Can a **star** be **smaller than** the **black hole** of the same mass?

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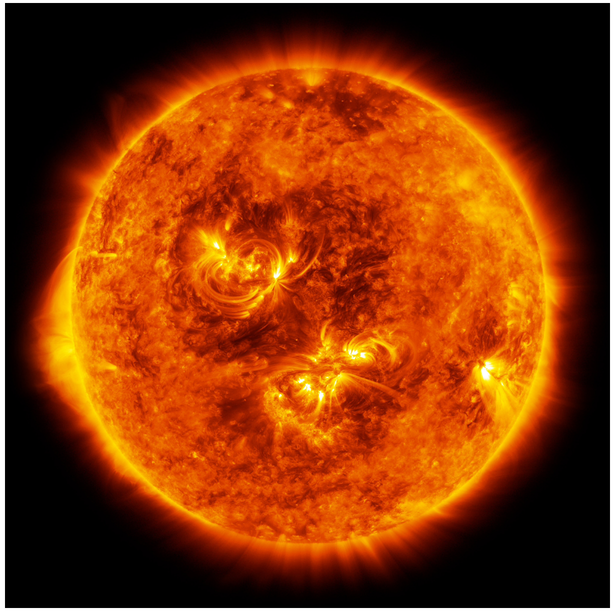
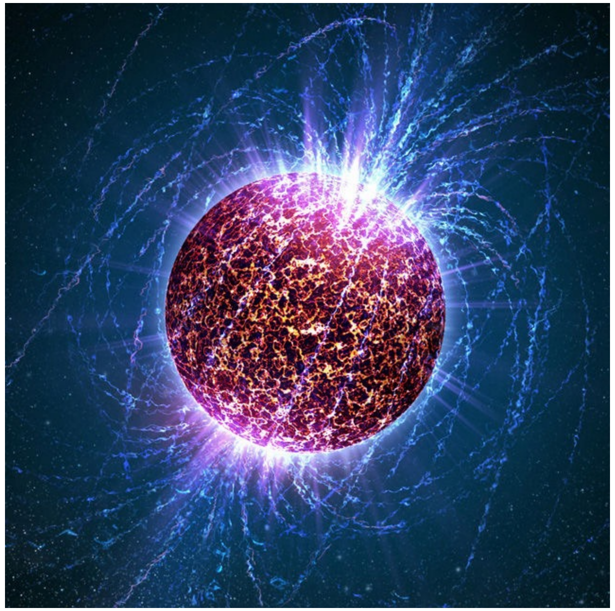
Size of an object with $M_\odot$ in GR





Sun	Neutron star (Pulsar)	Black hole
700,000 km	10 km	3 km
$\mathcal{C} = 2\times10^{-6}$	$\mathcal{C} \approx 0.15$	$\mathcal{C} = 0.5$

Compactness $\mathcal{C} = M/R$

Geometrized unit (G=c=1)

