Moment of inertia for axisymmetric neutron stars in the standard model extension

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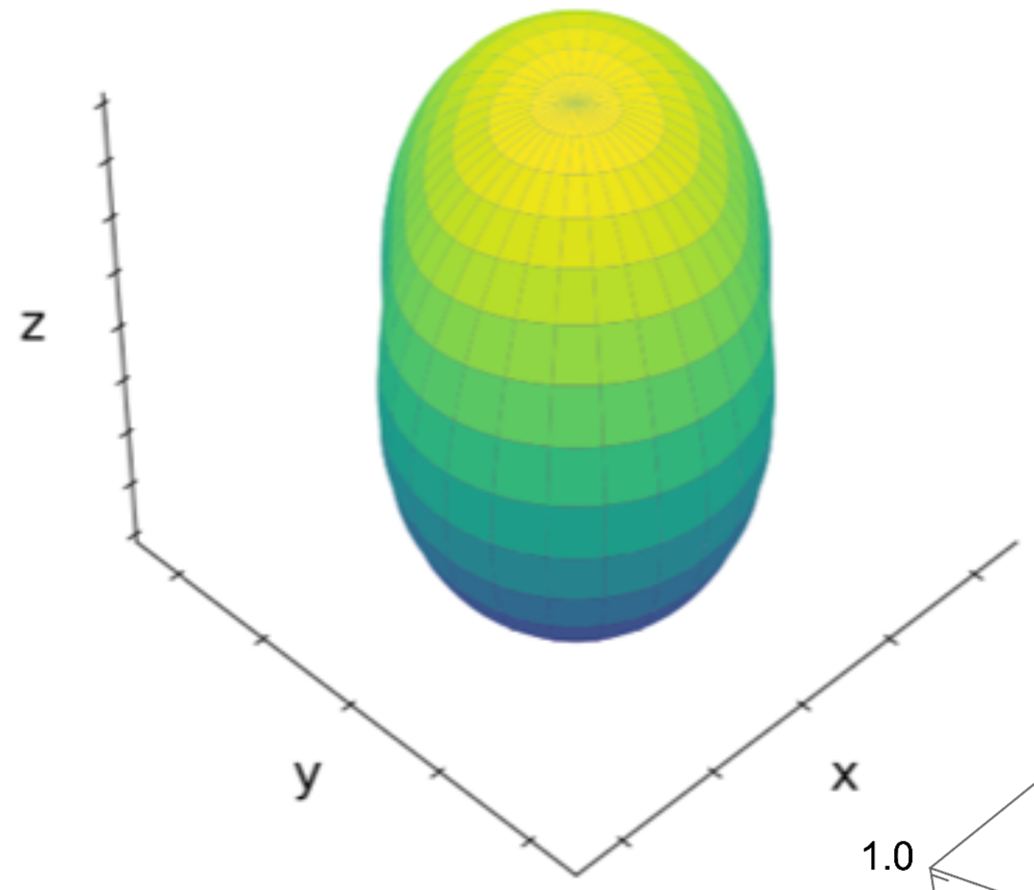
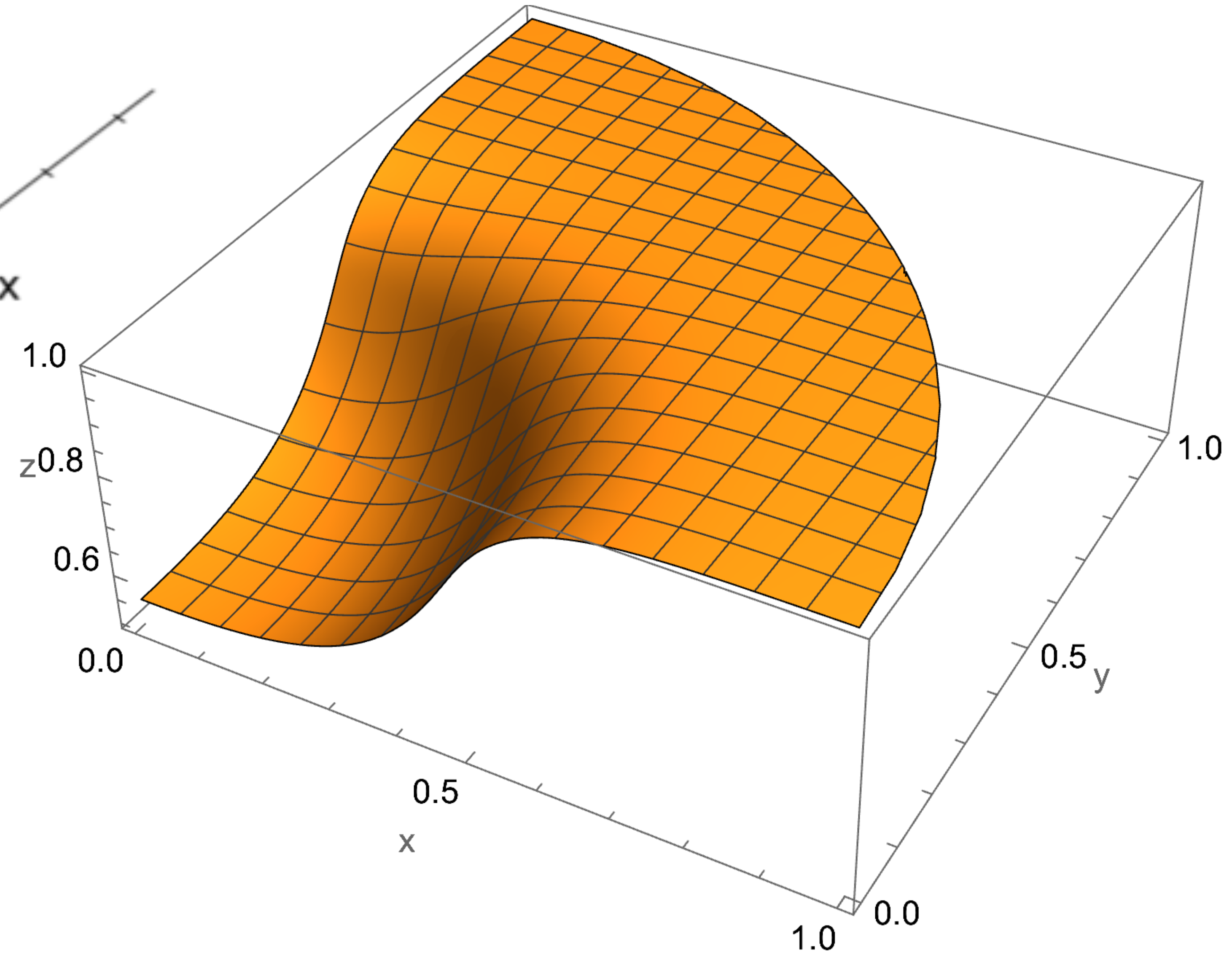
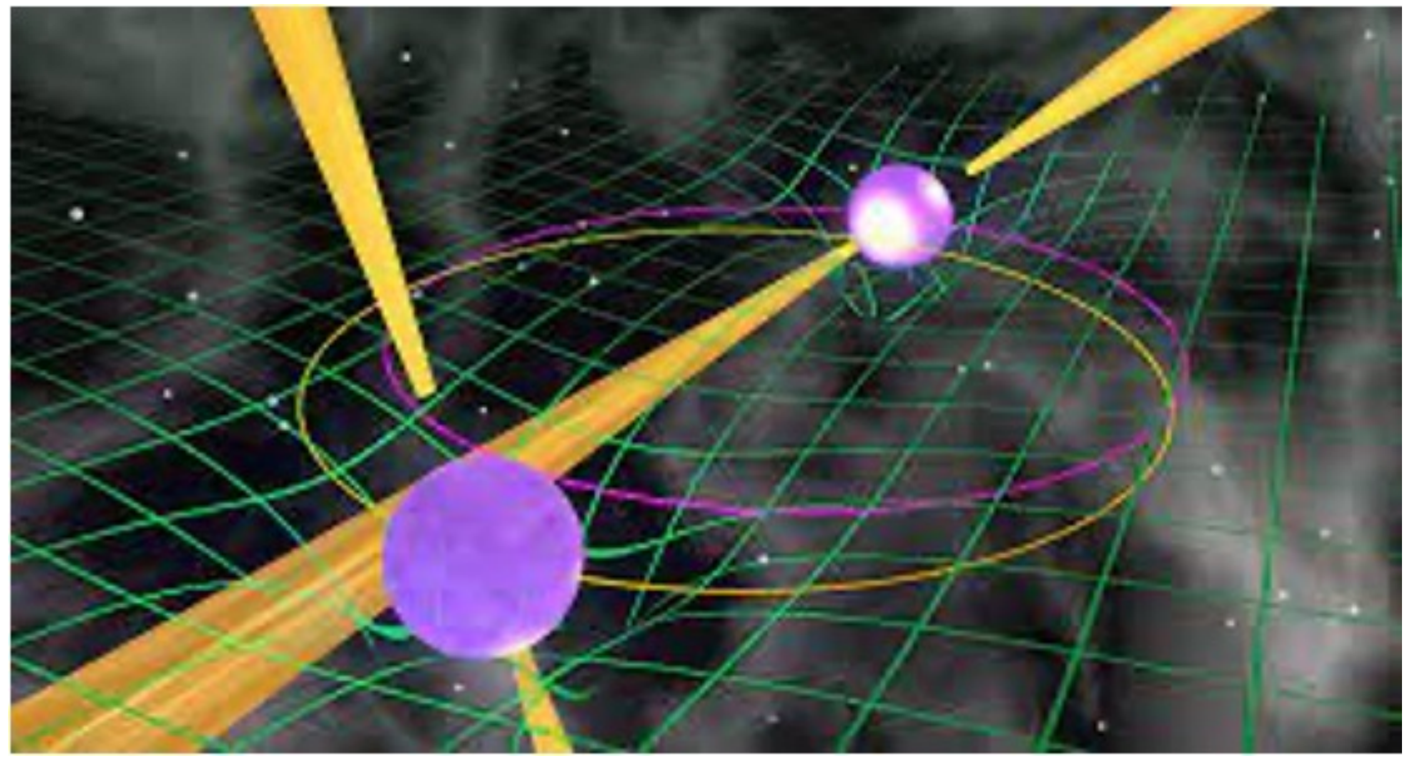




