

**Quarks and Compact Stars (QCS2023) Sept. 22-26, 2023, Yangzhou** 

# **The equations of state of compact star from machine learning**

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**11/20/13 23/09/2023 Jinniu Hu** 



## ! **Introduction**

- ! **The hadron-quark crossover from Gaussian Process**
- ! **The EOS of neutron star from deep neural network**
- ! **The phase transitions from DNN**
- ! **Summary**

# **The radii and masses**



#### **Shapiro delay measurement**



**The massive neutron star PSR J1614-2230 (1.928±0.017 M**⊙**), P. B. Demorest, et al., Nature. 467(2010)108 E. Fonseca et al., Astrophys. J. 832, 167 (2016). PSR J0348+0432 (2.01±0.04 M**⊙**), P. J. Antoniadis et al., Science 340, 1233232 (2013). PSR J0740+6620 (2.08±0.07 M**⊙**) H. T. Cromartie et al., Nat. Astron. 4, 72 (2020) M. C. Miller et al. Astrophys. J. Lett. 918(2021)L28 PSR J0952+0607 (2.35±0.17 M**⊙**) R. W. Romani et al. Astrophys. J. Lett. 934(2022)L17**

#### **Neutron Star Interior Composition Explorer**



#### **The NICER Measurement PSR J0740+6620 (2.08±0.07 M**⊙**,**

 **12.35±0.75 km) H. T. Cromartie et al., Nat. Astron. 4, 72 (2020) M. C. Miller et al. Astrophys. J. Lett. 918(2021)L28 PSR J0030+0451 (1.44±0.15M**⊙**,** 

 **13.02±1.24 km) M. C. Miller et al. Astrophys. J. Lett. 887(2019)L42**

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### **The data of neutron star**





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**D. Farrell, et al. J. Cosm. Astro. Phys. 2(2023)016** 



Jinniu Hu



#### ➢ **Parametric Bayesian inference**

 **F. Özel, G. Baym, and T. Güver, Phys. Rev. D 82 (2010) 101301(R) A. W. Steiner, J. M. Lattimer, and E. Brown, Astrophys. J 722(2010)33 D. Alvarez-Castillo, et al. Eur. Phys. J. A 52 (2016) 69 Z. Miao, J. L. Jiang, A. Li, and L. W. Chen, Astrophys. J. Lett. 917 (2021) L22** 

# ➢ **Nonparametric Bayesian inference P.Landry and R. Essick, Phys. Rev. D 99 (2019) 084049**

 **P.Landry, R. Essick, and K. Chatziioannou, Phys. Rev. D 101 (2020) 123007 M. Han, J. Jiang, S. Tang, Y. Fan, Astrophys. J. 919 (2021) 11**

# ➢ **Support Vector Machine P. Magierski and P. H. Heenen, Phys. Rev. C 65(2002)045804**

➢ **Deep neutral network Y. Fujimoto, K. Fukushima, K. Murase, Phys. Rev. D, 98 (2018) 023019** 

 **Y. Fujimoto, K. Fukushima, K. Murase, JHEP, 2021 (2021) 1** 

 **D. Farrell, et al. J. Cosm. Astro. Phys. 2(2023)016** 

 **L. Guo, J. Xiong, Y. Ma, Y. Ma, arXiv:2309.11227**

**……**



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# **Quark-Hadron pasta phase**



**2. CONSTRUCTION:** Red. 2. CONSTRUCTION:  $RED$ : quark matter<br>**2. CONSTRUCTION:**  $BLUE$ : hadron matt BLUE: hadron matter



### **Maxwell Gibbs 3-Window**

- Clear phase boundary.
- No mix phase.



- Clear phase boundary.
- With mix phase.