Buchdahl's theorem

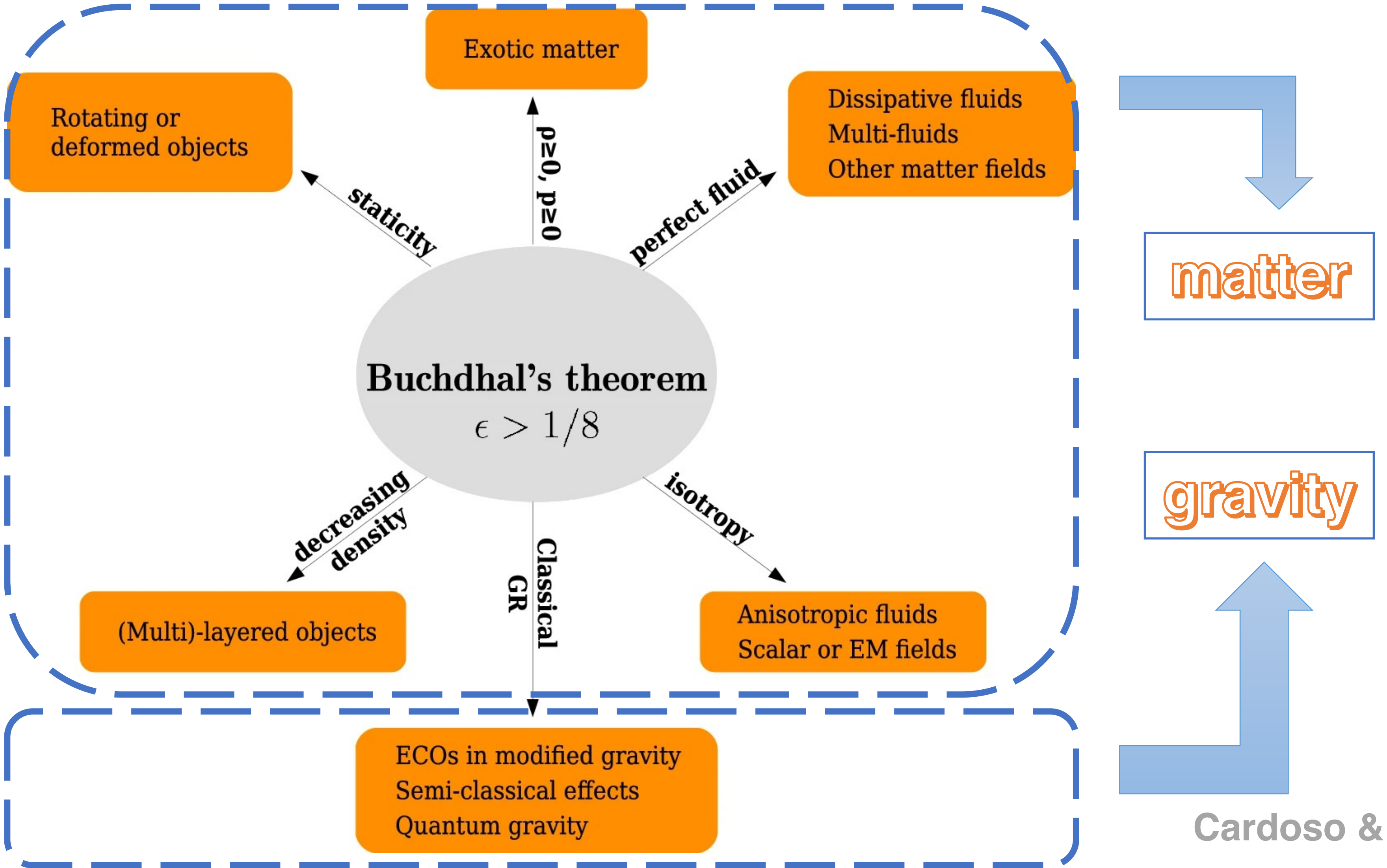
The radius of a static spherically symmetric star with mass M cannot be smaller than $9M/4$ under certain reasonable assumptions.

Assumption about matter

Assumption about gravity

Buchdahl, Phys. Rev. 116, 1027 (1959)

Buchdahl's theorem deconstructed



How can a star be smaller than the black hole of the same mass?

