Variational equation of state for spin-polarized nuclear matter

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Outline

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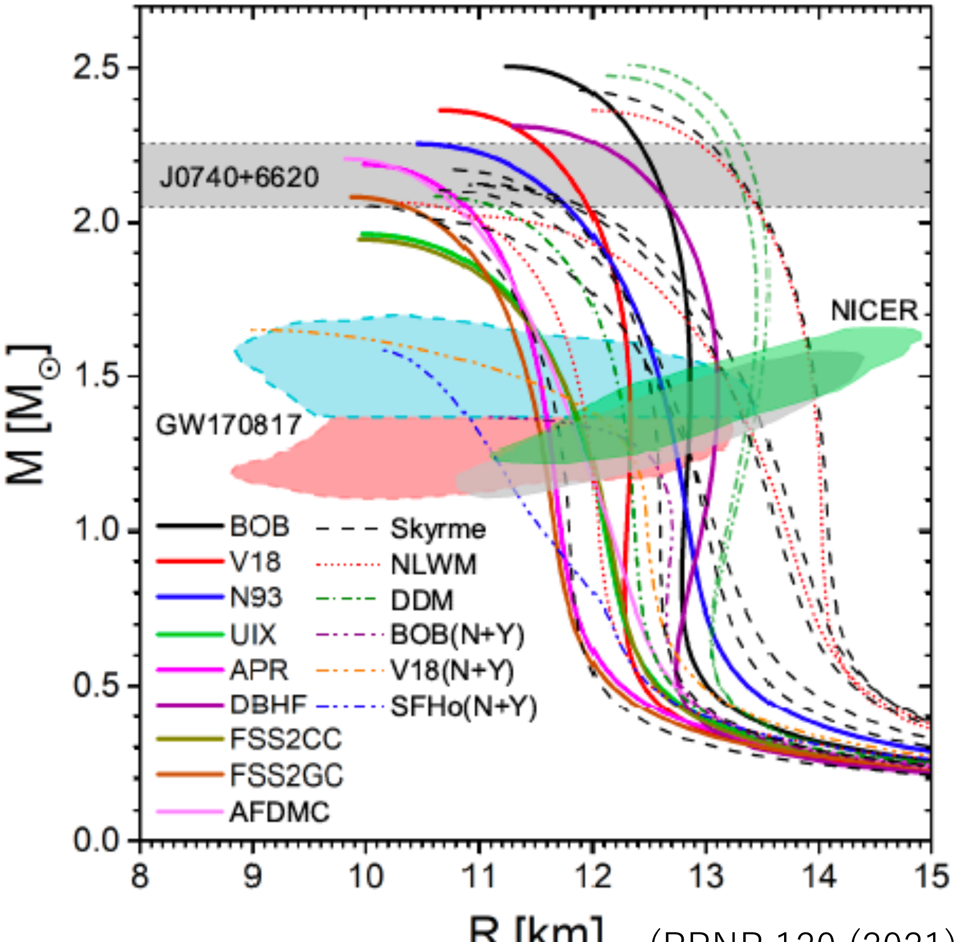
1. Introduction

Neutron Stars (NS)

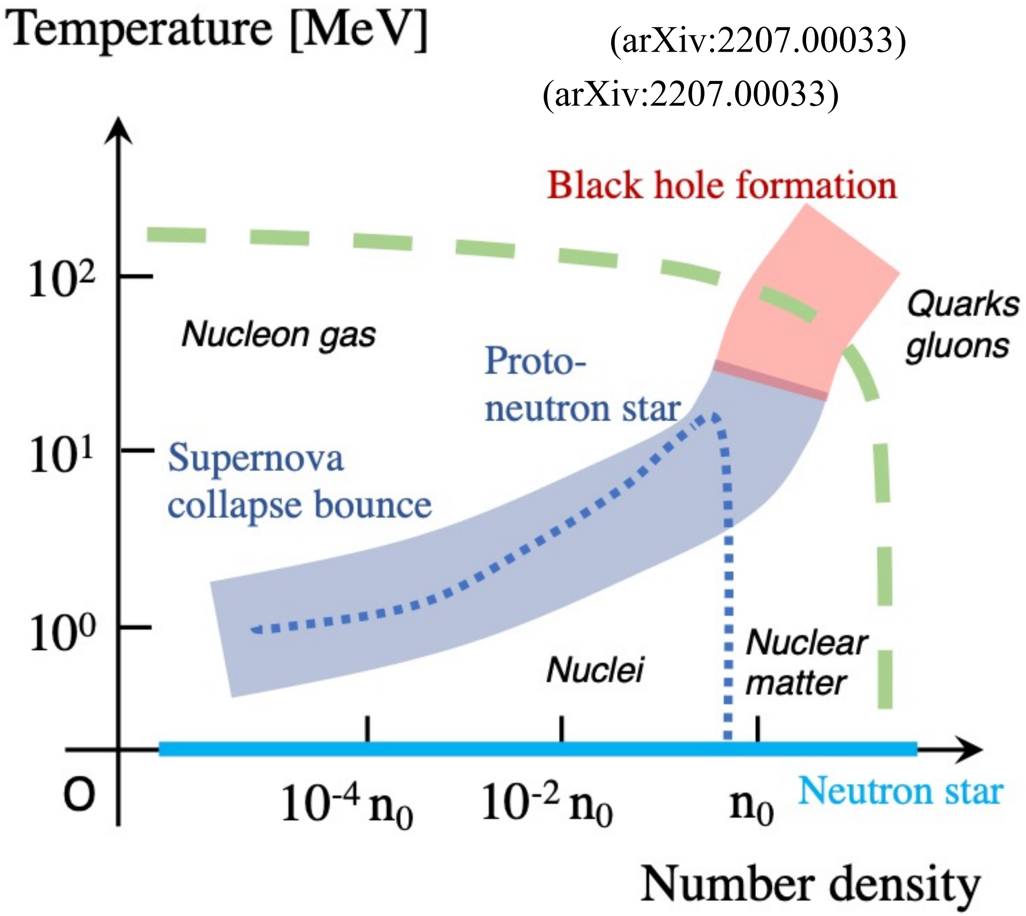
- $T = 0$ MeV, $Y_p \sim 0.1$
- Various EOS has been proposed.



(PPNP 120 (2021) 103879)

Supernovae / NS mergers

- Wide range of T, Y_p, n_B
- Limited number of EOSs are applicable.



