Oscillations of rapidly rotating quark stars in general relativity

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based on collaboration with
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Why the study of oscillation modes of compact stars is important?

Oscillations of compact stars

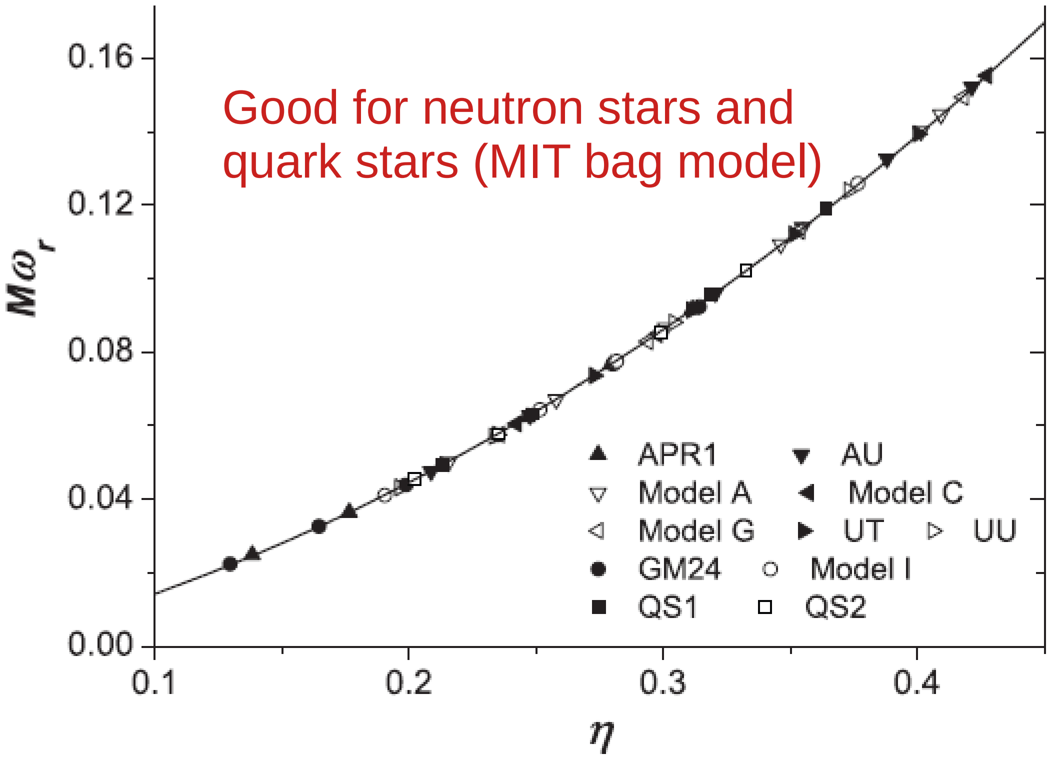
- Oscillations of compact stars carry rich information about the stellar interior and can be used to probe the properties of nuclear matter

Already discussed a few times yesterday in this workshop!

- Different oscillation modes
  f-mode ("fundamental") ← Focus in this talk
  p-modes ("pressure")
  interfacial modes
  r-modes
  torsional modes
  shear modes
  ….

Gravitational wave asteroseismology (…… in the future)

Universal relation between f-mode and moment of inertia



$$\eta \equiv \sqrt{\frac{M^3}{I}}$$

Lau, Leung & LML, ApJ, 714, 1234 (2010)

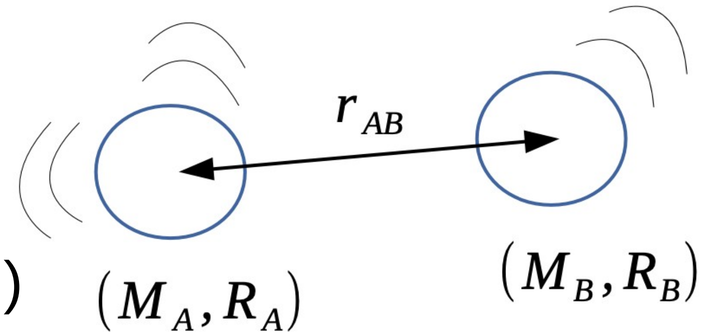
- Similar universal relation also exists for the imaginary part of the frequency (due to GW emission). M, I and R can be inverted from the "observed" signals to high accuracy (~1% level).

Relevance to GW measurement (…. now)

- When $r_{AB} \sim R_A$, orbital frequency approaches f-mode frequency, the system is approaching resonance and more complex tidal response of the stars.

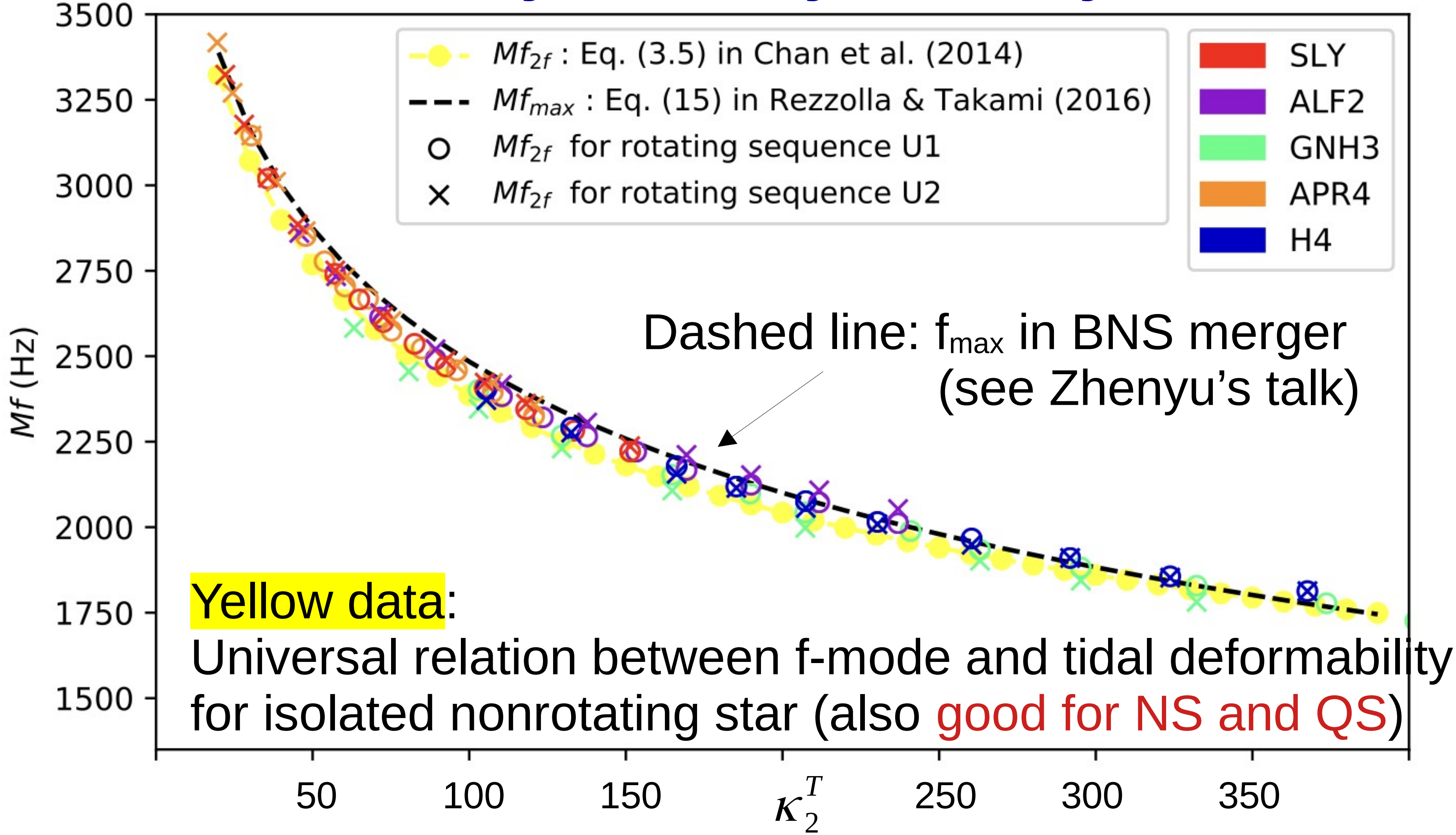
Dynamical tidal effects

- GW measurement depends on fast and accurate waveform modeling.
- Analytical waveform models including the dynamical tidal effects have been proposed and used. (eg, Hinderer et al. PRL, 116, 181101 (2016))



