Phase Structure of QCD matter under extreme conditions and holographic GW

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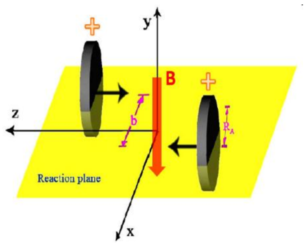
Quarks and Compact Stars , Yangzhou University, Sept. 22-26, 2023

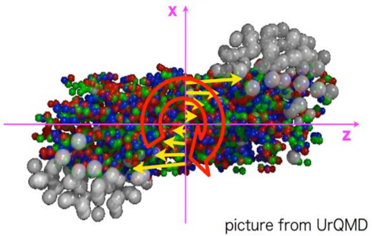
Outlines

- **Introduction and motivation**
- **Phase structure under rotation & Magnetic field**
- **Eos Ns structure**
- **GW from 1stOPT**
- **Summary**

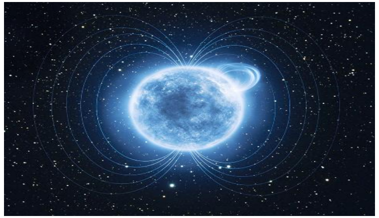
Phase structure under new extrem condition



Strongest EM fields

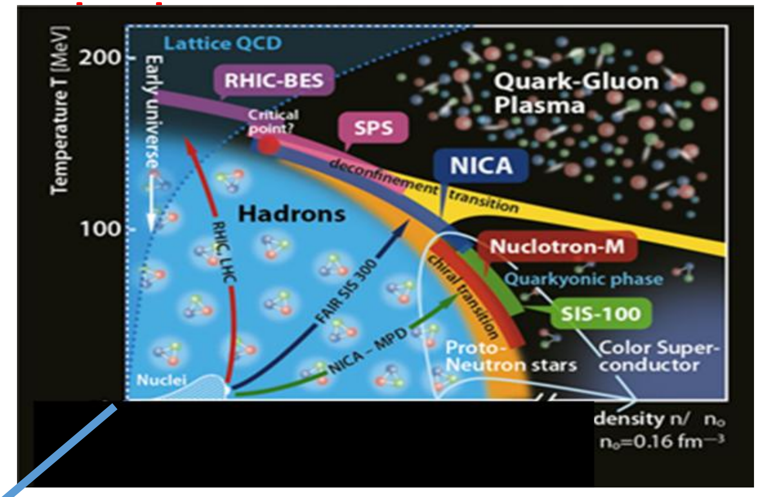


Largest local rotation



What are their effects on the QCD phase

Explore the new dimensions of the QCD phase diagram



B, ω, E, ni…

New theoretical techniques needed!

Lattice QCD

difficulty with Finite baryon density, Real time dynamics Continuum

Phenom. models: (p)NJL、(p)QMC…

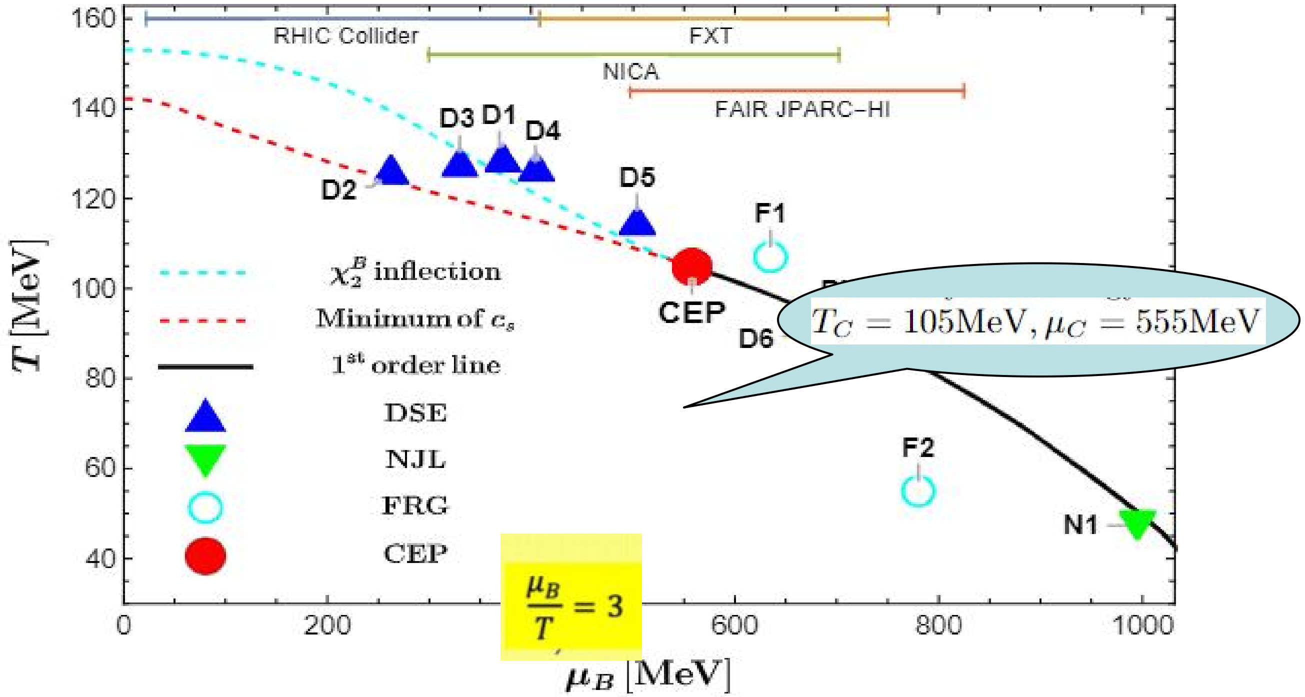
Field Theory: HD(T)L , pQCD , xPT, DSE

Functional Renormalization Group, Sum rules …

AdS/CFT, AdS/QCD

Predictions of QCD phase diagram

A. Bazavov, etc.Phys. Rev. D 95 (2017) no.5, 054504 [arXiv:1701.04325 [hep-lat]].



(DSE)： 2109.09935 [hep-ph], 1607.01675 [hep-ph], 1011.2876 [nucl-th], 1403.3797 , 1405.4762 , 2002.07500

(NJL, PNJL)： arXiv:1801.09215 [hep-ph]], Nucl. Phys. A 504 (1989), 668-684

(FRG)； Fu, Pawloski, Rennecke, PRD101(2020)； 1909.02991

Zhang, Hou, Kojo, Qin, PRD96（2017）1709.05654 [hep-ph]].

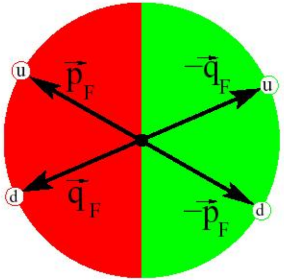
Phase structures in CSC

- BSC-like pairing

J=0: 2SC: u_r, d_r, u_g, d_g

CFL: all flavor and color

M. Alford, K. Rajagopal and F. Wilczek, NPB 537, 443 (1999)



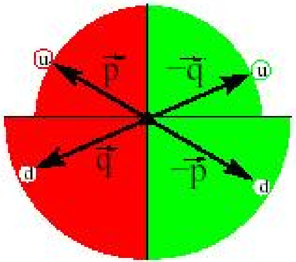
J=1: CSL

T. Schaefer, PRD 62, 094007 (2000)
A. Schmitt, PRD 71, 054016 (2005)

- Non-BCS pairing

gapless CSC

LOFF



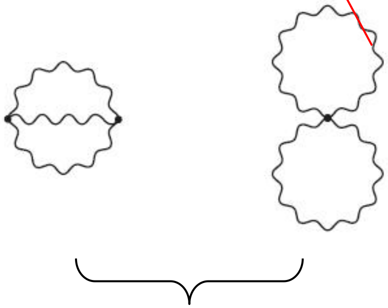
Shovkovy and M. Huang, PLB 546, 205 (2003)
M. Alford et al., PRL 92, 222001 (2004)
M. Alford et al., PRD 63, 074016 (2001)
…….

