

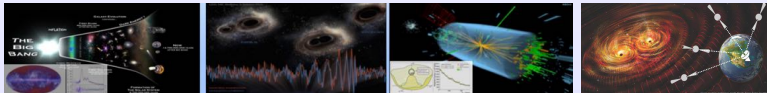
Deformed Hellings-Downs curve and angular correlation from Ultralight Dark Matter

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National Astronomical Observatories,
Chinese Academy of Sciences

Phys.Rev.D 110 (2024) 044052 by R.G. Cai, J.R. Zhang & Y.L. Zhang
Phys.Rev.D 106 (2022) 066006, by S.C Sun, X.Y. Yang & Y.L. Zhang

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Motivation: new physics in ultra-low energy

GW & ULDM

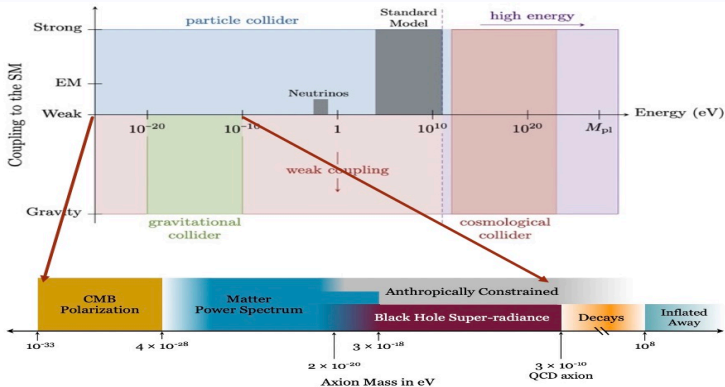
Ultralight DM

Axion-like DM

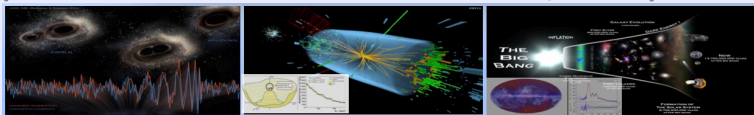
GW and DM

spin-0 DM

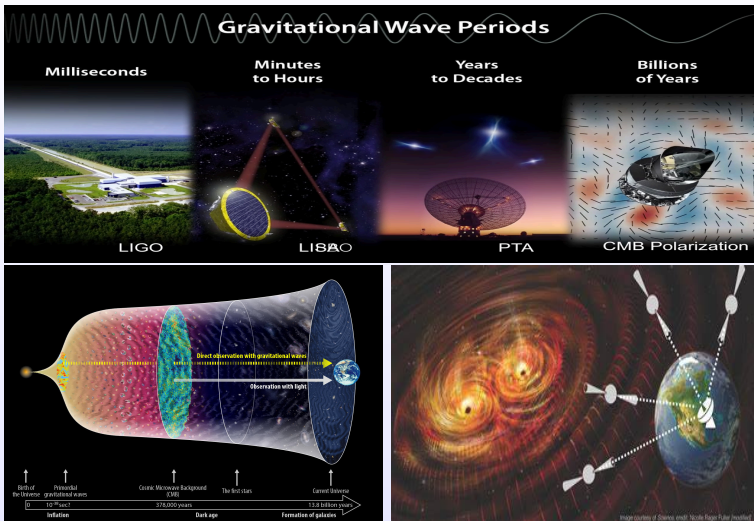
spin-2 DM



[cf. Baumann-Chia-Porto-Stout, Gravitational Collider Physics, 2019]



多波段引力波探测



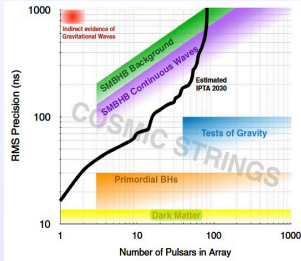
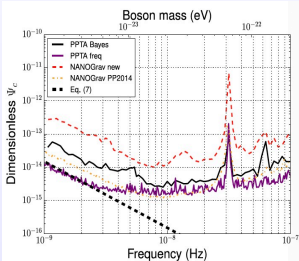
[cf. LISA/ Gravitational Waves Initiative]



