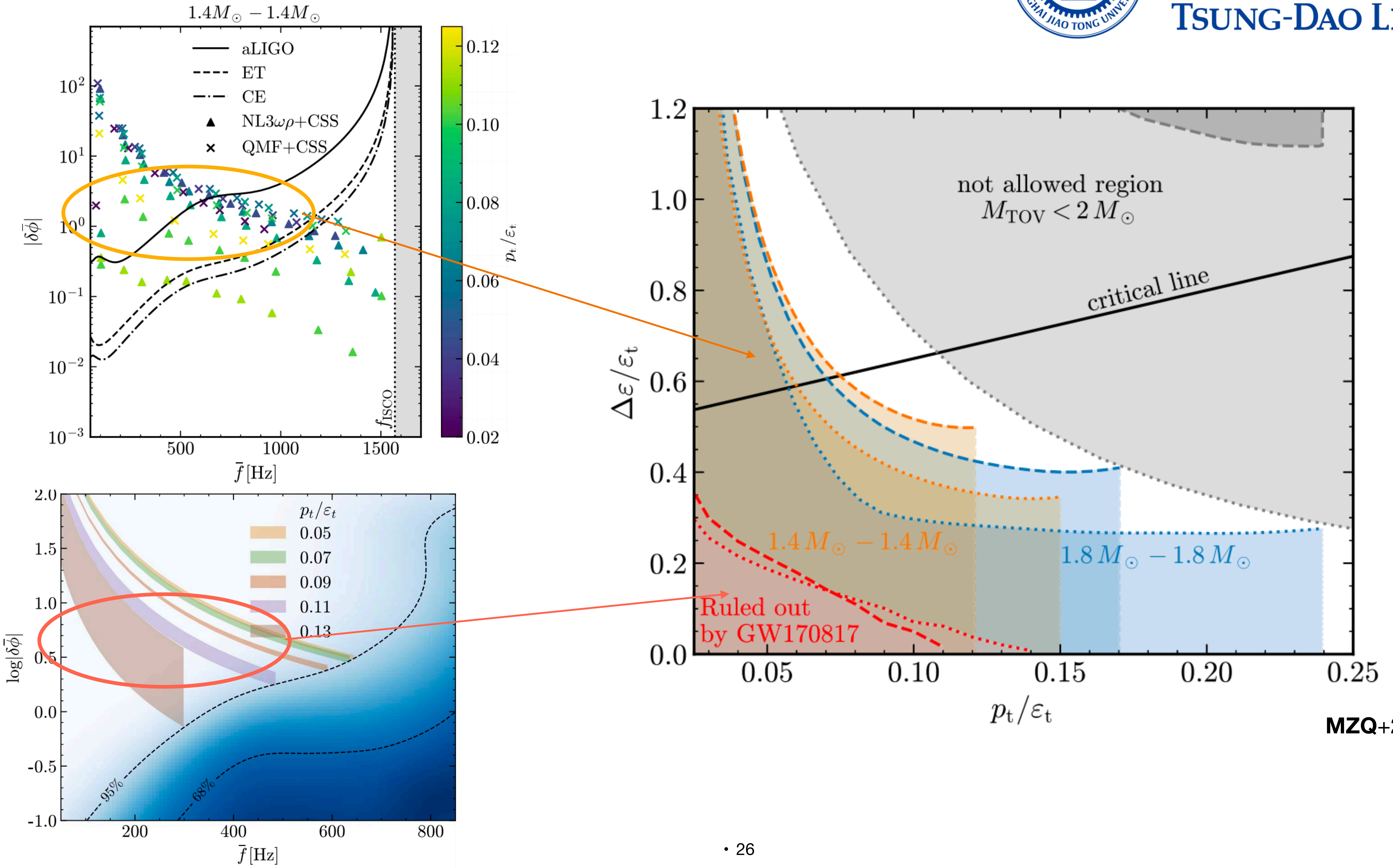
❖Constraints on EOS



MZQ+2024

Summary

✓Neutron star seismology, especially the tidal seismology, could serve as a probe for the phase state of high density NS cores.

✓A case study of the g-mode resonance in GW170817 data has ruled out the possibility of a weak phase transition taking place at low density.

Thank you !