Non-Abelian Vortex Lattice

--- Color Magnetism—

Ref: arXiv:1311.2399 JHEP

Eiji Nakano (Kochi)

w/ M. Kobayashi (Kyoto) and M. Nitta (Keio U)

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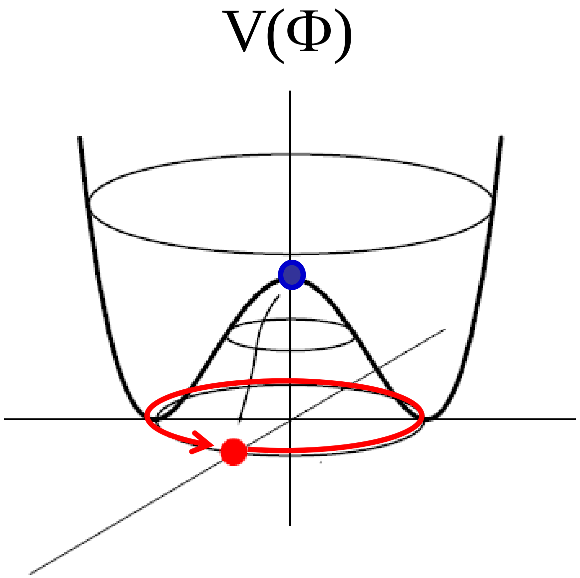
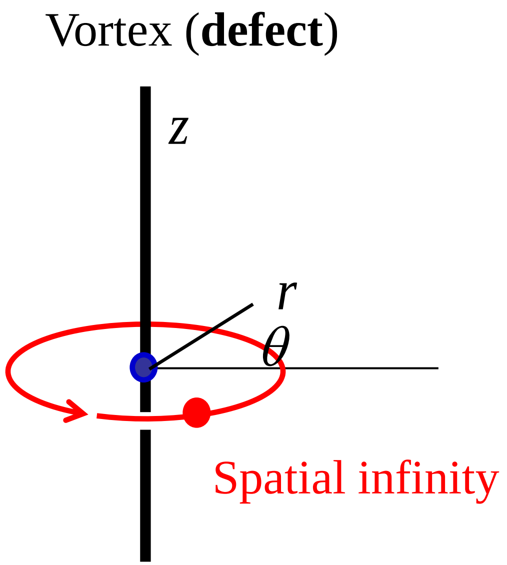
1. Review on Abelian U(1) vortices

U(1) vortex, e.g., in *U(1)* Higgs model:

$$L = \left|D\Phi\right|^2 + m^2\left|\Phi\right|^2 - \lambda\left|\Phi\right|^4 - \tfrac{1}{4}FF$$



$$\boxed{\Phi = e^{i\theta} f(r)}$$

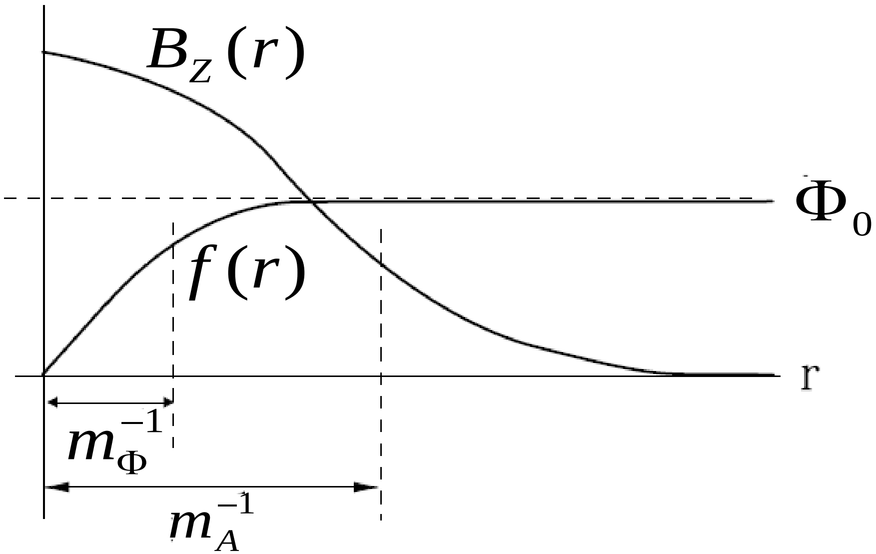


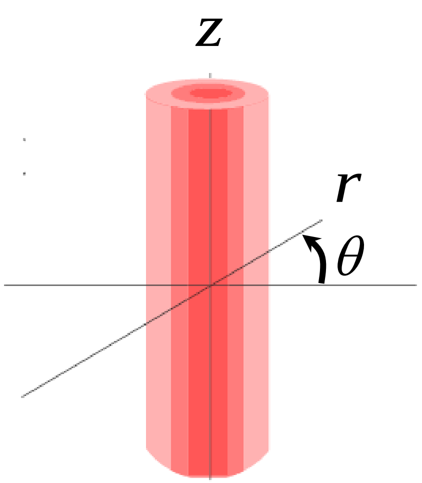
1st homotopy $\pi_1\left(S^1\right) = Z \;\Rightarrow$ quantized vortecity

- Topological soliton w/SSB
- String like object (1+3d)
- Flux if gauged

e.g., Gauged *U(1)* Vortex by A.A.Abrikosov ('57) and Nielsen-Olesen ('73)

$$\boxed{\Phi(r,\theta) = e^{i\theta} f(r), \;\; A_\theta(r) = \frac{h(r)}{er}}$$





Penetration depth : $m_A^{-1} = (e\Phi_0)^{-1}$

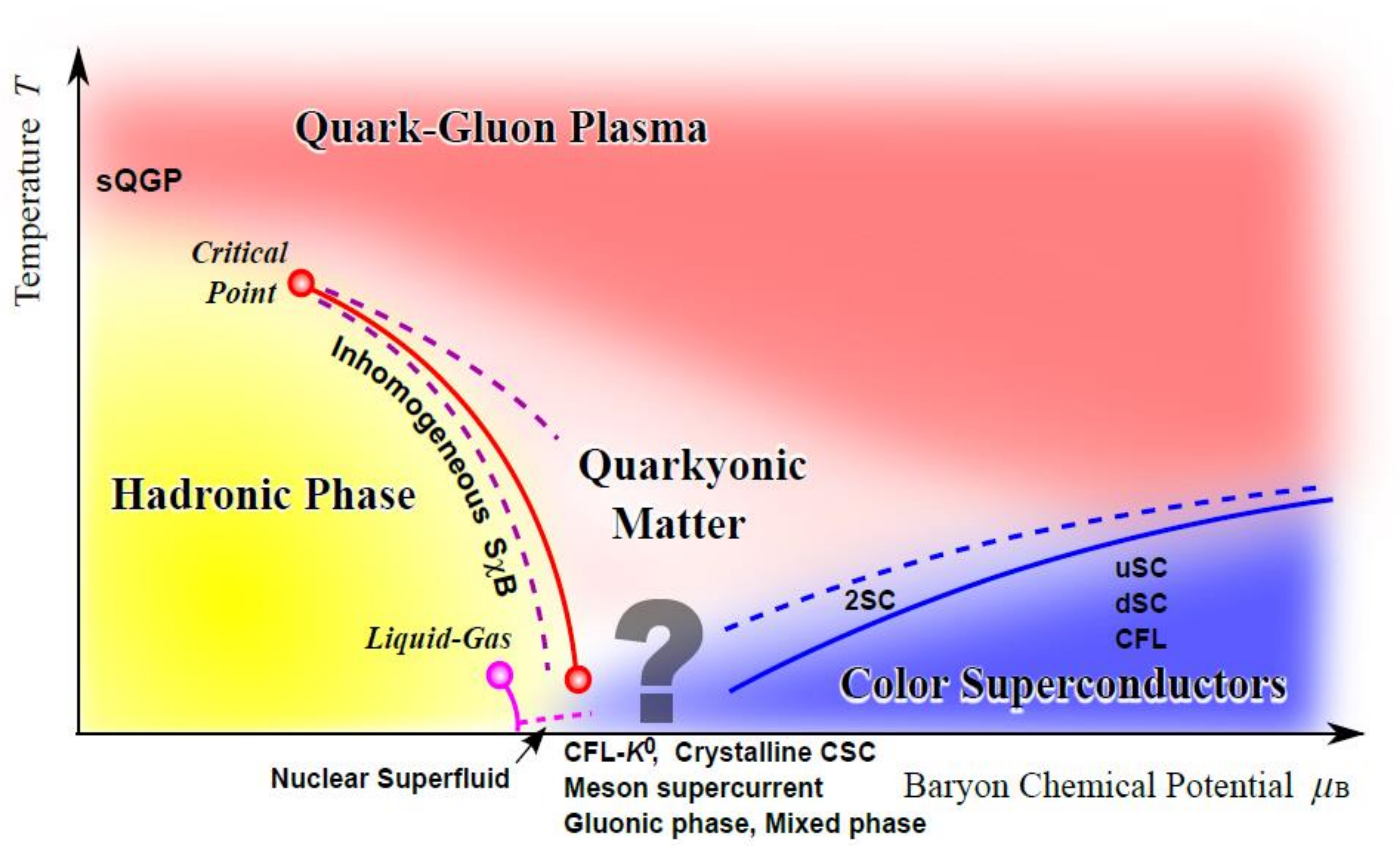
Coherence length : $m_\Phi^{-1} = (\sqrt{\lambda}\Phi_0)^{-1}$