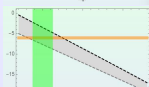
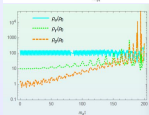
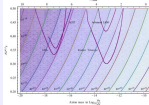
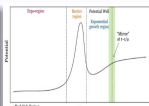
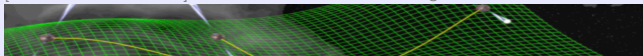
超轻暗物质和多波段引力波探测

类轴子型超轻暗物质模型和参数空间

- PTA & SKA 波段: 超轻 Fuzzy 暗物质 ($\sim$ nHz)
- LISA-Taiji-Tianqin & Ligo-Virgo-Kagra ($\sim$ mHz - kHz)
- 轴子星 & 快速射电暴 & 引力波暴等 ($\sim$ GHz)



[cf. X. J. Zhu et al, 2018] Parkes PTA constraints on ultralight scalar-field dark matter



Axion annihilation and Stochastic GWs

GW & ULDM

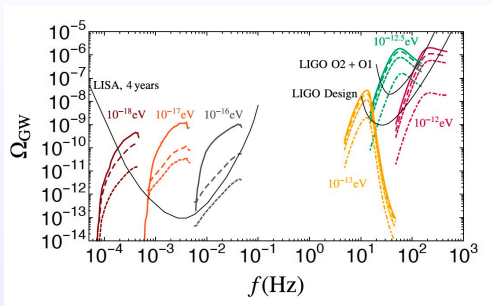
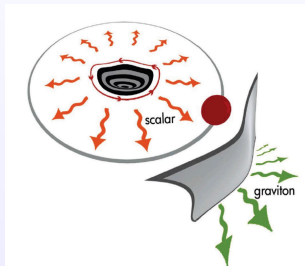
Ultralight DM

Axion-like DM

GW and DM

spin-0 DM

spin-2 DM



- Axion annihilation $\vartheta + \vartheta \rightarrow hh$, Strain $h \sim 10^{-21} - 10^{-32}$.
- Stochastic GW [cf. Brito-Cardoso-Pani, Superradiance 2020]

Energy level transition and Monochromatic GW

GW & ULDM

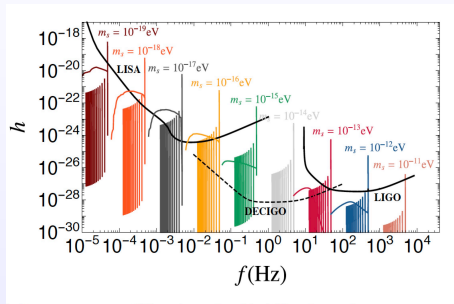
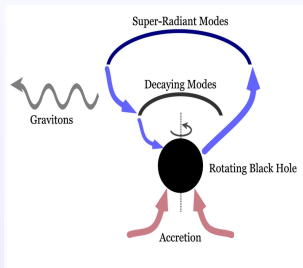
Ultralight DM

Axion-like DM

GW and DM

spin-0 DM

spin-2 DM



- Energy transition $\vartheta^+ \rightarrow \vartheta^- + h$, Strain $h \sim 10^{-19} - 10^{-27}$
- Monochromatic GW [cf. Brito-Cardoso-Pani, Superradiance 2020]