- Input parameters needed: tidal deformabilities λ and f-mode frequencies
- Universal relation between λ and f-mode can be used to reduce model parameters [Chan, Sham, Leung, LML, PRD, 90, 124023 (2014)]

Relevance to a question raised in Zhenyu's talk yesterday



Ng, Cheong, LML, Li, ApJ, 915, 108 (2021)

Computation of oscillation modes

- Nonrotating stars:

Perturbed scalar variables: $\delta \rho = f(r) Y_{lm}(\theta, \phi) e^{i\omega t}$

$$\delta G_{\mu\nu} = 8\pi \delta T_{\mu\nu} \quad ; \quad \delta\left(\nabla_{\mu} T^{\mu\nu}\right) = 0$$

↓

boundary value problem

For spherical stars, the radial eigenfunctions and mode frequencies are degenerate in the index *m* (only need to consider *m* = 0).

- Rotating stars:

$$\delta \rho = f(r, \theta) e^{i(m\phi + \omega t)}$$

Degeneracy in *m* is lifted due to rotation (similar to Zeeman effect)

- We shall focus on the $l = |m| = 2$ f-modes in this talk.

- For rapidly rotating stars, it is more convenient (or the only way?) to use a hydrodynamics code to extract the oscillation modes

Add a “suitable” perturbation to an initial rotating star

↓

Hydrodynamics code

Options:

Exact GR / approximated GR / fixed spacetime (no gravity perturbation)

+

Nonlinear hydro / Linearized hydro

↓

Time evolution

↓

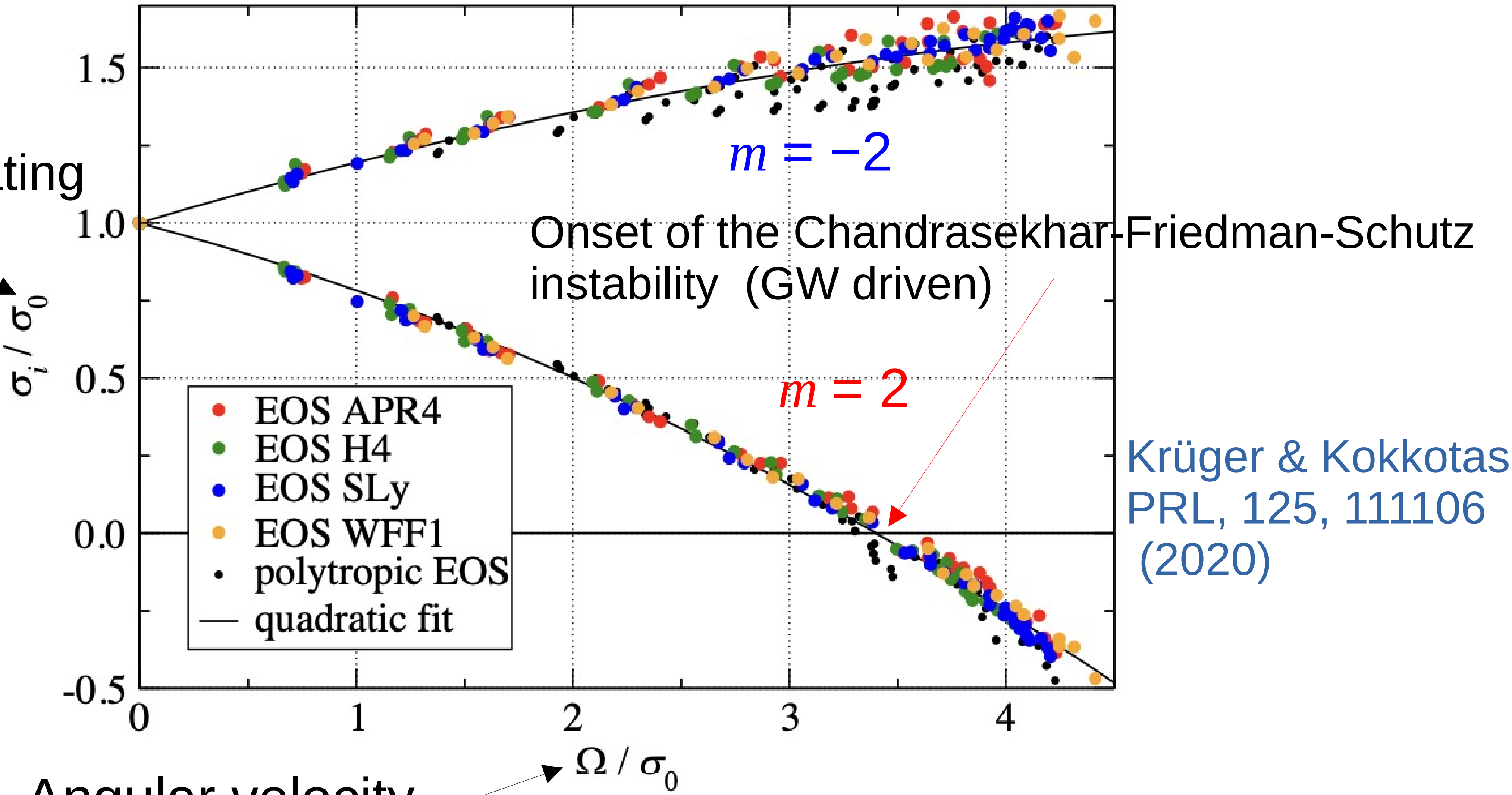
Perform Fourier transform for fluid variables to extract oscillation modes

What is latest result for rotating neutron stars?

Universal relations for the f-mode frequencies (observed in the inertial frame) for sequences of constant central energy density

