Functional renormalization group for a self-bound object

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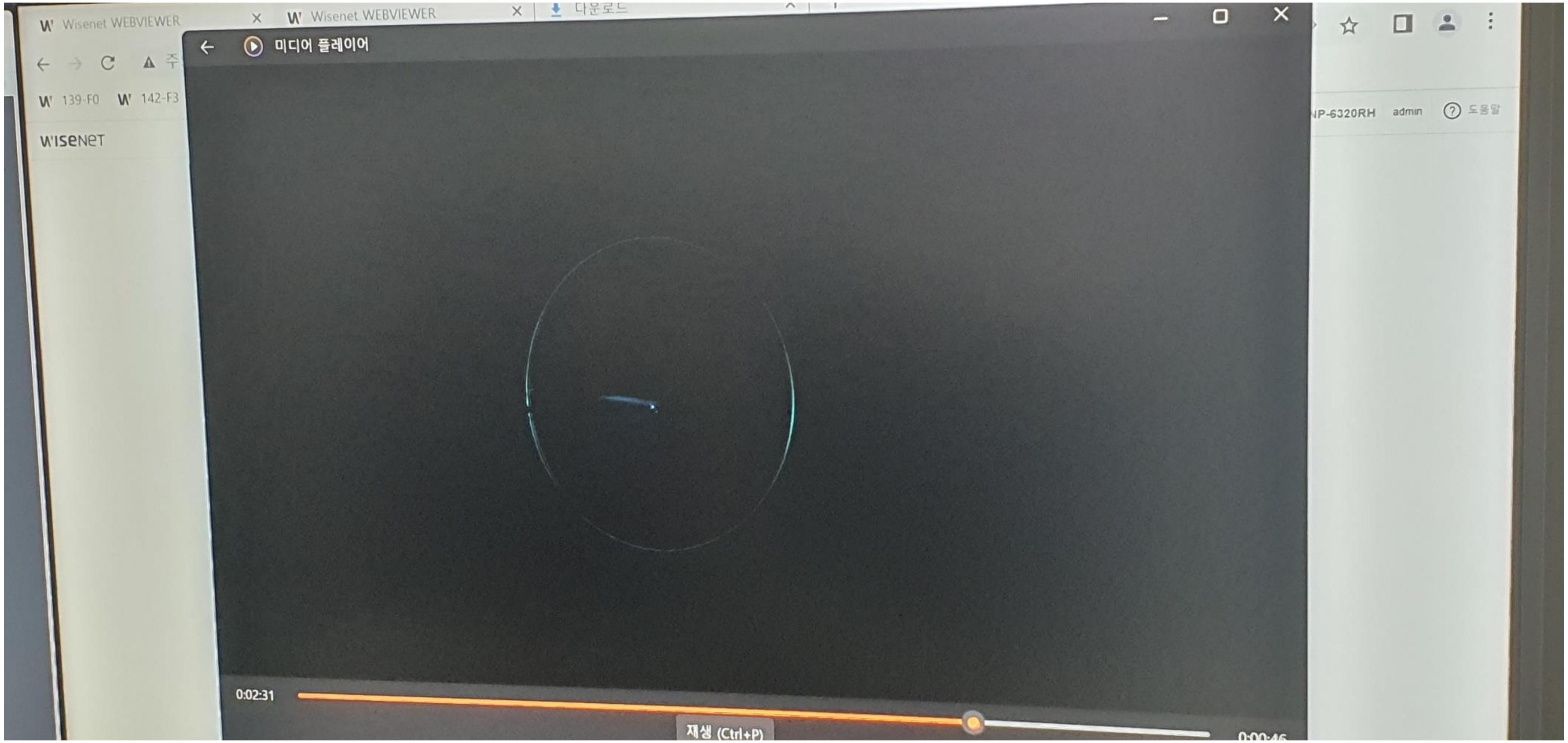
CENTER FOR EXOTIC NUCLEAR STUDIES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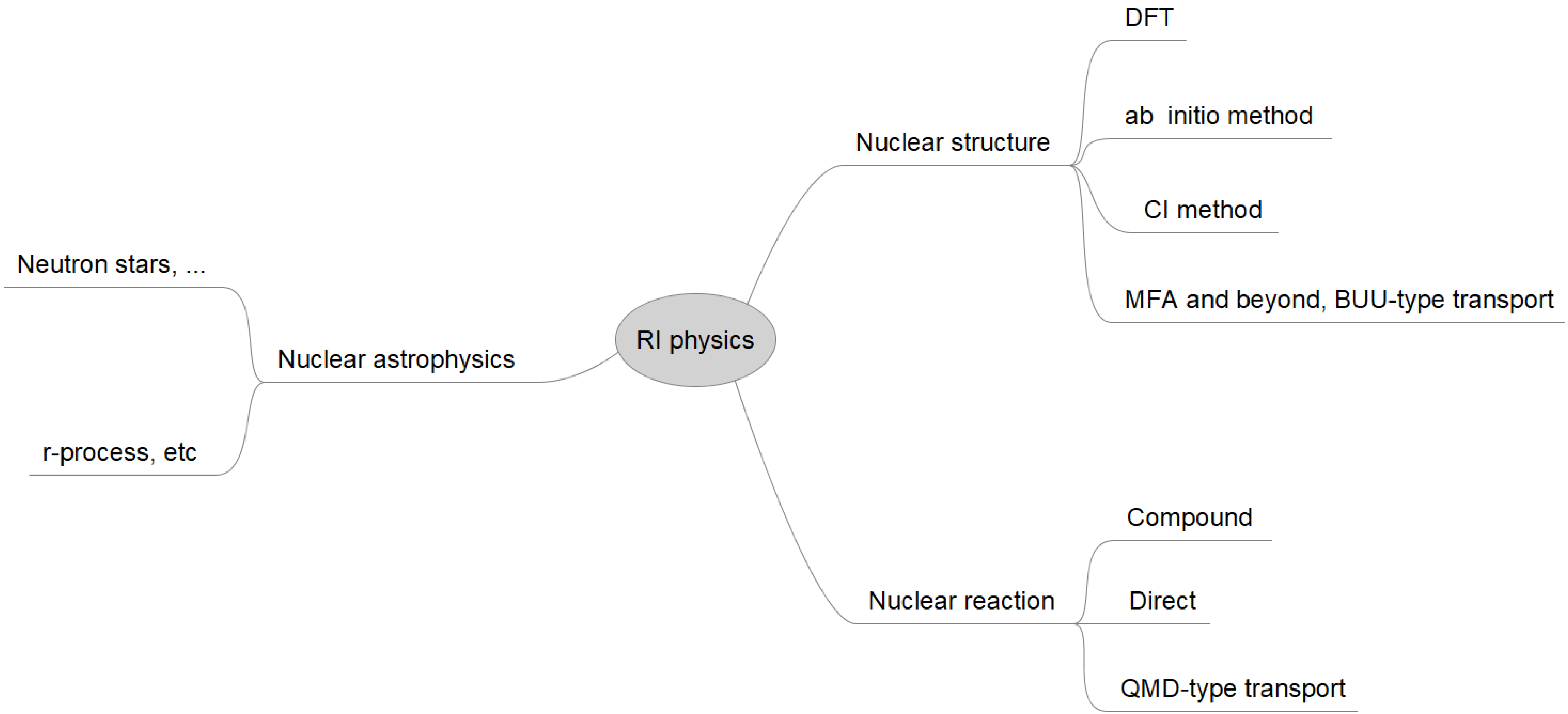
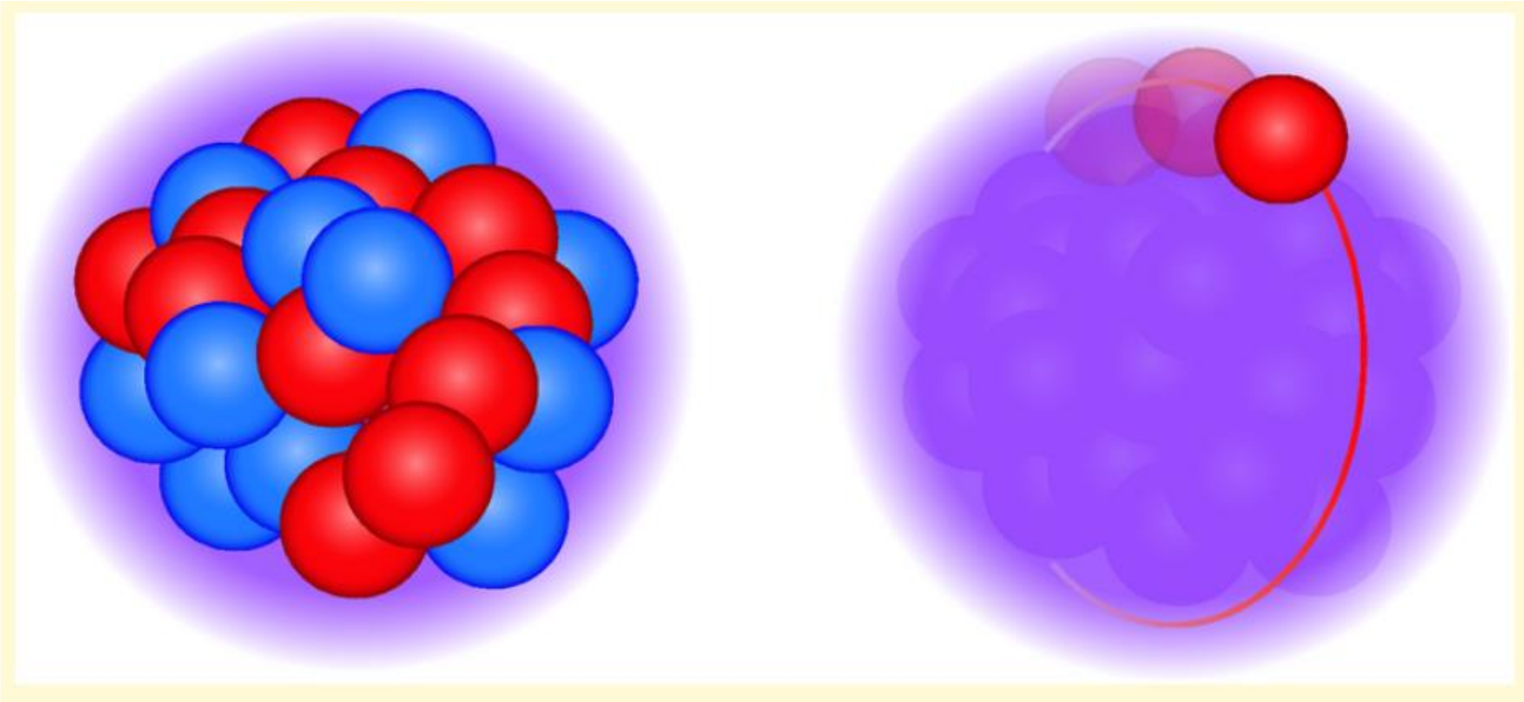
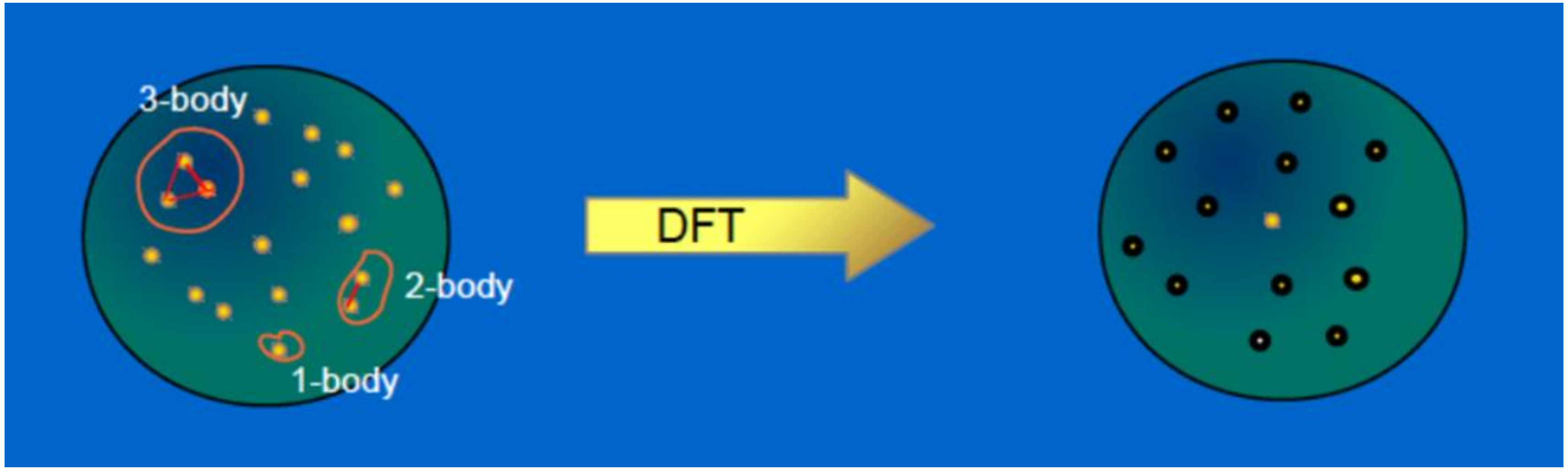

