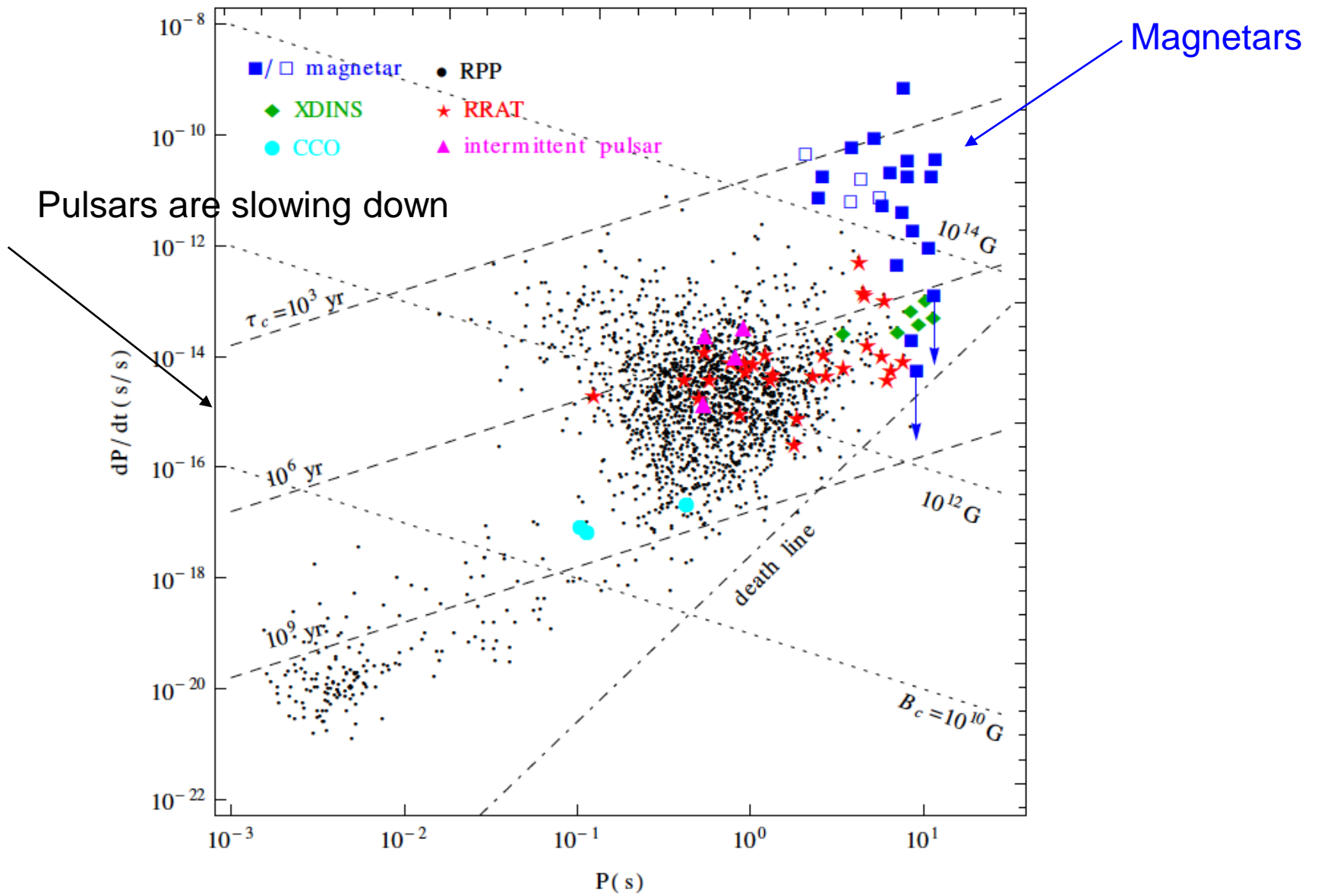


# Large polar caps for twisted magnetosphere of magnetars

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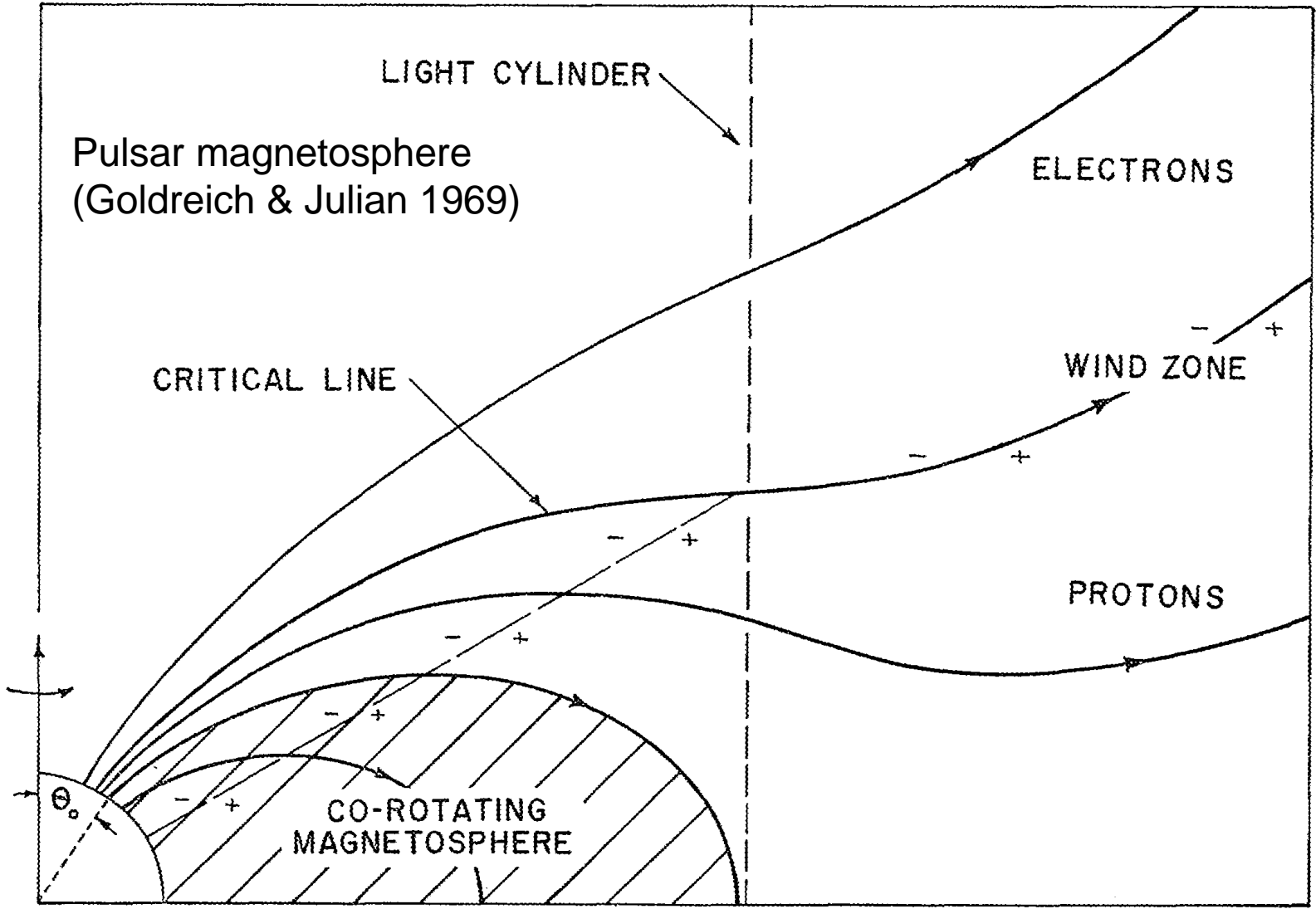
2019