Quantized flux : $n = \dfrac{e}{2\pi}\oint A_i \mathrm{d}x^i = \dfrac{e}{2\pi} B_z \in \pi_1[U(1)] = Z$

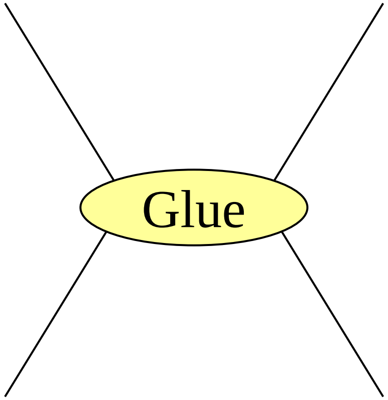
鳴門の渦(Non topological)

2. Color-Flavor Locking (CFL) in dense QCD

QCD phase structure K. Fukushima & T. Hatsuda, 10



Color Flavor Locking phase (Alford, Rajagopal, Wilczek '99)



: quark-quark interaction at $N_F = N_C = 3$

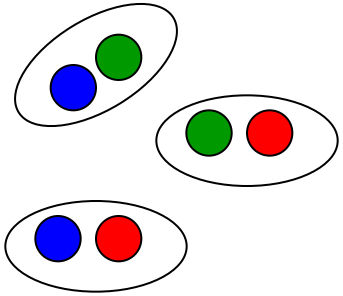
attractive in **color anti-symmetric** channels

$\Rightarrow$ quark cooper pairing

$$\left\langle \psi_i^{\alpha} C \gamma_5 \psi_j^{\beta} \right\rangle = \varepsilon^{\alpha\beta\gamma} \varepsilon^{ijk} \Phi_{\gamma k}$$

Color: $3 \times 3 = \bar{3} + 6$

Flavor: $3 \times 3 = \bar{3} + 6$



$m_u = m_d = m_s = 0$

Color-Flavor locking (CFL) : $\Phi_{\gamma k} = \Delta \delta_{\gamma k}$

Spontaneous symmetry breaking in CFL

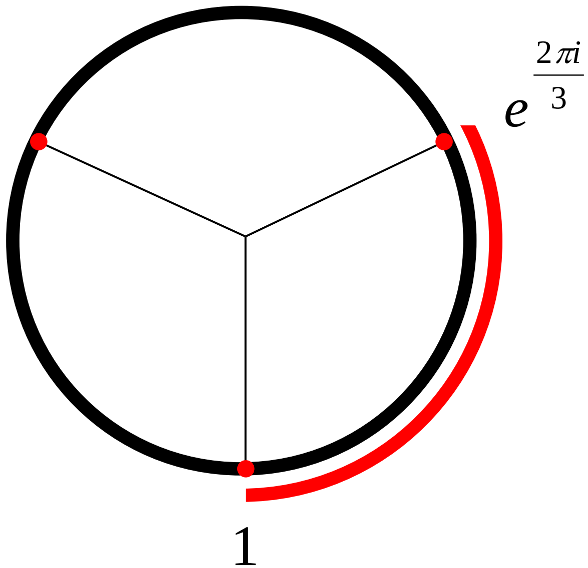
$$e^{i\alpha}U_c \Phi U_f^{-1} \;=\; \Delta\, e^{i\alpha}U_c \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} U_f^{-1}$$

$U(1)_B$

$$G = \frac{SU(3)_C \times SU(3)_F \times U(1)_B}{Z(3)_{C+B} \times Z(3)_{F+B}} \quad \Rightarrow \quad H = \frac{SU(3)_{C+F} \times Z(3)_{C-F+B}}{Z(3)_{C+B} \times Z(3)_{F+B}}$$

$$G/H = \frac{SU(3)_{C-F} \times U(1)_B}{Z(3)_{C-F+B}} = U(3)$$

1st homotopy in CFL phase: $\pi_1\left[\dfrac{SU(3)_{C-F}\times U(1)_B}{Z(3)_{C-F+B}}\right]=Z$

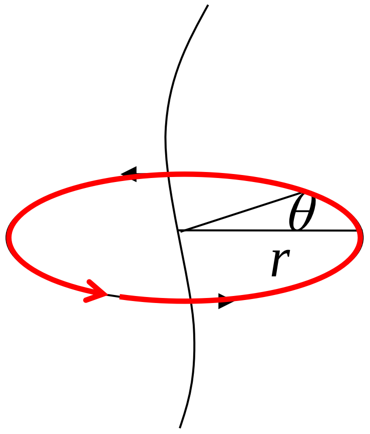


$U(1)_B$ circle and $SU(3)$ rotation

Fundamental mapping:

$$\Phi(r,\theta)\propto \exp i\left[\frac{1}{3}-\sqrt{\frac{2}{3}}\lambda_8\right]\theta$$

$$=\begin{pmatrix} e^{i\theta} & & \\ & 1 & \\ & & 1 \end{pmatrix}$$



$0\le\theta\le 2\pi$

NAV in CFL =Semi-Superfluid vortex

3 type of mapping for single-valued Φ :

$$\Phi(r,\theta) \propto \exp i\left[\frac{1}{3} - i\sqrt{\frac{2}{3}}\left(Q_3\lambda_3 + Q_8\lambda_8\right)\right]\theta$$

$$\Rightarrow \begin{pmatrix} e^{i\theta} & & \\ & 1 & \\ & & 1 \end{pmatrix} \text{ or } \begin{pmatrix} 1 & & \\ & e^{i\theta} & \\ & & 1 \end{pmatrix} \text{ or } \begin{pmatrix} 1 & & \\ & 1 & \\ & & e^{i\theta} \end{pmatrix}$$

- *SU(3)* part: three color fluxes

$$(Q_3\,,Q_8) = \underset{\text{Red}}{(0,1)}, \quad \underset{\text{Green}}{\left(\frac{\sqrt{3}}{2},-\frac{1}{2}\right)}, \quad \underset{\text{Blue}}{\left(-\frac{\sqrt{3}}{2},-\frac{1}{2}\right)}$$

