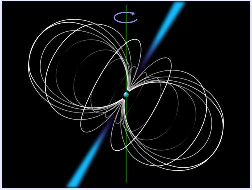
The structure of strange stars with a new quark mass scaling

Cheng-Jun Xia (夏铖君)

State Key Laboratory of Theoretical Physics,
Institute of Theoretical Physics,
Chinese Academy of Sciences, Beijing

October, 22, 2014



Collaborators

Shi-Wu Chen (陈世武) China Information Technology Security Evaluation Center.

Jia-Xun Hu (候佳旬) Institute of High Energy Physics, CAS.

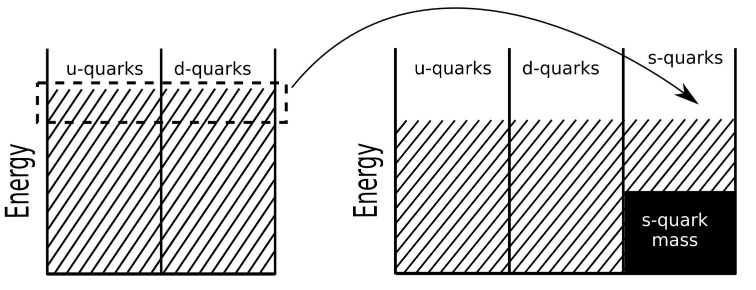
Zhen-Yan Lu (陆振烟) University of Chinese Academy of Sciences.

Guang-Xiong Peng (彭光雄) University of Chinese Academy of Sciences;
Institute of High Energy Physics, CAS.

Jian-Feng Xu (徐建峰) University of Chinese Academy of Sciences.

Contents

Strange Quark Matter (SQM)



Robert Jensen, Searches for Strange Quark Matter, March 2006

- Bodmer first suggested a low energy nuclear state called "collapsed nuclei". (Bodmer1971_PRD4-1601)
- Witten reported on the stability of strange quark matter (SQM) consisting of approximately equal numbers of *u*, *d* and *s* quarks, suggesting that SQM could indeed be stable even at zero external pressure. (Witten1984_PRD30-272)

Phenomenological models

In principle, the properties of SQM can be studied based on quantum chromodynamics (QCD). However, due to the known difficulties in the nonperturbative region, phenomenological models are essential for the study of SQM, e.g.

MIT bag model: The vacuum has a constant energy density, that is, the bag constant B provides a negative pressure to confine quarks.

Equiparticle model: The strong interaction is considered by adopting equivalent quark masses while the free energy density and particle number densities have the same form as a free particle system.

Other models: Nambu and Jona-Lasinio (NJL) model; Perturbation model; Quark-cluster model; Quasiparticle model; Global color symmetry model (GCM); Field correlator method; . . .

Equiparticle model

Quasiparticle approach: The medium created in ultrarelativistic nucleus-nucleus collisions interacts more strongly than hadron or string matter. (Peshier_Cassing2005_PRL94-172301)

1. The confinement is automatically achieved without an additional bag constant;
2. The mass scaling is related to the in-medium chiral condensate (Peng_Chiang_Yang_Li_Liu1999_PRC61-015201):

$$m_I = \frac{E_I}{\sum_i \left(\langle \bar{q}_i q_i \rangle - \langle \bar{q}_i q_i \rangle_0 \right)}, \qquad (1)$$

3. The results from more fundamental approaches (Lattice QCD and Perturbative QCD) can be incorporated.

Note: Self-consistent thermodynamic treatment is required.

Equiparticle model

Quasiparticle approach: The medium created in ultrarelativistic nucleus-nucleus collisions interacts more strongly than hadron or string matter. (Peshier_Cassing2005_PRL94-172301)

1. The confinement is automatically achieved without an additional bag constant;
2. The mass scaling is related to the in-medium chiral condensate (Peng_Chiang_Yang_Li_Liu1999_PRC61-015201):

$$m_{\mathrm{I}} = \frac{E_{\mathrm{I}}}{\sum_i \left(\langle \bar{q}_i q_i \rangle - \langle \bar{q}_i q_i \rangle_0\right)}; \quad (1)$$

3. The results from more fundamental approaches (Lattice QCD and Perturbative QCD) can be incorporated.

Note: Self-consistent thermodynamic treatment is required.