CJT effective action of QCD

$$\Gamma[\bar{D},\bar{S}] = \frac{1}{2}\{Tr\ln\bar{D}^{-1} + Tr(D^{-1}\bar{D}-1) - Tr\ln\bar{S}^{-1} - Tr(S^{-1}\bar{S}) - 2\Gamma_2[\bar{D},\bar{S}]$$

2-loop approximation

Order of g² μ^4



Powers of T

Stationary points

$$\left.\frac{\delta\Gamma}{\delta\bar{\mathcal{D}}}\right|_{\bar{\mathcal{D}}=\mathcal{D},\bar{\mathcal{S}}=\mathcal{S}} = 0\,, \qquad \left.\frac{\delta\Gamma}{\delta\bar{\mathcal{S}}}\right|_{\bar{\mathcal{D}}=\mathcal{D},\bar{\mathcal{S}}=\mathcal{S}} = 0$$

$$\Downarrow \qquad\qquad \Downarrow$$

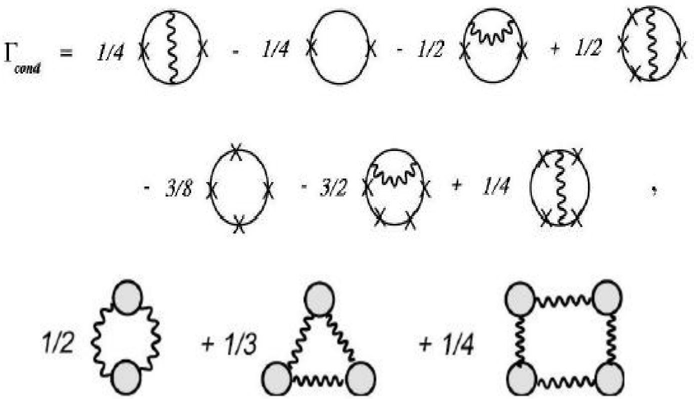
$$\mathcal{D}^{-1} = D^{-1} + \Pi[\mathcal{S}] \qquad \mathcal{S}^{-1} = \mathcal{S}_0^{-1} + \Sigma$$

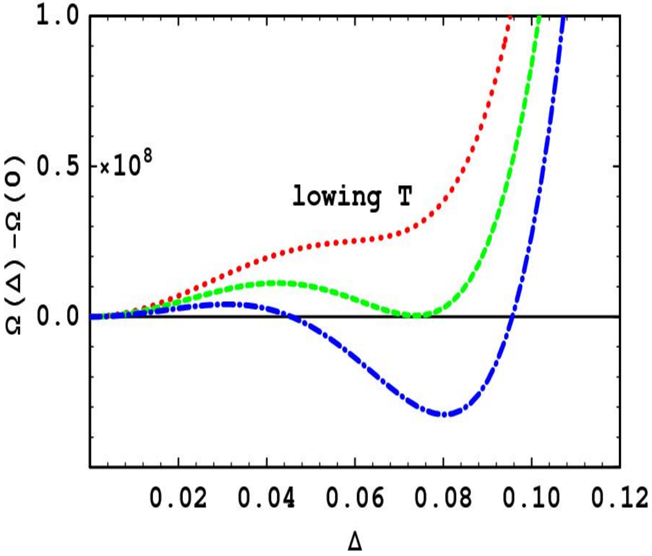
$$\Gamma_2[\bar{\mathcal{D}},\bar{\mathcal{S}}] = -\frac{1}{2}\,\mathrm{Tr}\{\bar{\mathcal{D}}\,\Pi[\bar{\mathcal{S}}]\}$$

D. Rischke Prog. Part. Nucl. Phys. 52 197 (2004)

Gauge field fluc. induce 1st order PT of CSC in dense QCD

Ginnakis, Hou, Ren, Rischke, PRL 93 (04) ; PRD73 (06)



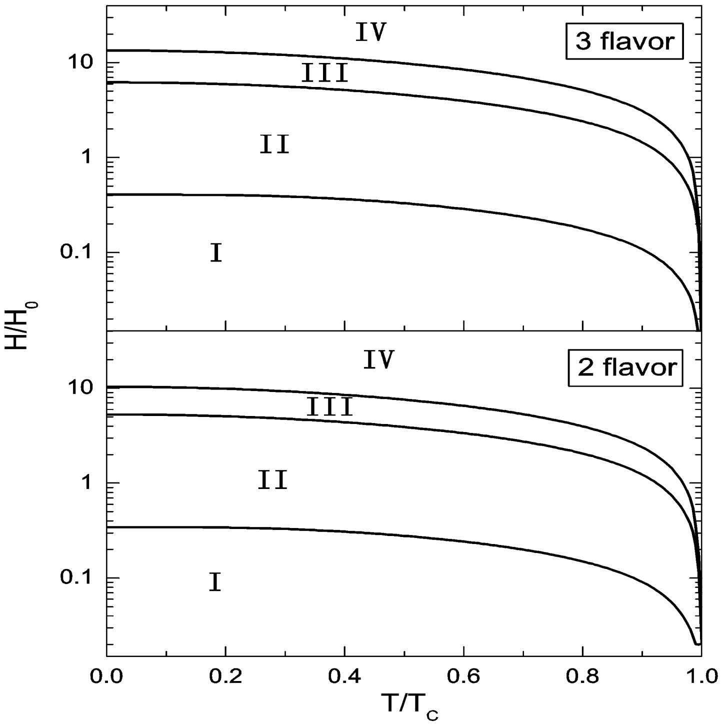


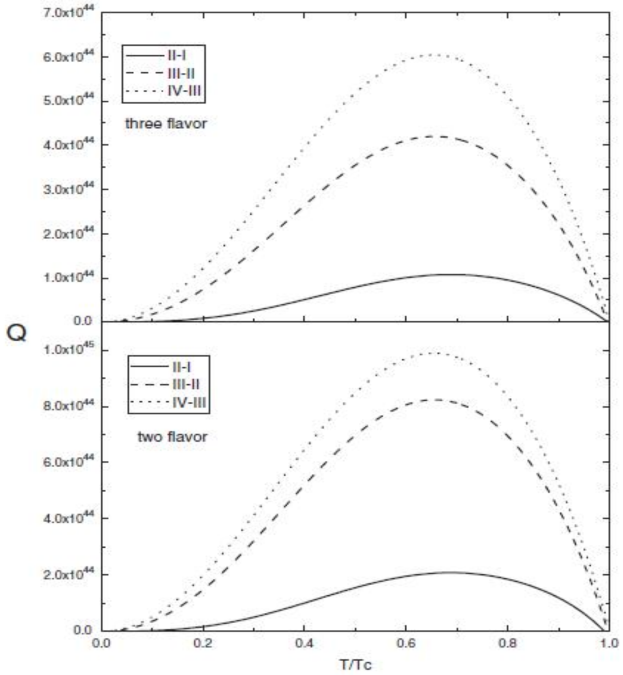
Introduction of Δ^3 term in free energy by flucts. Inducing 1st order PT in stead of 2nd order PT in MFA

Nonspherical states in dense QCD with B

	I	II	III	IV	$T_C(10^{-1}$ MeV)
Two-flavor	CSL_u, CSL_d	$(\text{polar})_u$, $(\text{planar})_d$	$(\text{normal})_u$, $(\text{polar})_d$	$(\text{normal})_u$, $(\text{normal})_d$	1.35
Three-flavor	CSL_u, $CSL_{d,s}$	$(\text{polar})_u$, $(\text{planar})_{d,s}$	$(\text{normal})_u$, $(\text{polar})_{d,s}$	$(\text{normal})_u$, $(\text{normal})_{d,s}$	0.49

H0=5.44×10^14 G, 1.97×10^14 G





Feng, Hou, Ren,Wu, PRL 105(2010)

Wu, He, Hou, Ren, PRD84 (2011)

FRGE and phase structure

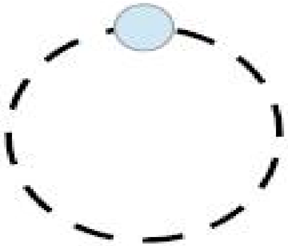
FRG flow equation

- For continuum field theory
- Non-perturbative
- (known) microscopic laws → complex macroscopic phenomena
- Flow from classical action S[φ] to effective action Γ[φ]
- Scale dependent effective action $\Gamma_k[\varphi]$

Wetterich, PLB301, 90 (1993).