RI physics
Nuclear structure
DFT
ab initio method
CI method
MFA and beyond, BUU-type transport
Nuclear astrophysics
Neutron stars, ...
r-process, etc
Nuclear reaction
Compound
Direct
QMD-type transport

Mean field Approximation





Atomic Data and Nuclear Data Tables 121–122 (2018) 1–215

Contents lists available at ScienceDirect

Atomic Data and Nuclear Data Tables

journal homepage: www.elsevier.com/locate/adt

The limits of the nuclear landscape explored by the relativistic continuum Hartree–Bogoliubov theory

X.W. Xia [a], Y. Lim [b,c], P.W. Zhao [d,e], H.Z. Liang [f], X.Y. Qu [a,g], Y. Chen [d,h], H. Liu [d], L.F. Zhang [d], S.Q. Zhang [d], Y. Kim [c], J. Meng [d,a,i,*]

Atomic Data and Nuclear Data Tables 144 (2022) 101488

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journal homepage: www.elsevier.com/locate/adt

Nuclear mass table in deformed relativistic Hartree–Bogoliubov theory in continuum, I: Even–even nuclei

Kaiyuan Zhang [a], Myung-Ki Cheoun [b], Yong-Beom Choi [c], Pooi Seong Chong [d], Jianmin Dong [e,f], Zihao Dong [a], Xiaokai Du [a], Lisheng Geng [g,h], Eunja Ha [i], Xiao-Tao He [j], Chan Heo [d], Meng Chit Ho [d], Eun Jin In [k,l], Seonghyun Kim [b], Youngman Kim [m], Chang-Hwan Lee [c], Jenny Lee [d], Hexuan Li [a], Zhipan Li [n], Tianpeng Luo [a], Jie Meng [a,*], Myeong-Hwan Mun [b,o], Zhongming Niu [p], Cong Pan [a], Panagiota Papakonstantinou [m], Xinle Shang [e,f], Caiwan Shen [q], Guofang Shen [g], Wei Sun [n], Xiang-Xiang Sun [r,s], Chi Kin Tam [d], Thaivayongnou [g], Chen Wang [j], Xingzhi Wang [a], Sau Hei Wong [d], Jiawei Wu [j], Xinhui Wu [a], Xuewei Xia [t], Yijun Yan [e,f], Ryan Wai-Yen Yeung [d], To Chung Yiu [d], Shuangquan Zhang [a], Wei Zhang [h], Xiaoyan Zhang [p], Qiang Zhao [k,a], Shan-Gui Zhou [s,u,v,w], DRHBc Mass Table Collaboration

Fluctuations

- **Nucleons, though massive, can fluctuate near the Fermi surface as p-h excitations.**
- **The pion and sigma mesons can fluctuate.**
- **Vector mesons such as omega mesons may not fluctuate because they are massive. Only as mean field.**

Beyond the mean field approximation: FRG

The FRG combines functional approach with the renormalization group idea and deals with the fluctuations not all at once but successively from scale to scale or shell by shell.

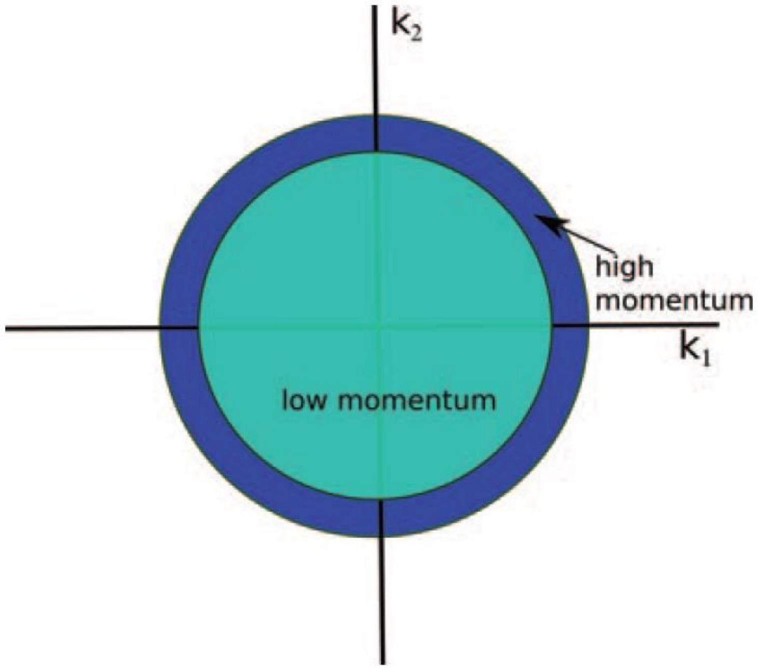
FRG is a good way to handle the fluctuations and a handy tool for those who are familiar with relativistic formalism.

My goal is to use the FRG method for exotic nuclei, neutron stars and heavy ion collisions using an Walecka type model.
For example, one can use a FRG-improved Walecka type model, and then what comes out with RCHB and DRHBc. For DRHBc with the PC-PK1 density functional, one has to first derive the FRG equations including various contact terms in the Lagrangian.

Renormalization group method with different goals

- To remove infinities (UV divergences)
- To describe the scale dependence of physical parameters
- To re-sum the perturbation expansion in QFT
- To solve strongly coupled theories
- …

Wilsonian (effective action) approach



It is about how a theory changes as we scale down the momentum scale.

In practice, however, non-local interactions are generated and a derivative expansion is not possible.

The Wetterich equation, the scale dependence (or flow) of the effective action, provides a better analytical and numerical accessibility and stability and practical realization of the Wilson (-Kadanoff) RG idea.

The average action Γ_k is a simple generalization of the effective action, with the distinction that only fluctuations with momenta $q^2 \gtrsim k^2$ are included.

Γ_k interpolates between the classical action S and the effective action Γ as k is lowered from the ultraviolet cutoff Λ to zero: $\lim_{k\to\Lambda} \Gamma_k = S$, $\lim_{k\to 0} \Gamma_k = \Gamma$.