Tong & Wang (2014)

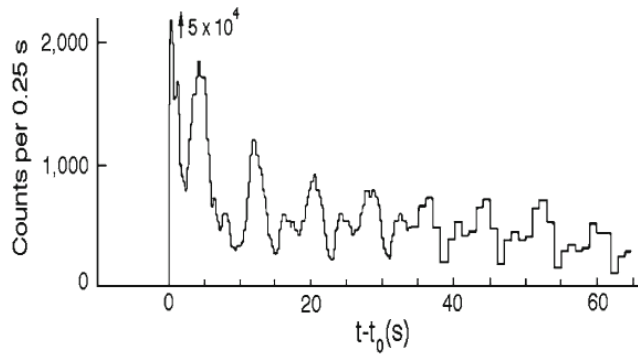
# Generic picture

1. Pulsars have a magnetosphere, where there is particle acceleration and subsequent radiation process--> pulse profile
  2. When flowing out, this particle component will also take away the rotational energy of the pulsar --> spin down
- Dipole radiation+particle component: wind braking model (Xu & Qiao 2001; Kou & Tong 2015)

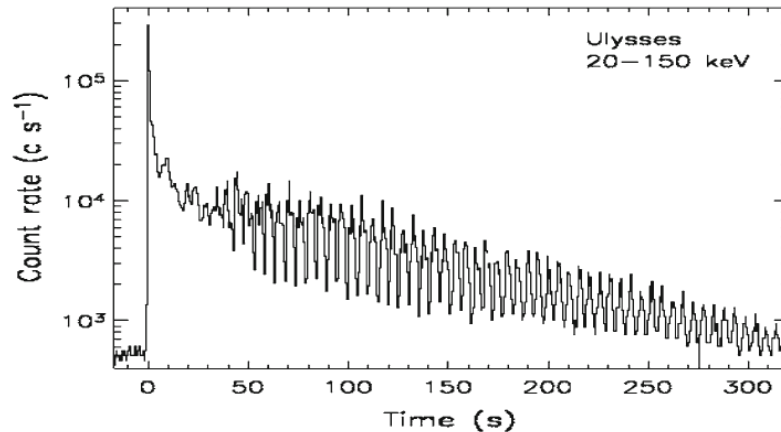


# Traditional magnetar model (Mereghetti 2008)

- Magnetar =
  1. young NS (SNR & MSC)
  2.  $B_{\text{dip}} > B_{\text{QED}} = 4.4 \times 10^{13} \text{ G}$  (**braking**)
  3.  $B_{\text{mul}} = 10^{14} - 10^{15} \text{ G}$  (burst and super-Eddington luminosity and persistent emission)

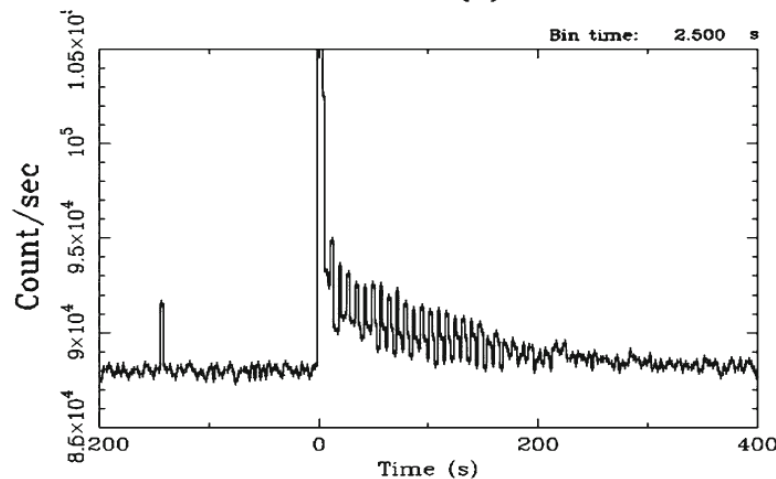


Giant flares of magnetars  
(Mereghetti 2008):  
1. Spike+pulsating tail  
(hundreds of seconds)



2.  $10^4$  times super-Eddington  
during the tail  
( $10^{42}$  erg  $s^{-1}$ )

Explanation:  $10^{15}$  G magnetic  
field as the energy power and  
cause of super-Eddington  
luminosity

