The equations of state of compact star from machine learning

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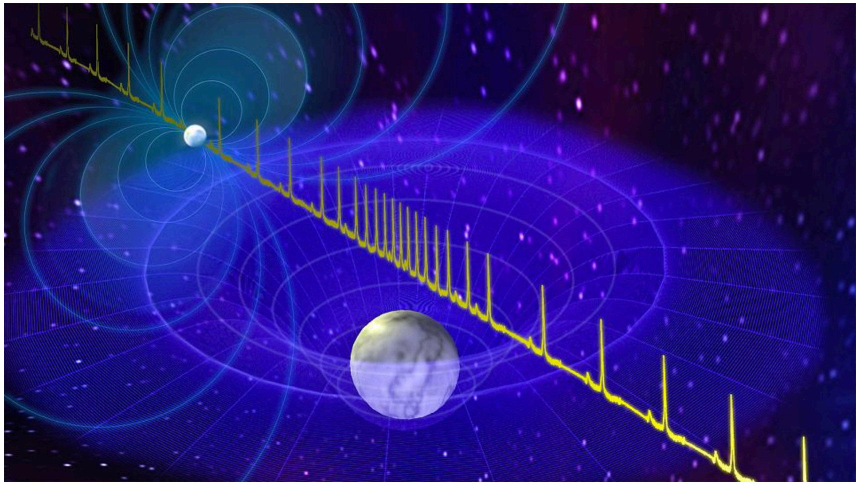
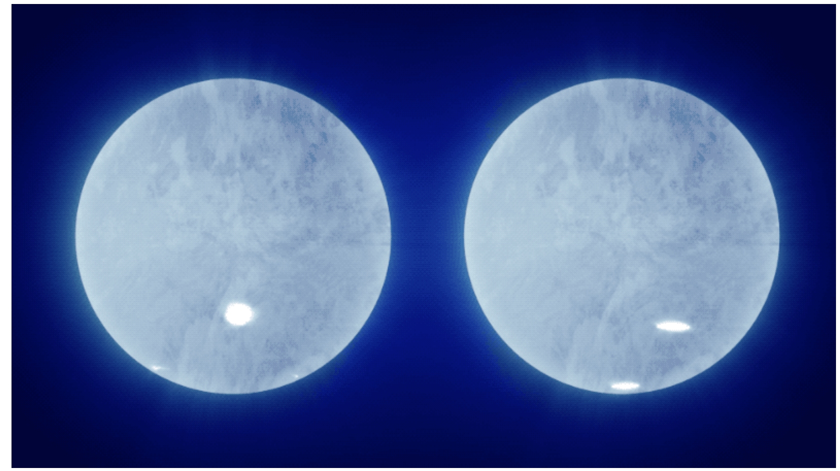
- Introduction
- The hadron-quark crossover from Gaussian Process
- The EOS of neutron star from deep neural network
- The phase transitions from DNN
- Summary

The radii and masses

Shapiro delay measurement



Neutron Star Interior Composition Explorer



The massive neutron star

PSR J1614-2230 (1.928±0.017 M⊙),

P. B. Demorest, et al., Nature. 467(2010)108

E. Fonseca et al., Astrophys. J. 832, 167 (2016).

PSR J0348+0432 (2.01±0.04 M⊙),

P. J. Antoniadis et al., Science 340, 1233232 (2013).

PSR J0740+6620 (2.08±0.07 M⊙)

H. T. Cromartie et al., Nat. Astron. 4, 72 (2020)

M. C. Miller et al. Astrophys. J. Lett. 918(2021)L28

PSR J0952+0607 (2.35±0.17 M⊙)

R. W. Romani et al. Astrophys. J. Lett. 934(2022)L17

The NICER Measurement

PSR J0740+6620 (2.08±0.07 M⊙, 12.35±0.75 km)

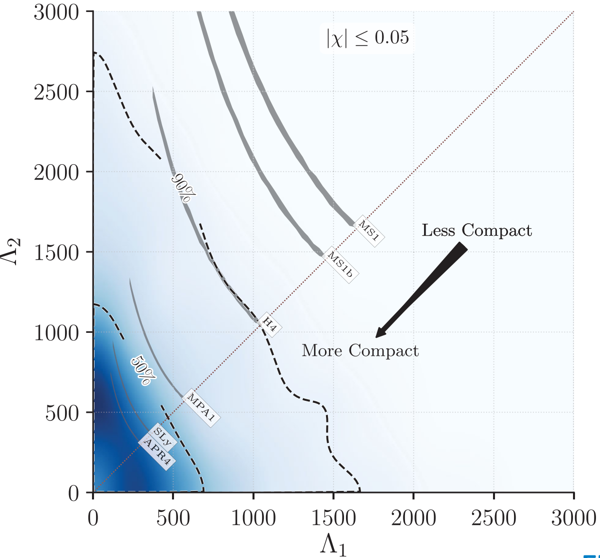
H. T. Cromartie et al., Nat. Astron. 4, 72 (2020)

M. C. Miller et al. Astrophys. J. Lett. 918(2021)L28

PSR J0030+0451 (1.44±0.15M⊙, 13.02±1.24 km)

M. C. Miller et al. Astrophys. J. Lett. 887(2019)L42

The data of neutron star



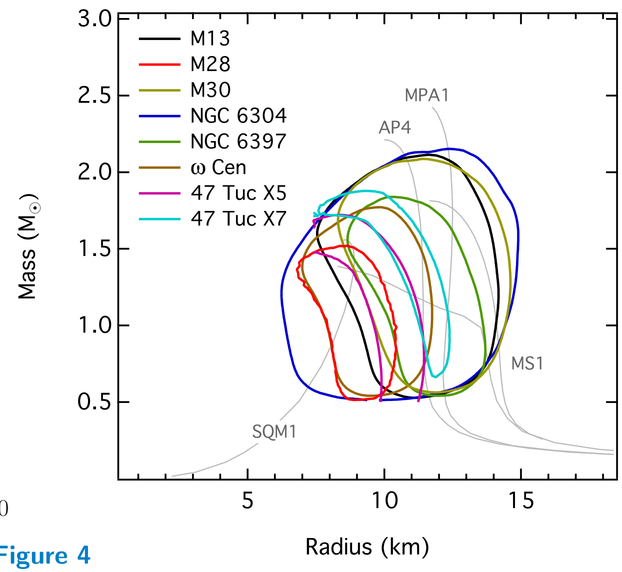
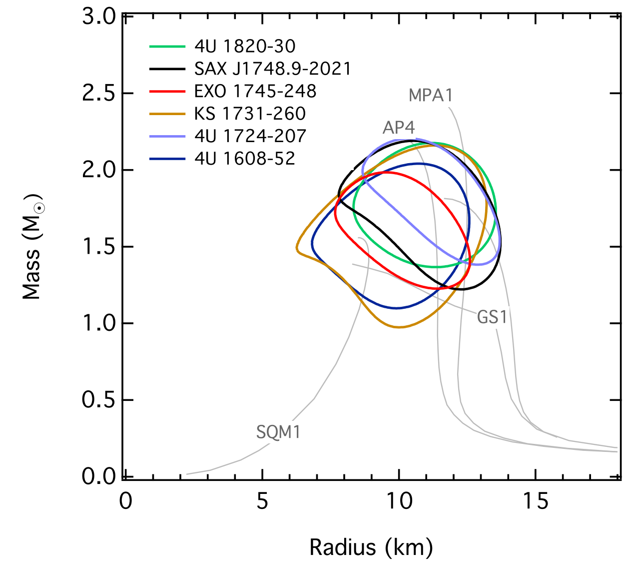
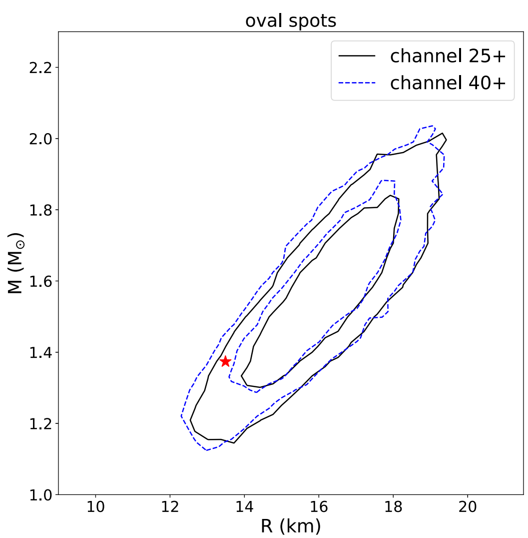
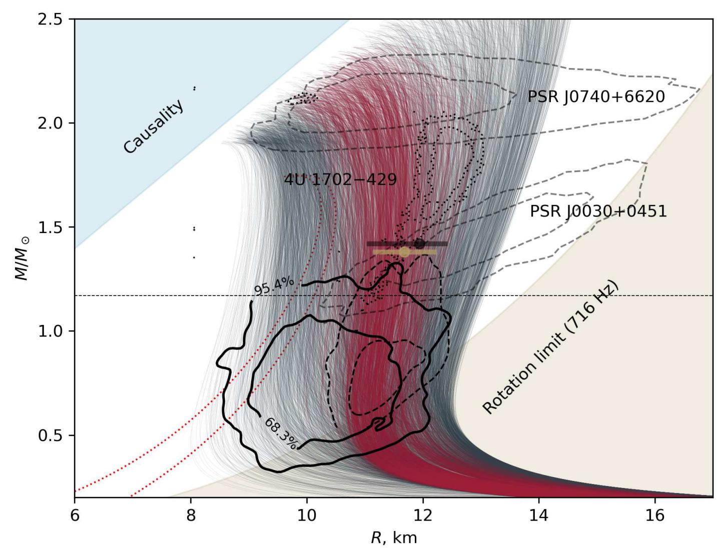
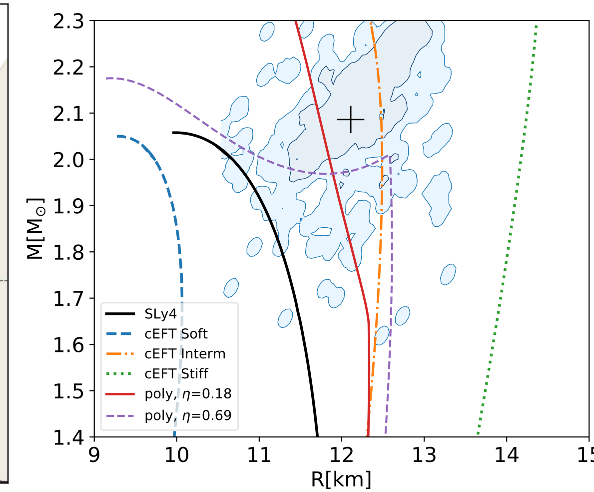