(arXiv:2207.00033)
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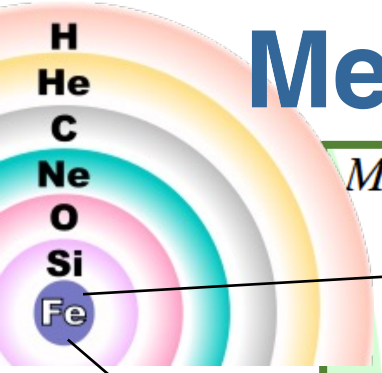
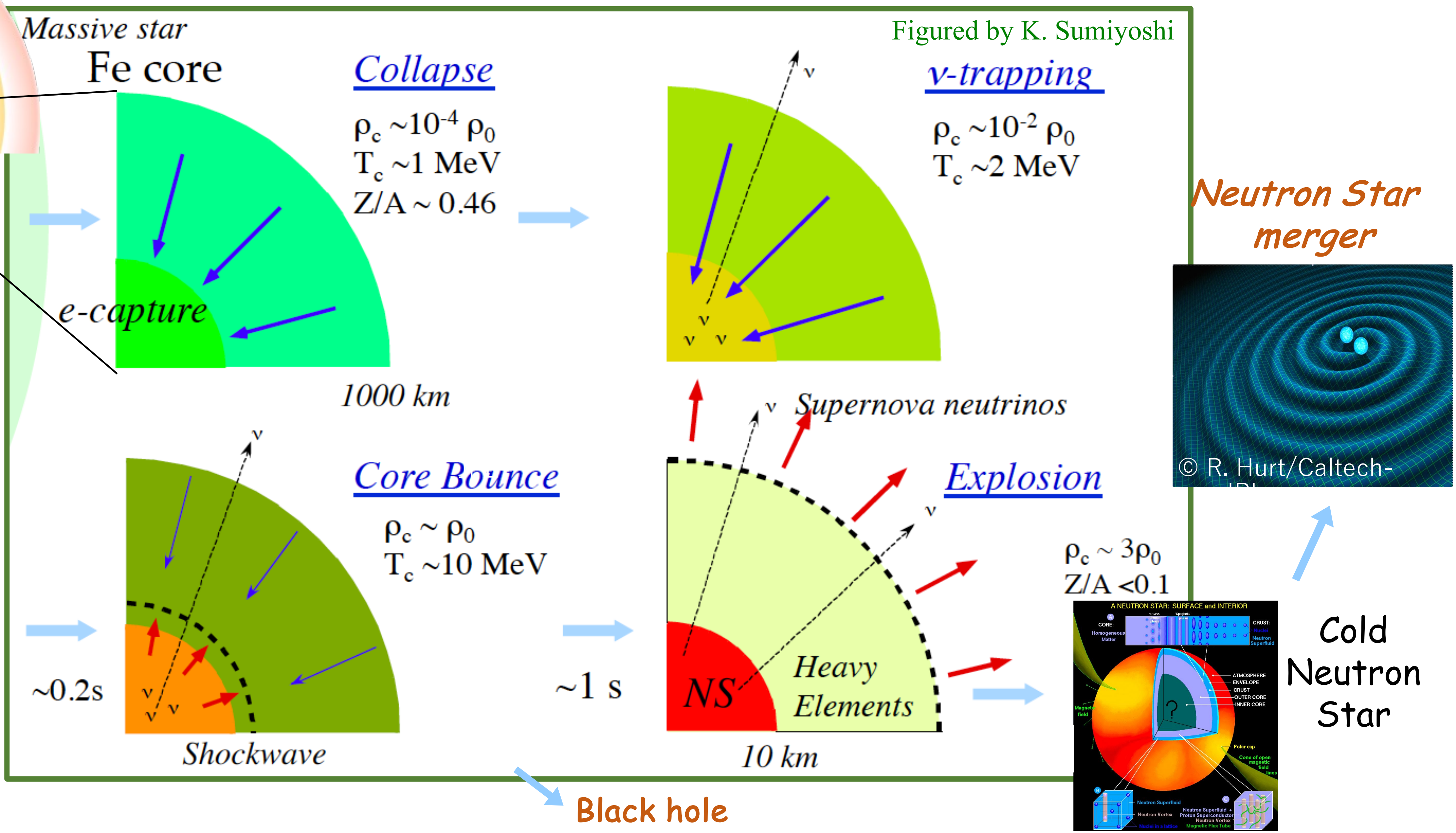
Mechanism of core-collapse supernovae





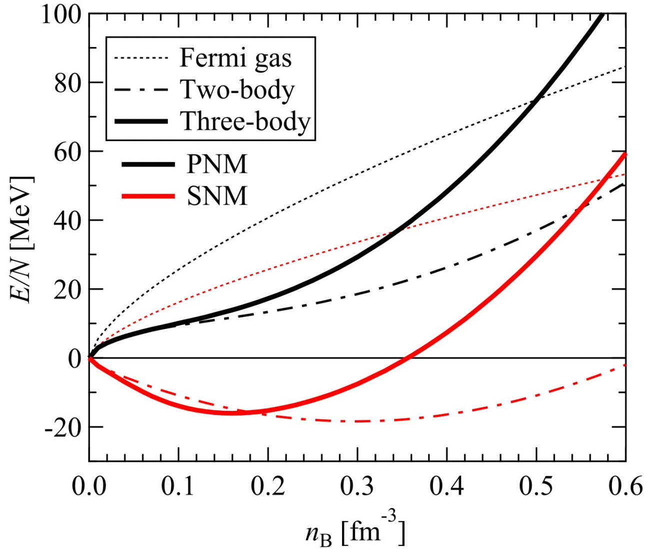
Variational EOS table for supernova simulations

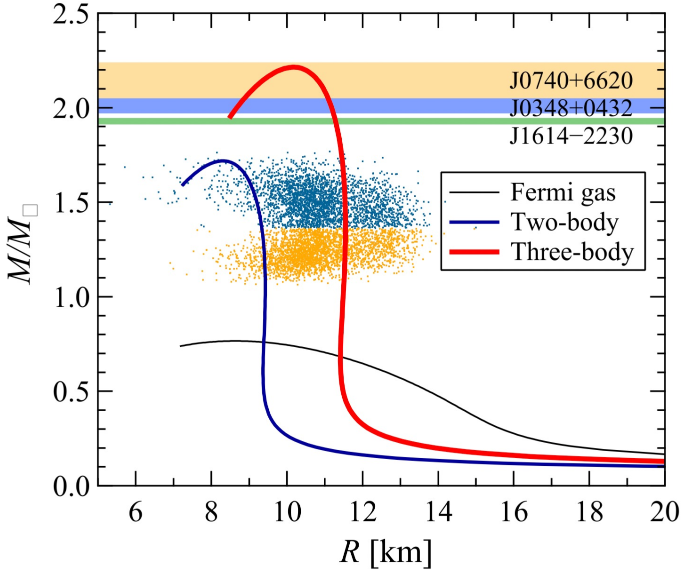
Supernova EOS with realistic nuclear forces

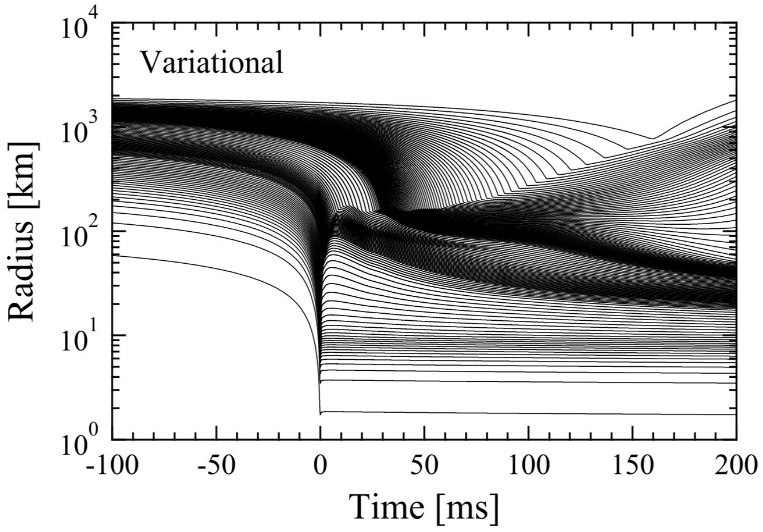
(**HT**, K. Nakazato, Y. Takehara, S. Yamamuro, H. Suzuki, and M. Takano, NPA961 (2017) 78)

http://www.np.phys.waseda.ac.jp/EOS/

- This is **the ONLY microscopic nuclear EOS** for astrophysical simulations based on realistic nuclear forces (AV18 + UIX).