3. Non-Abelian Vortex (NAV) in CFL phase

NAV solution of GL Lagrangian for CFL phase

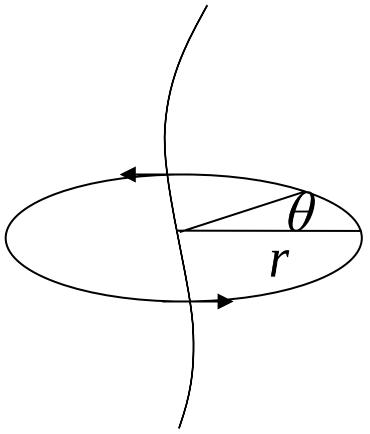
Pisarski ’00, Iida-Baym ’01, Giannakis-Ren ‘02

$$L_{GL} = \mathrm{tr}|D\Phi|^2 - m^2 \mathrm{tr}|\Phi|^2 - \lambda_1 \left(\mathrm{tr}|\Phi|^2\right)^2 - \lambda_2 \mathrm{tr}|\Phi|^4$$
$$- \frac{1}{4} FF$$

Barachandran-Digal-Matsuura 06, EN-Matsuura-Nitta ‘08

Vortex ansatz

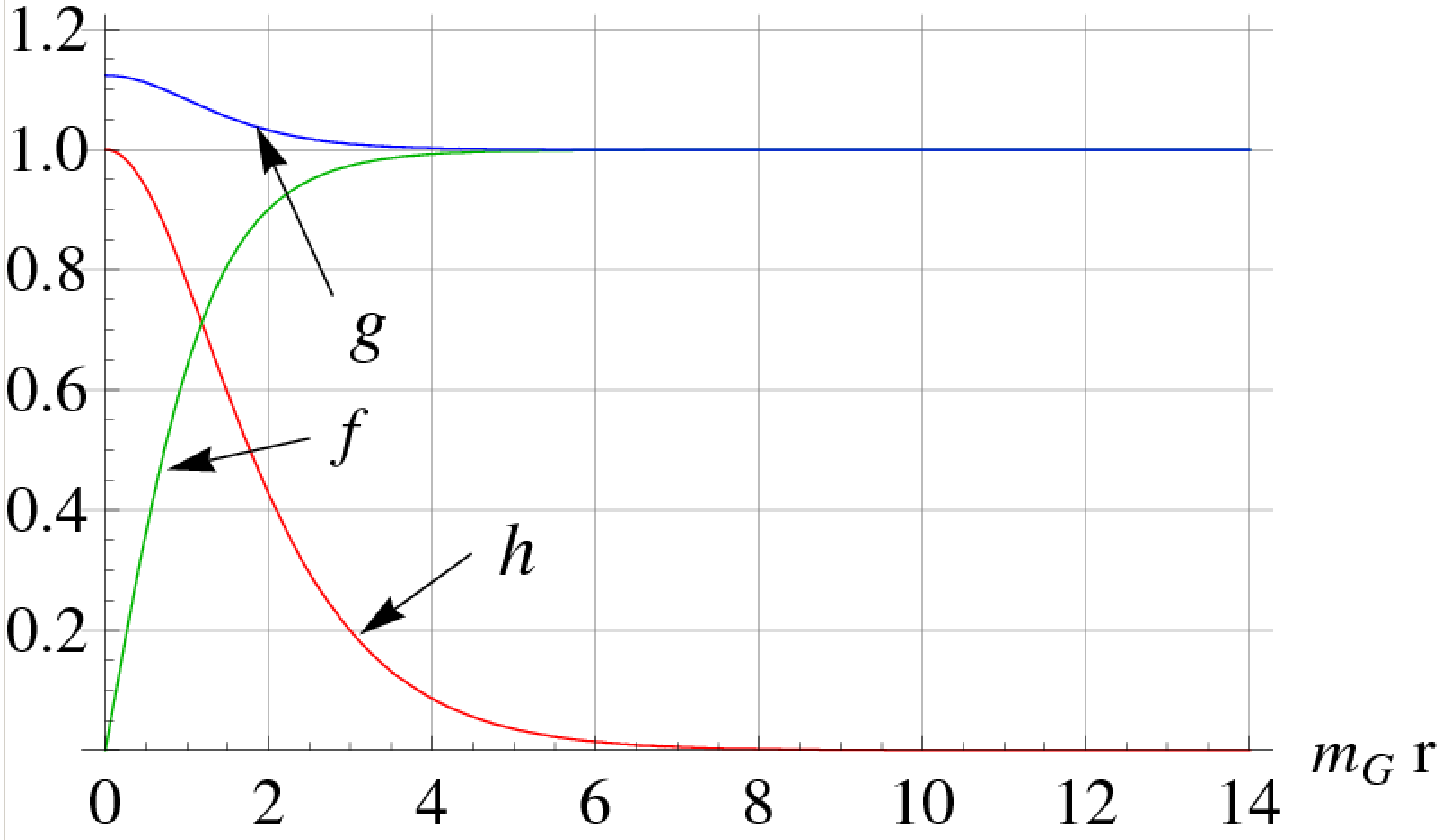
$$\left\{ \begin{array}{l} \Phi(r,\theta) = \Phi_0 \begin{pmatrix} e^{i\theta} f(r) & & \\ & g(r) & \\ & & g(r) \end{pmatrix}, \\ A_\theta^8(r) = -\sqrt{\frac{2}{3}} \frac{1}{g_s} \frac{h(r)}{r}, \end{array} \right.$$



Flux: $F^8 = \frac{1}{r} \frac{d}{dr}\left[r A_\theta^8 \right]$

Numerical solution

$$\Phi(r,\theta)=\Delta\begin{pmatrix} e^{i\theta}f(r) & & \\ & g(r) & \\ & & g(r)\end{pmatrix},\quad A_\theta^8(r)=\frac{\xi}{g_s}\frac{h(r)}{r},$$



4. Localized NG mode and EFT

Orientation moduli (NG modes)

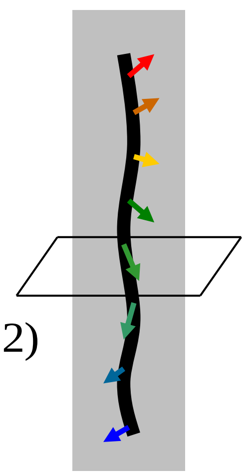
EN-Matsuura-Nitta '08, Eto-EN-Nitta '09

$$\Phi(r,\theta)=\begin{pmatrix} e^{i\theta}f(r) & & \\ & g(r) & \\ & & g(r) \end{pmatrix}$$

NAV string further breaks H :

$$H=SU(3)_{C+f}$$

$$\rightarrow H'=SU(2)_{C+f}\times U(1)_{C+f}$$



$CP(2)$

Moduli space (NG mode) along NAV string:

$$H/H'=\frac{SU(N)_{C+F}}{SU(N-1)_{C+F}\times U(1)_{C+F}}\simeq CP(N-1)\,|_{N=3}$$

1+1 dim Effective Field Theory of *CP(2)* moduli,

a la Chiral perturbation theory

$$\text{Vortex solution rotated with } U \in SU(3)_{C+F}$$

$$\Phi(U) = U\Phi U^{-1}, \quad A_{x,y}(U) = UA_{x,y}U^{-1}$$

Identify *CP(2)* modes:

$$\Phi \propto \#I_3 + \#\lambda_8 \;\; \Rightarrow \boxed{U\lambda_8 U^{-1} \equiv \phi\phi^\dagger - \frac{1}{3} I_3}$$

Homogen. coordinate: $\phi(t,z) = (\phi_1, \phi_2, \phi_3)^T \in CP(2)$

$$\phi^\dagger\phi = 1, \quad e^{i\alpha}\phi \sim \phi$$

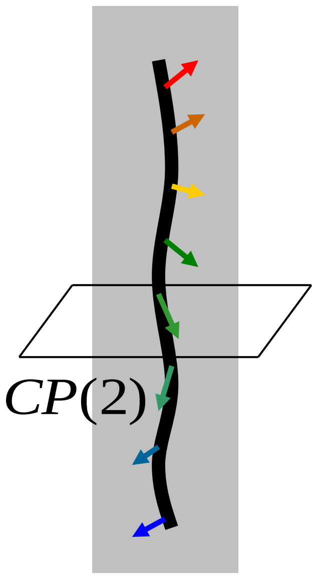
Induced *t* and *z* components: $A_{0,z} = \dfrac{i\rho_{0,z}(r)}{g}\left[\phi\phi^+, \partial_{0,z}\left(\phi\phi^+\right)\right]$