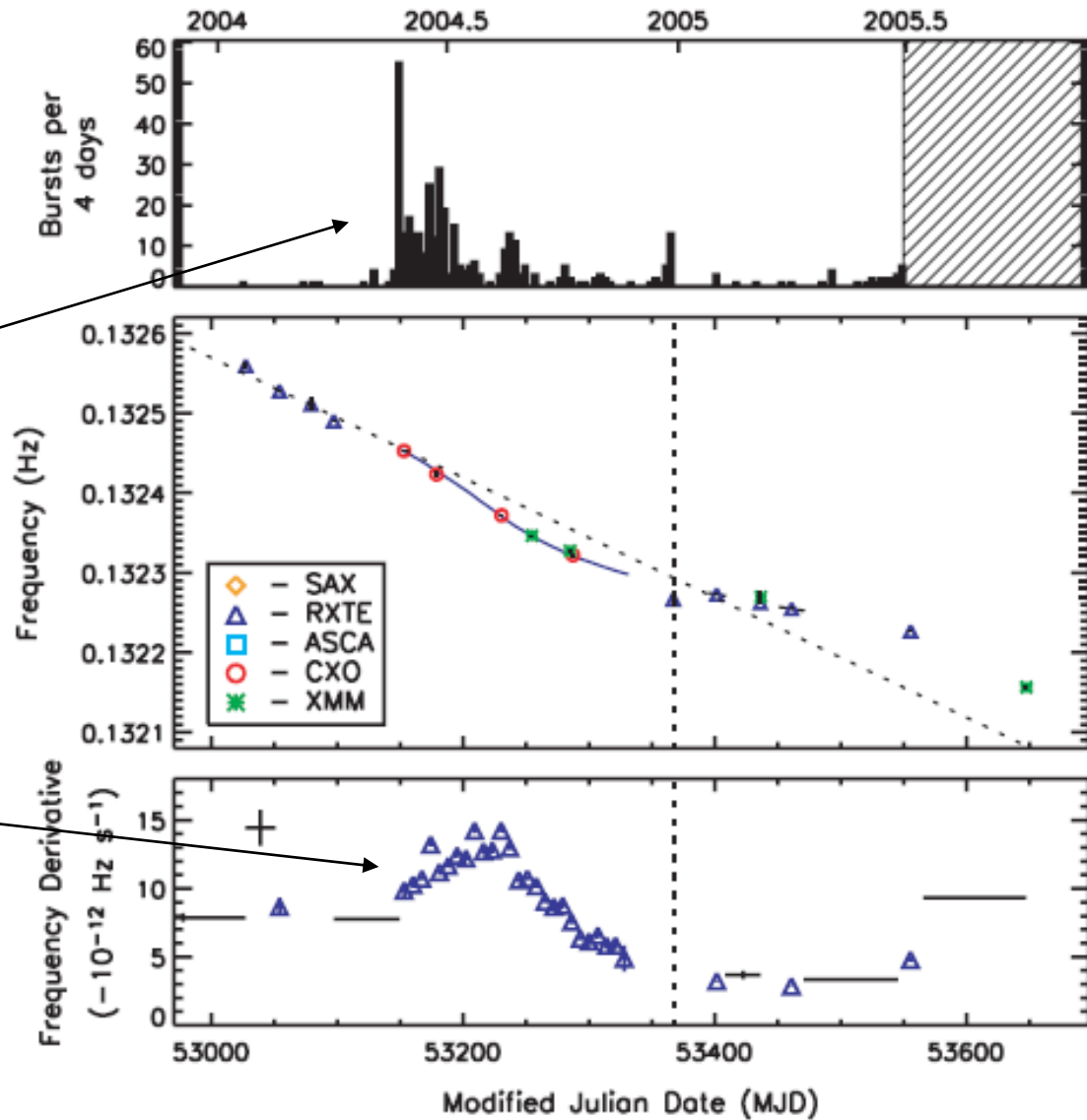


# Restless magnetars:

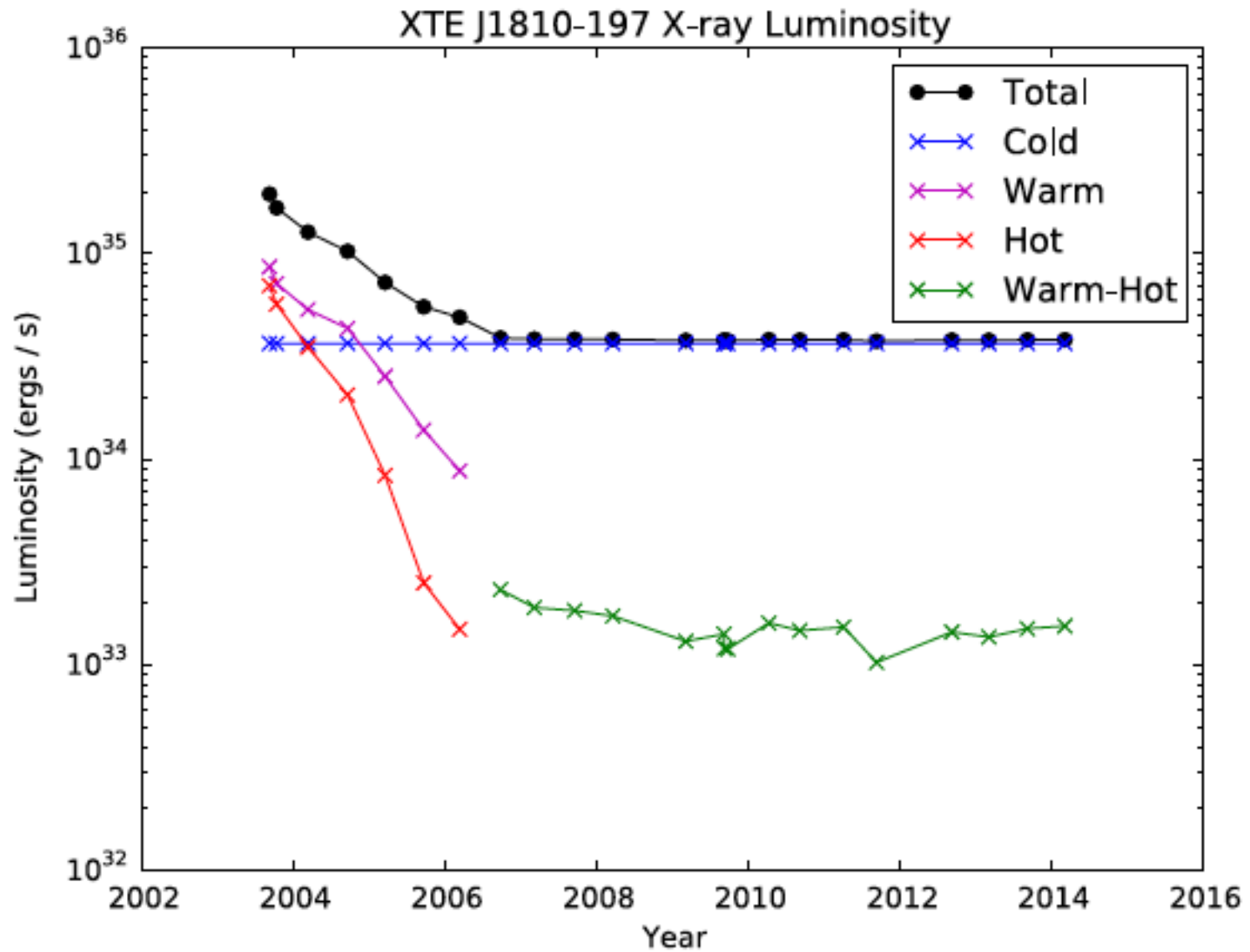
Woods+ 2007

bursts

A period of enhanced spindown