Wetterich Equation

$$Z[J] = \int D\phi \, e^{-S[\phi] + J\cdot\phi}$$

$$J\cdot\phi = \int d^4x \, J_a(x) \, \phi_a(x)$$

$$\langle \phi^n \rangle = \frac{1}{Z} \frac{\delta^n Z}{\delta^n J} = \frac{1}{Z} \int D\phi \, \phi^n e^{-S + \phi\cdot J}$$

$$W[J] = \ln Z[J]$$

↳ Schwinger functional

$$G = \frac{\delta^2 W}{\delta^2 J} = \frac{\delta}{\delta J}\left(\frac{1}{Z}\frac{\delta Z}{\delta J}\right)$$

$$= \frac{1}{Z}\frac{\delta^2 Z}{\delta^2 J} - \frac{1}{Z^2}\frac{\delta Z}{\delta J}\frac{\delta Z}{\delta J}$$

$$= \langle \phi\phi \rangle - \langle \phi \rangle \langle \phi \rangle$$

$$\equiv \langle \phi\phi \rangle_c$$

Introduce a cutoff ΔS_k that vanishes in the IR.

$$W_k[J] = \ln Z_k[J]$$
$$= \ln \int D\phi \, e^{-S[\phi] + J\cdot\phi - \Delta S_k[\phi]}$$

k: renormalization scale, we are probing.

$$\Delta S_k[\phi] = \frac{1}{2}\phi \cdot R_k \cdot \phi$$

$$= \frac{1}{2}\int_{x,y} \phi_a(x) R_{k,ab}(x,y) \phi_b(y)$$

↳ momentum dependent mass
UV and IR regulator!

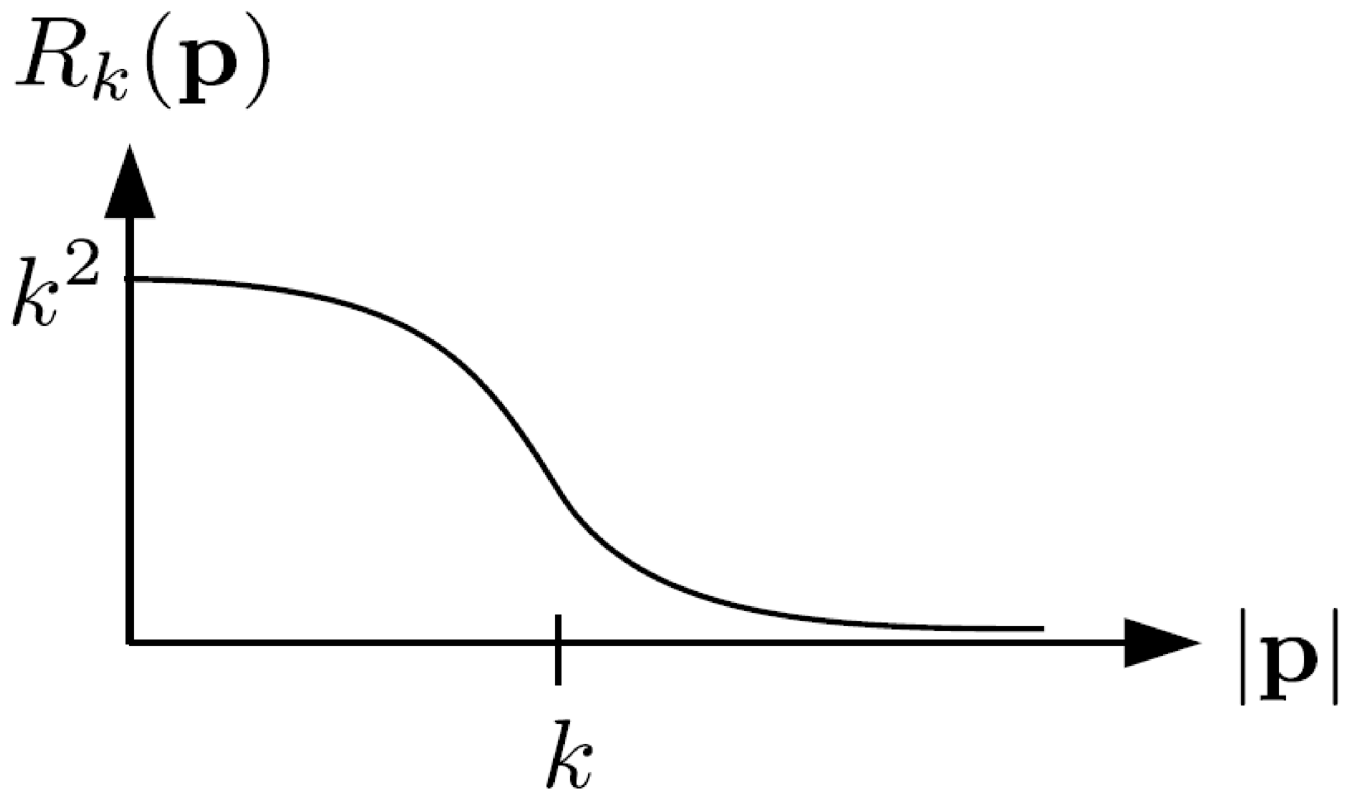
At fixed J,

$$\partial_k W_k[J] = -\frac{1}{Z_k}\int D[\phi] (\partial_k \Delta S_k[\phi]) \, e^{-S + J\cdot\phi - \Delta S_k}$$

$$= -\frac{1}{2}\langle \phi \, \partial_k R_k \phi \rangle$$

Using $\langle\phi\phi\rangle = \langle\phi\phi\rangle_c + \langle\phi\rangle\langle\phi\rangle$,
↳ φ

$$= -\frac{1}{2}\left(\langle\phi\phi\rangle_c + \varphi\varphi \right) \partial_k \cdot R_k$$



Momentum-dependent mass-like term that gives a mass of order k^2 to the low-energy modes and thus suppresses their fluctuations.

$$\langle \phi\phi \rangle_c \equiv W_k^{(2)}$$

$$= \frac{\delta W_k}{\delta^2 J} = \frac{\delta \varphi}{\delta J}$$

Now we arrive at Polchinski's equation.

$$\partial_k W_k[J] = -\frac{1}{2} \mathrm{Tr} \left[W_k^{(2)} \partial_k R_k \right]$$

$$-\frac{1}{2} \varphi (\partial_k R_k) \varphi$$

(Tr →) Integration over x ($\sim p$) and summation over a, b.

$$\mathrm{Tr}\left[(\partial_k R_k) W_k^{(2)} \right]$$

$$= \int_{x,y} W_{k,ab}^{(2)}(x,y)\, \partial_k R_{k,ab}(x,y)$$

Effective action

$(J \longleftrightarrow \varphi)$ why $\phi_{cl} \neq \phi_{cl} + \delta\varphi^{\text{1-loop}}$, etc

is a function in the full QFT

gives the exact value of $\langle \phi \rangle \Rightarrow \varphi$

$$\hat{\Gamma}_k[\varphi] = J \cdot \varphi - W_k[J]$$

$$\frac{\partial \hat{\Gamma}_k}{\partial \varphi} = J_k$$

$$\frac{\partial^2 \hat{\Gamma}_k}{\partial^2 \varphi} = \frac{\partial J_k}{\partial \varphi} \quad \left(= \tilde{\Gamma}_k^{(2)} \right)$$

↳ inverse propagator

$$? \quad \left(\tilde{\Gamma}_k^{(2)} \, W_k^{(2)} \right)_{ab} (x, y)$$

$$= \int_z \frac{\delta J_c(z)}{\delta \varphi_a(x)} \frac{\delta \varphi_b(y)}{\delta J_c(z)}$$

$$= \frac{\delta \varphi_b(y)}{\delta \varphi_a(x)} \rightarrow \delta_{ab}\, \delta(x-y)$$

$$\therefore \quad W_k^{(2)} = \left(\tilde{\Gamma}_k^{(2)} \right)^{-1}$$

$$= \left(\Gamma_k^{(2)} + R_k \right)^{-1}$$

for fixed φ

$$\partial_k \hat{\Gamma}_k = \varphi \partial_k J - \partial_k W_k[J]$$

$$- \frac{\delta W}{\delta J} \frac{\partial J}{\partial k} \quad (\varphi \leftarrow \frac{\delta W}{\delta J})$$

$$= - \partial_k W_k [J]$$

$$\Gamma_k[\varphi] = \hat{\Gamma}_k[\varphi] - \Delta S_k$$

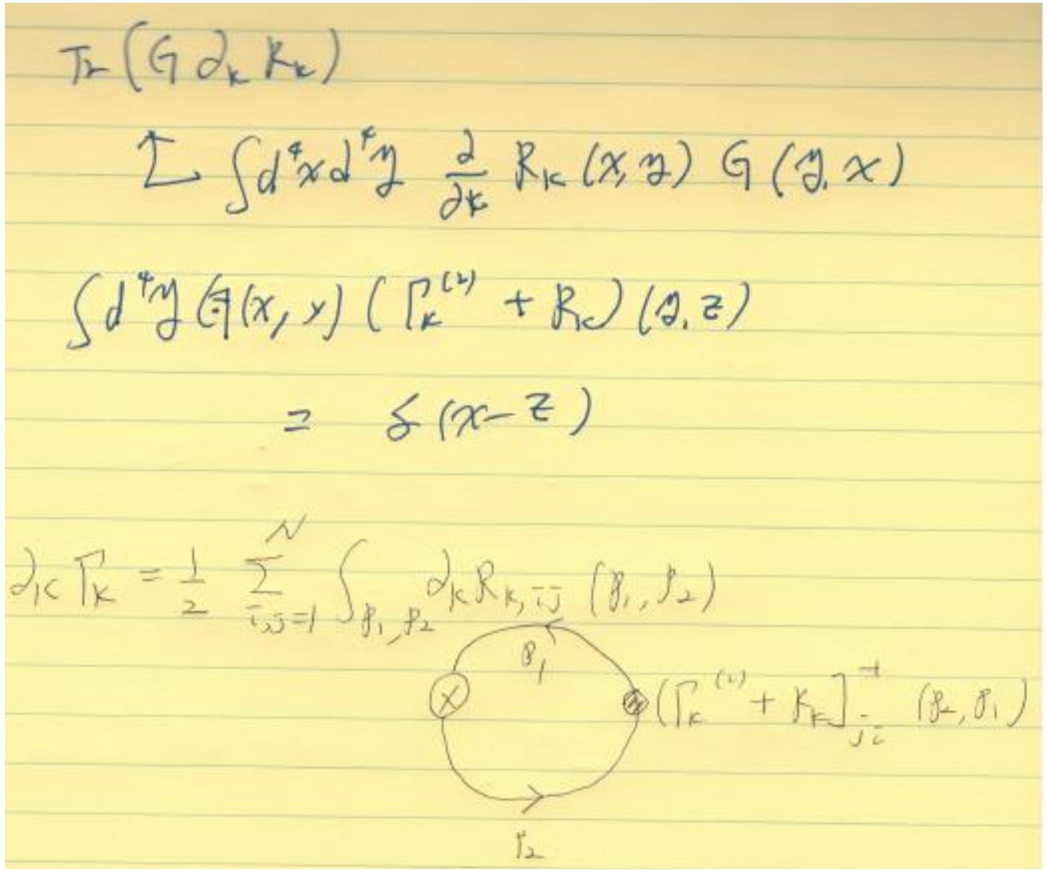
$$\partial_k \Gamma_k[\varphi] = - \partial_k W_k \Big|_J - \frac{1}{2} \varphi (\partial_k R_k) \varphi$$

$$\left(- \frac{1}{2} \mathrm{Tr}[W_k^{(2)} \partial_k R_k] - \frac{1}{2} \varphi (\partial_k R_k) \varphi \right)$$

