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Neutron Stars in Bumblebee Theory

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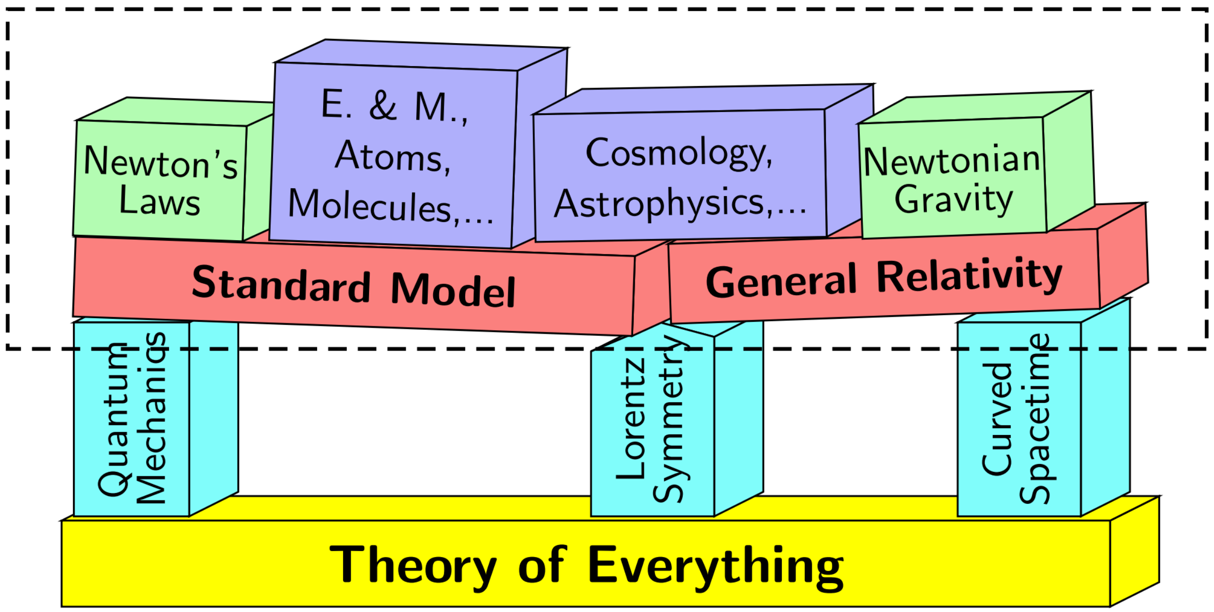
Bumblebee Theory

[V. Kostelecký (2004)]

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{2\kappa} + \frac{\xi}{2\kappa} B^\mu B^\nu R_{\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} - V \right) + S_m[g_{\mu\nu}, \Psi_m]$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \qquad V = V(B^\mu B_\mu - b^2)$$

SME = general description of LV and low energies



[c.f. M. Mewes]

$$\mathcal{L}_{SME} = \mathcal{L}_{GR} + \mathcal{L}_{SM} + \mathcal{L}_{LV}$$

$$\mathcal{L}_{LV} = uR + s^{\mu\nu} R^T_{\mu\nu} + t^{\kappa\lambda\mu\nu} R_{\kappa\lambda\mu\nu}$$

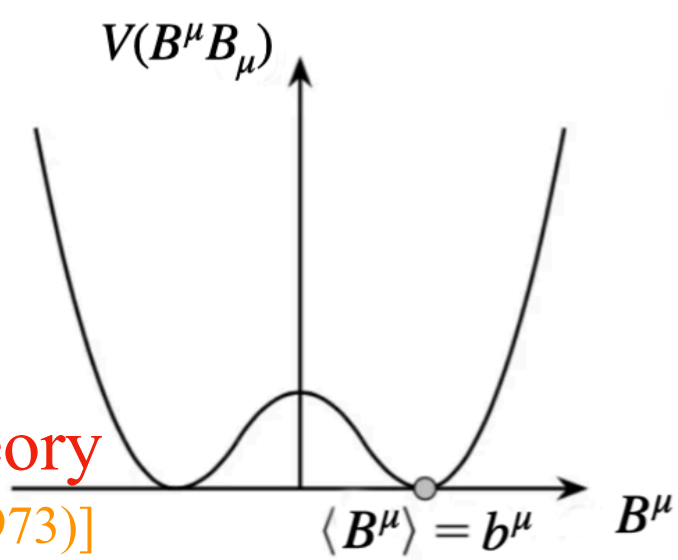
$$u_B = \frac{1}{4} g^{\mu\nu} B_\mu B_\nu \qquad s^{\mu\nu}_B = \xi B^\mu B^\nu - g^{\mu\nu} u_B$$

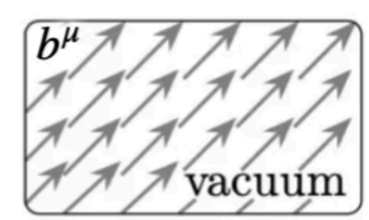
$$V|_{B_\mu = b_\mu} = 0 \qquad \left. \frac{dV}{d(B^\lambda B_\lambda)} \right|_{B_\mu = b_\mu} = 0$$

$$V = \frac{\lambda}{2}(B^\mu B_\nu - b^2)$$

Hellings-Nordtvedt theory

[R. Hellings & K. Nordtvedt (1973)]





