

Coexistence of kaon condensation and  
hyperons in hadronic matter  
and its relevance to quark matter

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# 1. Introduction

## Multi-strangeness system in hadronic matter

In neutron-stars

hyperonic matter

( $\Lambda$ ,  $\Sigma$ ,  $\Xi$ , ...  
in the ground state)

Kaon condensation

Strange matter  
(u, d, s quark matter)

- Softening of EOS
- Rapid cooling of neutron stars

• Coexistence of antikaons and hyperons

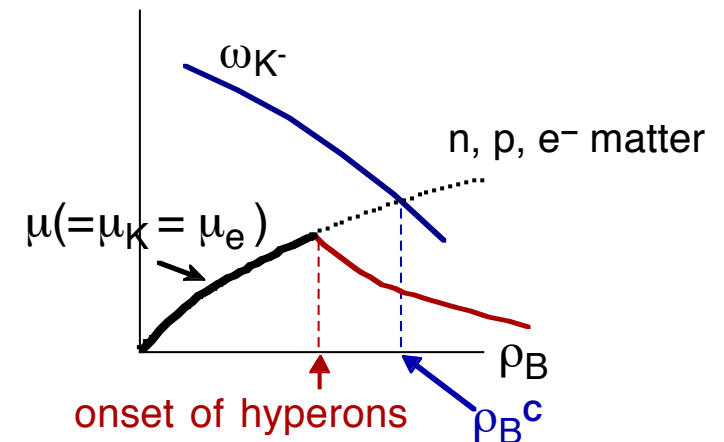
(Relativistic Mean-Field theory )

[P.J.Ellis, R.Knorren and M.Prakash, Phys. Rev. C52(1995), 3470.  
J. Schaffner and I.N.Mishustin, Phys. Rev. C53(1996), 1416.]

• Possible appearance of kaons from hyperon matter depends on kaon-baryon interactions

( possibility of third family )

[S. Banik and D. Bandyopadhyay,  
Phys. Rev. C 63 (2001) 035802; C64 (2001) 055805.]



## ( Quark Meson Coupling models)

[D. P. Menezes, P. K. Panda, C. Providencia, Phys. Rev. C 72 (2005) 035802. ]

[C. Y. Ryu, C. H. Hyun, S. W. Hong, and B. T. Kim, Phys. Rev. C 75 (2007), 055804. ]

## ( effect of $\delta$ meson)

[G.Y.Shao, Y.X.Liu, Phys. Rev. C82, 055801(2010).]

## (effective chiral Lagrangian + phenomenological Baryon-Baryon int.)

[ T. Muto, Nucl. Phys. A754 (2005) 350; Phys. Rev. C77, 015810 (2008). ]

• Most of the models including coexistent phase of Kaon condensation and hyperons predict

$$M_{\max} < 2 M_{\odot}$$

Observations

$$M(\text{PSR J1614-2230}) = 1.97 \pm 0.04 M_{\odot}$$

$$M(\text{PSR J0348+0432}) = (2.01 \pm 0.04) M_{\odot}$$

[ P. Demorest, T.Pennucci, S. Ransom, M. Roberts and J.W.T.Hessels, Nature 467 (2010) 1081.]

[J. Antoniadis et al., Science 340, 6131 (2013).]

How to reconcile theories with observations ?

We consider

coexistence of kaon condensation with hyperonic matter  
in neutron stars

based on the RMF model

coupled with nonlinear effective chiral Lagrangian,

which is the same interaction model as used in studying

multi-antikaonic nuclear bound states with hyperon-mixing  
for finite nuclei

[T. Muto, T. Maruyama and T. Tatsumi,  
Phys. Rev. C79, 035207 (2009). ]

[T. Muto, T. Maruyama and T. Tatsumi,  
Genshikaku Kenkyu 57 Supplement 3, 230(2013).]