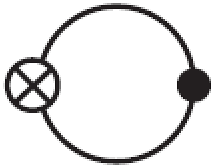
$$= + \frac{1}{2} \mathrm{Tr}\left[W_k^{(2)} \partial_k R_k \right]$$

$$= \frac{1}{2} \mathrm{Tr}\left[(\Gamma_k^{(2)} + R_k)^{-1} \partial_k R_k \right]$$

< Wetterich eq. >

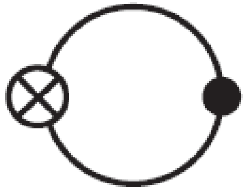


$$k\,\frac{\partial \Gamma_k}{\partial k} =$$



$$= \frac{1}{2}\,\mathrm{Tr}\,\frac{k\,\frac{\partial R_k}{\partial k}}{\Gamma_k^{(2)} + R_k},$$

$$k\frac{\partial\Gamma_k[\Phi]}{\partial k} = \frac{1}{2}\,\mathrm{Tr}\left[k\frac{\partial R_k}{\partial k}\cdot\left(\Gamma_k^{(1,1)}[\Phi]+R_k\right)^{-1}\right] = \frac{1}{2}$$



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Progress in Particle and Nuclear Physics 93 (2017) 69–107

Contents lists available at ScienceDirect

Progress in Particle and Nuclear Physics

journal homepage: www.elsevier.com/locate/ppnp

Review

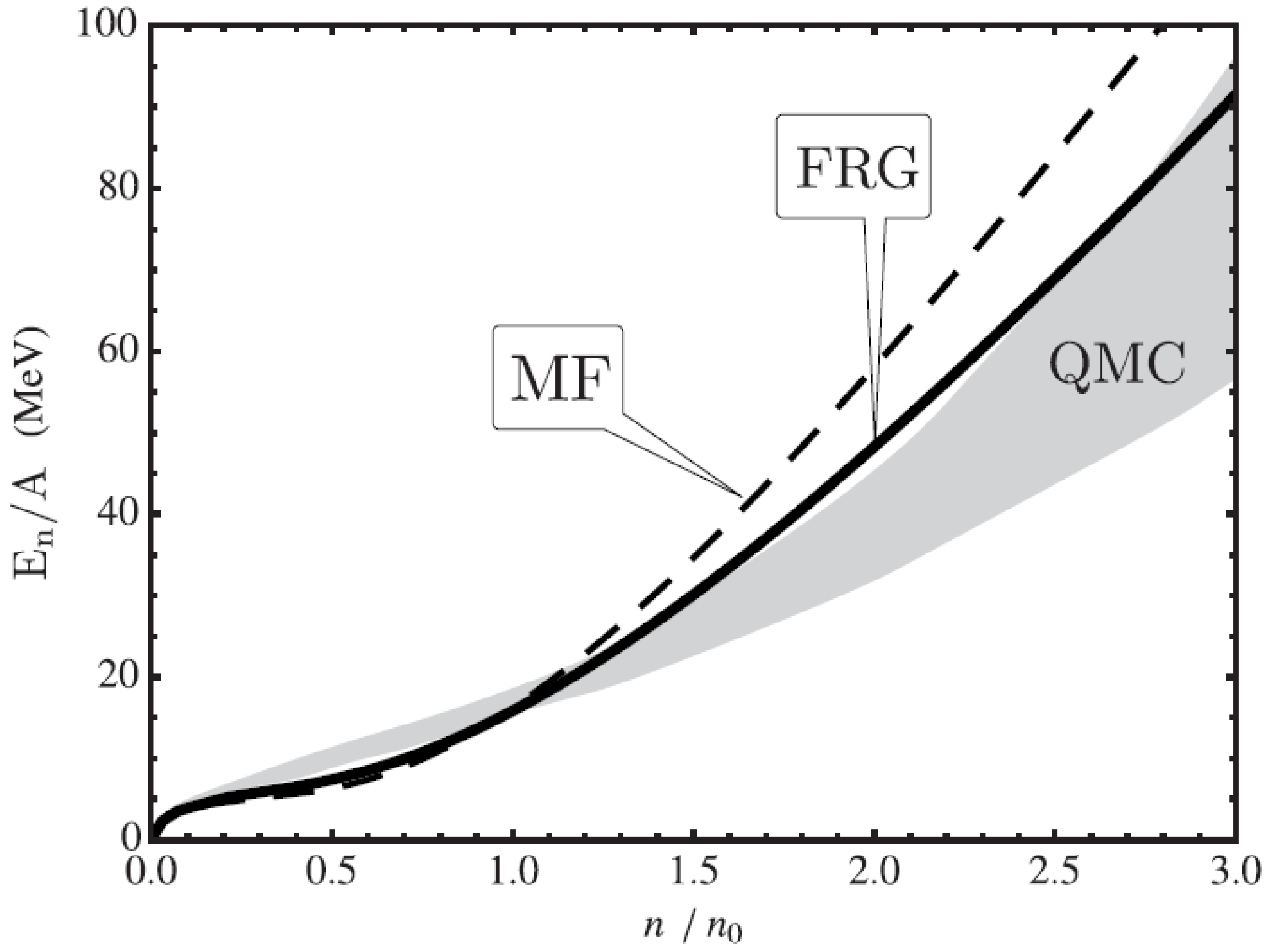
Functional renormalization group studies of nuclear and neutron matter

Matthias Drews [a], Wolfram Weise [a,b,*]

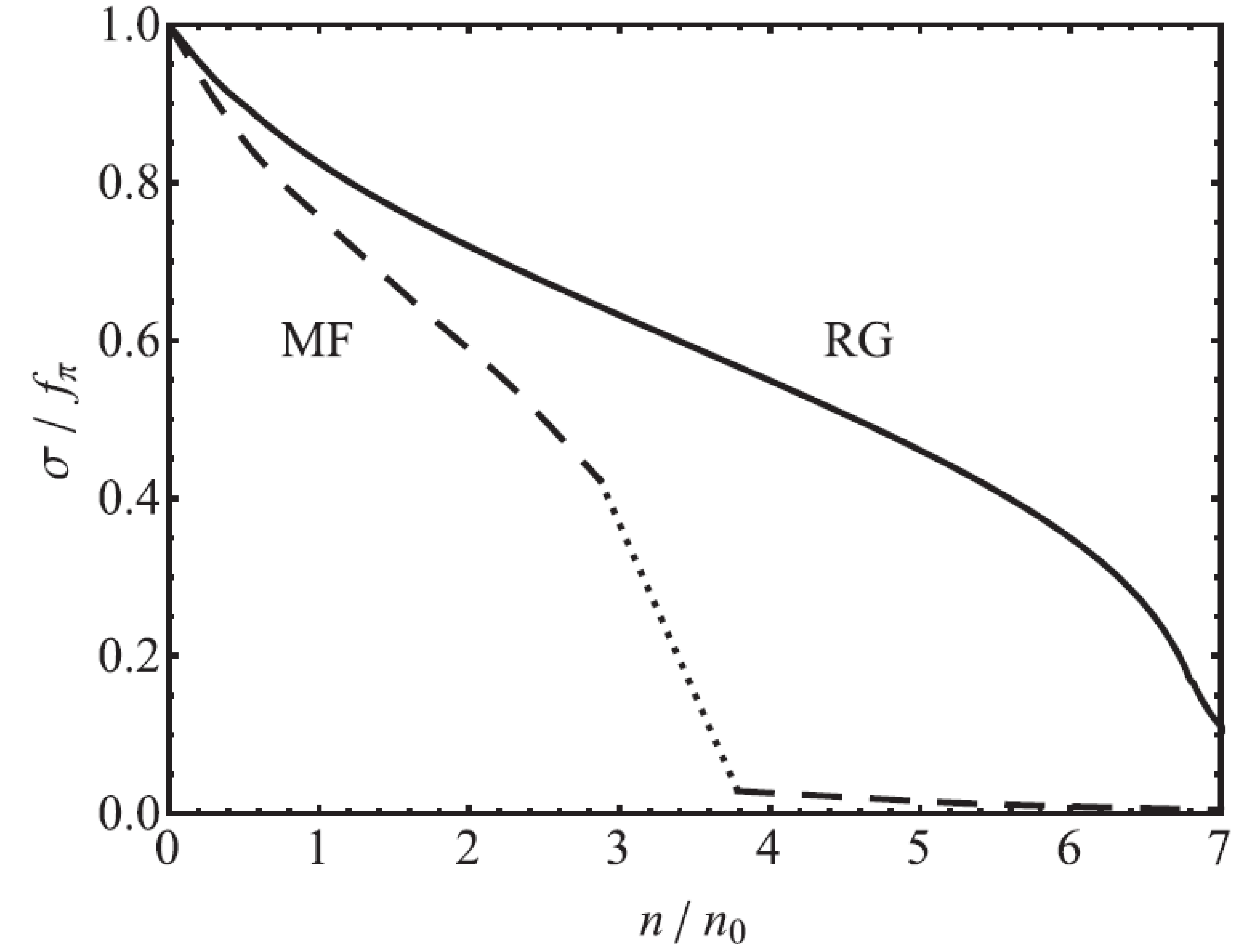
Chiral nucleon–meson model

$$\mathcal{L}_0(N,\sigma,\boldsymbol{\pi}) = \bar{N}\left[i\gamma_\mu\partial^\mu - g(\sigma + i\gamma_5\,\boldsymbol{\tau}\cdot\boldsymbol{\pi})\right]N + \frac{1}{2}(\partial_\mu\sigma)(\partial^\mu\sigma) + \frac{1}{2}(\partial_\mu\boldsymbol{\pi})\cdot(\partial^\mu\boldsymbol{\pi}) - \mathcal{U}(\sigma,\boldsymbol{\pi}).$$