Putting all into GL Lagrangian and integrate x, y components :

$$L_{eff}^{CP(2)}[\phi] = \int dxdy\, L_{GL}[A_\mu^a(\phi), \Phi(\phi)]$$

$$= \boxed{\sum_{\alpha=0,z} C_\alpha \left[\partial_\alpha \phi^\dagger \partial^\alpha \phi + \left(\phi^\dagger \partial_\alpha \phi\right)\left(\phi^\dagger \partial^\alpha \phi\right)\right]}$$

1+1 dim CP(2) Non-linear sigma model



$C_{0,z}[\rho_{0,z}, f, g, h] \sim O(1)$

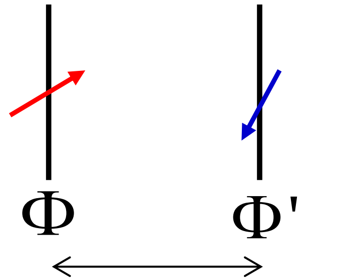
FINITE $\Rightarrow$ normalizable mode

5. Vortex 2-body force

Vortex interaction mediated by scalar, gauge and NG bosons:

- Scalars and Gluons are all massive $\Rightarrow$ short-range int.
- $U(1)_B$ phonon is massless $\Rightarrow$ long-range int.

Long-range force from free energy of 2-body system



$$r >> m_G^{-1}, m_\Phi^{-1}$$

Abrikosov ansatz : $\Phi_{tot} = \Phi.\Phi', \quad A_{tot}^{\mu} = A^{\mu} + A'^{\mu}$

GL potential : $E_{\mathrm{int}}(r) = E(\Phi_{tot}) - E(\Phi) - E(\Phi')$

Force between vortices : $-\dfrac{\mathrm{d}E_{\mathrm{int}}(r)}{\mathrm{d}r} \cong +\dfrac{1}{3}\dfrac{2\pi}{r}$

Repulsive force ! like a global *U(1)* vortex,
But reduced by 1/3.

Fate of $U(1)_B$ Vortex (Iida-Baym 02, Fobes-Zhitnitsky 02):

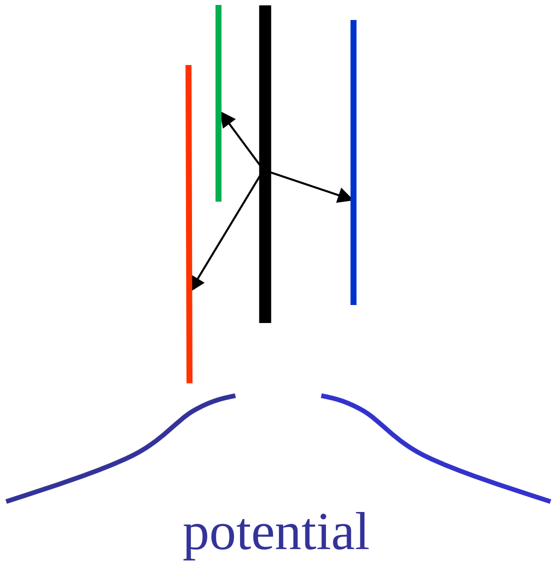
$$U(1)_B\,\text{Vortex} = \text{diag}(e^{i\theta}, e^{i\theta}, e^{i\theta}),$$

Decay!

$$3\,\text{NAVortices} = \text{diag}(e^{i\theta},1,1),$$

$$\text{diag}(1,e^{i\theta},1),$$

$$\text{diag}(1,1,e^{i\theta}),$$



⇒Stability of NAV in CFL phase.

6. NAV Lattice and EFT

Kobayashi-Nakano-Nitta ‘13

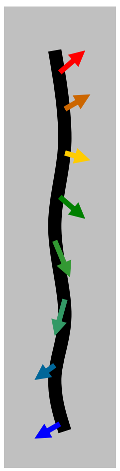
4. Vortex lattice

(rotating system)

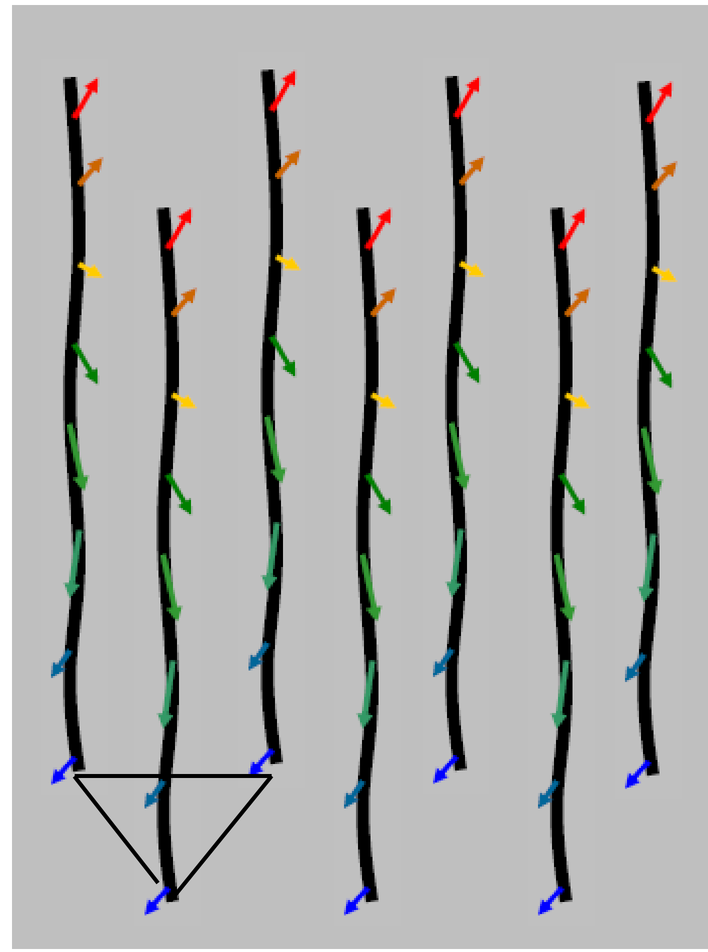
Repulsive Long-Range force

⇒ Triangle lattice (Abrikosov)

Single vortex



No Ordering [CMW 定理]



Color exchange force ⇒ Color ordering?

CP(N-1) Color ``Spin-Spin`` interaction **(Auzzi-Eto-Vinci 08)**

$$V_{\text{int}} = \sum_{vorex\, site,\, i \neq j} \left(\phi_i^{\ +} \lambda_A \phi_i \right) G(i-j) \left(\phi_j^{\ +} \lambda_A \phi_j \right)$$

$$G(r) \sim -\frac{Exp(-m_S r)}{\sqrt{m_S r}} + \frac{Exp(-m_g r)}{\sqrt{m_g r}}$$

Scalar exchange（−）　Gluon exchange（+）

In CFL phase $m_S << m_g \Rightarrow$ Color Ferromagnetism!

$$\boxed{\phi_0^T = (1,0,0)}$$

Rotating CFL $\Rightarrow$ Color Ferro $\Rightarrow$ Color Magnons (CP(2))

$$\boxed{\begin{aligned} H = \int dz \sum_{\langle i,j\rangle ,A} \Big[- J_{xy} S_{i,A} S_{j,A} \\ + K_3 \{ |\partial_z \phi_i|^2 + (\phi_i^\dagger \partial_z \phi_i)^2 \} \Big] \\ S_{i,A} := \phi_i^\dagger T_A \phi_i , \quad J_{xy} := \Delta^2 G(L) \end{aligned}}$$

Taking long wavelength limit, using *Fierz* transformation:

$$\sum_A (T_A)^a_b (T_A)^c_d = \delta^a_d \delta^c_b / 2 - \delta^a_b \delta^c_d / 2N$$

$$\sum_A (\phi_i^\dagger T_A \phi_i)\ (\phi_j^\dagger T_A \phi_j)$$

Derivative exp.

$$\to \quad -\frac{L^2}{2} \left[|\nabla \phi|^2 + \left(\phi^\dagger \nabla \phi\right)^2 + \mathcal{O}\left(\nabla^4\right) \right]$$

EFT on Lattice = Anisotropic 3+1d *CP(N-1)* NLSM

$$L_{CP2} = \sum_{\alpha=t,x,y,z} \mathrm{C}_\alpha \left(\partial_\alpha \phi^+ \partial_\alpha \phi + \phi^+ \partial_\alpha \phi \phi^+ \partial_\alpha \phi \right)$$

$$\mathrm{C}_x = \mathrm{C}_y \neq \mathrm{C}_z$$

Magnon dispersion relation:

$$\boxed{\omega_p^2 = C_{x,y}(p_x^2 + p_y^2) + C_z p_z^2}$$

in CFL phase, propagating 10km in ~1 min.