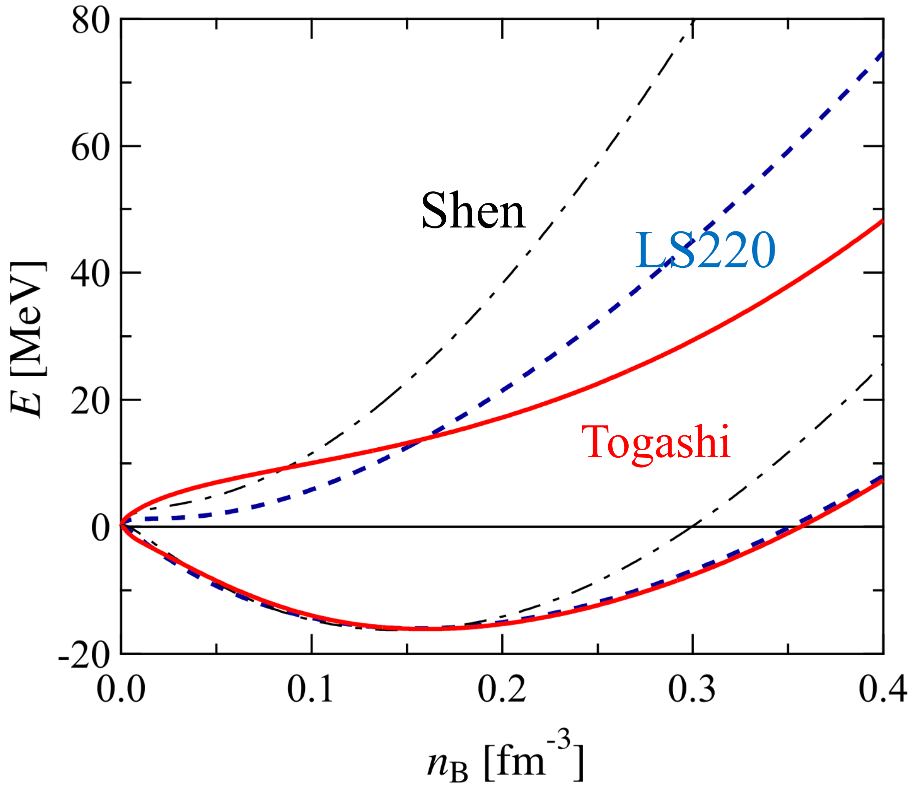


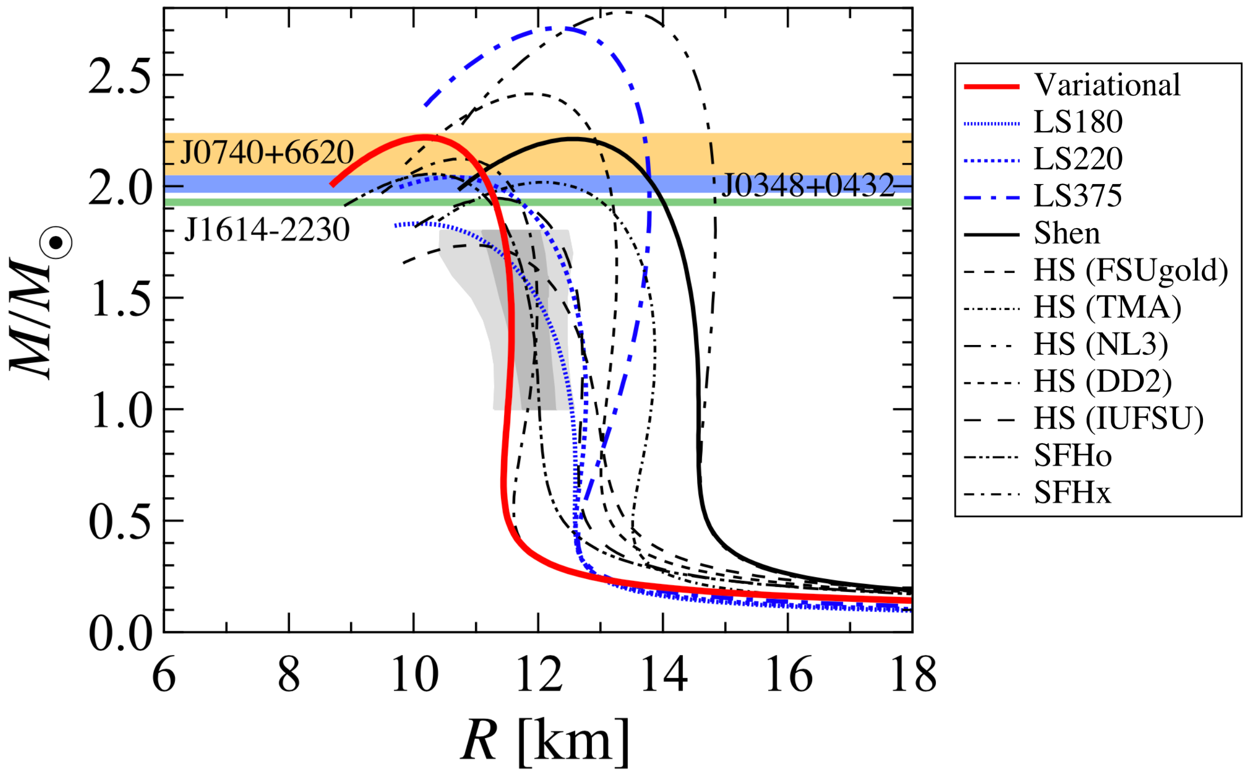




Extend to the EOS for spin-polarized nuclear matter to calculate the Neutrino reaction rates in a self-consistent manner