Ultralight dark matter and gravitational wave

GW & ULDM

Ultralight DM
Axion-like DM

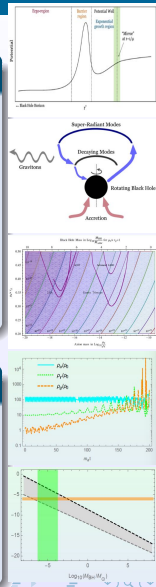
GW and DM
spin-0 DM
spin-2 DM

GW & EM signals from axion like DM

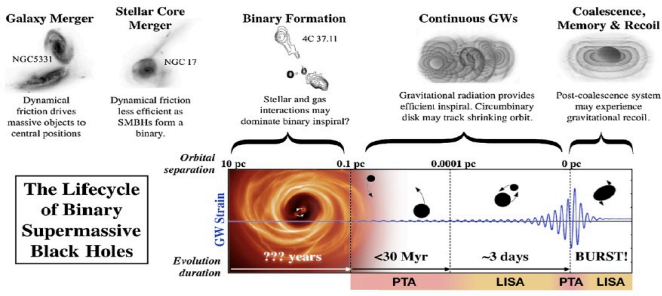
- Axion annihilation $\vartheta + \vartheta \rightarrow h$ (Stochastic GW)
Energy transition $\vartheta^+ \rightarrow \vartheta^- + h$ (Monochromatic)
- Superradiance $\alpha \equiv \frac{R_{BH}}{\lambda_{\vartheta}} \simeq \left(\frac{M_{BH}}{M_{\odot}} \right) \left(\frac{m_{\vartheta}}{10^{-10} \text{eV}} \right)$
- Fast Radio Burst from Axion $\sim \vartheta \tilde{F}\tilde{F} (\vartheta \rightarrow \gamma\gamma)$
- GW burst from Axion $\sim \vartheta R\tilde{R} (\vartheta \rightarrow hh)$

GW detection and Ultra-light DM

- Tabletop exp: QCD axion & GW burst ($\sim \text{GHz}$)
- LISA & LVK: BH Superradiance ($\sim \text{mHz} - \text{kHz}$)
- PTA & SKA : Ultra-light DM ($\sim \text{nHz}$)
- LSS & CMB : DE & Modified gravity ($\sim \text{nnHz}$)



脉冲星测时残差和暗物质效应



[cf. Burke-Spolaor, *et al.*, “The astrophysics of nanohertz gravitational waves”]

Cold Dark Matter induced Pulsar timing residual

- Shapiro delay: Propagation through a distribution of dark-matter sub-structure (integrated Sachs–Wolfe effect)
- Doppler delay: Acceleration of the Earth or pulsar caused by the nearby passage of dark matter clump.

The timing residuals of nHz gravitational waves

GW & ULDM

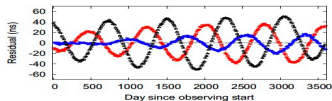
Ultralight DM

Axion-like DM

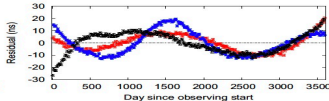
GW and DM

spin-0 DM

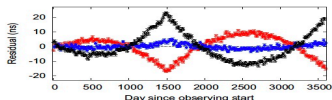
spin-2 DM



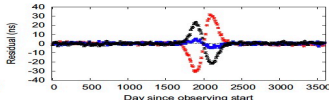
(a) Continuous wave



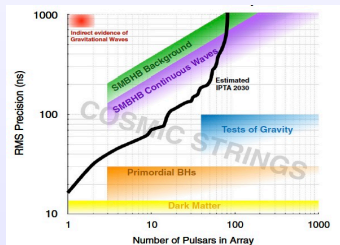
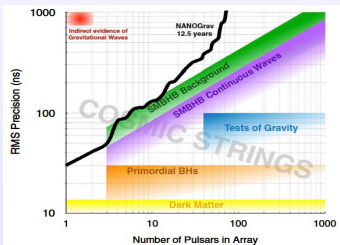
(b) Background



(c) Burst with Memory



(d) Burst

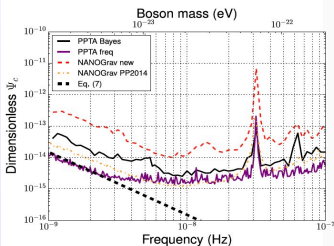


[cf. Burke-Spolaor, *et al.*, "The astrophysics of nanohertz gravitational waves"]

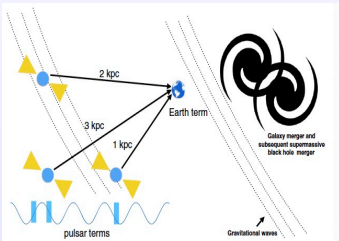
Timing residuals & Ultralight dark matter

PTA & Ultra light dark matter

- Fuzzy like DM $f_c = \frac{m}{\pi} \simeq 4.8 \text{ nHz} \left(\frac{m}{10^{-23} \text{ eV}} \right)$
- Coherence length: $\lambda_{\text{dB}} = \frac{2\pi\hbar}{mv} \simeq 4 \text{ kpc} \left(\frac{10^{-23} \text{ eV}}{m} \right) \left(\frac{10^{-3}}{v} \right)$
- Pulsar timing residuals: $R_c(f) = \frac{1}{\sqrt{3}} \frac{h_c(f)}{2\pi f} \left(\frac{f_s}{f} \right)^{1/2}$