EoM & Ansatz

Equations of motion:

$$G_{\mu\nu} = \kappa(T^{\mathrm{m}}_{\mu\nu} + T^{\mathrm{vec}}_{\mu\nu} + T^{\xi}_{\mu\nu})$$

$$\nabla_\nu B^{\mu\nu} - \frac{\xi}{\kappa} R^{\mu\nu} B_\nu = 0$$

Static and spherical ansatz:

$$\mathrm{d}s^2 = -\,\mathrm{e}^{2\nu(r)}\mathrm{d}t^2 + \mathrm{e}^{2\mu(r)}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2)$$

$$b_\mu = (b_t(r), b_r(r), 0, 0)$$

$$\mathrm{e}^{2\mu(r)} \rightarrow \left(1 - \frac{2Gm(r)}{r}\right)^{-1}$$

Two classes of solutions:

$$\text{Class I}: \quad b_r = 0$$

$$\text{Class II}: \quad R_{rr} = 0$$

Reduced geometric unit

$\kappa = 1 \quad G = 1/8\pi$

Integration

At center

$$m = \sum_{i=0}^{\infty} m_i r^i, \quad \nu = \sum_{i=0}^{\infty} \nu_i r^i, \quad b_t = \sum_{i=0}^{\infty} b_i r^i,$$

$$p = \sum_{i=0}^{\infty} p_i r^i, \quad \rho = \sum_{i=0}^{\infty} \rho_i r^i .$$

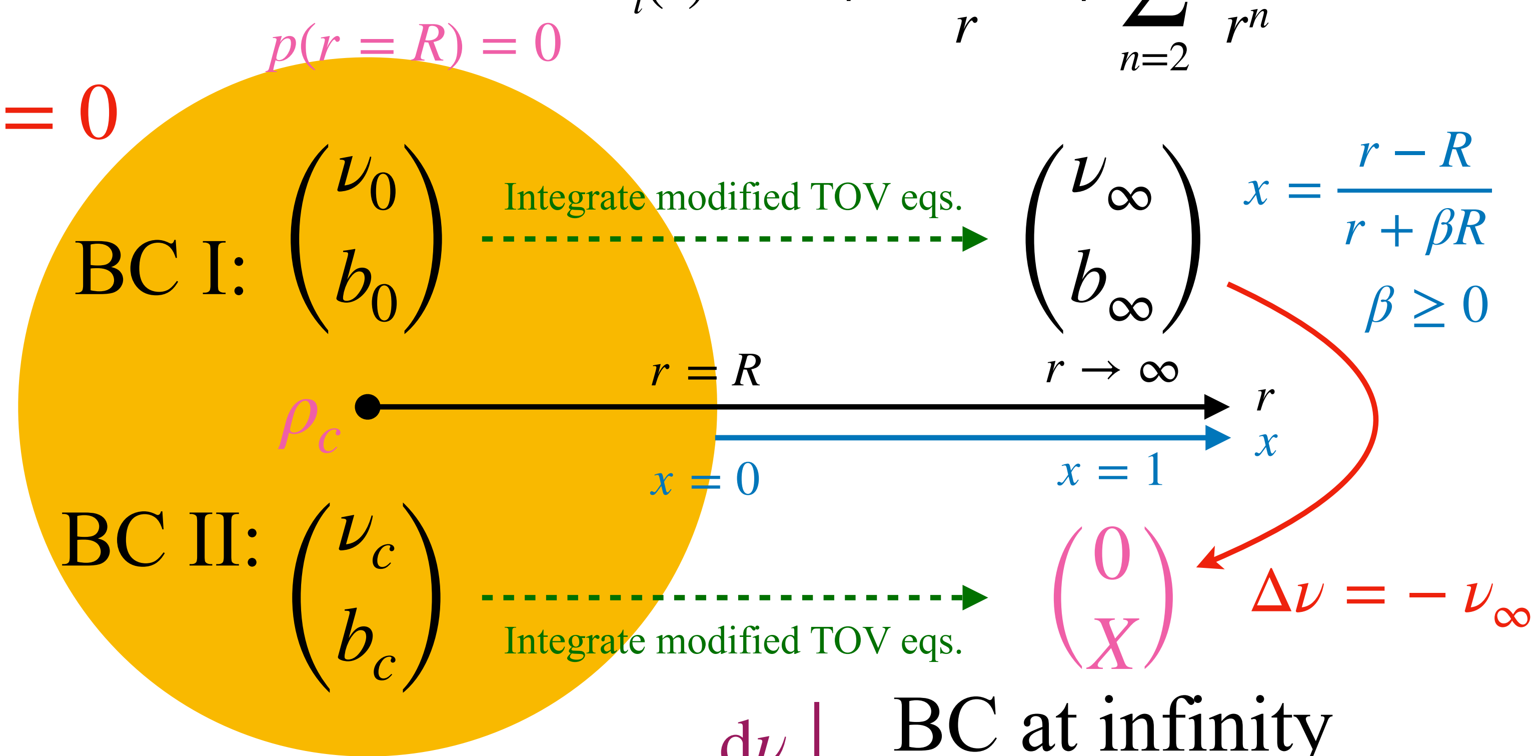
$\nu_1 = 0 \quad b_1 = 0 \quad \rho_1 = p_1 = 0$

At infinity

$$\nu(r) = \sum_{n=1}^{\infty} \frac{\nu_{-n}}{r^n},$$

$$m(r) = M + \sum_{n=1}^{\infty} \frac{m_{-n}}{r^n},$$

$$b_t(r) = X + \frac{2\sqrt{\pi}Q}{r} + \sum_{n=2}^{\infty} \frac{b_{-n}}{r^n} .$$