2. EOS effects on core-collapse supernovae

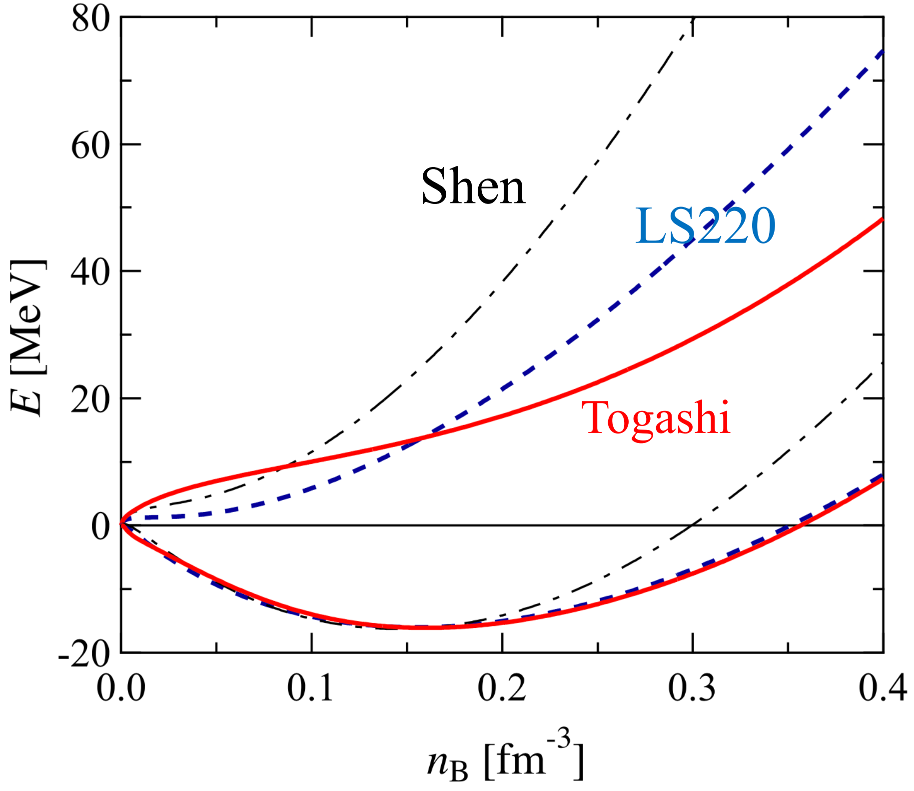


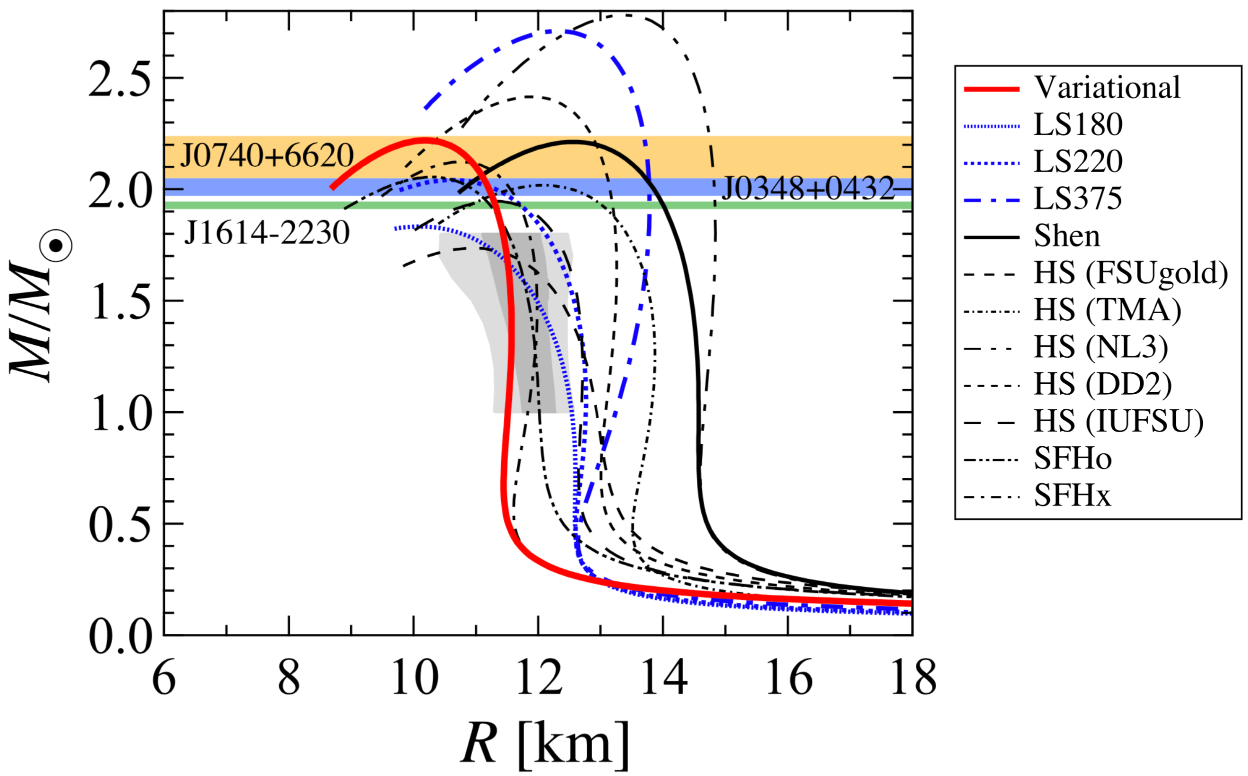


【Uniform phase】 【Non-uniform phase】

- **LS EOS**: Skyrme Hartree-Fock + Compressible Liquid Drop model
- **Shen EOS**: Relativistic Mean Field (TM1) + Thomas-Fermi calculation
- **Togashi EOS:** Variational method (AV18+UIX) + Thomas-Fermi calculation

2. EOS effects on core-collapse supernovae





EOS	n_0 [fm^{-3}]	E_0 [MeV]	K_0 [MeV]	S_0 [MeV]	L_0 [MeV]	$R_{1.4}$ [km]	M_{max} [$M_\odot$]
Togashi	0.160	16.1	245	29.1	38.7	11.6	2.21
Shen	0.145	16.3	281	36.9	110.8	14.5	2.23
LS220	0.155	16.0	220	28.6	73.8	12.7	2.06
Empirical	0.15 – 0.17	15.8 – 16.2	220 – 260	28 – 35	35 – 100	11 – 13	> 2.0

Application to core-collapse simulations

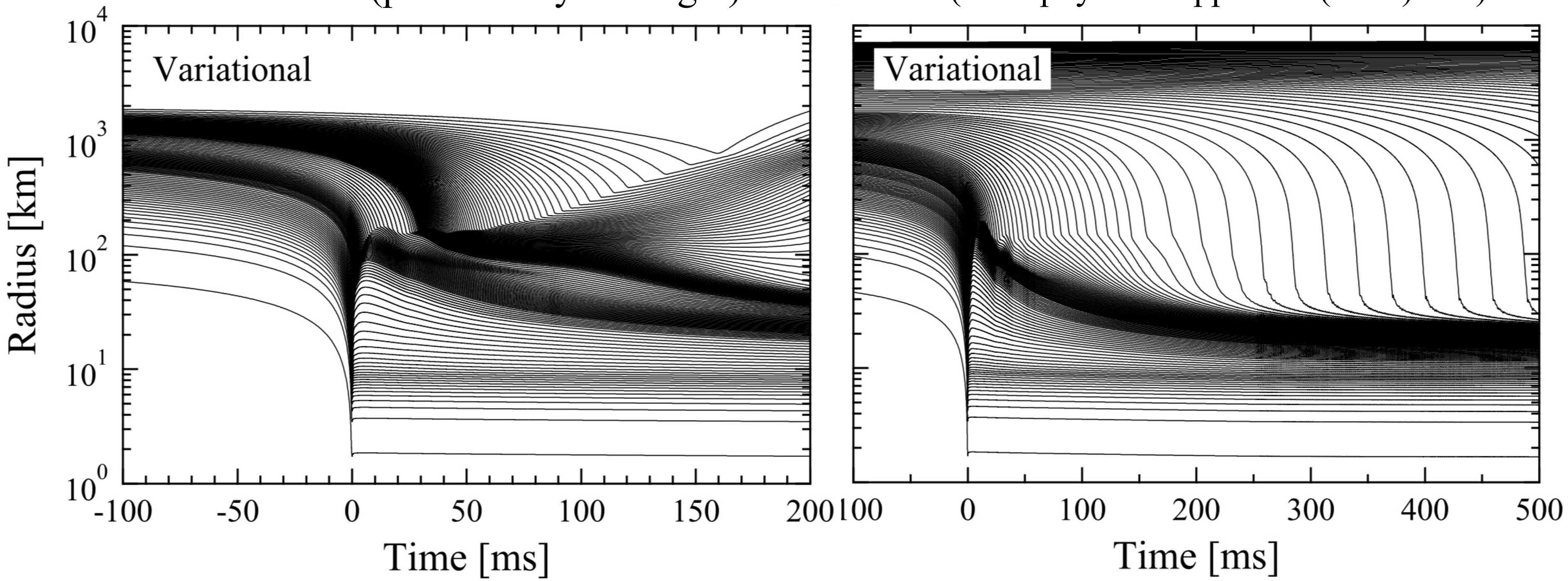
1D neutrino-radiation hydrodynamics simulations

(Nakazato, Sumiyoshi & HT, PASJ 73 (2021) 639)

- **EOS: Togashi / Shen / LS220 / LS180**
- Progenitor model : 9.6 $M_\odot$ / 15 $M_\odot$ / 30 $M_\odot$
- Neutrino Transport: Directly solve the Boltzmann equation

Progenitor model : 9.6 $M_\odot$ (provided by A. Heger)

Progenitor model: WW 15$M_\odot$ (Astrophys. J. Suppl. 101 (1995) 181)

