GRAVITATIONAL WAVES FROM MAGNETAR GLITCHES AND ANTIGLITCHES

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Image credit: Ryuunosuke Takeshige

MOTIVATION – GLITCHES AND ANTIGLITCHES OBSERVATIONS

SGR 1935+2154: first magnetar localised to within the Milky Way ($d \sim 9$ kpc), has repeating FRBs

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Younes et al. (2023)

- $\frac{\Delta\nu}{\nu} = -5.8 \times 10^{-6}$
- 3 FRBs detected 3 days later, all within a single rotation ($P \approx 3.25$ s, $\nu \approx 0.308$ Hz)
- A few hours later, a pulsed radio signal was observed by FAST for at least 20 days [Zhu et al., in press]
- $\frac{\Delta\dot{\nu}}{\dot{\nu}} \approx +0.2$

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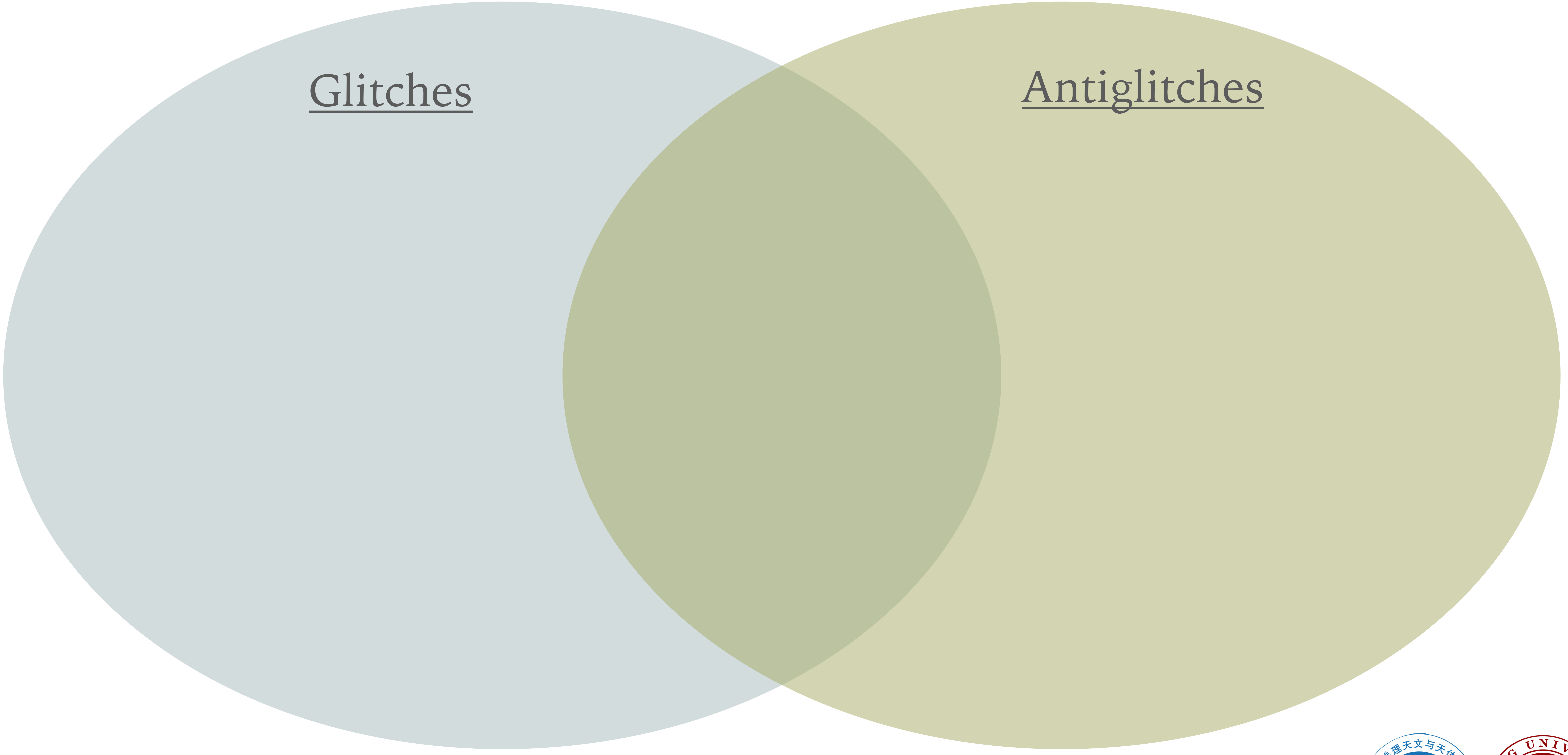
Ge et al. (submitted)

- $\frac{\Delta\nu}{\nu} = +6.4 \times 10^{-5}$
- FRB detected 3 days later, possibly weaker FRBs even later
- Information about pulsed radio signal not reported
- $\frac{\Delta\dot{\nu}}{\dot{\nu}} \approx -4.4$ [Weihua Wang's talk]

EXISTING MODELS FOR GLITCHES AND ANTIGLITCHES



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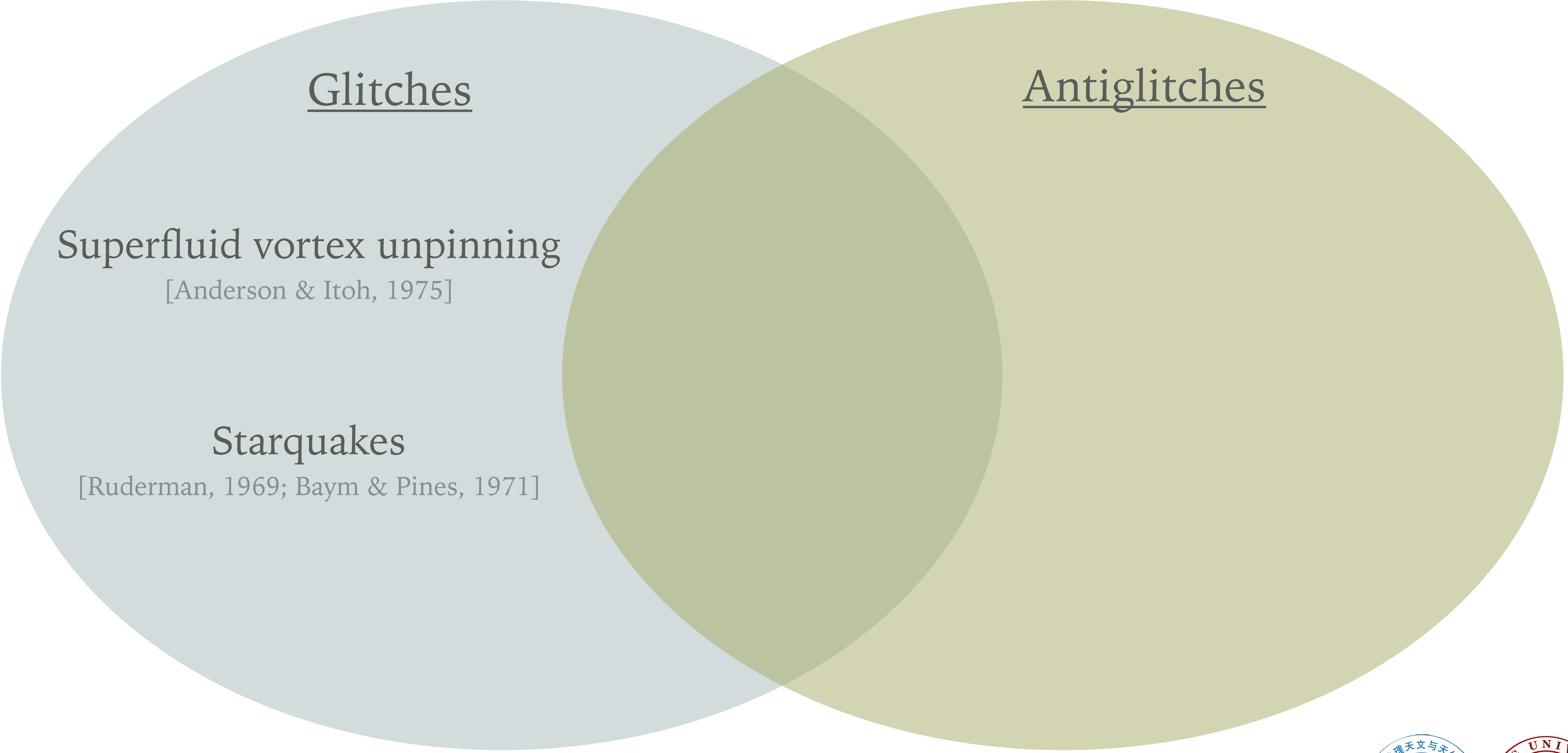

EXISTING MODELS FOR GLITCHES AND ANTIGLITCHES

Glitches

Superfluid vortex unpinning
[Anderson & Itoh, 1975]

Starquakes
[Ruderman, 1969; Baym & Pines, 1971]

Antiglitches

Enhanced particle wind
[Tong, 2014; Younes et al., 2023]

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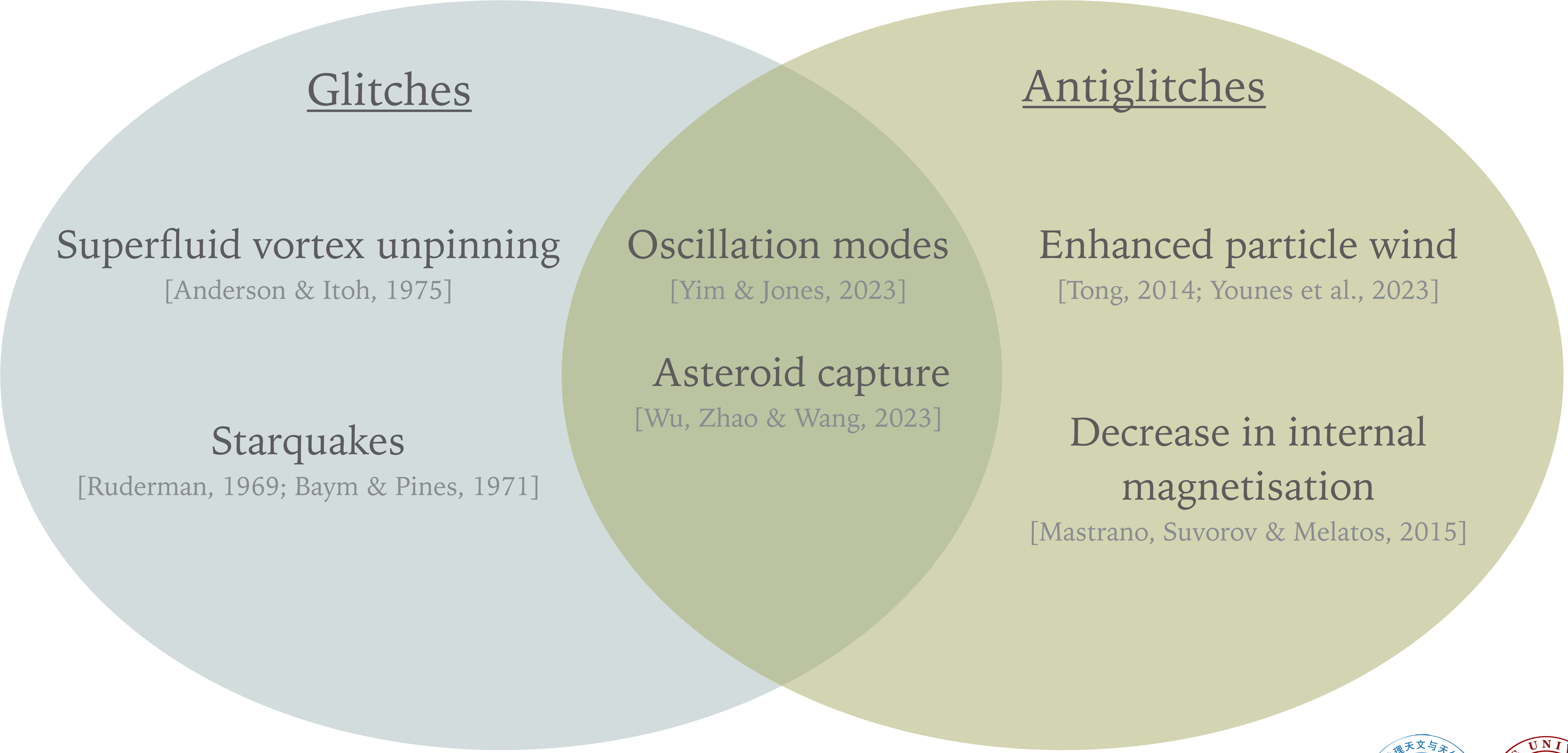