Ref: Y. Dong, Z. Hu, R. Xu and L. Shao, PRD 108,104039 (2023)

Moment of inertia

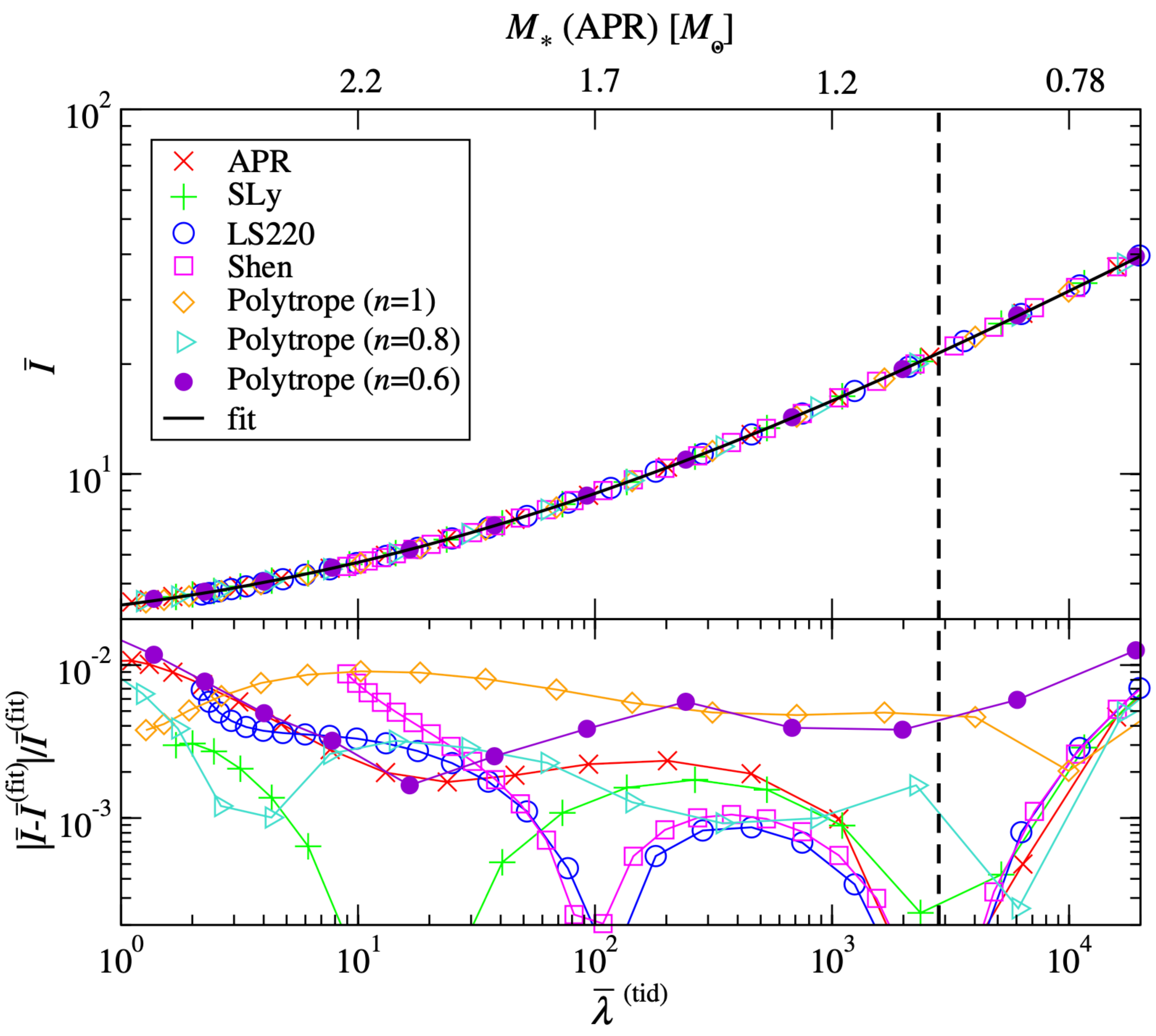
- **Moments of inertia (MOI) of neutron star (NS)**
  - Build relations between **angular momentum** and **angular velocity**



Credited: ScienceNews

Moment of inertia

- **Moments of inertia (MOI) of neutron star (NS)**
  - Build relations between **angular momentum** and **angular velocity**
- **For Theories**
  - **Structure of NS:** MOI differs under different equation of state (EOS)
  - **Universal relations (I-Love-Q)**: Insensitivity to EOS to help us learn gravity