• No clear phase boundary

#### **Courtesy from Bai Zhan's slide**

#### **23/09/2023 Jinniu Hu** 8

exist at present, we will consider a phenomenological "interpolation" between the H-EOS and  $t_{\rm B}$  is always uniform of each phase. In our case  $\sim$  $\frac{W}{\sqrt{W}}$ *µ*<sub>*N*</sub> $\alpha$ <sup>*N*</sup><sub>*c*</sub>  $\sim$  $\frac{1}{\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac{1}{2}}\sqrt{1-\frac$ 

#### **The Gaussian process in 3-windows**  $\mathbb{R}$  as a first step . Such an interpretation is certainly not unique: Here we consider the constant of  $\mathbb{R}$ simplest possibilities, *P*-interpolation and "-interpolation as described below. **The Gaussian process in 3-windows** (KAN) 4 3 given baryon density. the dominant pairing in the quarkyonic matter is, instead of  $\alpha$  not operations do not over  $\alpha$  over  $\alpha$  of the stiffe of the state sta  $\lim_{n \to \infty} \frac{1}{n}$  iii s-windows  $\lim_{n \to \infty} \frac{1}{n}$

#### **The Pressure interpolated method FRESSUTE INTETPOLATED METHOD**<br>F. Baym. et al., Rep. Prog. Phys. 8  $\overline{\phantom{a}}$ chiral participations of the pairs of the condensation condensities and condensities and chi-<br>Condensation condensations condensations and condensations condensations and chiconstruction (see !gure 14 and related discussion above) are a interpolated method  $\alpha$  unit-unitation. Furthermore, the unit-**Edge construction can encompass 12018)056902**<br>**G. Baym, et al., Rep. Prog. Phys. 81(2018)056902**

 $P(\rho)$  =  $P_H(\rho)w_-(\rho) + P_Q(\rho)w_+(\rho),$  (400)  $w_{\pm}(\rho) = \frac{1}{2}$  $\bigg(1 \pm \tanh\left(\frac{\rho - \bar{\rho}}{\Gamma}\right)$  $\binom{2}{2}$  ,  $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$   $\frac{1}{2}$  $\mathbf{E}(\lambda)$ *P*  $= \frac{1}{2} (1 \pm i)^2$  $\left(1 \pm \tanh\left(\frac{\rho-\rho}{\Gamma}\right)\right), \qquad \begin{array}{c} \frac{\varepsilon}{8} \\ \frac{\varepsilon}{8} \\ \frac{\varepsilon}{2} \end{array} \qquad \qquad \begin{array}{c} \end{array}$  $(0)$ ,  $(400 - 400)$  $P(\rho$  $\overline{2}$  $P_H(\rho)w_-(\rho)+P_Q(\rho)w_+(\rho), \hspace{1cm}$  400). And  $\hspace{0.1cm} \left.\begin{matrix} \rho\end{matrix}\right\}$ equation of  $\left(\rho-\bar{\rho}\right)$  is the state of  $\left(\rho-\bar{\rho}\right)$  $r=\frac{1}{2} \left(1 \pm \tanh \left(\frac{\pi}{\Gamma}\right)\right), \qquad \frac{1}{2} \left(1 \pm \frac{1}{2} \right)$ 

**The energy interpolated method**  where **P***P* and **P***N* and *N* and the matter and the matter and the matter and the quark matter and the quark matter and the quark matter and the set of the se  $\mu = \mu_{BL}$   $\mu' = \mu_{BL}$   $\mu' = \mu_{BL}$ *N*

$$
\varepsilon(\rho) = \varepsilon_H(\rho) w_-(\beta) + \varepsilon_Q(\rho \frac{\partial w_+}{\partial \rho_+}(\rho) + \Delta \varepsilon \qquad g(\rho) = \frac{2}{\Gamma} (\varepsilon^X + e^{-X})^{-2},
$$
  
\n
$$
\Delta \varepsilon = \rho \int_{\bar{\rho}}^{\rho} (\varepsilon_H(\rho \theta) + \varepsilon_Q(\rho \theta)) \frac{g(\rho')}{\rho'} d\rho' \qquad \eta_{\mathbf{X}}(\mu_{\mathbf{BL}}) \frac{\rho \cdot 2\eta}{\Gamma}.
$$