EXISTING MODELS FOR GLITCHES AND ANTIGLITCHES

Glitches

Superfluid vortex unpinning
[Anderson & Itoh, 1975]

Starquakes
[Ruderman, 1969; Baym & Pines, 1971]

Glitches and antiglitches

Oscillation modes
[Yim & Jones, 2023]

Asteroid capture
[Wu, Zhao & Wang, 2023]

Trapped ejecta
[Yim et al., this work]

Antiglitches

Enhanced particle wind
[Tong, 2014; Younes et al., 2023]

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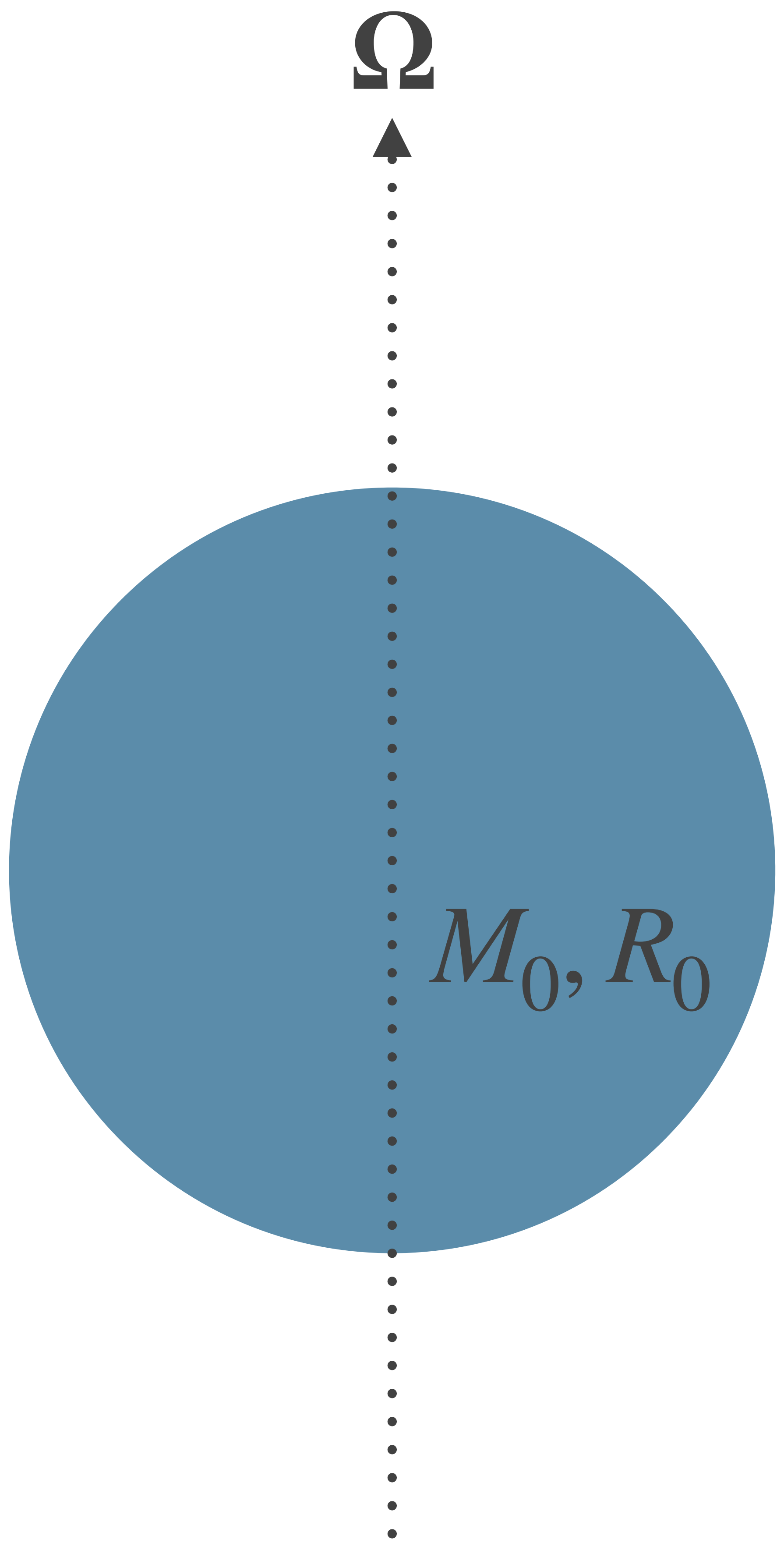
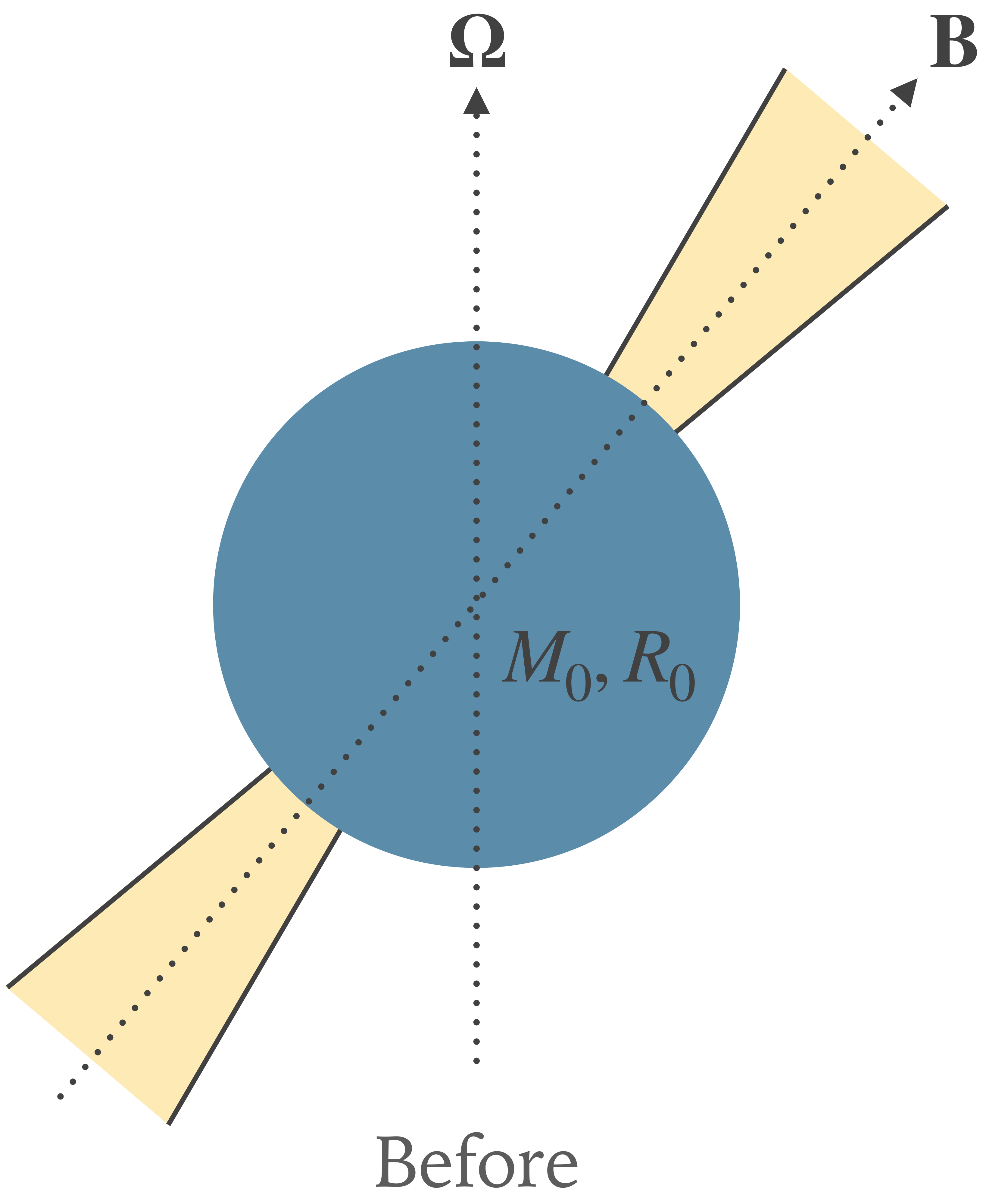
[Mastrano, Suvorov & Melatos, 2015]