Time transformation

$$t \to e^{-\Delta\nu} t$$

$$\mathbf{g} : g_{tt} \to e^{2\Delta\nu} g_{tt}$$

$$\mathbf{b} : b_t \to e^{\Delta\nu} b_b$$

System is invariant under

$\nu \to \nu + \Delta\nu, \ b_t \to e^{\Delta\nu} b_t$

$$M = 8\pi(1+\beta)R \left.\frac{d\nu}{dx}\right|_{x=1}$$

$$Q = -\frac{1+\beta}{2\sqrt{\pi}} R \left.\frac{db_t}{dx}\right|_{x=1}$$

$M \to M$

$X \to e^{\Delta\nu} X$

$Q \to e^{\Delta\nu} Q$

Back to GR

$$m_3 = \frac{4\pi}{3}\frac{3p_0 b_0^2(2\kappa-\xi)\xi + 2e^{2\nu_0}\kappa\rho_0}{2e^{2\nu_0}\kappa + \xi(\xi-2\kappa)b_0^2},$$
$$b_2 = \frac{\kappa}{6}\frac{e^{2\nu_0}(3p_0+\rho_0)}{2e^{2\nu_0}\kappa + \xi(\xi-2\kappa)b_0^2}\xi b_0,$$
$$\nu_2 = \frac{\kappa^2}{6}\frac{e^{2\nu_0}(3p_0+\rho_0)}{2e^{2\nu_0}\kappa + \xi(\xi-2\kappa)b_0^2},$$
$$p_2 = -\frac{\kappa^2}{6}\frac{e^{2\nu_0}(3p_0+\rho_0)(p_0+\rho_0)}{2e^{2\nu_0}\kappa + \xi(\xi-2\kappa)b_0^2}.$$

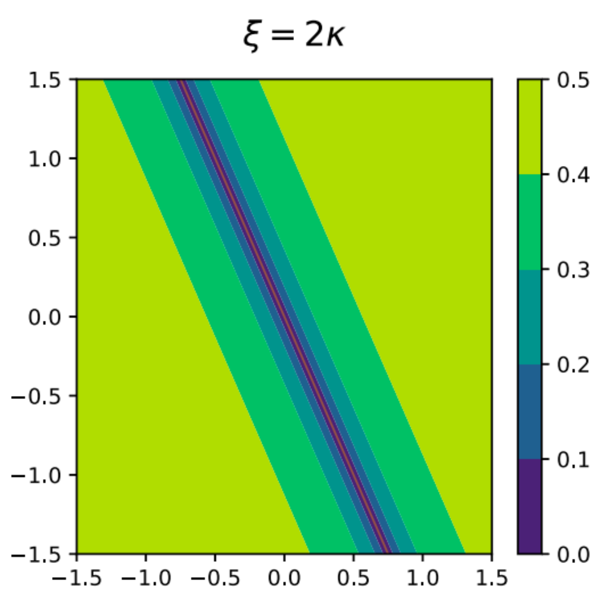
$b_0 = 0$ $\Rightarrow$ $\xi = 0, 2\kappa$

$$m_3 = \frac{4\pi}{3}\rho_0$$
$$\nu_2 = \frac{2\pi}{3}G(3p_0+\rho_0)$$
$$p_2 = -\frac{2\pi}{3}G(3p_0+\rho_0)(p_0+\rho_0)$$
$$b_2 = \frac{1}{6}(3p_0+\rho_0)b_0$$

Einstein-Maxwell theory ($\xi = 0$)

$$b_t(r) = b_0 + \int_0^r \frac{q_0}{r'^2} e^{\mu(r')+\nu(r')}\mathrm{d}r'$$

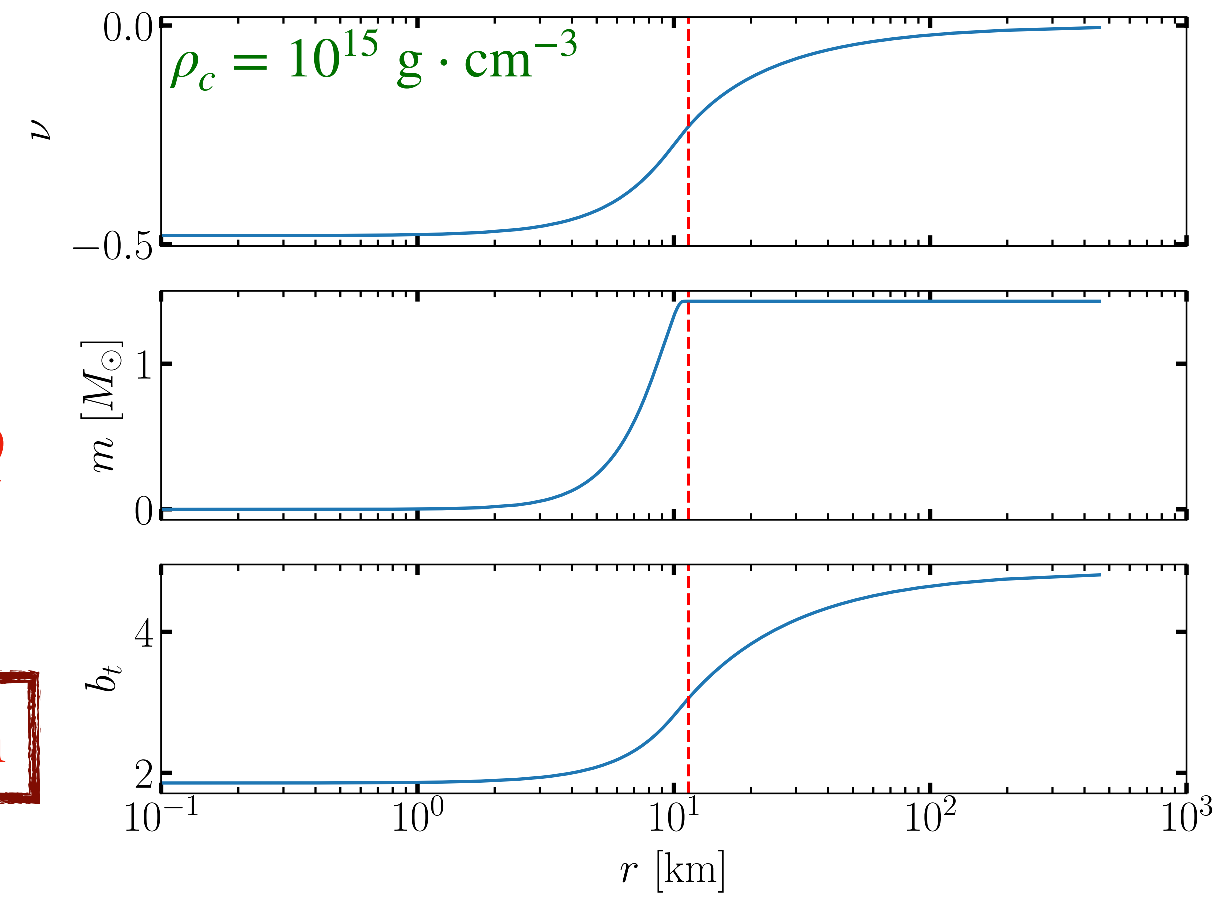
Stealth Schwarzschild ($\xi = 2\kappa$)