7. Summary and Outlook

Summary

- Dense QCD ⇒ CFL phase
- Rotating CFL ⇒ NAV lattice ⇒ ***Color Ferromagnetism***
- Color magnon（NG modes）

EFT ＝ Anisotropic 3+1d *CP(N-1)* NLSM

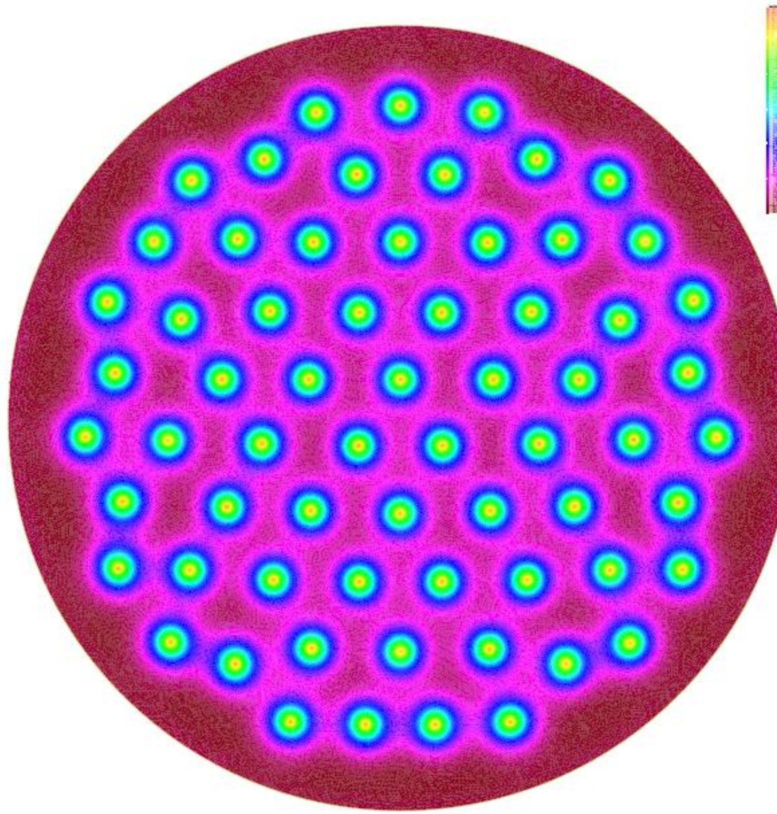
$$L_{CP2} = \sum_{\alpha=t,x,y,z} \mathrm{C}_\alpha \left(\partial_\alpha \phi^+ \, \partial_\alpha \phi + \phi^+ \partial_\alpha \phi \phi^+ \partial_\alpha \phi \right)$$

Outlook

- Dense vortex lattice/Electromag./Thermodynamics/Transport
- *Color anti-Ferro* ⇒ order parameter space

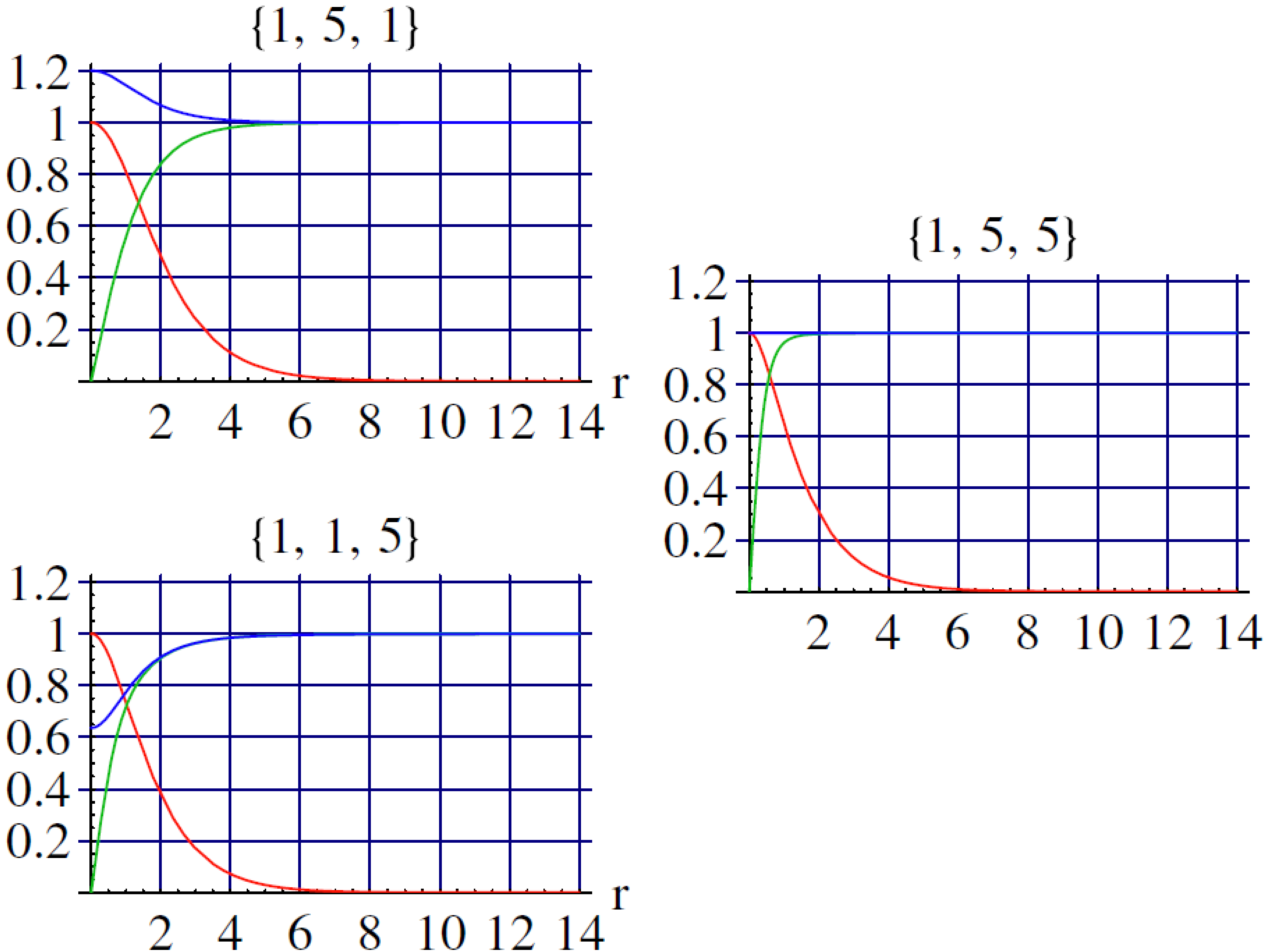
Thank you for attention!

Cross-section: Vortex lattice



Courtesy from Cipriani, Vinci, and Nitta

Possibly in *Neutron star core*?