Frequency for nonrotating models

Frequency in inertial frame

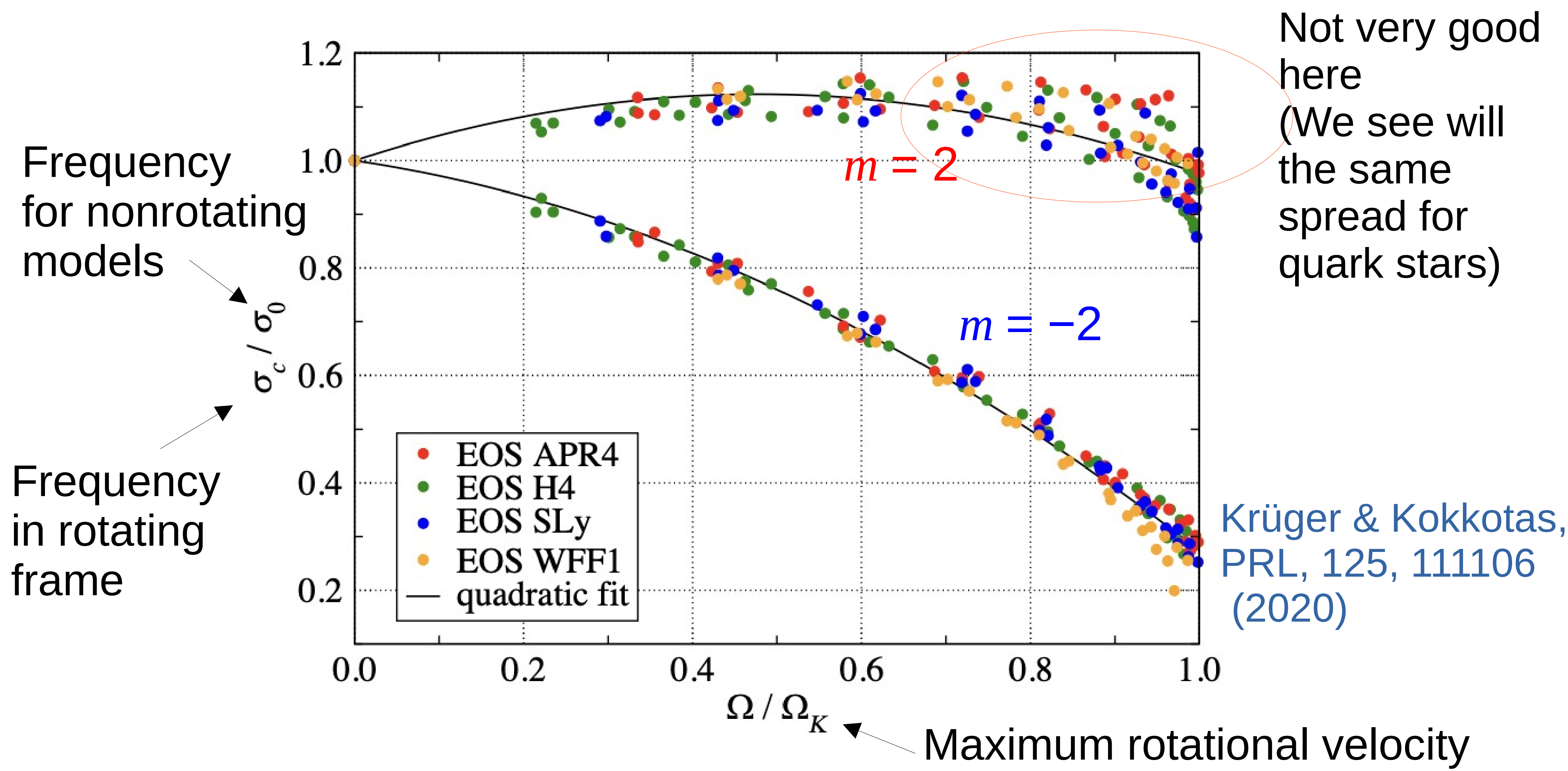


Krüger & Kokkotas, PRL, 125, 111106 (2020)

Angular velocity

Method: linearized GR hydrodynamics

Universal relations for the f-mode frequencies (observed in the rotating frame) for sequences of constant baryon mass



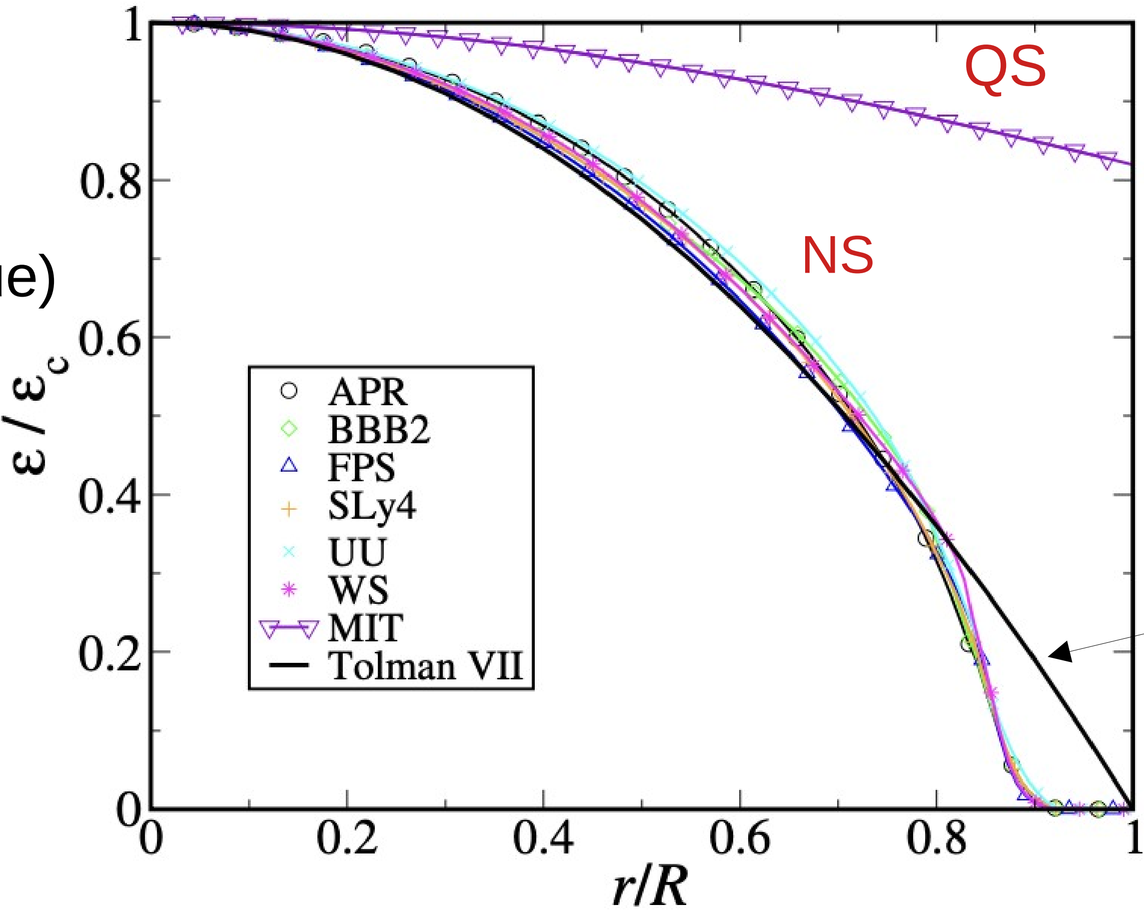
Note: There is another instability onset ($\sigma_c = 0$) driven by viscosity.
Realistic neutron star models cannot achieve that onset point.

How about quark stars?

- Oscillation modes of nonrotating quark stars are well studied.
- For rapidly rotating quark stars, using hydrodynamics code to study their oscillations is more difficult comparing to neutron stars.

(Nonrotating) neutron stars vs quark stars

Energy density (normalized by the central value)



Finite (and high) surface density!

Compactness M/R = 0.1 (fixed)

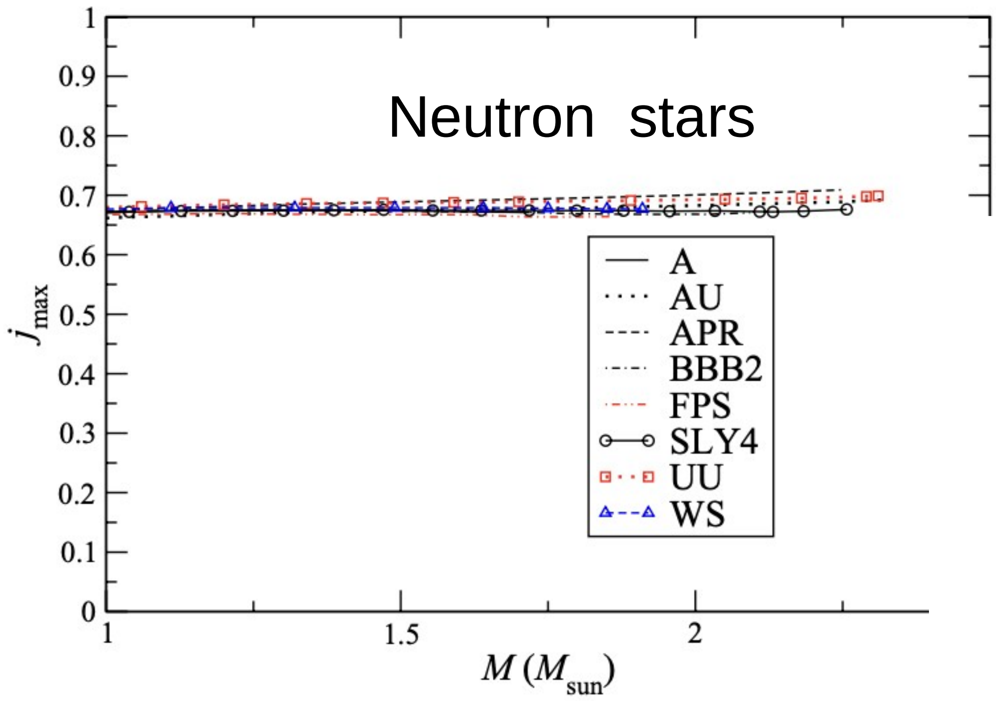
$\delta = 1$

$$\epsilon = \epsilon_c \left[1 - \delta \left(r / R \right)^2 \right]$$

Sham, Chan, LML, Leung
ApJ, 798, 121 (2015)