Talk in this WS: K. Youngman

$$\partial_k \Gamma_k = \frac{1}{2}$$



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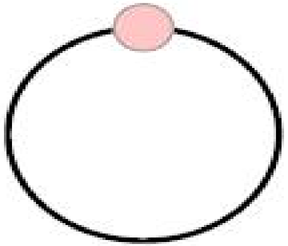
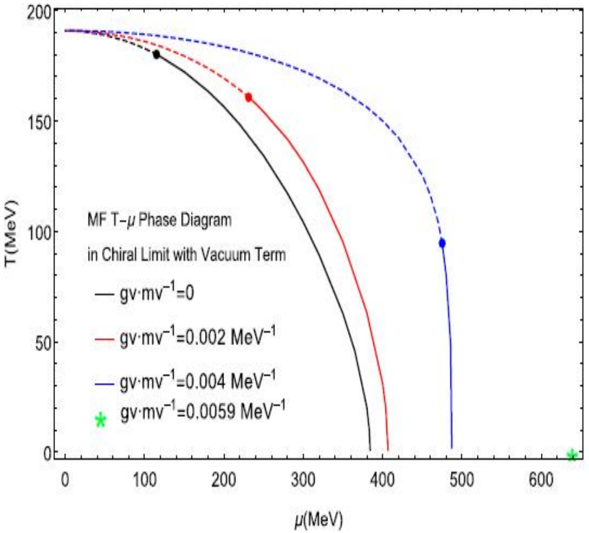
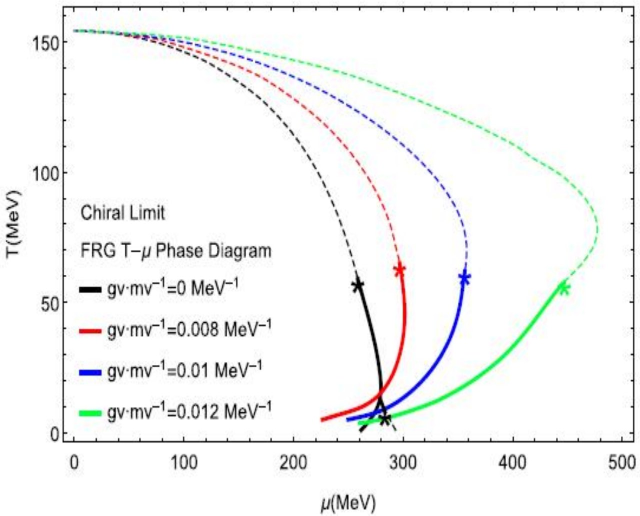


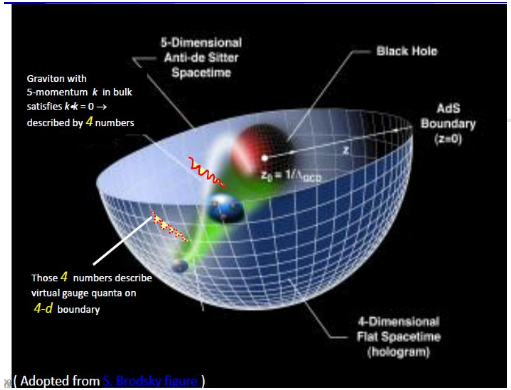
FRGE study of phase diagram: Flucts on CEP

Zhang, Hou , Kojo, Qin,   PRD96 (2017)





AdS/CFT correspondence

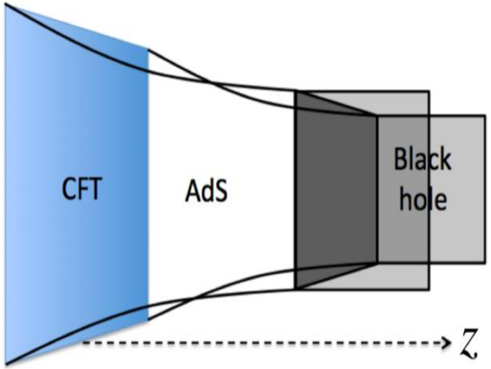


4dim. Large-Nc strongly coupled
SU(Nc) N=4 SYM (finite T).

Maldacena '97 ↕ conjecture

Witten '98

Type II B Super String theory on AdS5-BH×S5



Some complicated Field theory calculations become simple "geometric' problems in higher dimensions

Phase Structure of hQCD with magnetic field

The Einstein-Maxwell-dilaton(EMD) action *:

$$S = -\frac{1}{16\pi G_5}\int d^5x\sqrt{-g}[R - \frac{f_1(\phi)}{4}F_{(1)MN}F^{MN} - \frac{f_2(\phi)}{4}F_{(2)MN}F^{MN} - \frac{1}{2}\partial_M\phi\partial^M\phi - V(\phi)],$$

chemical potential B field breaking conformal sym.

The metric:

$$ds^2 = \frac{L^2 e^{S(z)}}{z^2}[-g(z)dt^2 + dx_1^2 + e^{B^2z^2}(dx_2^2 + dx_3^2) + \frac{dz^2}{g(z)}],$$

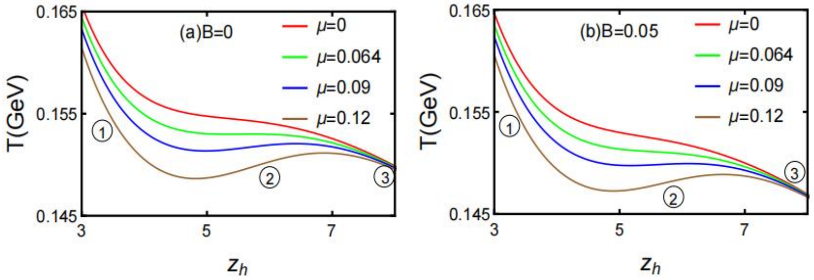
$$A_t(z) = \mu[1 - \frac{\int_0^z d\xi \frac{\xi e^{-B^2\xi^2}}{f_1(\xi)\sqrt{S(\xi)}}}{\int_0^{z_h} d\xi \frac{\xi e^{-B^2\xi^2}}{f_1(\xi)\sqrt{S(\xi)}}}] = \widetilde{\mu}\int_z^{z_h} d\xi \frac{\xi e^{-B^2\xi^2}}{f_1(\xi)\sqrt{S(\xi)}},$$