{1, 5, 1}
1.2
1
0.8
0.6
0.4
0.2
2 4 6 8 10 12 14
r
{1, 5, 5}
1.2
1
0.8
0.6
0.4
0.2
2 4 6 8 10 12 14
r
{1, 1, 5}
1.2
1
0.8
0.6
0.4
0.2
2 4 6 8 10 12 14
r

How about force at intermediate range?

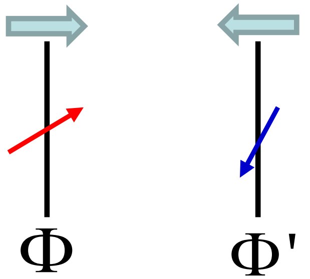
Dual transformation： clear to see interaction between vorticity tensor and bulk particle (gluons and U(1) phonon)

Hirono et al ‘11

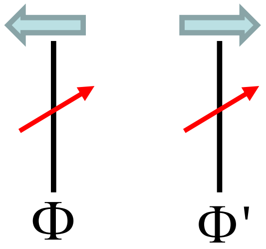
$$L_{coupling} = g_a \int \mathrm{d}\sigma^a_{\nu\mu} B^a_{\nu\mu}$$

$$\Downarrow$$

$$\mathrm{E} \sim \int d\vec{X}_1 d\vec{X}_2 \phi_1^\dagger T^a \phi_1 \ \phi_2^\dagger T^a \phi_2 \ \frac{\exp(-M_G|\vec{X}_1 - \vec{X}_2|)}{\sqrt{|\vec{X}_1 - \vec{X}_2|}}$$



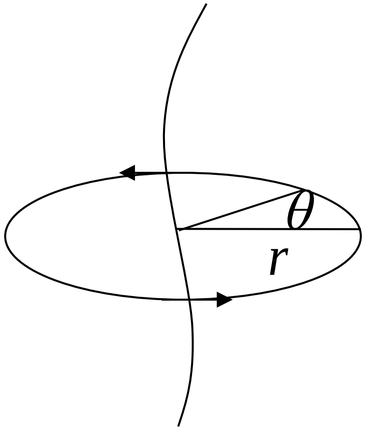
Most attractive
for the different orientation



Most repulsive
for the same orientation

NAV解 in GLラグランジアン

$$L_{GL} = \mathrm{tr}|D\Phi|^2 - m^2 \mathrm{tr}|\Phi|^2 - \tfrac{1}{4} F_a F_a - \lambda_1 \left(\mathrm{tr}|\Phi|^2\right)^2 - \lambda_2 \mathrm{tr}|\Phi|^4$$



$$\Phi = \Delta \begin{pmatrix} e^{i\theta} f & & \\ & g & \\ & & g \end{pmatrix}, or \quad \Delta \begin{pmatrix} g & & \\ & e^{i\theta} f & \\ & & g \end{pmatrix}, or \quad \Delta \begin{pmatrix} g & & \\ & g & \\ & & e^{i\theta} f \end{pmatrix}$$

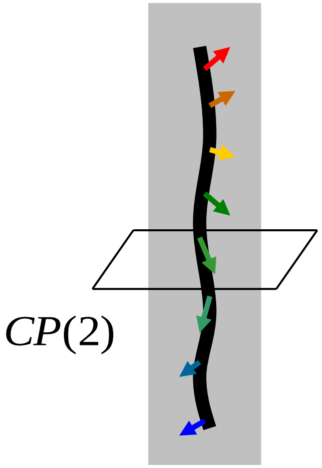
カラー磁束: Red, or Green, or Blue

遠方漸近形 $\rightarrow \Delta e^{i\frac{\theta}{3}} \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}$

3. NG modes

$$\Phi = \begin{pmatrix} e^{i\theta} f & & \\ & g & \\ & & g \end{pmatrix}$$

$$H = SU(3)_{C+f}$$
$$\to H' = SU(2)_{C+f} \times U(1)_{C+f}$$



渦糸に局在したNGモード ＝ カラー磁束

$$H/H' = \frac{SU(N)_{C+F}}{SU(N-1)_{C+F} \times U(1)_{C+F}} \cong CP(N-1)|_{N=3}$$

有効理論：1+1 dim CP(N-1)非線形シグマモデル

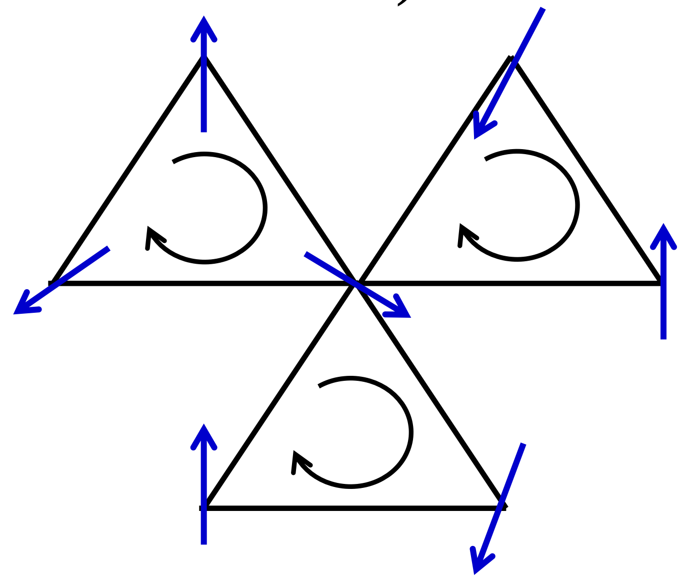
$$L^{eff}_{CP(N-1)} = \sum_{\alpha=t,z} C_\alpha \left(\partial_\alpha \phi^+ \partial_\alpha \phi + \phi^+ \partial_\alpha \phi \phi^+ \partial_\alpha \phi \right)$$

$$\phi^T = (\phi_1, \phi_2, \phi_3)$$

(Eto-Nakano-Nitta 09)

Non-BPS 渦糸格子に関する一般化

- カラー反強磁性の秩序変数空間（OPS)

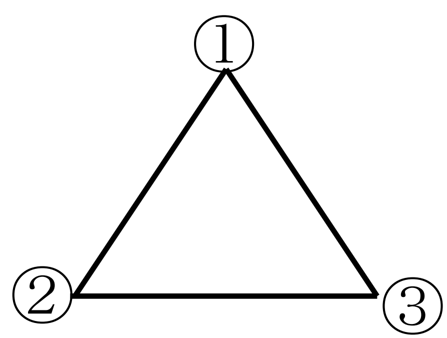


N=2

$$CP(1) \sim S^2$$
$$OPS = S \ltimes S^2 / Z_2 \sim SO(3)$$

N=3

$$CP(2)$$
$$OPS = CP(1) \ltimes CP(2) \sim \frac{SU(3)}{U(1)^2}$$



N≧4

$$OPS \sim \frac{SU(N)}{SU(N-3) \times U(1)^3}$$

①(1,0,0), ②(0,1,0), ③(0,0,1)

■ Taking quantum effect and θ-vacua into account,

D'adda-Luscher-DiVecchia '79, Witten '80, '98

$$L_{CP(N-1)} = \sum_{\alpha=t,z} C_\alpha \left(\partial_\alpha \phi^+ \, \partial_\alpha \phi + \phi^+ \partial_\alpha \phi \phi^+ \partial_\alpha \phi\right) - \sigma\left(\phi^+ \phi - 1\right)$$

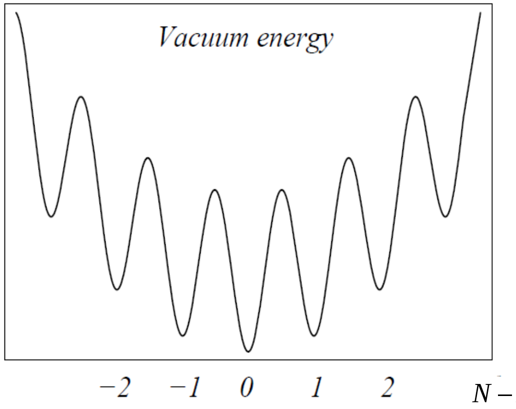
$\rightarrow$ Leading order at Large N_C (at the end $N_C = 3$)

$$L^{Quant}_{CP(N-1)} = \left(\partial_\alpha - iA_\alpha\right)\phi^+ \left(\partial^\alpha + iA^\alpha\right)\phi - M^2 \phi^+ \phi - \frac{N_C}{48\pi M^2} F^2_{\alpha\beta}$$

$$A_\alpha = \phi^+ \partial_\alpha \phi, \;\; M \sim \Delta e^{-c(\mu/\Delta)^2}$$