Equiparticle model

Quasiparticle approach: The medium created in ultrarelativistic nucleus-nucleus collisions interacts more strongly than hadron or string matter. (Peshier_Cassing2005_PRL94-172301)

1. The confinement is automatically achieved without an additional bag constant;
2. The mass scaling is related to the in-medium chiral condensate (Peng_Chiang_Yang_Li_Liu1999_PRC61-015201):

$$m_{\mathrm{I}} = \frac{E_{\mathrm{I}}}{\sum_i \left(\langle \bar{q}_i q_i \rangle - \langle \bar{q}_i q_i \rangle_0\right)}; \tag{1}$$

3. The results from more fundamental approaches (Lattice QCD and Perturbative QCD) can be incorporated.

Note: Self-consistent thermodynamic treatment is required.

Equiparticle model

Quasiparticle approach: The medium created in ultrarelativistic nucleus-nucleus collisions interacts more strongly than hadron or string matter. (Peshier_Cassing2005_PRL94-172301)

1. The confinement is automatically achieved without an additional bag constant;
2. The mass scaling is related to the in-medium chiral condensate (Peng_Chiang_Yang_Li_Liu1999_PRC61-015201):

$$m_{\mathrm{I}} = \frac{E_{\mathrm{I}}}{\sum_i \left(\langle \bar{q}_i q_i \rangle - \langle \bar{q}_i q_i \rangle_0\right)}; \tag{1}$$

3. The results from more fundamental approaches (Lattice QCD and Perturbative QCD) can be incorporated.

Note: Self-consistent thermodynamic treatment is required.

Equiparticle model

Quasiparticle approach: The medium created in ultrarelativistic nucleus-nucleus collisions interacts more strongly than hadron or string matter. (Peshier_Cassing2005_PRL94-172301)

1. The confinement is automatically achieved without an additional bag constant;
2. The mass scaling is related to the in-medium chiral condensate (Peng_Chiang_Yang_Li_Liu1999_PRC61-015201):

$$m_{\mathrm{I}} = \frac{E_{\mathrm{I}}}{\sum_i \left(\langle \bar{q}_i q_i \rangle - \langle \bar{q}_i q_i \rangle_0\right)}; \tag{1}$$

3. The results from more fundamental approaches (Lattice QCD and Perturbative QCD) can be incorporated.

Note: Self-consistent thermodynamic treatment is required.

Equiparticle model

Quasiparticle approach: The medium created in ultrarelativistic nucleus-nucleus collisions interacts more strongly than hadron or string matter. (Peshier_Cassing2005_PRL94-172301)

1. The confinement is automatically achieved without an additional bag constant;
2. The mass scaling is related to the in-medium chiral condensate (Peng_Chiang_Yang_Li_Liu1999_PRC61-015201):

$$m_{\mathrm{I}} = \frac{E_{\mathrm{I}}}{\sum_i \left(\langle \bar{q}_i q_i \rangle - \langle \bar{q}_i q_i \rangle_0\right)}; \tag{1}$$

3. The results from more fundamental approaches (Lattice QCD and Perturbative QCD) can be incorporated.

Note: Self-consistent thermodynamic treatment is required.

Thermodynamics of equiparticle model

The equivalent mass for quark flavor i is $m_i = m_i(n_u, n_d, n_s, T)$. For given T, V and n_i, the free energy density is then $F = F(T, V, \{n_i\}, \{m_i\})$.
Note that F and n_i have the same form as a free particle system.

Standard thermodynamics:

$$\mathrm{d}F = -S\mathrm{d}T + \left(-P - F + \sum_i \mu_i n_i\right)\frac{\mathrm{d}V}{V} + \sum_i \mu_i \mathrm{d}n_i \ . \qquad (2)$$

Equiparticle model:

$$\mathrm{d}F = \left[\frac{\partial\Omega_0}{\partial T} + \sum_i \frac{\partial\Omega_0}{\partial m_i}\frac{\partial m_i}{\partial T}\right]\mathrm{d}T + \frac{\partial\Omega_0}{\partial V}\mathrm{d}V + \sum_i\left[\mu_i^* + \sum_j \frac{\partial\Omega_0}{\partial m_j}\frac{\partial m_j}{\partial n_i}\right]\mathrm{d}n_i. \qquad (3)$$

Entropy density: $S = -\frac{\partial\Omega_0}{\partial T} - \sum_i \frac{\partial\Omega_0}{\partial m_i}\frac{\partial m_i}{\partial T}$;

Pressure: $P = -F + \sum_i \mu_i n_i - V\frac{\partial\Omega_0}{\partial V}$;

Chemical potential: $\mu_i = \mu_i^* + \sum_j \frac{\partial\Omega_0}{\partial m_j}\frac{\partial m_j}{\partial n_i}$.