- ➔ Repulsive effects to make the EOS stiff at high densities
- Baryon potentials composed from kaon-baryon interactions and baryon-baryon interactions
  - Consistency with observation of massive neutron stars

## 2. Outline of the model 2-1. Baryon-Baryon interaction

Baryons: ( $p, n, \Lambda, \Sigma^-, \Xi^-$ )

Mesons:  $\sigma, \omega, \rho, \sigma^*, \phi$

Relativistic mean-field theory

$$\begin{aligned} \mathcal{L}_{B,M} = & \sum_B \bar{B}(i\gamma^\mu D_\mu - m_B^*)B + \frac{1}{2} (\partial^\mu \sigma \partial_\mu \sigma - m_\sigma^2 \sigma^2) - U(\sigma) + \frac{1}{2} (\partial^\mu \sigma^* \partial_\mu \sigma^* - m_{\sigma^*}^2 \sigma^{*2}) \\ & - \frac{1}{4} \omega^{\mu\nu} \omega_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu - \frac{1}{4} R^{\mu\nu} R_{\mu\nu} + \frac{1}{2} m_\rho^2 R^\mu R_\mu - \frac{1}{4} \phi^{\mu\nu} \phi_{\mu\nu} + \frac{1}{2} m_\phi^2 \phi^\mu \phi_\mu \\ & - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}, \end{aligned} \quad m_B^*(r) = m_B - g_{\sigma B} \sigma(r) - g_{\sigma^* B} \sigma^*(r)$$

parameters

$$D^\mu \equiv \partial^\mu + ig_{\omega B} \omega^\mu + ig_{\rho B} \vec{\tau} \cdot \vec{R}^\mu + ig_{\phi B} \phi^\mu + iQA^\mu$$

--- NN interaction --- gross features of normal nuclei and nuclear matter

- saturation properties of nuclear matter ( $\rho_0 = 0.153 \text{ fm}^{-3}$ )
- binding energy of nuclei and proton-mixing ratio
- density distributions of p and n

$g_{\sigma N}$   
 $g_{\omega N}, g_{\rho N}$

--- vector meson couplings for Y --- SU(6) symmetry

$$\begin{aligned} g_{\omega\Lambda} = g_{\omega\Sigma^-} = 2g_{\omega\Xi^-} = \frac{2}{3}g_{\omega N} & \quad g_{\phi\Lambda} = g_{\phi\Sigma^-} = \frac{1}{2}g_{\phi\Xi^-} = -\frac{\sqrt{2}}{3}g_{\omega N} \\ g_{\rho\Lambda} = 0 \quad g_{\rho\Sigma^-} = 2g_{\rho\Xi^-} = 2g_{\rho N} & \end{aligned}$$

--- scalar meson couplings for Y --- **Hyperon potentials deduced from hypernuclear experiments**  
 ( analysis of  $\Lambda$  single-particle orbitals )

$$U_{\Lambda}^N(\rho_0) = -g_{\sigma\Lambda}\sigma + g_{\omega\Lambda}\omega_0 = -27 \text{ MeV} \rightarrow g_{\sigma\Lambda} = 3.84$$

• ( $K^-$ ,  $\pi^{\pm}$ ) at BNL  $\rightarrow T=3/2$  state: strongly repulsive

[ J. Dabrowski, Phys. Rev. C60 (1999), 025205. ]  $V_{\Sigma^-}(k_{\Sigma}) = V_0(k_{\Sigma}) - \frac{1}{2}V_1(k_{\Sigma}) \cdot \frac{Z - N}{A}$   
 23.5 MeV    80.4 MeV

• ( $\pi$ ,  $K^+$ ) at KEK

[ H. Noumi et al., Phys. Rev. Lett. 89 (2002), 072301; ibid 90(2003), 049902(E). ]

• analysis of  $\Sigma^-$  atoms : repulsive [ C. J. Batty, E. Friedman, A. Gal, Phys. Rep. 287 (1997), 385. ]

$$U_{\Sigma^-}^N(\rho_0) = -g_{\sigma\Sigma^-}\sigma + g_{\omega\Sigma^-}\omega_0 = 23.5 \text{ MeV} \text{ repulsive case } \rightarrow g_{\sigma\Sigma^-} = 2.28$$

$$U_{\Xi^-}^N(\rho_0) = -g_{\sigma\Xi^-}\sigma + g_{\omega\Xi^-}\omega_0 = -16 \text{ MeV} \rightarrow g_{\sigma\Xi^-} = 2.0$$

[ T. Fukuda et al., Phys. Rev. C58 (1998), 1306. ,  
 P. Khaustov et al., Phys. Rev. C61 (2000), 054603. ]

$$g_{\sigma^*N} = g_{\sigma^*\Lambda} = g_{\sigma^*\Sigma^-} = g_{\sigma^*\Xi^-} = 0$$