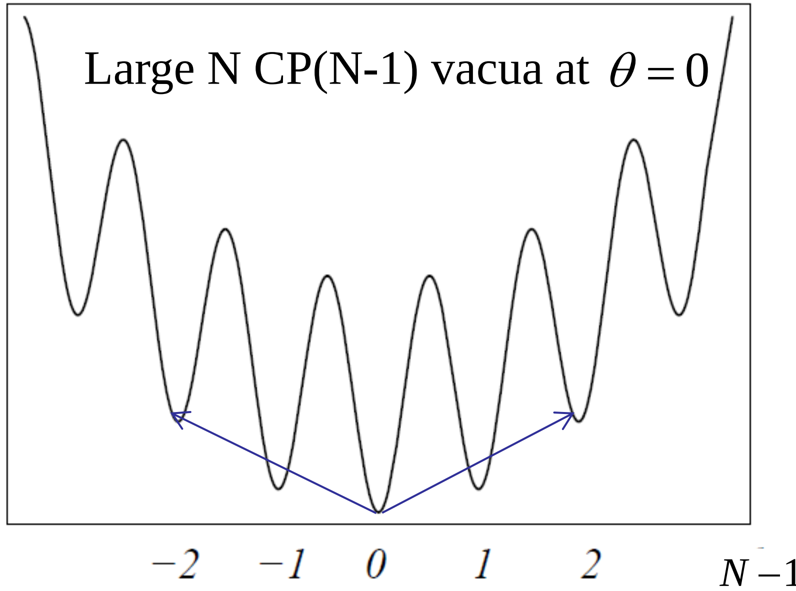
■ Energy with periodicity and reflection of θ

$$E(\theta)|_{\theta=0} = E_0 + c\frac{k^2}{N_C}, k = 0,1,2,\cdots N_C - 1$$



$\Rightarrow$ Kink-antiKink excitation in *CP(2) in world sheet EFT*

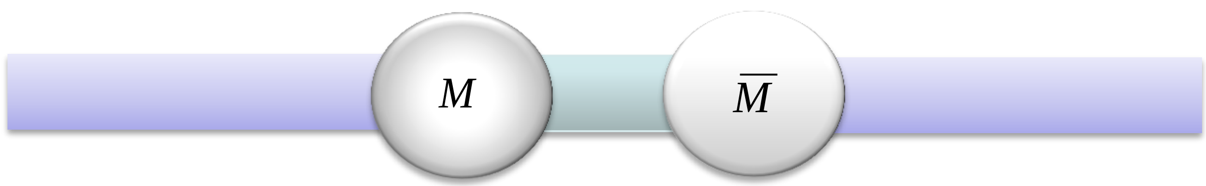
= looks Monopole - antiMonopole bound in bulk :



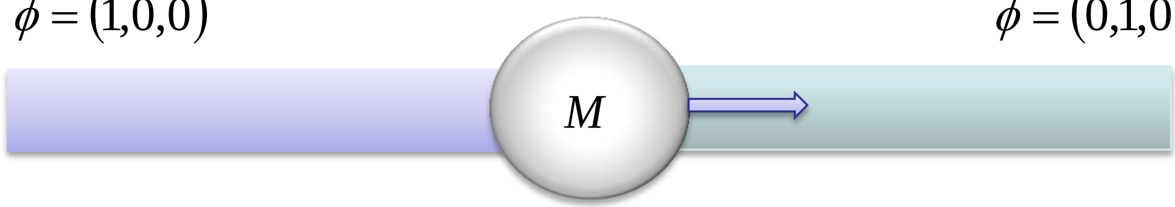
Gorsky-Shifman-Yung ’11
Eto-Nitta-Yamamoto ’11

Kink-antiKink

Confined monopoles (in Higgs phase)



$\phi = (1,0,0)$ $\phi = (0,1,0)$



$(Q_3, Q_8) = (1,0)$ $SU(3)$ fundamental

Short summary

- Non-Abelian vortex in CFL phase
- NG modes --- Localized and EFT
- 2-body forces (to many-body physics):

1) Long range >> m_Φ^{-1}, m_A^{-1}
massless $U(1)_B$ phonon exchange
⇒ Universal Repulsion ⇒ Stability

2) Intermediate range > m_Φ^{-1},
massive gluon exchange
⇒ Attraction for different orientations
Repulsion for same orientations

3) Short range < m_Φ^{-1}
- massive scalar exchange ?
- role of monopole excitations?
- genuine 3-body force?

Other topics

■ Realistic situations

Quark masses & Beta equilibrium effects on zero modes

$\Rightarrow$ specific orientation (CP(2) quantum effect swept out?)

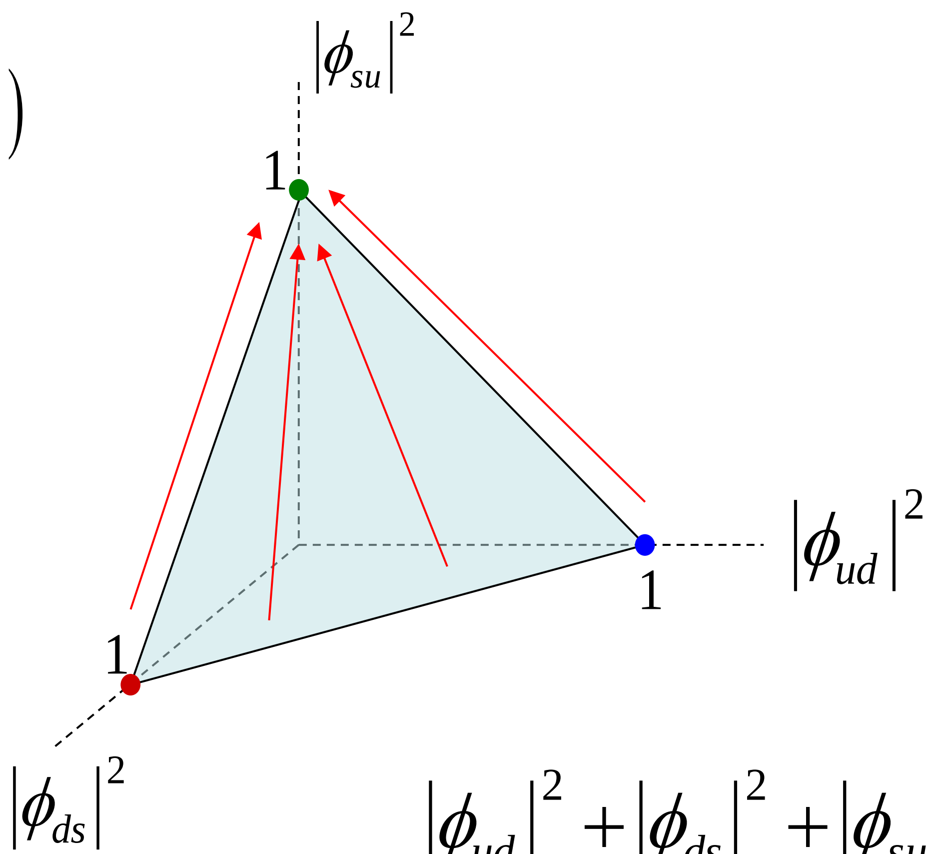
Eto et al ’10, Gorsky et al ’11

e.g., $V_{CP} \sim \varepsilon\left(\left|\phi_3\right|^2 - 1\right),$

or $V_{CP} \sim \varepsilon\left(\left|\phi_3\right|^2 - \left|\phi_2\right|^2\right)$