C.-J. Xia, G.-X. Peng, S.-W. Chen, Z.-Y. Lu & J.-F. Xu
Phys. Rev. D, 2014, 89, 105027

History of quark mass scaling

Equivalent mass for quark flavor i: $m_i = m_{i0} + m_{\mathrm{I}}$.

1. Originally it was suggested to be an inversely linear scaling: $m_{\mathrm{I}} = \frac{B}{3n}$; (Fowler_Raha_Weiner1981_ZPC9-271)
2. Based on this scaling, in 2002 Zhang and Su included finite temperature in the scaling: $m_{\mathrm{I}} = \frac{B}{3n}\left[1 - \left(\frac{T}{T_c}\right)^2\right]$; (Zhang_Su2002_PRC65-035202)
3. Based on the in-medium chiral condensates and linear confinement, an inversely cubic scaling was derived by Peng *et al.*: $m_{\mathrm{I}} = \frac{D}{n^{1/3}}$; (Peng_Chiang_Yang_Li_Liu1999_PRC61-015201)
4. Adopting the similar method, Wen *et al.* extended this scaling to including finite temperature: $m_{\mathrm{I}} = \frac{D}{n^{1/3}}\left[1 - \frac{T}{T_c} \exp\left([illegible] \right)\right]$; (Wen_Zhong_Peng_Shen_Ning2005_PRC72-015204)
5. The one-gluon-exchange interaction was further included by Chen *et al.*: $m_{\mathrm{I}} = \frac{D}{n^{1/3}} - C n^{1/3}$; (Chen_Gao_Peng2012_CPC36-947)
6. The quark matter symmetry energy was considered by Chu and Chen: $m_{\mathrm{I}} = \frac{D}{n^{1/3}} - \tau_i \delta D_I n^{\alpha} e^{-\beta n}$; (Chu_Chen2014_ApJ780-135)

History of quark mass scaling

Equivalent mass for quark flavor i: $m_i = m_{i0} + m_\mathrm{I}$.

1. Originally it was suggested to be an inversely linear scaling: $m_\mathrm{I} = \frac{B}{3n}$; (Fowler_Raha_Weiner1981_ZPC9-271)
2. Based on this scaling, in 2002 Zhang and Su included finite temperature in the scaling: $m_\mathrm{I} = \frac{B}{3n}\left[1 - \left(\frac{T}{T_\mathrm{c}}\right)^2\right]$; (Zhang_Su2002_PRC65-035202)
3. Based on the in-medium chiral condensates and linear confinement, an inversely cubic scaling was derived by Peng *et al.*: $m_\mathrm{I} = \frac{D}{n^{1/3}}$; (Peng_Chiang_Yang_Li_Liu1999_PRC61-015201)
4. Adopting the similar method, Wen *et al.* extended this scaling to including finite temperature: $m_\mathrm{I} = \frac{D}{n^{1/3}}[1 - \frac{8T}{\lambda T_c}\exp\left(-\frac{\lambda T_c}{T}\right)]$; (Wen_Zhong_Peng_Shen_Ning2005_PRC72-015204)
5. The one-gluon-exchange interaction was further included by Chen *et al.*: $m_\mathrm{I} = \frac{D}{n^{1/3}} - Cn^{1/3}$; (Chen_Gao_Peng2012_CPC36-947)
6. The quark matter symmetry energy was considered by Chu and Chen: $m_\mathrm{I} = \frac{D}{n^{1/3}} - \tau_i \delta D_I n^\alpha e^{-\beta n}$; (Chu_Chen2014_ApJ780-135)

History of quark mass scaling

Equivalent mass for quark flavor i: $m_i = m_{i0} + m_{\rm I}$.