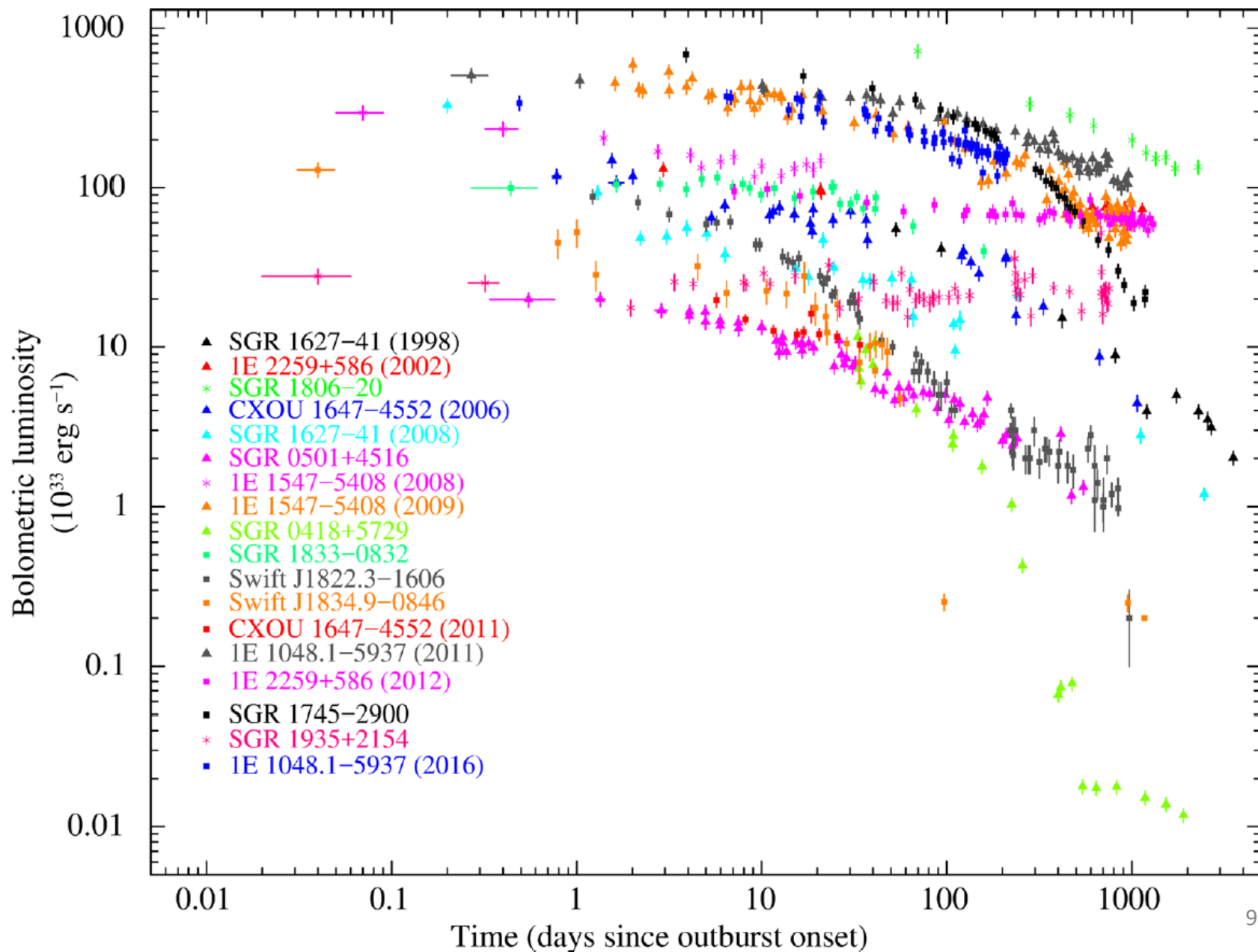


outburst decay over 10 years  
Alford & Halpern 2016

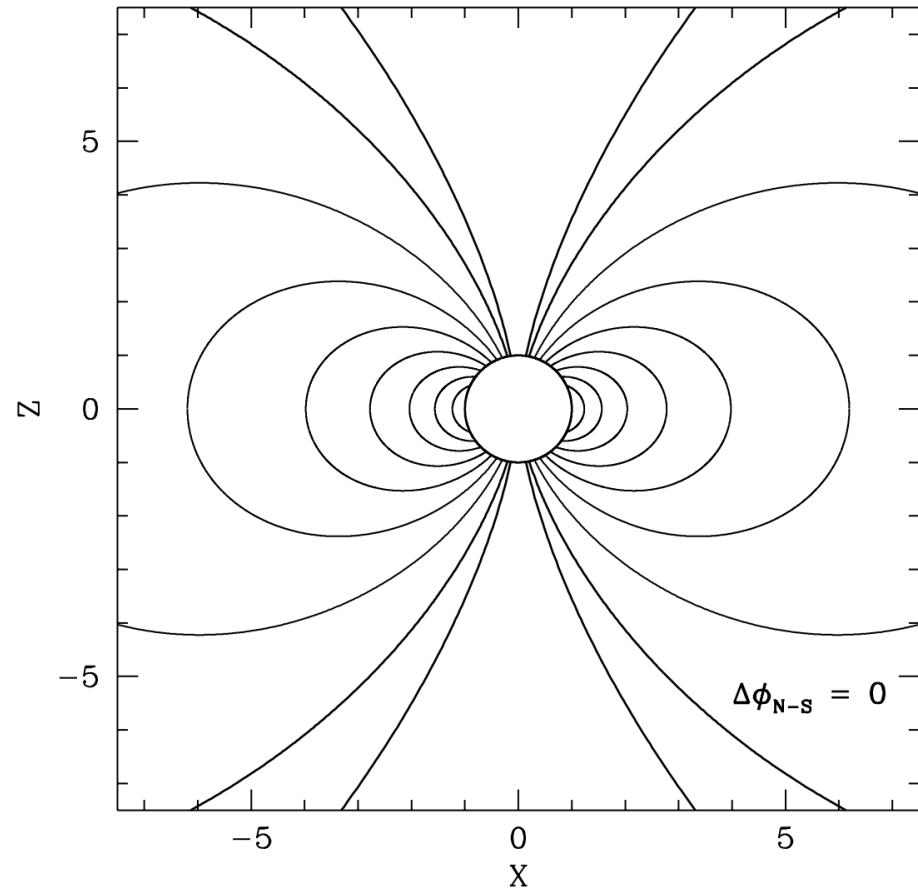
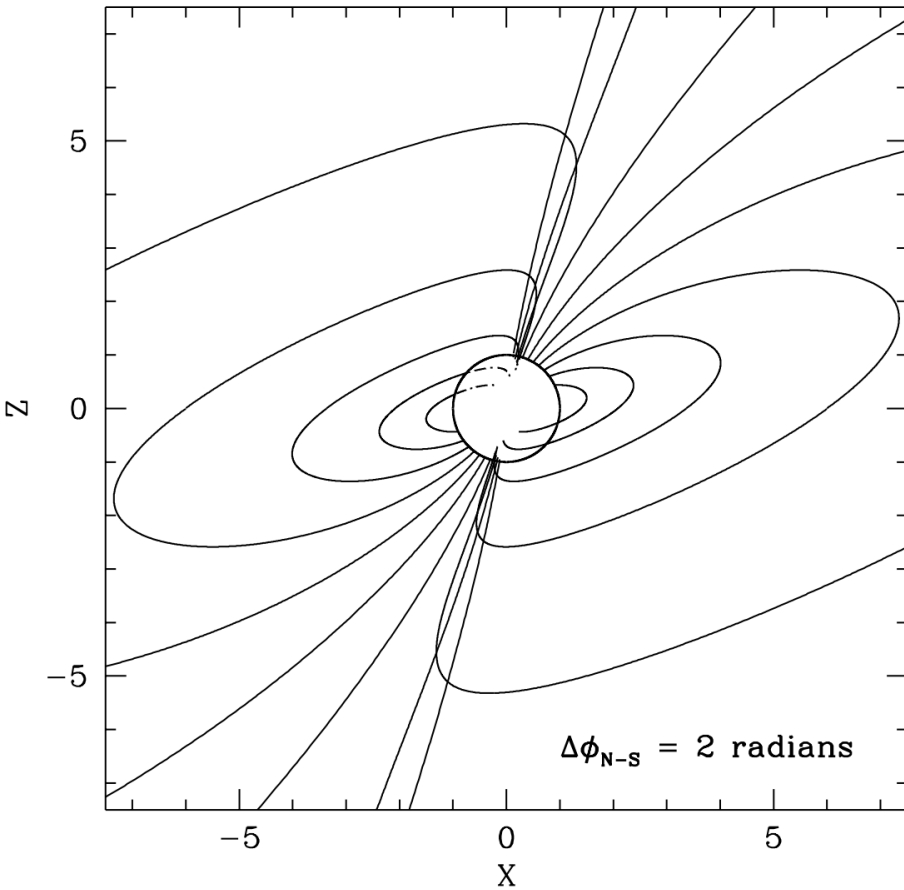




