北京大学
PEKING UNIVERSITY

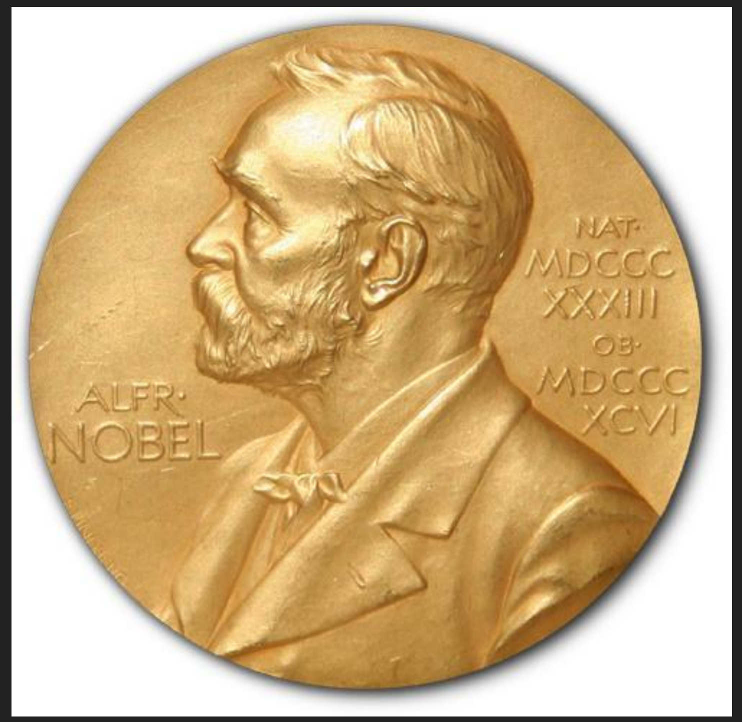
Measuring the Spin of Sgr A* with Pulsar Timing

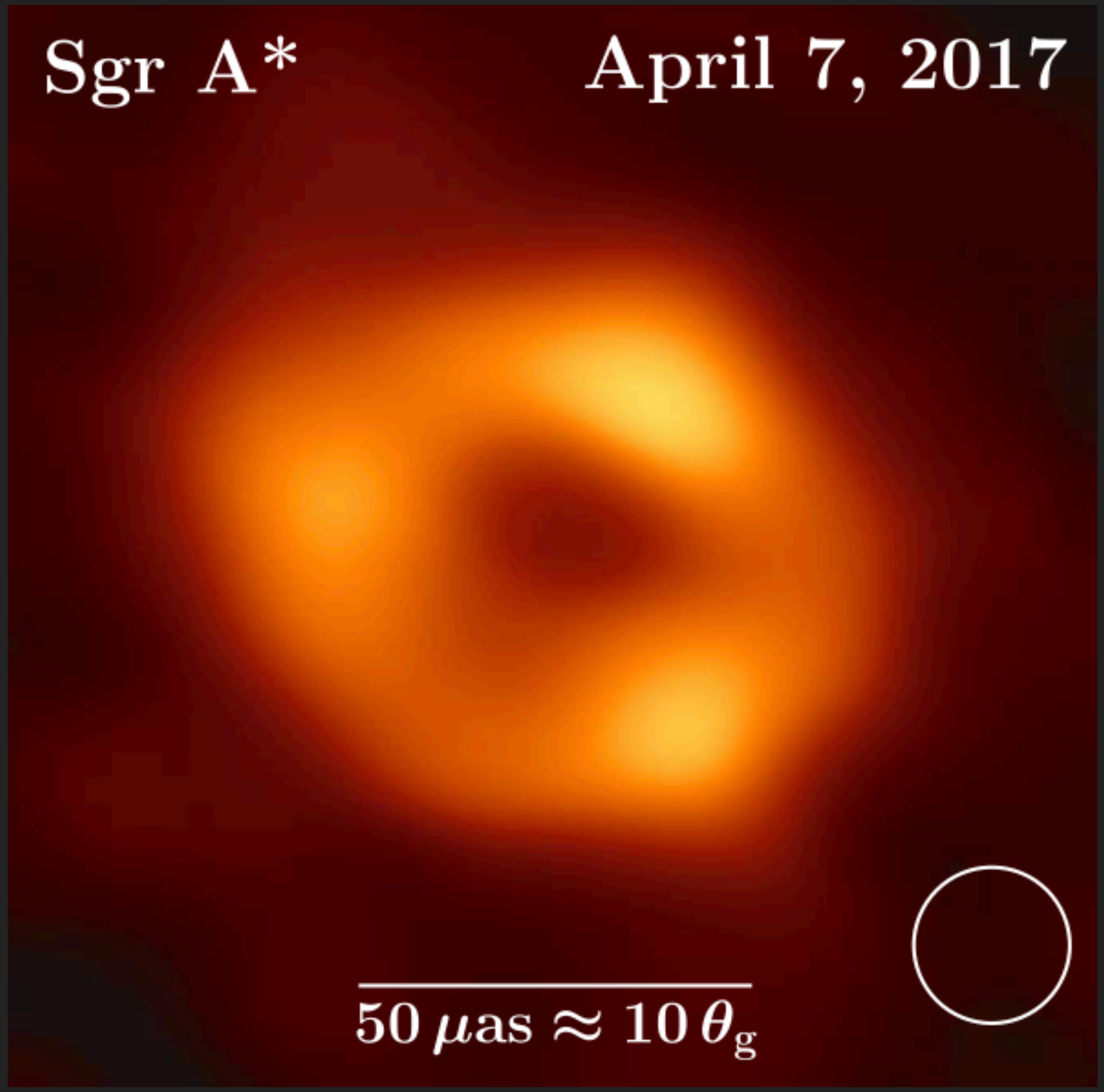
Speaker: Zexin Hu
Supervisor: Lijing Shao