## 2-2 $\bar{K} - B, \bar{K} - \bar{K}$ interactions

[ D. B. Kaplan and A. E. Nelson,  
Phys. Lett. B 175 (1986) 57. ]

$SU(3)_L \times SU(3)_R$  chiral effective Lagrangian

$$\mathcal{L} = \frac{1}{4} f^2 \text{Tr} \partial^\mu \Sigma^\dagger \partial_\mu \Sigma + \frac{1}{2} f^2 \Lambda_{\chi SB} (\text{Tr} M (\Sigma - 1) + \text{h.c.})$$

Baryons

$$+ \text{Tr} \bar{\Psi} (i \not{\partial} - m_B) \Psi + \text{Tr} \bar{\Psi} i \gamma^\mu [V_\mu, \Psi] + D \text{Tr} \bar{\Psi} \gamma^\mu \gamma^5 \{A_\mu, \Psi\}$$

$\Psi \longrightarrow$  (p, n,  
 $\Lambda, \Xi^-, \Sigma^-)$

$$+ F \text{Tr} \bar{\Psi} \gamma^\mu \gamma^5 [A_\mu, \Psi] + a_1 \text{Tr} \bar{\Psi} (\xi M^\dagger \xi + \text{h.c.}) \Psi$$

$$+ a_2 \text{Tr} \bar{\Psi} \Psi (\xi M^\dagger \xi + \text{h.c.}) + a_3 (\text{Tr} M \Sigma + \text{h.c.}) \text{Tr} \bar{\Psi} \Psi,$$

$$\left[ M = \text{diag}(m_u, m_d, m_u) \right]$$

Meson fields ( $K^\pm$ )

$$\Sigma \equiv e^{2i\Pi/f}$$

Vector current  $V^\mu = \frac{1}{2} (\xi^\dagger \partial^\mu \xi + \xi \partial^\mu \xi^\dagger)$

$$\Pi = \pi_a T_a = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & K^+ \\ 0 & 0 & 0 \\ K^- & 0 & 0 \end{pmatrix}$$

Axial-vector current  $A^\mu = \frac{i}{2} (\xi^\dagger \partial^\mu \xi - \xi \partial^\mu \xi^\dagger)$

( $\xi \equiv \Sigma^{1/2} = e^{i\pi_a T_a/f}$ )