Credited: K. Yagi & N. Yunes, 2013, PRD

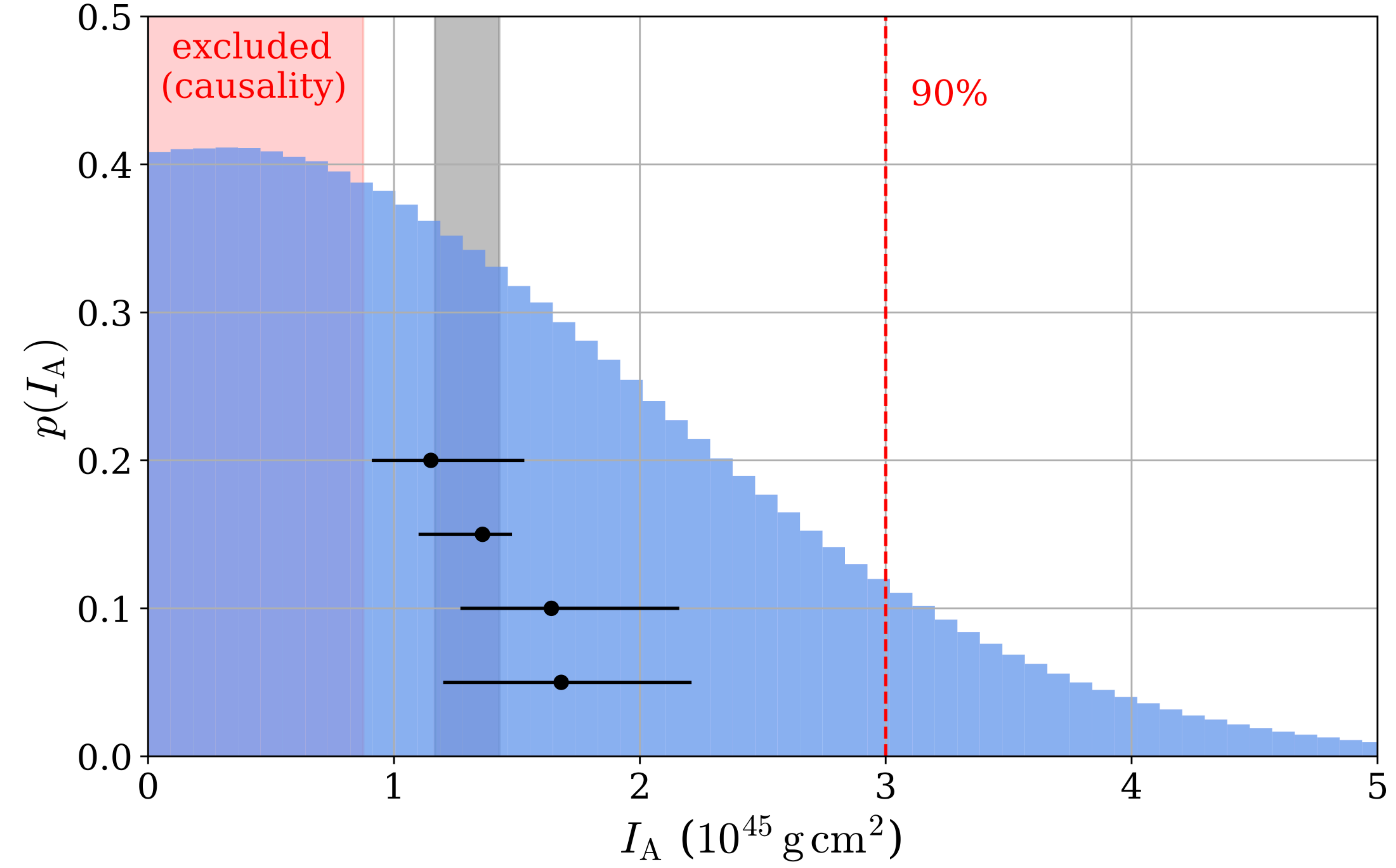
J. B. Hartle, 1967, ApJ

J. B. Hartle & K. S. Thorne, 1968 ApJ

Moment of inertia

- **For observations**
  - **High precision pulsar timing**: **direct detection** of MOI from Lense-Thirring effect in pulsar timing

H. Hu et al., 2020, MNRAS



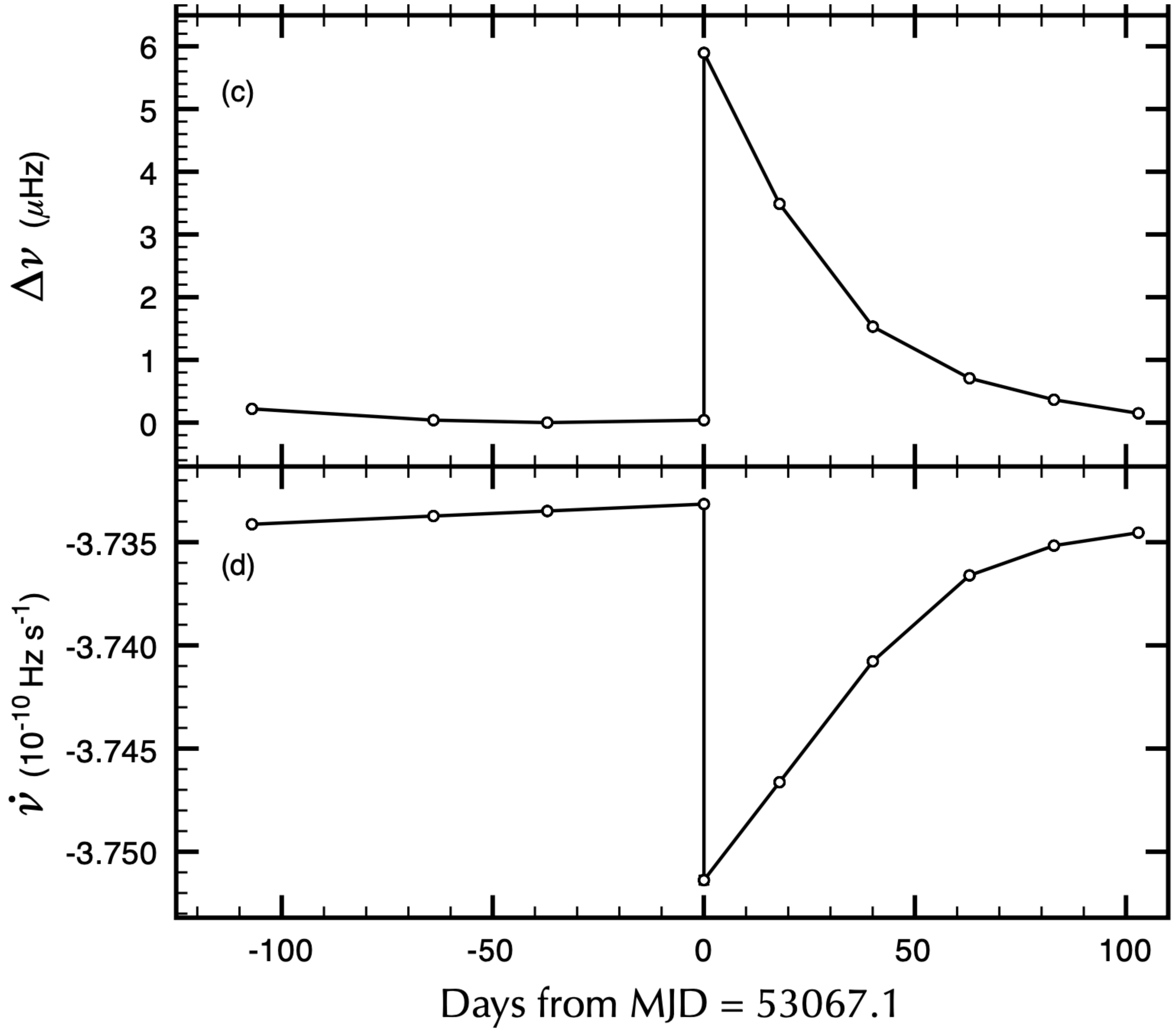
Credited: M. Kramer et al., 2022, PRX

Moment of inertia

- **For observations**
  - **High precision pulsar timing**: **direct detection** of MOI from Lense-Thirring effect in pulsar timing
    H. Hu et al., 2020, MNRAS
  - **Glitch**: timing irregularities in pulsar timing observation
    - Changes in MOI will result in variations in angular velocity