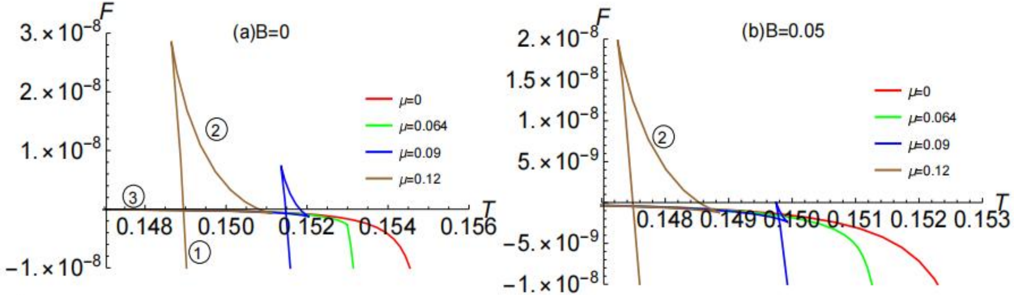
Dilaton field ：

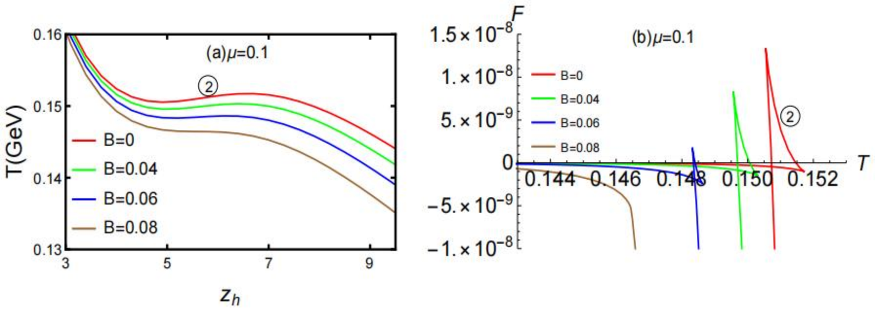
$$\phi(z) = \int dz\sqrt{-\frac{2}{z}(3zA''(z) - 3zA'(z)^2 + 6A'(z) + 2B^4z^3 + 2B^2z)} + K_5.$$

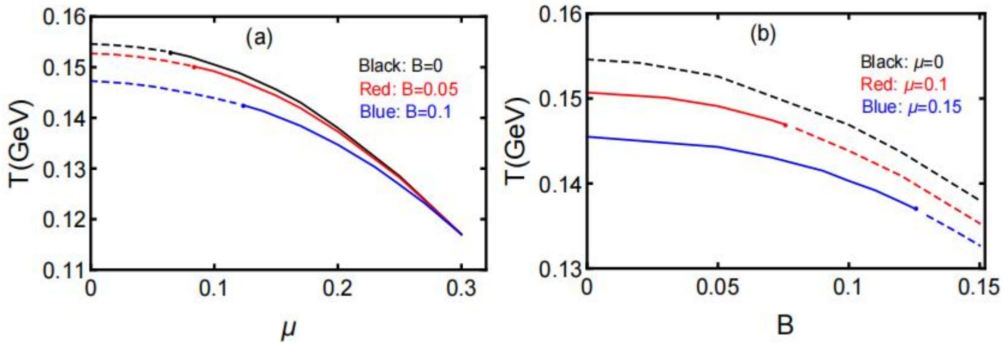
*Hardik Bohra a, David Dudal et al, Anisotropic string tensions and IMC from a dynamical AdS/QCD model.PLB 801 (2020) 135184.

Phase Structure with magnetic field









1） **μ makes cross over to 1st order PT；B vice verse；**

2） inverse Magnetic catalysis。

Zhou-Run Zhu, De-fu Hou,, (arXiv:2305.12375).

Zhou-Run Zhu, Jun-Xia Chen, Xian-Ming Liu, De-fu Hou, Eur.Phys.J.C 82 (2022) 6,560.

QCD Phase Diagram with rotation

- **Holographic model:**

[1].Phys.Rev.D 106 (2022) 12, L121902 • e-Print: 2201.02004

- **The action:**

$$S_M = \frac{1}{2\kappa_N^2}\int d^5x\sqrt{-g}\left[\mathcal{R} - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - \frac{Z(\phi)}{4}F_{\mu\nu}F^{\mu\nu} - V(\phi)\right],$$

where the potential and kinetic functions read

$$V(\phi) = -12\cosh\left[c_1\phi\right] + \left(6c_1^2 - \frac{3}{2}\right)\phi^2 + c_2\phi^6,$$

Capturing the behavior of EOS at zero chemical potential.

$$Z(\phi) = \frac{1}{1+c_3}\text{sech}[c_4\phi^3] + \frac{c_3}{1+c_3}e^{-c_5\phi}.$$

Capturing the flavor dynamic.

- **The metric:**

$$ds^2 = -e^{-\eta(r)}f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(dx_1^2 + dx_2^2 + dx_3^2),$$

$$\phi = \phi(r), \qquad A_t = A_t(r),$$

- **The Hawking temperature and entropy density:**

$$T = \frac{1}{4\pi}f'(r_h)e^{-\eta(r_h)/2} \qquad s = \frac{2\pi}{\kappa_N^2}r_h^3.$$

effective Newton constant (κ_N^2)

Phase diagram @2+1 flavor

- **Holographic model *with rotation*:**
  - **To introduce the rotation effect, we split the 3-dimensional space into two parts as** $\mathcal{M}_3 = \mathbb{R} \times \Sigma_2$. **Then the metric becomes to**

$$ds^2 = -f(r)e^{-\eta(r)}dt^2 + \frac{dr^2}{f(r)} + r^2\ell^2 d\theta^2 + r^2 d\sigma^2$$

where $d\sigma^2$ denotes the line element of Σ_2.

  - **We assume the system that has an angular velocity ω with a fixed radius ℓ, and consider the following local Lorentz boost**

[5]. JHEP 07 (2021) 132 • e-Print: 2010.14478 [6].Phys.Rev.D 97 (2018) 2, 024034 • e-Print: 1707.03483
[7].JHEP 04 (2017) 092 • e-Print: 1702.02416 [8].Gen.Rel.Grav. 42 (2010) 1571-1583 • e-Print: 0911.2831

$$t \to \frac{1}{\sqrt{1-\omega^2\ell^2}}(\hat{t} + \omega\ell^2\hat{\theta}), \qquad \theta \to \frac{1}{\sqrt{1-\omega^2\ell^2}}(\hat{\theta} + \omega\hat{t}) .$$

  - **The corresponding metric can be written as**

$$d\hat{s}^2 = g_{\mu\nu}d\hat{x}^\mu d\hat{x}^\nu = -N(r)d\hat{t}^2 + \frac{dr^2}{f(r)} + R(r)(d\hat{\theta} + Q(r)d\hat{t})^2 + r^2 d\sigma^2 ,$$

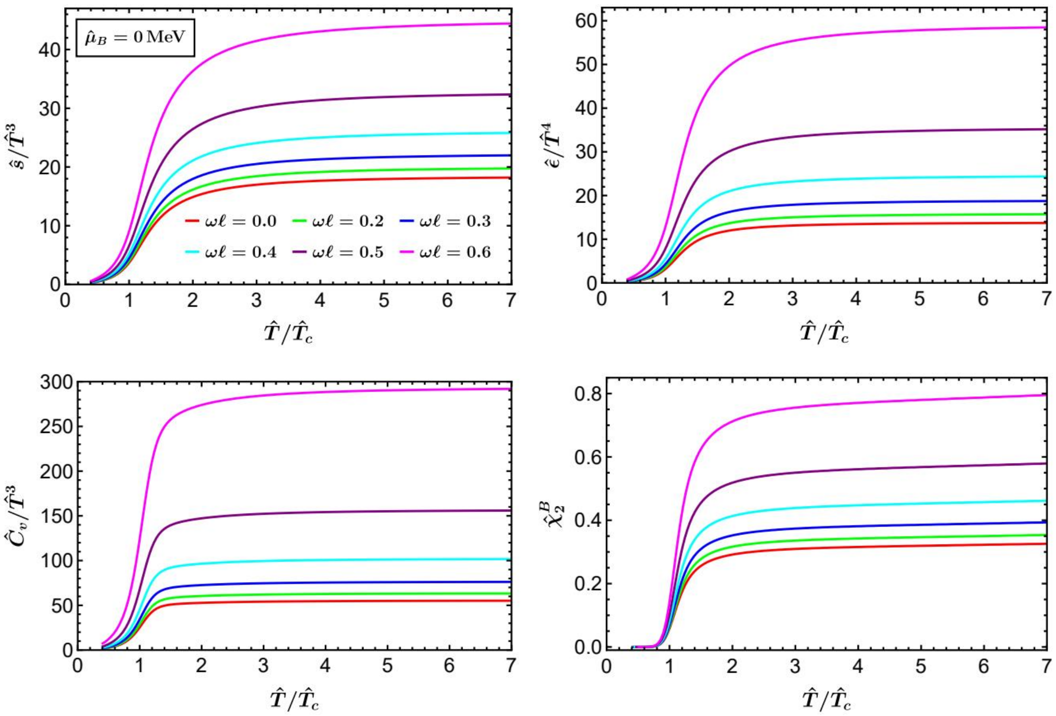
$$N(r) = \frac{r^2 f(r)\left(1-\omega^2\ell^2\right)}{r^2 e^{\eta(r)} - \omega^2\ell^2 f(r)} ,$$

$$R(r) = \frac{r^2\ell^2 - \omega^2\ell^4 f(r)e^{-\eta(r)}}{1-\omega^2\ell^2} ,$$

$$Q(r) = \frac{\omega\left(f(r) - r^2 e^{\eta(r)}\right)}{\omega^2\ell^2 f(r) - r^2 e^{\eta(r)}} .$$