1. Originally it was suggested to be an inversely linear scaling: $m_{\rm I} = \frac{B}{3n}$; (Fowler_Raha_Weiner1981_ZPC9-271)
2. Based on this scaling, in 2002 Zhang and Su included finite temperature in the scaling: $m_{\rm I} = \frac{B}{3n}\left[1 - \left(\frac{T}{T_{\rm c}}\right)^2\right]$; (Zhang_Su2002_PRC65-035202)
3. Based on the in-medium chiral condensates and linear confinement, an inversely cubic scaling was derived by Peng *et al.*: $m_{\rm I} = \frac{D}{n^{1/3}}$; (Peng_Chiang_Yang_Li_Liu1999_PRC61-015201)
4. Adopting the similar method, Wen *et al.* extended this scaling to including finite temperature: $m_{\rm I} = \frac{D}{n^{1/3}}[1 - \frac{8T}{\lambda T_c}\exp\left(-\frac{\lambda T_c}{T}\right)]$; (Wen_Zhong_Peng_Shen_Ning2005_PRC72-015204)
5. The one-gluon-exchange interaction was further included by Chen *et al.*: $m_{\rm I} = \frac{D}{n^{1/3}} - Cn^{1/3}$; (Chen_Gao_Peng2012_CPC36-947)
6. The quark matter symmetry energy was considered by Chu and Chen: $m_{\rm I} = \frac{D}{n^{1/3}} - \tau_i \delta D_I n^{\alpha} e^{-\beta n}$; (Chu_Chen2014_ApJ780-135)

History of quark mass scaling

Equivalent mass for quark flavor i: $m_i = m_{i0} + m_{\rm I}$.

1. Originally it was suggested to be an inversely linear scaling: $m_{\rm I} = \frac{B}{3n}$; (Fowler_Raha_Weiner1981_ZPC9-271)
2. Based on this scaling, in 2002 Zhang and Su included finite temperature in the scaling: $m_{\rm I} = \frac{B}{3n}\left[1 - \left(\frac{T}{T_{\rm c}}\right)^2\right]$; (Zhang_Su2002_PRC65-035202)
3. Based on the in-medium chiral condensates and linear confinement, an inversely cubic scaling was derived by Peng *et al.*: $m_{\rm I} = \frac{D}{n^{1/3}}$; (Peng_Chiang_Yang_Li_Liu1999_PRC61-015201)
4. Adopting the similar method, Wen *et al.* extended this scaling to including finite temperature: $m_{\rm I} = \frac{D}{n^{1/3}}[1 - \frac{8T}{\lambda T_c}\exp\left(-\frac{\lambda T_c}{T}\right)]$; (Wen_Zhong_Peng_Shen_Ning2005_PRC72-015204)
5. The one-gluon-exchange interaction was further included by Chen *et al.*: $m_{\rm I} = \frac{D}{n^{1/3}} - Cn^{1/3}$; (Chen_Gao_Peng2012_CPC36-947)
6. The quark matter symmetry energy was considered by Chu and Chen: $m_{\rm I} = \frac{D}{n^{1/3}} - \tau_i \delta D_I n^\alpha e^{-\beta n}$; (Chu_Chen2014_ApJ780-135)

History of quark mass scaling

Equivalent mass for quark flavor i: $m_i = m_{i0} + m_{\mathrm{I}}$.

1. Originally it was suggested to be an inversely linear scaling: $m_{\mathrm{I}} = \frac{B}{3n}$; (Fowler_Raha_Weiner1981_ZPC9-271)
2. Based on this scaling, in 2002 Zhang and Su included finite temperature in the scaling: $m_{\mathrm{I}} = \frac{B}{3n}\left[1 - \left(\frac{T}{T_{\mathrm{c}}}\right)^2\right]$; (Zhang_Su2002_PRC65-035202)
3. Based on the in-medium chiral condensates and linear confinement, an inversely cubic scaling was derived by Peng *et al.*: $m_{\mathrm{I}} = \frac{D}{n^{1/3}}$; (Peng_Chiang_Yang_Li_Liu1999_PRC61-015201)
4. Adopting the similar method, Wen *et al.* extended this scaling to including finite temperature: $m_{\mathrm{I}} = \frac{D}{n^{1/3}}[1 - \frac{8T}{\lambda T_c} \exp\left(-\frac{\lambda T_c}{T}\right)]$; (Wen_Zhong_Peng_Shen_Ning2005_PRC72-015204)
5. The one-gluon-exchange interaction was further included by Chen *et al.*: $m_{\mathrm{I}} = \frac{D}{n^{1/3}} - Cn^{1/3}$; (Chen_Gao_Peng2012_CPC36-947)
6. The quark matter symmetry energy was considered by Chu and Chen: $m_{\mathrm{I}} = \frac{D}{n^{1/3}} - \tau_i \delta D_I n^{\alpha} e^{-\beta n}$; (Chu_Chen2014_ApJ780-135)

History of quark mass scaling

Equivalent mass for quark flavor i: $m_i = m_{i0} + m_{\rm I}$.