N. Andersson et al., 2003, PRL
X. Y. Lai et al., 2023, MNRAS

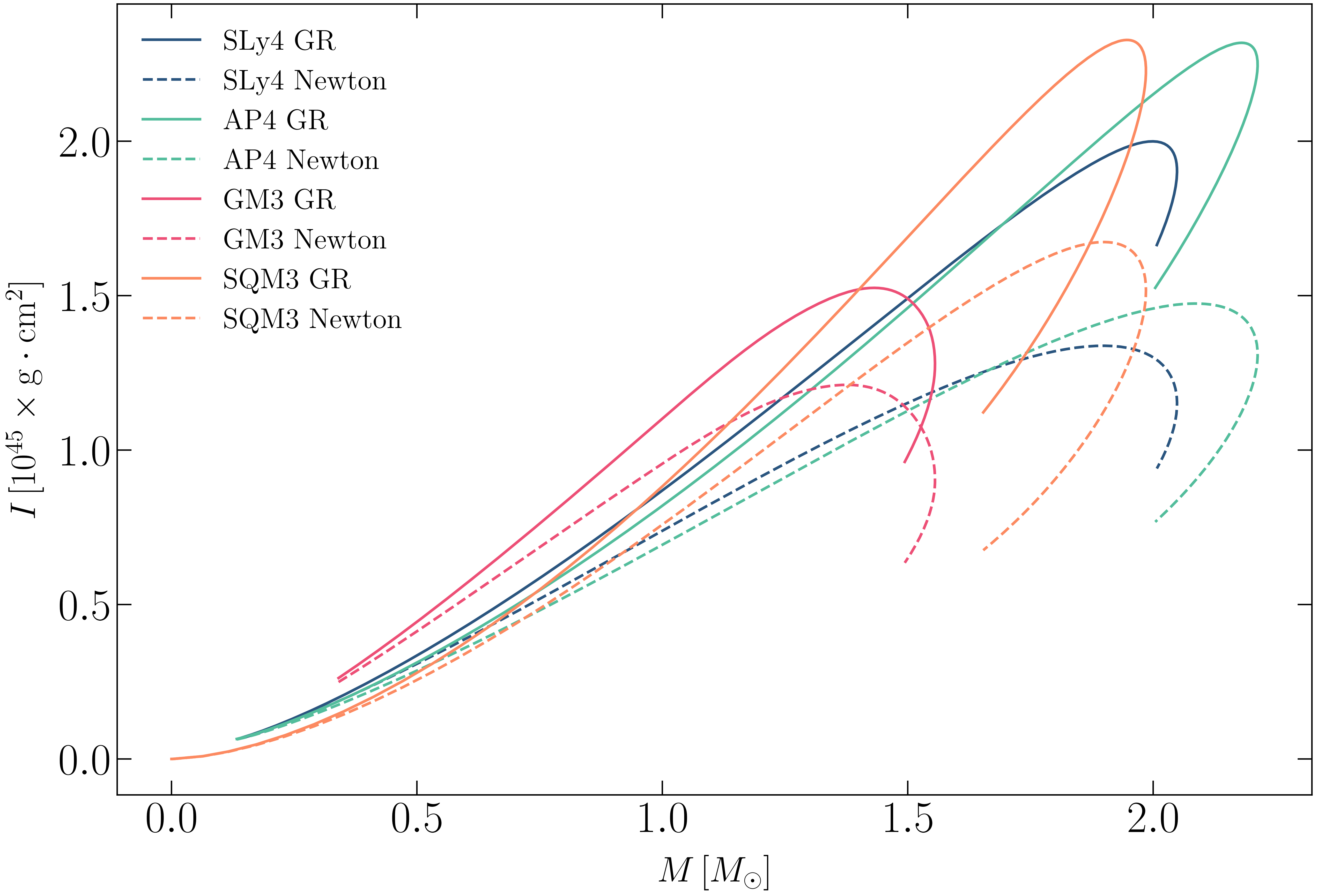
Glitch data of PSR B0531+21 (Crab pulsar)



Credited: C. M. Espinoza et al., 2010, MNRAS

Calculation of moment of inertia

- **Moments of inertia of spherically symmetric neutron stars in GR**
  - Solve the perturbed **Einstein's field equation**
  - For same density distribution, Newton or GR give **different results**



J. B. Hartle, 1967, ApJ
J. B. Hartle & K. S. Thorne, 1968 ApJ

Calculation of moment of inertia

- **Moments of inertia of spherically symmetric neutron stars in GR**
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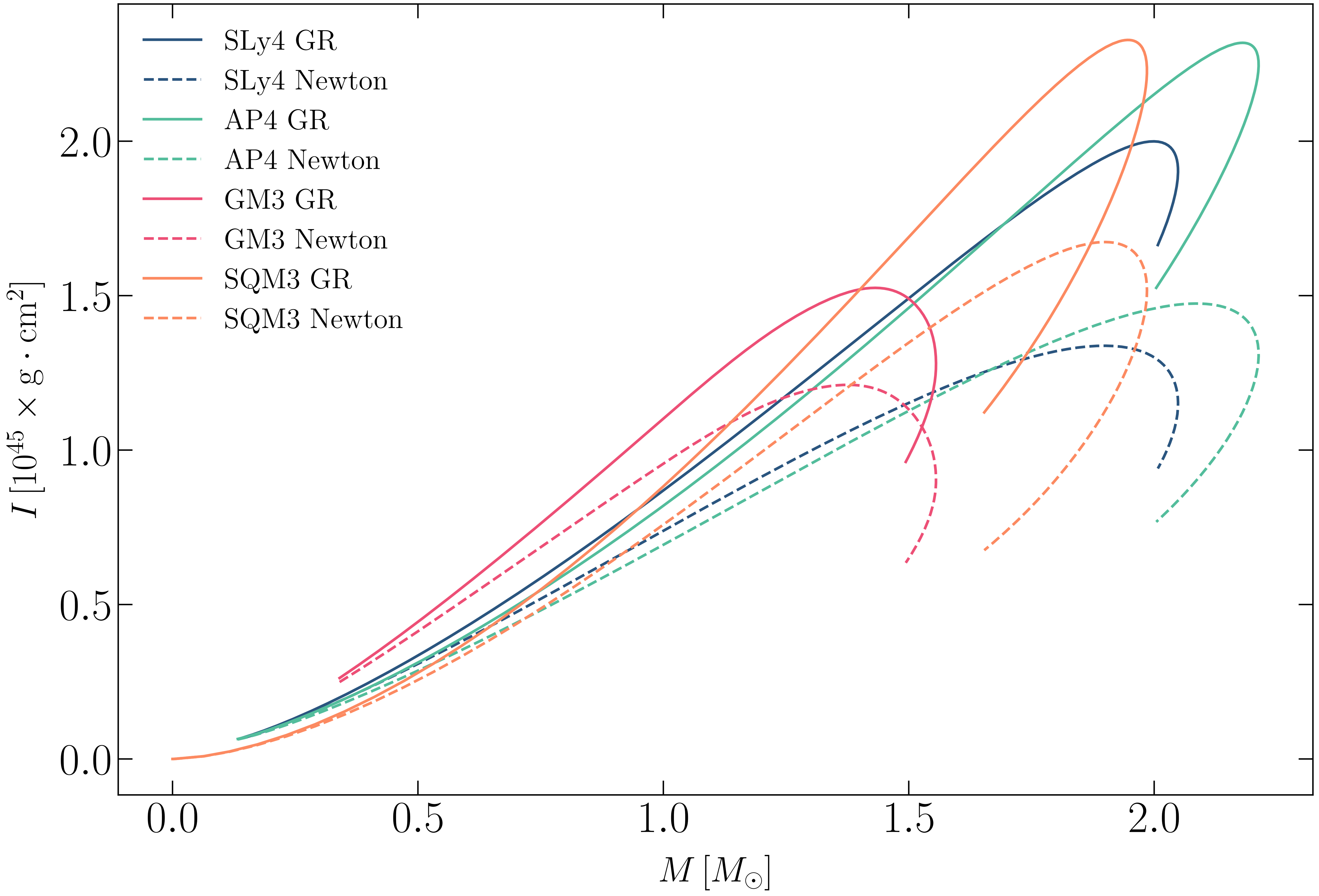
A consistent approach to calculate MOI of axisymmetric NS numerically



J. B. Hartle, 1967, ApJ
J. B. Hartle & K. S. Thorne, 1968 ApJ

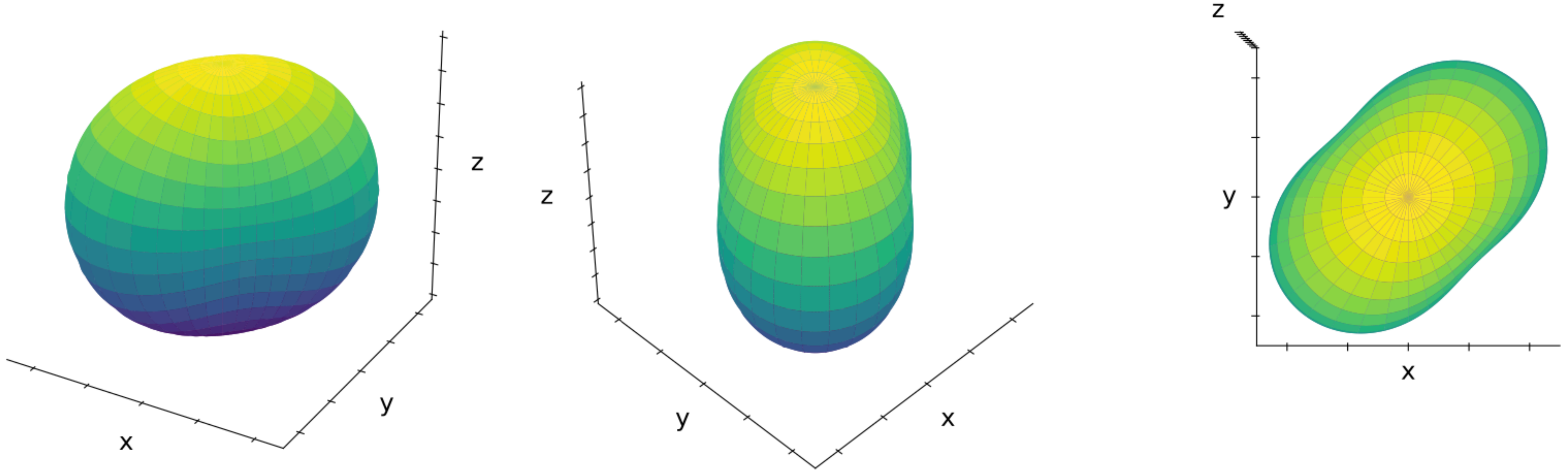
Standard-Model Extension (SME)