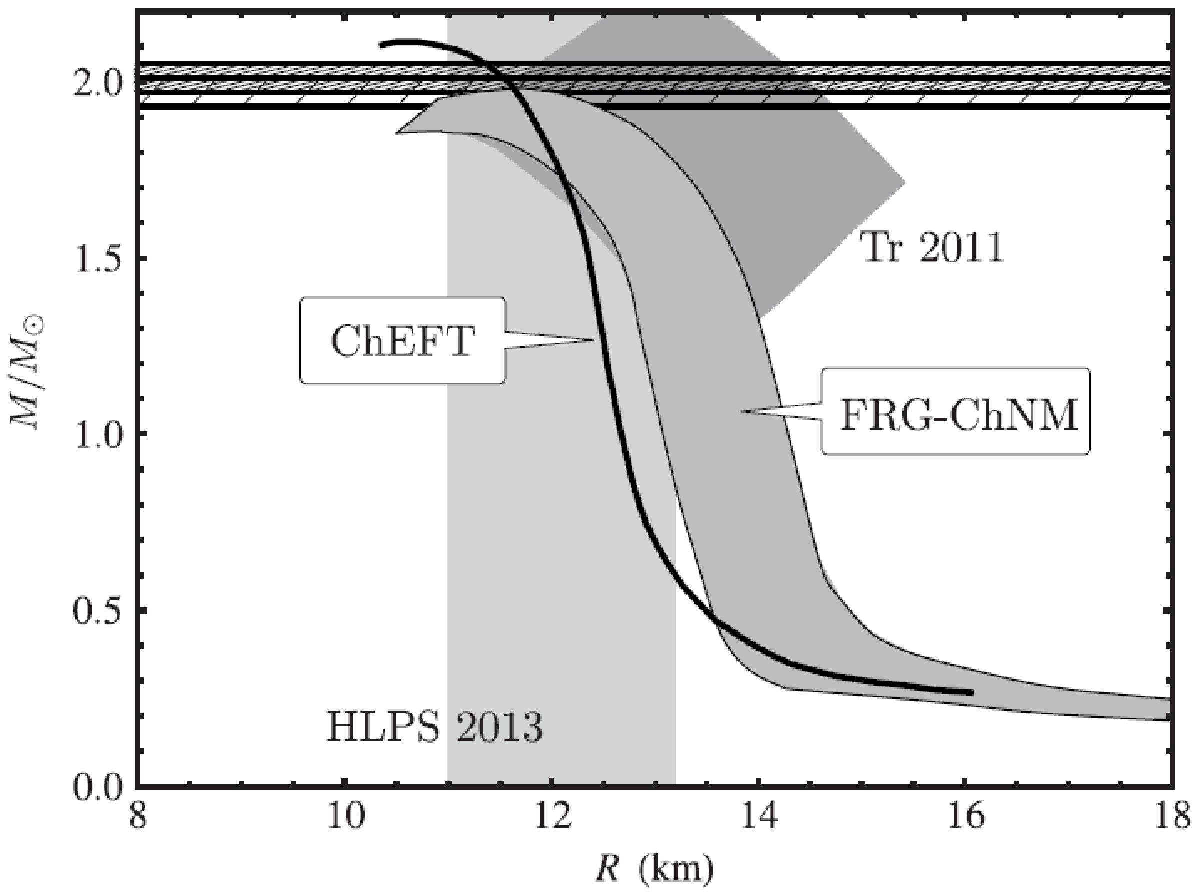
The equation of state for pure neutron matter at $T = 0$ with $E_{sym} = 32$ MeV



M. Drews, W. Weise / Progress in Particle and Nuclear Physics 93 (2017) 69–107



1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
5
6
7
σ / f_π
n / n_0
MF
RG



Mass–radius relation of neutron stars.

As a warm-up, ...

PHYSICAL REVIEW D **96,** 114029 (2017)

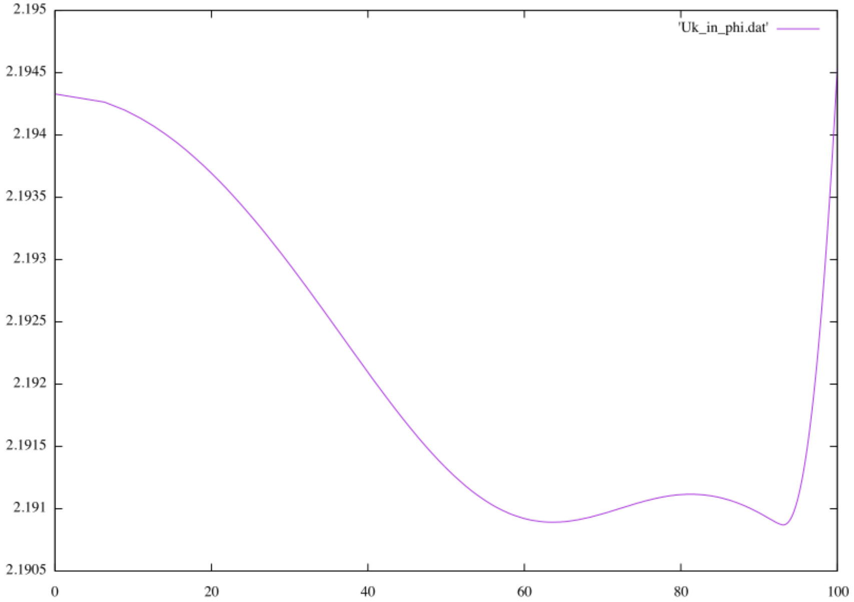
Functional renormalization group study of the quark-meson model with ω meson

Hui Zhang,[†] Defu Hou,[*] Toru Kojo,[‡] and Bin Qin[§]
Institute of Particle Physics (IOPP) and Key Laboratory of Quark and Lepton Physics (MOE), Central China Normal University, Wuhan 430079, China

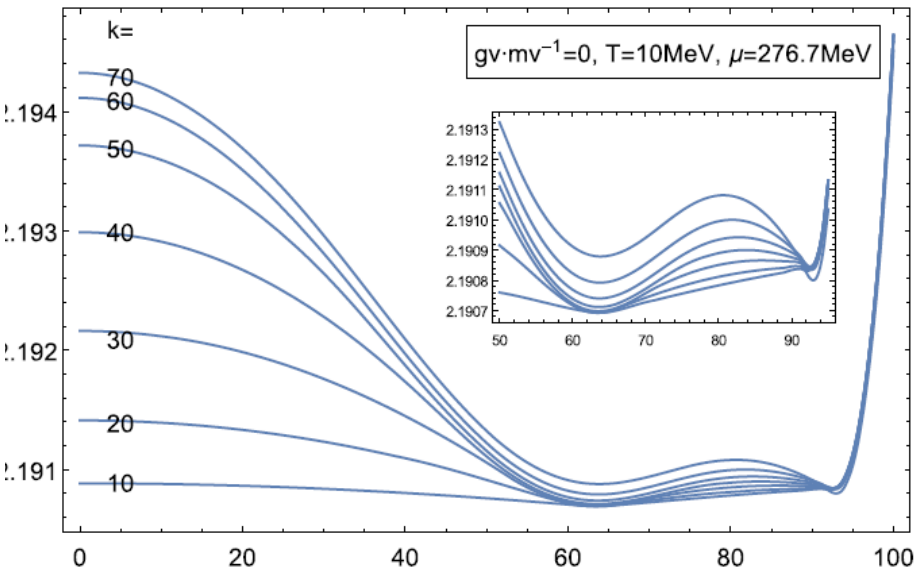
$$\mathcal{L} = \bar{\psi}[i\gamma_\mu\partial^\mu - g_s(\sigma + i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}) - g_v\gamma_\mu\omega^\mu + \mu\gamma_0]\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - U(\sigma,\boldsymbol{\pi},\omega),$$

$$U(\sigma,\boldsymbol{\pi},\omega) = \frac{\lambda}{4}(\sigma^2 + \boldsymbol{\pi}^2 - f_\pi^2)^2 - \frac{m_v^2}{2}\omega_\mu\omega^\mu$$