TRAPPED EJECTA MODEL



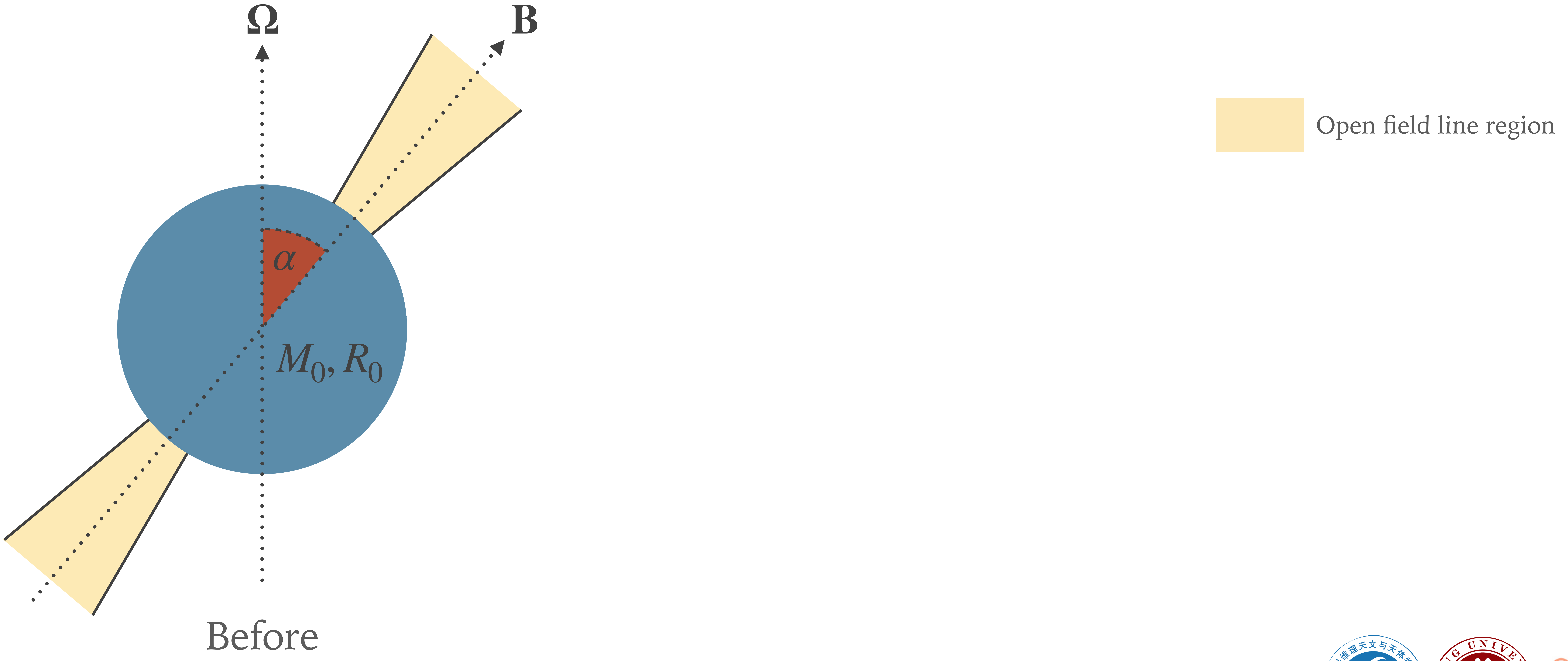
Before

TRAPPED EJECTA MODEL



Open field line region

TRAPPED EJECTA MODEL



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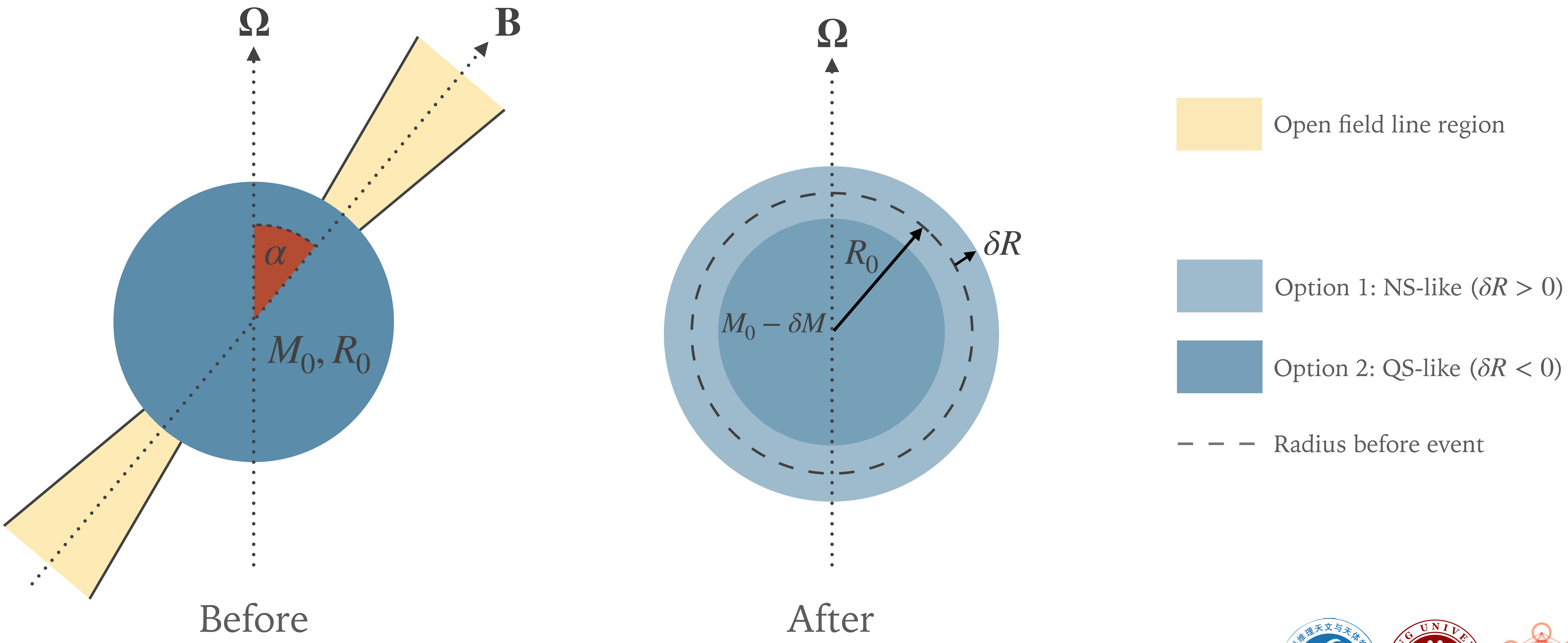
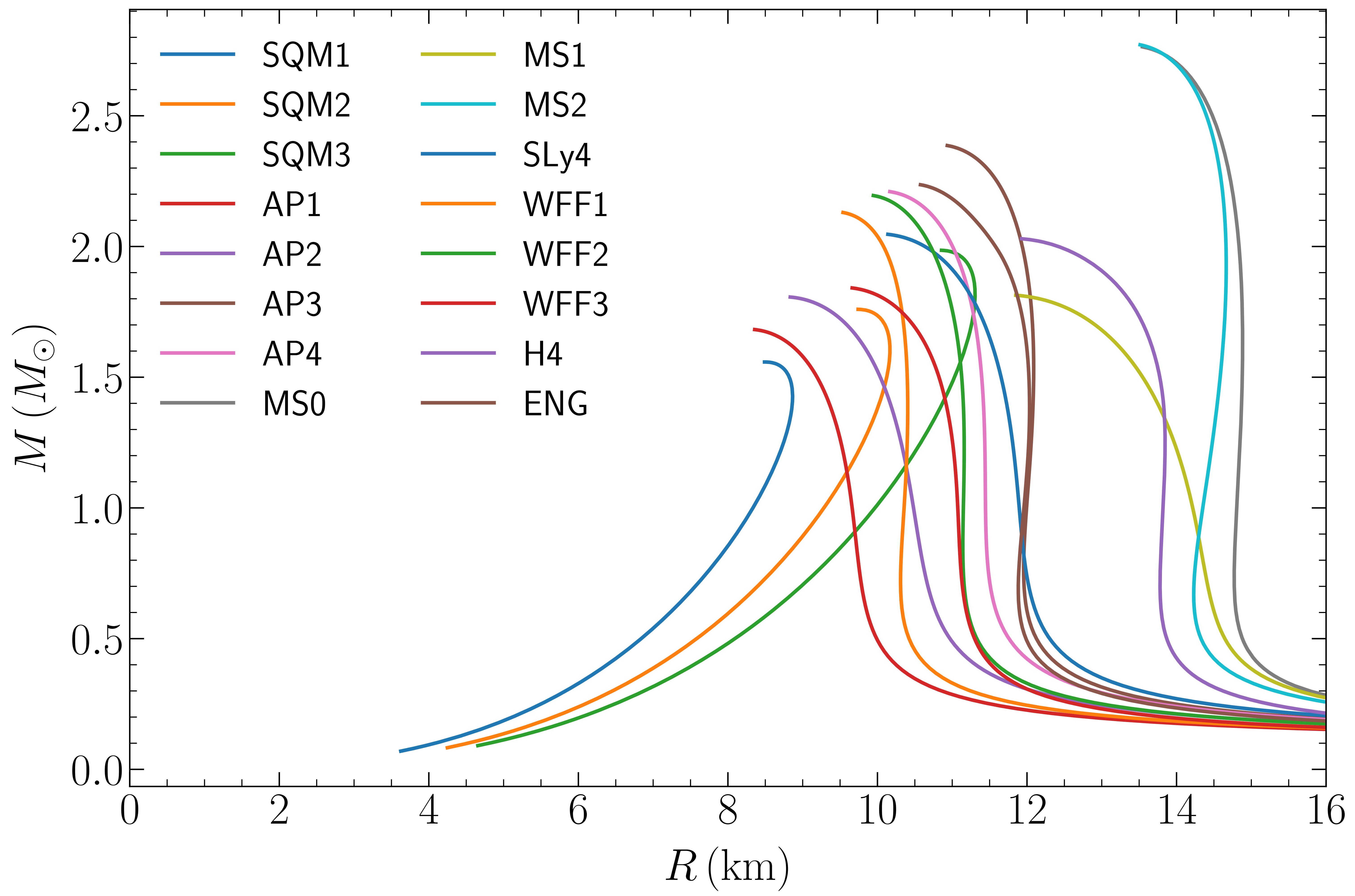


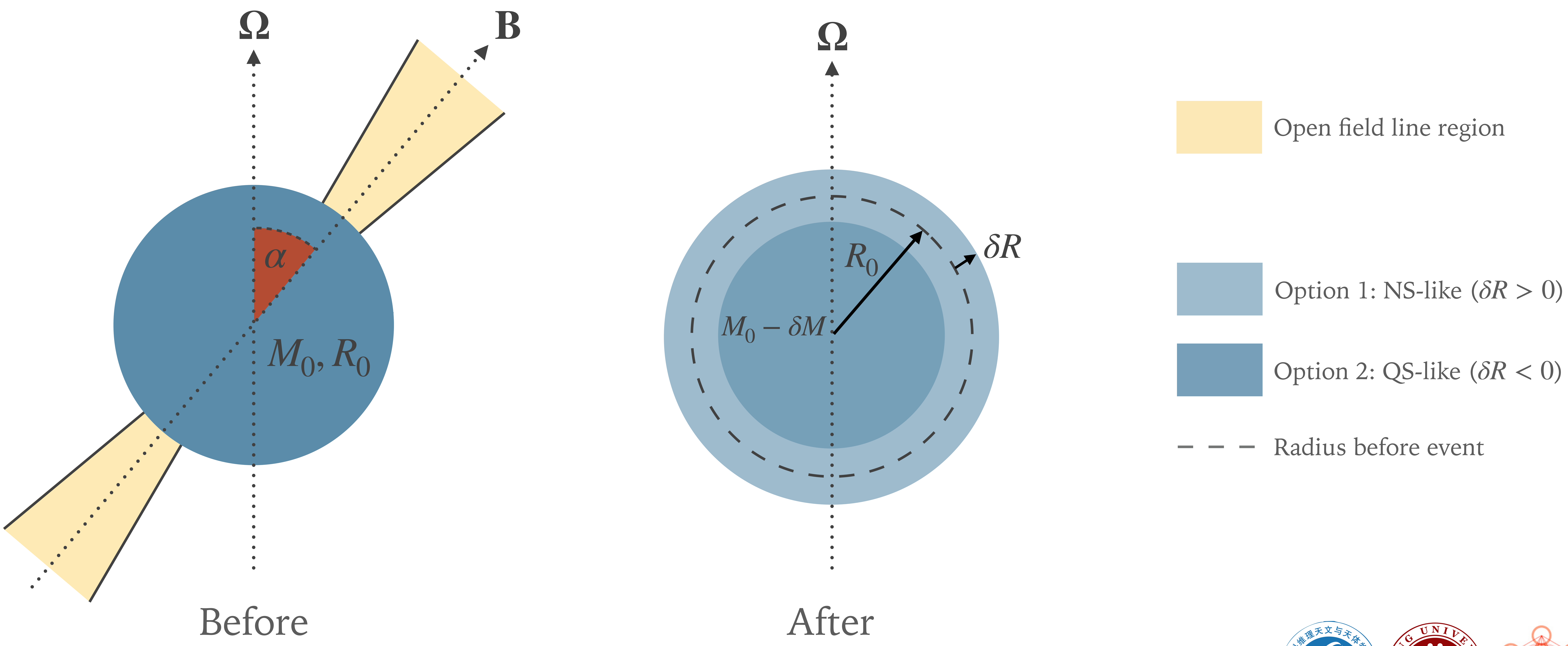
MASS VS RADIUS



Generally:

- NSs have $\dfrac{dM}{dR} < 0$
- QSs have $\dfrac{dM}{dR} > 0$

TRAPPED EJECTA MODEL



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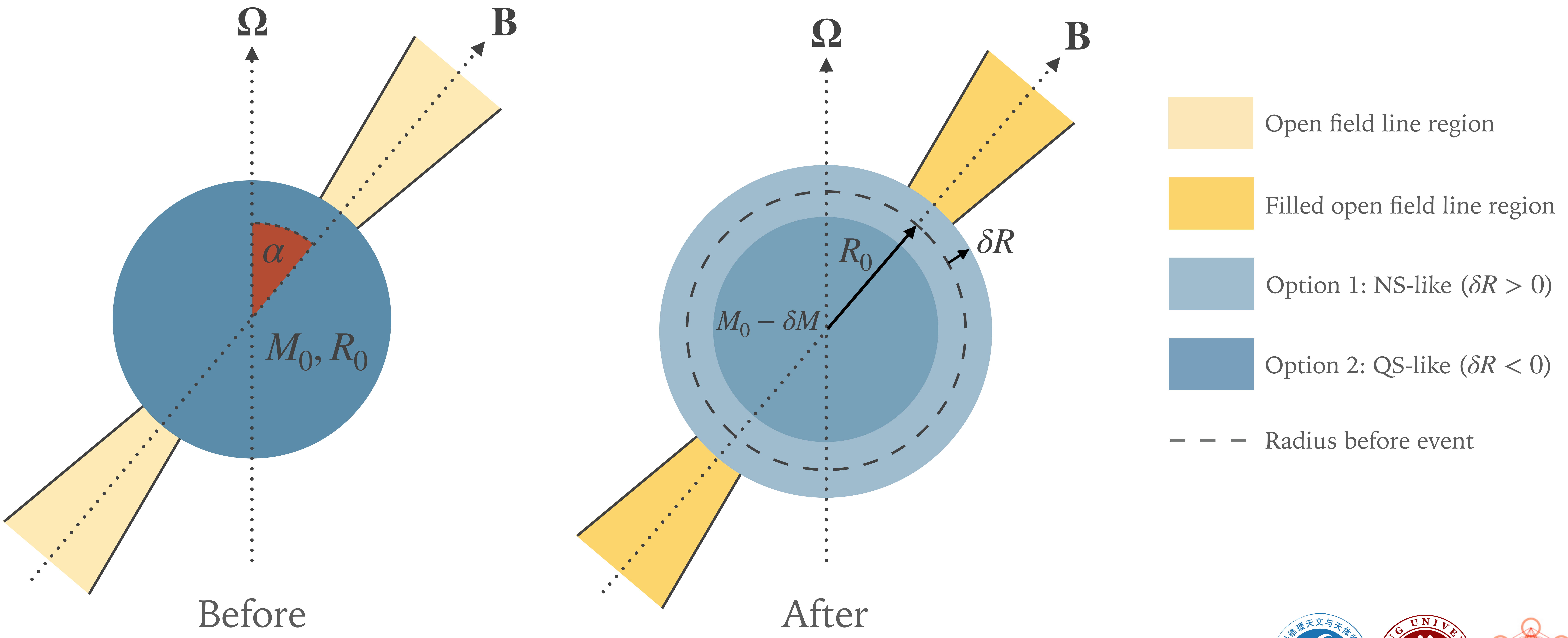

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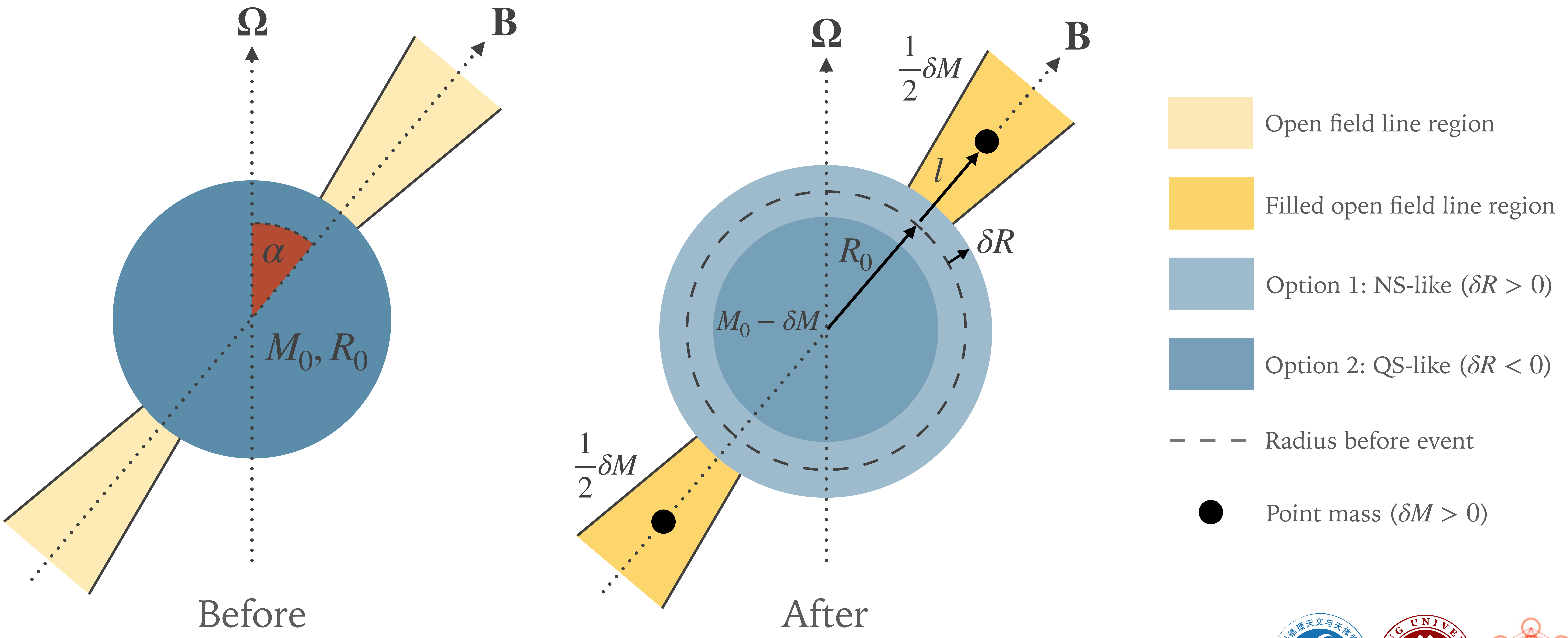

KEY MODEL ASSUMPTIONS

- Conservation of angular momentum
- Open field line region rigidly coupled to magnetar
- Ejecta held near polar cap region (e.g. via higher order magnetic multipoles)
- Ejecta can be treated as a point mass particles held at $R_0 + l$ from the origin
- Angle between rotational and magnetic axes does not change

$$\frac{\Delta\nu}{\nu_0} = -\frac{\Delta I}{I_0}$$

MODEL MECHANISM

$$I_{system} = I_{magnetar} + I_{ejecta}$$

	Antiglitch	Glitch
ΔI_{system}	> 0	< 0
ΔI_{ejecta}	> 0	> 0
Requirement:		
$\Delta I_{magnetar}$	$-\Delta I_{ejecta} < \Delta I_{magnetar} < 0$	$\Delta I_{magnetar} < -\Delta I_{ejecta}$

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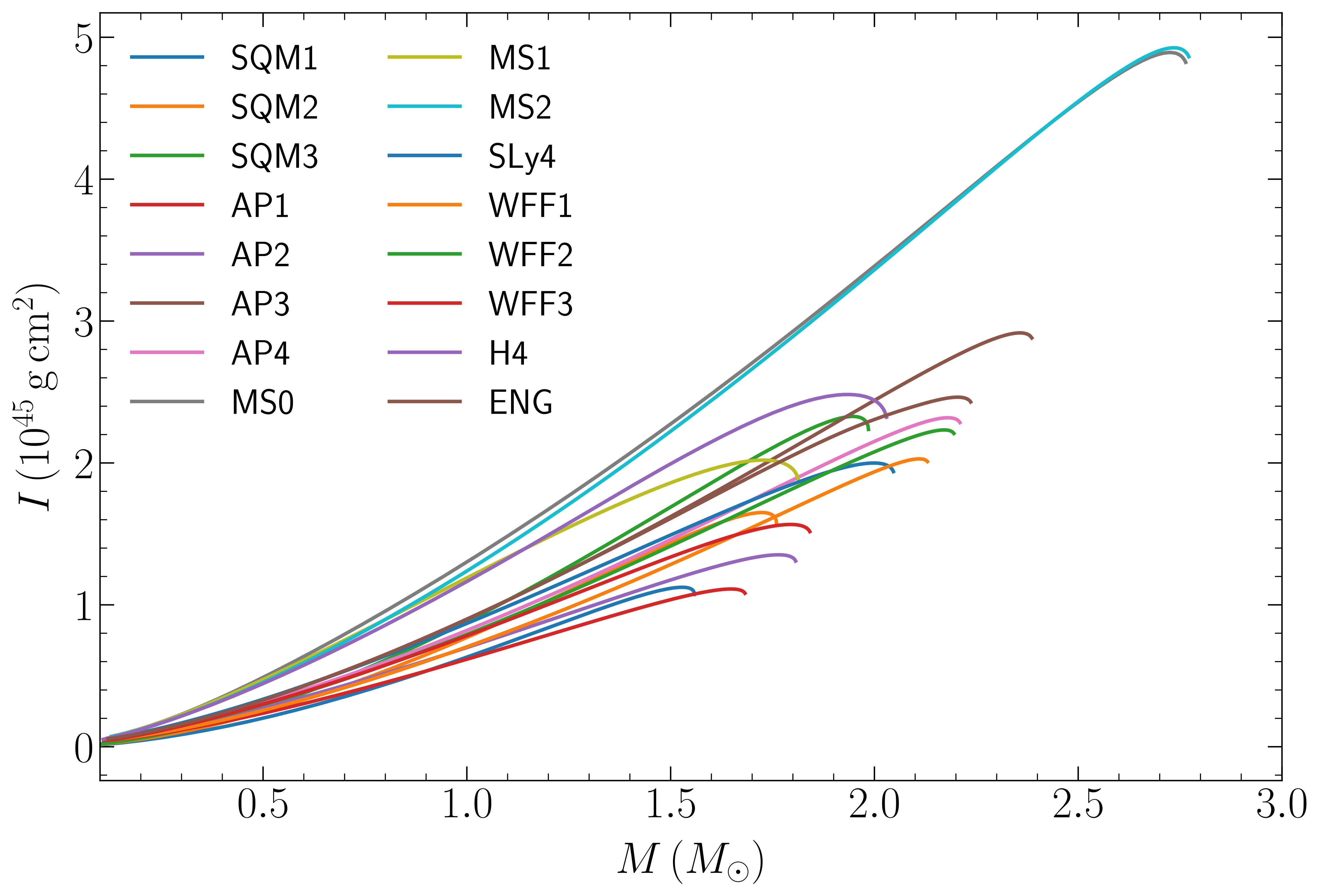