- Effective field theory that involve operators for Lorentz violation

$$\mathscr{L}_{\mathrm{LV}}^{(4)} = \frac{1}{16\pi G}\left(-uR + s^{\mu\nu}R^{T}_{\mu\nu} + t^{\alpha\beta\gamma\delta}C_{\alpha\beta\gamma\delta}\right)$$

Q. G. Bailey & V. A. Kostelecký, 2006, PRD

- The neutron stars have a preferred direction



Credited: R. Xu et al., 2020, PLB

Calculations

MOI for spherical NSs in GR

MOI for axisymmetric NSs in SME

Results

Moment of inertia in GR

- Stationary and axially symmetric metric,

$$ds^2 = -H^2dt^2 + Q^2dr^2 + r^2K^2\left[d\theta^2 + \sin^2\theta(d\varphi - Ldt)^2\right]$$

- Field equation,

$$R^t_\varphi = 8\pi T^t_\varphi$$

4-velocity

$$u^r = u^\theta = 0, \quad u^\varphi = \Omega u^t$$

Slowly rotation,

$$L(r,\theta) = \omega(r,\theta) + \mathcal{O}(\Omega^3)$$

J. B. Hartle, 1967, ApJ
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- Further (partial differential equation, PDE),

$$\frac{1}{r^4}\frac{\partial}{\partial r}\left(r^4 j\frac{\partial\bar{\omega}}{\partial r}\right) + \frac{4}{r}\frac{dj}{dr}\bar{\omega} + \frac{e^{(\lambda-\nu)/2}}{r^2}\frac{1}{\sin^3\theta}\frac{\partial}{\partial\theta}\left(\sin^3\theta\frac{\partial\bar{\omega}}{\partial\theta}\right) = 0$$

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Frame dragging,

$$\bar{\omega} = \Omega - \omega(r,\theta)$$

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Moment of inertia in GR

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- For spherical NS in GR,

$$\frac{1}{r^4}\frac{d}{dr}\left(r^4 j\frac{d\bar{\omega}}{dr}\right) + \frac{4}{r}\frac{dj}{dr}\bar{\omega} = 0$$

4-velocity

$$u^r = u^\theta = 0, \quad u^\varphi = \Omega u^t$$

Slowly rotation,

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Frame dragging,

$$\bar{\omega} = \Omega - \omega(r,\theta)$$

Spherical NS in GR,

$$\bar{\omega}(r) = \Omega - \frac{2J}{r^3}$$

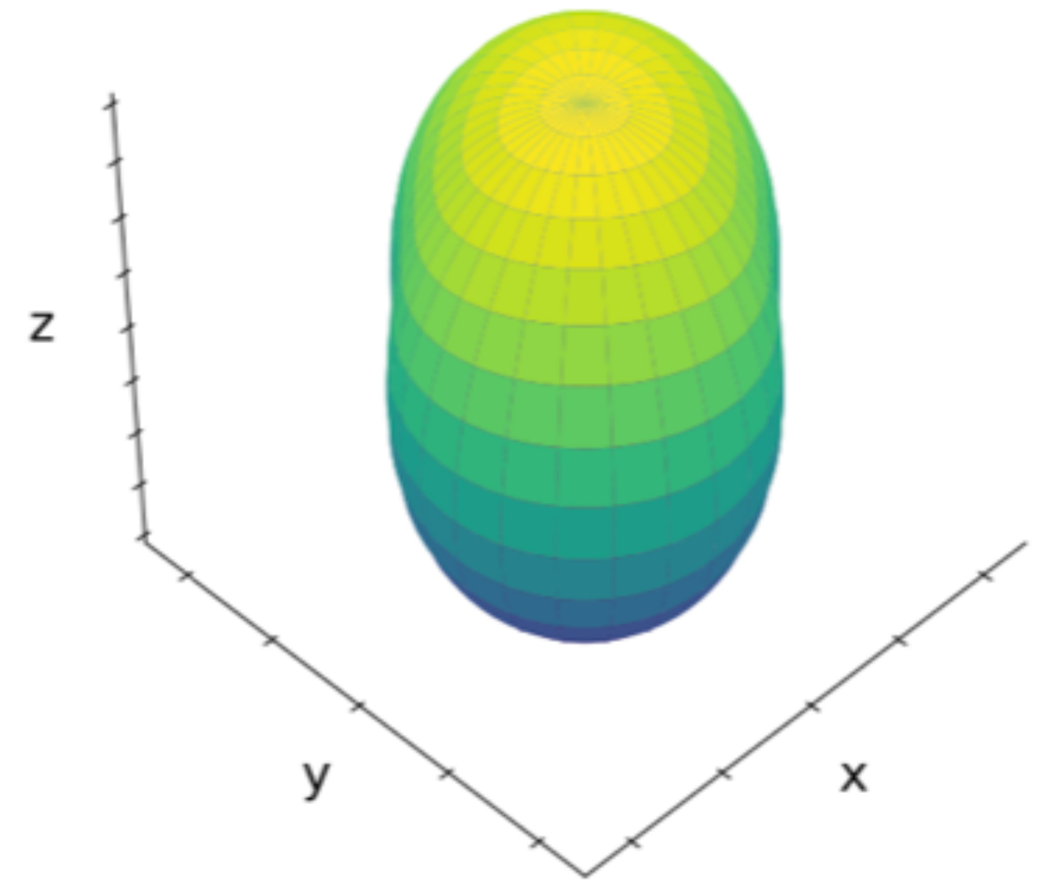
$$I = J/\Omega$$

J. B. Hartle, 1967, ApJ
J. B. Hartle & K. S. Thorne, 1968 ApJ

Moment of inertia in SME

- We still want to calculation this field equation, but there's some difference
  - **Difference 1: field equation has another term**
    - We should solve $R^t_\varphi = 8\pi T^t_\varphi - V^t_\varphi$, where $V^\mu_\nu = \bar{s}^{\alpha\beta} g^{\mu\delta} G_{\delta\alpha\beta\nu}$

Q. G. Bailey et al., 2014, PRD



R. Xu et al., 2020, PLB

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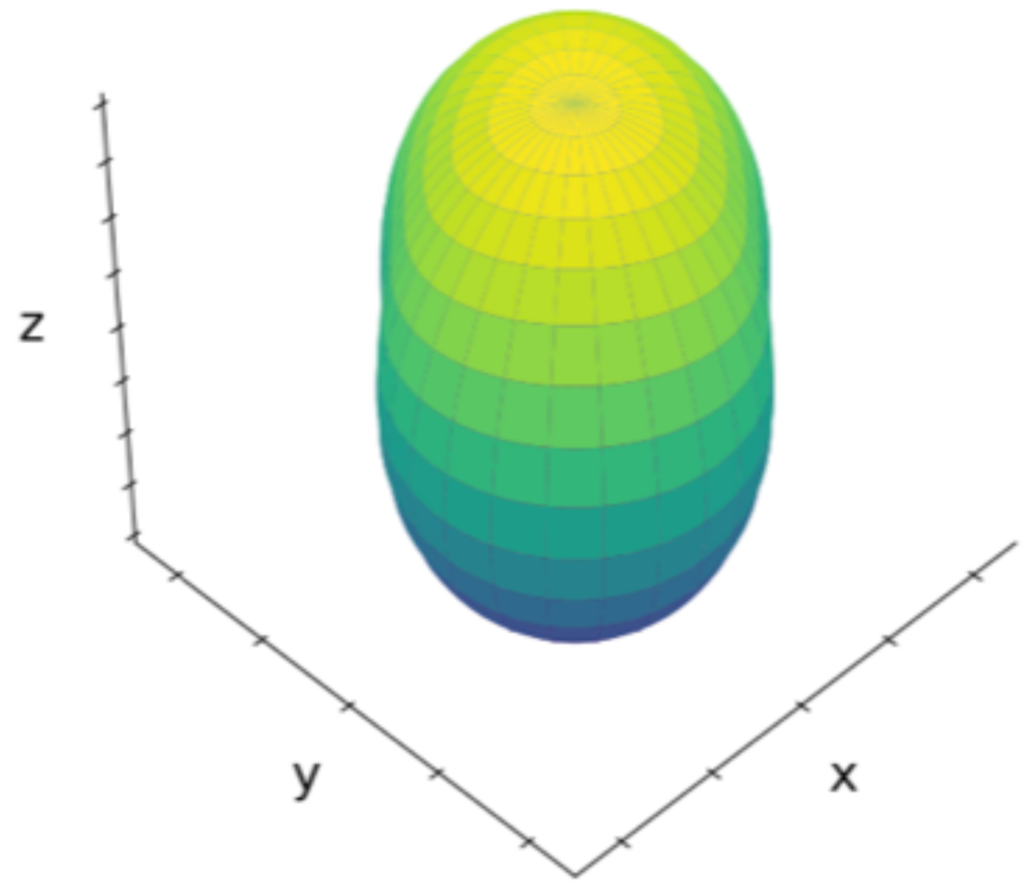
Q. G. Bailey et al., 2014, PRD