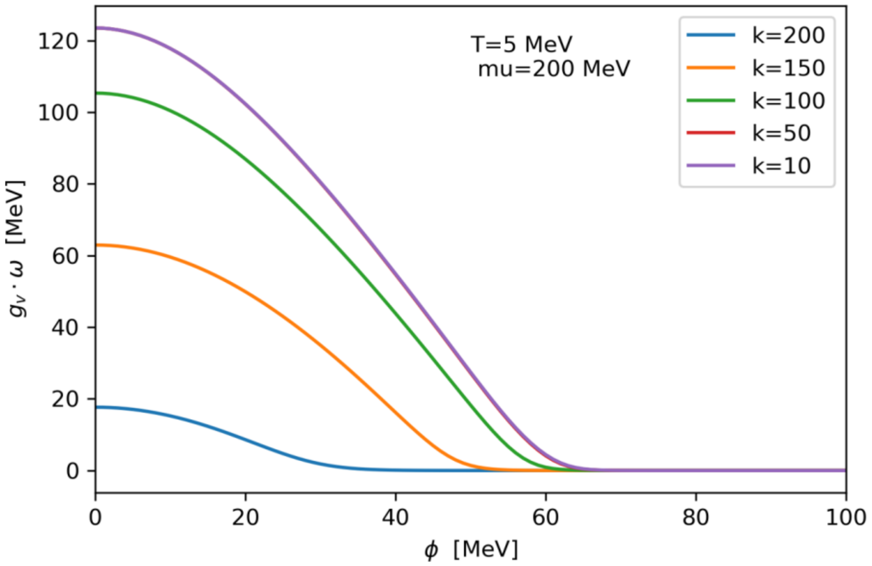
Y.-M. Kim, C.-H. Lee, S. Jeon and YK



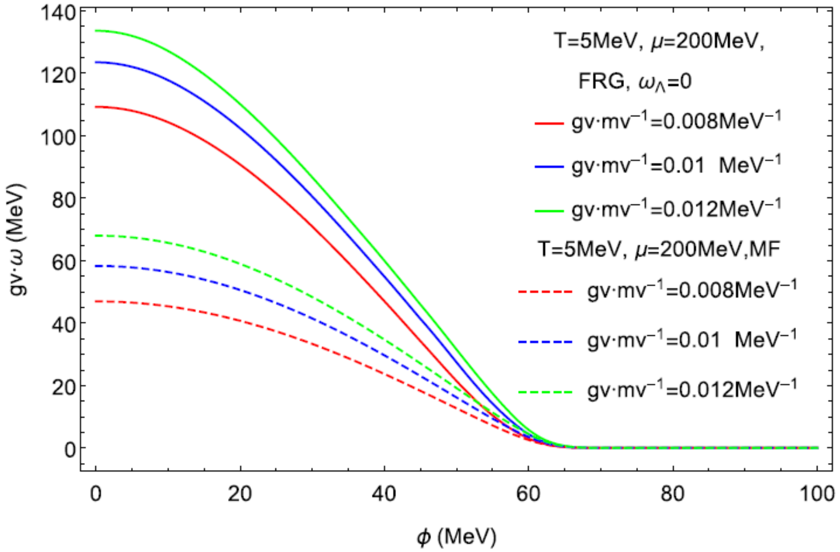
PHYSICAL REVIEW D **96,** 114029 (2017)



Y.-M. Kim, C.-H. Lee, S. Jeon and YK



HUI ZHANG, DEFU HOU, TORU KOJO, and BIN QIN



An extended Walecka model

$$\mathcal{L} = \bar{\psi}[i\not\partial - M_N + g_\sigma \phi - g_\omega \not\omega - g_\rho \not\vec{b}]\psi\,,$$
$$+ \frac{1}{2}(\partial_\mu \phi \partial^\phi - m_\sigma^2 \phi^2) - U(\phi)$$
$$+ \frac{1}{2}m_\omega^2 \omega^2 + \frac{1}{2}m_\rho^2 \vec{b}\cdot\vec{b} - \frac{1}{2}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}G_{\mu\nu}G^{\mu\nu}\,,$$

$$U(\phi) = \tfrac{1}{3}a\phi^3 + \tfrac{1}{4}b\phi^4.$$

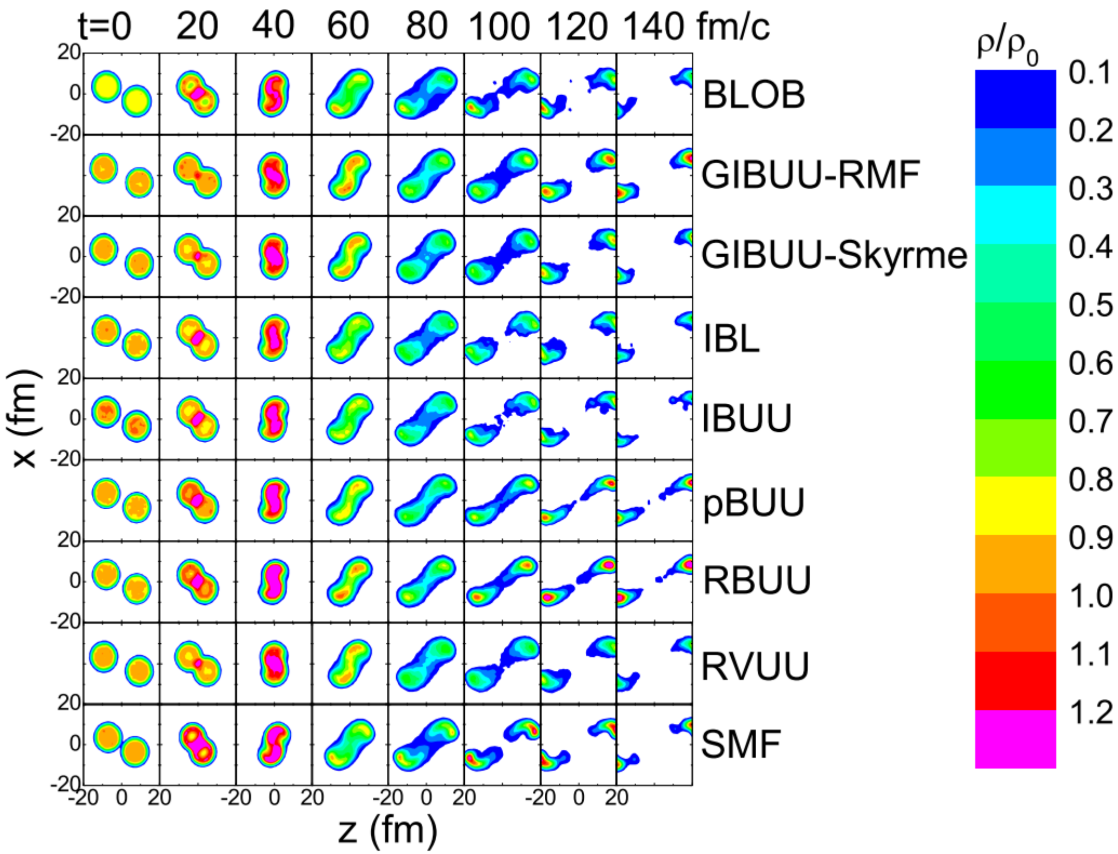
Here, the sigma is not a chiral partner of the pion.

B. Liu, V. Greco, V. Baran, M. Colonna and M. Di Toro, Phys. Rev. C **65**, 045201 (2002)

PHYSICAL REVIEW C **93**, 044609 (2016)

Understanding transport simulations of heavy-ion collisions at 100A and 400A MeV: Comparison of heavy-ion transport codes under controlled conditions

Jun Xu,[1,*] Lie-Wen Chen,[2,†] ManYee Betty Tsang,[3,‡] Hermann Wolter,[4,§] Ying-Xun Zhang,[5,‖] Joerg Aichelin,[6] Maria Colonna,[7] Dan Cozma,[8] Pawel Danielewicz,[3] Zhao-Qing Feng,[9] Arnaud Le Fèvre,[10] Theodoros Gaitanos,[11] Christoph Hartnack,[6] Kyungil Kim,[12] Youngman Kim,[12] Che-Ming Ko,[13] Bao-An Li,[14] Qing-Feng Li,[15] Zhu-Xia Li,[5] Paolo Napolitani,[16] Akira Ono,[17] Massimo Papa,[18] Taesoo Song,[19] Jun Su,[20] Jun-Long Tian,[21] Ning Wang,[22] Yong-Jia Wang,[15] Janus Weil,[19] Wen-Jie Xie,[23] Feng-Shou Zhang,[24] and Guo-Qiang Zhang[1]



$$
\begin{aligned}
\mathcal{L} = {} & \bar{\psi}\left[i\gamma^{\mu}\partial_{\mu} - M - g_{\sigma}\sigma - g_{\omega}\gamma^{\mu}\omega_{\mu} - g_{\rho}\gamma^{\mu}\vec{\tau}\cdot\vec{\rho}_{\mu} - e\gamma^{\mu}A_{\mu}\frac{1-\tau_3}{2}\right]\psi \\
& + \frac{1}{2}\partial^{\mu}\sigma\partial_{\mu}\sigma - U_{\sigma}(\sigma) - \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + U_{\omega}(\omega_{\mu}) - \frac{1}{4}\vec{R}^{\mu\nu}\cdot\vec{R}_{\mu\nu} + U_{\rho}(\vec{\rho}_{\mu}) \\
& - \frac{1}{4}F^{\mu\nu}F_{\mu\nu},
\end{aligned}
$$

Meng J, Toki H, Zhou S G, et al. Relativistic continuum Hartree-Bogoliubov theory for ground-state properties of exotic nuclei. Prog Part Nucl Phys, 2006, 57: 470–563

We use the extended Walecka model since there are a lot of results from the MFA to be compared with our FRG results. Downside is that we have more parameters compared to the models with chiral symmetry.