**P 2 2 高** 

("*H*(⇢<sup>0</sup>

) "*Q*(⇢<sup>0</sup>

and is shown by the shaded area on the horizontal axis. The filled circles denote the onset  $\sim$  strangeness degrees of  $\sim$  strange  $\sim$  strang **THE SUPERING STATE INTERACTION CONTROL**  $\mathbb{E}[\mathbb{$  $\mathcal{L}(\mathcal{L}(\mathcal{A}) \setminus \mathcal{A})$  is not the asymptotic form of the asymptotic form of the  $\mathcal{L}(\mathcal{A})$ **P(** $\frac{1}{2}$ ) and  $\frac{1}{2}$ . Therefore, naive extrapolation of H-EOS and  $\frac{1}{2}$ their application would miss essential physics. To see the sti $\mathcal{L}$ 

The chemical potential interpolated method construction. The hadronic equation of state is used only at  $\overline{\phantom{a}}$  ine chemi l notantial internalated method i. Poloninal linoi polatoa

$$
\mathcal{P}(\mu_{\text{BL}}) = P_{\text{H}}(\mu_{\text{BL}}), \frac{\partial \mathcal{P}}{\partial \mu_{\text{B}}}\Big|_{\mu_{\text{BL}}} = \frac{\partial P_{\text{H}}}{\partial \mu_{\text{B}}}\Big|_{\mu_{\text{BL}}} , \dots
$$

$$
\mathcal{P}(\mu_{\text{BU}}) = P_{\text{Q}}(\mu_{\text{BU}}), \frac{\partial \mathcal{P}}{\partial \mu_{\text{B}}}\Big|_{\mu_{\text{BU}}} = \frac{\partial P_{\text{Q}}}{\partial \mu_{\text{B}}}\Big|_{\mu_{\text{BU}}}, \dots
$$

**23/09/2023 Jinniu Hu** 9 which guarantees the thermodynamic consistency. Note that the energy per baryon from th  $\frac{1}{2}$  baryons. rB − 0.5 **fm, document** perco

Jinniu Hu a matter of choice. Matter of choice. Matter of choice. Matter of choice.  $\mathcal{A}$ 

 $\frac{18}{12}$ 

*,* (4.6)

### **The Gaussian process**

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**Assume the function**  to satisfy Assume that: **Assume the function** K. Huang, J. N. Hu, Y. Zhang, and H. Shen, Astrophys. J. 935(2022)88  $\mathcal{F}(\mathcal{L})$  is a function with the following properties,  $\mathcal{L}(\mathcal{L})$   $\mathcal{L}(\mathcal{L})$   $\mathcal{L}(\mathcal{L})$  $y = f(x)$ .

To satisfy  
\n
$$
\begin{bmatrix} f(x_1) \\ f(x_1) \\ \vdots \\ f(x_n) \end{bmatrix} \sim \mathcal{N} \left( \begin{bmatrix} \mu(x_1) \\ \mu(x_2) \\ \vdots \\ \mu(x_n) \end{bmatrix}, \begin{bmatrix} \kappa(x_1, x_1) & \kappa(x_1, x_2) & \dots & \kappa(x_1, x_n) \\ \kappa(x_2, x_1) & \kappa(x_2, x_2) & \dots & \kappa(x_2, x_n) \\ \vdots & \vdots & \ddots & \vdots \\ \kappa(x_n, x_1) & \kappa(x_n, x_2) & \dots & \kappa(x_n, x_n) \end{bmatrix} \right)
$$
\nThe

observation data is *f*(*xn*) *µ*(*xn*) **The observation data is**   $\mathcal{H}$  to do inference with the Gaussian process? *•* Observed data:

$$
(\mathbf{x}_1,y_1), (\mathbf{x}_2,y_2), \cdots, (\mathbf{x}_n,y_n)
$$

For simplicity we will use *µ* = 0. The prediction value of f is

$$
\begin{bmatrix} f(x_1) \\ f(x_2) \\ \vdots \\ f(x_n) \end{bmatrix} \sim \mathcal{N} \left( \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} \kappa(x_1, x_1) & \kappa(x_1, x_2) & \dots & \kappa(x_1, x_n) \\ \kappa(x_2, x_1) & \kappa(x_2, x_2) & \dots & \kappa(x_2, x_n) \\ \vdots & \vdots & \ddots & \vdots \\ \kappa(x_n, x_1) & \kappa(x_n, x_2) & \dots & \kappa(x_n, x_n) \end{bmatrix} \begin{bmatrix} \kappa(x_1, x_2) \\ \kappa(x_2, x_3) \\ \vdots \\ \kappa(x_n, x_n) \end{bmatrix} \right)
$$

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∪ Inniu *K*(X*,* X) *K*(*x*?*,* X)



#### **It can be use the matrix notation**  *f*(*x*⇤) 0 (*x*⇤*, x*1) (*x*⇤*, x*2) *...* (*xn, x*⇤) (*x*⇤*, x*⇤) It can be use the matrix notation

$$
\begin{bmatrix} \mathbf{y} \\ f(x_{\star}) \end{bmatrix} \sim \mathcal{N}\left( \begin{bmatrix} \mathbf{0} \\ 0 \end{bmatrix}, \begin{bmatrix} K(\mathbf{X}, \mathbf{X}) & K(x_{\star}, \mathbf{X}) \\ K(\mathbf{X}, x_{\star}) & K(x_{\star}, x_{\star}) \end{bmatrix} \right)
$$

## **where the mean function is zero for notational simplicity.**  The distribution of prediction point can be obtained

 $f(x_{\star})$  |  $\mathbf{v} \sim \mathcal{N}$  |  $K(x_{\star}, \mathbf{X})K(\mathbf{X}, \mathbf{X})$   $^{-1}\mathbf{v}$ ,  $K(x_{\star}, x_{\star}) - K(x_{\star}, \mathbf{X})K(\mathbf{X}, \mathbf{X})$   $^{-1}K(\mathbf{X}, x_{\star})$  |  $f(x_{\star}) | \mathbf{y} \sim \mathcal{N} \left( K(x_{\star}, \mathbf{X}) K(\mathbf{X}, \mathbf{X})^{-1} \mathbf{y}, K(x_{\star}, x_{\star}) - K(x_{\star}, \mathbf{X}) K(\mathbf{X}, \mathbf{X})^{-1} K(\mathbf{X}, x_{\star}) \right)$ 





#### **The Hadron and Quark phases**  $2014$ a), the nucleons interact with each other by exchanging  $\mathbf{r}$ ron and Quark phases vector-isovector meson meson  $\overline{\phantom{a}}$



1

1

1

#### **The Lagrangian of Hadron phase**   $\blacksquare$  interaction between the unit is involved in and  $\rho$  mesons is involved in and  $\rho$ **2 Lagrangian of He**  $\overline{\phantom{a}}$  $n$  $of$   $H$ Lagrangia 4 agrangian of Hadi