[cf. X. Xue, X. J. Zhu et al. 2018] & [cf. Burke-Spolaor, et al. 2019]



Oscillation of fuzzy dark matter

GW & ULDM

Ultralight DM

Axion-like DM

GW and DM

spin-0 DM

spin-2 DM

DM oscillation induced time residual

- Metric: $ds^2 = -(1 + 2\Phi) dt^2 + [(1 - 2\Psi) \delta_{ij} + h_{ij}] dx^i dx^j$.
- e.g. the scalar field $\phi(x, t) = \phi(x) \cos[mt + \theta_0(x)]$,
- Oscillating potential $\Psi \simeq \bar{\Psi}(x) + \Psi_\phi \cos[2(mt + \theta_0(x))]$
- Doppler effect: $z_\phi(t) \equiv \frac{\omega_0 - \omega_\phi(t)}{\omega_0} \simeq \Psi(x_\phi, t_\phi) - \Psi(x_0, t_0)$.
- Timing residual in the pulse $R_\phi(t) = \int_0^t z_\phi(t') dt'$
- **Strain** $h_\phi = 2\sqrt{3} \Psi_\phi = \frac{\sqrt{3}}{4M_{pl}^2} \frac{\rho_\phi}{m^2} \simeq 5.2 \times 10^{-17} \alpha_0 \left(\frac{f_{yr}}{f}\right)^2$,
- **GW Timing residual** $R_c(f) \equiv \sqrt{\frac{S_c(f)}{T_s}} = \frac{1}{\sqrt{3}} \frac{h_c(f)}{2\pi f} \left(\frac{f_s}{f}\right)^{1/2}$

[cf. Burke-Spolaor, "Pulsar timing signal from ultralight scalar DM" JCAP(2014)]

Pulsar timing constraints on ULDM

GW & ULDM

Ultralight DM

Axion-like DM

GW and DM

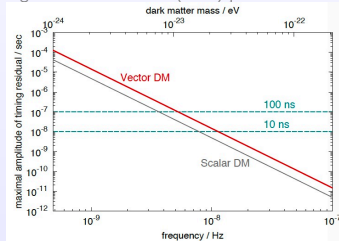
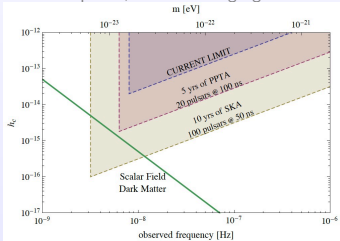
spin-0 DM

spin-2 DM

DM oscillation induced time residual

- Spin-0: massive scalar field $\mathcal{L}_{(0)} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2$
- Spin-1: massive vector field $\mathcal{L}_{(1)} = -\frac{1}{4}F^2 - \frac{1}{4}m^2A^2$

[cf. Burke-Spolaor, "Pulsar timing signal from ultralight scalar DM" JCAP(2014)]



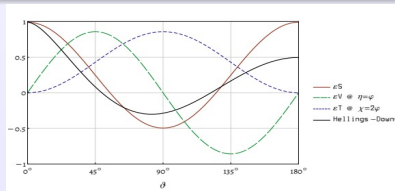
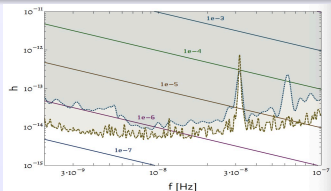
[cf. Nomura-Itoy-Soda, "Pulsar timing residual induced by ultralight vector DM" PRD(2020)]

Pulsar timing constraints on spin-2 ULDM

spin-2 ultralight fields

- Spin-2: massive tensor field(Fierz-Pauli): Bi-metric gravity,

$$\mathcal{L}_{(2)} = \frac{1}{2} M_{\mu\nu} \mathcal{E}^{\mu\nu\rho\sigma} M_{\rho\sigma} - \frac{1}{4} m^2 (M_{\mu\nu} M^{\mu\nu} - M^2)$$
- The oscillating solution $M_{ij} = \mathcal{M} \cos [mt + \theta_2(x)] \varepsilon_{ij}$
- Effective metric perturbations: $\tilde{g}_{ij} = \delta_{ij} + \frac{\alpha_2}{M_{pl}} M_{ij}$
- The redshift $z(t) = \frac{\omega(t) - \omega_0}{\omega_0} = \frac{\alpha_2}{2M_{pl}} \int dt \omega_0 \partial_t M_{ij} n^i n^j$