Figure 4









The data-driven methodologies

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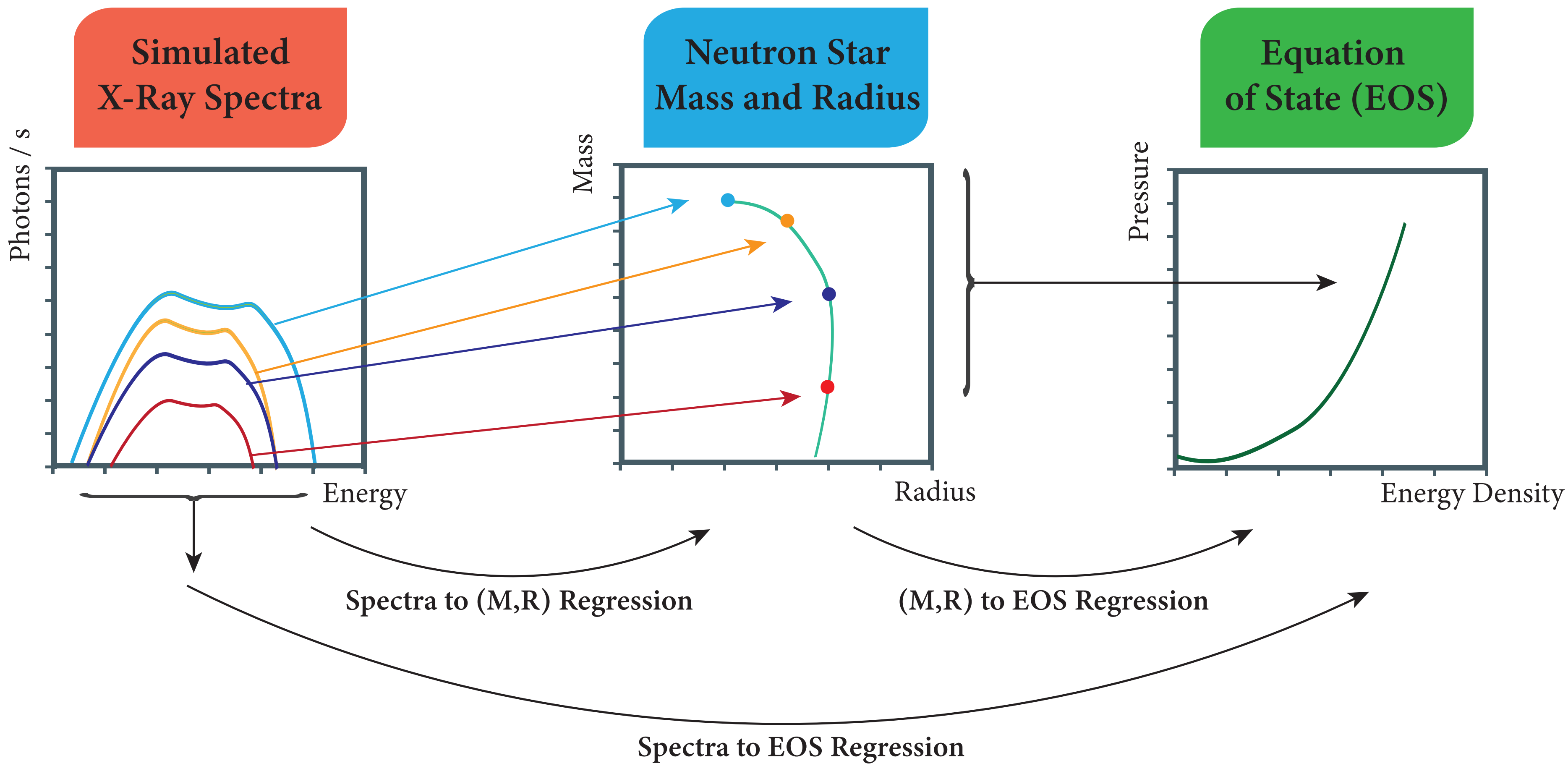
D. Farrell, et al. J. Cosm. Astro. Phys. 2(2023)016



The data-driven methodologies

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- **Parametric Bayesian inference**

  F. Özel, G. Baym, and T. Güver, Phys. Rev. D 82 (2010) 101301(R)
  A. W. Steiner, J. M. Lattimer, and E. Brown, Astrophys. J 722(2010)33
  D. Alvarez-Castillo, et al. Eur. Phys. J. A 52 (2016) 69
  Z. Miao, J. L. Jiang, A. Li, and L. W. Chen, Astrophys. J. Lett. 917 (2021) L22

- **Nonparametric Bayesian inference**

  P.Landry and R. Essick, Phys. Rev. D 99 (2019) 084049
  P.Landry, R. Essick, and K. Chatziioannou, Phys. Rev. D 101 (2020) 123007
  M. Han, J. Jiang, S. Tang, Y. Fan, Astrophys. J. 919 (2021) 11

- **Support Vector Machine**

  P. Magierski and P. H. Heenen, Phys. Rev. C 65(2002)045804

- **Deep neutral network**

  Y. Fujimoto, K. Fukushima, K. Murase, Phys. Rev. D, 98 (2018) 023019
  Y. Fujimoto, K. Fukushima, K. Murase, JHEP, 2021 (2021) 1
  D. Farrell, et al. J. Cosm. Astro. Phys. 2(2023)016
  L. Guo, J. Xiong, Y. Ma, Y. Ma, arXiv:2309.11227

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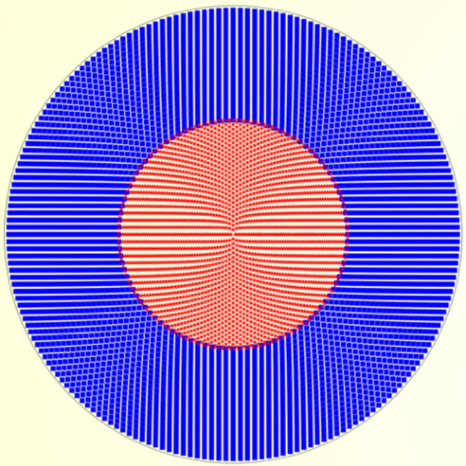
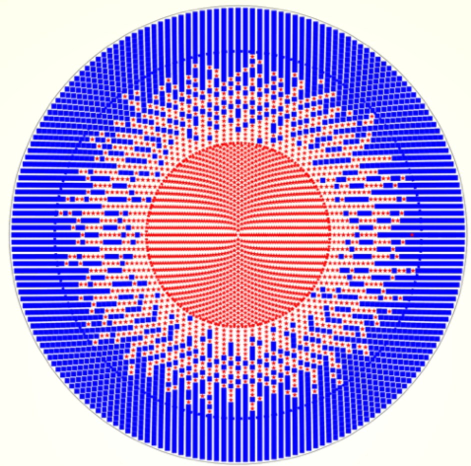
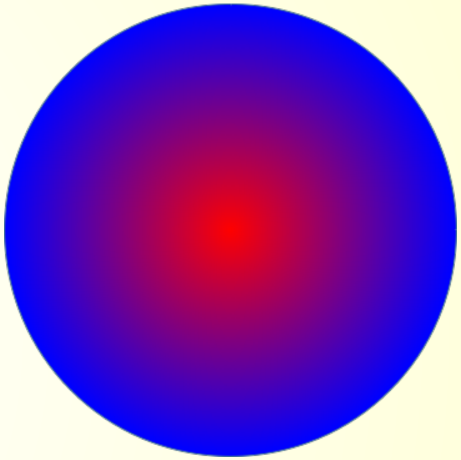
Quark-Hadron pasta phase







Maxwell

- Clear phase boundary.
- No mix phase.

Gibbs

- Clear phase boundary.
- With mix phase.