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→ Dependent on equation of state (EOS)

MOMENT OF INERTIA

- The fractional change in moment of inertia, to first order in the small quantities $\delta M \ll M_0$ and $\delta R \ll R_0$, is found to be

$$\frac{\Delta I}{I_0} \approx 2\left(\frac{\delta R}{R_0}\right) - \left(\frac{\delta M}{M_0}\right) + \frac{5}{2}\left(\frac{\delta M}{M_0}\right)\left(1+\frac{l}{R_0}\right)^2 \sin^2\alpha$$

- We can try to rewrite the first term in terms of δM, but δR is different for QSs and NSs → Treat QSs and NSs separately

QUARK STARS

- Quark stars act in the "naïve" sense, where decreasing its mass (shown by $\delta M > 0$) also decreases its radius

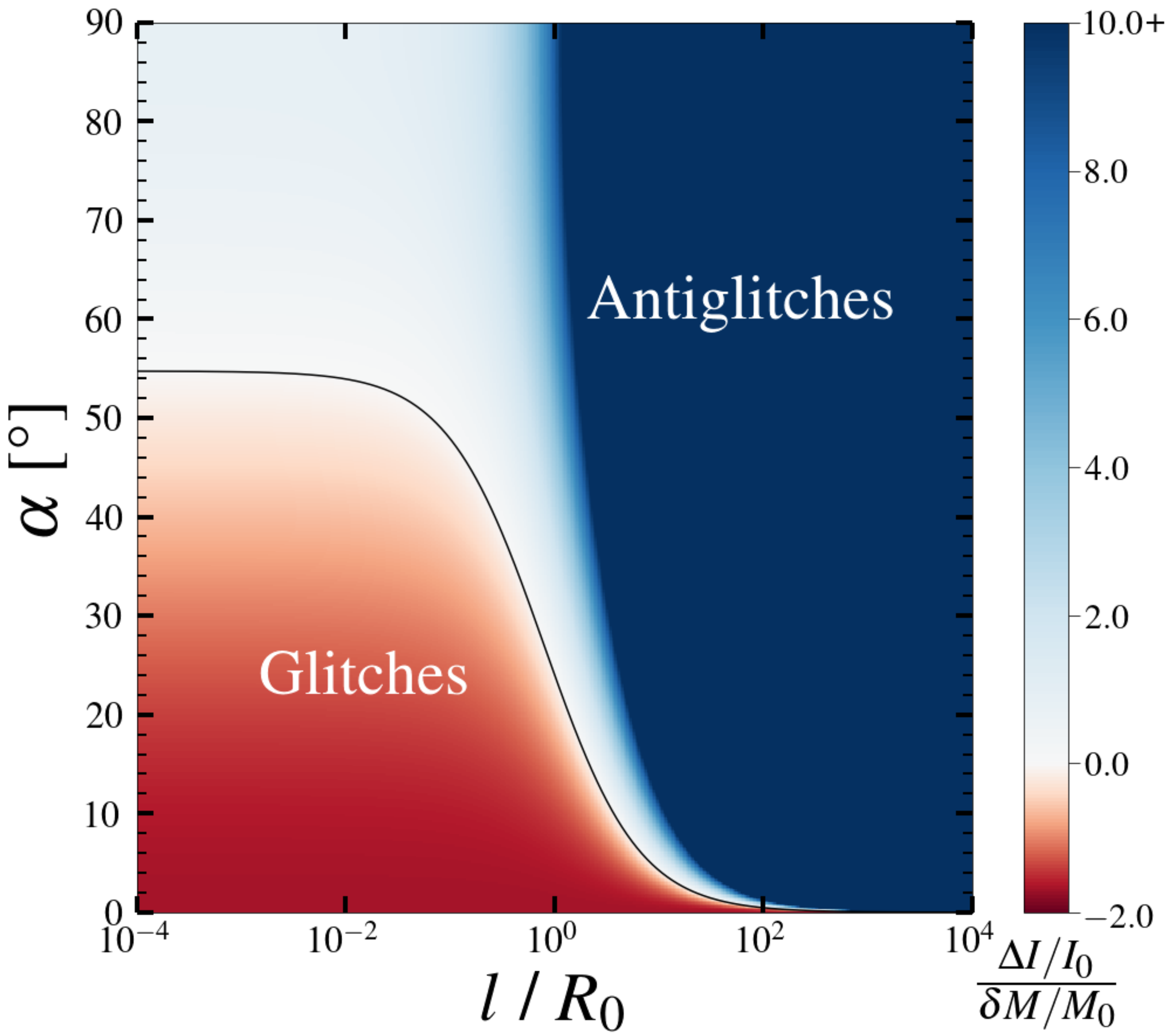
$$\delta M \approx -4\pi R_0^2 \bar{\rho} \delta R \quad \rightarrow \quad \frac{\delta R}{R_0} = -\frac{1}{3}\frac{\delta M}{M_0}$$

- Putting this into the expression for the fractional change in moment of inertia gives

$$\frac{\Delta I_{QS}}{I_0} \approx \left(\frac{\delta M}{M_0}\right)\left[\frac{5}{2}\left(1 + \frac{l}{R_0}\right)^2 \sin^2\alpha - \frac{5}{3}\right]$$

- The sign of the square brackets determines if we get a glitch ($[\ldots] < 0$) or antiglitch ($[\ldots] > 0$) irrespective of how large δM is

QUARK STARS



$$\frac{\Delta I_{QS}}{I_0} \approx \left(\frac{\delta M}{M_0}\right)\left[\frac{5}{2}\left(1+\frac{l}{R_0}\right)^2 \sin^2\alpha - \frac{5}{3}\right]$$

$$\text{For } \frac{l}{R_0} \to 0, \ \alpha_0 = \sin^{-1}\left(\sqrt{\frac{2}{3}}\right) \approx 54.7°$$

NEUTRON STARS

- When neutron stars lose mass (shown by $\delta M > 0$), its radius increases or remains zero

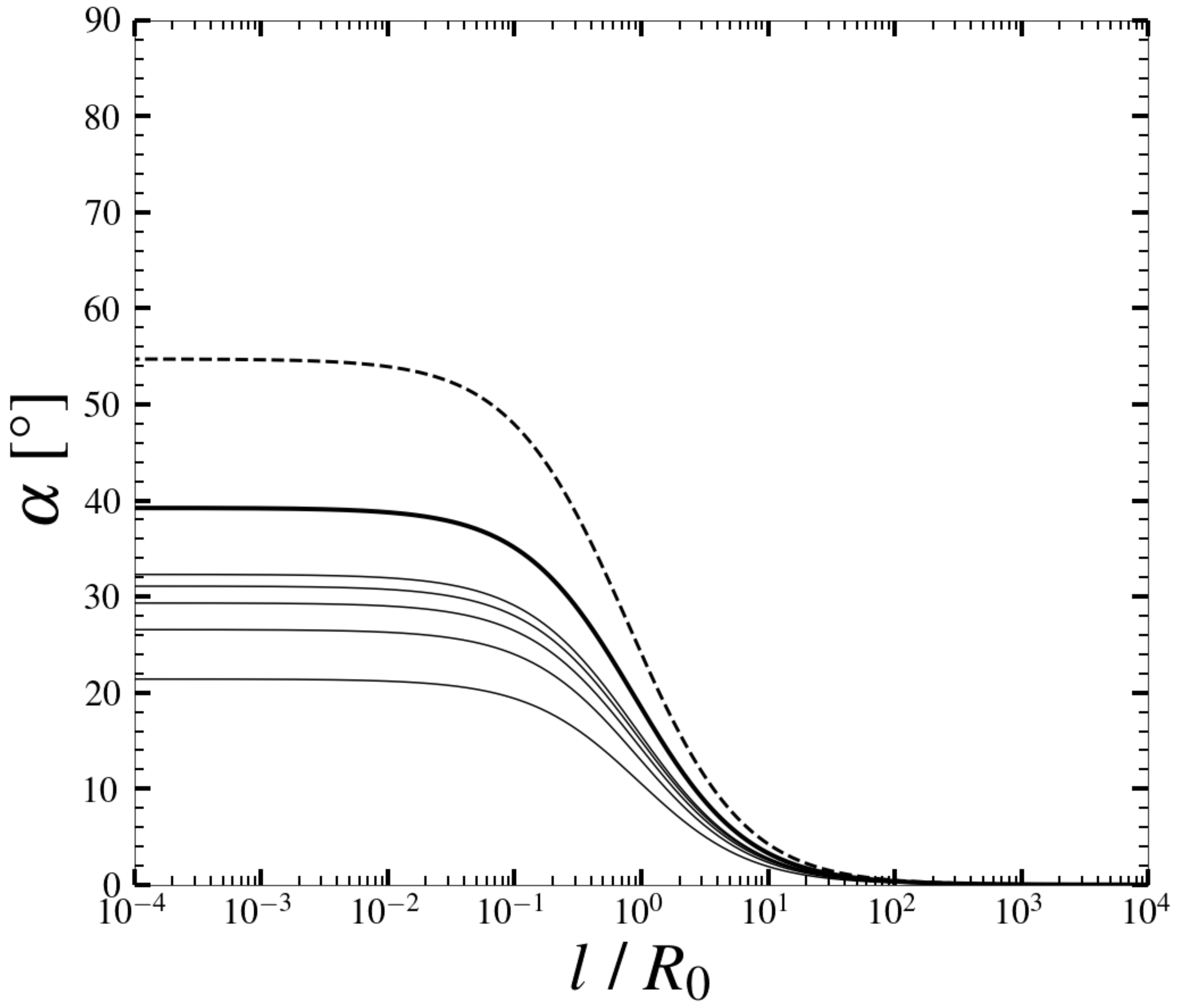
$$\frac{\delta R}{R_0} = \gamma \frac{\delta M}{M_0}$$

where $\gamma \geq 0$ and parametrises our ignorance of the EOS. Note QSs have $\gamma = -\frac{1}{3}$.