  - **Difference 2: axisymmetric deformation of neutron stars**
    - $\rho^{(1)}(r,\theta) = -\dfrac{1}{6}\bar{s}^{zz}(3\cos^2\theta - 1) r \rho^{(0)}(r)_{,r}$
    - $P^{(1)}(r,\theta) = -\dfrac{1}{6}\bar{s}^{zz}(3\cos^2\theta - 1) r P^{(0)}(r)_{,r}$



R. Xu et al., 2020, PLB

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Q. G. Bailey et al., 2014, PRD

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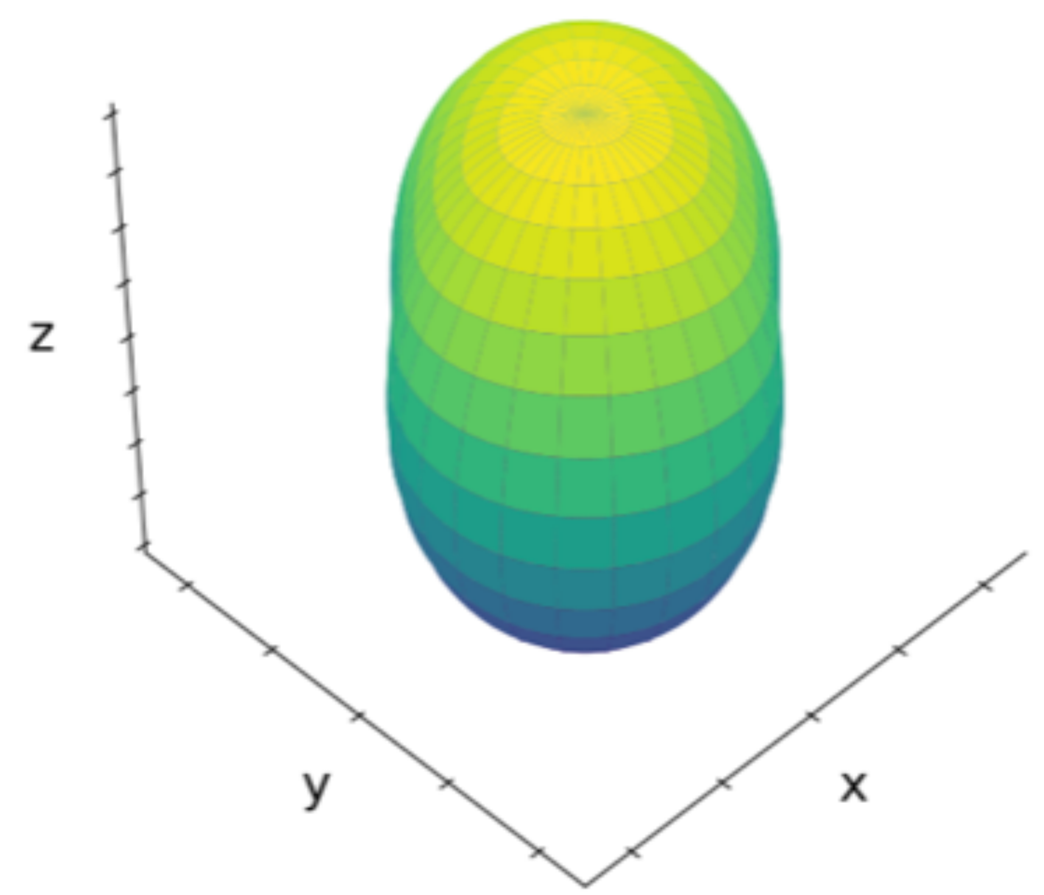
$$\boxed{\frac{1}{r^4}\frac{\partial}{\partial r}\left(r^4 j \frac{\partial \bar{\omega}}{\partial r}\right) + \frac{4}{r}\frac{\mathrm{d}j}{\mathrm{d}r}\bar{\omega} + \frac{e^{(\lambda-\nu)/2}}{r^2}\frac{1}{\sin^3\theta}\frac{\partial}{\partial \theta}\left(\sin^3\theta \frac{\partial \bar{\omega}}{\partial \theta}\right) = S_1'(r,\theta) + S_2'(r,\theta)}$$



R. Xu et al., 2020, PLB

Solve PDE numerically

$$\boxed{\frac{1}{r^4}\frac{\partial}{\partial r}\left(r^4 j \frac{\partial \bar{\omega}}{\partial r}\right) + \frac{4}{r}\frac{\mathrm{d}j}{\mathrm{d}r}\bar{\omega} + \frac{e^{(\lambda-\nu)/2}}{r^2}\frac{1}{\sin^3\theta}\frac{\partial}{\partial\theta}\left(\sin^3\theta \frac{\partial\bar{\omega}}{\partial\theta}\right) = S_1'(r,\theta) + S_2'(r,\theta)}$$