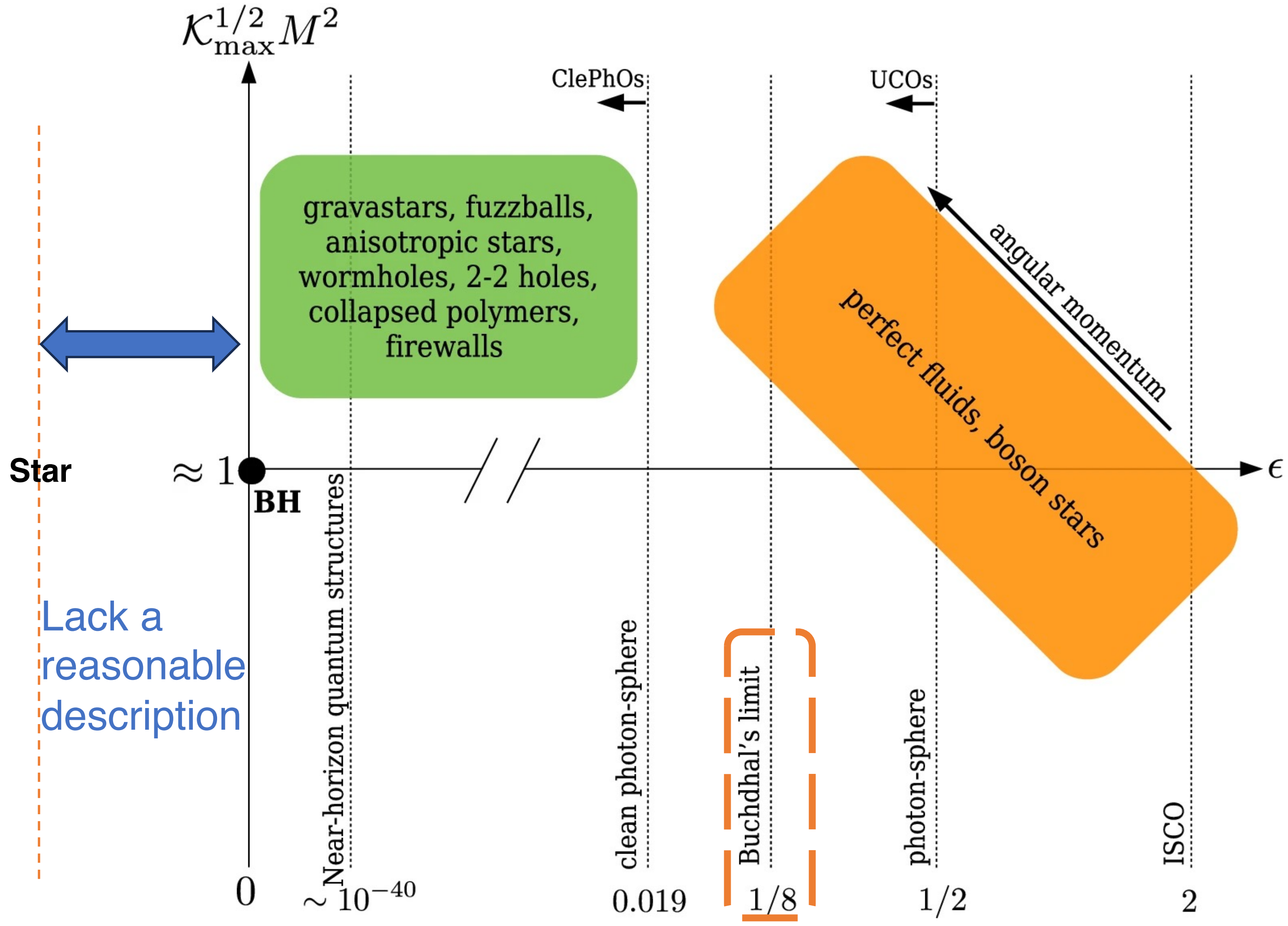
Cardoso & Pani, Living Rev. Rel. 22, 4 (2019)

How about violate assumption of matter?

Answer is: ❌

Birkhoff's theorem in GR:

- the unique static, spherically symmetric, asymptotically flat vacuum solution of GR is the Schwarzschild metric



Cardoso & Pani, Living Rev. Rel. 22, 4 (2019)