$$\mathcal{L} = \bar{\psi}[i \not\partial - M_N + g_\sigma \phi - g_\omega \not\omega - g_\rho \not\vec{b}]\psi ,$$
$$+ \frac{1}{2}(\partial_\mu \phi \partial^\phi - m_\sigma^2 \phi^2) - U(\phi)$$
$$+ \frac{1}{2} m_\omega^2 \omega^2 + \frac{1}{2} m_\rho^2 \vec{b} \cdot \vec{b} - \frac{1}{2} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} G_{\mu\nu} G^{\mu\nu} ,$$

$$U(\phi) = \tfrac{1}{3} a \phi^3 + \tfrac{1}{4} b \phi^4 .$$

B. Liu, V. Greco, V. Baran, M. Colonna and M. Di Toro, Phys. Rev. C **65**, 045201 (2002)

$$k\frac{\partial U_{k,\chi}}{\partial k} = \frac{k^5}{12\pi^2}\left\{\frac{1+2n_B(E_\phi)}{E_\phi} - \sum_{i=n,p} 4\frac{1-\sum_{r=\pm 1} n_F(E_N - r\mu_{i,\text{eff}})}{E_N}\right\},$$

$$E_\phi = \sqrt{k^2+m_\sigma^2} \qquad E_N = \sqrt{k^2+(M_N-g_\phi\sigma)^2} \text{ with } \sigma =< \phi >.$$

$$g_\omega\omega_{0,k} = \frac{f_\omega}{3\pi^2}\int_k^\Lambda dp\frac{p^4}{E_\text{N}}\sum_{r=\pm 1}\frac{\partial}{\partial\mu}\left[n_\text{F}(E_\text{N}-r\mu_p^\star)+(E_\text{N}-r\mu_n^\star)\right],$$

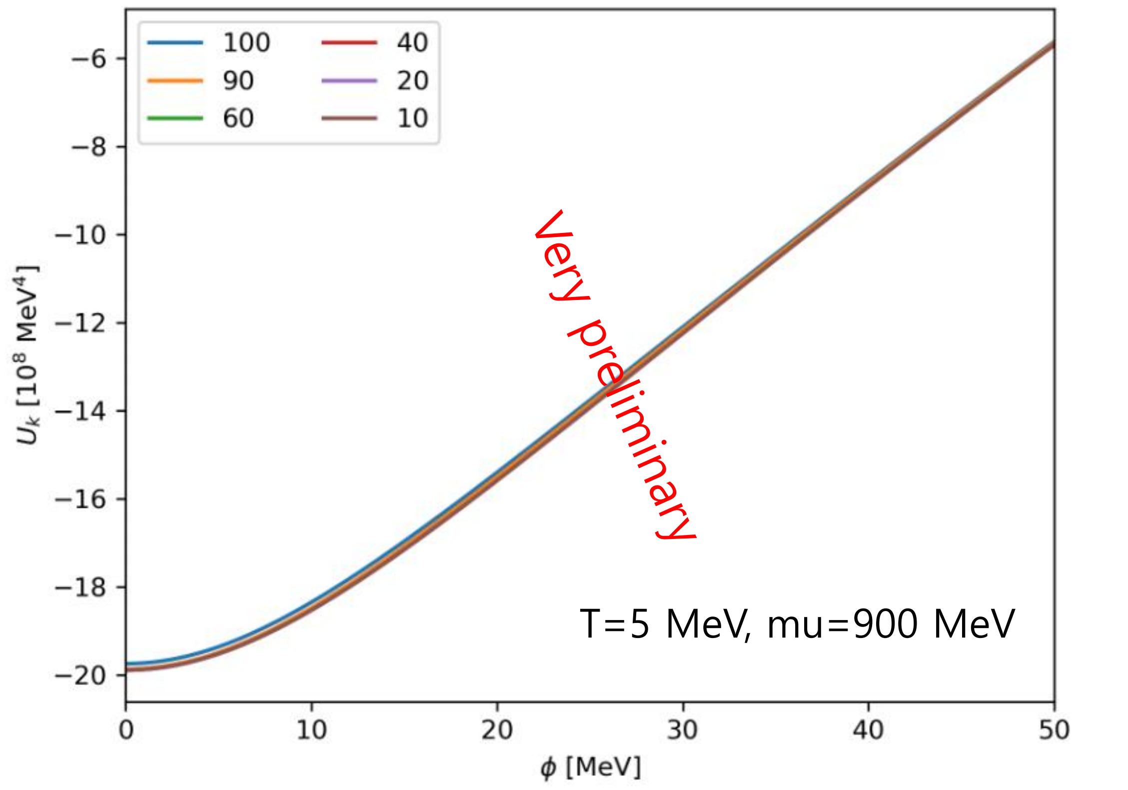
$$g_\rho\rho_{0,k} = \frac{f_\rho}{3\pi^2}\int_k^\Lambda dp\frac{p^4}{E_\text{N}}\sum_{r=\pm 1}\frac{\partial}{\partial\mu}\left[n_\text{F}(E_\text{N}-r\mu_p^\star)-(E_\text{N}-r\mu_n^{star})\right]$$

$$f_\omega \equiv g_\omega^2/m_\omega^2 \text{ and } f_\rho \equiv g_\rho^2/m_\rho^2.$$

$$\mu_i^\star = \mu_i - g_\omega \omega_0 \mp g_\rho b_0 \text{ (} - \text{ proton, } + \text{ neutron)}$$

The boundary conditions are: $g_\omega \omega_0(\Lambda, \mu, T) = 0$, $g_\rho \rho_0(\Lambda, \mu, T) = 0$

and $U_\Lambda^\phi = \frac{1}{3}\tilde{a}\phi^3 + \frac{1}{4}\tilde{b}\phi^4$.



Y. M. Kim, C.-H. Lee, S. Jeon and YK, in progress

Coming back to the linear sigma model with quarks, we seek for udQM.

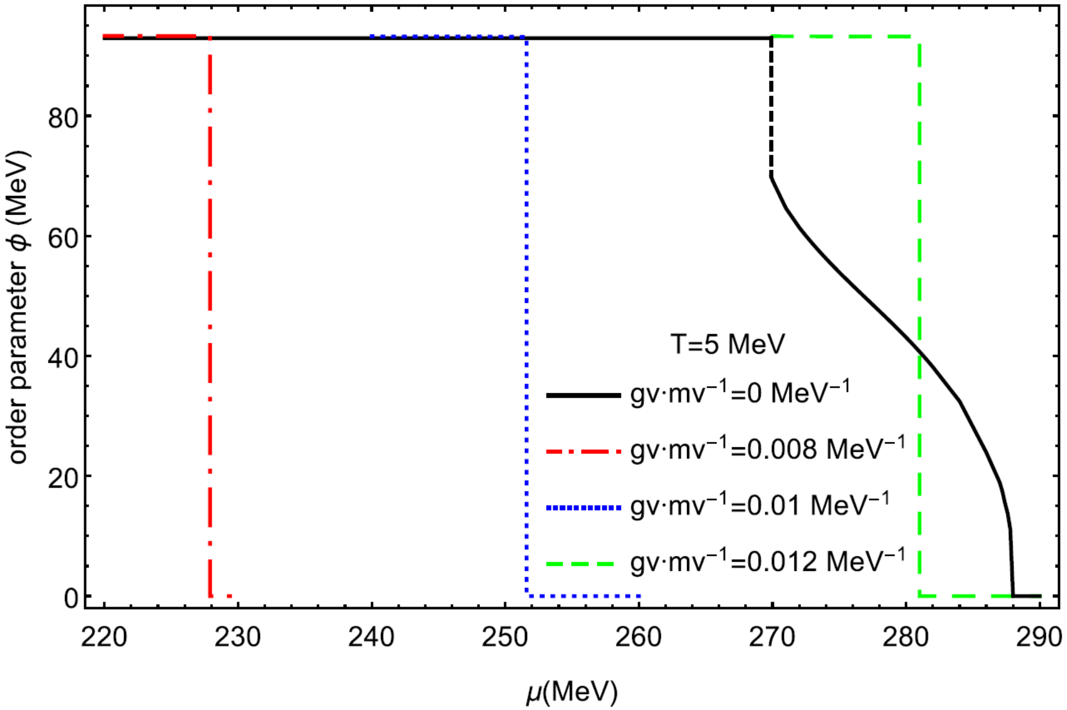
PHYSICAL REVIEW D **96,** 114029 (2017)

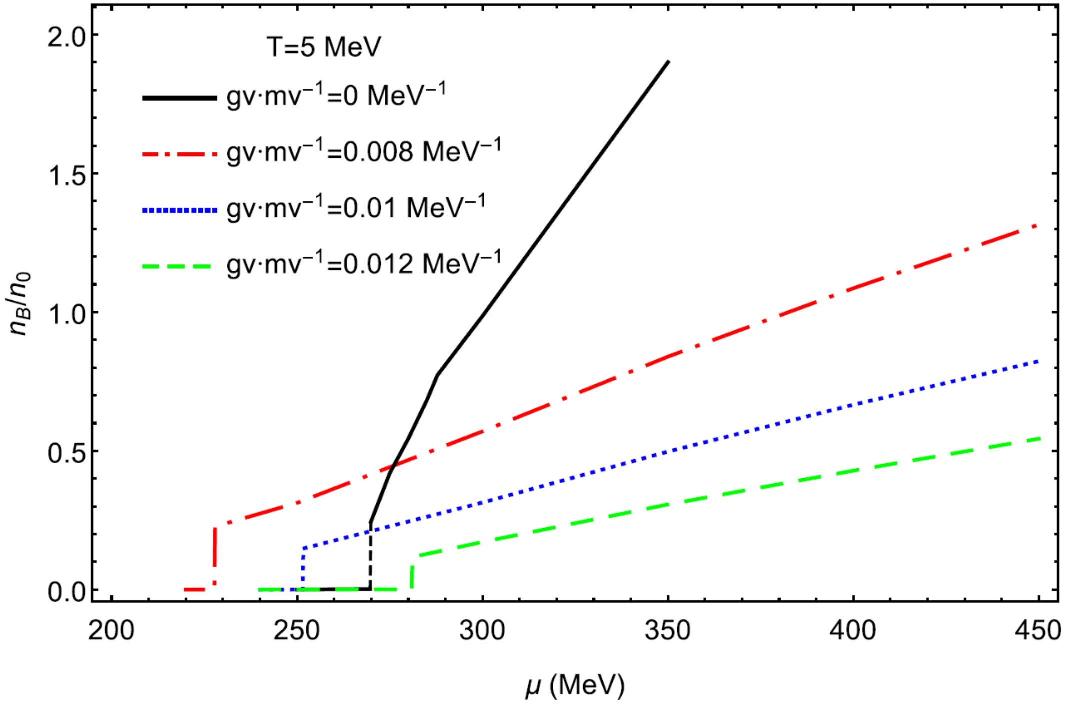
Functional renormalization group study of the quark-meson model with ω meson