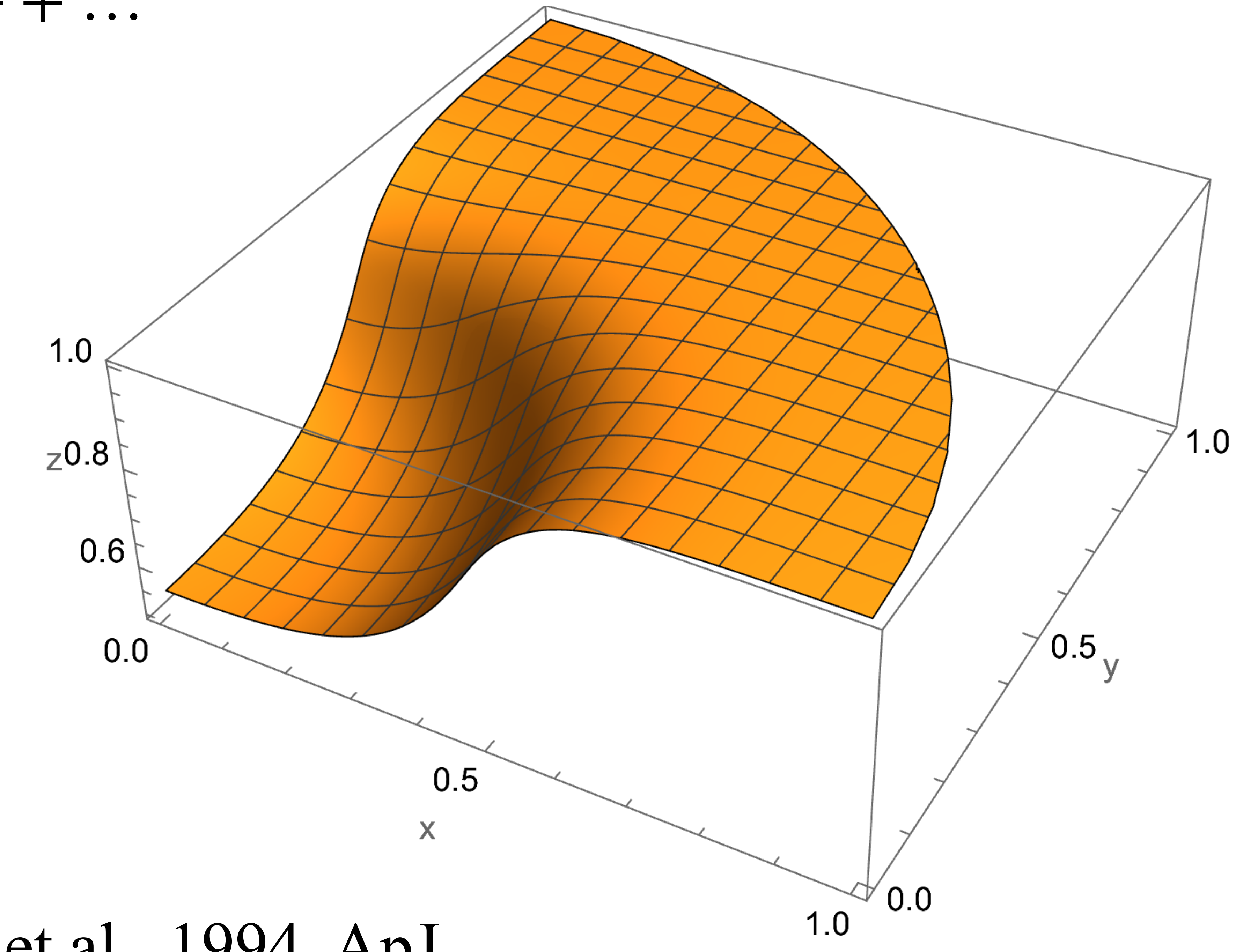
- **How to get MOI?**

$$\bar{\omega}(r) = \Omega - \frac{2J}{r^3}$$

  - Far field behaviors of $\bar{\omega}$

$$\bar{\omega}(r,\theta) = A + \frac{B(\theta)}{r^3} + \frac{C(\theta)}{r^4} + \ldots$$

  - Solve PDE for $r \in (0,\infty)$
- **Finite element method**
  - Numerically, integrating r to infinity is impossible
  - Change of variables $(r,\theta) \to \left(x \equiv \dfrac{r\cos\theta}{r+R}, y \equiv \dfrac{r\sin\theta}{r+R}\right)$
    - Improve accuracy greatly



G. B. Cook et al., 1994, ApJ

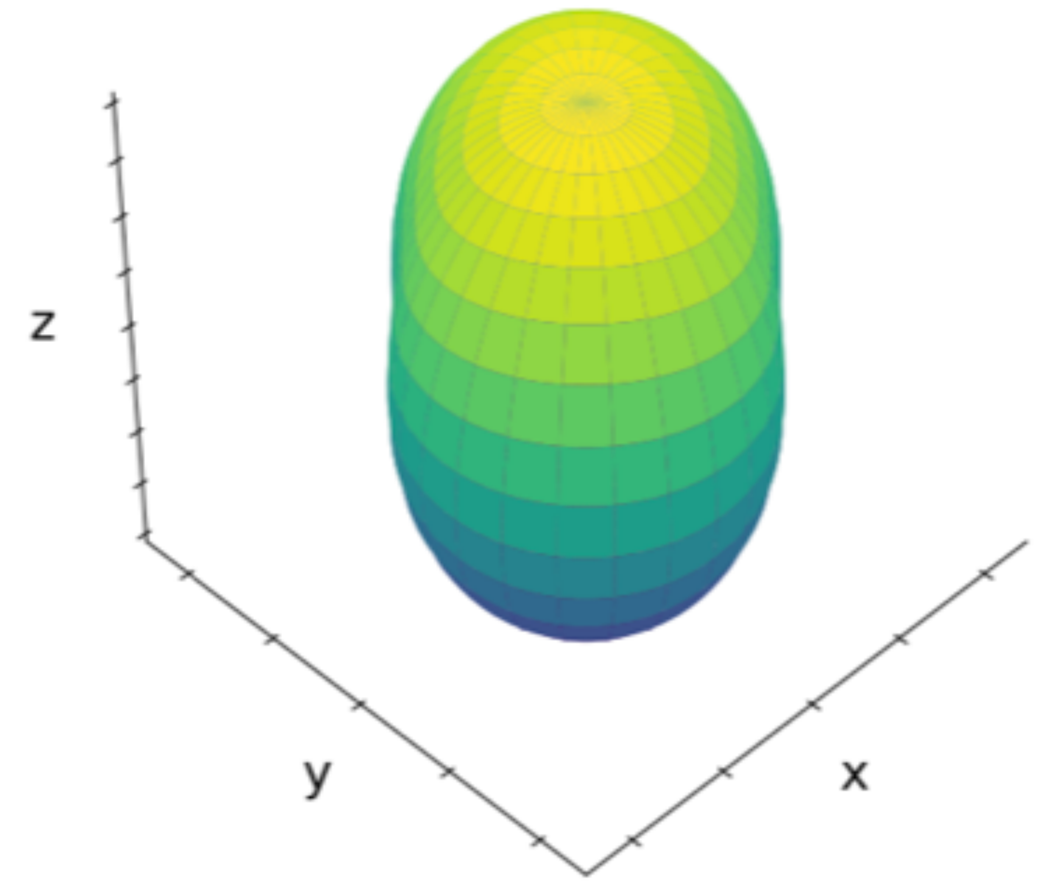
Solve PDE numerically

- **Process of numerical solving**
  - Derive the modified PDE of MOI
  - Change of variables

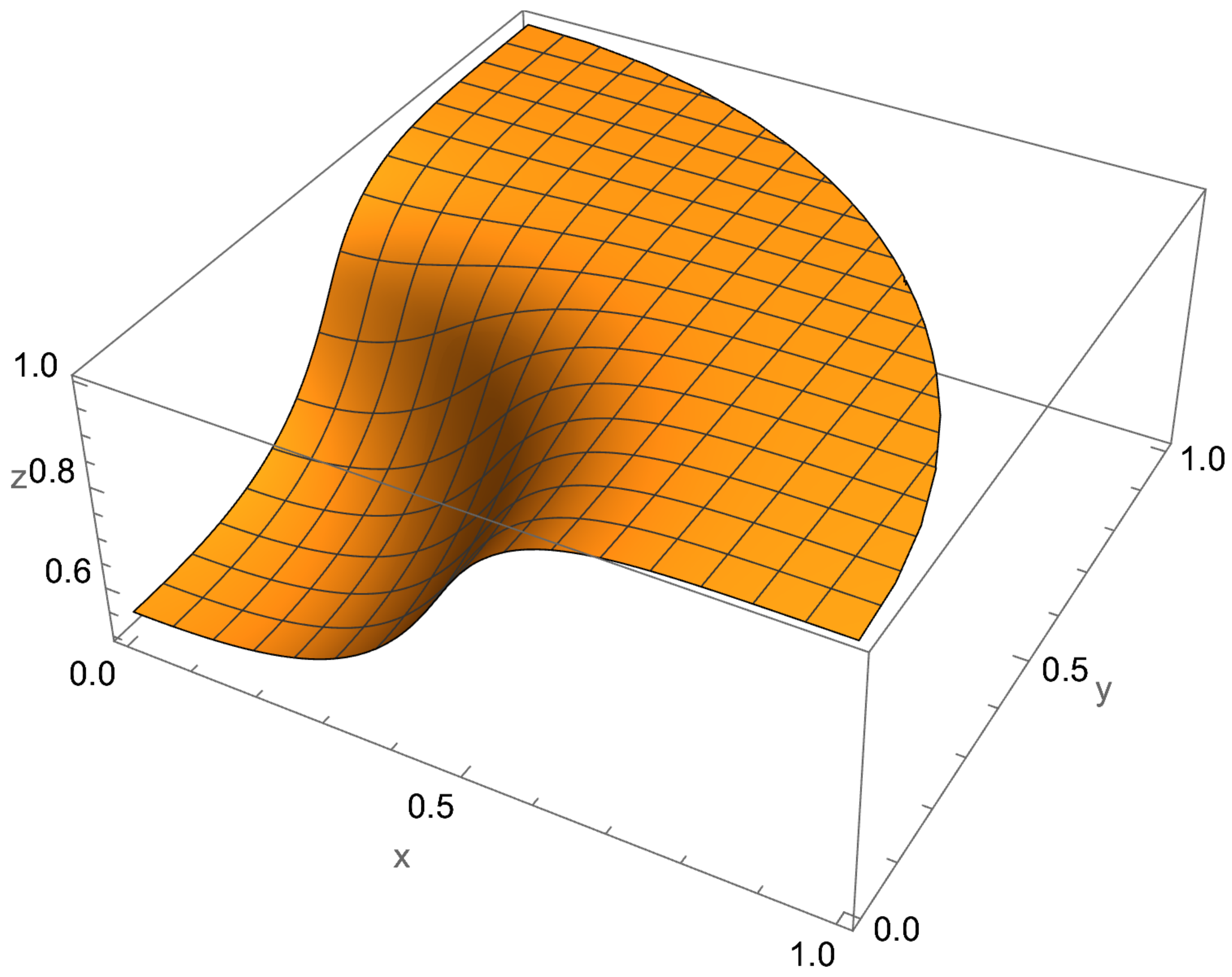
$$(r,\theta) \to \left(x \equiv \frac{r\cos\theta}{r+R}, y \equiv \frac{r\sin\theta}{r+R} \right)$$