3-Window

- No clear phase boundary

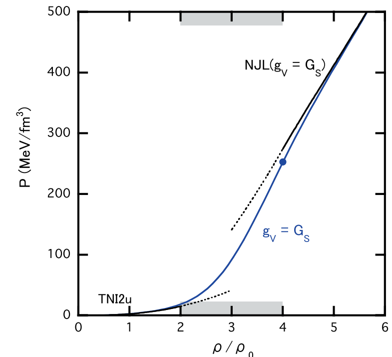
Courtesy from Bai Zhan's slide

The Gaussian process in 3-windows

The Pressure interpolated method

G. Baym, et al., Rep. Prog. Phys. 81(2018)056902

$$
\begin{aligned}
P(\rho) &= P_H(\rho)w_-(\rho) + P_Q(\rho)w_+(\rho),\\
w_\pm(\rho) &= \frac{1}{2}\left(1 \pm \tanh\left(\frac{\rho-\bar{\rho}}{\Gamma}\right)\right),
\end{aligned}
$$



The energy interpolated method

$$
\begin{aligned}
\varepsilon(\rho) &= \varepsilon_H(\rho)w_-(\rho) + \varepsilon_Q(\rho)w_+(\rho) + \Delta\varepsilon\\
\Delta\varepsilon &= \rho\int_{\bar{\rho}}^{\rho}(\varepsilon_H(\rho') - \varepsilon_Q(\rho'))\frac{g(\rho')}{\rho'}d\rho'
\end{aligned}
$$

$$
\begin{aligned}
g(\rho) &= \frac{2}{\Gamma}(e^X + e^{-X})^{-2},\\
X &= \frac{\rho-\bar{\rho}}{\Gamma}.
\end{aligned}
$$

The chemical potential interpolated method

$$
\mathcal{P}(\mu_{\rm BL}) = P_{\rm H}(\mu_{\rm BL}),\quad \left.\frac{\partial\mathcal{P}}{\partial\mu_{\rm B}}\right|_{\mu_{\rm BL}} = \left.\frac{\partial P_{\rm H}}{\partial\mu_{\rm B}}\right|_{\mu_{\rm BL}},\cdots
$$

$$
\mathcal{P}(\mu_{\rm BU}) = P_{\rm Q}(\mu_{\rm BU}),\quad \left.\frac{\partial\mathcal{P}}{\partial\mu_{\rm B}}\right|_{\mu_{\rm BU}} = \left.\frac{\partial P_{\rm Q}}{\partial\mu_{\rm B}}\right|_{\mu_{\rm BU}},\cdots.
$$

The Gaussian process

Assume the function K. Huang, J. N. Hu, Y. Zhang, and H. Shen, Astrophys. J. 935(2022)88

$$y = f(x).$$

to satisfy

$$\begin{bmatrix} f(x_1) \\ f(x_1) \\ \vdots \\ f(x_n) \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu(x_1) \\ \mu(x_2) \\ \vdots \\ \mu(x_n) \end{bmatrix}, \begin{bmatrix} \kappa(x_1,x_1) & \kappa(x_1,x_2) & \dots & \kappa(x_1,x_n) \\ \kappa(x_2,x_1) & \kappa(x_2,x_2) & \dots & \kappa(x_2,x_n) \\ \vdots & \vdots & \ddots & \vdots \\ \kappa(x_n,x_1) & \kappa(x_n,x_2) & \dots & \kappa(x_n,x_n) \end{bmatrix} \right)$$

The observation data is

$$(\mathbf{x}_1, y_1), (\mathbf{x}_2, y_2), \cdots, (\mathbf{x}_n, y_n)$$

The prediction value of f is

$$\left[\begin{array}{c} f(x_1) \\ f(x_2) \\ \vdots \\ f(x_n) \\ \hline f(x_*) \end{array}\right] \sim \mathcal{N}\left(\left[\begin{array}{c} 0 \\ 0 \\ \vdots \\ 0 \\ \hline 0 \end{array}\right], \left[\begin{array}{cccc|c} \kappa(x_1,x_1) & \kappa(x_1,x_2) & \dots & \kappa(x_1,x_n) & \kappa(x_1,x_*) \\ \kappa(x_2,x_1) & \kappa(x_2,x_2) & \dots & \kappa(x_2,x_n) & \kappa(x_2,x_*) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \kappa(x_n,x_1) & \kappa(x_n,x_2) & \dots & \kappa(x_n,x_n) & \kappa(x_n,x_*) \\ \hline \kappa(x_*,x_1) & \kappa(x_*,x_2) & \dots & \kappa(x_n,x_*) & \kappa(x_*,x_*) \end{array}\right] \right)$$

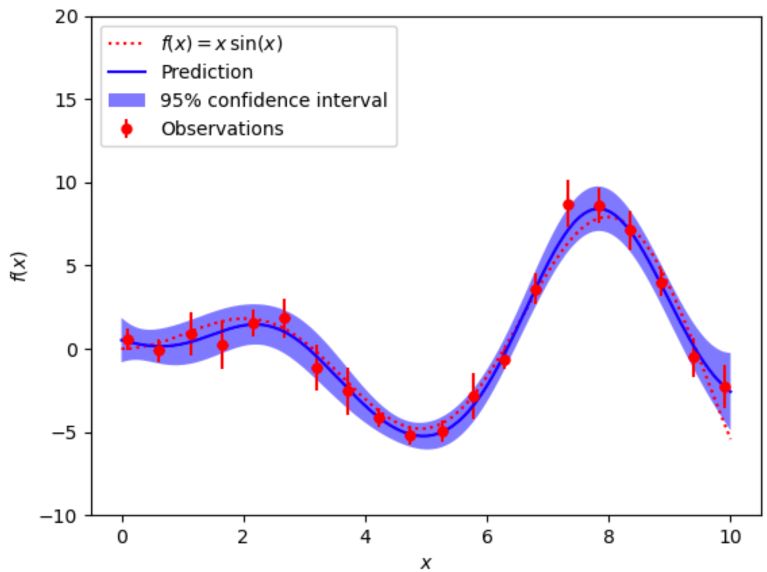
It can be use the matrix notation

$$\begin{bmatrix} \mathbf{y} \\ f(x_\star) \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mathbf{0} \\ 0 \end{bmatrix}, \begin{bmatrix} K(\mathbf{X},\mathbf{X}) & K(x_\star,\mathbf{X}) \\ K(\mathbf{X},x_\star) & K(x_\star,x_\star) \end{bmatrix}\right)$$

where the mean function is zero for notational simplicity.

The distribution of prediction point can be obtained

$$f(x_\star)\,|\,\mathbf{y} \sim \mathcal{N}\left(K(x_\star,\mathbf{X})K(\mathbf{X},\mathbf{X})^{-1}\mathbf{y}, K(x_\star,x_\star) - K(x_\star,\mathbf{X})K(\mathbf{X},\mathbf{X})^{-1}K(\mathbf{X},x_\star)\right)$$



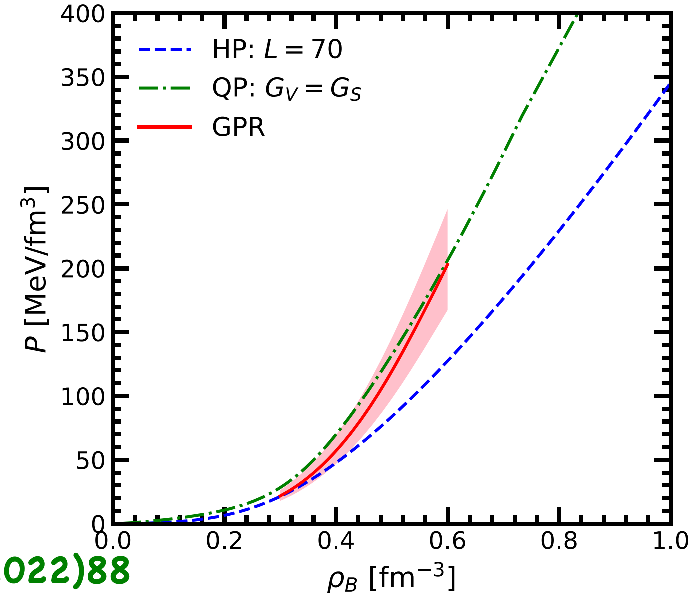
The Lagrangian of Hadron phase

$$
\begin{aligned}
\mathcal{L}_{\rm RMF} = & \sum_{i=n,p} \bar{\psi}_i \Big\{ i\gamma^\mu \partial_\mu - (M_i - g_\sigma \sigma) \\
& - \gamma^\mu \Big(g_\omega \omega_\mu + \frac{g_\rho}{2} \vec{\tau} \vec{\rho}_\mu \Big) \Big\} \psi_i \\
& + \frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 \\
& - \frac{1}{4} W^{\mu\nu} W_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu + \frac{1}{4} c_3 (\omega^\mu \omega_\mu)^2 \\
& - \frac{1}{4} \vec{R}^{\mu\nu} \vec{R}_{\mu\nu} + \frac{1}{2} m_\rho^2 \vec{\rho}^\mu \vec{\rho}_\mu + \Lambda_{\rm v} (g_\omega^2 \omega^\mu \omega_\mu)(g_\rho^2 \vec{\rho}^\mu \vec{\rho}_\mu),
\end{aligned}
$$

L(MeV)	50.0	70.0	90.0	110.0
g_ρ	12.8202	10.3150	9.3559	8.8192
$\Lambda_{\rm v}$	0.0420	0.0220	0.0098	0.0011
$E_{\rm sym}(\rho_0)$(MeV)	31.68	33.94	35.74	37.27
$R_{\rm skin}^{208}$ (fm)	0.1739	0.2278	0.2571	0.2770

The Lagrangian of Quark phase

$$
\begin{aligned}
\mathcal{L}_{\rm NJL} = & \bar{q}(i\gamma^\mu \partial_\mu - m) q + G_S \sum_{a=0}^{8} [(\bar{q}\lambda_a q)^2 \\
& + (\bar{q} i \gamma_5 \lambda_a q)^2] - G_V (\bar{q}\gamma^\mu q)^2 \\
& - K \{\det[\bar{q}(1+\gamma_5) q) + \det(\bar{q}(1-\gamma_5) q]\},
\end{aligned}
$$