Hui Zhang,[†] Defu Hou,[*] Toru Kojo,[‡] and Bin Qin[§]
Institute of Particle Physics (IOPP) and Key Laboratory of Quark and Lepton Physics (MOE), Central China Normal University, Wuhan 430079, China

$$\mathcal{L} = \bar{\psi}[i\gamma_\mu\partial^\mu - g_s(\sigma + i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}) - g_v\gamma_\mu\omega^\mu + \mu\gamma_0]\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - U(\sigma,\boldsymbol{\pi},\omega),$$

$$U(\sigma,\boldsymbol{\pi},\omega) = \frac{\lambda}{4}(\sigma^2 + \boldsymbol{\pi}^2 - f_\pi^2)^2 - \frac{m_v^2}{2}\omega_\mu\omega^\mu$$

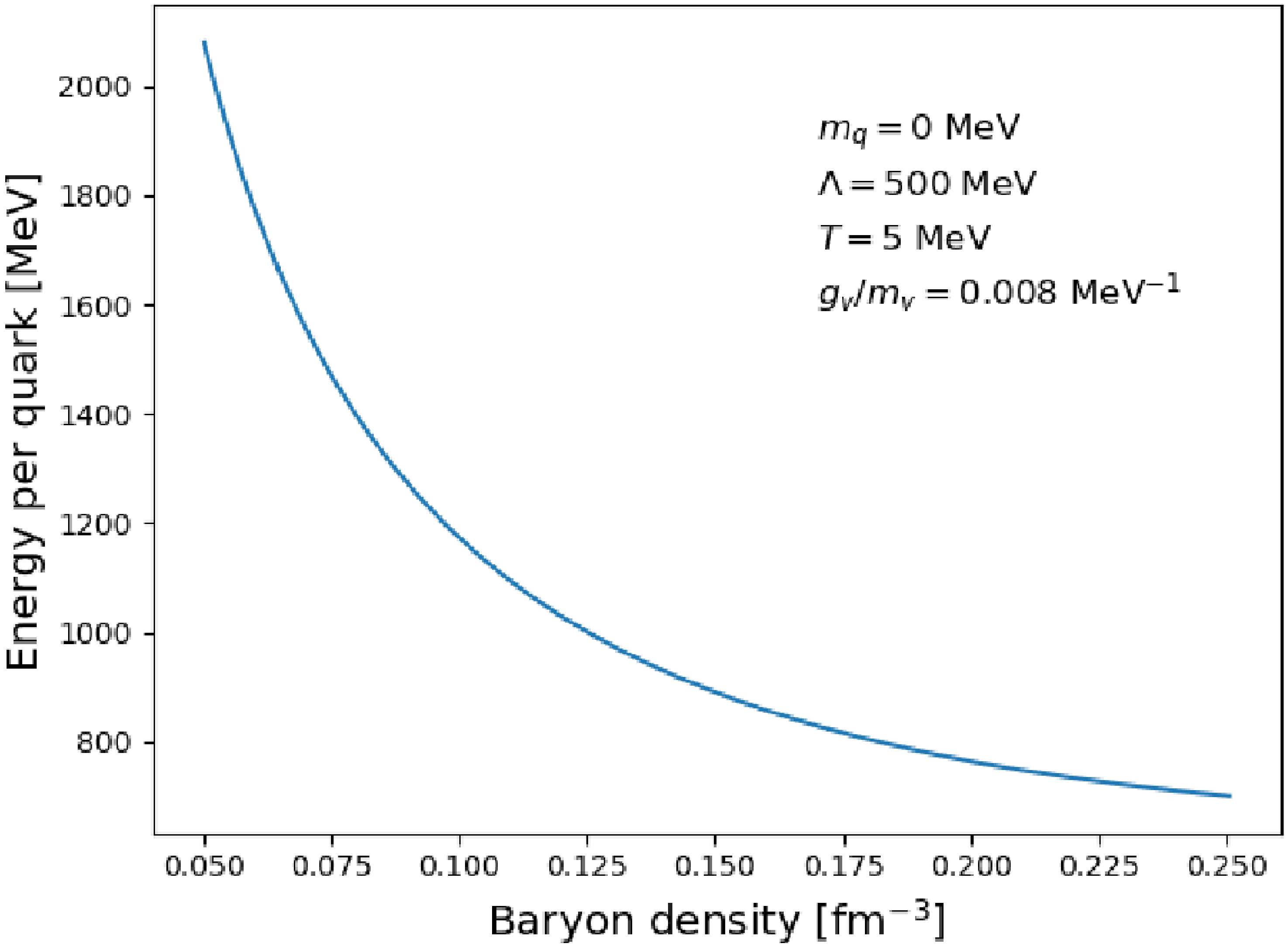




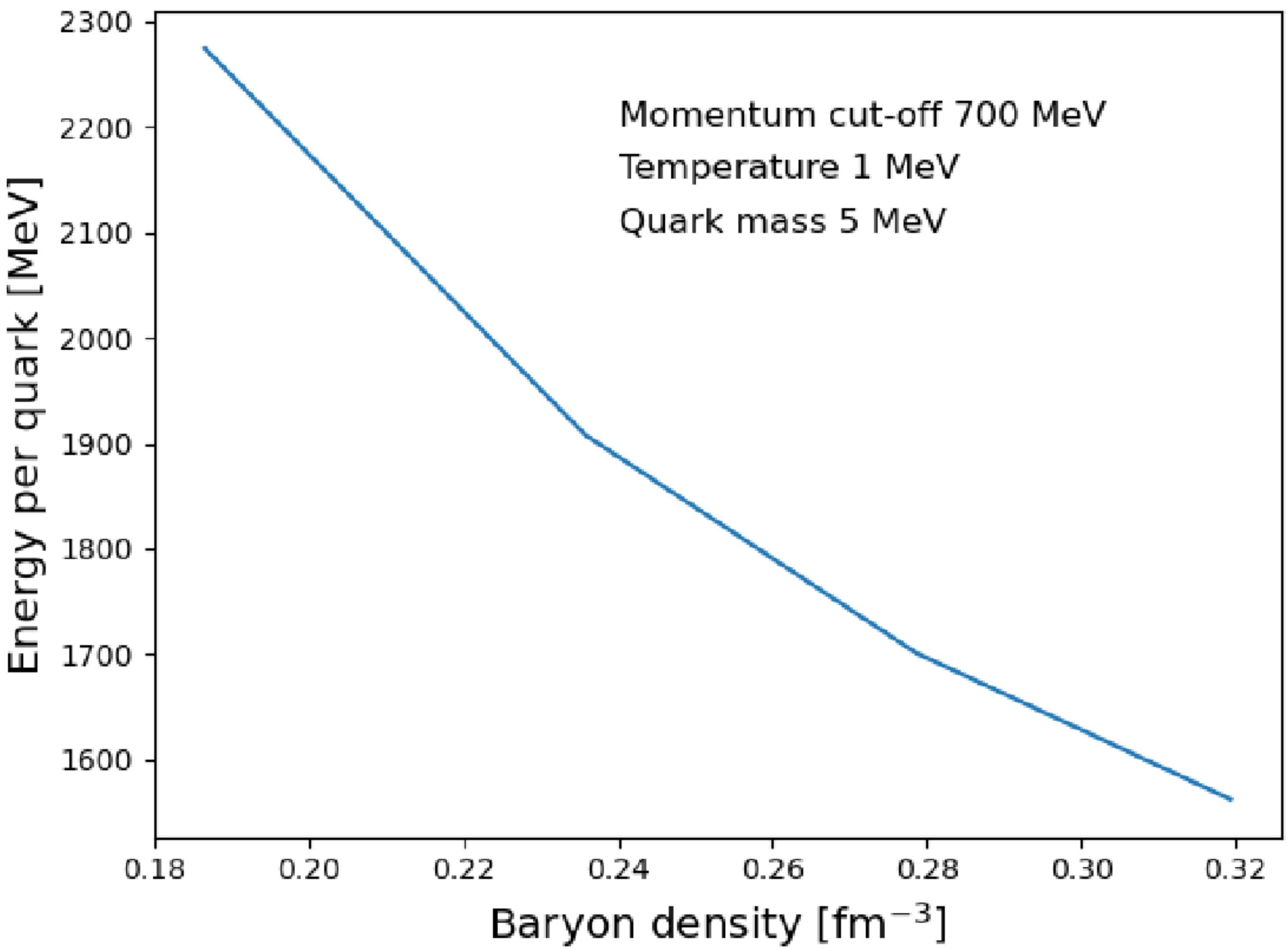
This behavior looks somewhat unnatural to us, and we expected that the introduction of the repulsive density-density interactions would tame this problem.

Our FRG results do not follow our expectation; what we found is that the low temperature first order phase transition in the FRG is induced by fluctuations, rather than number density as in the MF case, so that the structure of the low temperature boundaries remains similar for different values of vector couplings.

H. Zhang, D. Hou, T. Kojo and B.~Qin, Phys. Rev. D 96, 114029 (2017)



Qiang Zhao, Y.-M. Kim, YK, etal



Momentum cut-off 700 MeV
Temperature 1 MeV
Quark mass 5 MeV
Energy per quark [MeV]
Baryon density [fm^{-3}]
2300
2200
2100
2000
1900
1800
1700
1600
0.18
0.20
0.22
0.24
0.26
0.28
0.30
0.32

Summary

- The FRG method has been widely used in dense nuclear matter to deal with fluctuations. (also in various fields in physics such as electroweak phase transitions, ultra-cold atoms, etc.)
- We have developed a numerical code to solve the FRG equations in the extended Walecka model, but yet no relevant parameter ranges for nuclear matter.
- We are trying to find udQM …