1. Originally it was suggested to be an inversely linear scaling: $m_{\rm I} = \frac{B}{3n}$; (Fowler_Raha_Weiner1981_ZPC9-271)
2. Based on this scaling, in 2002 Zhang and Su included finite temperature in the scaling: $m_{\rm I} = \frac{B}{3n}\left[1-\left(\frac{T}{T_{\rm c}}\right)^2\right]$; (Zhang_Su2002_PRC65-035202)
3. Based on the in-medium chiral condensates and linear confinement, an inversely cubic scaling was derived by Peng *et al.*: $m_{\rm I} = \frac{D}{n^{1/3}}$; (Peng_Chiang_Yang_Li_Liu1999_PRC61-015201)
4. Adopting the similar method, Wen *et al.* extended this scaling to including finite temperature: $m_{\rm I} = \frac{D}{n^{1/3}}[1 - \frac{8T}{\lambda T_c}\exp\left(-\frac{\lambda T_c}{T}\right)]$; （Wen_Zhong_Peng_Shen_Ning2005_PRC72-015204）
5. The one-gluon-exchange interaction was further included by Chen *et al.*: $m_{\rm I} = \frac{D}{n^{1/3}} - Cn^{1/3}$; (Chen_Gao_Peng2012_CPC36-947)
6. The quark matter symmetry energy was considered by Chu and Chen: $m_{\rm I} = \frac{D}{n^{1/3}} - \tau_i \delta D_I n^{\alpha} e^{-\beta n}$; (Chu_Chen2014_ApJ780-135)

History of quark mass scaling

Equivalent mass for quark flavor i: $m_i = m_{i0} + m_{\mathrm{I}}$.

1. Originally it was suggested to be an inversely linear scaling: $m_{\mathrm{I}} = \frac{B}{3n}$; (Fowler_Raha_Weiner1981_ZPC9-271)
2. Based on this scaling, in 2002 Zhang and Su included finite temperature in the scaling: $m_{\mathrm{I}} = \frac{B}{3n}\left[1-\left(\frac{T}{T_{\mathrm{c}}}\right)^2\right]$; (Zhang_Su2002_PRC65-035202)
3. Based on the in-medium chiral condensates and linear confinement, an inversely cubic scaling was derived by Peng *et al.*: $m_{\mathrm{I}} = \frac{D}{n^{1/3}}$; (Peng_Chiang_Yang_Li_Liu1999_PRC61-015201)
4. Adopting the similar method, Wen *et al.* extended this scaling to including finite temperature: $m_{\mathrm{I}} = \frac{D}{n^{1/3}}[1-\frac{8T}{\lambda T_c}\exp\left(-\frac{\lambda T_c}{T}\right)]$; (Wen_Zhong_Peng_Shen_Ning2005_PRC72-015204)
5. The one-gluon-exchange interaction was further included by Chen *et al.*: $m_{\mathrm{I}} = \frac{D}{n^{1/3}} - Cn^{1/3}$; (Chen_Gao_Peng2012_CPC36-947)
6. The quark matter symmetry energy was considered by Chu and Chen: $m_{\mathrm{I}} = \frac{D}{n^{1/3}} - \tau_i \delta D_I n^{\alpha} \mathrm{e}^{-\beta n}$; (Chu_Chen2014_ApJ780-135)

A new quark mass scaling

Adopting the similar method of Peng2005_NPA747-75, namely expanding the equivalent mass to a Laurant series of the holistic Fermi momentum ν, and take the leading term in both directions:

$$m_{\mathrm{I}} = \frac{a_{-1}}{\nu} + a_1\nu \ , \tag{4}$$

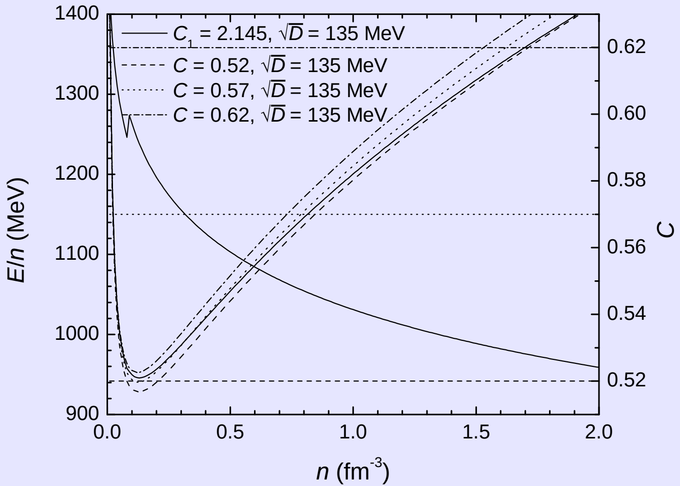
here the first term corresponds to the linear confinement, while the second term is responsible for the leading-order perturbative interactions. The mass scaling is then given by

$$m_i = m_{i0} + \frac{D}{n^{1/3}} + Cn^{1/3}, \tag{5}$$

where $C = C_1 a_1 \approx C_1\sqrt{\frac{2}{3}\alpha}$. Since the strong coupling runs logarithmically, the running rate is thus much slower and the parameter C can be taken as constant. According to the analytic coupling constant, the maximum value of α is $1/\beta_0$, then we have

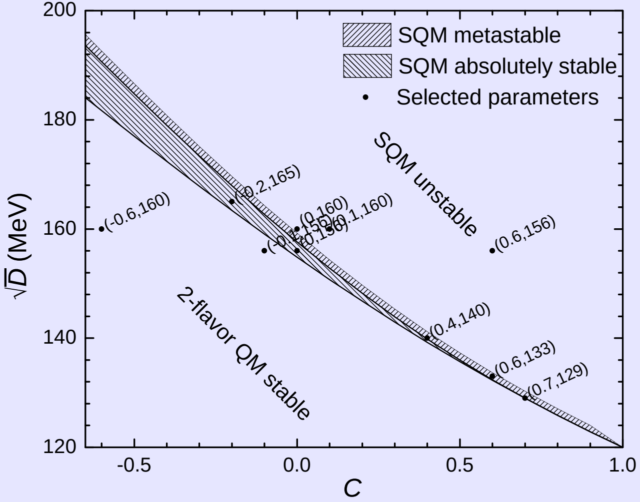
$$C < \left(\frac{3\pi^2}{N_{\mathrm{f}}}\right)^{1/3}\sqrt{\frac{2}{3\beta_0}} \approx 1.1676. \tag{6}$$

Variation range of C



1. The fluctuation of the energy per baryon is small with running coupling;
2. The one-gluon-exchange interaction reduces C at lower densities.

The range of the parameters D and C



1. The current quark masses are taken as $m_{u0} = 5$ MeV, $m_{d0} = 10$ MeV and $m_{s0} = 100$ MeV;
2. The solid dots are the combinations of parameters we are about to use.

Calculate the properties of SQM

The weak equilibrium conditions:

$$\mu_u + \mu_e = \mu_d = \mu_s. \tag{7}$$

The charge neutrality condition:

$$\frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s - n_e = 0. \tag{8}$$

The baryon number conservation:

$$n = \frac{1}{3}\left(n_u + n_d + n_s\right). \tag{9}$$

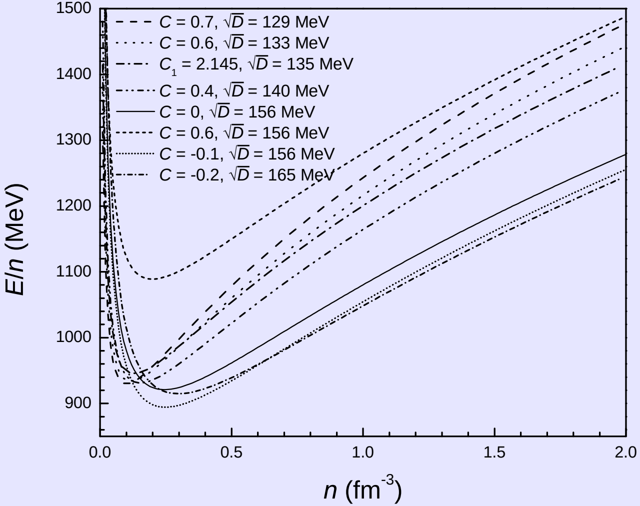
Pressure and chemical potential

Based on the newly obtained mass scaling (5), taking the temperature $T = 0$ and volume $V \to \infty$, the pressure and chemical potential can be obtained by