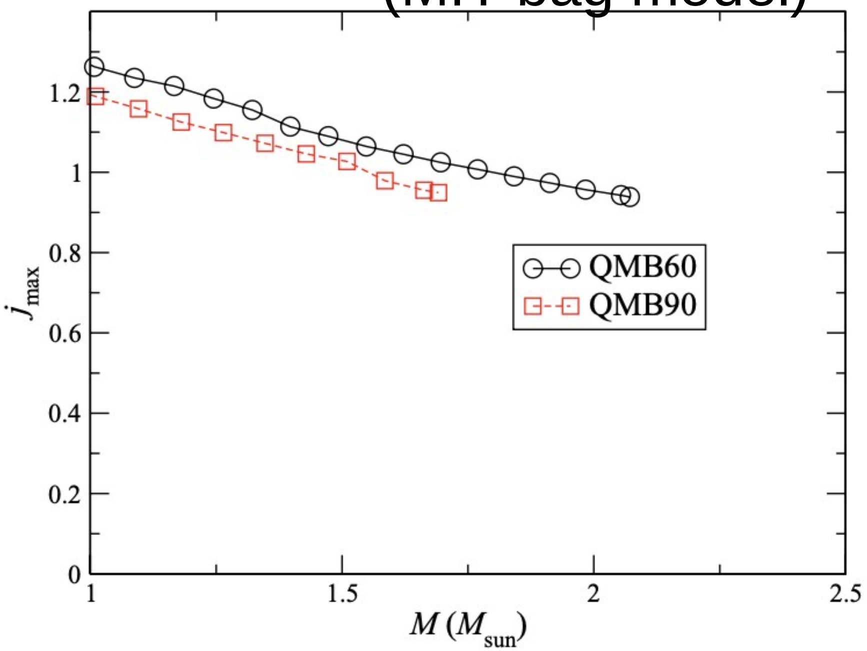
(Rotating) neutron stars vs quark stars

Dimensionless spin parameter $j = \frac{cJ}{G M^2}$



Rotating black hole: $j_{max} = 1$

Quark stars
(MIT bag model)



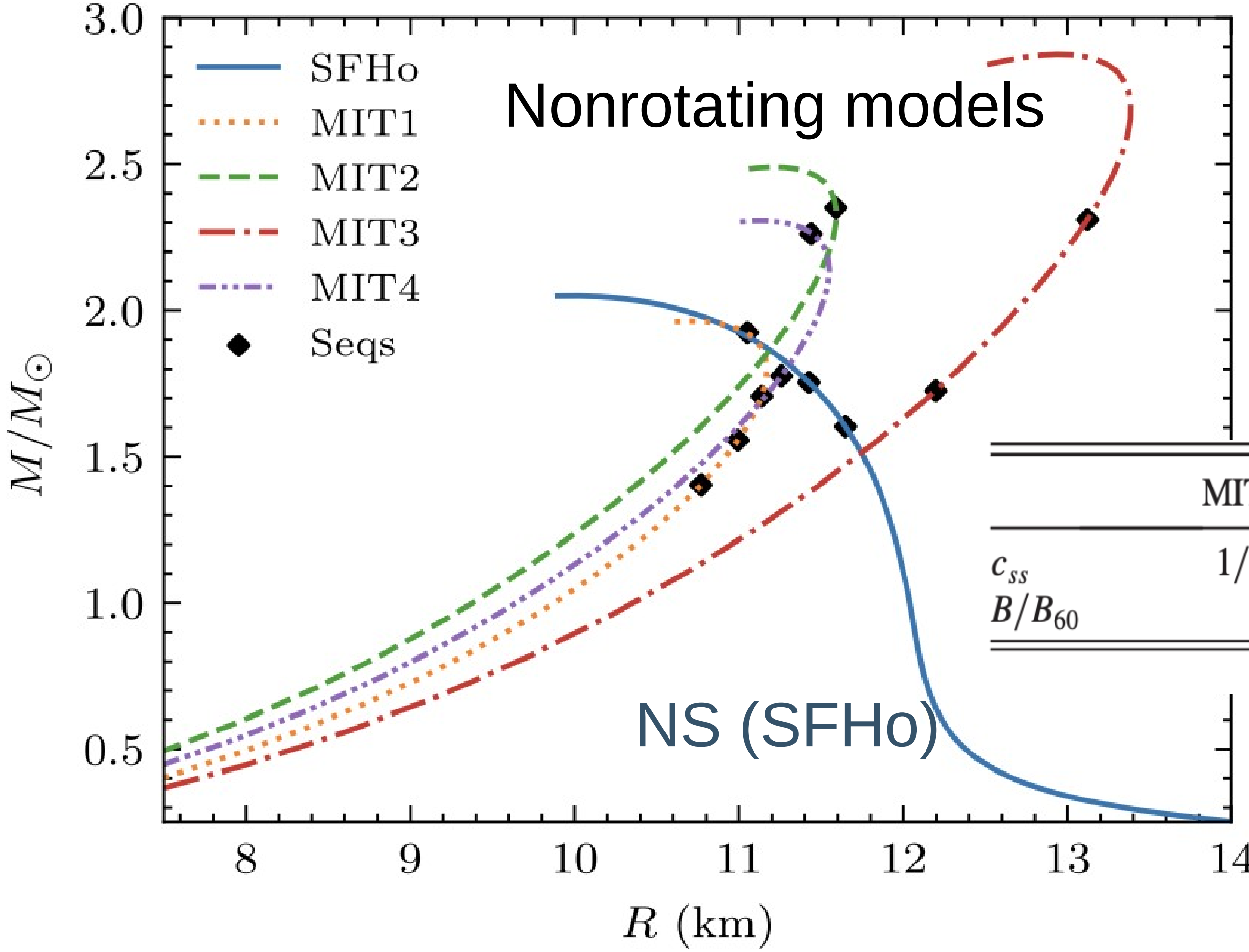
Lo & LML, ApJ, 728, 12 (2011)

Our recent study of the oscillation modes of rapidly rotating quark stars

Chen & LML, PRD, 108, 064007 (2023)

Kenneth Chen

Our quark star models



Parametrized model:

$$P = c_{ss}\,\epsilon - (1 + c_{ss})\,B$$

	MIT1	MIT2	MIT3	MIT4
c_{ss}	1/3	1	2/3	1/2
B/B_{60}	1	3	3/2	3/2

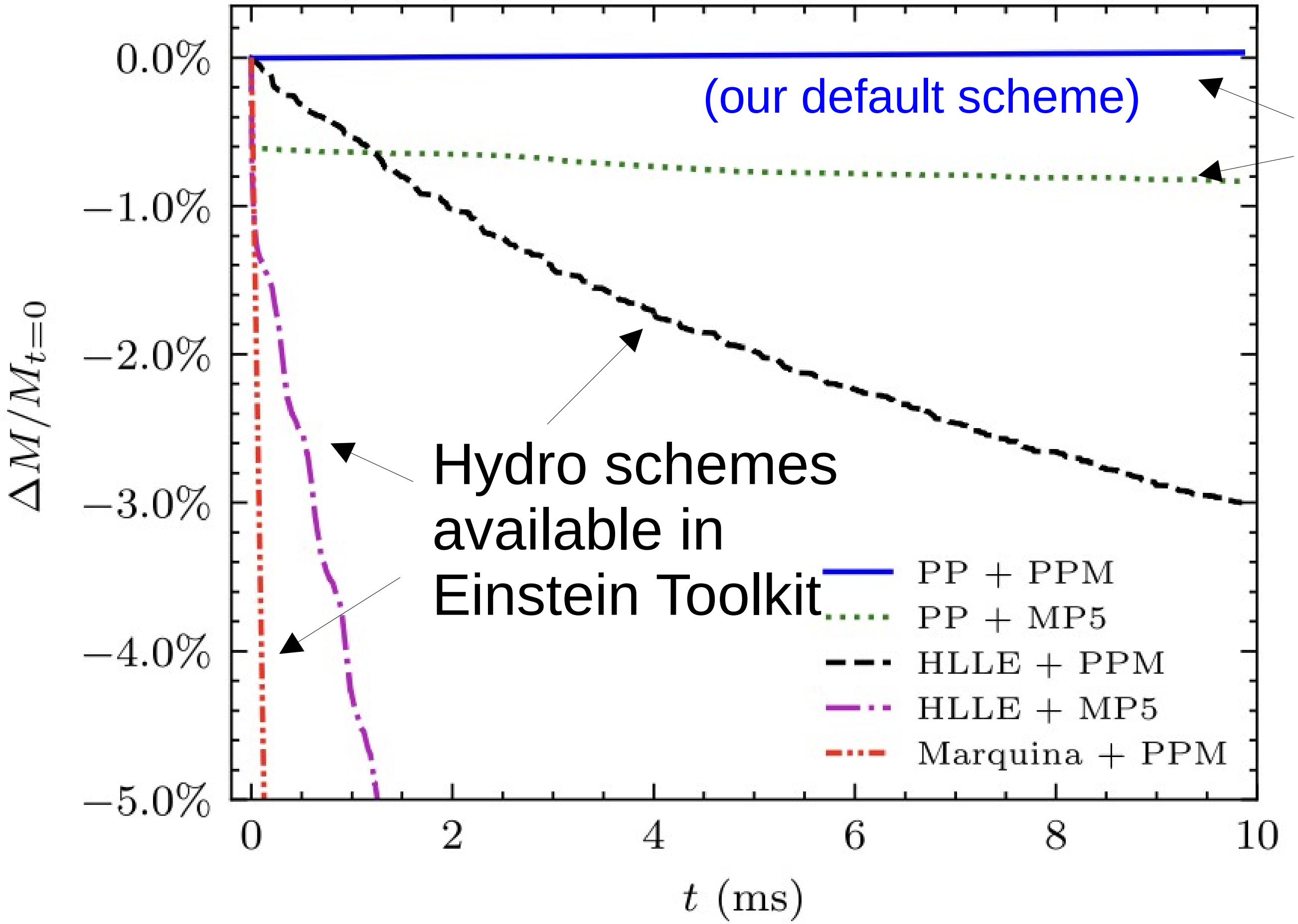
B_{60} = 60 MeV/fm^3

Rapidly rotating quark stars: Full GR hydrodynamics modelling

- Use open-source *LORENE* to generate initial rotating quark stars
- Use *Einstein Toolkit* to perform standard formulation for spacetime evolution
- We need to implement our own hydro scheme (Positivity-preserving Riemann solver) to handle the surface density discontinuity of quark stars
- Special treatment of a “dust” atmosphere outside the star

LORENE: http://www.lorene.obspm.fr/
Einstein Toolkit http://einsteintoolkit.org/

Mass conservation



~0.03% deviation

Our hydro method (Positivity preserving)

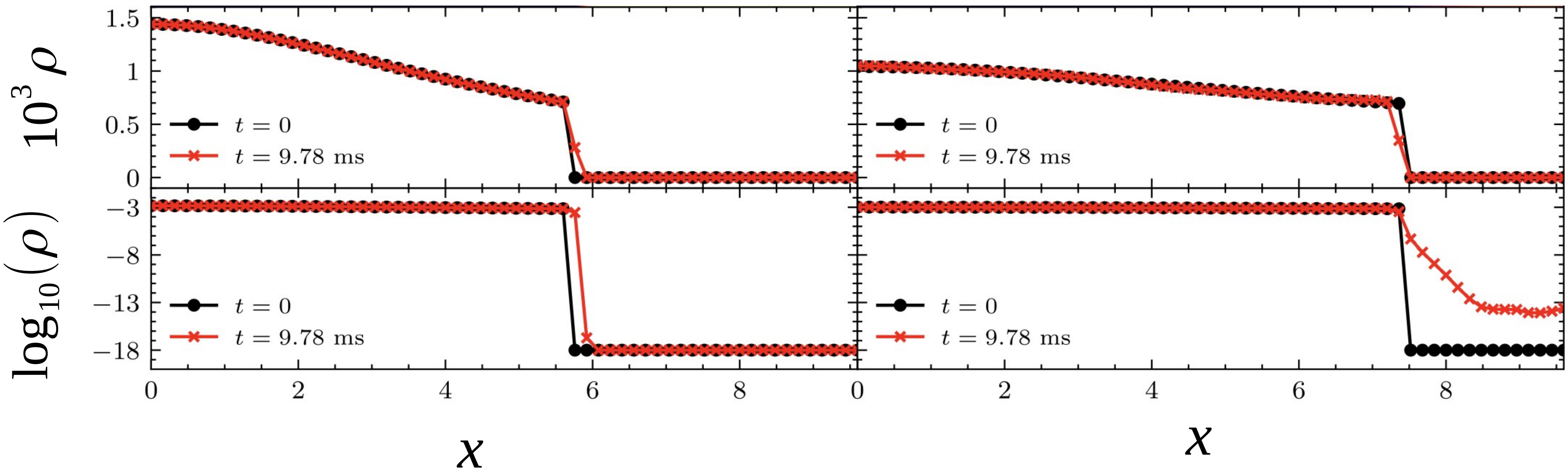
Basically, the scheme preserves the positivity of density and pressure near the surface

Einstein Toolkit does not work out of the box for quark stars!