$GMX + \sqrt{\pi}Q = 0$

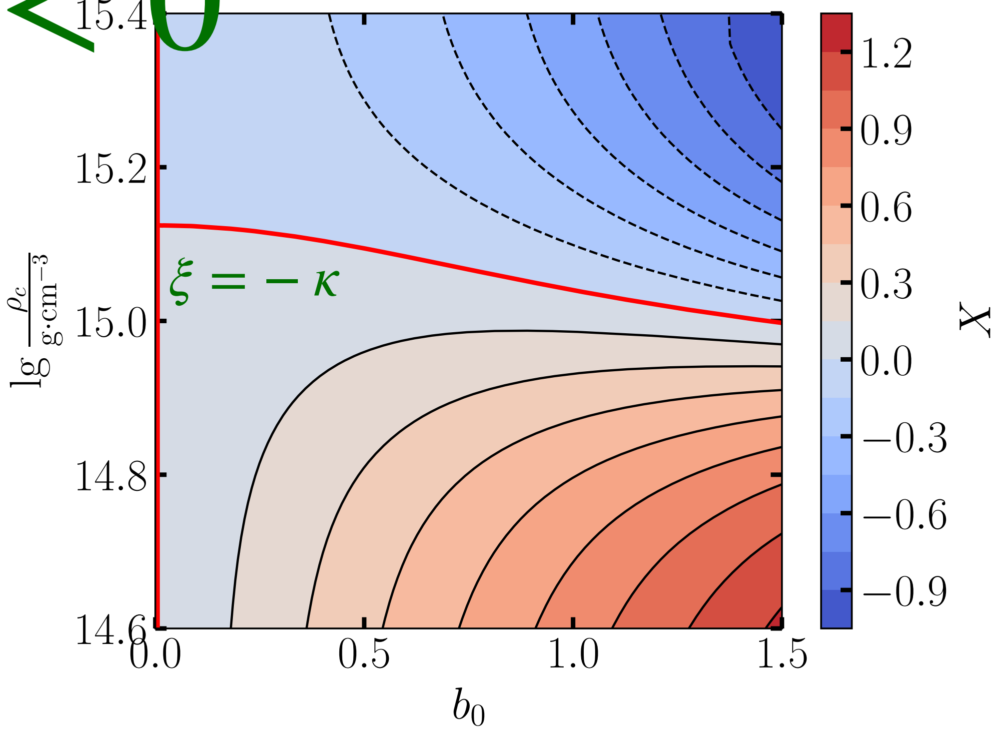
Only 2 of M, X, Q are independent!