[cf. Armaleo-Nacir-Urbanb, "Pulsar timing array constraints on spin-2 ULDM" JCAP(2020)
 Gromov-Son, "Bimetric Theory of Fractional Quantum Hall States" PRX(2017)]

Gravitational effects & coupling effects

GW & ULDM

Ultralight DM

Axion-like DM

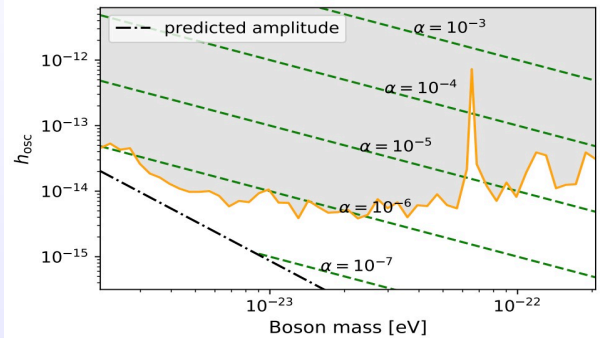
GW and DM

spin-0 DM

spin-2 DM

$$S = \frac{M_{\text{Pl}}^2}{1 + \alpha^2} \int d^4x \left[\sqrt{|g|} R(g) + \alpha^2 \sqrt{|f|} R(f) - 2 \frac{\alpha^2 M_{\text{Pl}}^2}{1 + \alpha^2} \sqrt{|g|} V(g, f; \beta_n) \right] + \int d^4x \sqrt{|g|} \mathcal{L}_m(g, \Psi)$$

$$S^{(2)} = \int d^4x \sqrt{|\bar{g}|} \left[\mathcal{L}_{\text{GR}}^{(2)}(\mathcal{G}) + \mathcal{L}_{\text{FP}}^{(2)}(M) - \frac{1}{M_{\text{Pl}}} (\mathcal{G}_{\mu\nu} - \alpha M_{\mu\nu}) T^{\mu\nu}(\Psi) \right],$$



[cf. Y. M. Wu, Z. C. Chen, Q. G. Huang, JCAP **09**, 021 (2023)]

"Pulsar timing residual induced by ultralight tensor dark matter,"]

Angular correlation & Hellings-Downs curves

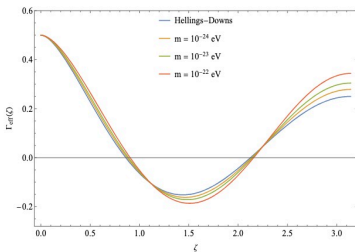


FIG. 2: Effective cross-correlation curves with $\alpha = 10^{-6}$ and mass ranging from 10^{-24} to 10^{-22} . It can be seen that in this range, the deformation of spin-2 dark matter on the Hellings-Downs curve is relatively small.

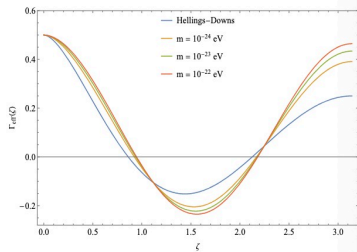


FIG. 3: Effective cross-correlation curves with $\alpha = 10^{-5.5}$. The deformation is very strong in this range, suggesting that if the coupling constant α is above this magnitude, existing ultralight mass spin-2 dark matter would have considerable effects on the deformation of the Hellings-Downs curve at corresponding frequency.

$$S = \frac{M_{\text{Pl}}^2}{1 + \alpha^2} \int d^4x \left[\sqrt{|g|} R(g) + \alpha^2 \sqrt{|f|} R(f) - 2 \frac{\alpha^2 M_{\text{Pl}}^2}{1 + \alpha^2} \sqrt{|g|} V(g, f; \beta_n) \right] + \int d^4x \sqrt{|g|} \mathcal{L}_m(g, \Psi)$$

$$S^{(2)} = \int d^4x \sqrt{|\bar{g}|} \left[\mathcal{L}_{\text{GR}}^{(2)}(\mathcal{G}) + \mathcal{L}_{\text{FP}}^{(2)}(M) - \frac{1}{M_{\text{Pl}}} (\mathcal{G}_{\mu\nu} - \alpha M_{\mu\nu}) T^{\mu\nu}(\Psi) \right],$$