K. Huang, J. N. Hu, Y. Zhang, and H. Shen, Astrophys. J. 935(2022)88

The speed of sound

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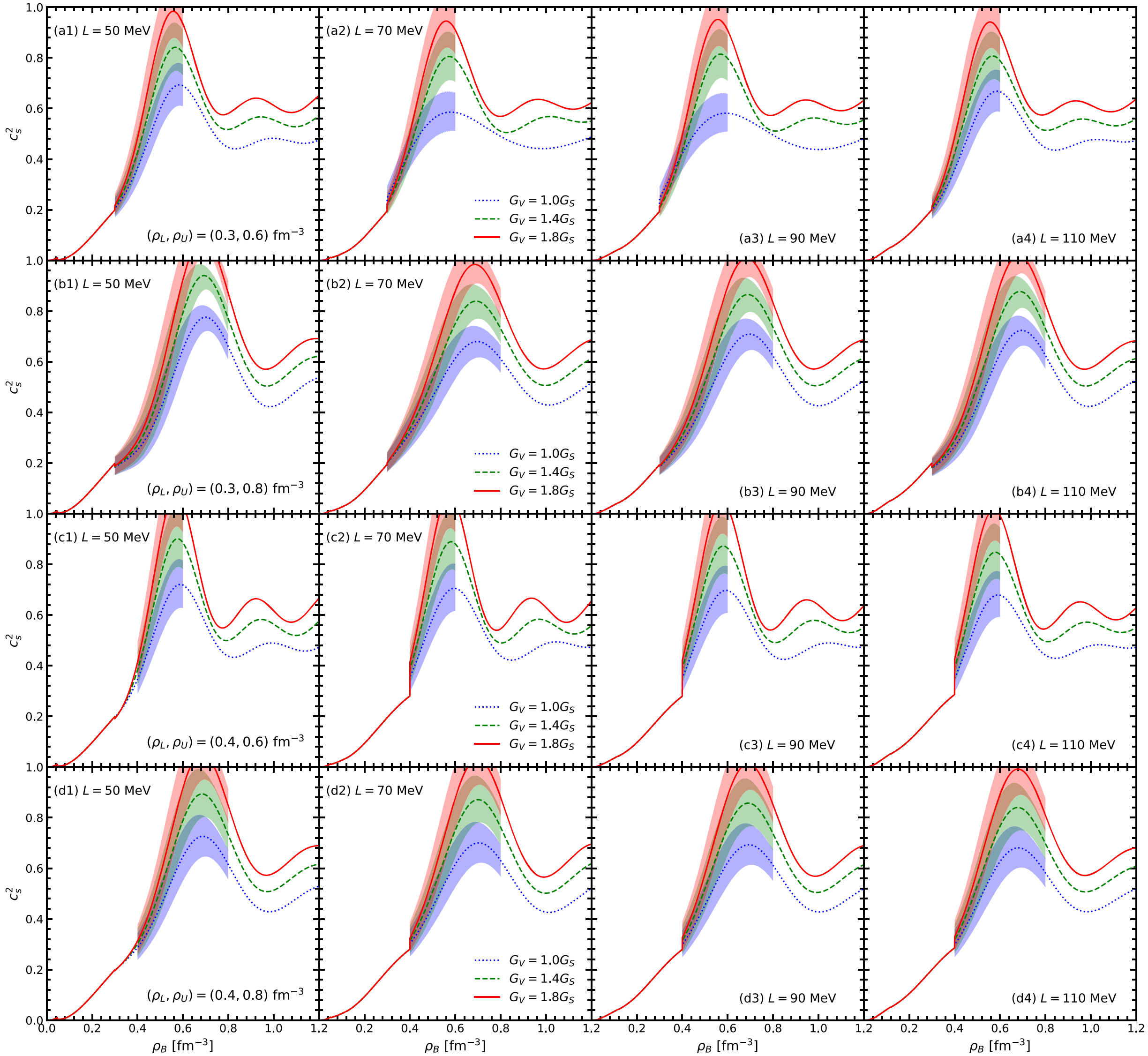