Stability of rest-mass density profiles

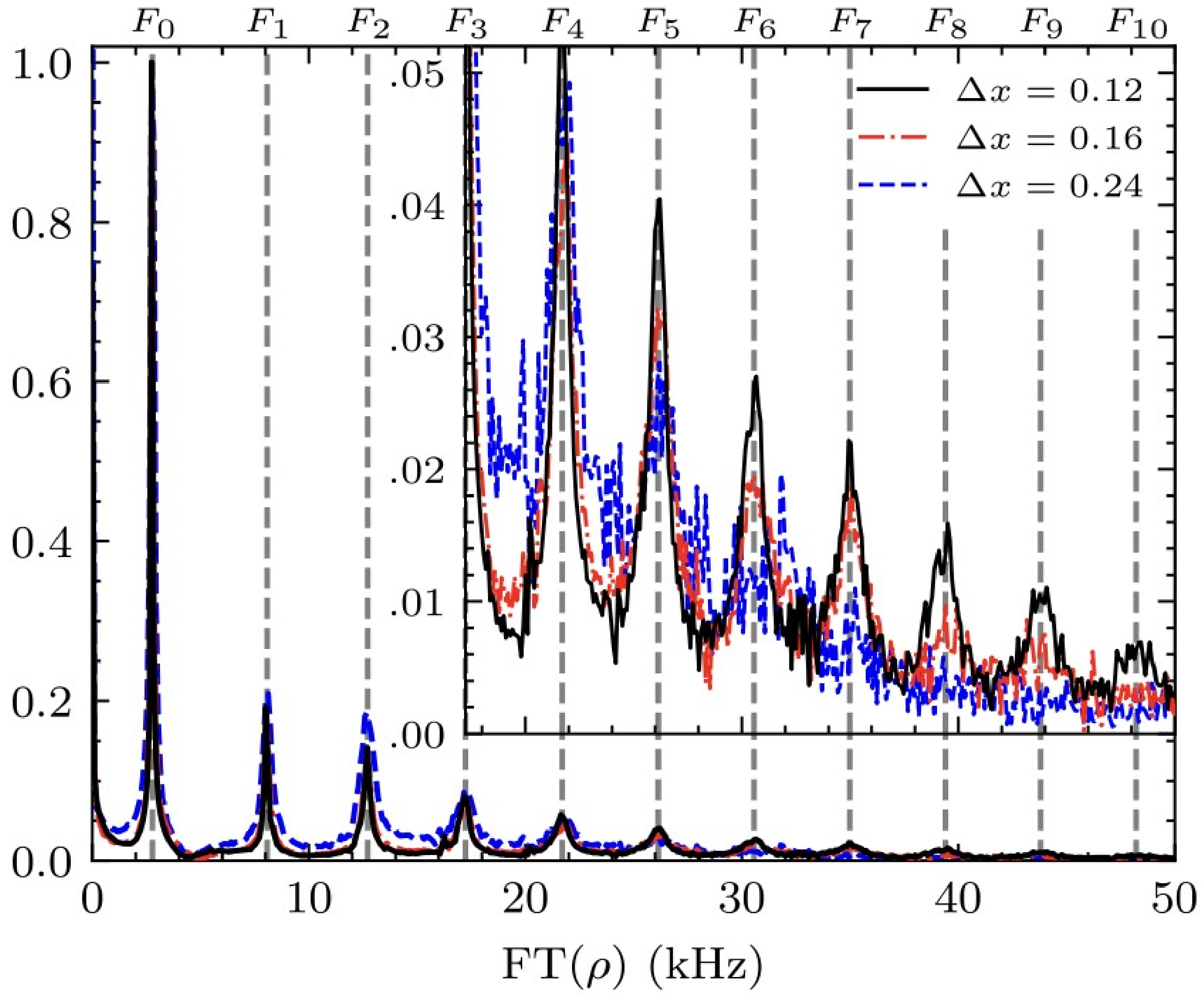
Rotation frequency = 300 Hz

Rotation frequency = 1200 Hz (~max frequency)



(Code's length unit: 1 ≈ 1.47 km)

Radial oscillation modes of nonrotating quark star

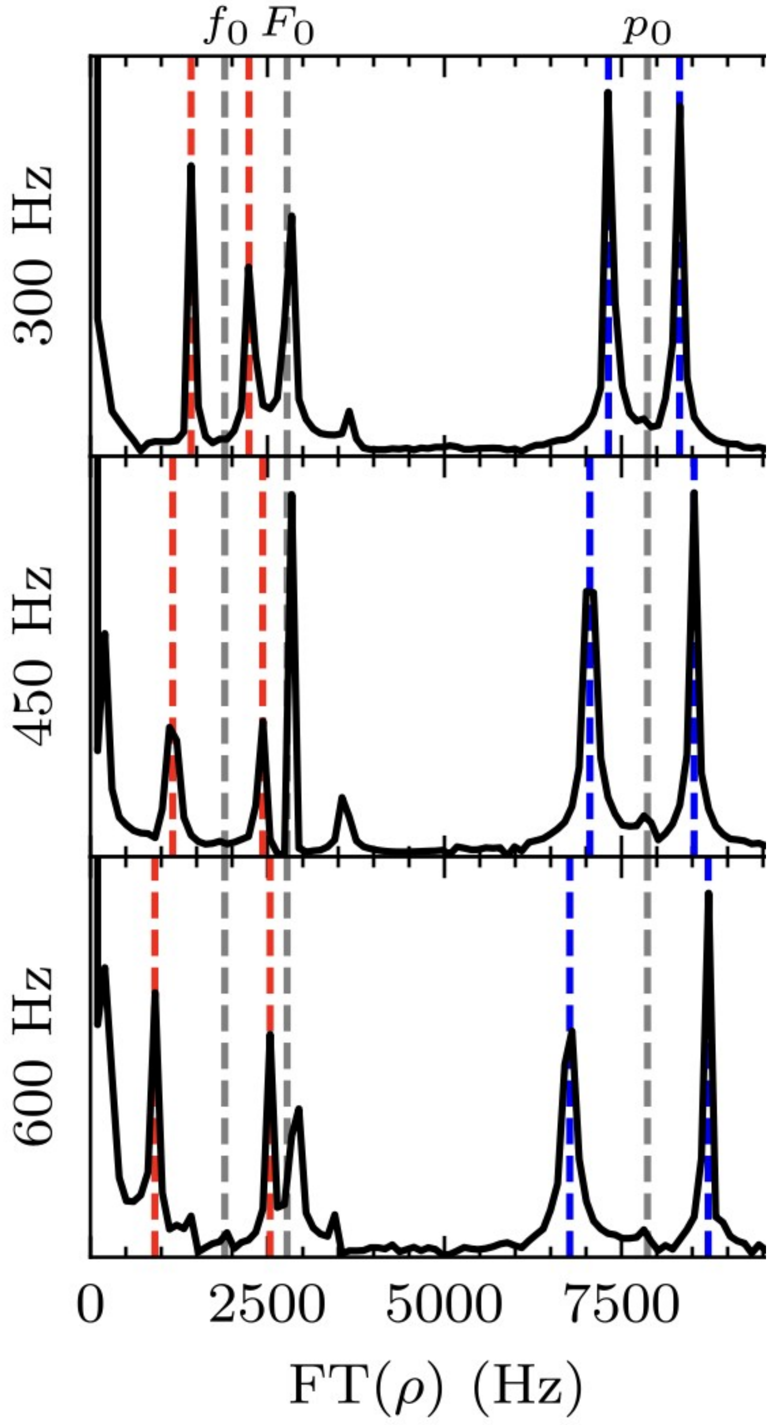


f_0 = f-mode (nonrotating)