- The fractional change in moment of inertia for a NS system is therefore

$$\frac{\Delta I_{NS}}{I_0} \approx \left(\frac{\delta M}{M_0}\right)\left[\frac{5}{2}\left(1+\frac{l}{R_0}\right)^2 \sin^2\alpha + (2\gamma - 1)\right]$$

NEUTRON STARS



Above curve = Antiglitch

Below curve = Glitch

From bottom to top, the curves represent:

$$\gamma = \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}$$

Bold: $\gamma = 0$

Dashed: Quark star

For $\gamma = 0$ and $\frac{l}{R_0} \rightarrow 0, \ \alpha_0 = \sin^{-1}\left(\sqrt{\frac{2}{5}}\right) \approx 39.2^\circ$

GRAVITATIONAL WAVES

- The ejecta held above the magnetic poles causes a time-varying mass quadrupole moment → gravitational wave radiation
- The moment of inertia tensor changes but since angular momentum is conserved, the angular velocity vector must evolve → biaxial precession
- Gravitational wave luminosity and torque calculated using quadrupole formulae

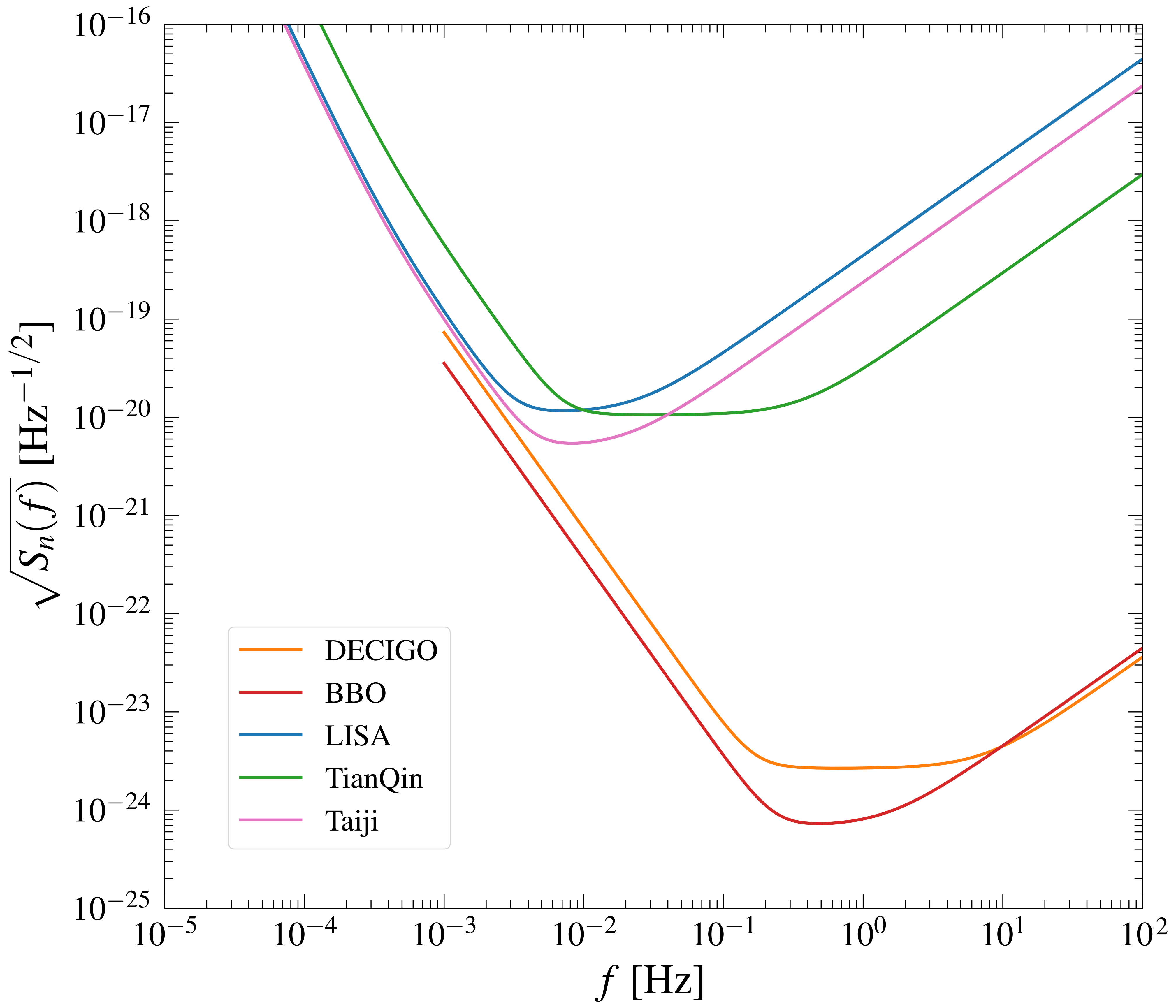
$$\dot{E}_{GW} = \frac{8}{5}\frac{G}{c^5}M_0^2R_0^4\Omega^6\left(\frac{\delta M}{M_0}\right)^2\left(1+\frac{l}{R_0}\right)^4\sin^2\alpha\left[\cos^2\alpha+16\sin^2\alpha\right]$$

$$\dot{J}_{GW} = \frac{8}{5}\frac{G}{c^5}M_0^2R_0^4\Omega^5\left(\frac{\delta M}{M_0}\right)^2\left(1+\frac{l}{R_0}\right)^4\sin^2\alpha\left[\cos^2\alpha+16\sin^2\alpha\right]$$

$$\dot{E}_{GW} = \Omega\dot{J}_{GW}$$

PROPERTIES OF GRAVITATIONAL WAVES

- Gravitational waves are emitted at $f_{GW} \approx \nu$ and $f_{GW} \approx 2\nu$ for a duration equal to the time between the glitch/antiglitch event and the onset of pulsed radio emission

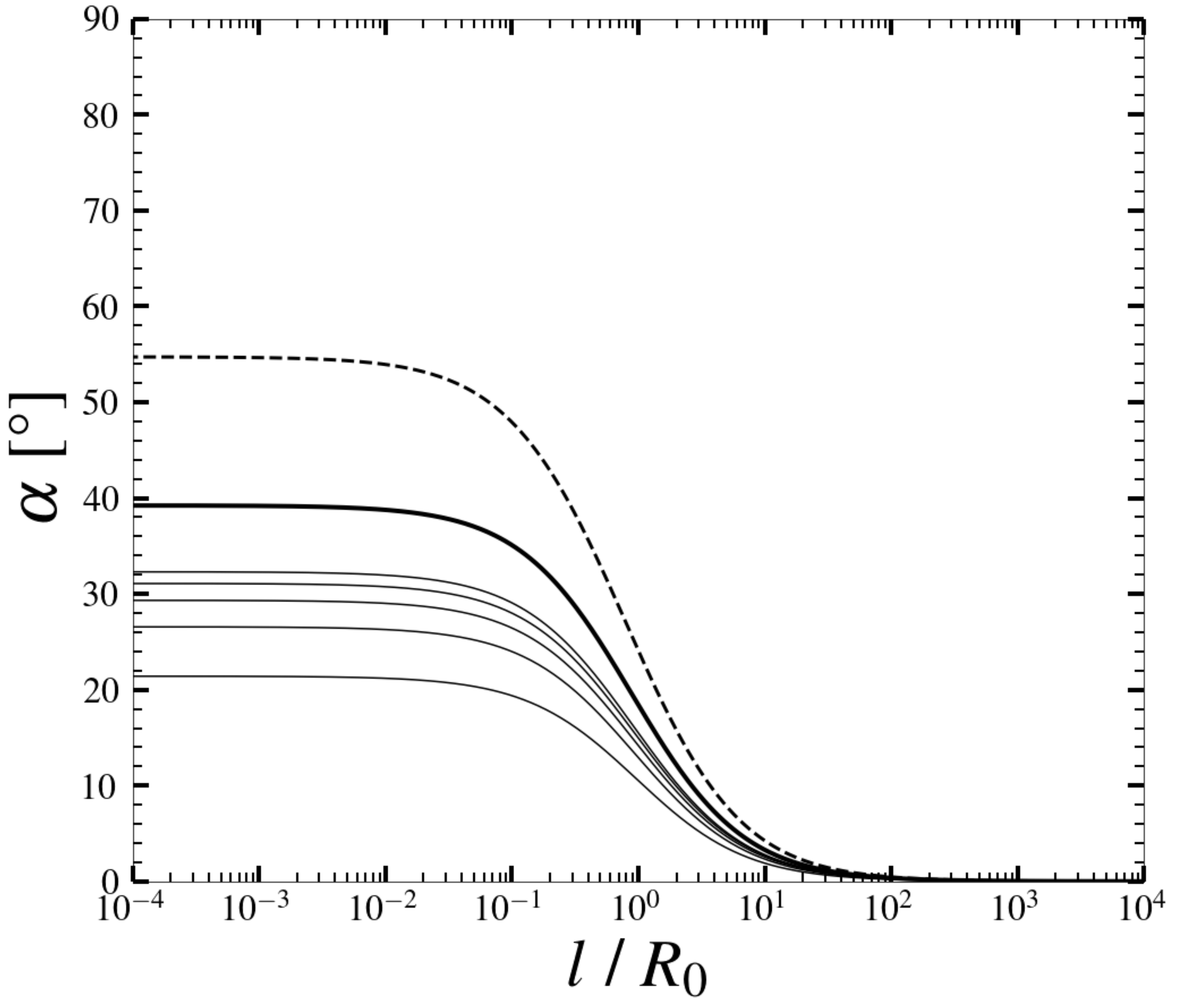


- $T_{GW} \sim 4$ d for the SGR 1935+2154 antiglitch
- Most relevant GW detectors would be future space-based detectors, especially DECIGO and Big Bang Observer (recall $\nu \approx 0.308$ Hz for SGR 1935+2154)

DETECTABILITY OF GRAVITATIONAL WAVES

Signal to noise ratio: $\rho_{2\nu} \sim \frac{A}{\sqrt{2}} \frac{\sqrt{T_{\mathrm{GW}}}}{\sqrt{S_{\mathrm{n}}(2\nu)}} \sin^2\alpha \qquad \rho_{\nu} \sim \frac{A}{\sqrt{2}} \frac{\sqrt{T_{\mathrm{GW}}}}{\sqrt{S_{\mathrm{n}}(\nu)}} \sin\alpha\cos\alpha$

where $A = -\frac{2}{d}\frac{G}{c^4}\Omega^2 M_0 R_0^2 \left(\frac{\Delta\nu}{\nu_0}\right) \frac{\left(1+\frac{l}{R_0}\right)^2}{\frac{5}{2}\left(1+\frac{l}{R_0}\right)^2 \sin^2\alpha + (2\gamma - 1)}$