SKA Pulsar Science Symposium (SPSS) 2024
Kunming Yunnan, 2024.07.13

Introduction

- Sgr A*, the supermassive black hole (SMBH) in our galactic center (GC)
- $M \sim 4.3 \times 10^6 M_\odot$, $R_{GC} \sim 8$ kpc
- $r_g \sim 0.04$ AU $\sim 5\,\mu$as
- ✱ Monitoring the S-stars (Ghez et al. 2008, Genzel et al. 2010)





The EHT Collaboration 2022

Measuring the properties of the SMBH ⇒

✓ Gravity tests
✓ BH physics
✓ Environments in the GC

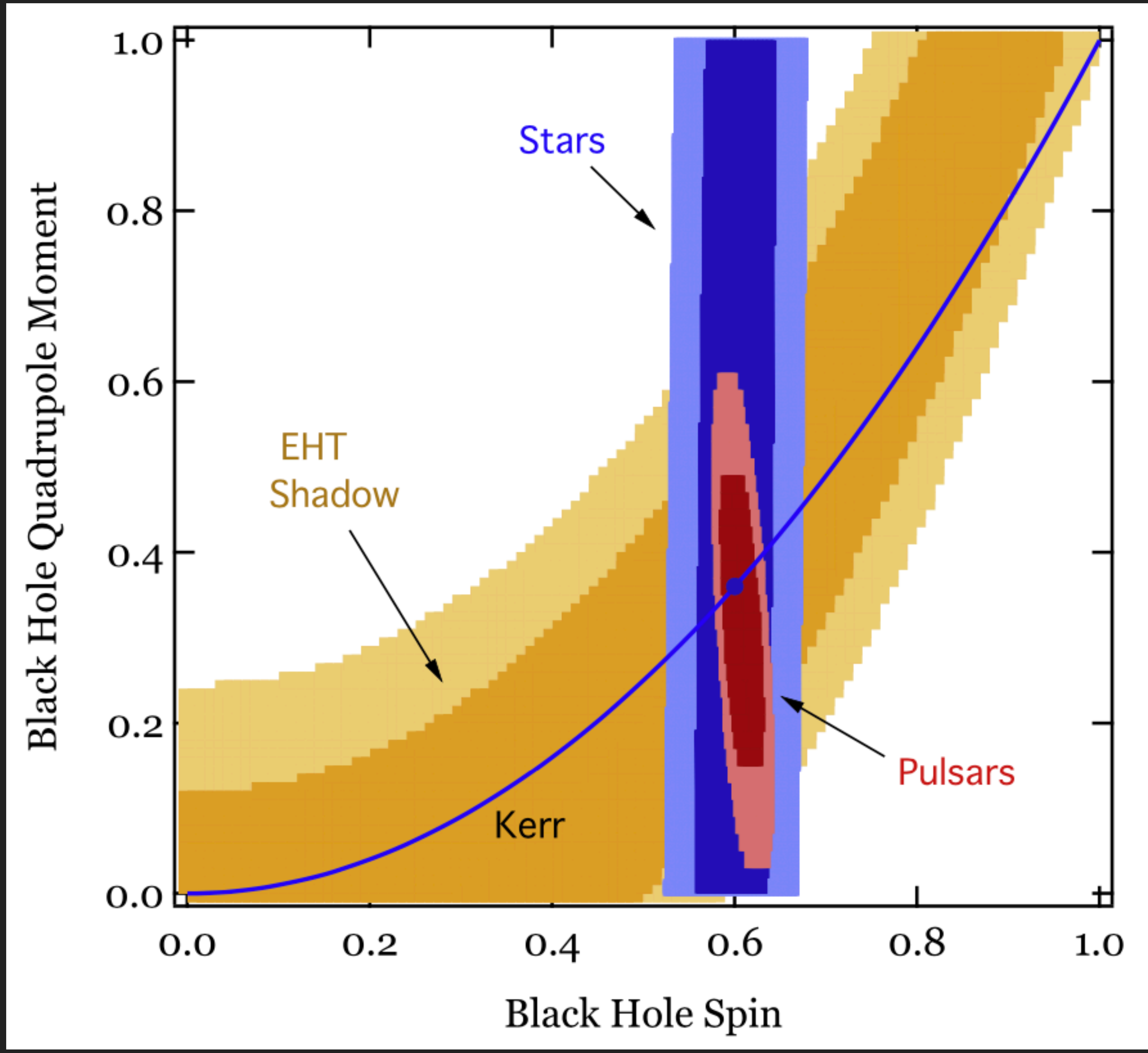
Testing the no-hair theorem

- Cosmic censorship conjecture (Penrose 1979)