F_0 = Fundamental radial mode

p_0 = pressure mode

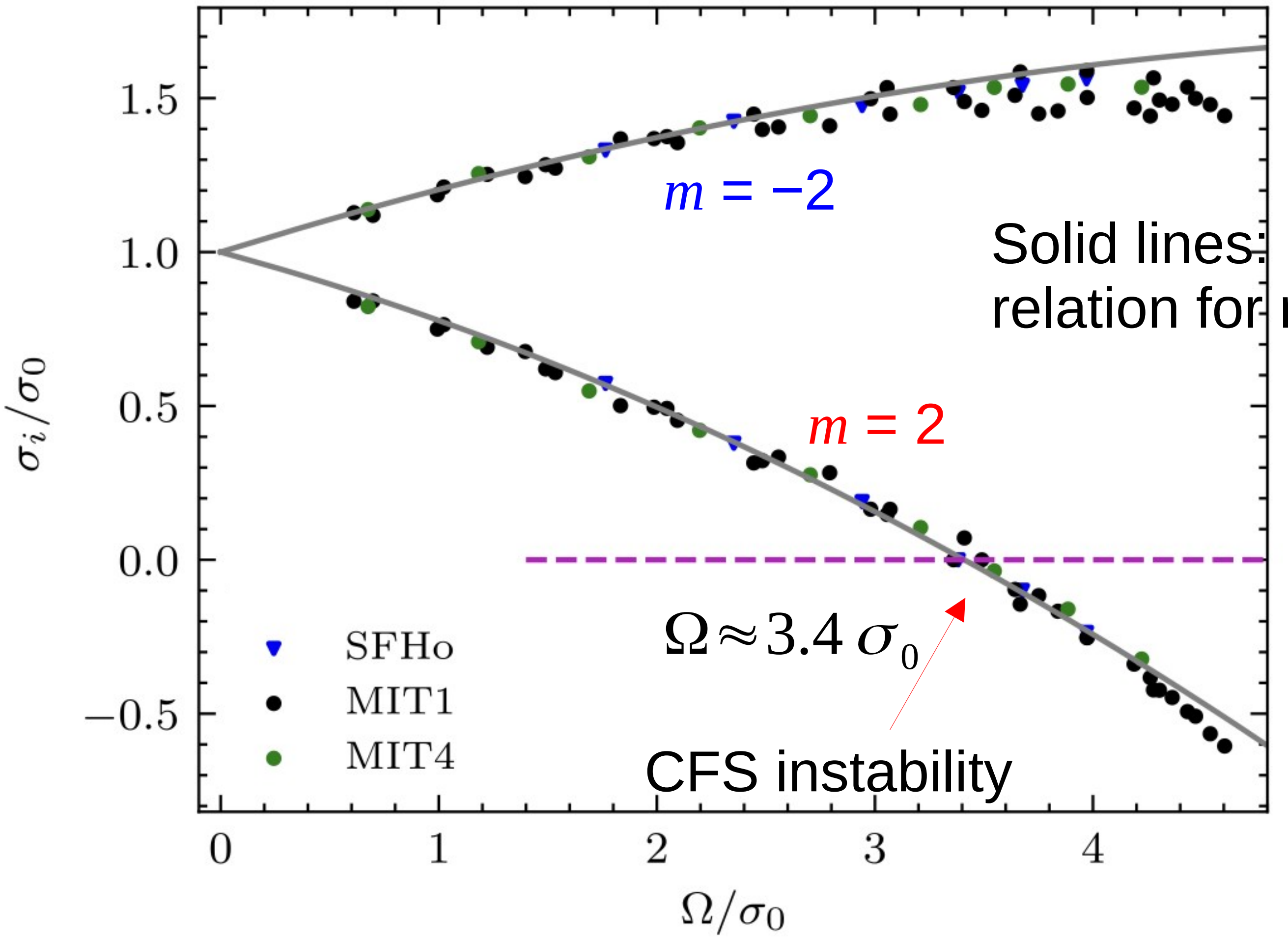
Rotation frequency



Left red (blue) lines:
m = 2 mode

Right red (blue) lines:
m = −2 mode

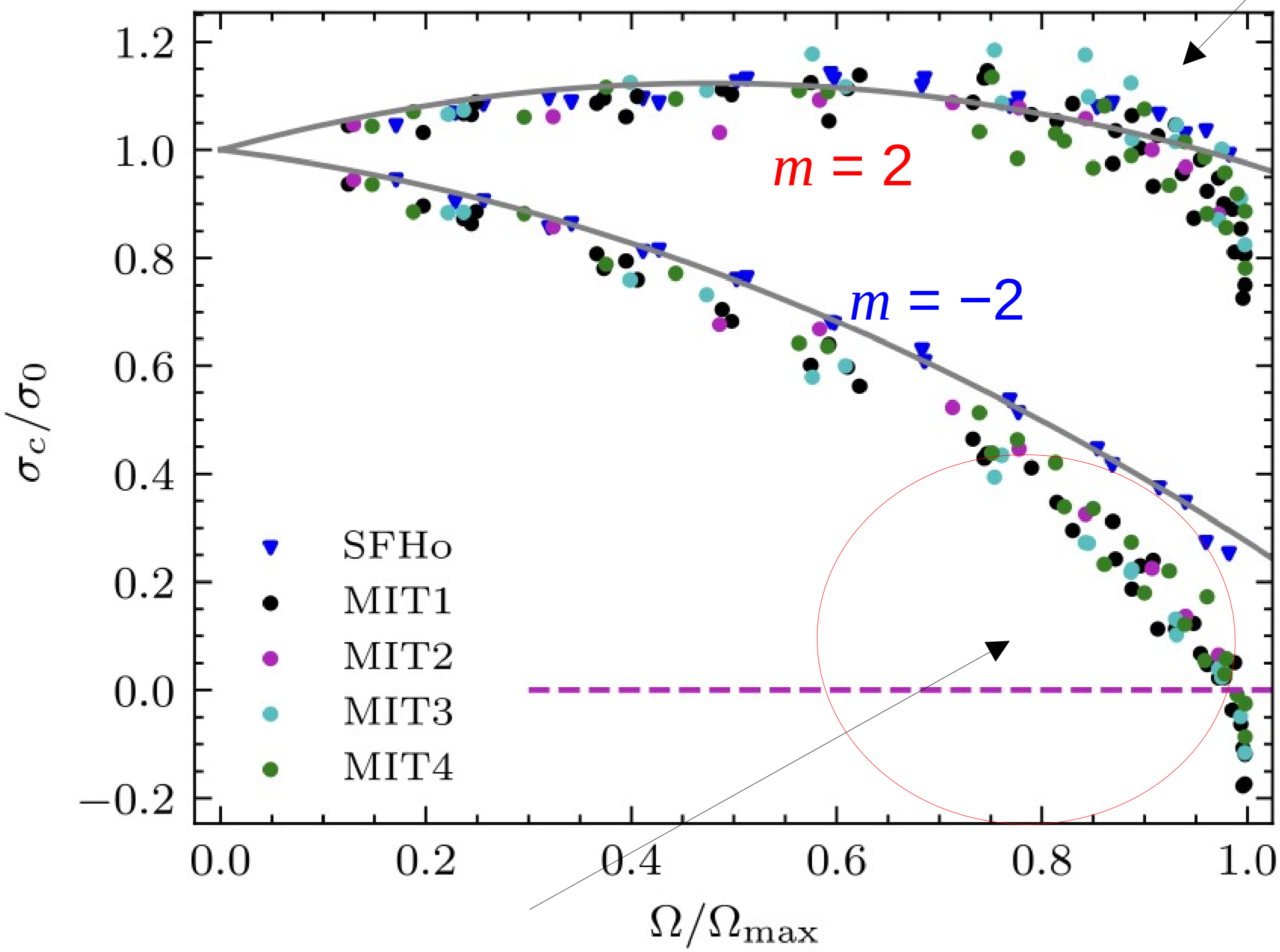
f-mode frequencies (observed in the inertial frame) for sequences of constant central energy density



Solid lines: Original universal relation for neutron stars

Rapidly rotating quark stars still satisfy the same relations and the onset condition for CFS instability f-mode frequencies (observed in the rotating frame) for sequences of constant baryon mass