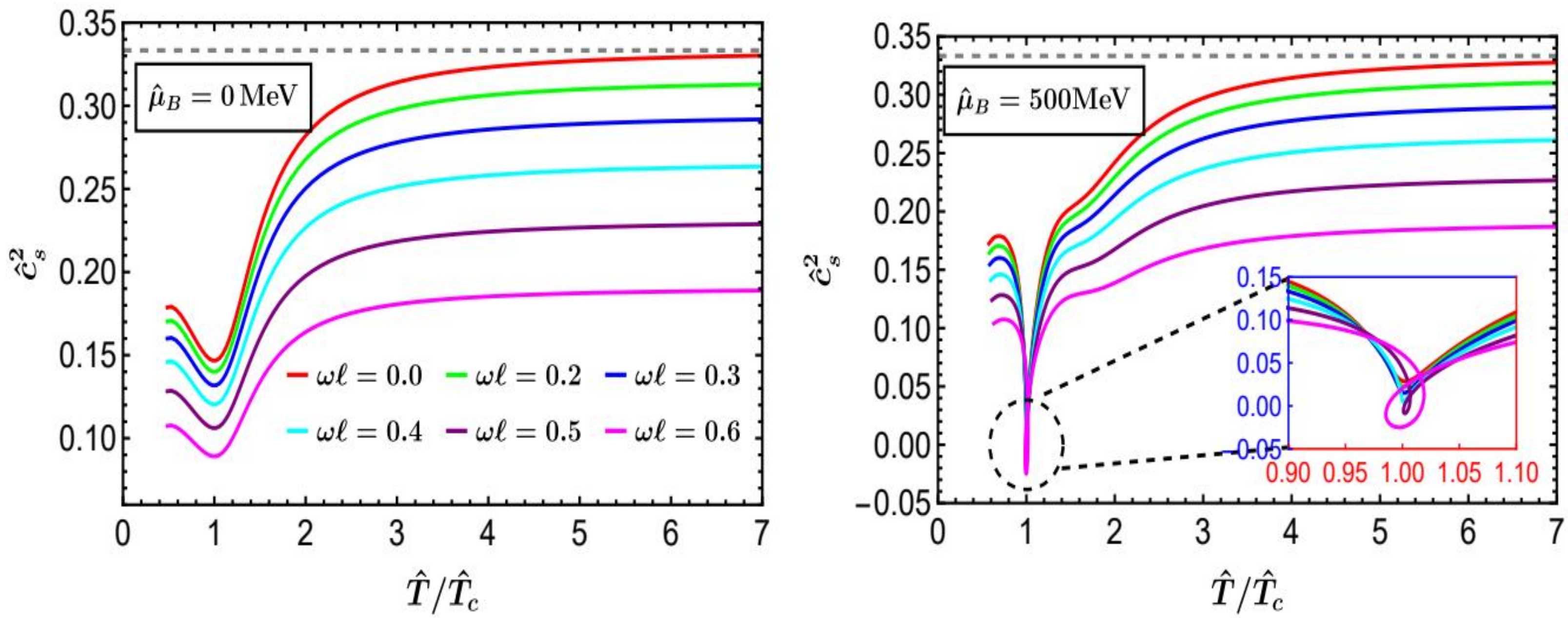
Phase diagram @2+1 flavor

- Thermodynamics *with rotation*:



Yan-Qing Zhao, Song He, Defu Hou, Li Li, Zhibin Li, JHEP 04 (2023) 115 • e-Print: 2212.14662

➢ Thermodynamics *with rotation*:

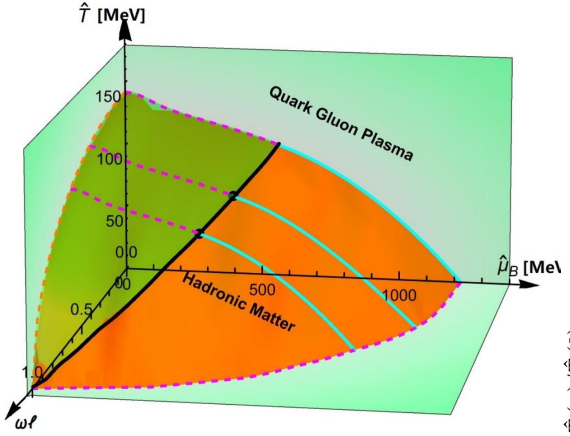


Yan-Qing Zhao, Song He, Defu Hou, Li Li, Zhibin Li, JHEP 04 (2023) 115 • e-Print: 2212.14662

➢ 2+1 flavor:

➢ Pure gluon($\hat{\mu}_B = 0$):

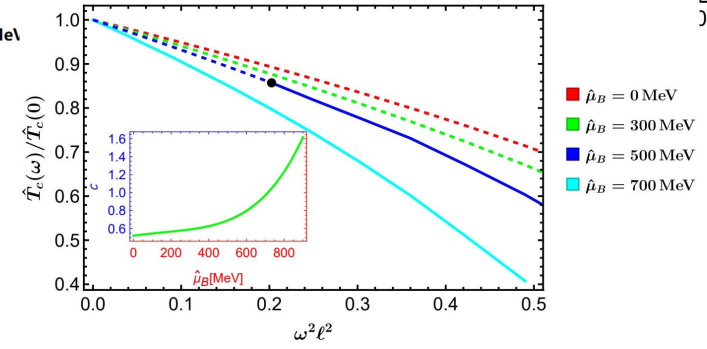
- ✓ **black solid line:** denoting the location of CEP.
- ✓ At high $\hat{T}$and small $\hat{\mu}_B$: **Being the smooth crossover.**
- ✓ At low $\hat{T}$and large $\hat{\mu}_B$: **Being 1st-order transition.**
- ✓ $\omega \uparrow$ ⟹ $\hat{T}_c \downarrow$, $\hat{\mu}_c \downarrow$, $\hat{T}_{cep} \downarrow$, $\hat{\mu}_{cep} \downarrow$.