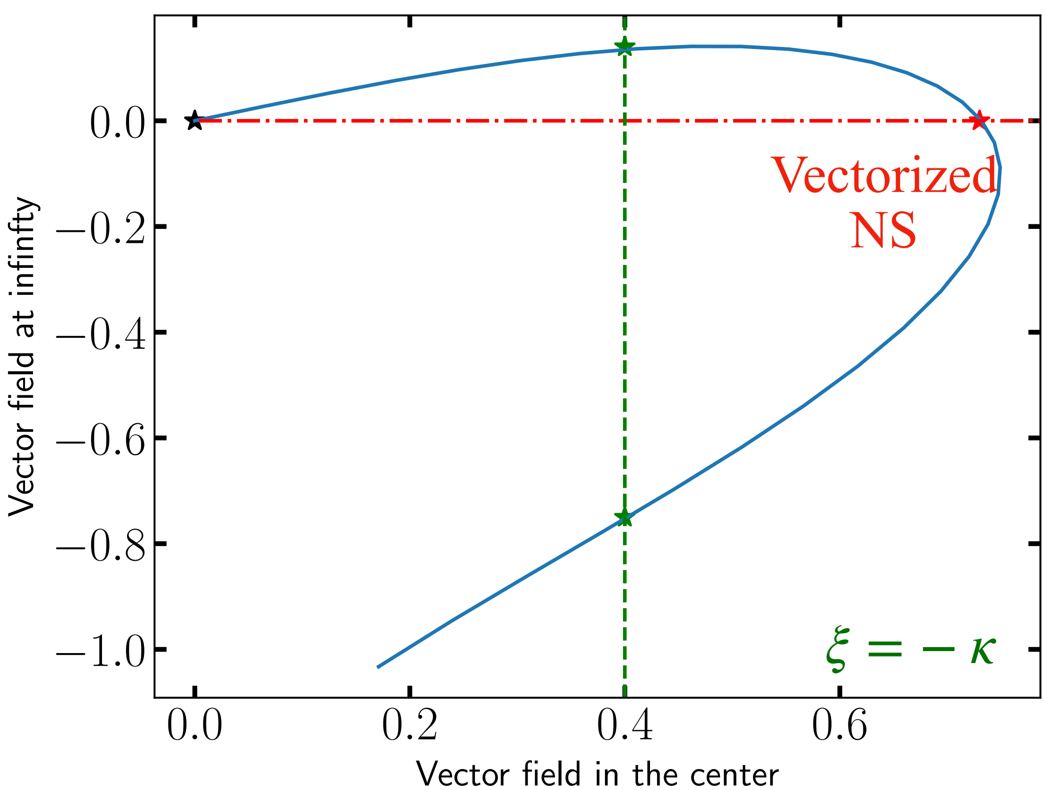
Brikhoff Theorem

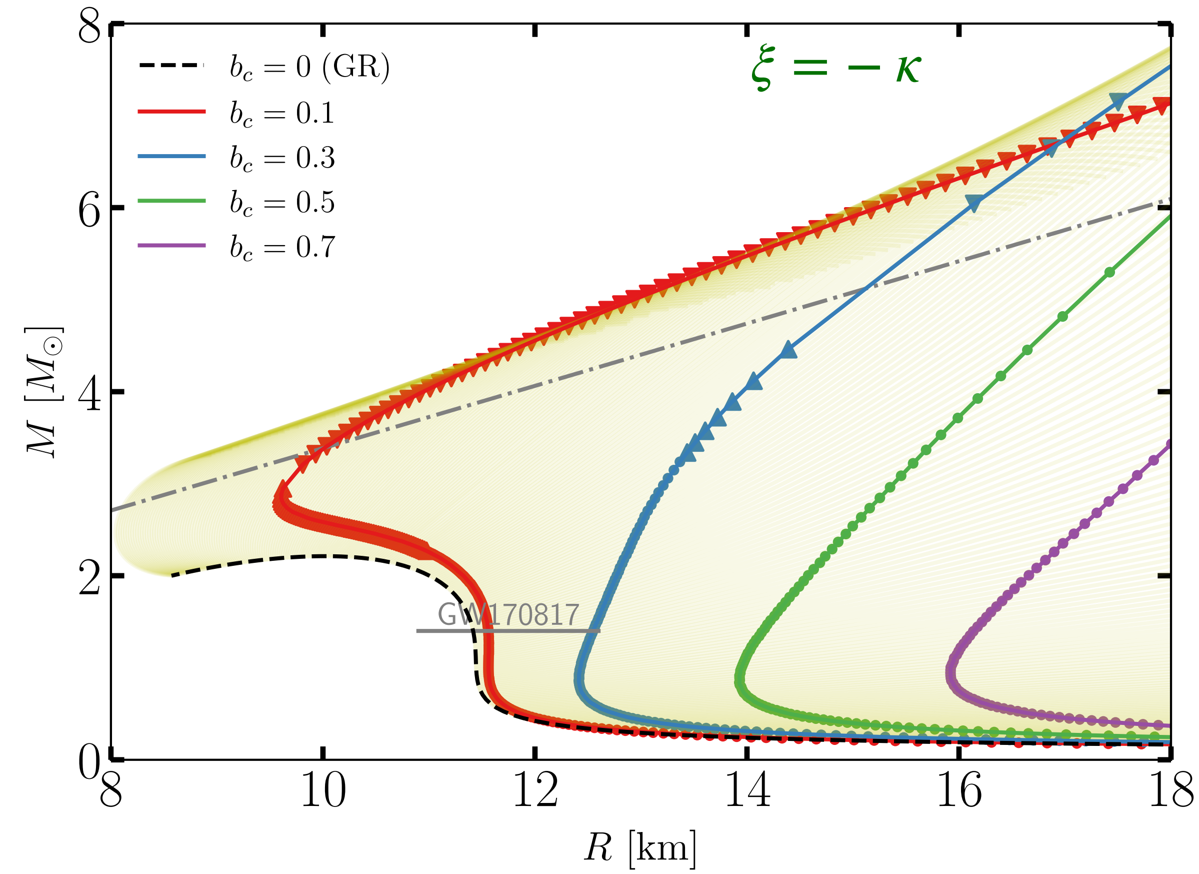


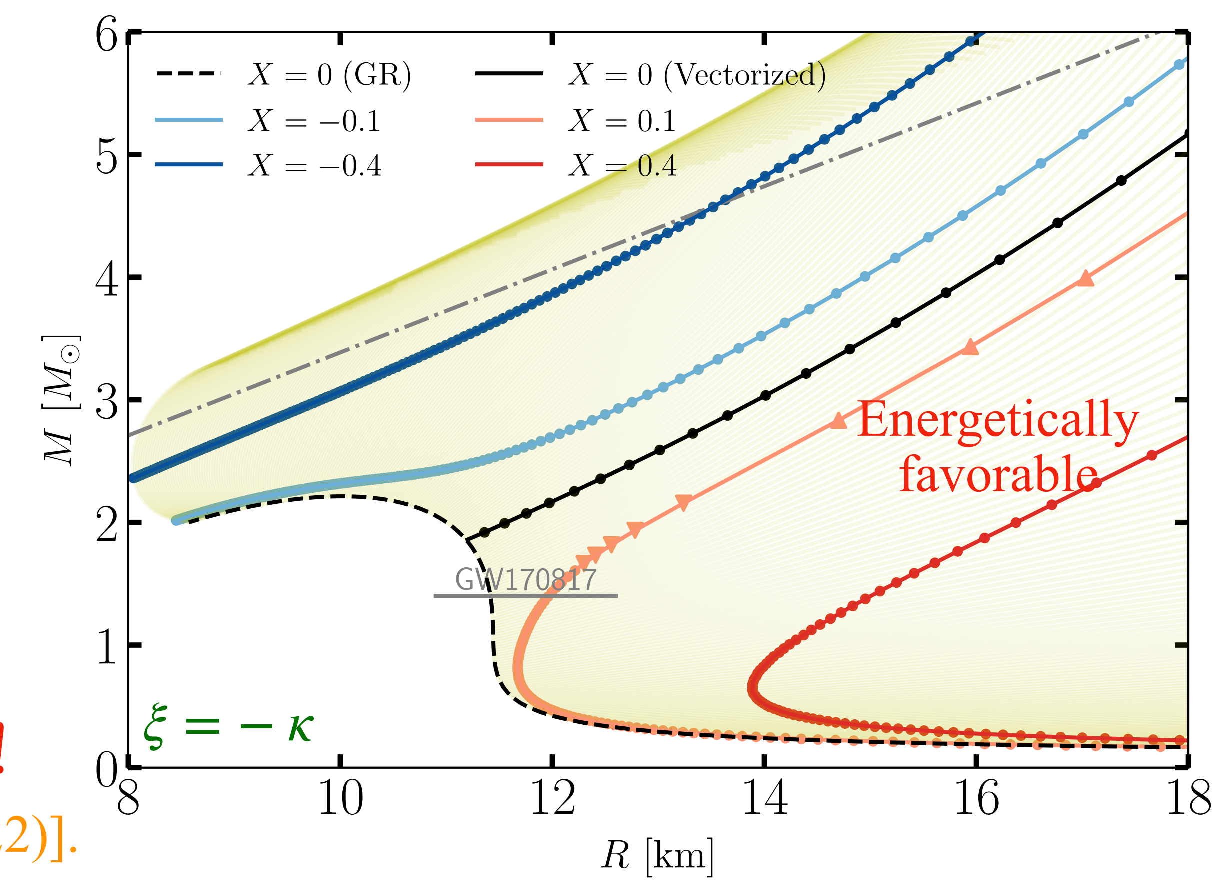