  - Solve (x, y) PDE to get numerical solution
  - Fitting with polynomial to get MOI with far field behaviors of $\bar{\omega}$

$$\bar{\omega}(r,\theta) = A + \frac{B(\theta)}{r^3} + \frac{C(\theta)}{r^4} + \dots$$

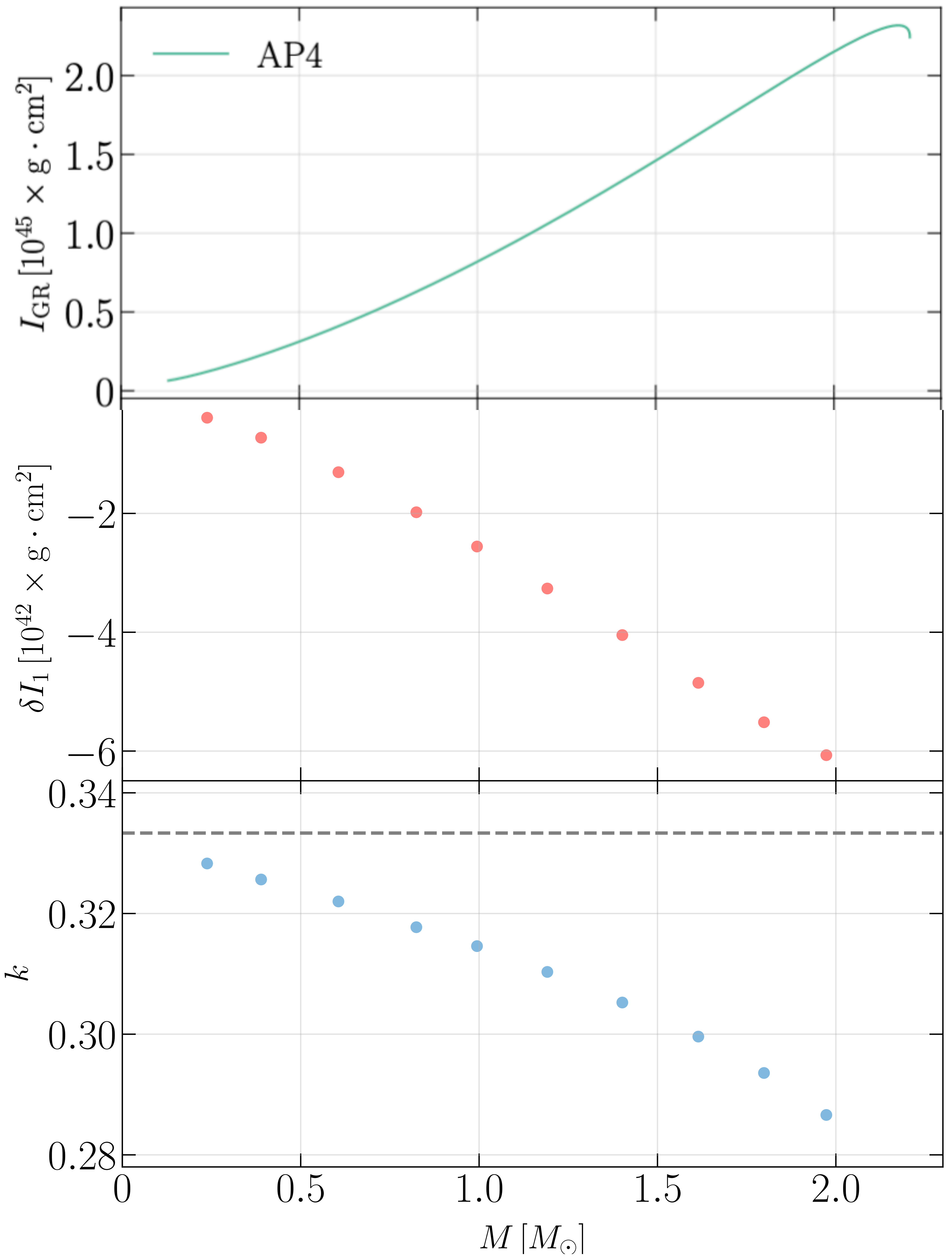


R. Xu et al., 2020, PLB



Results

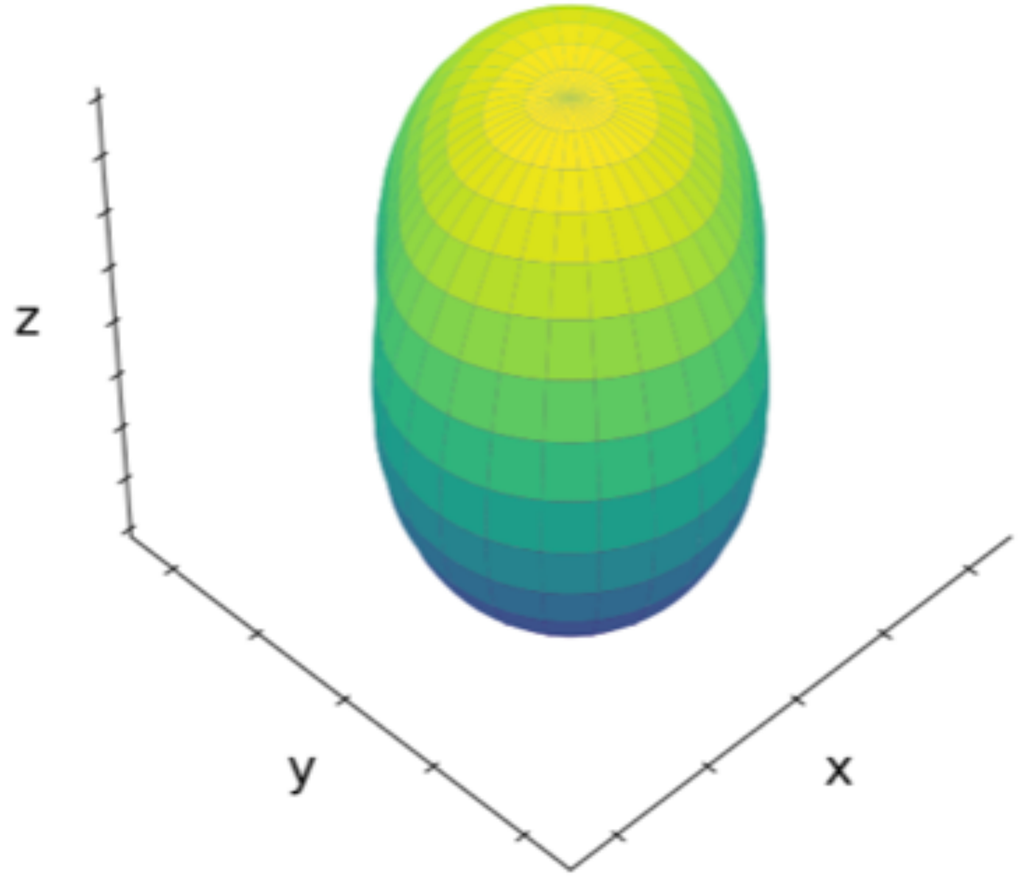
- Corrections to MOIs **caused by axisymmetric deformations**
  - $k = \delta I_1 / I$, for chosen deformations it is $1/3$
  - For deformation caused by Lorentz violation,
    - **Ratio $k = \delta I_1 / I$ from GR differs from Newtonian one**
    - **For a neutron star with $1.4\, M_\odot$, it is $8\,\%$**



Summary

- We develop a **consistent approach** to calculate moment of inertia for **axisymmetric** neutron stars in the Standard-Model Extension
  - **Finite element method**
  - **Ratio $k = \delta I_1 / I$ from GR differs from Newtonian one**
    - **For a neutron star with $1.4\,M_{\odot}$, it is $8\,\%$**

Thanks!



R. Xu et al., 2020, PLB