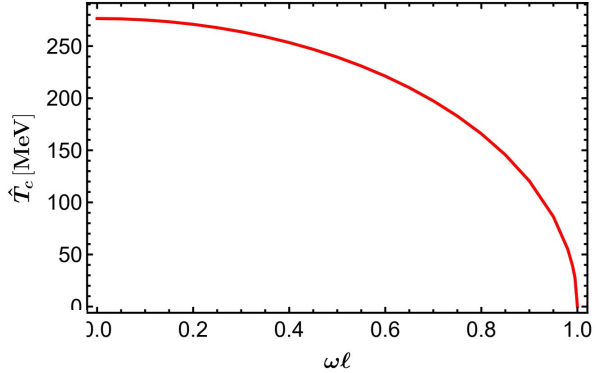
$$\hat{T}_c(\omega)/\hat{T}_c(0) \approx 1 - c\,\omega^2$$

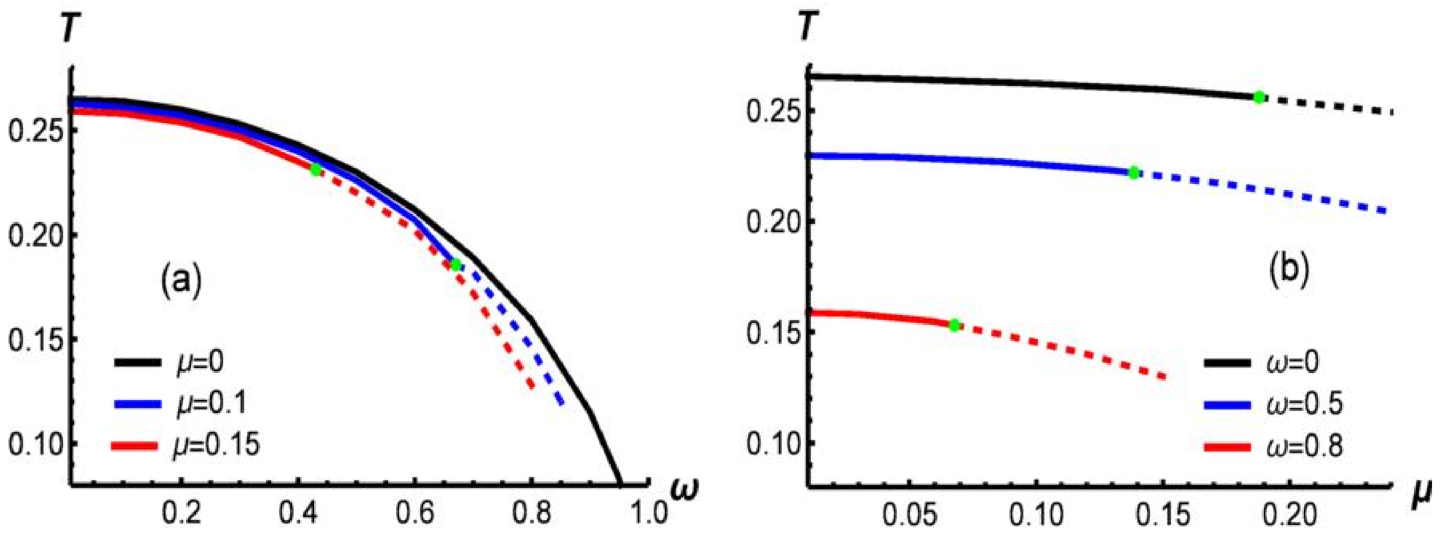
- ✓ At finite $\hat{\mu}_B$ **and smaller ω.**

The value of c depends on $\hat{\mu}_B$.



Analytically: $\hat{T}_c(\omega) = T_c\sqrt{1-\omega^2\ell^2}$

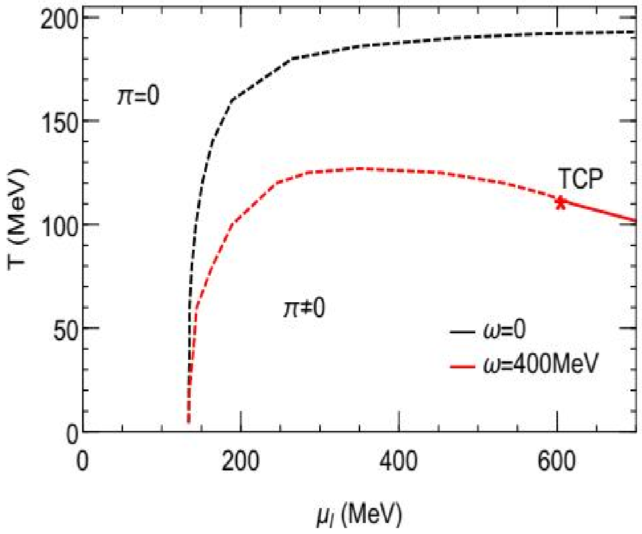


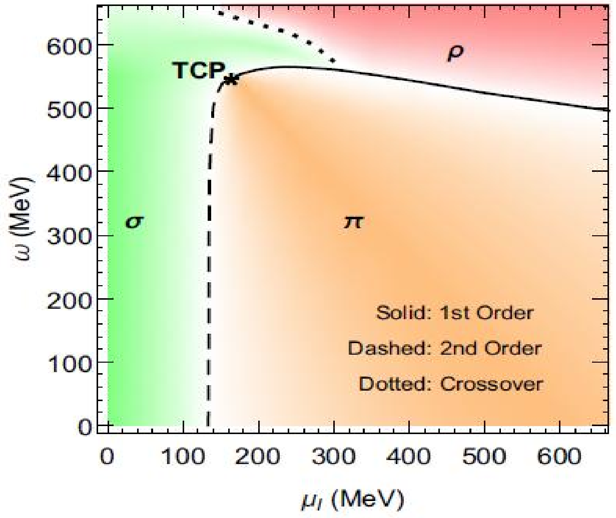


X.Chen, L. Zhang, D.N. Li, D. Hou , M.Huang ,JHEP 07 (2021) 132

New mesonic superfluid phase diagram

H. Zhang, DF Hou, JF Liao, NPA 1005 (2021) 121762



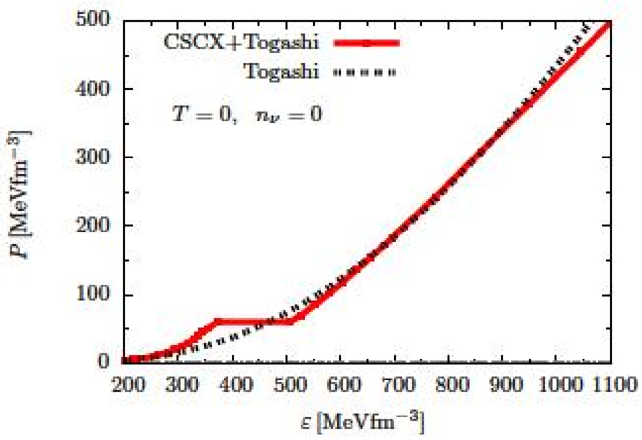


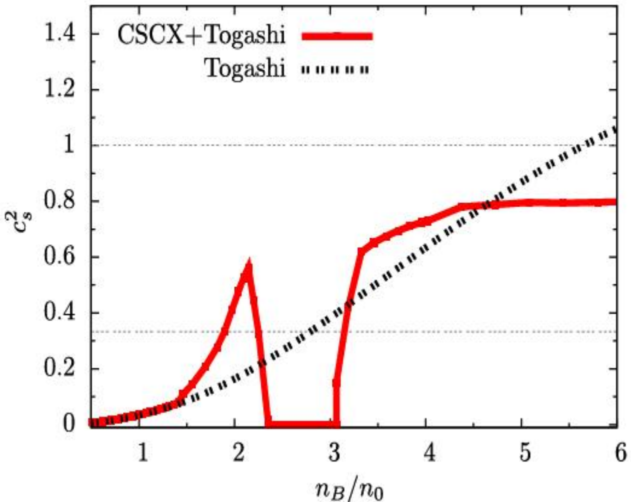
Rotation enhances spin 1 condensate， suppresses spin-zero condensates