#### **P** arangian of Quark phase the state  $\frac{1}{3}$ The Lagrangian of Quark phase  $\cup$

$$
\mathcal{L}_{\text{NJL}} = \overline{q}(i\gamma^{\mu}\partial_{\mu} - m)q + G_{S} \sum_{a=0}^{8} [(\overline{q}\lambda_{a}q)^{2} \qquad \sum_{\substack{\delta \in \mathbb{Z} \\ \delta \geq 2 \\ \Delta \epsilon}}^{R} (i\gamma^{\mu}\partial_{\mu} - m)q + G_{S} \sum_{a=0}^{8} [(\overline{q}\lambda_{a}q)^{2} \qquad \sum_{\substack{\delta \in \mathbb{Z} \\ \delta \geq 1 \\ \Delta}}^{R} (i\gamma^{\mu}\partial_{\mu} - m)q + G_{S} \sum_{a=0}^{8} [(\overline{q}\lambda_{a}q)^{2} \qquad \sum_{\substack{\delta \in \mathbb{Z} \\ \delta \geq 1 \\ \Delta}}^{R} (i\gamma^{\mu}\partial_{\mu} - m)q + G_{S} \sum_{a=0}^{8} [(\overline{q}\lambda_{a}q)^{2} \qquad \sum_{\substack{\delta \in \mathbb{Z} \\ \delta \geq 1}}^{R} (i\gamma^{\mu}\partial_{\mu} - m)q + G_{S} \sum_{a=0}^{8} [(\overline{q}\lambda_{a}q)^{2} \qquad \sum_{\substack{\delta \in \mathbb{Z} \\ \delta \geq 1}}^{R} (i\gamma^{\mu}\partial_{\mu} - m)q + G_{S} \sum_{a=0}^{8} [(\overline{q}\lambda_{a}q)^{2} \qquad \sum_{\substack{\delta \in \mathbb{Z} \\ \delta \geq 1}}^{R} (i\gamma^{\mu}\partial_{\mu} - m)q + G_{S} \sum_{a=0}^{8} [(\overline{q}\lambda_{a}q)^{2} \qquad \sum_{\substack{\delta \in \mathbb{Z} \\ \delta \geq 1}}^{R} (i\gamma^{\mu}\partial_{\mu} - m)q + G_{S} \sum_{a=0}^{8} [(\overline{q}\lambda_{a}q)^{2} \qquad \sum_{\substack{\delta \in \mathbb{Z} \\ \delta \geq 1}}^{R} (i\gamma^{\mu}\partial_{\mu} - m)q + G_{S} \sum_{a=0}^{8} [(\overline{q}\lambda_{a}q)^{2} \qquad \sum_{\substack{\delta \in \mathbb{Z} \\ \delta \geq 1}}^{R} (i\gamma^{\mu}\partial_{\mu} - m)q + G_{
$$

( ) *m c gg g g* 2 , *n p* 3 v *X X XS S S*  -  *X X S X* together with the current quark mass matrix m = diag(mu, md, *S* l<mark>ang, J. N. Hu, Y. Zhang, and H. Shen, Astrophys. J. 935(2</mark>02  $\sum_{i=1}^n$  is  $\sum_{i=1}^n$  and  $\sum_{i=1}^n$  order,  $\sum_{i=1}^n$  superpose of K. Huang, J. N. Hu, Y. Zhang, and H. Shen, Astrophys. J. 935(2022)88 <sub>or [fm<sup>-3</sup>]</sub>