$$\chi \equiv \frac{c}{G}\frac{S}{M^2} \leq 1$$

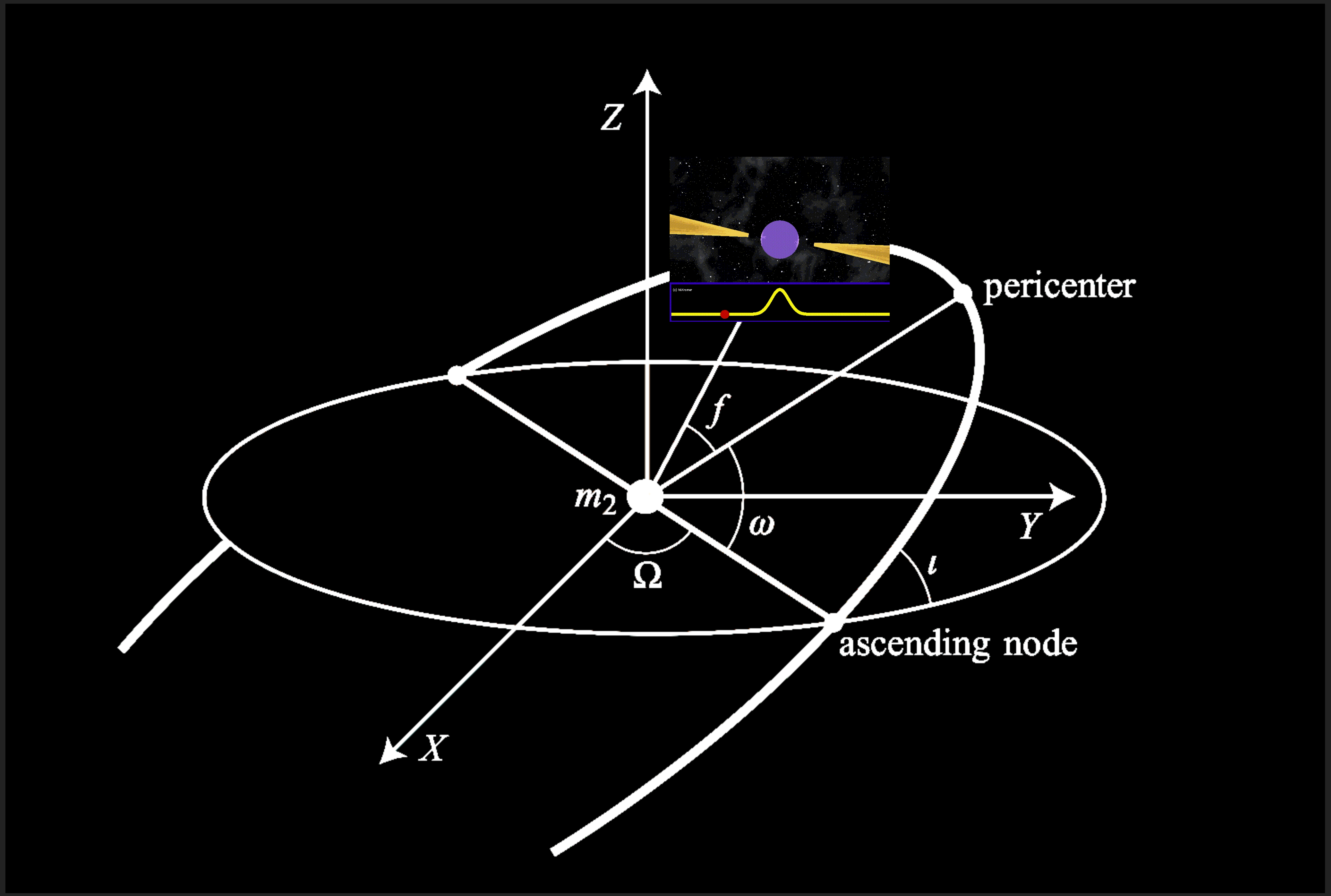
- No-hair theorem (Israel 1967, Carter 1971)