# Other magnetar outburst light curve: Coti Zelati et al. (2018)



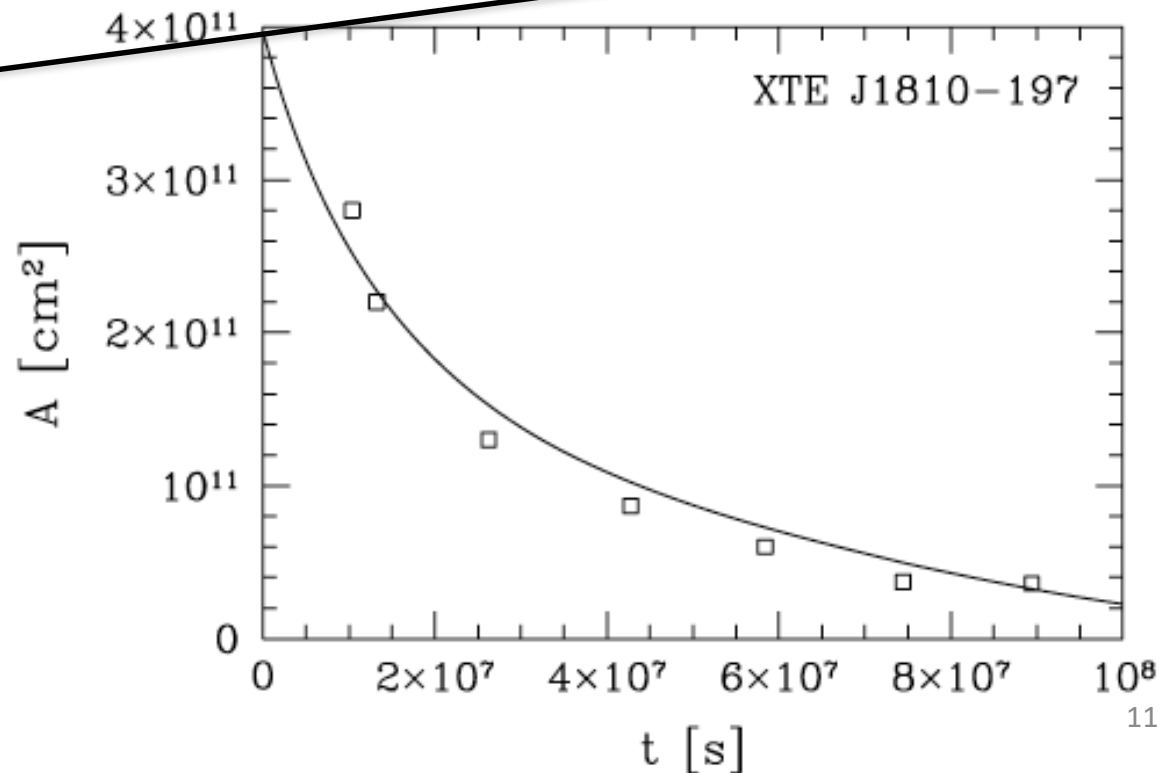
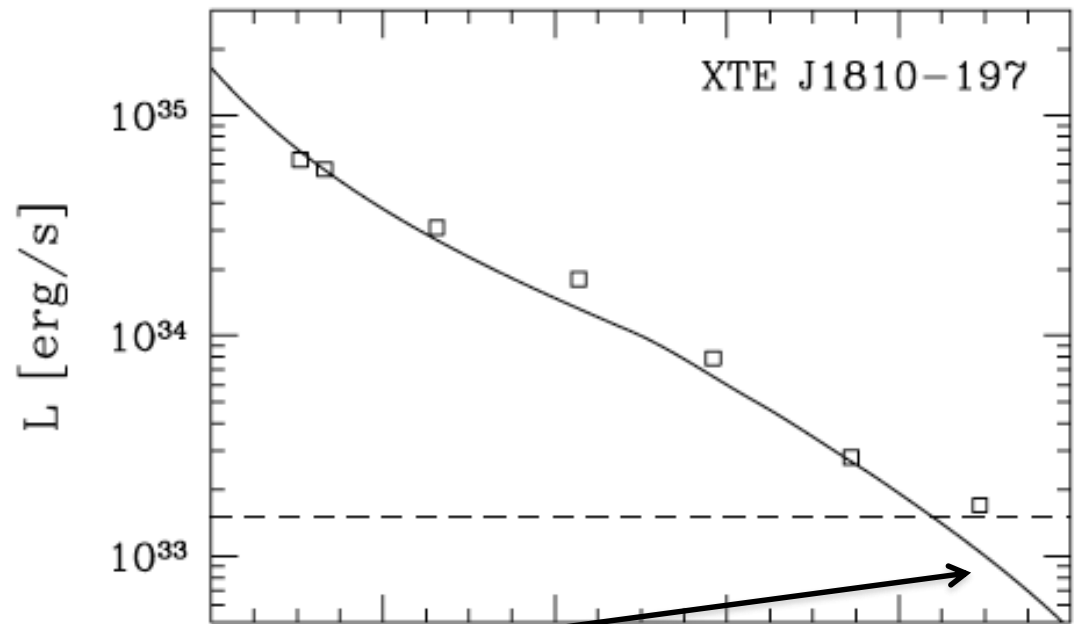
Magnetars may have twisted magnetosphere compared with that of normal pulsars (Thompson et al. 2002)



untwisting magnetosphere of magnetars and application to XTE J1810-197  
Beloborodov 2009

two shortcomings:

1. the central neutron star is non-rotating
2. it decays quickly



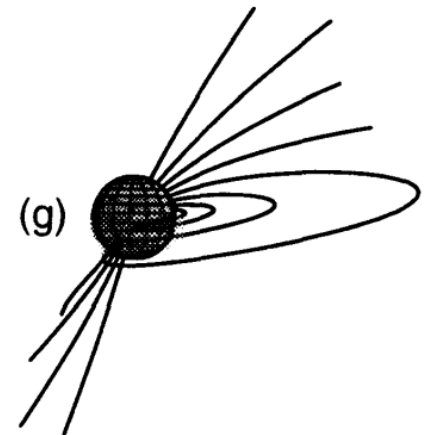
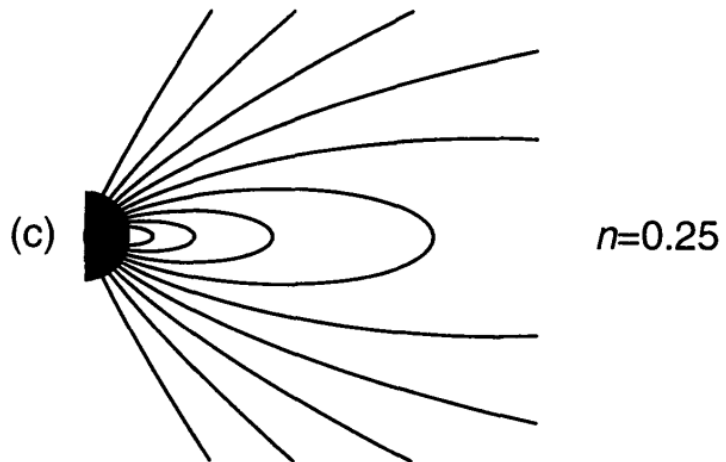
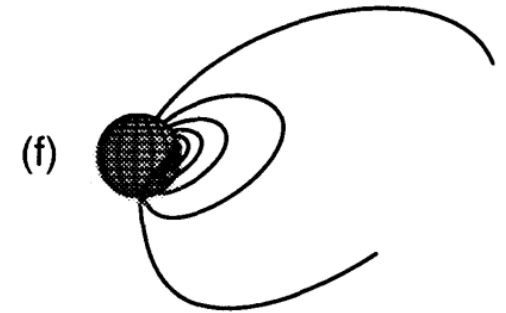
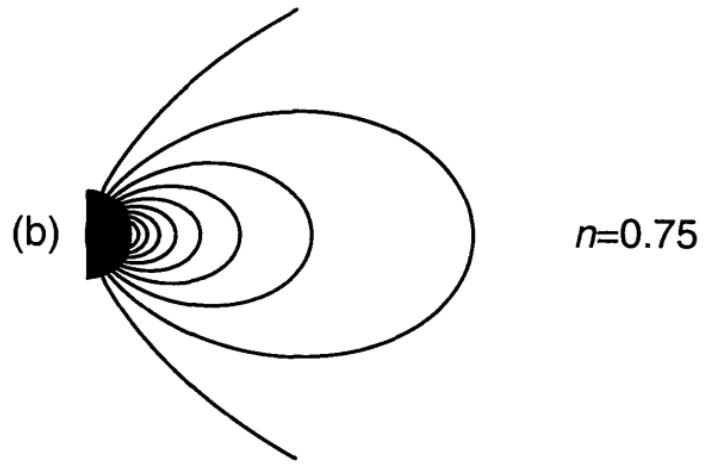
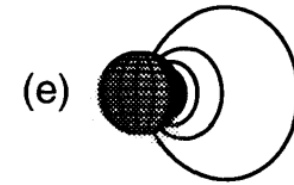
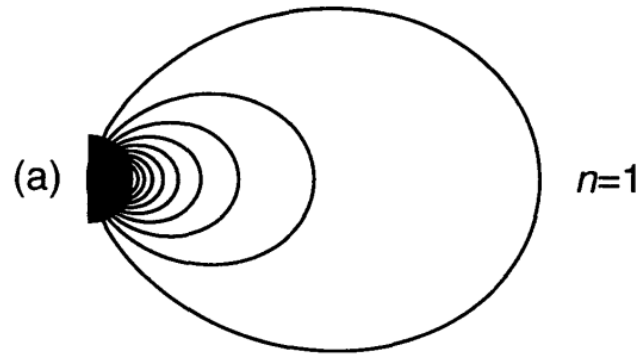
# More information