2

*m gg*

2

 $\mathcal{L} = \mathbf{I}$  $\mathcal{L} = \mathcal{L} \mathcal{L} = \mathcal{L} \mathcal{L}$  $\frac{1}{2}$  ( )  $\frac{1}{2}$  ( )  $\frac{1}{2}$  ( )  $\frac{1}{2}$ *qi mq G q q qi q G q q Kq q q q* det 1 det 1 , *a V* r. *L* – 70<br>D. *C* – *C*  $\sim$  2  $\sim$  2  $\sim$ 5 5 5 5 5 6 *H M HM H H H*  $\frac{1}{2}$   $\mathcal{L} = \mathcal{L} \left( \mathcal{L} \right)$ アンディー アール・エンジン where  $\mathcal{N}$  is the quark field with the flavors and three colors and the colors and three colors and the colors and three colors and three colors and to gether with the current  $\mathcal{U}$ . The term relation of  $\mathcal{L}$  $\mathcal{U}^{\mathcal{I}}$  are the Gell–Mann matrices with  $\mathcal{I}$  $M<sup>2</sup>$   $\frac{1}{2}$ tional vector and axial-vector interactions to produce universal  $\mathcal{L}$  $r = 0.2$  and  $r = 0.6$  are  $r = 31$  and  $r =$  $\overline{P}$  and  $\overline{P}$  and ,<br>।<br>।  $\mathcal{N} = \mathcal{N}$  ( ) and ( ) a  $\frac{1}{2}$  ,  $\frac{1}{2}$  ,  $\frac{1}{2}$  $300$   $\uparrow$ GPR ⎜ ⎟ ⎠ ⎝ *S*  $\rightarrow$  S  $300 \frac{L}{L}$  GPR  $\sum_{i=1}^{\infty}$  **F** the symmetry is  $\sum_{i=1}^{\infty}$  $\overline{a}$   $^{150}$   $\overline{b}$   $^{150}$   $\overline{d}$   $^{150}$  ence the  $\mathbb{Z}$  is linearly correlated with the neutron skinned with the neutron skinned sk  $\mathcal{D}$  of 208Pb. However, the uncertainty in the presentation in the presentation in the presentation in the presentation of  $\mathcal{D}$ measurements,  $\mathcal{L}$  $2\sqrt{2\pi\epsilon^2}$  , and the slope inferrent use  $2\sqrt{2\epsilon}$ neutron-rich systems, several new parameter systems, several new parameter sets based on the system of the system on the system of the sys

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 $v_{\rm c}$  mesons, including the scalar-isoscalar mesons, including the scalar-isoscalar meson ( $\sigma$ 

### **The speed of sound**



#### **The M-R relations of neutron star**





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**Y. Fujimoto, K. Fukushima, K. Murase, Phys. Rev. D, 98 (2018) 023019** 



#### **23/09/2023 Jinniu Hu** 17 dependent in the calculation of the calculation process from the calculation process from the calculation proc<br>Tinnius the calculation process from the calculation process from the calculation process from the calculation  $\bullet$  left with input fixig to the right with output for  $\bullet$  . The right with output fyig.  $\mathbf{b}$  fixed  $\mathbf{b}$

 $\mathbf{r}$ 

in the contract of the contract of the final output from the final output from the contract of the L-th layer. The contract of Jinniu Hu

# **The representation of EOS**  $\qquad \qquad \mathbb{R}^n$



The spectral representation  $M$   $Z_{\text{low}}$   $T$  N  $\mu$ ,  $V$   $Z_{\text{long}}$  and  $\mu$  shan Astrophys  $T$   $0.50(2023)194$ W. Zhou, J. N. Hu, Y. Zhang, and H. Shen, Astrophys. J. 950(2023)186<br>A**tation**  $\mathcal{L}$  small set of basis functions. In contrast, we are the small set of basis functions. In contrast, we are the small set of  $\mathcal{L}$ 

 $\phi = \log \left( c^2 \frac{d\varepsilon}{dn} \right)$  $\frac{dE}{dp} - 1$  $\sum$  $\phi = \phi(\log p)$ <br>by  $\phi = \phi(\log p)$  $\setminus$  dp  $\setminus$ 

 $1 + e^{\phi}$ 

$$
\phi = \phi(\log p)
$$

 $\int_0^t 1 + e^{\phi}$ 

 $\mathsf{nd}$  and  $\mathsf{d}$  in  $\mathsf$ **and** 



<sup>ð</sup>σðα<sup>Þ</sup>

; lðα<sup>Þ</sup>



### **The training data**



6 **W. Zhou, J. N. Hu, Y. Zhang, and H. Shen, Astrophys. J. 950(2023)186**



Figure 2. The generation range of -ln *p*. We will randomly

<sup>430</sup> tron stars less than 2*.*2*M* and the *M*-*R* relations that  $\overline{\phantom{a}}$ 

#### The EOSs from the neural network ne cubs from the neural 4 40 August 2014 12:00 PM rne neural network

blom 2010). The EOS parameters of the prior



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#### The mass-radius relation <u>sa scheme for the training series</u>  $\mathbf{F}$ <sub>18</sub> of present framework is independent of the generation  $\overline{\phantom{a}}$  ing the training straining data.