$$q \equiv \frac{c^4}{G^2}\frac{Q}{M^3} = -\chi^2$$



Psaltis et al. 2016

Measuring the spin of Sgr A* with pulsar timing

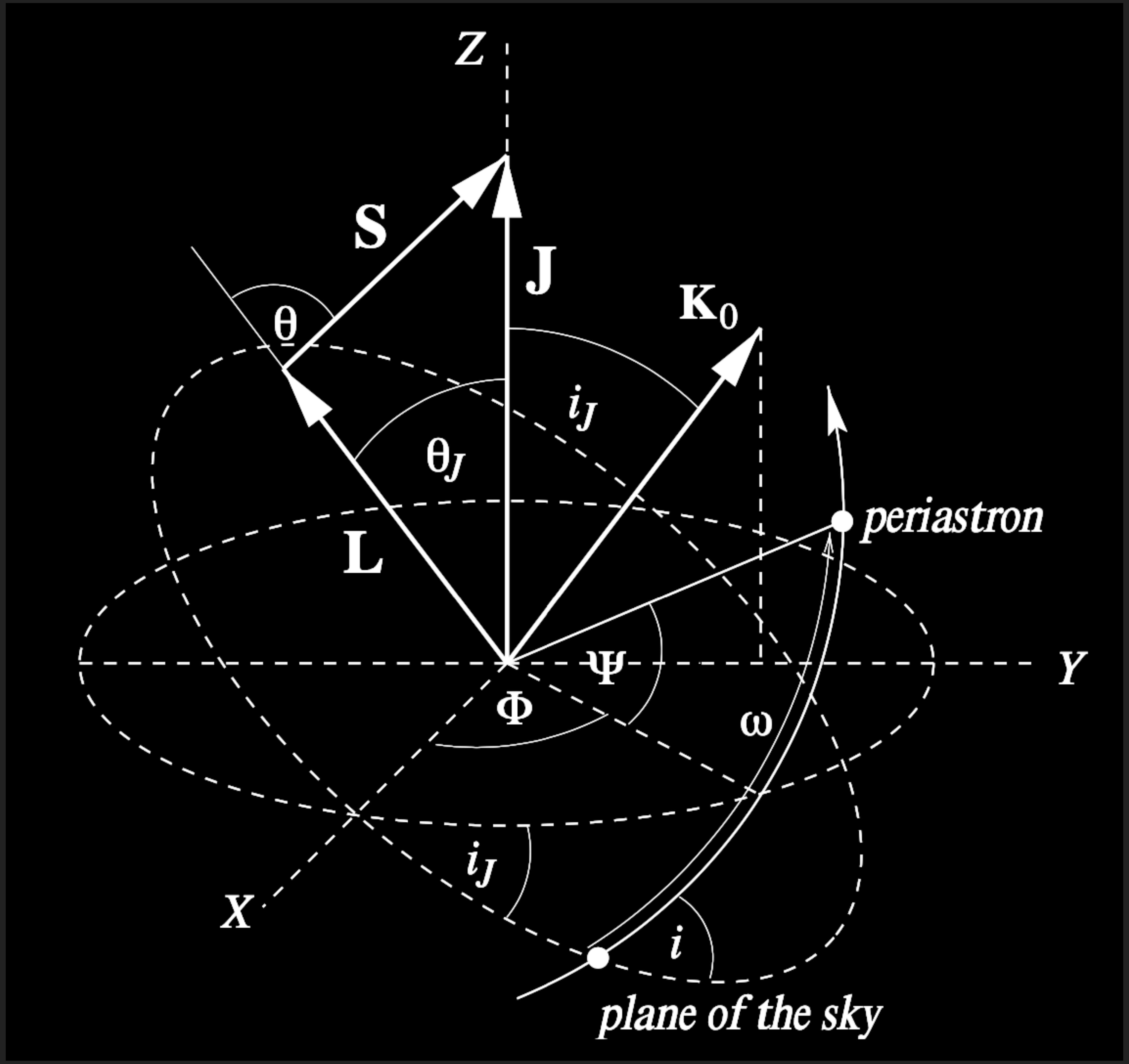


Credit: Poisson & Will

- Timing model (Damour & Taylor 1992)
- $t_b - t_0 = D^{-1}\left[T + \Delta_R + \Delta_E + \Delta_S + \Delta_A\right]$
- $N = N_0 + \nu T + \dot{\nu} T^2/2 + \cdots$
- $\left\{p^{\mathrm{K}}\right\} = \left\{P_b, T_0, e_0, \omega_0, x_0\right\}$
- $\left\{p^{\mathrm{PK}}\right\} = \left\{k, \gamma, \dot{P}_b, r, s, \delta_\theta, \dot{e}, \dot{x}\right\}$

Measuring the spin of Sgr A* with pulsar timing

- Spin-orbit coupling (Lense & Thirring 1918)
- $\dot{\vec{L}} = \vec{\Omega}_{\rm prec} \times \vec{L}\,,\ \dot{\vec{A}} = \vec{\Omega}_{\rm prec} \times \vec{A}$
- $\dot{\omega} = \dot{\omega}_M + \dot{\omega}_S + \dot{\omega}_Q$
- $\dot{x} = \dot{x}_S + \dot{x}_Q$



Wex & Kopeikin 1999

Wex & Kopeikin 1999, Liu et al. 2012

Numerical simulation

- The post-Newtonian equation of motion

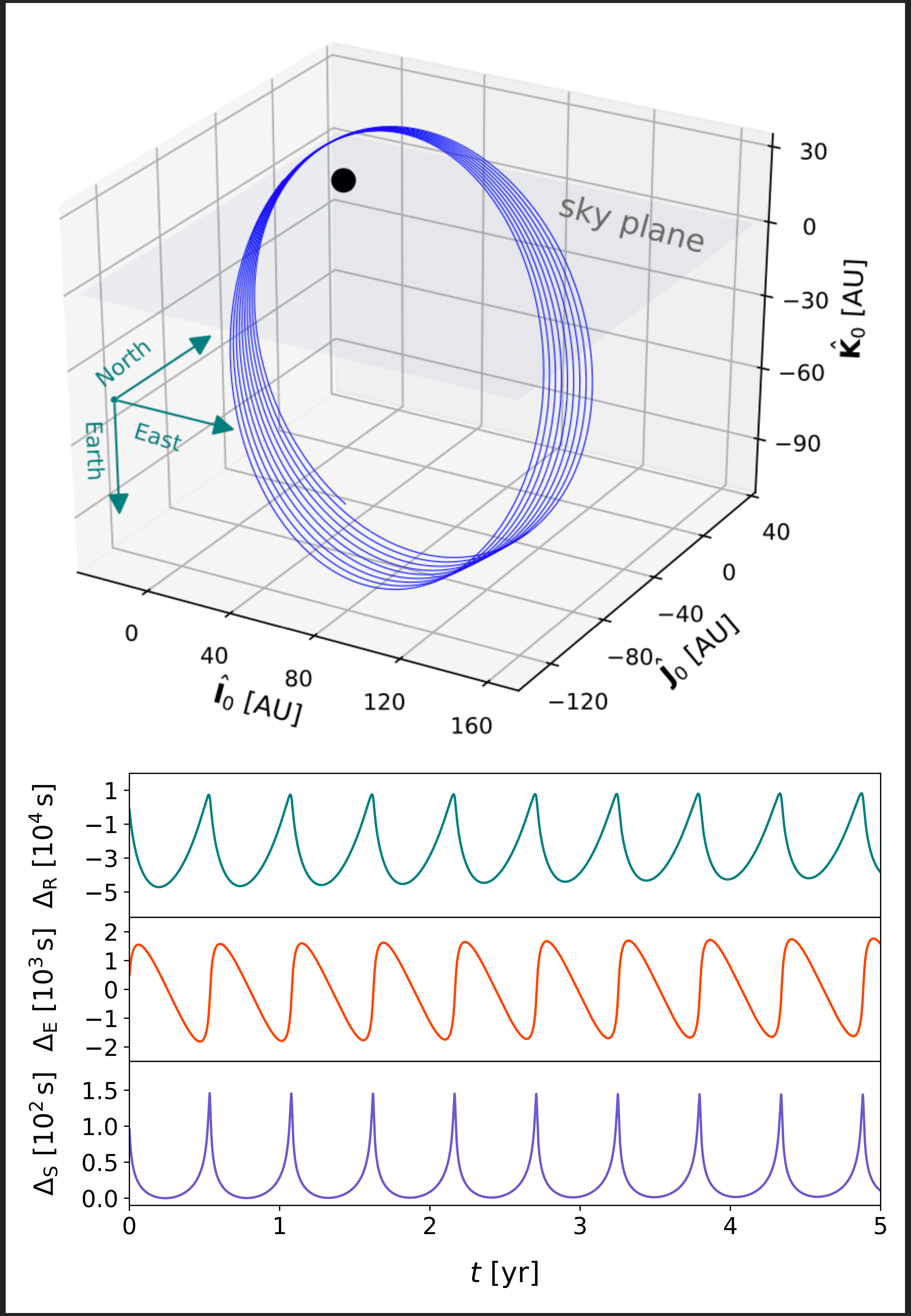
$$\ddot{\vec{r}} = -GM\hat{n}/r^2 + \ddot{\vec{r}}_{1\mathrm{PN}} + \ddot{\vec{r}}_{\mathrm{SO}} + \ddot{\vec{r}}_{\mathrm{Q}} + \cdots$$