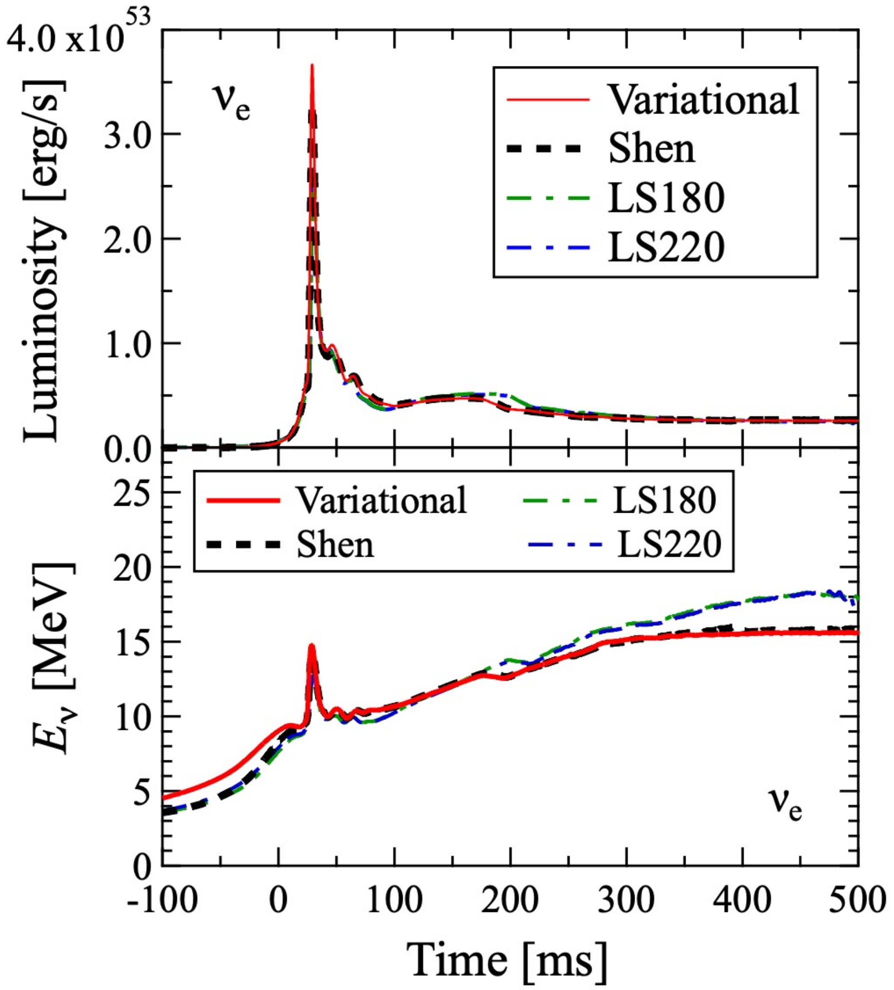


Radial trajectories of mass elements

Neutrino luminosity and average energy

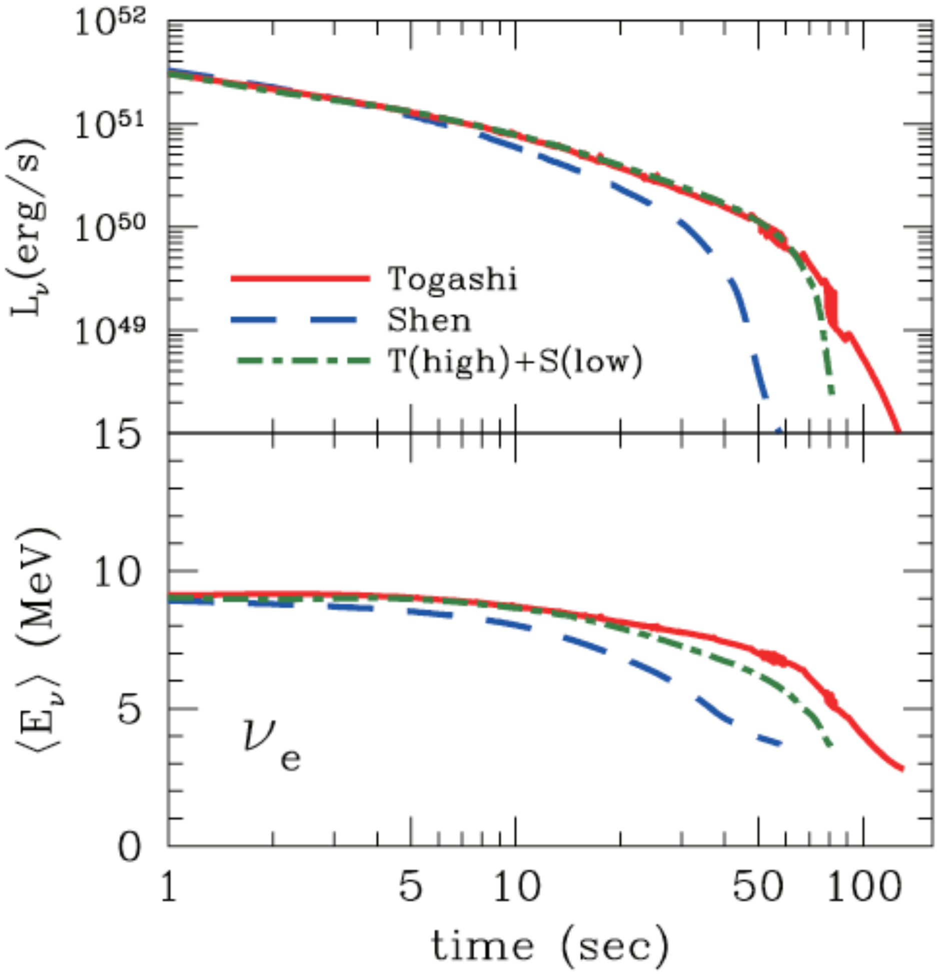
Progenitor model: WW $15M_\odot$

【Core-bounce stage】



(Nakazato, Sumiyoshi & HT, PASJ 73 (2021) 639)

【Proto-NS cooling stage】



(Nakazato, Suzuki &HT, PRC 97 (2018) 035804)

Neutrino reactions in supernova simulation

Nuclear weak interactions are also important ingredient for core-collapse simulations.

1. electron-type neutrino absorption on neutrons and its inverse,

$$\nu_e + n \longleftrightarrow e^- + p,$$

2. electron-type antineutrino absorption on protons and its inverse,

$$\bar{\nu}_e + p \longleftrightarrow e^+ + n,$$

3. neutrino scattering on nucleons,

$$\nu + N \longleftrightarrow \nu + N,$$

4. neutrino scattering on electrons,

$$\nu + e^- \longleftrightarrow \nu + e^-,$$

5. electron-type neutrino absorption on nuclei,

$$\nu_e + A \longleftrightarrow A + e^-,$$

6. neutrino coherent scattering on nuclei,

$$\nu + A \longleftrightarrow \nu + A,$$

7. electron-positron pair annihilation and creation,

$$e^- + e^+ \longleftrightarrow \nu + \bar{\nu},$$

8. plasmon decay and creation,

$$\gamma^* \longleftrightarrow \nu + \bar{\nu},$$

9. neutrino bremsstrahlung,

$$N + N' \longleftrightarrow N + N' + \nu + \bar{\nu}.$$

We calculate the neutrino scattering reaction rates in a nuclear medium
by using the EOS of spin-polarized nuclear matter with the cluster variational method.

Medium effects on neutrino-nucleon scattering

Neutrino scattering on nucleons in nuclear medium

Free-space: $$\frac{1}{N}\frac{d\sigma_0}{d\Omega} = \frac{G_F^2 E_\nu^2}{4\pi^2}\left(C_a^2(3-\cos\theta) + C_v^2(1+\cos\theta)\right)$$

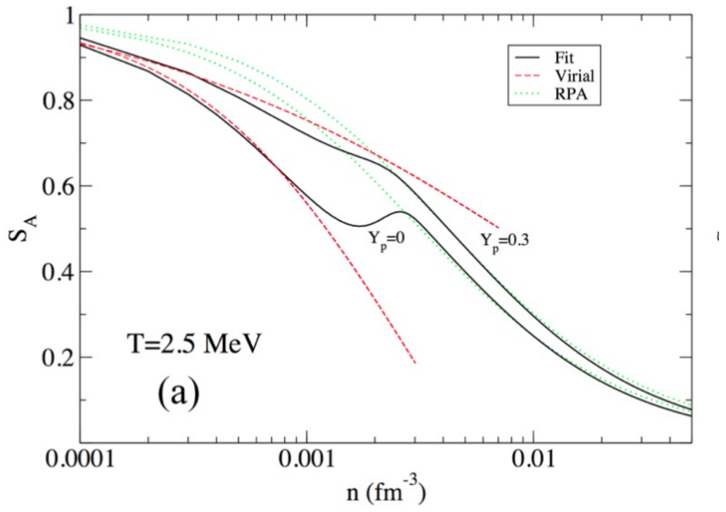
(C. J. Horowitz, PRD 65 (2002) 043001)
(C. J. Horowitz & A. Schwenk PLB 642 (2006) 326)
(C. J. Horowitz et al., PRC 95 (2017) 025801)

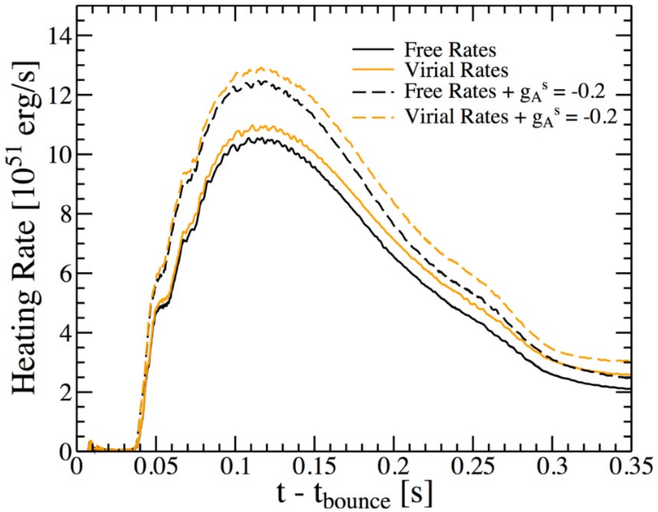
Scattering cross section In nucelar medium: $$\frac{1}{N}\frac{d\sigma}{d\Omega} = \frac{G_F^2 E_\nu^2}{16\pi^2}\left(g_a^2(3-\cos\theta)S_a(q) + (1+\cos\theta)S_v(q)\right).$$

Spin susceptibility

Density vector response function: $S_V(q=0) = \frac{T}{(\partial P/\partial n)_T}$.