 $\mathcal{S}_{\mathcal{S}}$  but in the final results of observables. This shows that the final results is shown that the final results is

 $\mathbf{S}(\mathbf{S})$  but in the final results of observables. This shows that the final results is shown that the final results is

### The tidal deformability





**23/09/2023 Jinniu Hu 22** nonparametric machine learning method, the observation

and compared to that from DDME1 and the values extracted to the values extracted to the values extracted to  $\sim$ 



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N piecewise polytropes representation

$$
p = K_i \varepsilon^{\Gamma_i} \quad (\varepsilon_{i-1} < \varepsilon < \varepsilon_i)
$$

The pressure values at the ith segment boundaries, p<sub>i</sub>, are **read as** 

$$
p_i = p_{i-1} + c_{s,i}^2 (\varepsilon_i - \varepsilon_{i-1}),
$$

c is the average speed of sound  $\boldsymbol{c}$ *c* is the average speed of sound

$$
\langle c_s^2 \rangle \equiv \int_{\varepsilon_{i-1}}^{\varepsilon_i} \frac{d\varepsilon}{\varepsilon_i - \varepsilon_{i-1}} c_s^2 = \int_{\varepsilon_{i-1}}^{\varepsilon_i} \frac{d\varepsilon}{\varepsilon_i - \varepsilon_{i-1}} \frac{\partial p}{\partial \varepsilon}
$$

$$
= \frac{1}{\varepsilon_i - \varepsilon_{i-1}} \int_{p_{i-1}}^{p_i} dp = \frac{p_i - p_{i-1}}{\varepsilon_i - \varepsilon_{i-1}} = c_{s,i}^2.
$$

*y*. Fujimoto, K. Fukushima, K. Murase, Phys. Rev. D, 98 (2018) 023019

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 $J$ *inniu* Hu

### **The speeds of sound from DNN**





FIG. 4: **W. Zhou, J. N. Hu, Y. Zhang, and H. Shen, in preparation**

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### **The EOS including the phase transition**



FIG. 2: **W. Zhou, J. N. Hu, Y. Zhang, and H. Shen, in preparation**

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FIG. 3: **W. Zhou, J. N. Hu, Y. Zhang, and H. Shen, in preparation**

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## ! **Introduction**

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## ! **Summary**

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**The hadron-quark crossover in neutron star was investigated by the Gaussian process**

**The nonparametric deep neural network with Gaussian process was applied to extract the equation of state of neutron star from the observations**

**The first-order hadron-quark phase transition was preliminarily discussed with deep neural network**

**The physics-informed neural networks will be considered.** 



# **Thank you very much for your attention!**



#### **Bayesian inference** To determine the most likely EoS, we can use the most likely EoS, we can use the MAP in the MAP in the MAP is t<br>The MAP is the MAP is produce informed D  $\overline{\phantom{a}}$ tional distribution, Proposition, Proposition, Proposition, Proposition, Proposition, Proposition, P

$$
f_{\text{MAP}}(\mathcal{D}) = \underset{\boldsymbol{\theta}}{\text{arg max}}[\Pr(\boldsymbol{\theta}) \Pr(\mathcal{D}|\boldsymbol{\theta})].
$$

#### $T_{\rm{max}}$  be interpreted as a and approximation of  $\epsilon$ Neural network: minimize **Meural**

$$
L[f] = \langle \ell[f] \rangle = \int d\theta d\mathcal{D} \Pr(\theta) \Pr(\mathcal{D}|\theta) \ell(\theta, f(\mathcal{D})).
$$

Neural network allows for more general choice of loss functions Bayesian inference assumes parametrized likelihood functions. is assumed to be Proposed to be posterior EoS distr<br>The posterior EoS distribution is a posterior EoS distribution is a posterior experimental to be proposed to b

[4] K. Fukushima and T. Hatsuda, Rep. Prog. Phys. 74, 014001