$(\rho_L, \rho_U) = (0.3, 0.6)$ fm^{-3}

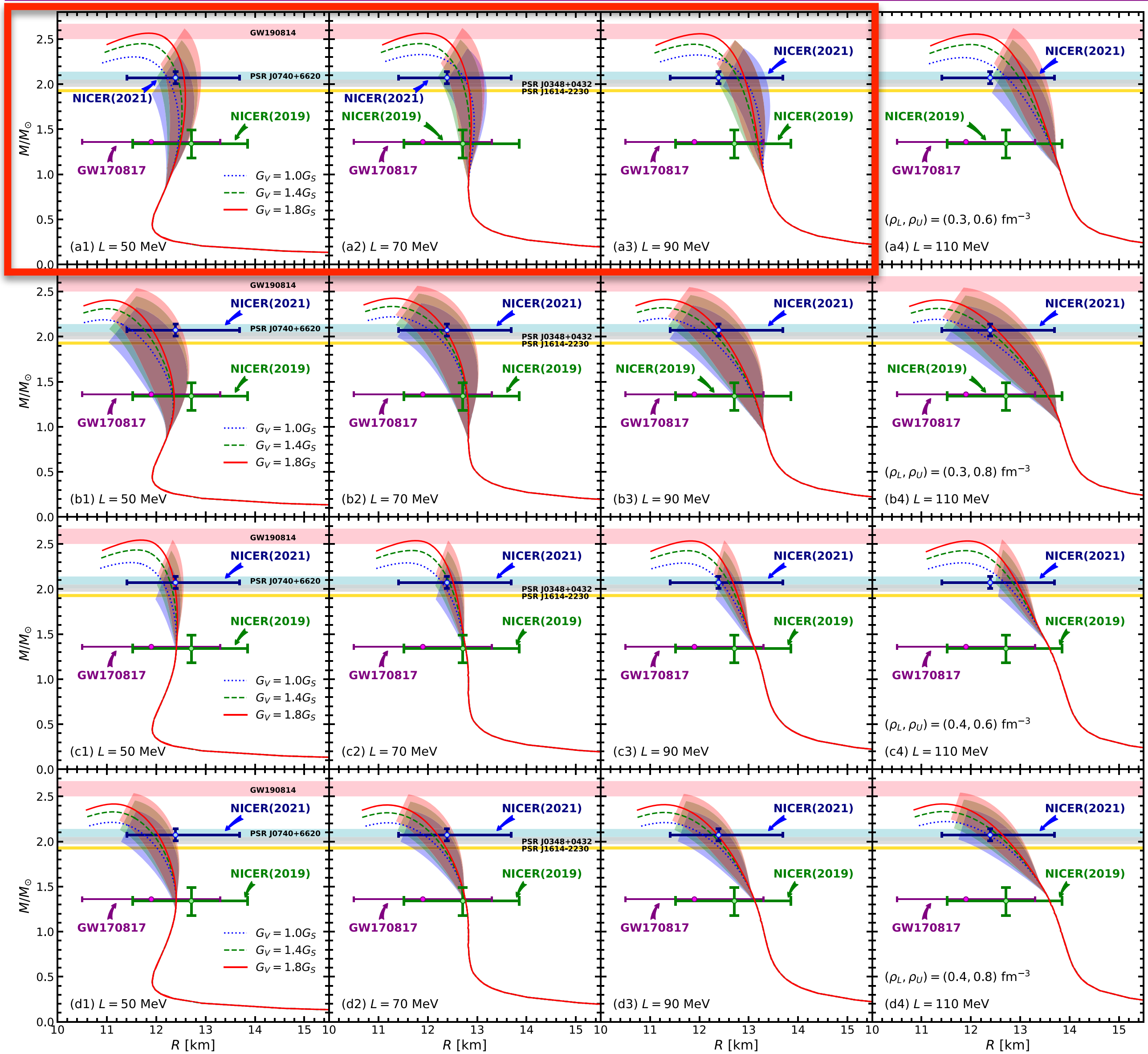
$(\rho_L, \rho_U) = (0.3, 0.8)$ fm^{-3}

$(\rho_L, \rho_U) = (0.4, 0.6)$ fm^{-3}

$(\rho_L, \rho_U) = (0.4, 0.8)$ fm^{-3}

K. Huang, J. N. Hu, Y. Zhang, and H. Shen, Astrophys. J. 935(2022)88

The M-R relations of neutron star



$(\rho_L, \rho_U) = (0.3, 0.6)$ fm^{-3}

$(\rho_L, \rho_U) = (0.3, 0.8)$ fm^{-3}

$(\rho_L, \rho_U) = (0.4, 0.6)$ fm^{-3}

$(\rho_L, \rho_U) = (0.4, 0.8)$ fm^{-3}

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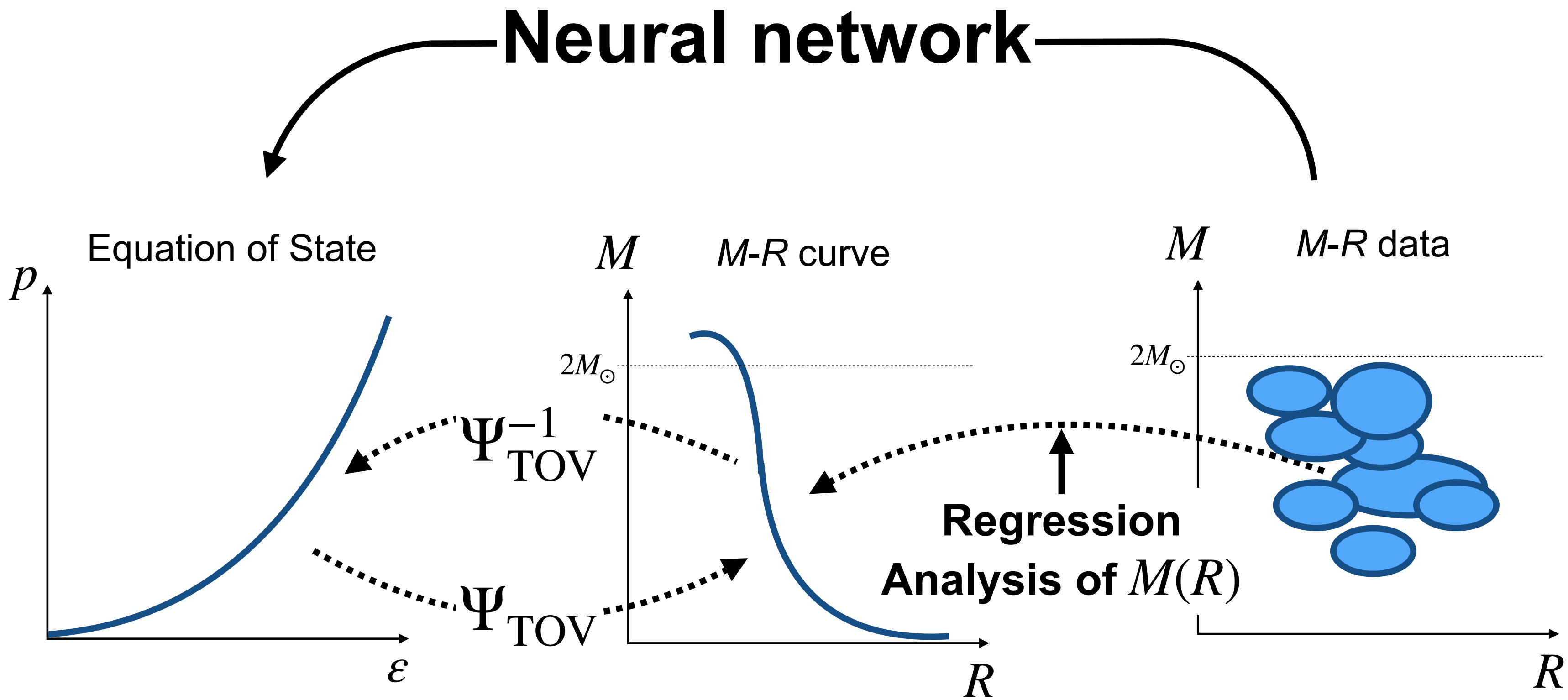
The Schematic of EOS from DNN



Y. Fujimoto, K. Fukushima, K. Murase, Phys. Rev. D, 98 (2018) 023019

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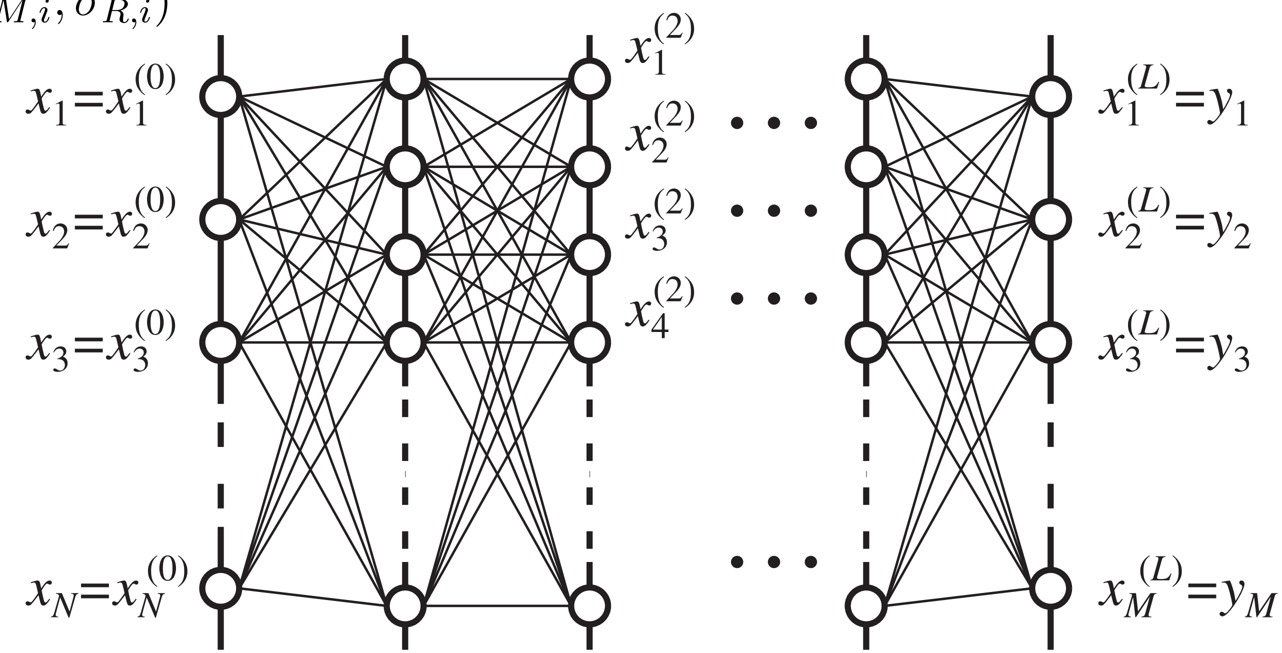
Input **Output**

$(M_i, R_i; \sigma_{M,i}, \sigma_{R,i})$

EOS



Finite number of variables

$$x_i^{(k+1)} = \sigma^{(k+1)}\left(\sum_{i=1}^{N_k} W_{ij}^{(k+1)} x_j^{(k)} + a_i^{(k+1)}\right),$$