- Lynden-Bell & Boily (1994), Wolfson (1995): self-similar solution
- Thompson et al. (2002), Pavan et al. (2009), Beloborodov (2009): twisted magnetosphere of magnetars (global or local)
- Glampedakis et al. (2014), Gourgouliatos (2008), Fujisawa & Kisaka (2014), Akgun et al. (2016), Pili et al. (2015), Kojima 2017 ....: magnetars with localized twist in the magnetosphere

# One wrong concept

- Magnetars have long period ( $P \sim 10\text{s}$ ), then their polar caps is very small ( $\sim 100\text{m}$ )
- Therefore, the open field line regions of magnetar magnetosphere are not considered. Magnetars have no open field lines, no polar caps in these models. **Magnetars (closed field line matters) and pulsars (open field line matters) are totally different and can not be unified.**
- This is wrong: magnetar may have twisted magnetic field. For a twisted magnetic field, magnetar may have large polar caps despite of their long period.

Wolfson 1995



Twist example: 麻花, 小蛮腰



图片来源：百度





油条： fried twisted dough sticks  
→ Small twist case





The magnetosphere of pulsars and magnetars may be in the force-free equilibrium state. Assuming axisymmetric condition, the magnetosphere is described by the Grad-Shafranov equation. In spherical coordinate, the Grad-Shafranov equation is:

$$\frac{\partial^2 A}{\partial r^2} + \frac{1 - x^2}{r^2} \frac{\partial^2 A}{\partial x^2} + F(A) \frac{dF}{dA} = 0, \quad (1)$$

where  $r$  is the dimensionless radial coordinate (in units of the neutron star radius),  $x = \cos \theta$  ( $\theta$  is the polar angle),  $A = A(r, \theta)$  is the flux function, and  $F(A)$  is a yet undetermined function (Wolfson 1995 and references therein). The

For self-similar solutions, the dimensionless flux function governed by the following ODE:

$$(1 - x^2)f''(x) + n(n + 1)f(x) + \lambda^2 \left(1 + \frac{1}{n}\right) f^{1+2/n}(x) = 0. \quad (4)$$

When  $\lambda = 0$  and  $n = 1$ , this corresponds to the magnetic dipole field. When  $\lambda$  is different from zero, this means the presence of toroidal field, and the magnetic field is twisted compared with the dipole case. The polar axis should be a

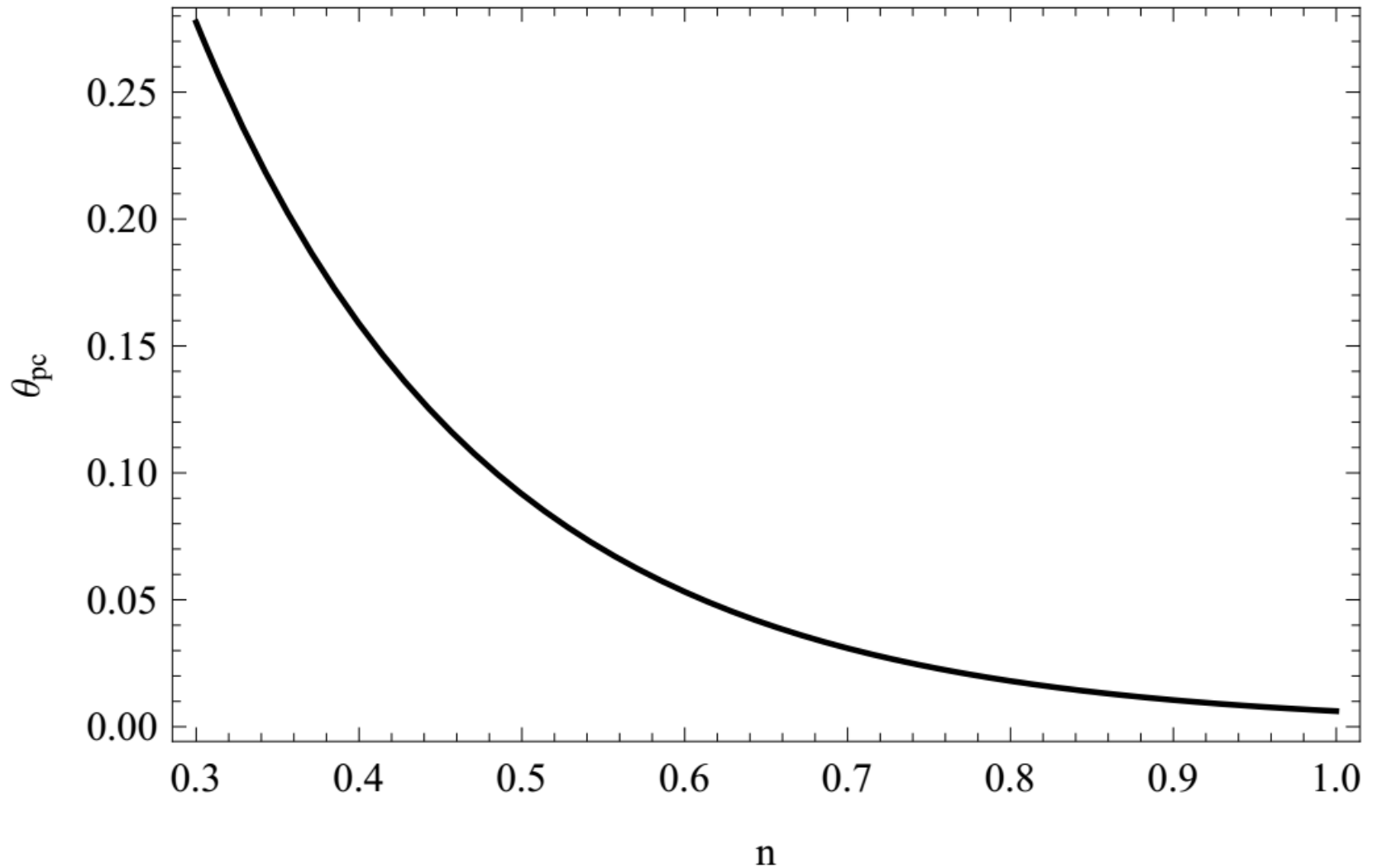
of the flux function. From equation (2), the radial dependence of the magnetic field will be  $B(r) \propto r^{-(2+n)}$ . In general, the radial