[R. Xu, D. Liang & L. Shao (2023)]

$\xi < 0$





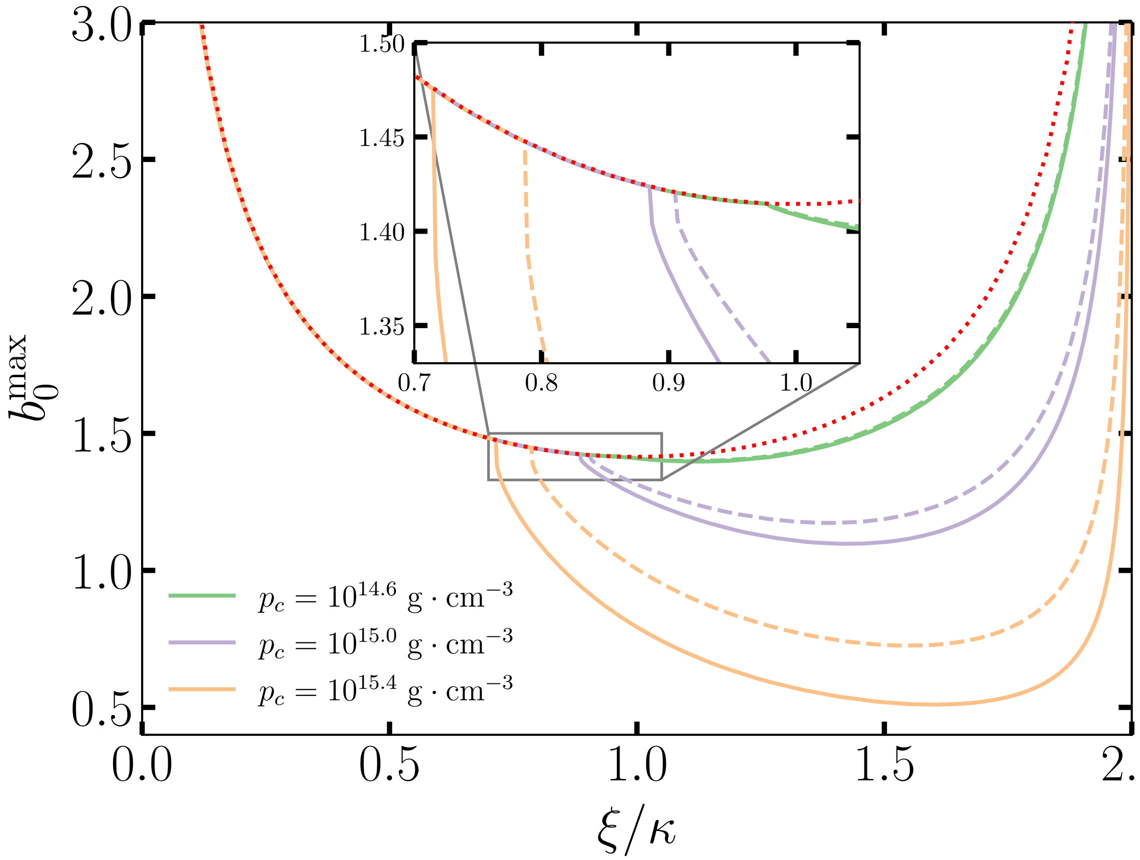




No spontaneous vectorization so far!