The representation of EOS

W. Zhou, J. N. Hu, Y. Zhang, and H. Shen, Astrophys. J. 950(2023)186

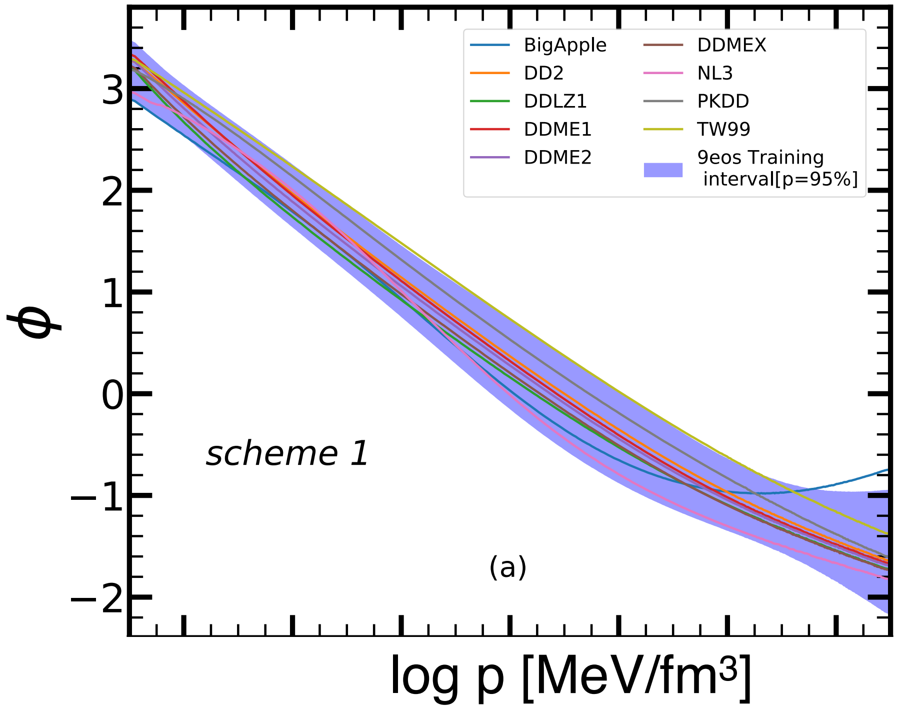
The spectral representation

$$\phi = \log\left(c^2 \frac{d\varepsilon}{dp} - 1\right).$$

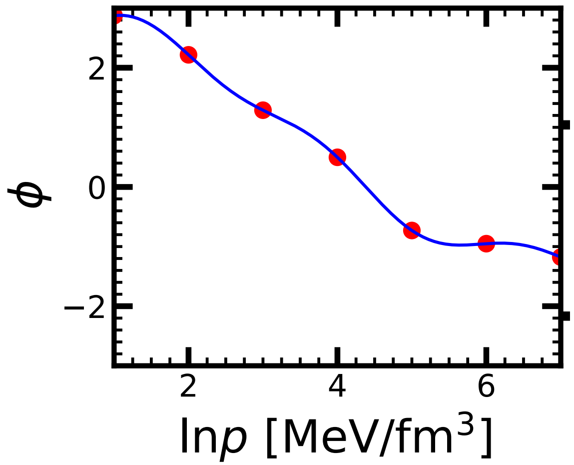
$$\phi = \phi(\log p)$$

and

$$\varepsilon = \int \frac{1 + e^{\phi}}{c^2} dp$$

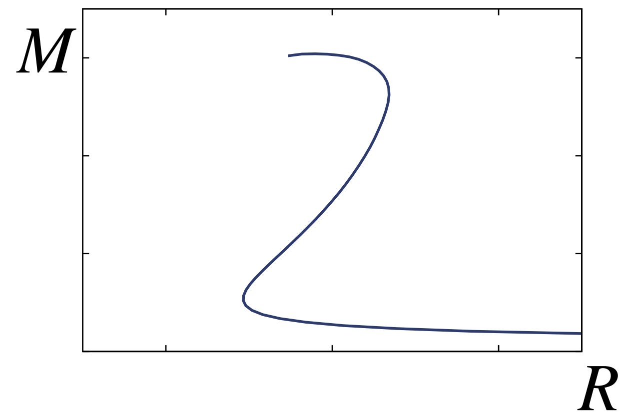


(1) Generate EoS randomly

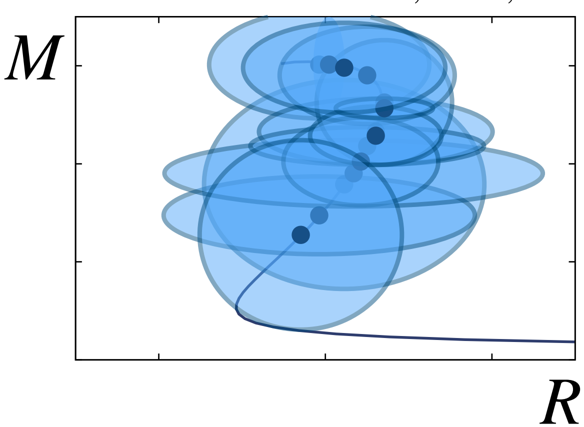


Solve TOV eq →

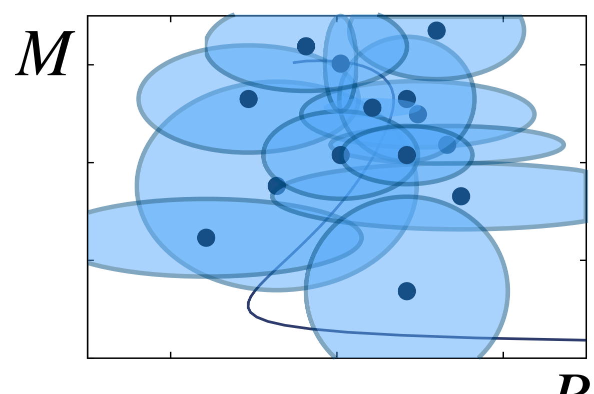
(2) Obtain M-R curve by solving TOV eq



(3) Sample 14 points on M-R curve. Each points are assigned with random errors $(\sigma_{M,i},\ \sigma_{R,i})$.



(4) Shift the points within the assigned errors $(\sigma_{M,i},\ \sigma_{R,i})$



$5,000,000$

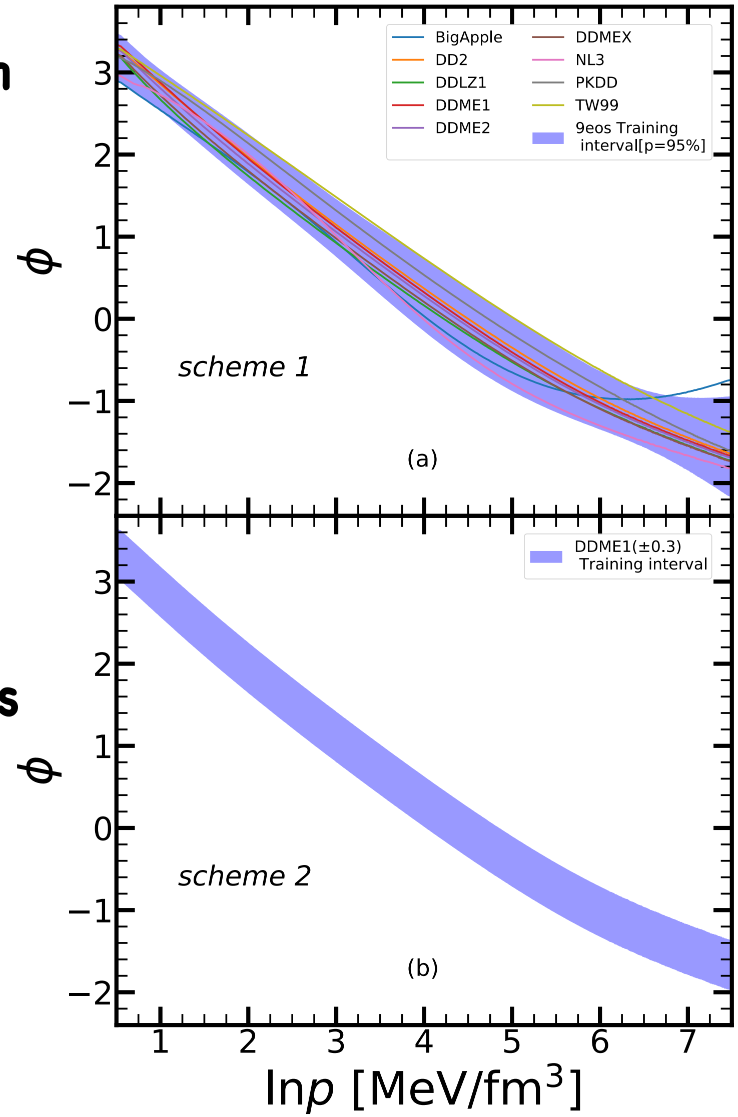
$(M_i + \Delta M_i,\ R_i + \Delta R_i,\ \sigma_{M_i},\ \sigma_{R_i})$

The training data

W. Zhou, J. N. Hu, Y. Zhang, and H. Shen, Astrophys. J. 950(2023)186

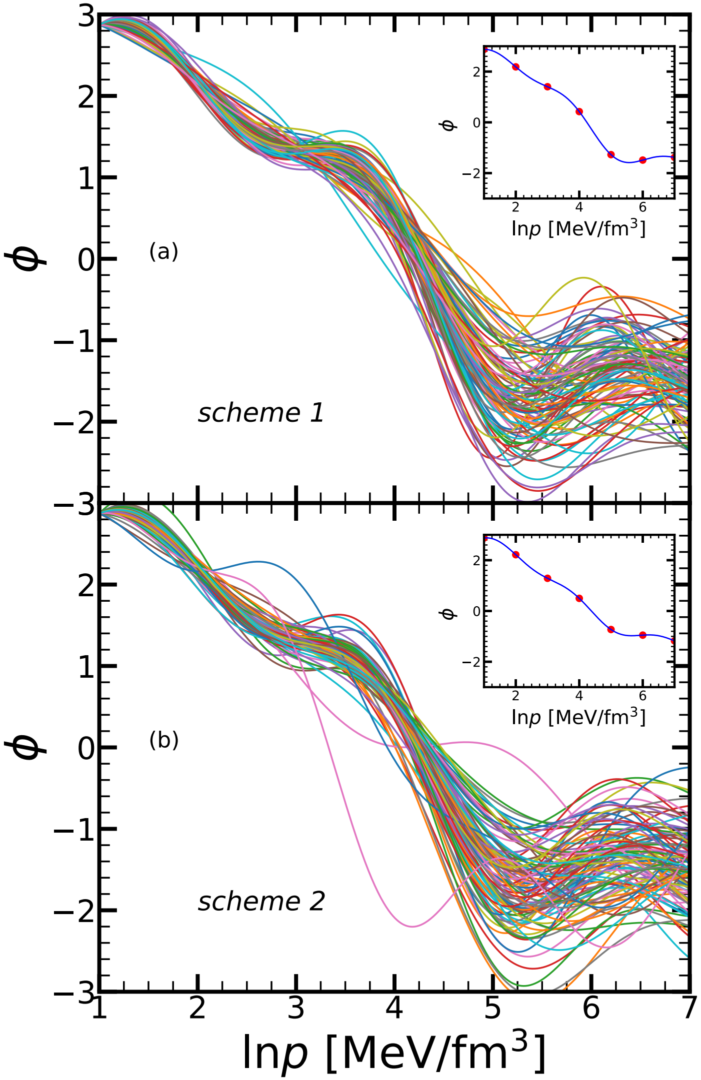
Prior scheme1: After obtaining the mean and variance of ϕ-ln p functions from nine RMF parameter sets, the 95% confidence interval of the variance was selected as the generation range of ϕ_i.

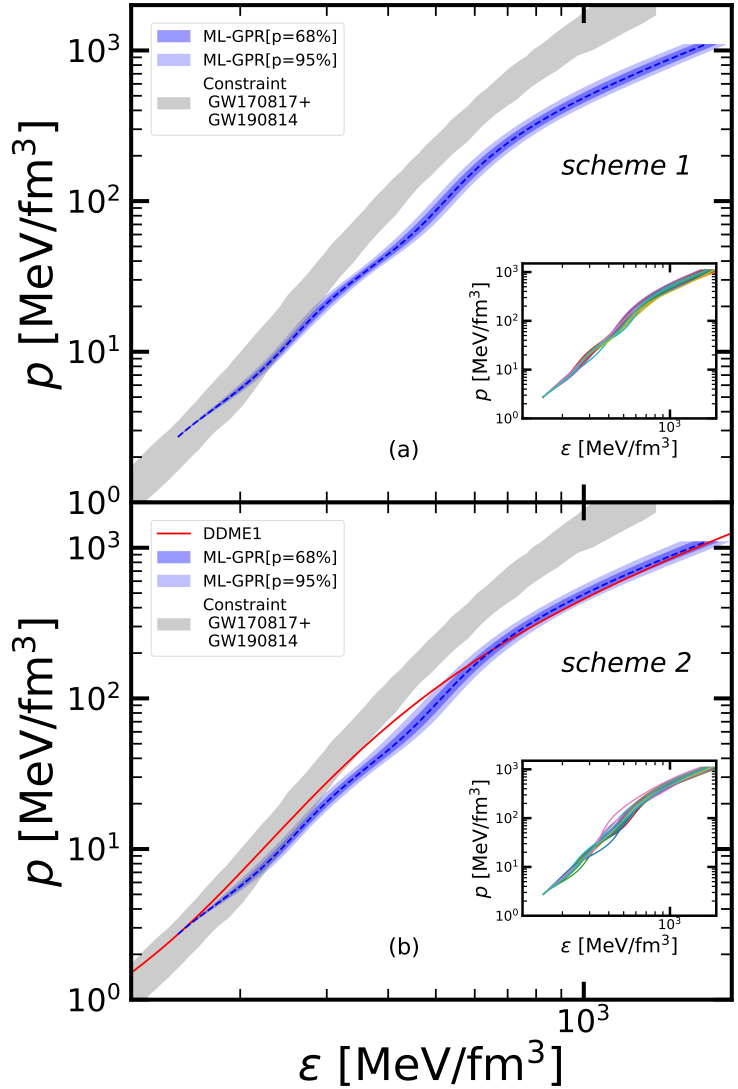
Prior scheme2: The ϕ-ln p function provided by DDME1 set was regarded as the standard, and $\phi \pm 0.3\phi$ are chosen as the upper and lower bounds of the generation range of ϕ_i.