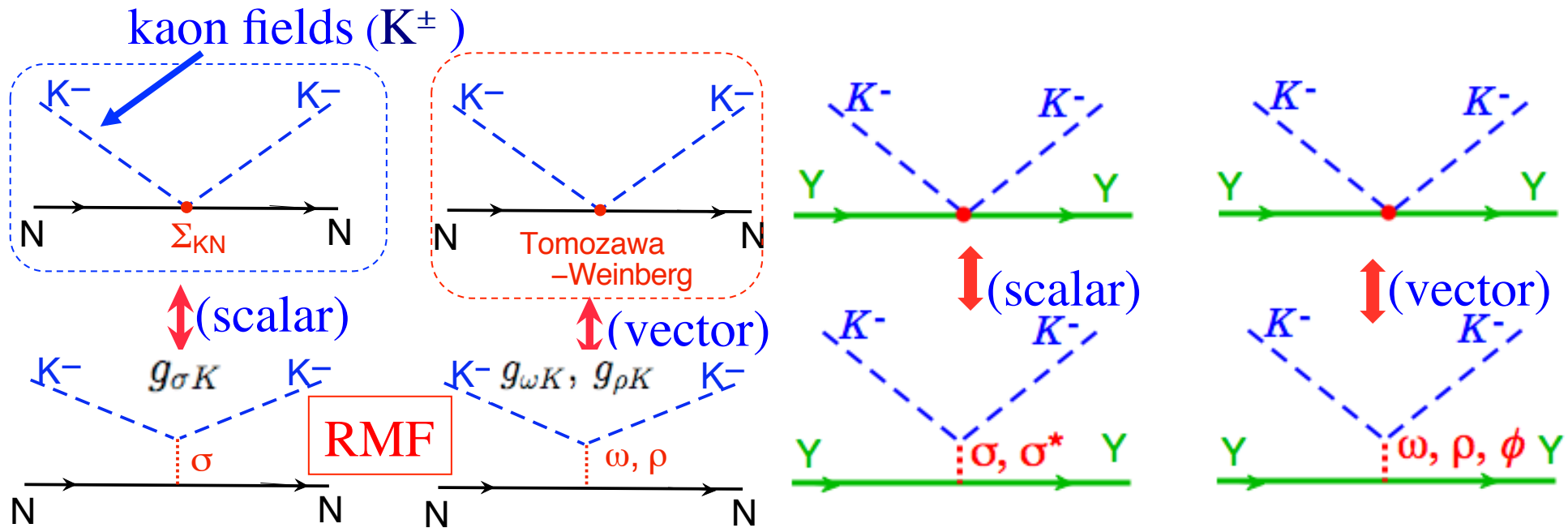
Classical K-field

$$K^\pm = \frac{f}{\sqrt{2}} \theta \exp(\pm i\mu_K t)$$

Meson decay constant

$$f = 93 \text{ MeV}$$

$\mu_K$ : kaon chemical potential



Kaonic part of the Lagrangian density

$$\begin{aligned}
 \mathcal{L}_{KB} = & \frac{1}{2} \left\{ 1 + \left( \frac{\sin \theta}{\theta} \right)^2 \right\} \partial^\mu K^+ \partial_\mu K^- + \frac{1 - \left( \frac{\sin \theta}{\theta} \right)^2}{2f^2 \theta^2} \left\{ (K^+ \partial_\mu K^-)^2 + (K^- \partial_\mu K^+)^2 \right\} \\
 & - \left\{ m_K^2 - 2m_K (g_{\sigma K} \sigma + g_{\sigma^* K} \sigma^*) \right\} \left( \frac{\sin(\theta/2)}{\theta/2} \right)^2 K^+ K^- \\
 & + i (g_{\omega K} \omega_0 + g_{\rho K} R_0 + g_{\phi K} \phi_0) \left( \frac{\sin(\theta/2)}{\theta/2} \right)^2 (K^+ \partial_\mu K^- - \partial_\mu K^+ K^-)
 \end{aligned}$$

S wave scalar int.  
 S wave vector int.



# parameters

--- vector meson couplings for Kaon ---

Corresponding to  
the Tomozawa-Weinberg term

$$X_0 \equiv g_{\omega K}\omega_0 + g_{\rho K}R_0 + g_{\phi K}\phi_0 \sim \frac{1}{2f^2} \left( \rho_p + \frac{1}{2}\rho_n - \frac{1}{2}\rho_{\Sigma^-} - \rho_{\Xi^-} \right)$$

$$2f^2 \left( \frac{g_{\omega K}}{m_\omega^2} g_{\omega N} + \frac{g_{\rho K}}{m_\rho^2} g_{\rho N} \right) = 1 \quad \text{for } p$$

$$2f^2 \left( \frac{g_{\omega K}}{m_\omega^2} g_{\omega N} - \frac{g_{\rho K}}{m_\rho^2} g_{\rho N} \right) = \frac{1}{2} \quad \text{for } n$$

$$2f^2 \left( \frac{g_{\omega K}}{m_\omega^2} g_{\omega \Lambda} + \frac{g_{\rho K}}{m_\rho^2} g_{\rho \Lambda} + \frac{g_{\phi K}}{m_\phi^2} g_{\phi \Lambda} \right) = 0 \quad \text{for } \Lambda$$

$$2f^2 \left( \frac{g_{\omega K}}{m_\omega^2} g_{\omega \Sigma^-} - \frac{g_{\rho K}}{m_\rho^2} g_{\rho \Sigma^-} + \frac{g_{\phi K}}{m_\phi^2} g_{\phi \Sigma^-} \right) = -\frac{1}{2} \quad \text{for } \Sigma^-$$

$$2f^2 \left( \frac{g_{\omega K}}{m_\omega^2} g_{\omega \Xi^-} - \frac{g_{\rho K}}{m_\rho^2} g_{\rho \Xi^-} + \frac{g_{\phi K}}{m_\phi^2} g_{\phi \Xi^-} \right) = -1 \quad \text{for } \Xi^-$$

$$g_{\omega K} = 3.05$$

$$g_{\rho K} = 2.00$$

$$g_{\phi K} = 7.33$$

quark – isospin  
counting rule  
SU(6) symmetry

$$g_{\omega K} = g_{\omega N}/3 = 2.90$$

$$g_{\rho K} = g_{\rho N} = 4.26$$


$$g_{\phi K} = 6.04/\sqrt{2}$$

--- scalar meson couplings for Kaon ---

$$g_{\sigma^*K} = 2.65/2 \quad : \text{Decay of } f_0(975)$$

$K^-$  optical potential depth :