Higher flux fraction for a twisted polar cap:

$$f(x_{\text{pc}}) = \left( \frac{R}{R_{\text{lc}}} \right)^n .$$

**Large polar caps for twisted magnetosphere of magnetars:**

$$\sin \theta_{\text{pc}} = \sqrt{\frac{(R/R_{\text{lc}})^n}{(15 + 17n)/32}} .$$



**Figure 4.** Polar cap angular radius of a magnetar with twisted dipole field.

# More calculations

- Magnetic free energy
- Particle outflow in the open field line regions of magnetars
- Untwisting of magnetar magnetosphere (due to magnetic energy release in the open field line regions)
- Decreasing X-ray luminosity during outburst, shrinking hot spot and temperature etc
- Geometry and spin-down torque: same geometry and enhanced spin-down
- Statistical properties of magnetars:

these two cases. Therefore, a correlation between the X-ray luminosity and hot spot area will be:  $L_x \propto A^\alpha$ , where the power law coefficient  $1 < \alpha < 2$  is expected. Long term flux

A general correlation between magnetar quiescent luminosity and dipole magnetic field is:  $L_{x,q} \propto B_p^2$ . For the case of a constant acceleration potential, the correlation will be:  $L_{x,q} \propto B_p$ . Therefore, the power law index between one and two is expected for  $L_{x,q} \propto B_p^\beta$ , where  $1 < \beta < 2$ . Obser-

Enhanced particle outflow due to a large polar cap:

$$\dot{E}_{p,\text{twist}} = \frac{\Omega^2 R^4 B_p^2}{2c} \sin^4 \theta_{pc}, \quad (19)$$

viewed as the strong particle flow case. When the twisted dipole field is relaxed back to the pure dipole case, the polar cap will return back to the dipole case. The corresponding particle luminosity (equation(19)) will also return back the dipole case. In this way, the magnetosphere of magnetars and normal pulsars may be unified together. This is merit of the above treatment.

According to energy conservation, the untwisting of the twisted dipole field is governed by the following equation

$$\frac{dE_{\text{mf}}}{dt} = -\dot{E}_{\text{p,twist}}. \quad (20)$$

It should be noted that the magnetic free energy is expressed in units of the magnetic energy of a pure dipole field.

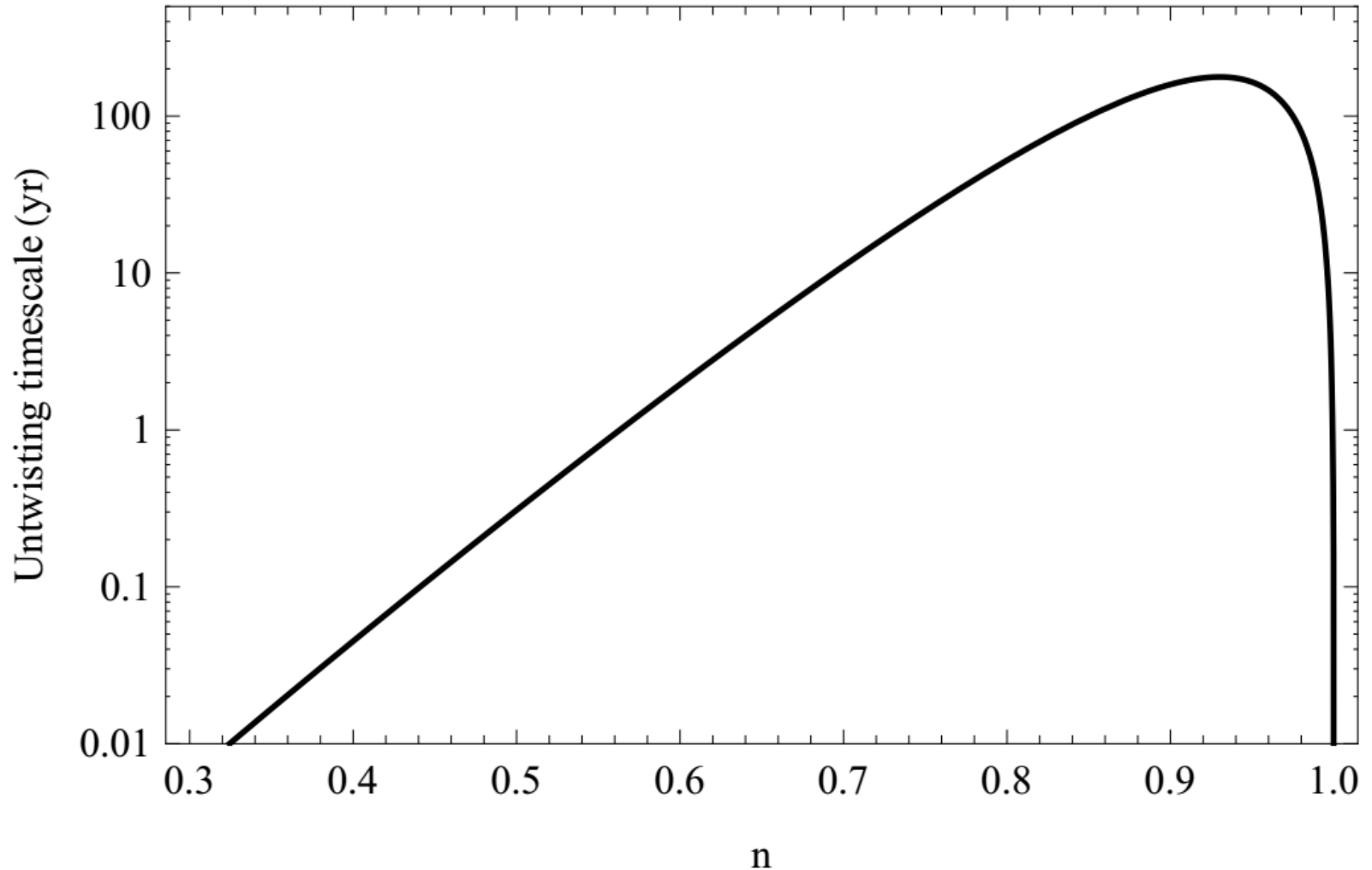
The particle luminosity should be of magnetic origin (instead of rotational origin, Lyutikov 2013). This is one basic assumption of our model. Justification of equation (20) from the energy conservation point of view is presented in the appendix. Different modeling of the particle luminosity will only result in quantitative differences.