Pressure: $P = -\Omega_0 + n\frac{\mathrm{d}m_\mathrm{I}}{\mathrm{d}n}\frac{\partial\Omega_0}{\partial m_\mathrm{I}};$

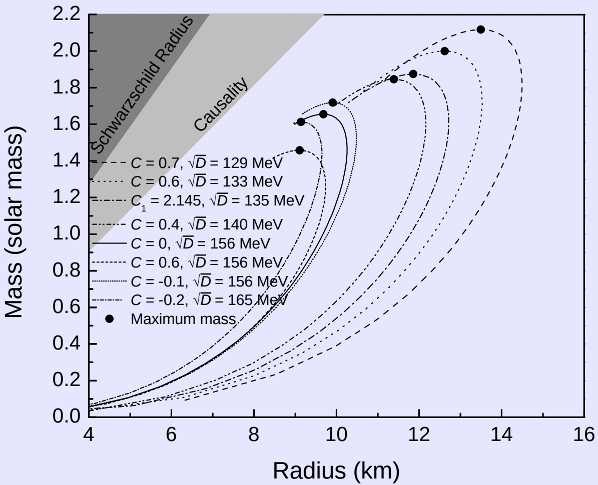
Chemical potential: $\mu_i = \mu_i^* + \frac{1}{3}\frac{\mathrm{d}m_\mathrm{I}}{\mathrm{d}n}\frac{\partial\Omega_0}{\partial m_\mathrm{I}}.$

Energy per baryon



1. The EOS becomes stiffer for parameters within the "SQM stable" area;
2. Increasing C or decreasing D also makes the EOS stiffer;
3. Zero pressure point can be smaller than nuclear saturation density, and quark hadron phase transition will occur.

Mass-radius relation of strange stars



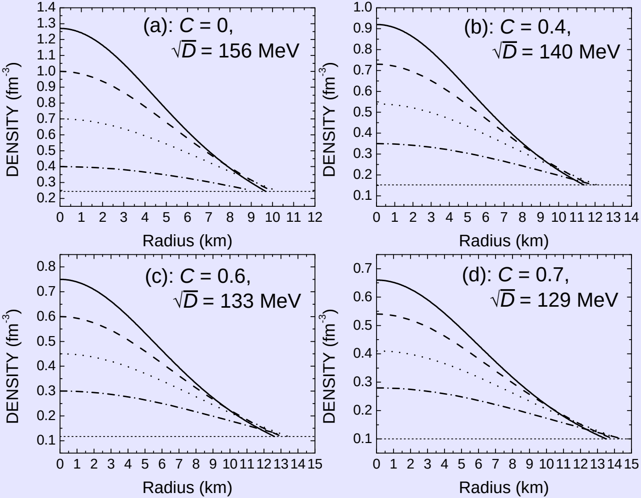
To obtain the structure of strange stars, we have used the Tolman-Oppenheimer-Volkov (TOV) Equation:

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -(P+E)\frac{G}{r^2}\frac{4\pi r^3 P + M}{1-\frac{2GM}{r}}$$

with

$$M(r) = \int_0^r 4\pi E r^2 \mathrm{d}r.$$

Density profiles



1. The uppermost curve corresponds to the largest acceptable central density;
2. The horizontal line corresponds to the surface density of the star;
3. The surface density gets even lower than nuclear saturation density, and the quark hadron phase transition should be considered.

Summary and Outlook

Summary

1. A new quark mass scaling with linear confinement and leading-order perturbative interactions is obtained by expanding the equivalent mass to a Laurant series and taking the leading terms in both directions;
2. With the new quark mass scaling and self-consistent thermodynamic treatment, we have studied the equation of state (EOS) of SQM with various combination of parameters;
3. Based on the EOS of SQM and TOV equation, we studied the structure of strange stars, where the masses and radii of PSR J1614-2230 and PSR J0348+0432 can be reproduced.

Outlook

1. Quark hadron phase transition and hybrid stars;
2. The effect of electric field on the structure of compact stars;
3. The effect of magnetic field on the structure of compact stars;
4. Compact stars at finite temperature with nonzero neutrino chemical potential;
5. . . .

Thank You!!!