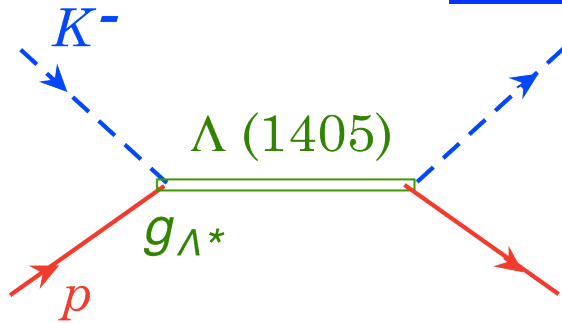
$$U_K = -(g_{\sigma K}\sigma + g_{\omega K}\omega_0) \text{ at } \rho_0$$



$$U_K = (-180, -120, -80) \text{ MeV}$$

(extremely, moderately, weakly) attractive

## Effect of $\Lambda(1405)$ and range terms



+  $O(\mu^2)$

$$d_p = (d_1 + d_2)/(2f^2) = d_{\Xi^-}$$

$$d_n = d_1/(2f^2) = d_{\Sigma^-}$$

$$d_\Lambda = (d_1 + 5d_2/6)/(2f^2)$$

Energy correction

$$\Delta\epsilon = -\frac{1}{2}(f\mu \sin\theta)^2 \left[ \rho_p^s \left\{ d_p + \frac{g_{\Lambda^*}^2}{2f^2} \frac{m_{\Lambda^*} - m_N - \mu}{(m_{\Lambda^*} - m_N - \mu)^2 + \gamma_{\Lambda^*}^2} \right\} + d_n \rho_n^s + d_\Lambda \rho_\Lambda^s + d_{\Sigma^-} \rho_{\Sigma^-}^s + d_{\Xi^-} \rho_{\Xi^-}^s \right]$$

Effective baryon masses

On-shell S-wave KN scattering lengths

$$a(K^-p) = \frac{1}{4\pi f^2(1 + m_K/m_N)} \left( \Sigma_{KN} + m_K + d_p f^2 m_K^2 + \frac{g_{\Lambda^*}^2}{2} \frac{m_K^2}{m_{\Lambda^*} - m_N - m_K - i\gamma_{\Lambda^*}} \right)$$

$$= (-0.67 + i0.64) \text{ fm}$$

KN sigma term  $\longleftrightarrow \Sigma_{KN} = \frac{2m_K f^2 g_{\sigma K} g_{\sigma N}}{m_\sigma^2}$

$$a(K^-n) = \frac{1}{4\pi f^2(1 + m_K/m_N)} \left( \Sigma_{KN} + \frac{1}{2}m_K + d_n f^2 m_K^2 \right)$$

$$= (0.37 + i0.60) \text{ fm}$$

$$a(K^+p) = \frac{1}{4\pi f^2(1 + m_K/m_N)} \left( \Sigma_{KN} - m_K + d_p f^2 m_K^2 + \frac{g_{\Lambda^*}^2}{2} \frac{m_K^2}{m_{\Lambda^*} - m_N + m_K - i\gamma_{\Lambda^*}} \right)$$

$$= -0.33 \text{ fm}$$

$$a(K^+n) = \frac{1}{4\pi f^2(1 + m_K/m_N)} \left( \Sigma_{KN} - \frac{1}{2}m_K + d_n f^2 m_K^2 \right)$$

$$= -0.16 \text{ fm}$$

$$d_p = (0.351 - \Sigma_{KN}/m_K)/(f^2 m_K)$$

$$d_n = (0.130 - \Sigma_{KN}/m_K)/(f^2 m_K)$$

$$g_{\Lambda^*} = 0.583 \quad \gamma_{\Lambda^*} = 12.4 \text{ MeV}$$

## 2-3. Thermodynamic potential

$$\Omega = \int d^3r \mathcal{H}(r) + \mu_Q \hat{Q} + \nu \hat{N}_B$$

$$\delta\Omega = 0 \quad \text{as} \quad \rho_a \rightarrow \rho_a + \delta\rho_a$$