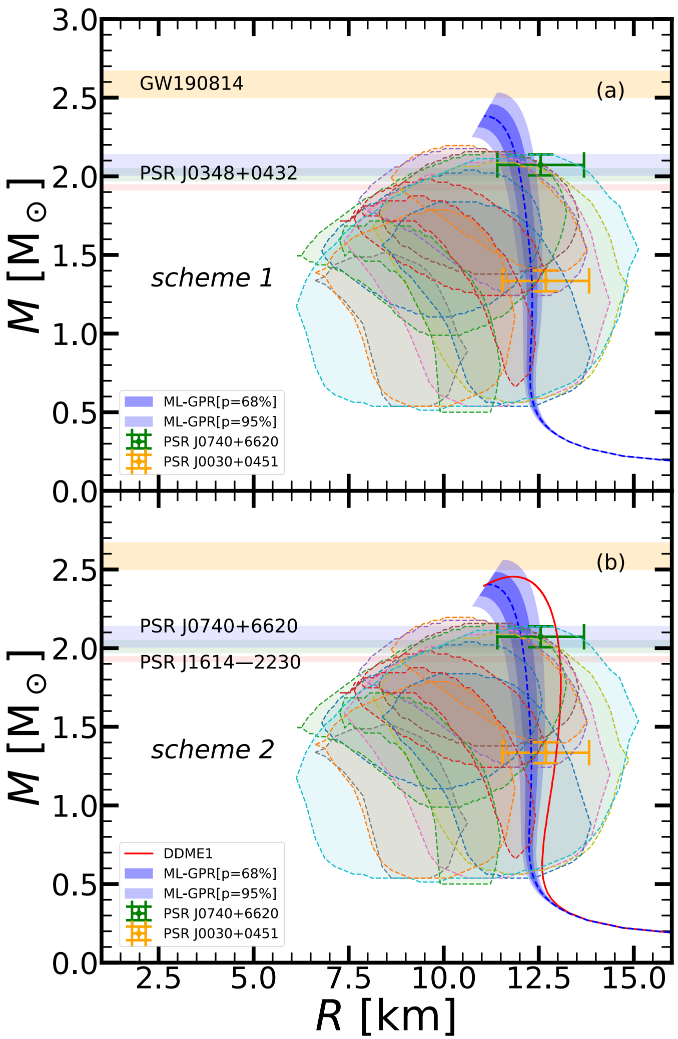
W. Zhou, J. N. Hu, Y. Zhang, and H. Shen, Astrophys. J. 950(2023)186

The EOSs from 200 NN models

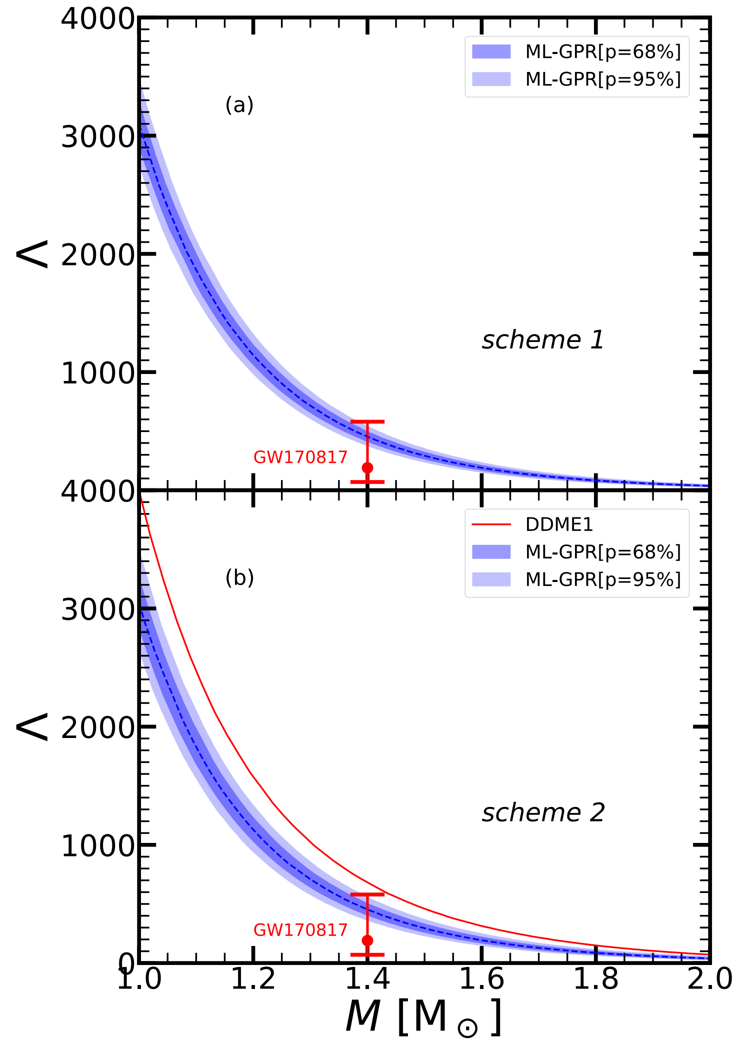




The mass-radius relation



The tidal deformability



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N piecewise polytropes representation

$$p = K_i \varepsilon^{\Gamma_i} \quad (\varepsilon_{i-1} < \varepsilon < \varepsilon_i)$$

The pressure values at the ith segment boundaries, p_i, are read as

$$p_i = p_{i-1} + c_{s,i}^2 (\varepsilon_i - \varepsilon_{i-1}) \, ,$$

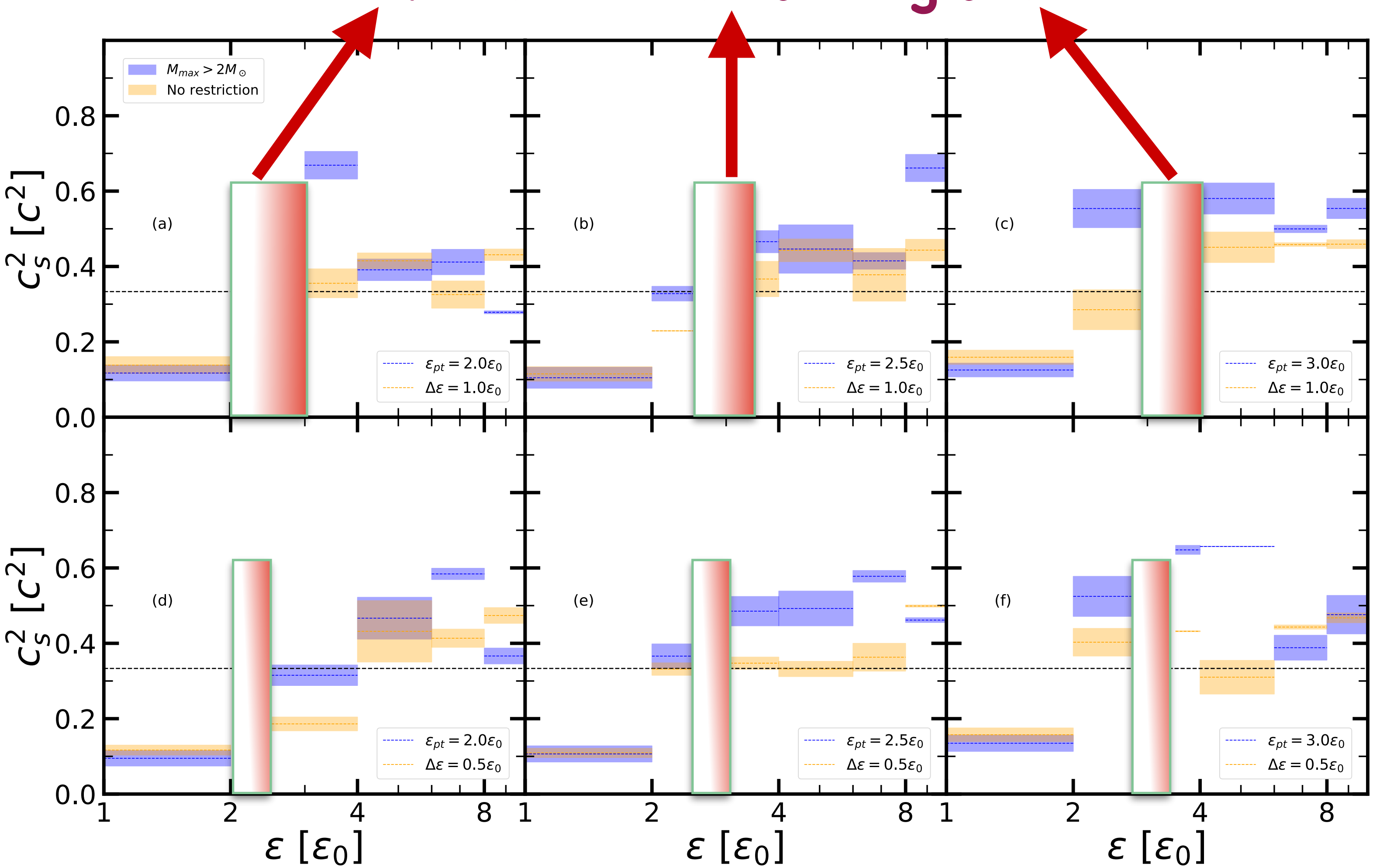
c is the average speed of sound

$$\langle c_s^2 \rangle \equiv \int_{\varepsilon_{i-1}}^{\varepsilon_i} \frac{d\varepsilon}{\varepsilon_i - \varepsilon_{i-1}} c_s^2 = \int_{\varepsilon_{i-1}}^{\varepsilon_i} \frac{d\varepsilon}{\varepsilon_i - \varepsilon_{i-1}} \frac{\partial p}{\partial \varepsilon}$$
$$= \frac{1}{\varepsilon_i - \varepsilon_{i-1}} \int_{p_{i-1}}^{p_i} dp = \frac{p_i - p_{i-1}}{\varepsilon_i - \varepsilon_{i-1}} = c_{s,i}^2 .$$