Spin axial response function: $S_a(q=0) = \frac{\chi}{\chi_F}$





3. Variational method for spin-polarized matter

Nuclear Hamiltonian

$$H = -\sum_{i=1}^{N} \frac{\hbar^2}{2m}\nabla^2 + \sum_{i<j}^{N} V_{ij} + \sum_{i<j<k}^{N} V_{ijk}$$

Argonne v18 (AV18) two-body potential

Urbana IX (UIX) three-body potential

AV18 potential: (PRC 51 (1995) 38)

$$V_{ij} = \sum_{t=0}^{1}\sum_{s=0}^{1}[V_{\mathrm{C}ts}(r_{ij}) + sV_{\mathrm{T}t}(r_{ij})S_{\mathrm{T}ij} + sV_{\mathrm{SO}t}(r_{ij})(\boldsymbol{L}_{ij}\cdot\boldsymbol{s}) + V_{\mathrm{qL}ts}(r_{ij})\left|\boldsymbol{L}_{ij}\right|^2 + sV_{\mathrm{qSO}t}(r_{ij})(\boldsymbol{L}_{ij}\cdot\boldsymbol{s})^2]P_{tsij}$$

UIX potential: (PRL 74 (1995) 4396)

$$V_{ijk} = V_{ijk}^{\mathrm{R}} + V_{ijk}^{2\pi}$$

Two-body correlation function

Jastrow wave function

$$\Psi = \mathrm{Sym}\left[\prod_{i<j} f_{ij}\right]\Phi_{\mathrm{F}}$$

Φ_{F}: The Fermi-gas wave function

Two-body correlation function:

$$f_{ij} = \sum_{t=0}^{1}\sum_{\mu}\sum_{s=0}^{1}\sum_{\nu}\left[f_{\mathrm{C}ts}^{\mu\nu}(r_{ij}) + s f_{\mathrm{T}t}^{\mu\nu}(r_{ij})S_{\mathrm{T}ij} + s f_{\mathrm{SO}t}^{\mu\nu}(r_{ij})(\boldsymbol{L}_{ij}\cdot\boldsymbol{s})\right]P_{tsij}^{\mu\nu}$$

t : Total isospin μ: 3rd component of t s : Total spin ν : 3rd component of s

E_2/N is the expectation value of H_2 in *the two-body cluster approximation*.

$$\frac{E_3}{N} = \frac{1}{N}\langle\sum_{i<j<k}^{N}\left[\alpha V_{ijk}^{\mathrm{R}} + \beta V_{ijk}^{2\pi}\right]\rangle_{\mathrm{F}} + \gamma n_{\mathrm{B}}^2 e^{-\delta n_{\mathrm{B}}}\left[1-(1-2Y_{\mathrm{p}})^2\right]$$

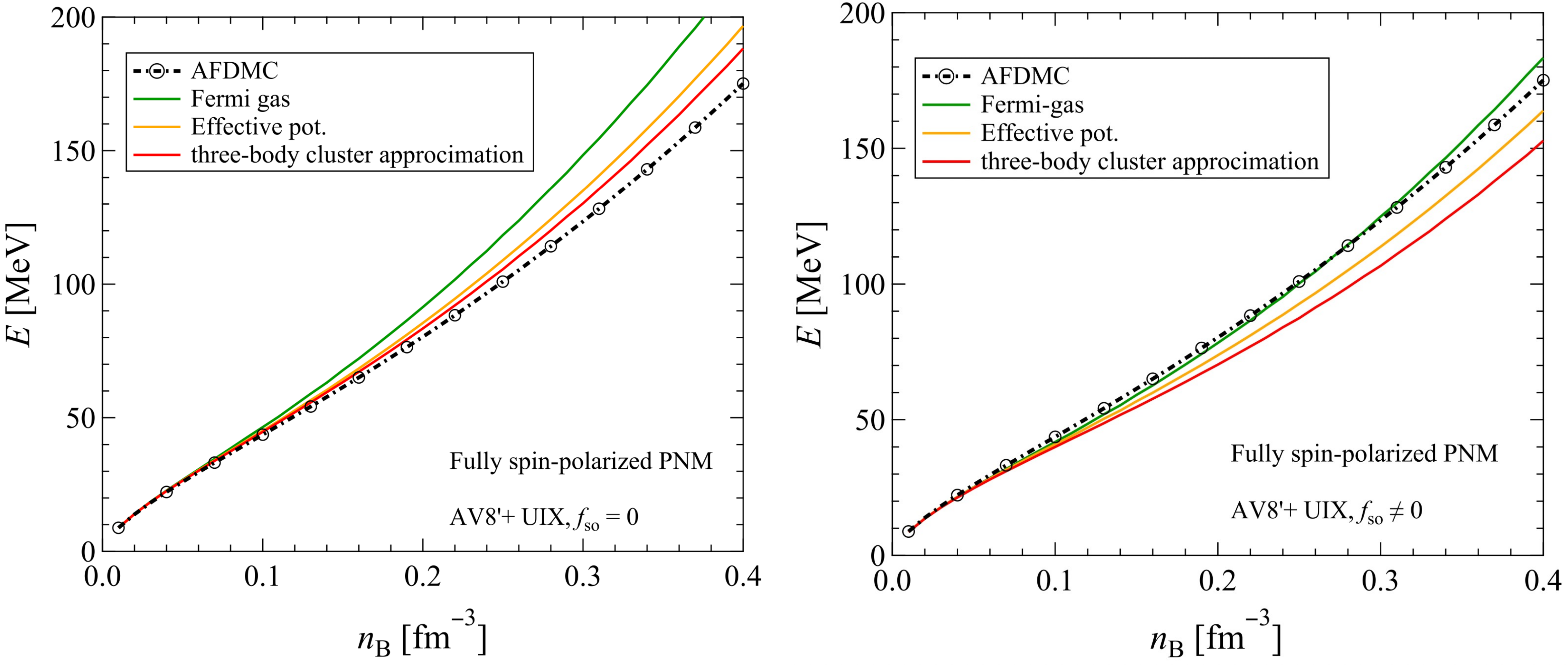
Modified expectation value of H_3 with Φ_{F}