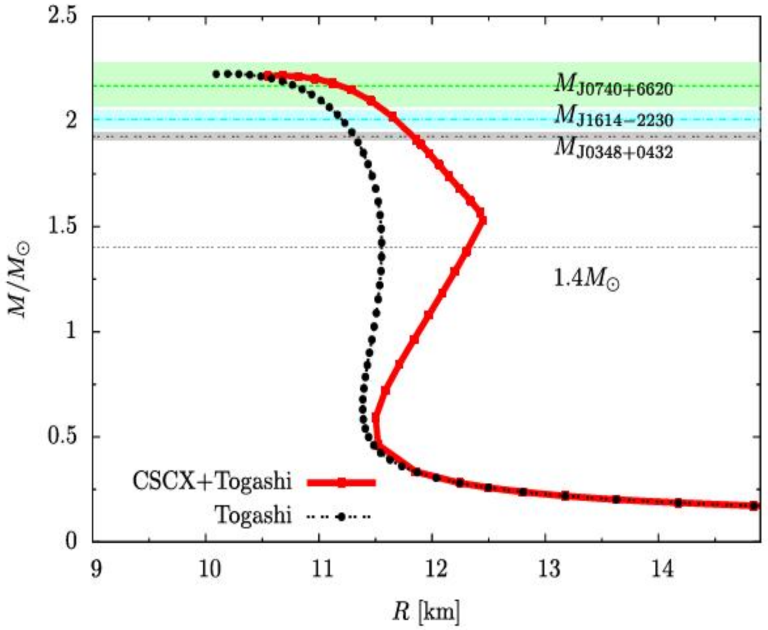
H. Zhang, DF Hou, JF Liao. CPC44, No. 11 (2020) 1

Rotational suppression of Pion superfluid

EOS of QCD Matter and Structure of NSs







T. Kojo， Hou, J. Okafor, H. Togashi, PRD 104 (2021) 063036

GW from Holographic QCD Phase Transition

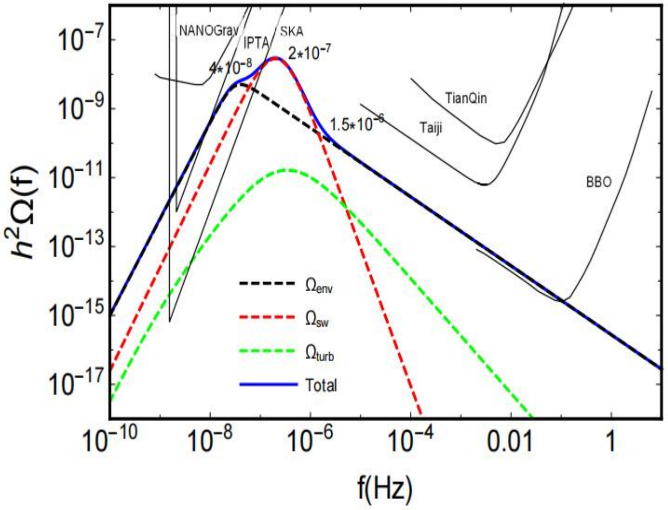
- GW
  - Astrophysics sources: Supernova explosion, merger of compact stars
  - Cosmology source
    - inflation , reheating
    - EW 1stOPT 、 **QCD PT**

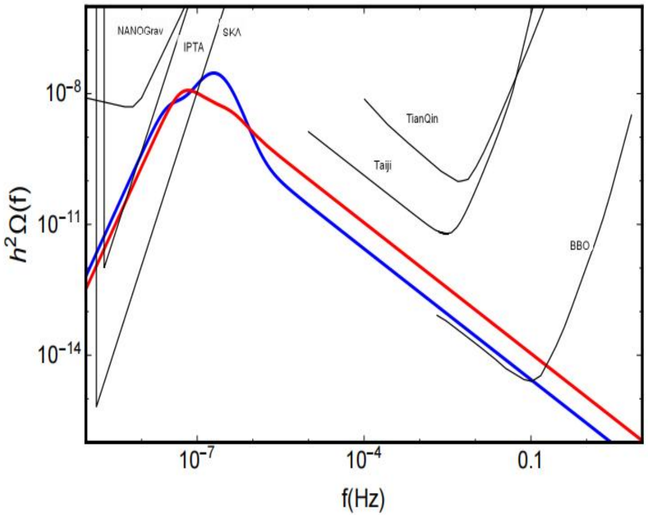
$$h^2\Omega(f) = h^2\Omega_{env}(f) + h^2\Omega_{sw}(f) + h^2\Omega_{turb}(f)\ ^{*}$$

$h^2\Omega_{env}(f)$	bubble collision
$h^2\Omega_{sw}(f)$	Sound wave
$h^2\Omega_{turb}(f)$	Turbulent

C. Caprini, M. Hindmarsh, S. Huber, T. Konstandin, J. Kozaczuk, G. Nardini, J. M. No, A. Petiteau, P. Schwaller and G. Servant, et al. JCAP 04 (2016), 001

Zhou-Run Zhu, Jun Chen, De-fu Hou,, Eur.Phys.J.A 58 (2022) 6,104.





1)max. Freq. is determined by the speed of sound

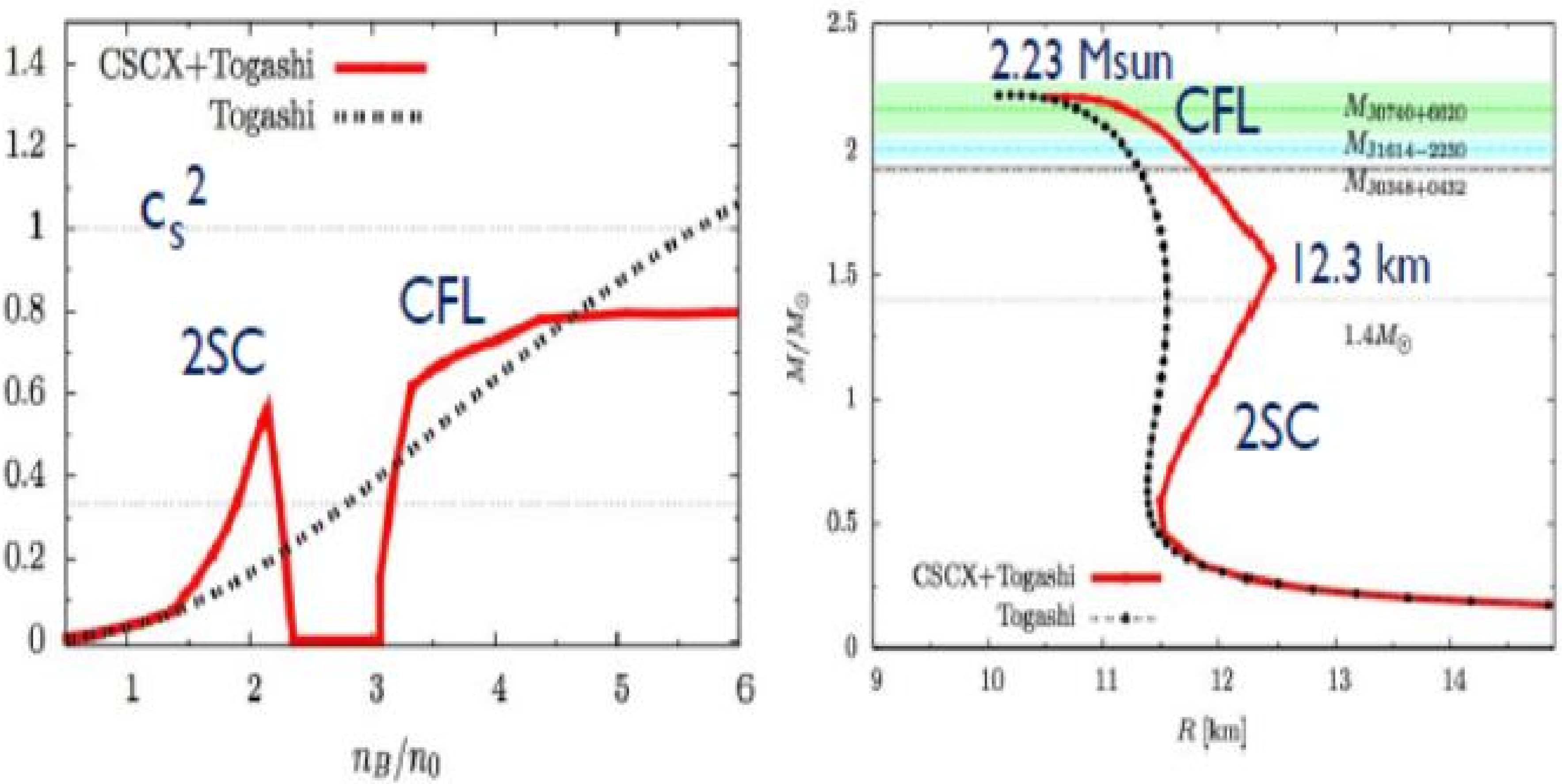
2)Gluon condensate suppresses the energy density of GW and the max. freq.