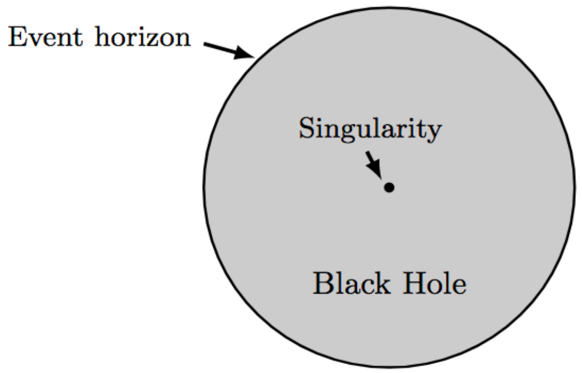
1st prerequisite

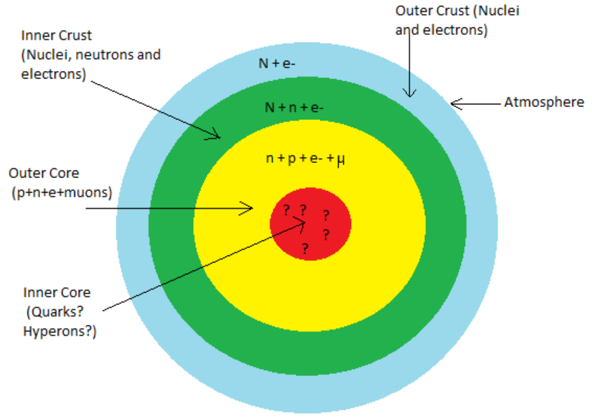
A theory of gravity possesses non-uniqueness of vacuum spherically symmetric solutions

the exterior vacuum region of a star can be reasonably described by non-black-hole spacetime solutions

Higher-order Gravity

the solution is a three-parameter family where the parameters depend on the mass distribution





Is the black hole solution unique?

Metric: $ds^2 = -h\,dt^2 + f^{-1}\,dt^2 + r^2(d\theta^2 + \sin^2\theta\,d\phi^2)$

$h = f$: Schwarzschild-like black hole

- $h = 1 - \frac{2M}{r} + \cdots$