$$(a = K^-, p, n, \Lambda, \Sigma^-, \Xi^-)$$

$$\mu_a = \frac{\partial \mathcal{E}}{\partial \rho_a}$$

$$\mu_K = \mu_e = \mu_Q$$

$$\mu_p = -(\mu_Q + \nu)$$

$$\mu_n = \mu_\Lambda = -\nu$$

$$\mu_{\Xi^-} = \mu_{\Sigma^-} = \mu_Q - \nu$$

- charge neutrality
- baryon number conservation

Chemical equilibrium  
for weak processes

$$\mu_K = \mu_e$$

$$\mu_n = \mu_p + \mu_e$$

$$\mu_n = \mu_\Lambda$$

$$\mu_n + \mu_e = \mu_{\Sigma^-}$$

$$\mu_\Lambda + \mu_e = \mu_{\Xi^-}$$

Onset condition of kaon condensation (assumed continuous phase transition)

Lowest  $K^-$  energy:  $\omega_{K^-} = \mu_{K^-} = \mu_e (= \mu_Q)$

## Equations of motion for meson fields

### Scalar mean fields

$$m_\sigma^2 \sigma = -\frac{dU}{d\sigma} + g_{\sigma N}(\rho_n^s + \rho_p^s) + g_{\sigma \Lambda} \rho_\Lambda^s + g_{\sigma \Sigma^-} \rho_{\Sigma^-}^s + 2f^2 g_{\sigma K} m_K (1 - \cos \theta)$$

$$m_{\sigma^*}^2 \sigma^* = g_{\sigma^* \Lambda} \rho_\Lambda^s + g_{\sigma^* \Sigma^-} \rho_{\Sigma^-}^s + 2f^2 g_{\sigma^* K} m_K (1 - \cos \theta)$$

### Vector mean fields

$$m_\omega^2 \omega_0 = g_{\omega N}(\rho_n + \rho_p) + g_{\omega \Lambda} \rho_\Lambda + g_{\omega \Sigma^-} \rho_{\Sigma^-} - 2f^2 g_{\omega K} \mu_K (1 - \cos \theta)$$

$$m_\rho^2 R_0 = g_{\rho N}(\rho_p - \rho_n) + g_{\rho \Lambda} \rho_\Lambda - g_{\rho \Sigma^-} \rho_{\Sigma^-} - 2f^2 g_{\rho K} \mu_K (1 - \cos \theta)$$