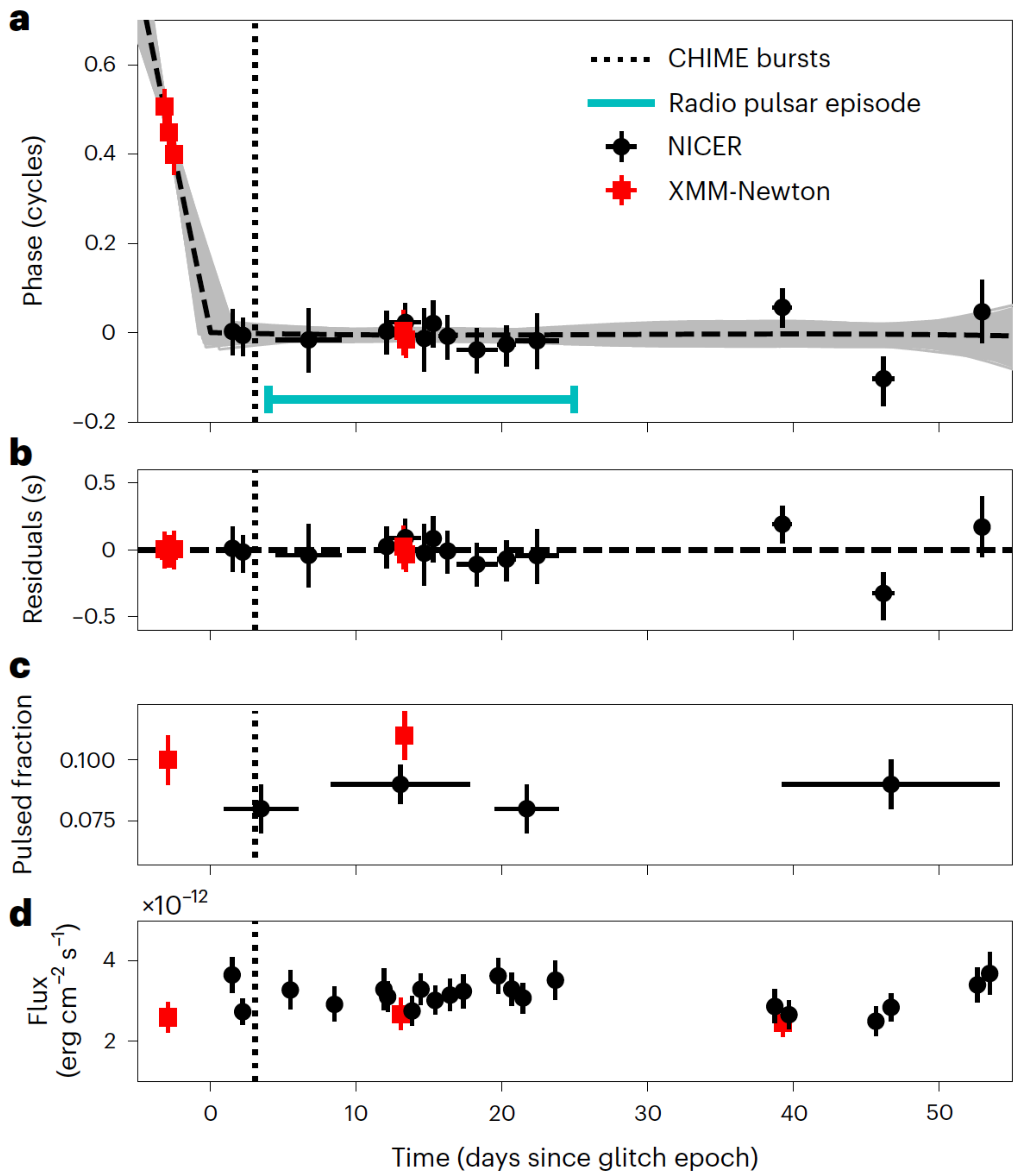


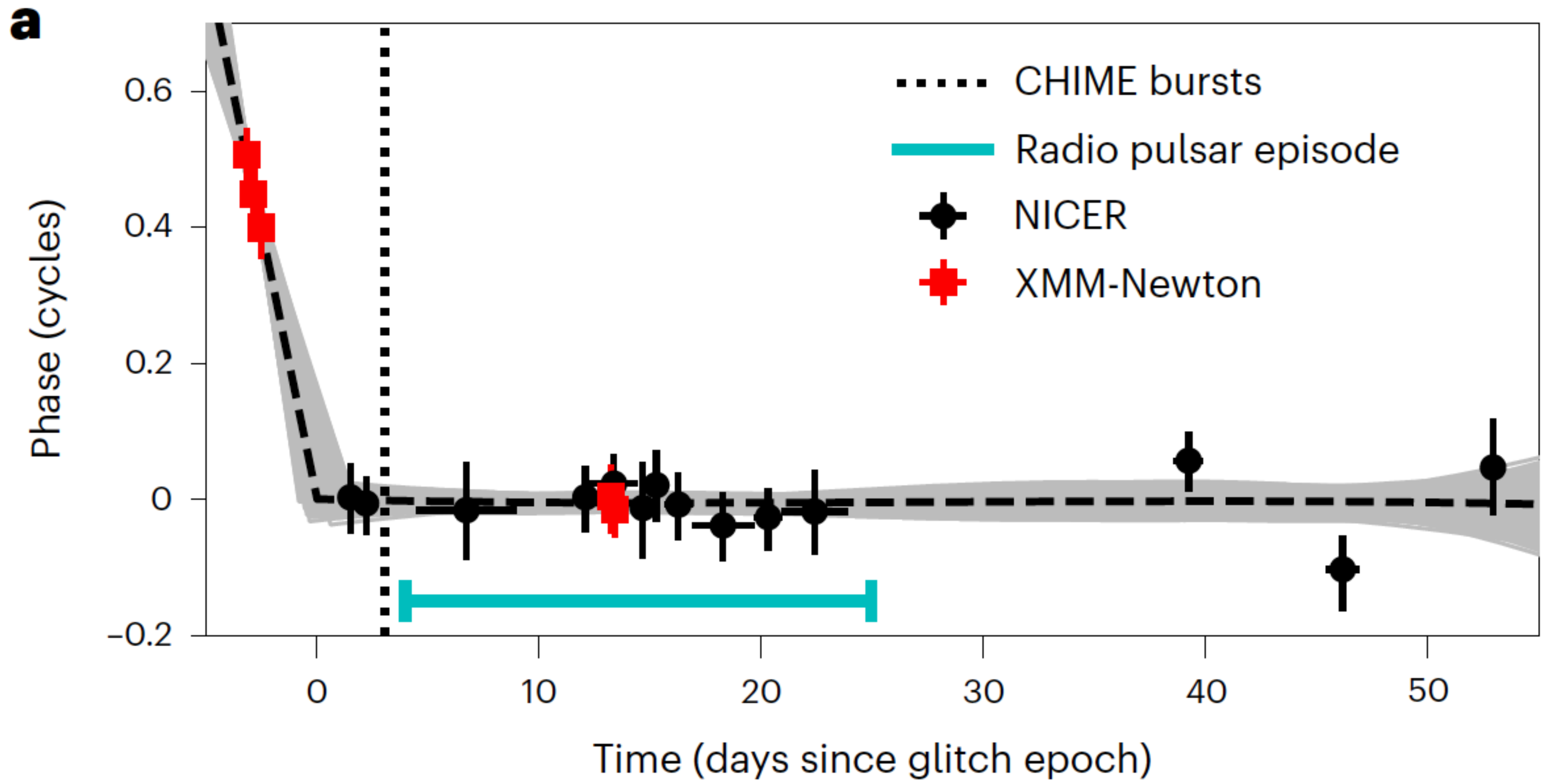
- Signal-to-noise ratio largest for nearby, rapidly rotating magnetars which exhibit large glitches
- Signal-to-noise ratio largest when close to the boundary line separating glitches and antiglitches

CONCLUSIONS AND FUTURE STEPS

- Created a simple model to simultaneously explain glitches and antiglitches which is testable with gravitational waves
- Gravitational waves from the trapped ejecta model are detectable with future space-based detectors so long as the magnetar is one (or a combination) of the following:
  - Sufficiently nearby
  - Rotating fast enough
  - Exhibits a large enough glitch/antiglitch
  - The combination of (α, l) is sufficiently close to the boundary line that separates glitches and antiglitches
- Future steps: relax assumptions of point masses, re-do calculation using realistic EOSs, incorporate FRB production into the model

EXTRA SLIDES – YOUNES ET AL. (2023)





EXTRA SLIDES – YOUNES ET AL. (2023)

Timeline

- Day -38 — 28th August 2020 — No detection of pulsed radio emission by FAST
- Day 0 — 5th October 2020 (±1 day) — Anti-glitch
- Day 3 — 8th October 2020, 02:23 UTC — 3 FRBs
- Day 3/4 — 8th/9th October 2020 — Pulsed radio emission observed by FAST
- Day 24 — 29th October 2020 — Last FAST observation of pulsed radio emission

EXTRA SLIDES - YOUNES ET AL. (2023)

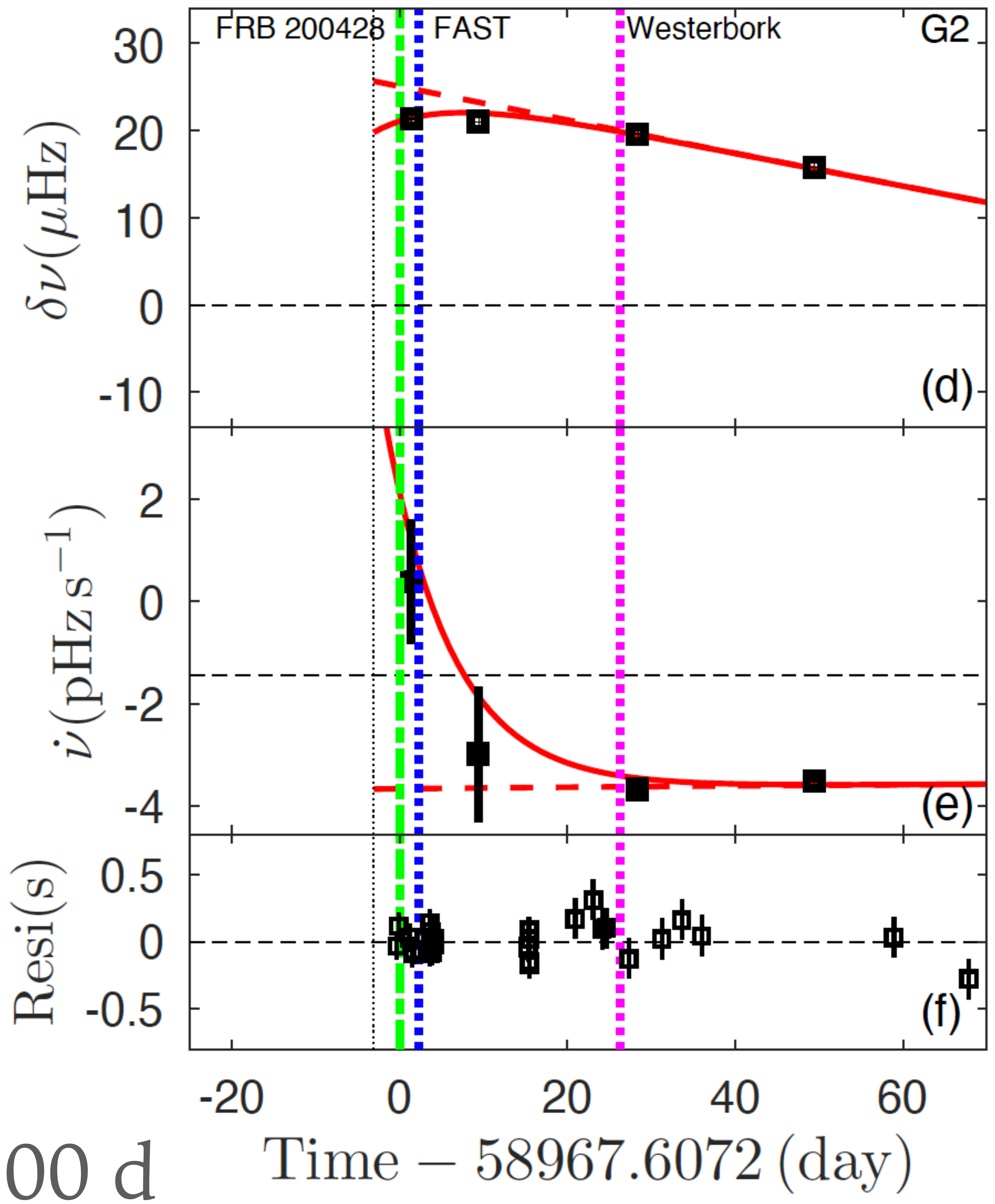
- Suggested an "ephemeral wind" as the reason for the antiglitch
- The strong wind "combs out the magnetic field lines" and the wind carries away angular momentum from the system