$h \neq f$: non-Schwarzschild black hole

- $h = 1 - \frac{2M}{r} + c_1 \frac{e^{-m_0 r}}{r} + c_2 \frac{e^{-m_2 r}}{r} + \cdots$

Stelle, Gen. Rel. Grav. 9 (1978) 353; Lü, Perkins, Pope & Stelle, Phys. Rev. Lett. 114 (2015) 171601

2nd prerequisite

The static black hole solutions are unique,
preferably the Schwarzschild-like black hole

A specific combination of higher-order curvature terms
leads $m_0 \to \infty$ and $m_2 \to \infty$

Quasi-topological gravity (QTG)

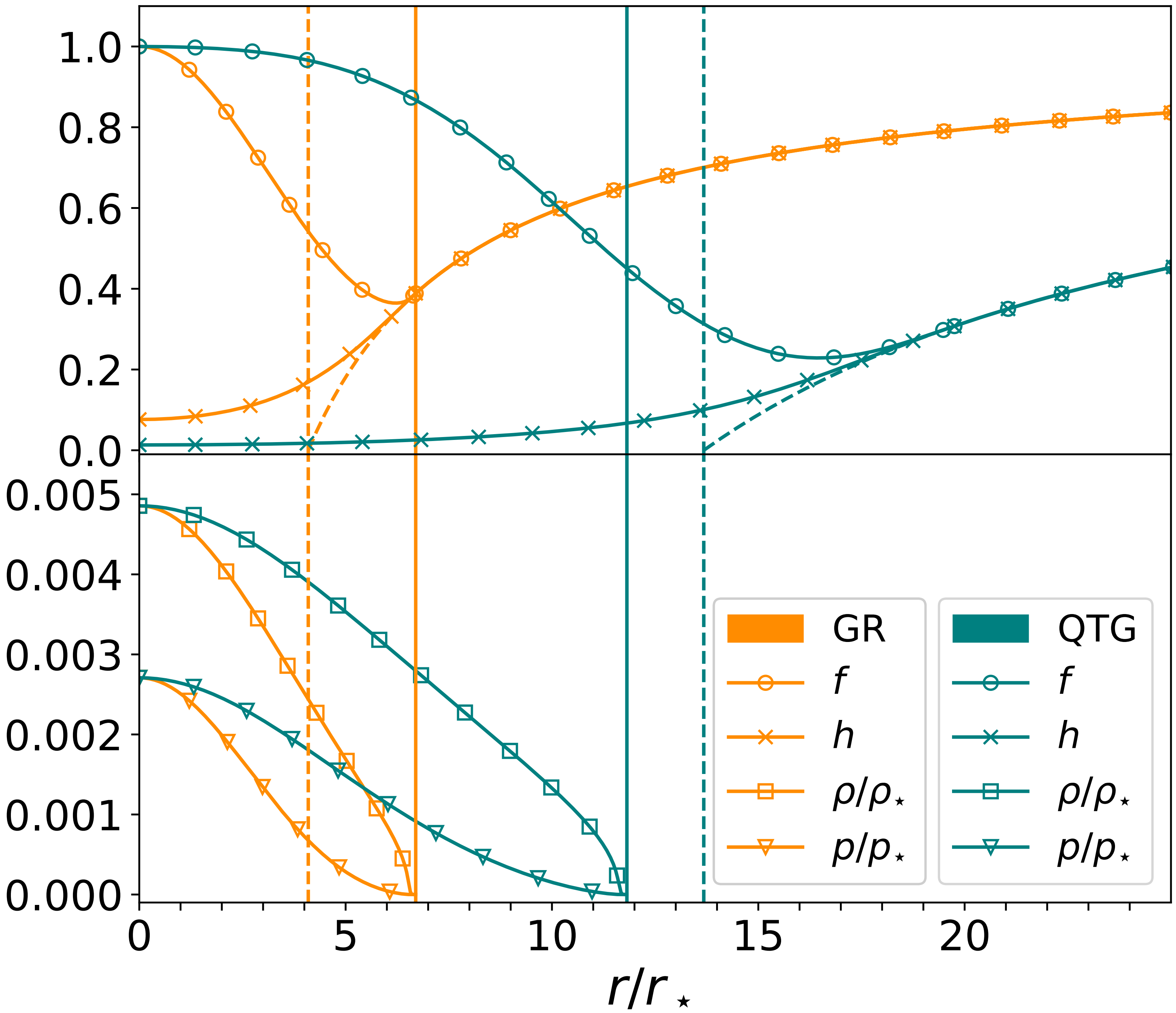
Bueno & Cano, PRD 94, 104005(2016) ;Hennigar, Kubiznak & Mann, PRD. 95, 104042 (2017) ; Li, Liu & Lü, JHEP 02, 166 (2018)