- Light propagation (Römer delay & Shapiro delay)

$$\Delta_{\mathrm{R}} + \Delta_{\mathrm{S}} = \hat{K}_0 \cdot \vec{r}/c - 2GM/c^3 \ln\left(r - \hat{K}_0 \cdot \vec{r}\right)$$

- Einstein delay

$$\Delta_{\mathrm{E}} = t - T = t - \int \mathrm{d}t \left(1 - GM/c^2 r - v^2/2c^2\right)$$

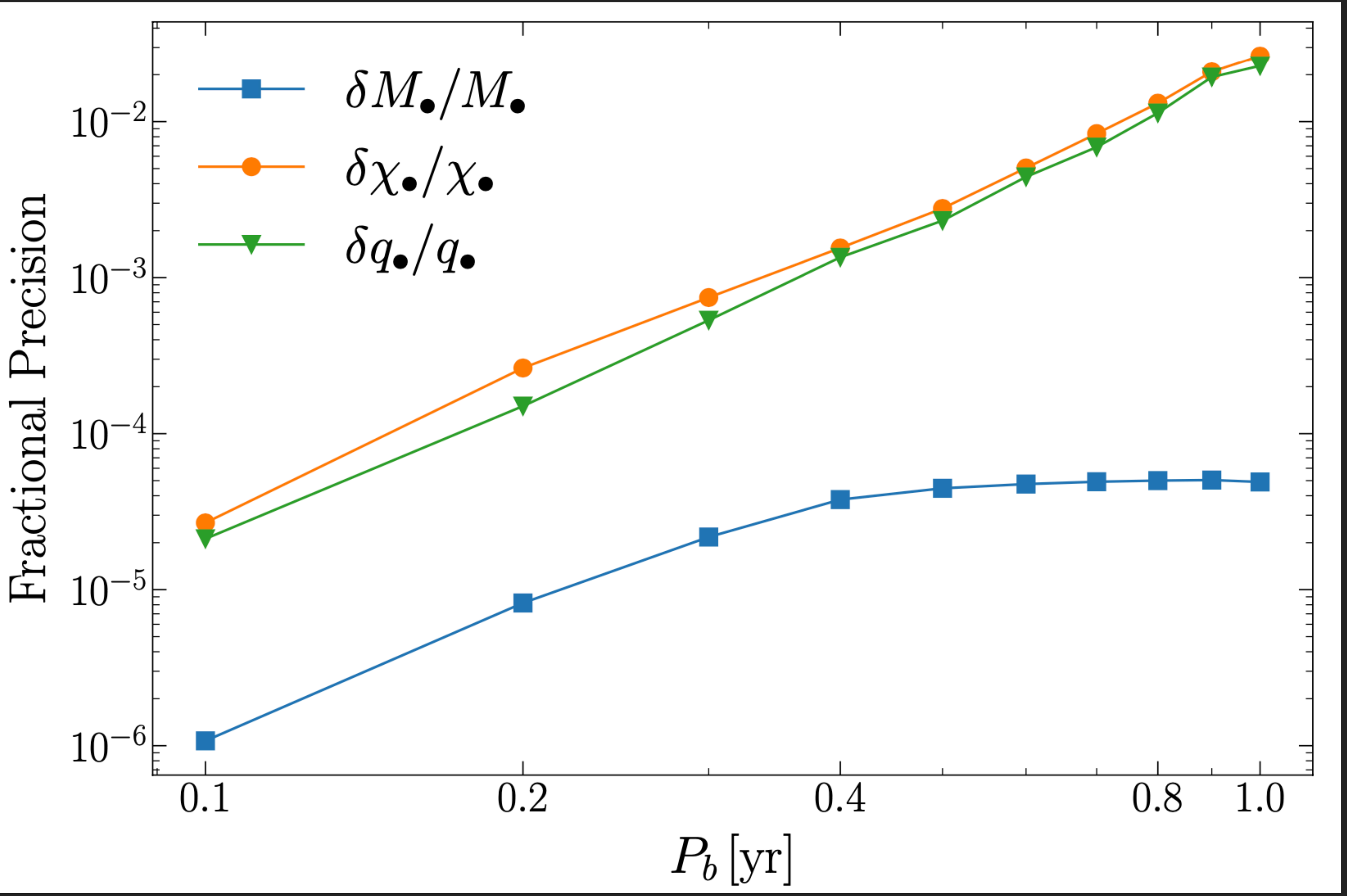


Measuring the spin of Sgr A* with pulsar timing

- $\mathscr{L} = \frac{1}{2} \sum_{a=1}^{N_{\rm TOA}} \frac{R_a(\Theta)^2}{\sigma_{\rm TOA}^2}, R_a(\Theta) = \left[N(t_a^{\rm TOA}; \Theta) - N_a\right]/\nu$

- $F_{\alpha\beta} = \left(C^{-1}\right)_{\alpha\beta} = \frac{\partial^2 \mathscr{L}}{\partial \Theta^\alpha \partial \Theta^\beta}$