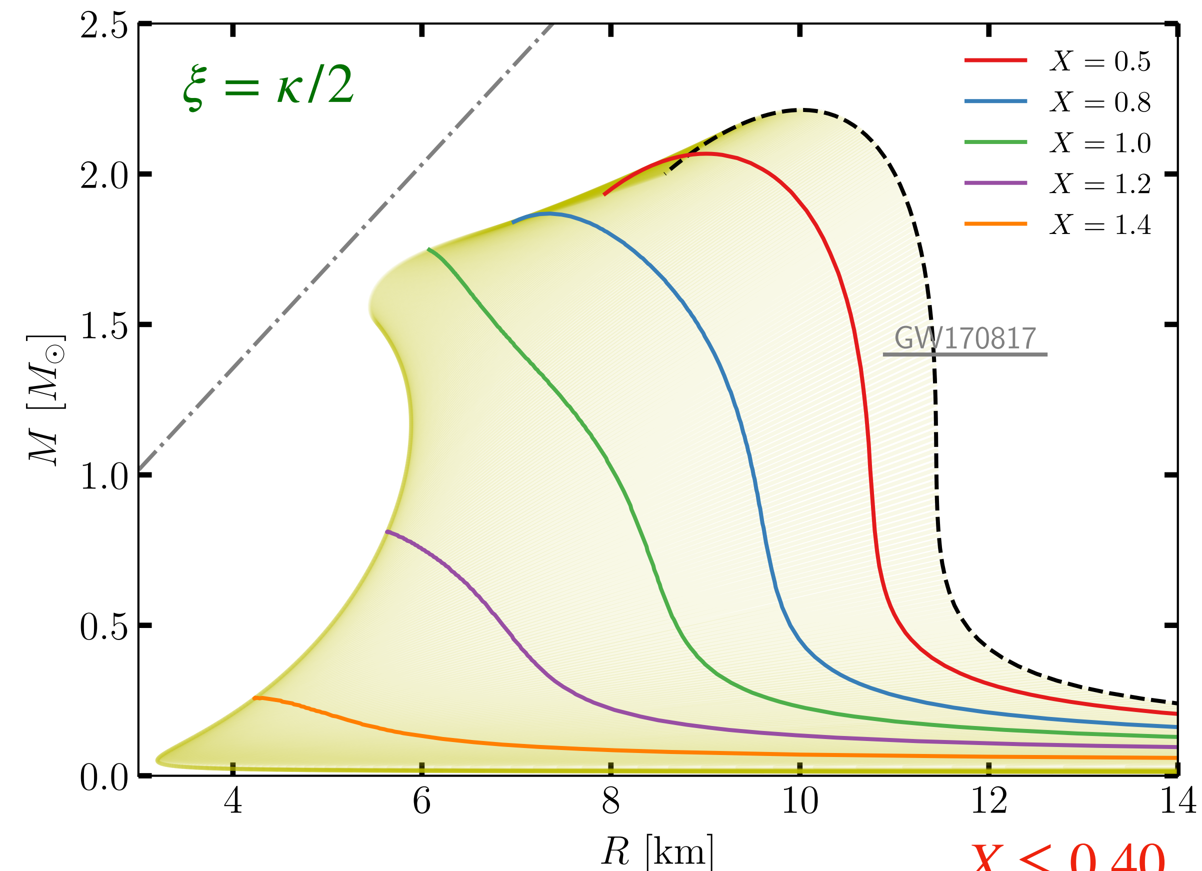
See [L. Annulli *et al*. (2019)] & [H. Silva *et al*. (2022)].