Setup

- Theory
  - $L = R + \lambda(R^3 - 6RR_{\mu\nu}R^{\mu\nu} + 8R^{\mu}_{\nu}R^{\nu}_{\gamma}R^{\gamma}_{\mu})$
- EOS
  - $p = E(\rho)$
- TOV
  - $h''' = F_1(r, \rho, h'', h', h, f'', f', f)$, $f''' = F_2(r, \rho, h'', h', h, f'', f', f)$, $\rho' = F_3(r, \rho, h', h)$
- Initial and asymptotical conditions
  - $h = h_0 + h_2 r^2, \quad f = 1 + f_2 r^2, \quad \rho = \rho_0, \qquad h(\infty) = f(\infty) = 1 - 2M/r$
- Relation of dimensions
  - $M_{\odot} \sim r_{\star} \sim p_{\star}^{-1/2} \sim \rho_{\star}^{-1/2} \sim \lambda_{\star}^{1/2}$

Solution

- $\rho_0 = 3\times10^{15}\mathrm{g/cm^3}$
- $\lambda = 500\ \lambda_\star$
- EOS: SLy
- GR: $2.05\ M_\odot$
- QTG: $6.84\ M_\odot$



Relations of $\mathcal{C} - \lambda$ & $M - R$

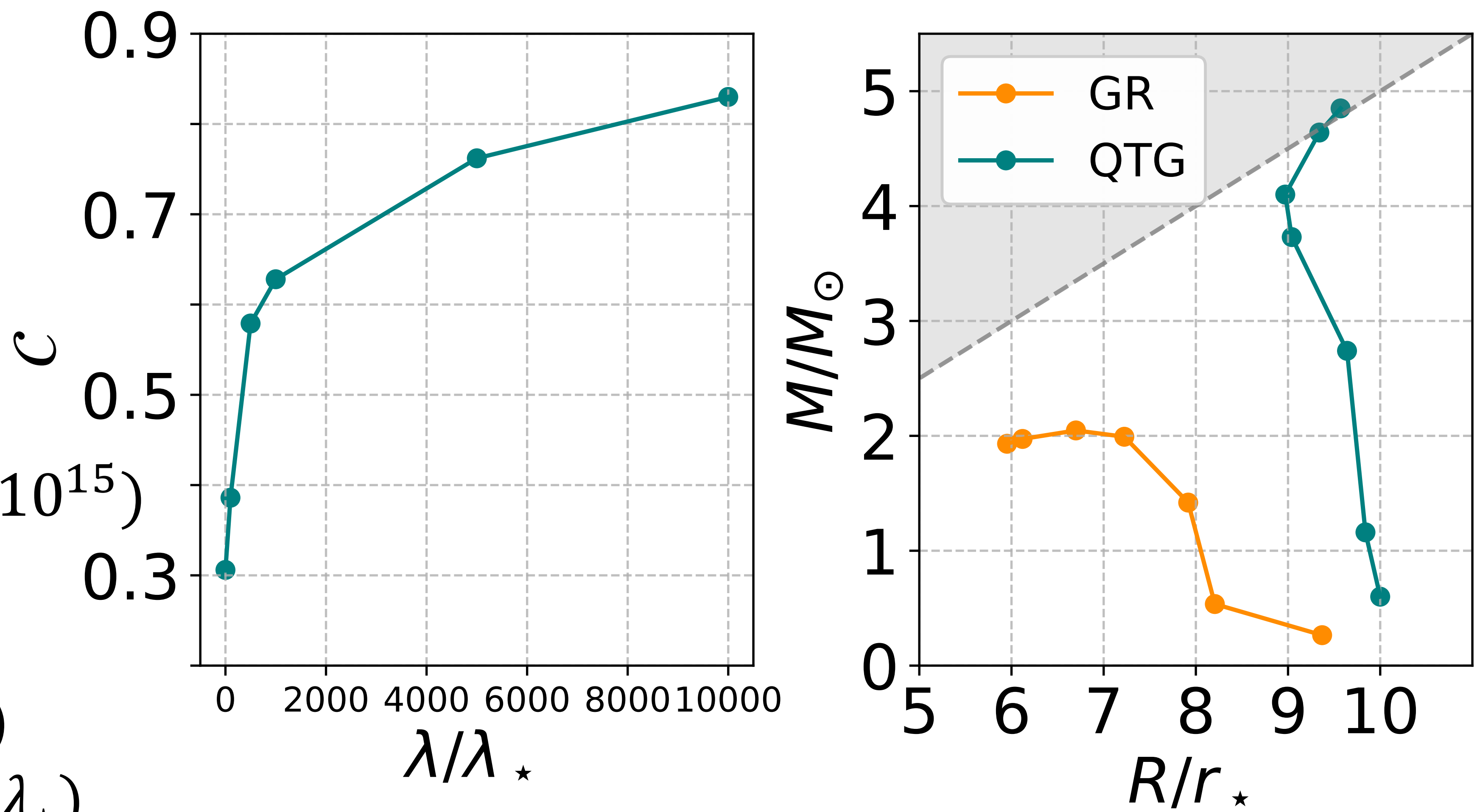
Left

- $\rho_0 = 3{\times}10^{15}\mathrm{g/cm^3}$
- EOS: SLy

Right

- $\lambda = 65\ \lambda_\star$
- $\rho_0 \in (3.5{\times}10^{14}, 6{\times}10^{15})$
- EOS: SLy

- Quark star (DSQS)
- $(2.97 M_\odot, 5.91 r_\star, 15\lambda_\star)$
- $\rho_0 = 1.67{\times}10^{16}\mathrm{g/cm^3}$



Summary and outlook

- A theory of gravity possesses non-uniqueness of vacuum spherically symmetric solutions, but the static black hole solution is unique, i.e. the Schwarzschild black hole

- Obtain solutions of stars smaller than black holes of the same mass
  - Understanding the strong-field properties of gravity
  - Black hole mimicking
    - Geodesic: Light rings ($1.48M$& $3M$), ISCO ($6M$), shadow
    - Dynamical: How about the evolution? Produce the GW? (Future research)

Thanks for your attention!