Summary

Strongly interacting matter under extreme conditions has rich structures

- **QCD Phase diagram under rotation and Magnetic field**
- **EoS and Structure of NS**
- **GWs from 1OPT with gluon condensate**

Thank you very much for your attention!



CSCX+Togashi
Togashi
c_s^2
2SC
CFL
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1
2
3
4
5
6
n_B/n_0
2.23 Msun
CFL
$M_{J0740+6620}$
$M_{J1614-2230}$
$M_{J0348+0432}$
12.3 km
$1.4M_\odot$
2SC
CSCX+Togashi
Togashi
$M/M_\odot$
2.5
2
1.5
1
0.5
0
9
10
11
12
13
14
R [km]

The related vacuum energy at the PT is

$$\epsilon_* = \left(-\Delta F(T) + T\frac{d\Delta F(T)}{dT}\right)\Big|_{T=T_*}$$

One key parameter in phase transition gravitational wave

$$\alpha = \frac{\epsilon_*}{\rho^*_{\rm rad}} = \frac{\epsilon_*}{\frac{\pi^2}{30} g_* T_*^4}$$

Hubble parameter at the temperature T_*

$$H_* = \sqrt{\frac{8\pi^3 g_*}{90}}\frac{T_*^2}{m_{pl}}$$

The contribution of the GW energy density from phase transition

$$h^2\Omega(f) = h^2\Omega_{en}(f) + h^2\Omega_{sw}(f) + h^2\Omega_{tu}(f)$$

$$h^2\Omega_{en}(f) = 3.5\times 10^{-5}\left(\frac{0.11v_b^2}{0.42+v_b^2}\right)\left(\frac{H_*}{\tau}\right)^2\left(\frac{\kappa\alpha}{1+\alpha}\right)^2\left(\frac{10}{g_*}\right)^{\frac{1}{3}} S_{en}(f)$$
$$h^2\Omega_{sw}(f) = 5.7\times 10^{-6}\left(\frac{H_*}{\tau}\right)\left(\frac{\kappa_v\alpha}{1+\alpha}\right)^2\left(\frac{10}{g_*}\right)^{\frac{1}{3}} v_b S_{sw}(f)$$
$$h^2\Omega_{tu}(f) = 7.2\times 10^{-4}\left(\frac{H_*}{\tau}\right)\left(\frac{\kappa_{tu}\alpha}{1+\alpha}\right)^{\frac{3}{2}}\left(\frac{10}{g_*}\right)^{\frac{1}{3}} v_b S_{tu}(f)$$

The spectral shapes of GWs are characterized by the numerical fits as

$$S_{en}(f) = \frac{3.8\left(\frac{f}{f_{en}}\right)^{2.8}}{1+2.8\left(\frac{f}{f_{en}}\right)^{3.8}}$$

$$S_{sw}(f) = \left(\frac{f}{f_{sw}}\right)^3\left(\frac{7}{4+3\left(\frac{f}{f_{sw}}\right)^2}\right)^{\frac{7}{2}}$$

$$S_{tu}(f) = \frac{\left(\frac{f}{f_{tu}}\right)^3}{\left(1+\frac{f}{f_{tu}}\right)^{\frac{11}{3}}\left(1+\frac{8\pi f}{h_*}\right)}$$

Peak frequency of each GW spectrum

$$f_{en} = 11.3\times 10^{-9}[\text{ Hz}]\left(\frac{f_*}{\tau}\right)\left(\frac{\tau}{H_*}\right)\left(\frac{T_*}{100\text{MeV}}\right)\left(\frac{g_*}{10}\right)^{\frac{1}{6}}$$

$$f_{sw} = 1.3\times 10^{-8}[\text{ Hz}]\left(\frac{1}{v_b}\right)\left(\frac{\tau}{H_*}\right)\left(\frac{T_*}{100\text{MeV}}\right)\left(\frac{g_*}{10}\right)^{\frac{1}{6}}$$

$$f_{tu} = 1.8\times 10^{-8}[\text{ Hz}]\left(\frac{1}{v_b}\right)\left(\frac{\tau}{H_*}\right)\left(\frac{T_*}{100\text{MeV}}\right)\left(\frac{g_*}{10}\right)^{\frac{1}{6}}$$

BH thermodynamics of hot dense hQCD

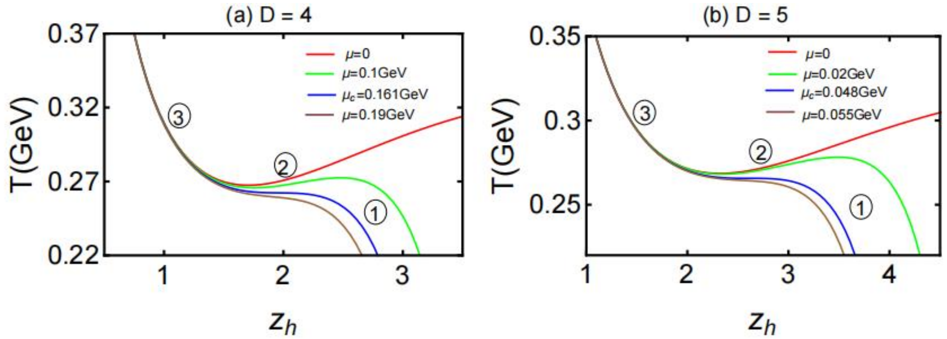


图16. D维下温度跟视界关系

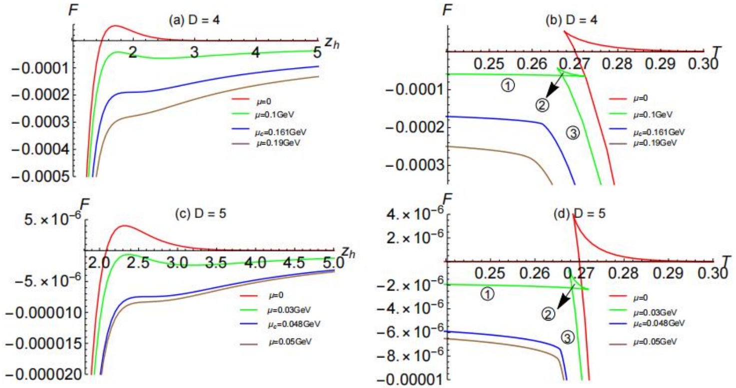


图17. D维下的自由能

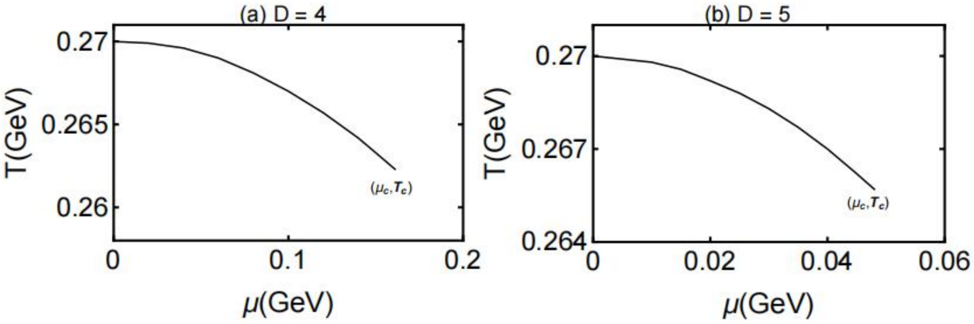


图18. D维下相图

1） 一阶相变附近温度、自由能非单调；平滑过渡附近单调；

2） 维度较低时，临界μ_{CEP}更大

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