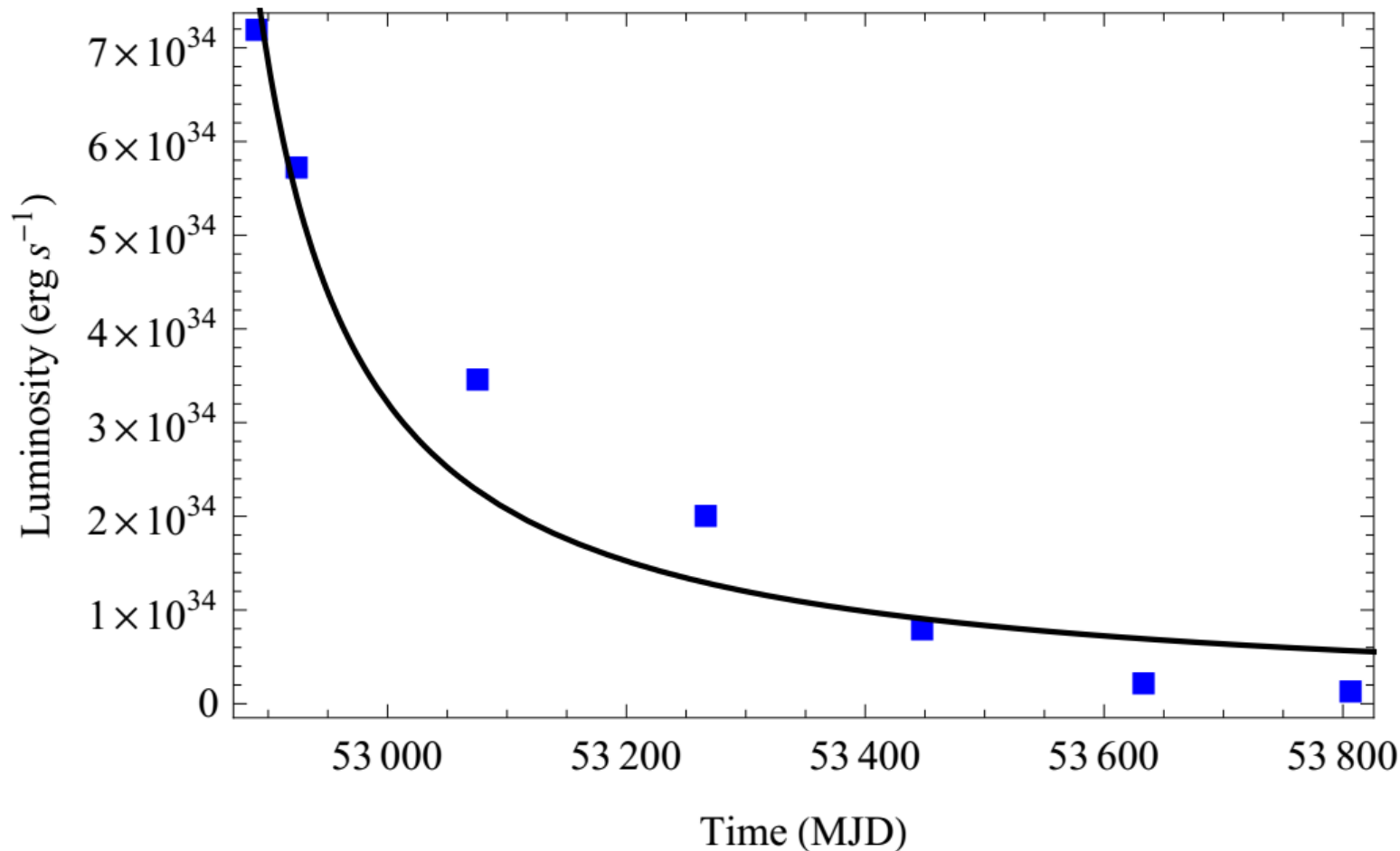
the open field lines regions. For a typical parameter  $n = 0.5$ , the total magnetic free energy is about  $E_{\text{mf}} \approx 10^{44} B_{14}^2$  erg, the particle luminosity is about  $\dot{E}_{\text{p,twist}} \approx 10^{37} B_{14}^2$  erg s<sup>-1</sup>. From equation (20), the magnetic energy decays with a typical timescale (i.e. untwisting timescale)

$$\tau(n) \equiv E_{\text{mf}} / \dot{E}_{\text{p,twist}} \sim 0.3 \text{ yr (for } n=0.5\text{)}. \quad (21)$$

Difference between magnetosphere and high-B pulsar?

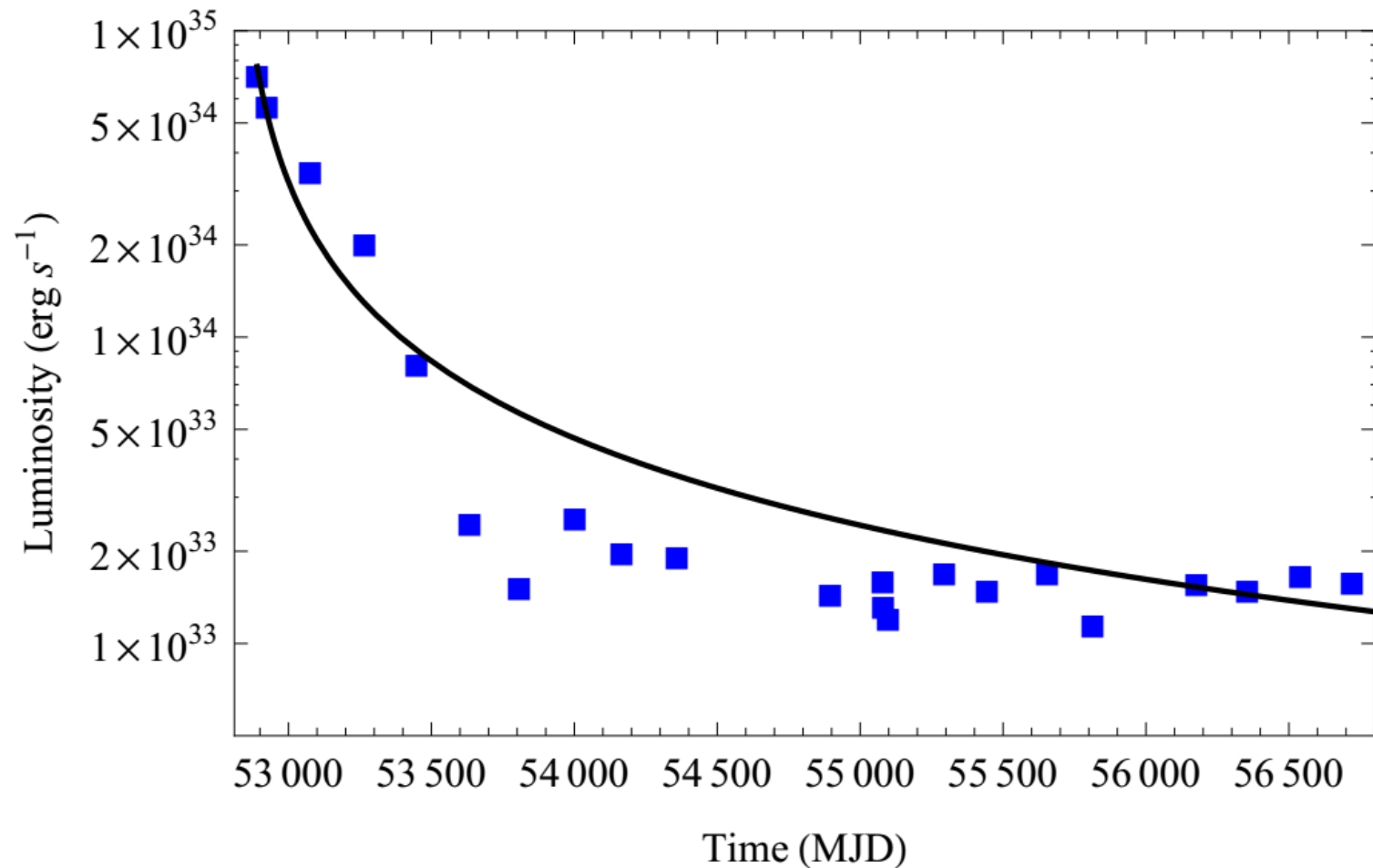


**Figure 6.** Magnetic field untwisting timescale as a function of  $n$ . The untwisting timescale peaks at about  $n = 0.93$ .



**Figure 7.** Luminosity of the hot spot component of XTE J1810–197. The blue squares are observations (Alford & Halpern 2016), the solid line is the model calculation.

## Long term magnetic energy release?



**Figure 8.** Long term luminosity decay of XTE J1810–197. Similar to Figure 7, the blue squares are observations (Alford & Halpern 2016), the solid line is the model calculation.

# Conclusions

- Magnetars may have large polar caps, in spite of their long period
- Due to the radial inflation of the twisted magnetic field
- May result in the untwisting of the magnetic field
- May explain the general behavior of magnetars during outbursts (e.g., long term magnetic energy release)

# Discussions

- Both global twisted magnetic field (our model) and local twist may occur
- Solving the “pulsar equation” with a twisted dipole boundary condition?
- Physical modeling of the particle density and acceleration potential?
- ...