Correction term

Total energy per nucleon $E/N = E_2/N + E_3/N$

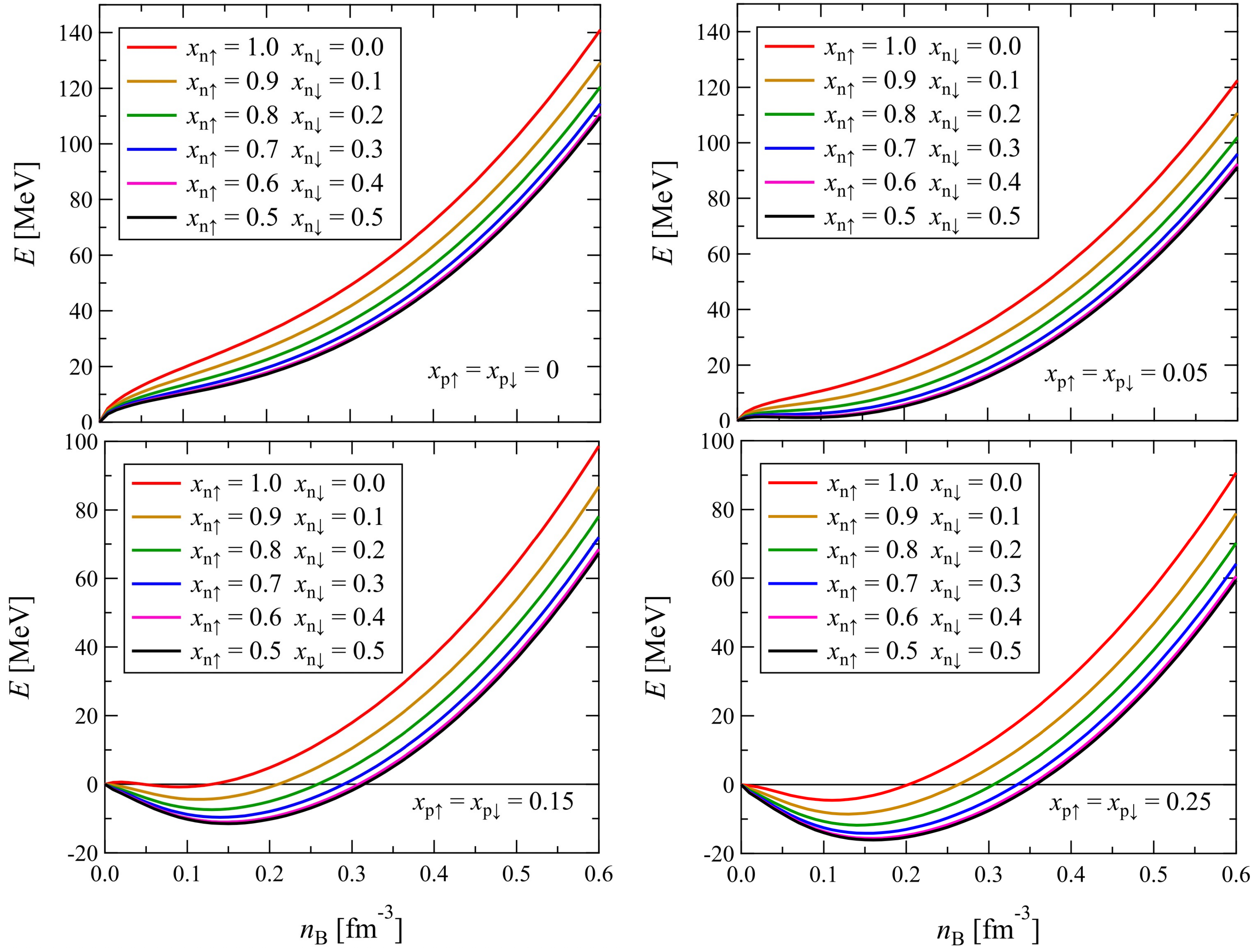
Comparison with the AFDMC method

AFDMC: **A**uxiliary **F**ield **D**iffusion **M**onte **C**arlo method



AFDMC: L.Riz, S. Gandolfi, and F. Pederiva., J. Phys. G 47 (2020) 0245106)

Energy per particle for spin-polarized matter

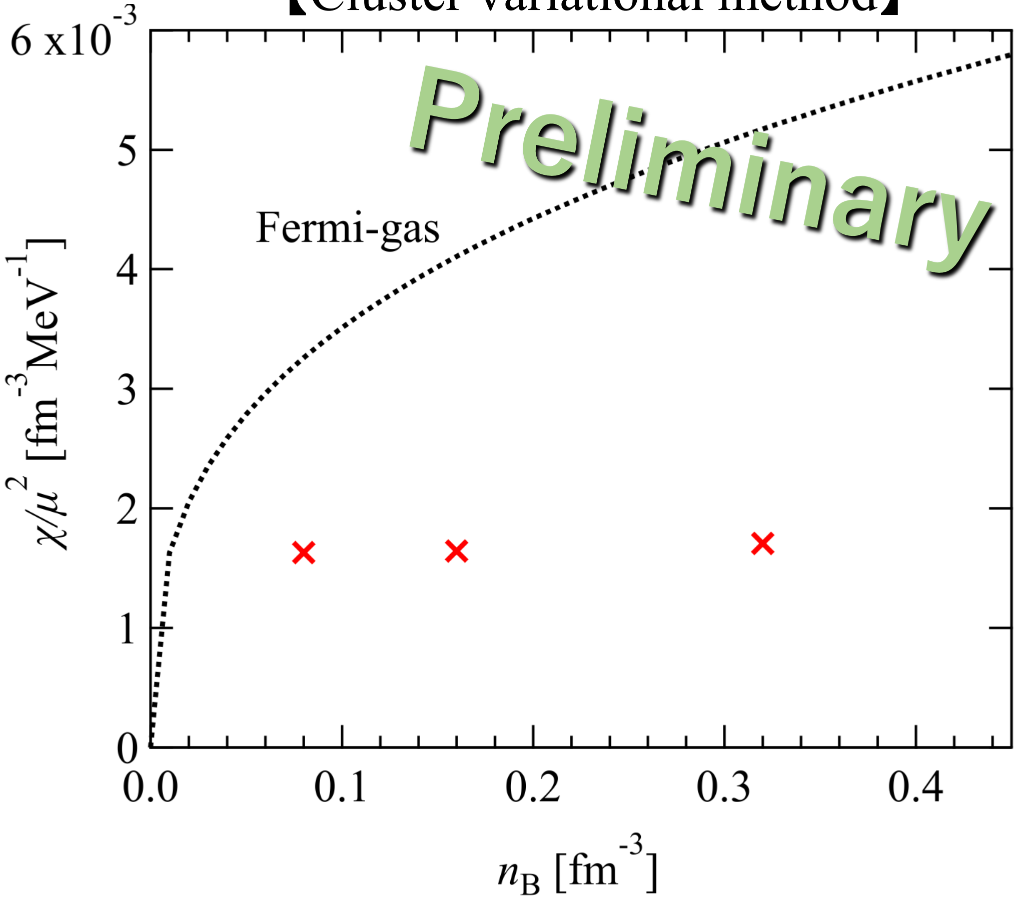


Spin-susceptibility for Pure Neutron Matter

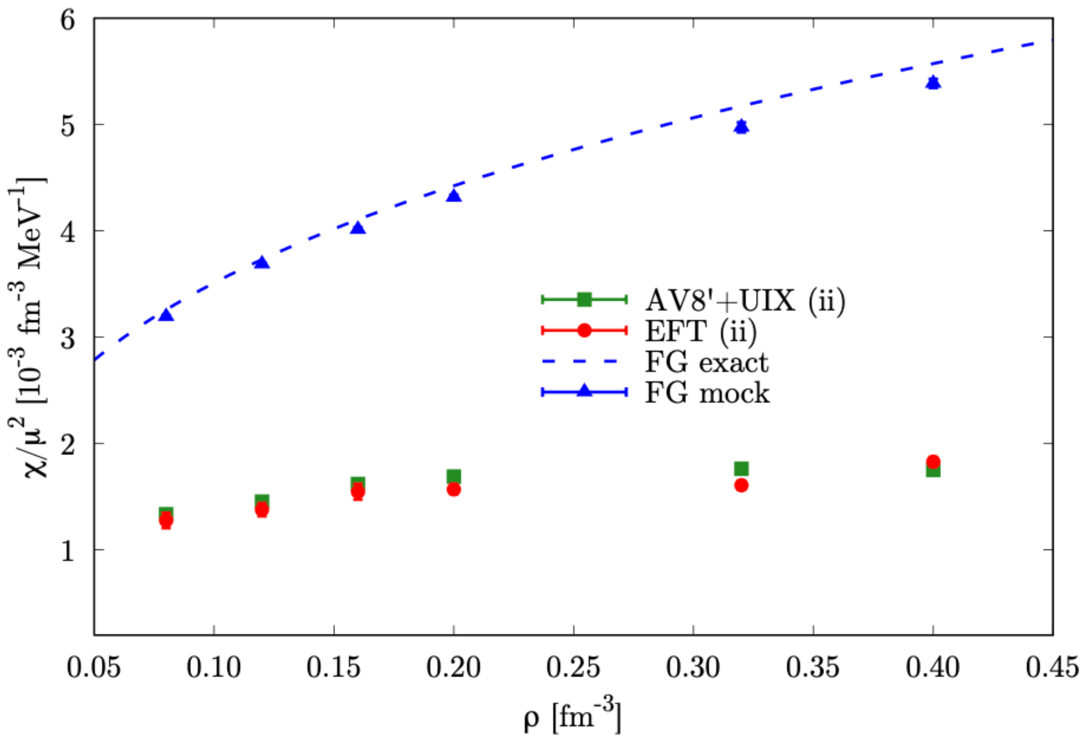
$$\text{Spin-susceptibility}: \chi = \frac{\mu^2 \rho}{\left(\frac{\partial^2 (E/N)}{\partial \Delta^2}\right)_{\Delta=0}},$$