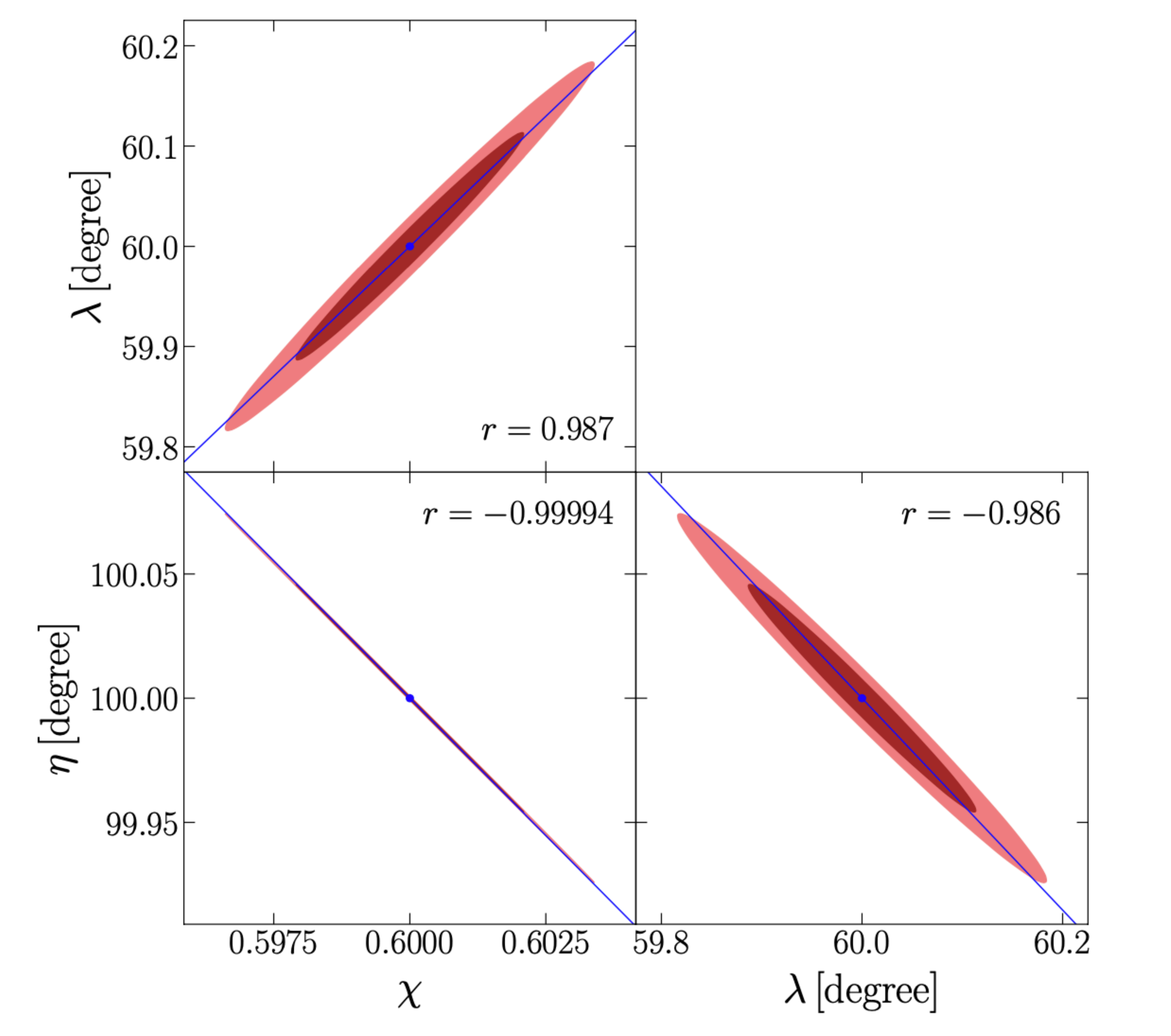
Pulsar with smaller orbital period gives better measurement precision in BH spin parameters



Hu, Shao & Zhang 2023

Degeneracies among spin parameters

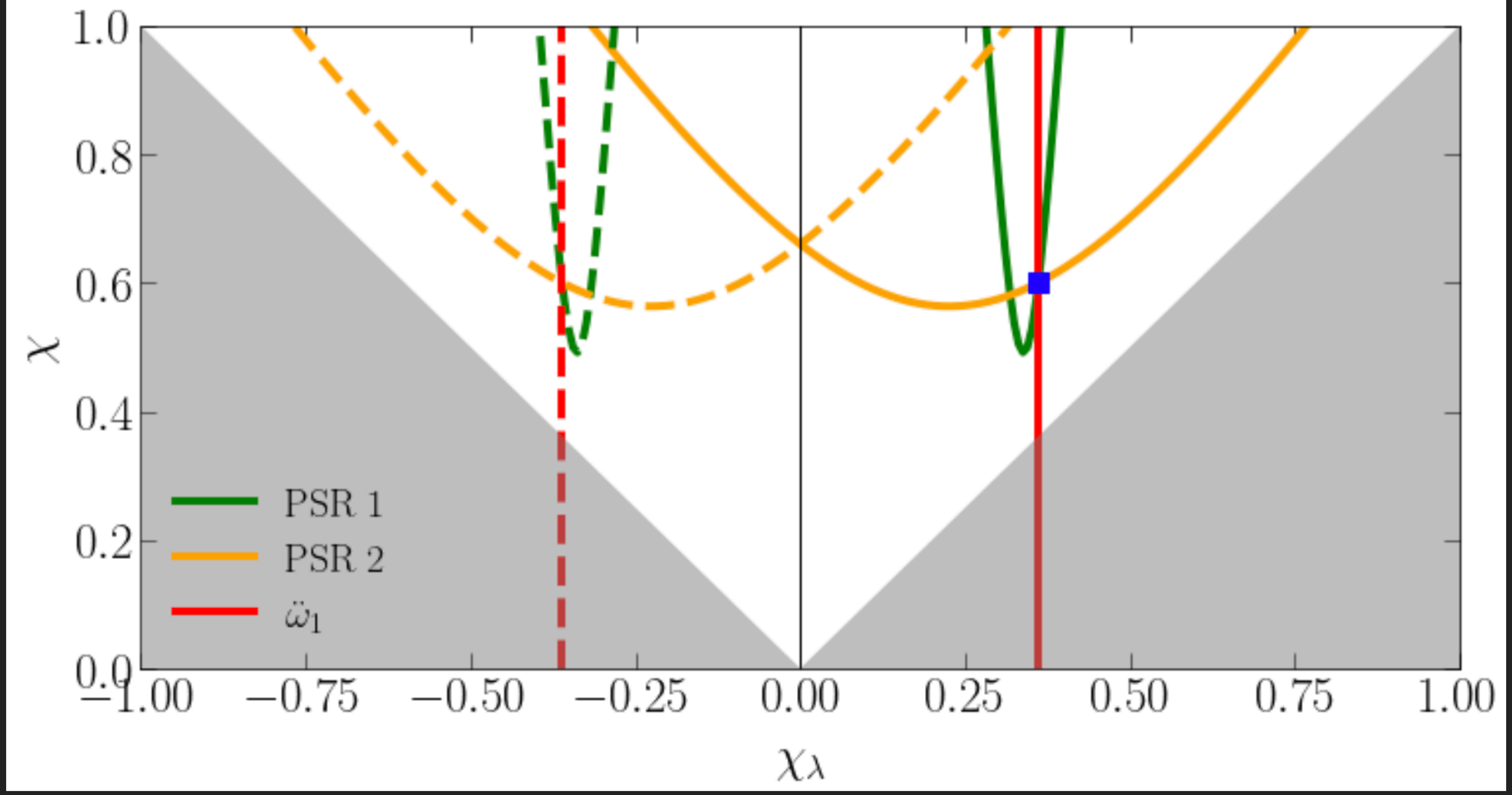
- 2 leading order observables $\{\dot{\omega}, \dot{x}\} < 3$ spin parameters $\{\chi, \lambda, \eta\}$
- Require the measurements of the second order derivatives/periodic effects/proper motion of the pulsar
- Combing the timing of another pulsar