$$\frac{\delta m}{M} \sim -\frac{P^2}{(\delta t)^2}\frac{M^2c^4}{B_p^2R^6}\left(\frac{\delta\Omega}{\Omega}\right)^3$$

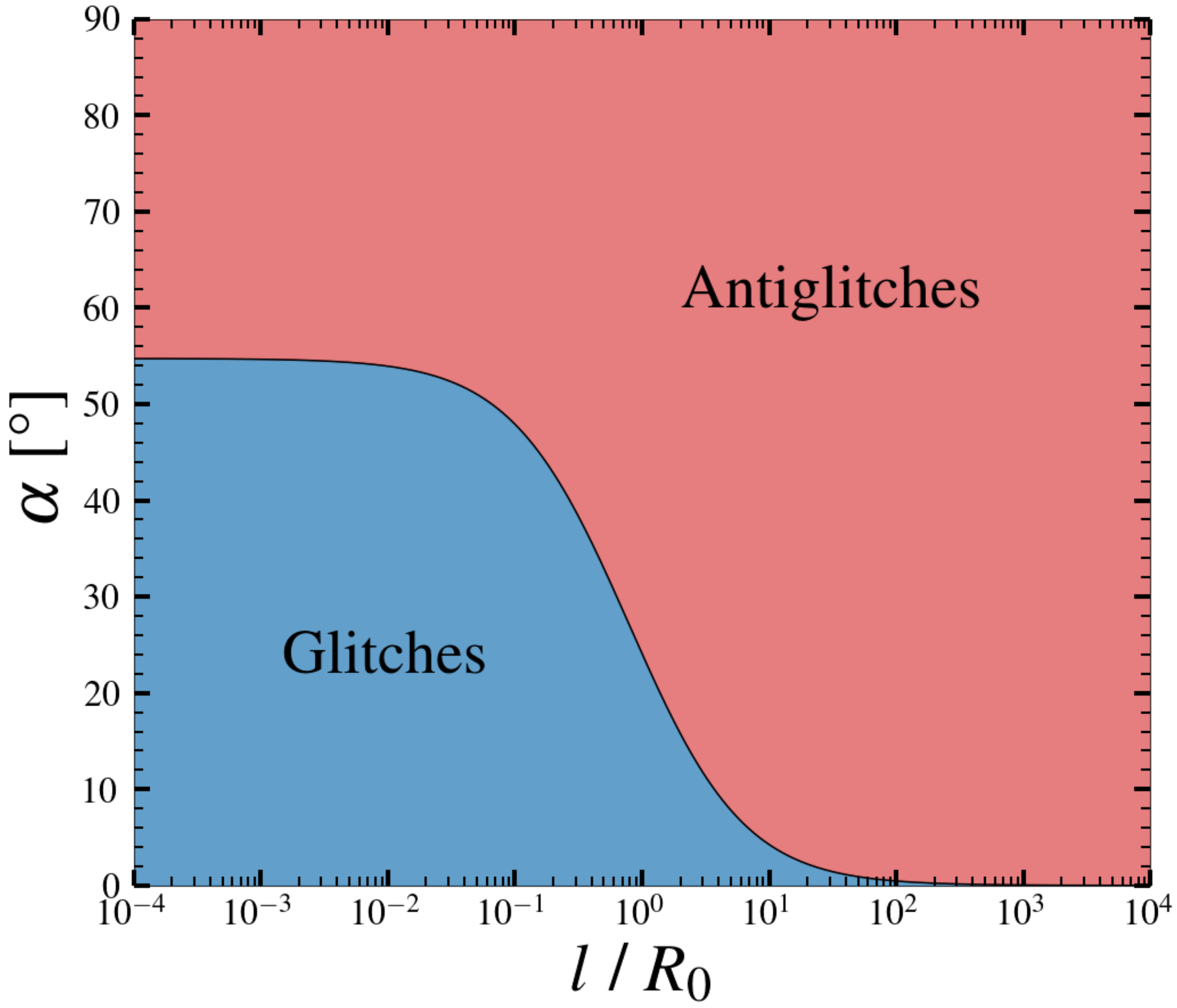
- For a wind lasting 10 hours, they found $\delta m \sim 10^{-10}M$ and for a wind lasting a few minutes, $\delta m \sim 10^{-6}M$
- The high opacity conditions during the wind prevents strong electric potential gaps, curvature radiation and electron-positron pair production
- Combing of the magnetic field lines may temporarily favour conditions for FRB production and pulsed radio emission

EXTRA SLIDES - GE ET AL. (SUBMITTED)

- Glitch observed on 25th April 2020 $\dfrac{\Delta\nu}{\nu} = 6.4 \times 10^{-5}$
- FRB 200428 detected 3 days after glitch, possibly more
- Change in pulse profile and X-ray burst observed coincident with FRB
- Large change in spin-down rate $\dfrac{\Delta\dot{\nu}}{\dot{\nu}} = -4.4$
- Glitch recovery modelled with $Q = 0.13$
- Fitting may be unreliable as there was no prior data for ~900 d



EXTRA SLIDES – QUARK STARS



Boundary given by

$$\alpha = \sin^{-1}\left(\sqrt{\frac{2}{3}}\left(1+\frac{l}{R_0}\right)^{-1}\right)$$

For $\dfrac{l}{R_0} \to 0,\ \alpha_0 = \sin^{-1}\left(\sqrt{\dfrac{2}{3}}\right) \approx 54.7°$

EXTRA SLIDES - NEUTRON STARS

- Again, the sign of the square brackets tells us if we get a glitch or antiglitch with the boundary determined by

$$\sin^2\alpha = \frac{\frac{2}{5} - \frac{4}{5}\gamma}{\left(1 + \frac{l}{R_0}\right)^2}$$

but $\sin^2\alpha$ must be bound between 0 and 1, which leads to the condition $0 < \gamma < \frac{1}{2}$.

- For a polytrope, $P = \kappa\rho^\Gamma = \kappa\rho^{1+\frac{1}{n}}$ where Γ is the adiabatic index and n is the polytropic index

$$\gamma = \frac{n-1}{3-n} = \frac{2-\Gamma}{3\Gamma - 4} \qquad \therefore \quad 1 < n < \frac{5}{3} \quad \text{and} \quad \frac{8}{5} < \Gamma < 2$$

EXTRA SLIDES - POLYTROPIC EQUATION OF STATE

- As a first approximation, we can use a polytropic EOS in the model

$$P = P(\rho) \;\rightarrow\; P = \kappa\rho^{\Gamma} = \kappa\rho^{1+\frac{1}{n}}$$ where Γ is the adiabatic index and n is the polytropic index

- Combine hydrostatic equilibrium with Poisson's equation (with a polytropic EOS) to get the Lane-Emden equation

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\xi}\right) = -\,\theta^n \qquad \text{where } \xi = \frac{r}{a} \text{ and } \theta^n = \frac{\rho}{\rho_{centre}}$$

- With appropriate boundary conditions, one can solve for $\theta = \theta(\xi)$

- At $\xi = \xi_1$, the density goes to zero so $\theta(\xi_1) = 0$ which gives us the NS radius, $R = a\xi_1$

EXTRA SLIDES - POLYTROPIC EQUATION OF STATE

- The mass of a NS can be found simply from

$$M = 4\pi \int_0^R r^2 \rho dr$$

- Converting to the dimensionless variables ξ and θ, one can utilise the Lane-Emden equation to carry out the integration which results in

$$M = -4\pi \left[\frac{(n+1)\kappa}{4\pi G}\right]^{\frac{3}{2}} \rho_{centre}^{\frac{3-n}{2n}} \xi_1^2 \frac{d\theta}{d\xi}(\xi_1)$$

- The radius is easily obtained from

$$R = a\xi_1 = \left[\frac{(n+1)\kappa}{4\pi G}\right]^{\frac{1}{2}} \rho_{centre}^{\frac{1-n}{2n}} \xi_1$$

EXTRA SLIDES - POLYTROPIC EQUATION OF STATE

➤ Eliminating the central mass density, we get the mass-radius relation for polytropes

$$M = -4\pi R^{\frac{3-n}{1-n}} \left[\frac{(n+1)\kappa}{4\pi G} \right]^{-\frac{n}{1-n}} \xi_1^{-\frac{1+n}{1-n}} \frac{d\theta}{d\xi}(\xi_1)$$

➤ The important point is that $M \propto R^{\frac{3-n}{1-n}}$, e.g. for $n = \frac{3}{2}$, we get $M \propto \frac{1}{R^3}$

➤ This relation allows us to calculate γ for polytropes

$$\delta M \approx -\frac{dM}{dR}\delta R \quad \rightarrow \quad \frac{\delta R}{R_0} = \left(\frac{n-1}{3-n} \right) \frac{\delta M}{M_0} \quad \rightarrow \quad \boxed{\gamma = \frac{n-1}{3-n} = \frac{2-\Gamma}{3\Gamma-4}} \quad \left(n \neq 3, \Gamma \neq \frac{4}{3} \right)$$

$$\boxed{0 < \gamma < \frac{1}{2} \quad \rightarrow \quad 1 < n < \frac{5}{3} \quad \rightarrow \quad \frac{8}{5} < \Gamma < 2}$$