$0 < \xi < 2\kappa$

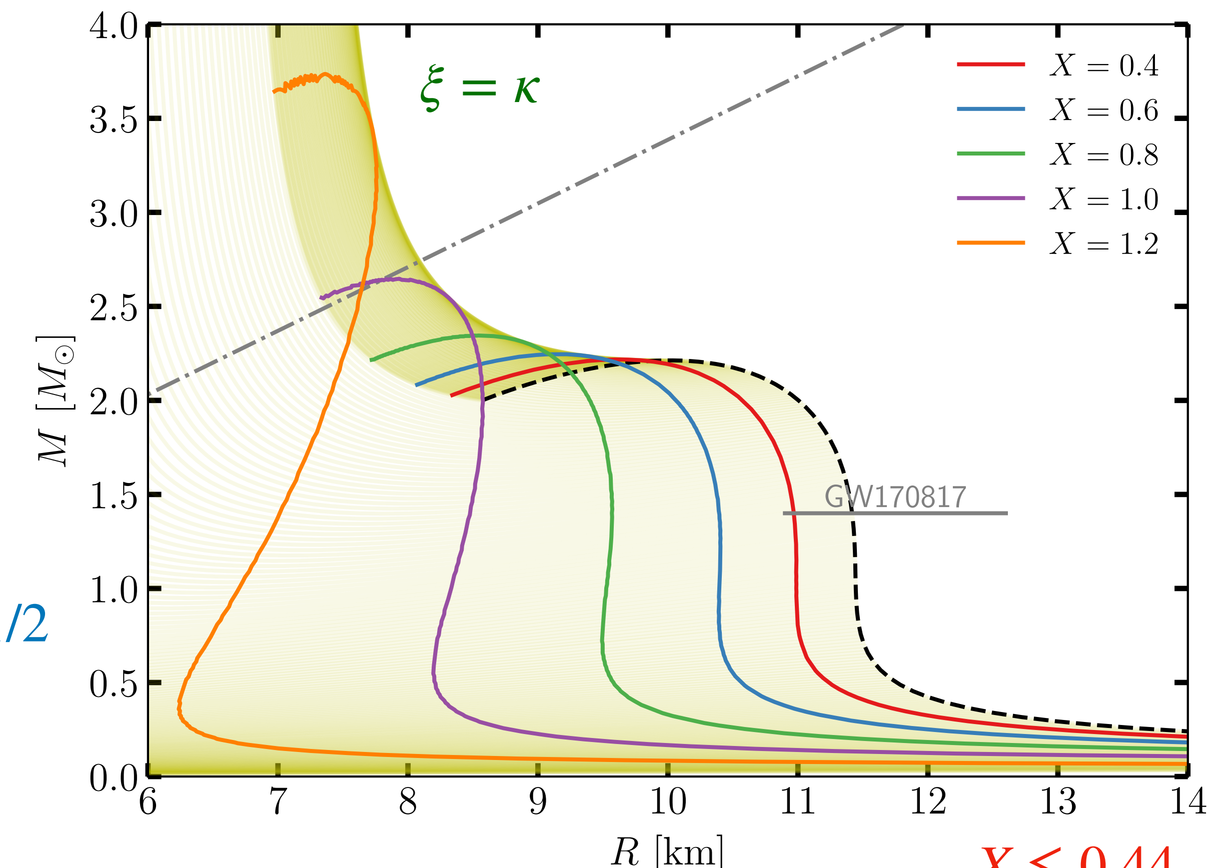


- Asymptotic behavior at center $\sim [2\kappa + \xi(\xi - 2\kappa)b_0^2]^{-1}$
- Asymptotic behavior at center $\sim [2\kappa + \xi(\xi - 2\kappa)X^2]^{-1}$

$$M, Q \to \infty \text{ as } X \to \left(\frac{2\kappa}{\xi(2\kappa - \xi)}\right)^{1/2}$$

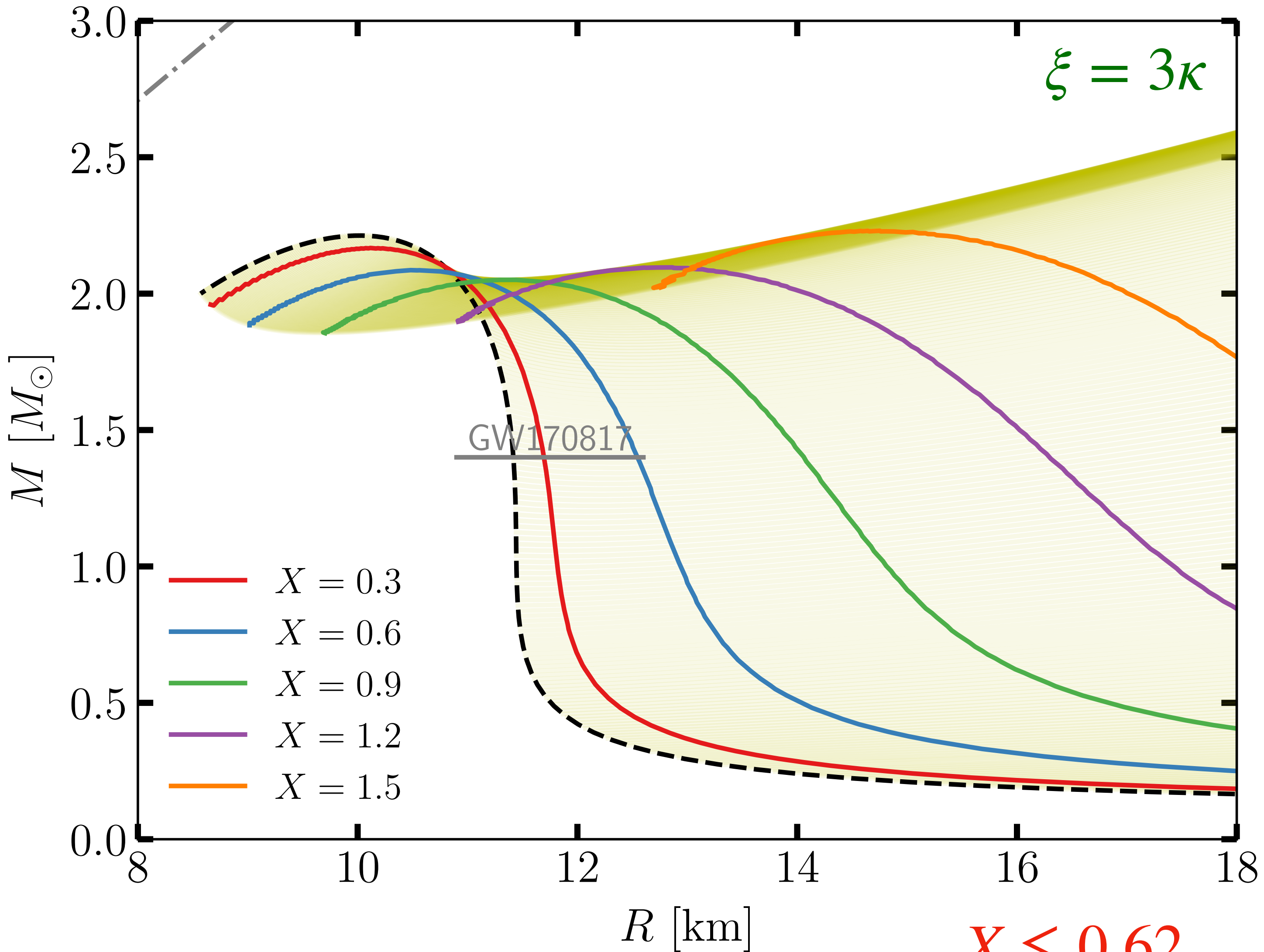


$X \lesssim 0.40$



$X \lesssim 0.44$

$\xi > 2\kappa$



$X \lesssim 0.62$

We are able to get both large radius and large mass in this range of non-minimal coupling without divergence.

Summary

- We systematically studied NSs in bumblebee theory
  - Differences between the well-studied scalar-tensor theory
- Some interesting cases
  - Large compactness ($\mathscr{C} > 0.5$) generally
  - Vectorized NSs when $\xi < 0$
  - Extremely massive (ADM mass) star with a relative small radius
  - Stealth Schwarzschild NS solutions
- Something to do
  - Perturbations on static spherical solutions
    - ✓ Tidal deformability — more constraints
    - ✓ Time-dependent perturbation — stability
  - Different possible potentials in the SME framework

Thank you!