The spread is the same as NS data

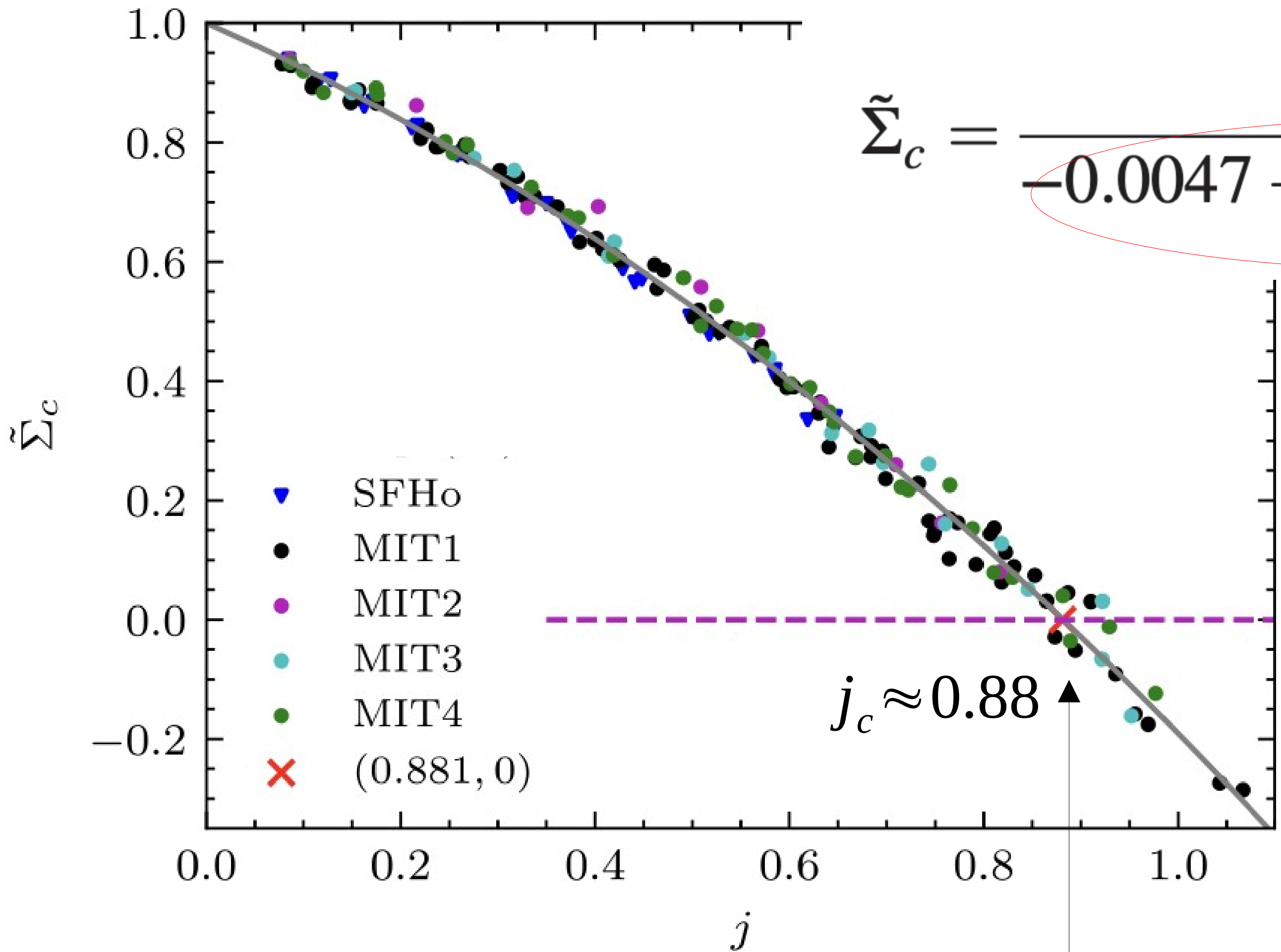


Solid lines:
Original universal relation for neutron stars

Quark star data can cross zero (onset of viscosity driven instability)

Rapidly rotating quark stars deviate from this relation significantly

"Unified" relation for neutron and quark stars



$$\tilde{\Sigma}_c = \frac{2\pi M \sigma_c}{-0.0047 + 0.133\eta + 0.575\eta^2}$$

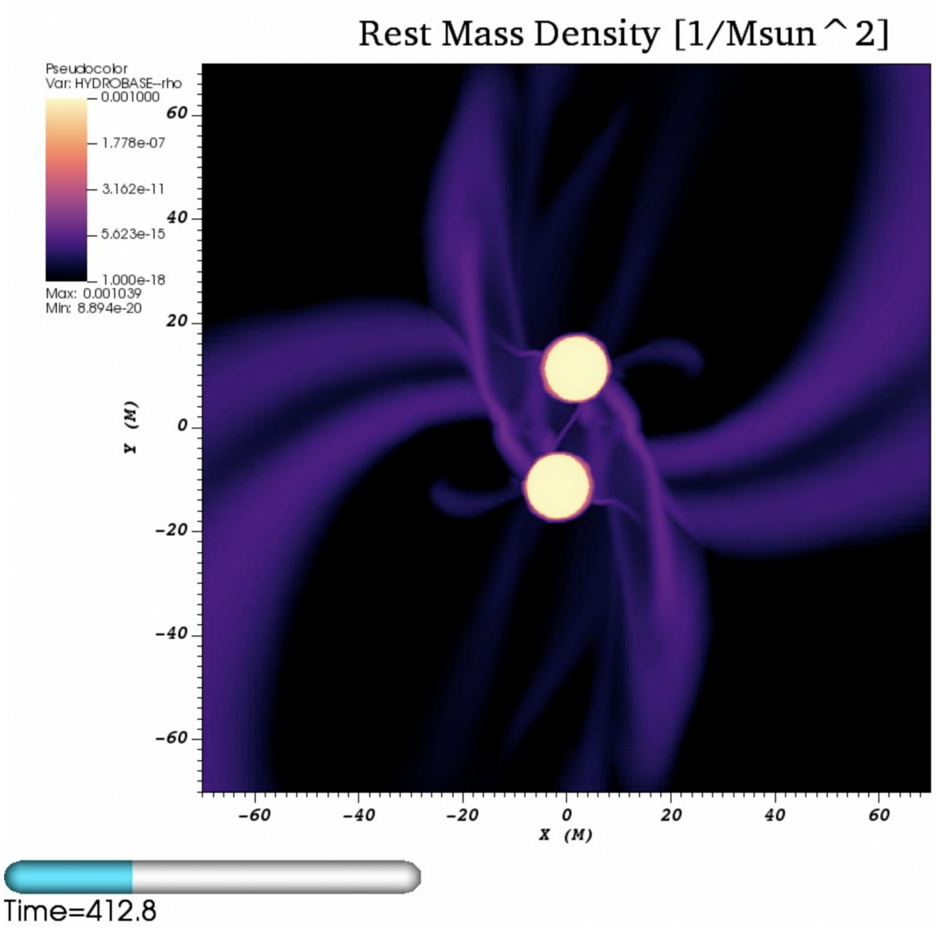
$$\eta \equiv \sqrt{\frac{M^3}{I}}$$

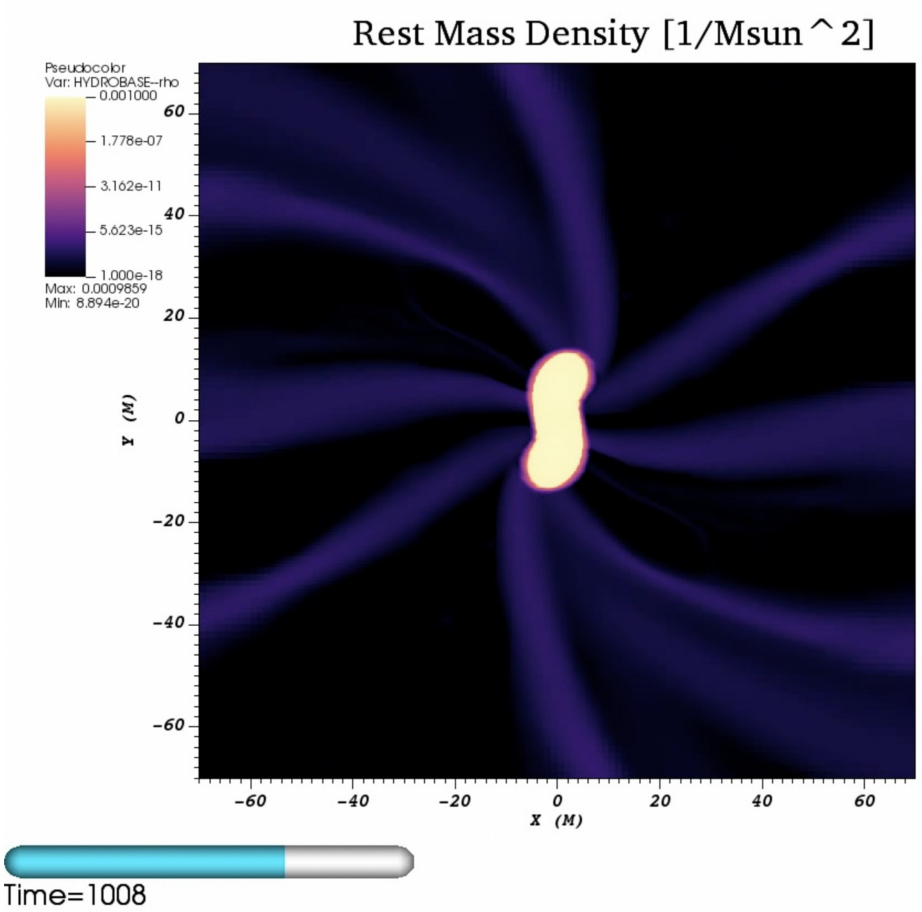
Motivated from nonrotating stars

Lau, Leung & LML, ApJ, 714, 1234 (2010)

Onset of viscosity-driven instability for quark stars
(Neutron stars cannot achieve such a high *j*)

What's next……..binary quark star merger? (Preliminary result)





Previous work: Zhu & Rezzolla PRD, 104, 083004 (2021)
Zhou et al. PRD, 106, 103030 (2022)

Summary

- We demonstrate our ability to evolve rapidly rotating quark stars in full GR simulations.
- We study the oscillation modes of rapidly rotating quark stars for the first time and investigate the onset of mode instabilities
- We have also considered the validity of universal relations found previously for rotating neutron stars.

Thank you!

Kenneth Chen:
I can now graduate……thank you!