[cf. R.G.Cai, J.R.Zhang, Y.L. Zhang, arXiv: 2402.03984,

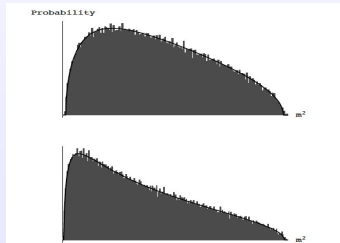
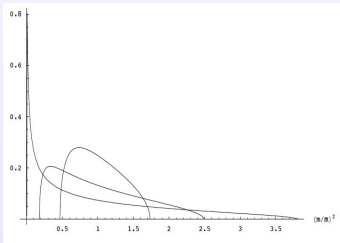
Angular correlation and deformed Hellings-Downs curve by spin-2 ultralight dark matter]

Multi fields: Marcenko-Pastur distribution

Mass spectrum and ultralight fields

- Marcenko-Pastur: $P_M(m^2) = \frac{\sqrt{(m^2 - m_-^2)(m_+^2 - m^2)}}{2\pi\beta m_0^2 m^2}$,
- Energy density: $\rho_\phi \equiv \int dm \tilde{\rho}(m) = \int dm \frac{1}{2} m^2 \tilde{\phi}(m)^2 P(m)$.
- Convenient choice: $\tilde{\rho}(m) \simeq \rho_\phi P(m)$, $\int dm P(m) = 1$.

[cf. Marcenko-Pastur, "Distributions of Eigenvalues for Some Sets of Random Matrices," (1967)]



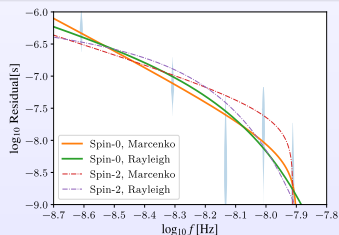
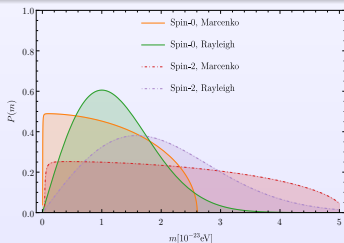
[cf. Easthera-McAllister, "Random Matrices and the Spectrum of N-flation" JCAP(2006)

Cai-Hu-Piao, "Entropy Perturbations in N-flation" PRD(2009)]

Phenomenological fitting results

Mass spectrum and ultralight fields

- Marcenko-Pastur: $P_M(m^2) = \frac{\sqrt{(m^2 - m_-^2)(m_+^2 - m^2)}}{2\pi\beta m_0^2 m^2}$,
- Rayleigh distribution: $P_\sigma(m) = \frac{m}{\sigma^2} e^{-\frac{m^2}{2\sigma^2}}$.



[Sun-Yang-Zhang, PRD(2022) "Pulsar Timing Residual induced by Wideband Ultralight Dark Matter"]

Corner Figures of Bayesian Fitting

GW & ULDM

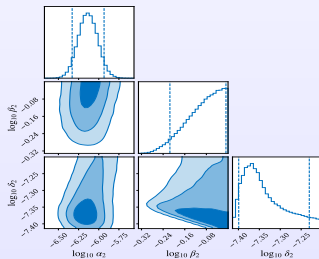
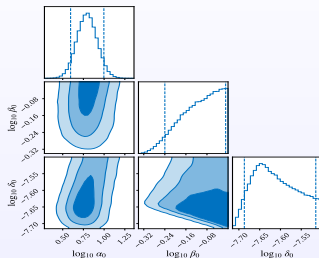
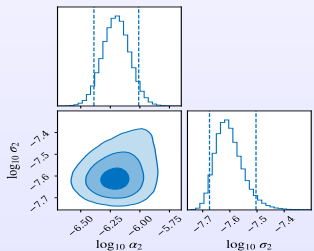
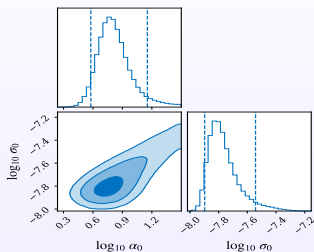
Ultralight DM