Y. Fujimoto, K. Fukushima, K. Murase, Phys. Rev. D, 98 (2018) 023019

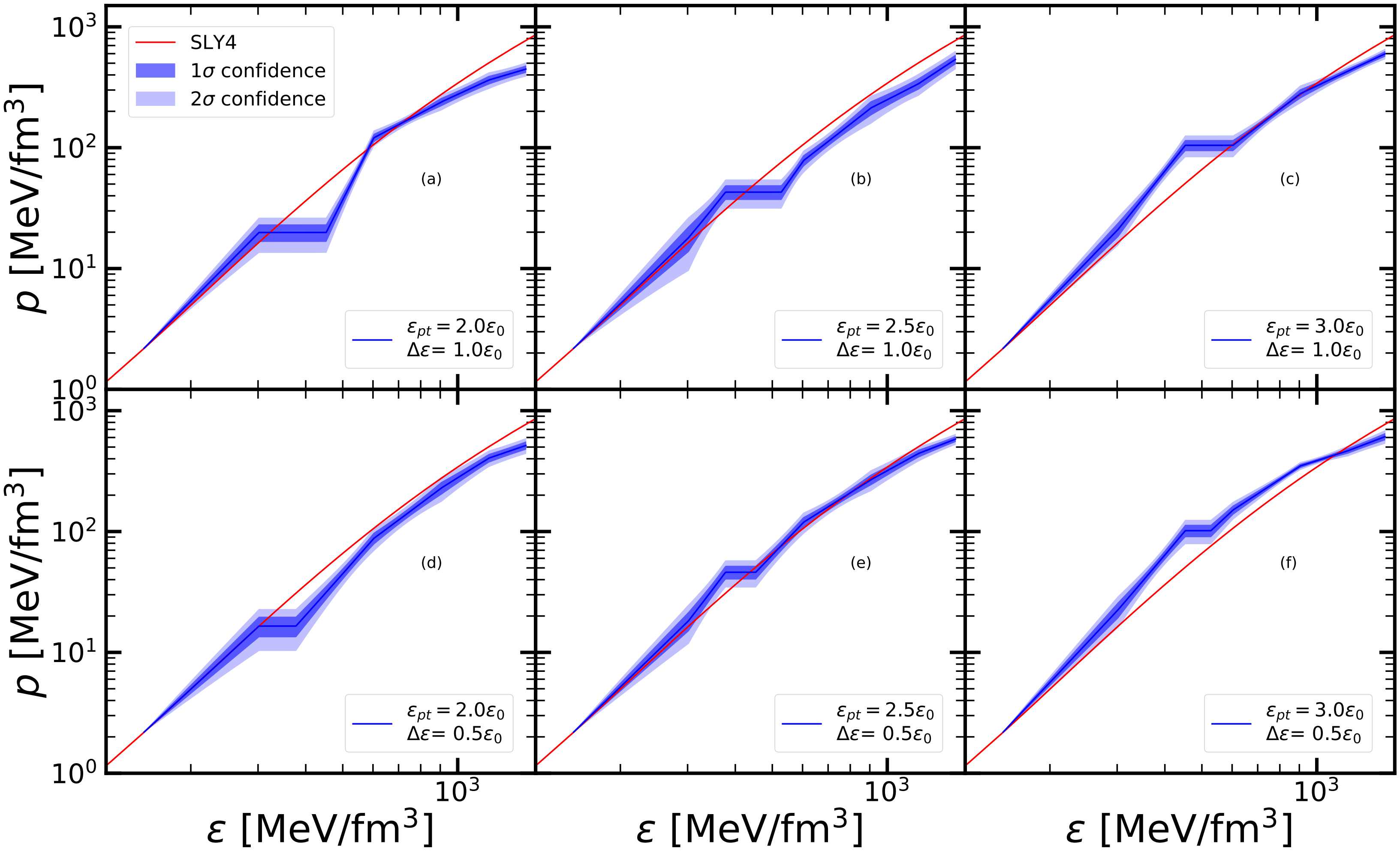
The speeds of sound from DNN

Phase transition regions



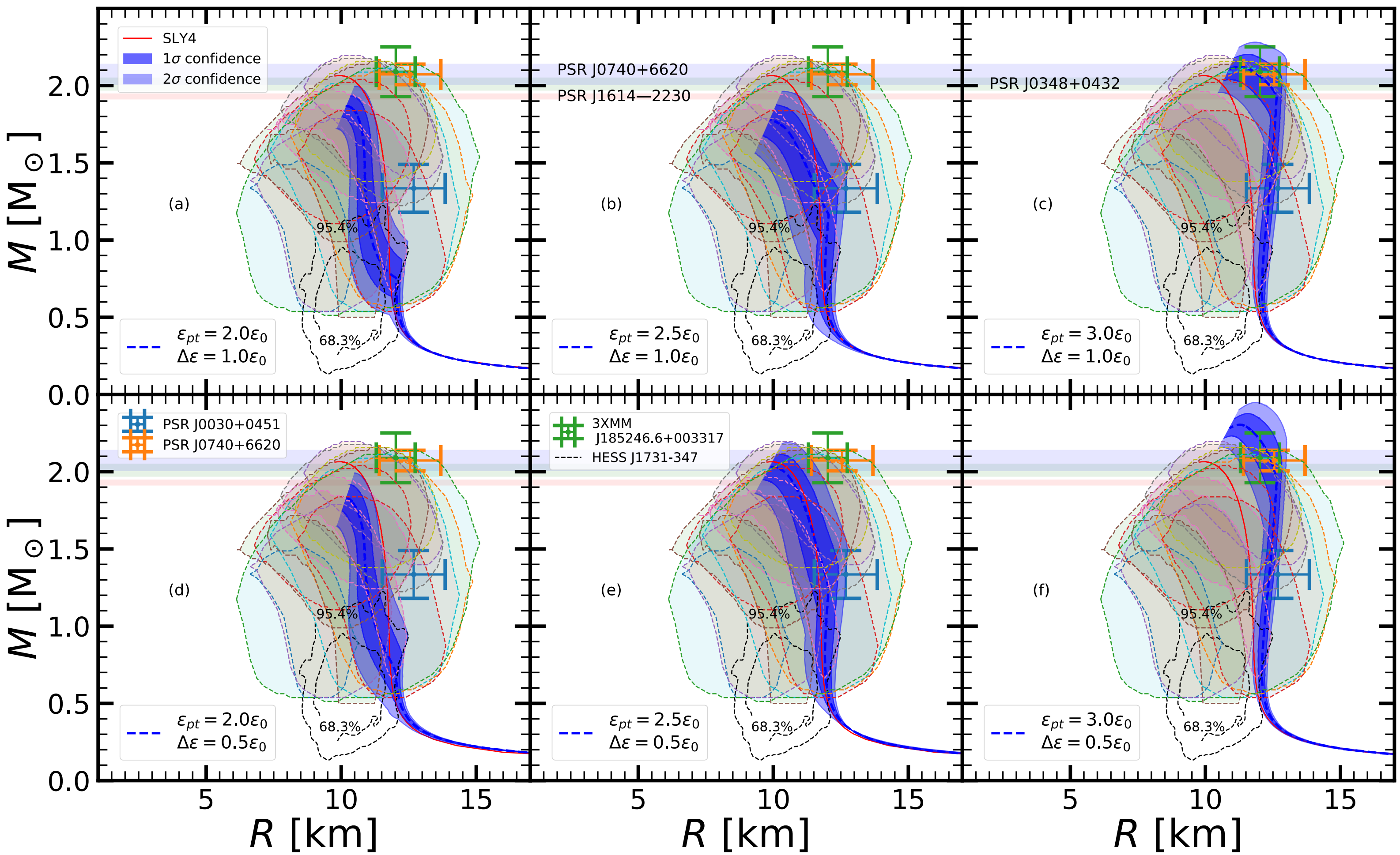
W. Zhou, J. N. Hu, Y. Zhang, and H. Shen, in preparation

The EOS including the phase transition



W. Zhou, J. N. Hu, Y. Zhang, and H. Shen, in preparation

The Mass-Radius relations with PT



W. Zhou, J. N. Hu, Y. Zhang, and H. Shen, in preparation

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Summary

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The hadron-quark crossover in neutron star was investigated by the Gaussian process

The nonparametric deep neural network with Gaussian process was applied to extract the equation of state of neutron star from the observations

The first-order hadron-quark phase transition was preliminarily discussed with deep neural network

The physics-informed neural networks will be considered.

Thank you very much for your attention!

Bayesian inference

$$f_{\mathrm{MAP}}(\mathcal{D}) = \arg\max_{\boldsymbol{\theta}}[\Pr(\boldsymbol{\theta})\Pr(\mathcal{D}|\boldsymbol{\theta})].$$

Neural network: minimize

$$L[f] = \langle \ell[f] \rangle = \int d\boldsymbol{\theta} d\mathcal{D} \Pr(\boldsymbol{\theta}) \Pr(\mathcal{D}|\boldsymbol{\theta}) \ell(\boldsymbol{\theta}, f(\mathcal{D})).$$

Neural network allows for more general choice of loss functions

Bayesian inference assumes parametrized likelihood functions.