$$\text{Spin-asymmetry}: \Delta = \rho_{\uparrow} - \rho_{\downarrow}$$

【Cluster variational method】



【AFDMC method】



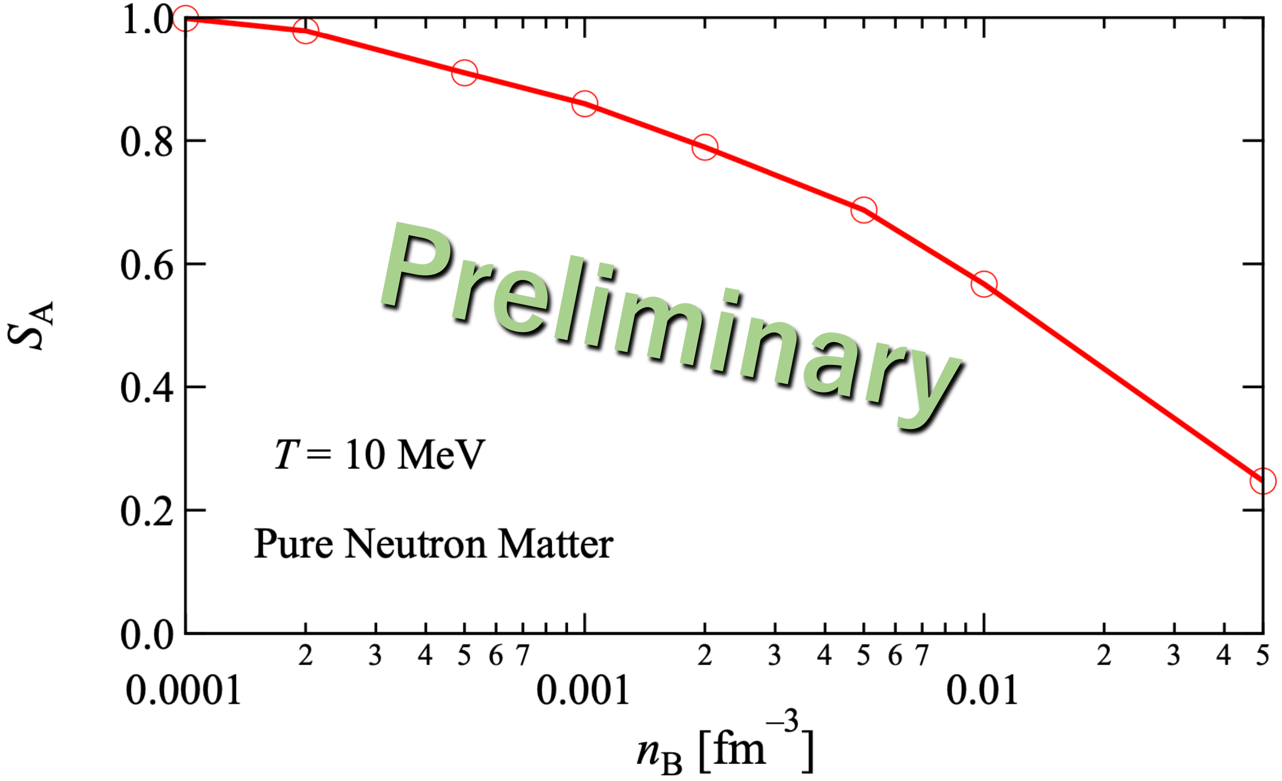
(AFDMC: L.Riz, et al., Particles 3 (2020) 706)

Axial response function for pure neutron matter

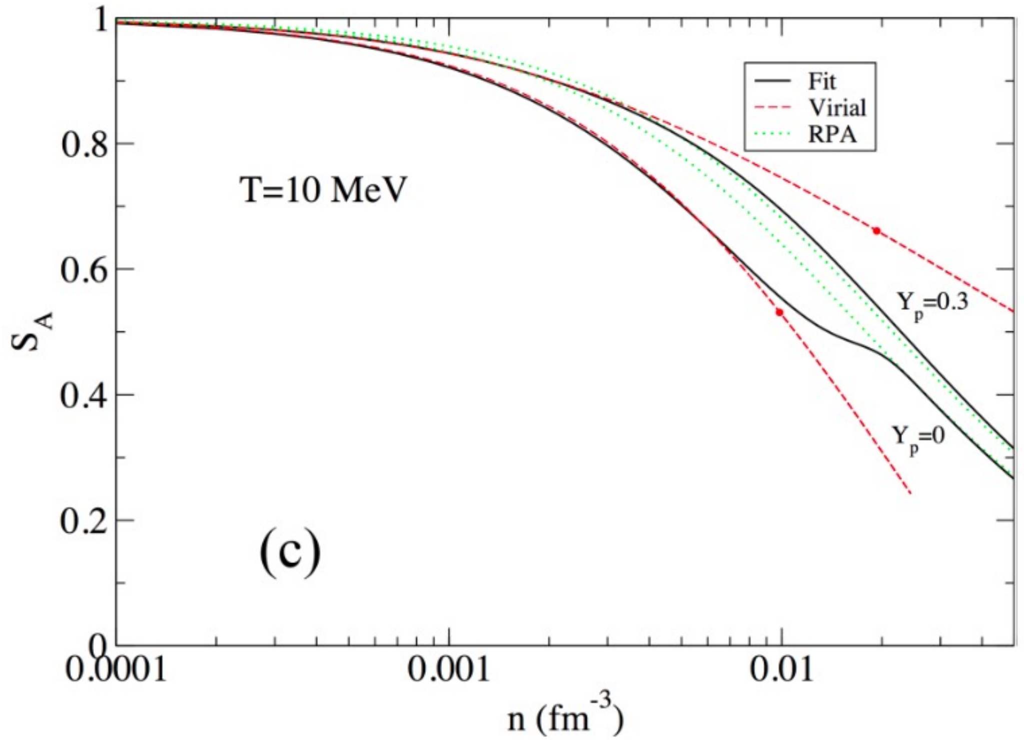
Neutrino scattering cross section in nuclear medium:

$$\frac{1}{N}\frac{d\sigma}{d\Omega} = \frac{G_F^2 E_\nu^2}{16\pi^2}\left(g_a^2(3-\cos\theta)S_a(q) + (1+\cos\theta)S_v(q)\right)$$

【Cluster variational method】



【Virial & RPA method】



(C. J. Horowitz et al., PRC 95 (2017) 025801)

Summary

We extend the variational EOS based on realistic nuclear forces (AV18 + UIX) to calculate the thermodynamic quantities for spin-polarized nuclear matter.

→ The obtained spin susceptibility is applied to calculations for the neutrino-nucleon scattering rates in a consistent manner.

- Energy for fully spin-polarized matter is in good agreement with the results by AFDMC.
- The obtained axial response function is reduced by nuclear medium effect.

Future Plans

- Systematic study of the axial response function for asymmetric matter
- Supernova simulations with the obtained neutrino-nucleon scattering cross sections