Axion-like DM

GW and DM

spin-0 DM

spin-2 DM



Phenomenological fitting results

The effective strain

- $h_c^\phi(f) = \frac{\alpha_0}{M_{pl}^2} \frac{\sqrt{3}\rho_{DM}}{4\pi f} P(\pi f)$
- $h_c^M(f) = \frac{\alpha_2}{M_{pl}} \frac{m\mathcal{M}P(m)}{\sqrt{5}} = \frac{\alpha_2}{M_{pl}} \frac{2\sqrt{\rho_M}}{\sqrt{5}} P(2\pi f).$

	Parameters	spin-0	spin-1	spin-2
Marcenko	α_i	$5.9^{+1.9}_{-1.3}$	$\sim 3\alpha_0$	$7.6^{+2.2}_{-1.7} \times 10^{-7}$
	$m_-^i/(10^{-23}\text{eV})$	$2.9^{+3.6}_{-0.3} \times 10^{-3}$	$\sim \delta_0(1 - \sqrt{\beta_0})$	$6.3^{+6.0}_{-1.7} \times 10^{-3}$
	$m_+^i/(10^{-23}\text{eV})$	$2.61^{+0.21}_{-0.01}$	$\sim \delta_0(1 + \sqrt{\beta_0})$	$5.08^{+0.02}_{-0.01}$
Rayleigh	α_i	$5.6^{+3.8}_{-1.0}$	$\sim 3\alpha_0$	$6.1^{+2.1}_{-1.3} \times 10^{-7}$
	$\sigma_i/(10^{-23}\text{eV})$	$1.0^{+0.4}_{-0.1}$	$\sim \sigma_0$	$1.6^{+0.3}_{-0.1}$

[Sun-Yang-Zhang, PRD(2022), "Pulsar Timing Residual induced by Wideband Ultralight Dark Matter"]

Deformed Hellings-Downs curve: GW v.s. ULDM

GW & ULDM

Ultralight DM
Axion-like DM

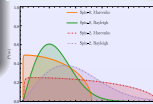
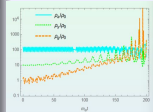
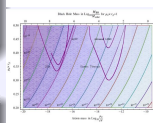
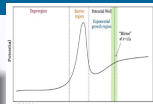
GW and DM
spin-0 DM
spin-2 DM

ULDM with spin 0,1,2 & GW effects

- Spin-2 ultralight dark matter & Bi-metric gravity
- **Wideband mass spectrum** extension of ULDM
- Search/constraint for ULDM with GW detection
- Angular correlation & **Deformed Hellings-Downs**

GW detection & Ultra-light DM with multi band

- Tabletop exp: GW burst & axion ($\sim$ kHz - GHz)
- LVK & LISA: BH Superradiance ($\sim$ mHz - kHz)
- PTA & SKA : Fuzzy dark matter ($\sim$ nHz- uHz)
- LSS & CMB : DE & Modified gravity ($\sim$ nHz)



超轻暗物质的多波段引力波探测

- 引力波宇宙学观测：修改引力效应 (nnHz)
- PTA-SKA 脉冲星测时：超轻暗物质 (nHz)
- LISA-Taiji-Tianqin & Ligo-Virgo-Kagra (mHz - kHz)
- 桌面实验 & 轴子星 & 引力波暴 (kHz - GHz)

黑洞的超辐射和引力波效应

- 超辐射和轴子云： $\alpha \equiv \frac{R_{BH}}{\lambda_\varphi} \simeq \left(\frac{M_{BH}}{M_\odot} \right) \left(\frac{m_\varphi}{10^{-10} \text{eV}} \right)$
- 能级跃迁： $\varphi^+ \rightarrow \varphi^- + h$ ，轴子湮灭： $\varphi + \varphi \rightarrow h$
- 轴子诱导的快速射电暴： $\sim \varphi \tilde{F}\tilde{F} (\varphi \rightarrow \gamma\gamma)$
- 轴子诱导的引力波暴： $\sim \varphi R\tilde{R} (\varphi \rightarrow hh)$

感谢各位老师同学！