Iida et al ’04,

$SU(3)_{C+L+R} \to U(1) \times U(1)$



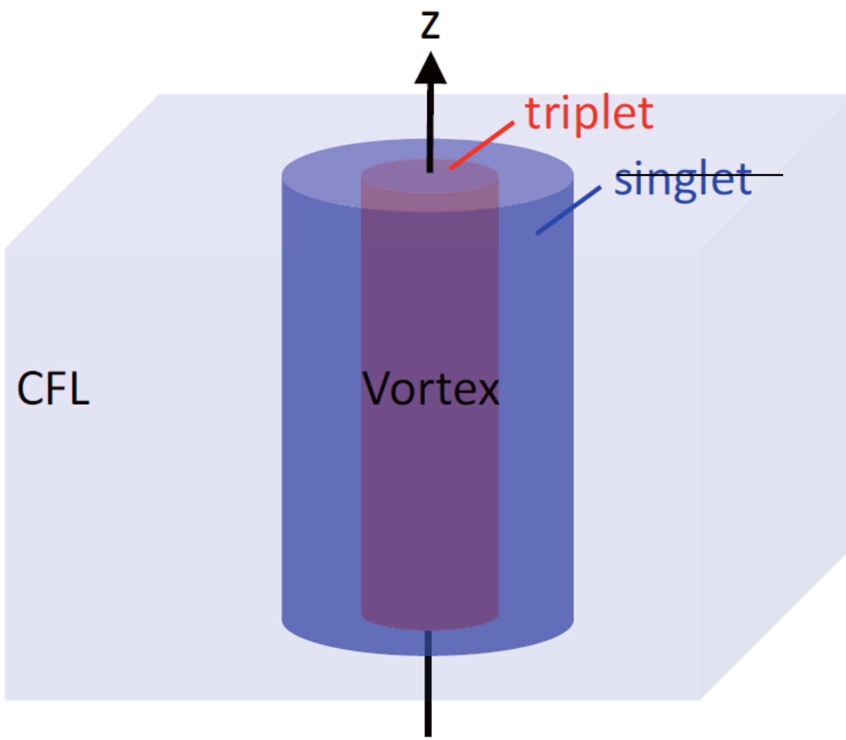
$$\left|\phi_{ud}\right|^2 + \left|\phi_{ds}\right|^2 + \left|\phi_{su}\right|^2 = 1$$

■ Fermion zero modes inside vortex core Yasui et al ‘10

Fujiwara et al ‘12

⇒statistical feature of vortex strings

NAVの非可換統計 (Yasui-Hirono-Itakura-Nitta 11-13)



Fermion wave functions :

Multiplets of $H' = SU(2)_{C+F} \times U(1)_{C+F}$

■ Global Non-Abelian vortex strings in Chiral transition:

$$G = SU(N)_L \times SU(N)_R \times U(1)_A$$
$$H = SU(N)_{L+R} \times Z_N$$

$$G/H = \frac{SU(N)\times U(1)}{Z_N} = U(N)$$

- **Order parameter always respects U(1)_B sym.**
- **In absence of chiral anomaly.**

$U(1)_A$ vortex string (η' string)

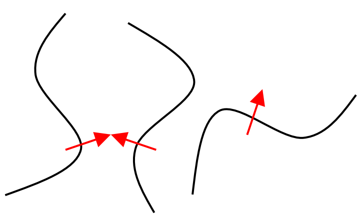
Interaction dependent on orientation (not normalizable):

$$f(\rho) = (1+\cos[\alpha])\frac{\pi}{N\rho}$$

$\mathrm{diag}(e^{i\theta},1,....,1),\quad \alpha = 0$

$\mathrm{diag}(1,e^{i\theta},1,....,1),\quad \alpha = \pi$

$\mathrm{diag}(e^{i\theta},1,....1),\quad \alpha = 2\pi N$



Relax only via NG boson radiation (might be observed in RHIC).

General expression of 3 fundamental vortices :

where $F \equiv f + 2g, \quad G \equiv f - g$

$$\Phi(r,\theta) = \exp\left[i\frac{\theta}{3} - i\sqrt{\frac{2}{3}}\left(Q_3\lambda_3 + Q_8\lambda_8\right)\theta \right]\left[\frac{F(r)}{3} - \sqrt{\frac{2}{3}}\left(Q_3\lambda_3 + Q_8\lambda_8\right)G(r) \right]$$

$$\Rightarrow \begin{pmatrix} e^{i\theta} f(r) & & \\ & g(r) & \\ & & g(r) \end{pmatrix} \text{ or } \begin{pmatrix} g(r) & & \\ & e^{i\theta} f(r) & \\ & & g(r) \end{pmatrix} \text{ or } \begin{pmatrix} g(r) & & \\ & g(r) & \\ & & e^{i\theta} f(r) \end{pmatrix}$$

SU(3) part: three color fluxes

$$(Q_3, Q_8) = (0,1) \text{ Red, } \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right) \text{ Blue, } \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right) \text{ Green}$$

$$\pi_1\left[SU(3)/Z(3)\right] = Z(3)$$

$$
-\sum_{\langle i,j\rangle,A} J_{xy}\left(\phi_i^\dagger T_A\phi_i\right)\left(\phi_j^\dagger T_A\phi_j\right)
$$

$$
\to\ \tilde{K}_{xy}\int \mathrm{d}x\mathrm{d}y\sum_{i=x,y}\left[|\partial_i\phi|^2+\left(\phi^\dagger\partial_i\phi\right)^2\right]
$$

以上とまとめると
渦糸格子上の有効理論 ＝ 非等方3+1d *CP(N-1)*非線形シグマモデル

$$
\mathcal{L}_{\mathrm{eff}}=\sum_{\mu=0}^{3}\tilde{K}_\mu\left[\partial^\mu\phi^\dagger\partial_\mu\phi+\left(\phi^\dagger\partial^\mu\phi\right)\left(\phi^\dagger\partial_\mu\phi\right)\right]
$$

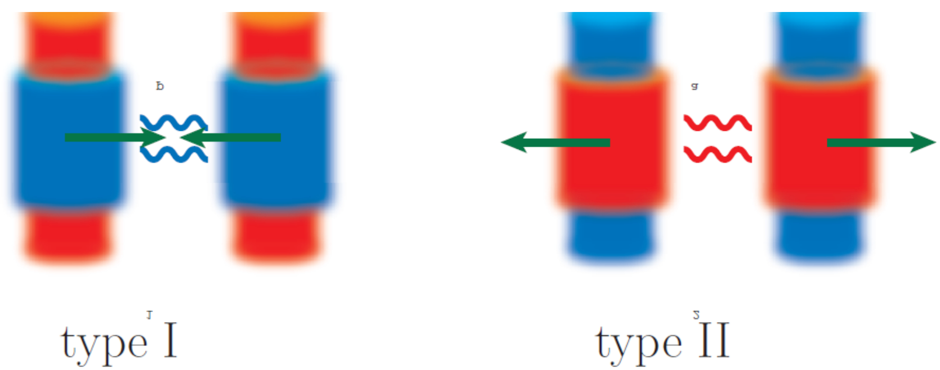
Long Range Interactions:

1) Local (gauged) Vortex Strings (3+1 dim)

Type I: $m_\Phi^{-1} > m_A^{-1} \Rightarrow$ attractive force $\Rightarrow$ unstable

Type II: $m_\Phi^{-1} < m_A^{-1} \Rightarrow$ repulsive force $\Rightarrow$ stable $\Rightarrow$ Lattice

Critical (BPS): $m_\Phi^{-1} = m_A^{-1} \Rightarrow$ no force $\Rightarrow$ moduli dynamics



2) Global Vortex Strings (3+1 dim)

Logarismic divergent energy: $E_{\mathrm{int}}(r) = 4\pi \log[\Lambda / r]$

Repulsive force: $F(r) = -\dfrac{\partial E_{\mathrm{int}}}{\partial r} = \dfrac{4\pi}{r}$