Measuring the spin with two pulsars

- Timing of each pulsar's $\{\dot{\omega}, \dot{x}\}$ gives a curve in the $\chi - \chi_\lambda$ plane

$$\frac{\mathscr{X}_1^2}{s_i^2(1-s_i^2)} + \frac{\mathscr{W}_1^2}{s_i^2(1+3s_i^2)} = \chi^2 - \frac{1+3s_i^2}{(1-3s_i^2)^2}\left(x_\lambda + \frac{3\mathscr{W}_1 c_i}{1+3s_i^2}\right)^2$$



$$\mathscr{X}_1 \equiv -\dot{x} s_i^2 (x\hat{\Omega})^{-1}$$

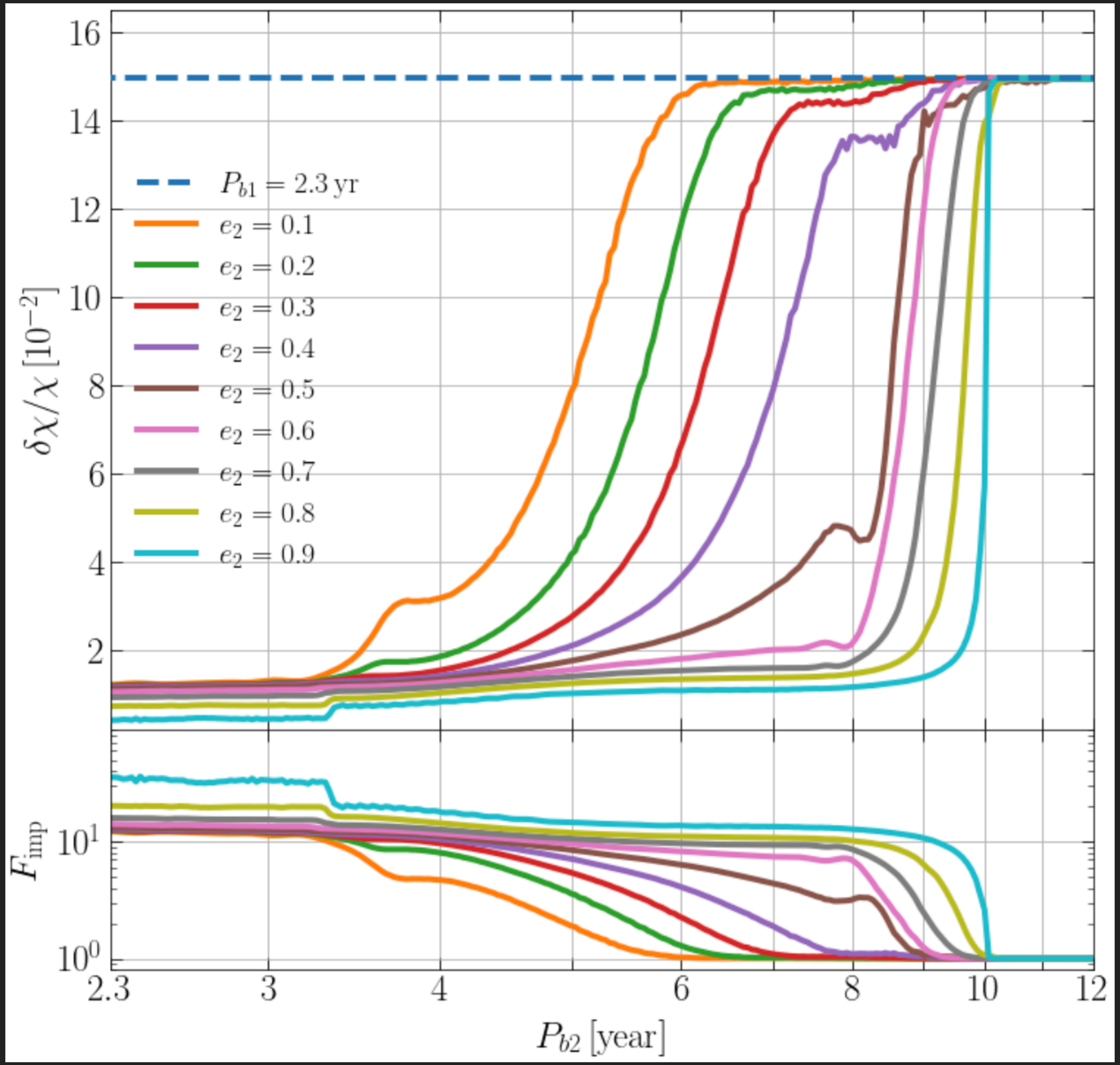
$$\mathscr{W}_1 \equiv (\dot{\omega} - \dot{\omega}_M) s_i^2 \hat{\Omega}^{-1}$$

In general, there can exist at most eight different solutions in the global spin parameter space due to the $i \leftrightarrow \pi - i$ ambiguity

Hu & Shao in preparation

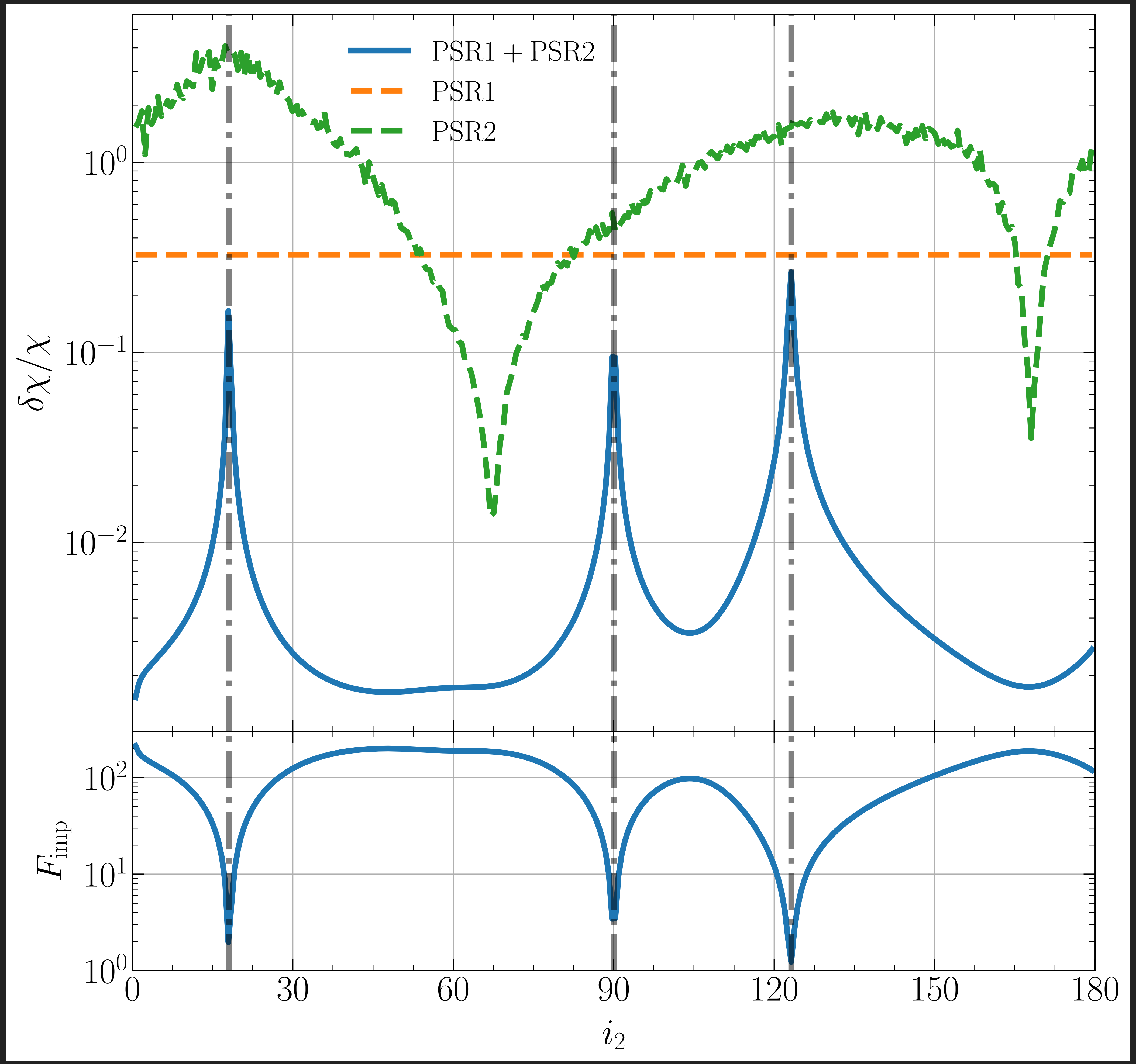
Measuring the spin with two pulsars

- Combing two pulsars with configurations that each of them cannot measure the spin of Sgr A* well individually
- $P_{b1} = 2.3\,\mathrm{yr}\,,\, e_1 = 0.8$
- An improvement factor of 10 can be reached after observing one periastron passage of the secondary pulsar



Hu & Shao in preparation

Measuring the spin with two pulsars



- The breaking of the degeneracy mainly dependents on the orbital inclinations of the two pulsars

$$\frac{\mathscr{X}_1^2}{s_i^2(1-s_i^2)} + \frac{\mathscr{W}_1^2}{s_i^2(1+3s_i^2)} = \chi^2 - \frac{1+3s_i^2}{(1-3s_i^2)^2}\left(x_\lambda + \frac{3\mathscr{W}_1 c_i}{1+3s_i^2}\right)^2$$

Hu & Shao in preparation

Summary

- Timing pulsars orbiting around Sgr A* can provide precise tests of the cosmic censorship conjecture and the no-hair theorem
- Combing the timing of two or more pulsars properly can improve the measurement precision of the spin of Sgr A* for one or two orders of magnitude

Thank you !!!