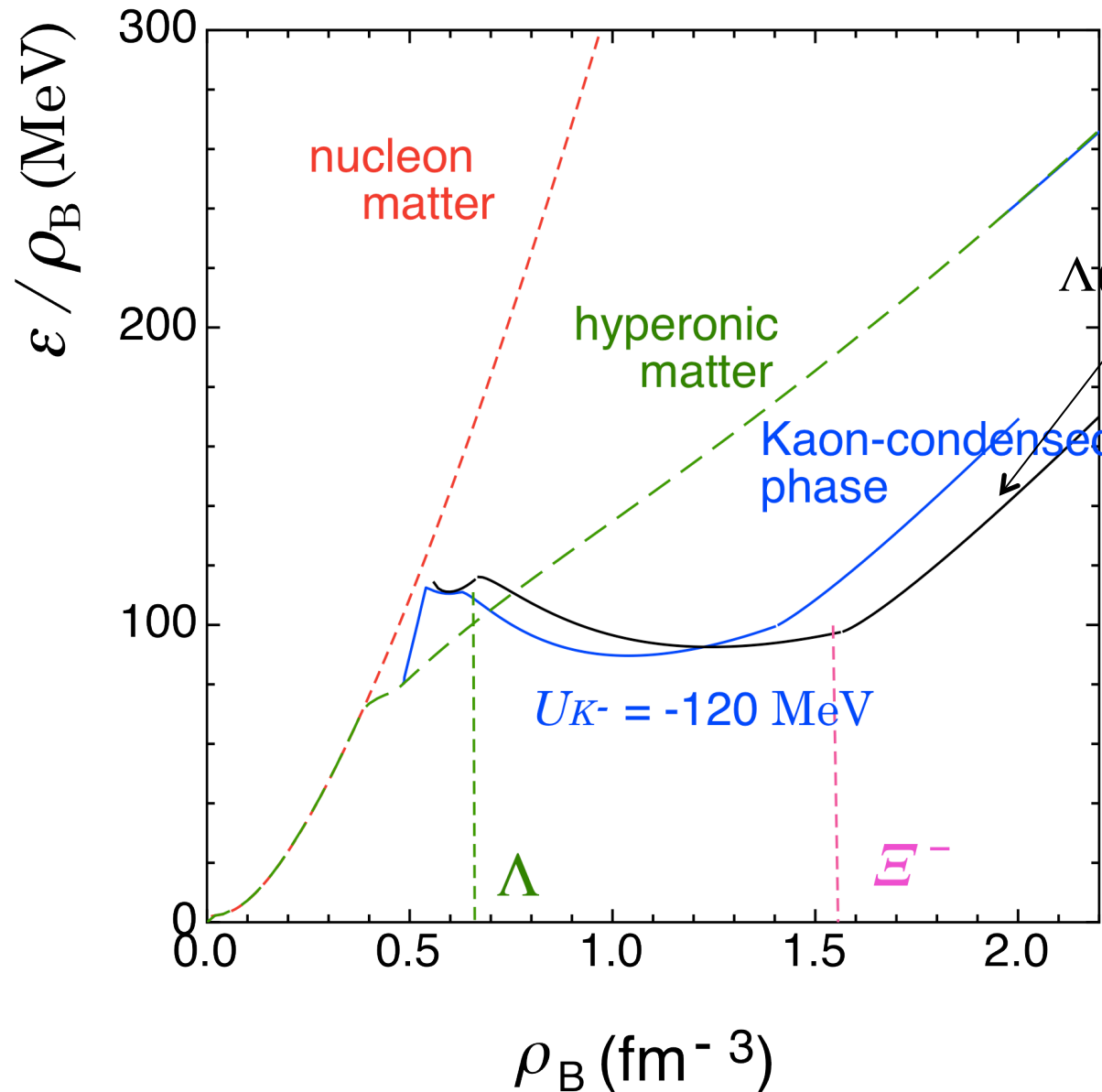
$$m_\phi^2 \phi_0 = g_{\phi \Lambda} \rho_\Lambda + g_{\phi \Sigma^-} \rho_{\Sigma^-} - 2f^2 g_{\phi K} \mu_K (1 - \cos \theta)$$

## 3. Numerical results

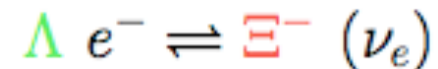
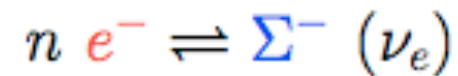
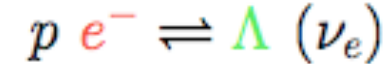
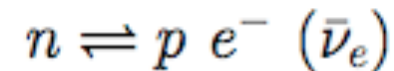
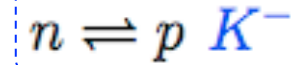
### 3. Results

### 3-1. EOS in $\beta$ -equilibrated matter

### Energy per particle



chemical equilibrium  
for weak processes





# $\overline{K}$ - Baryon interactions in $K^-$ field equation

$\delta\Omega/\delta\theta(r) = 0$   $K^-$  field equation

$$m_K^2 - 2g_{\sigma K}m_K\sigma - 2g_{\sigma^* K}m_K\sigma^* - 2\mu_K(g_{\omega K}\omega_0 + g_{\rho K}R_0 + g_{\phi K}\phi_0) - \mu_K^2 \cos\theta = 0$$

S-wave scalar int.

$$m_K^{*2} \equiv m_K^2 - 2g_{\sigma K}m_K\sigma - 2g_{\sigma^* K}m_K\sigma^*$$

$$( U_{K^-} = -g_{\sigma K}\sigma - g_{\omega K}\omega_0 )$$

S-wave vector int.

$$X_0 \equiv g_{\omega K}\omega_0 + g_{\rho K}R_0 + g_{\phi K}\phi_0$$

Chiral symmetry

## S-wave vector int.

$$\begin{aligned}\mu X_0 &= \mu(g_{\omega K}\omega_0 + g_{\rho K}R_0 + g_{\phi K}\phi_0) \\ &= \frac{\mu}{2f^2} \left( \rho_p + \frac{1}{2}\rho_n - \frac{1}{2}\rho_{\Sigma^-} - \rho_{\Xi^-} \right) \\ &\quad - 2 \left[ \left( \frac{g_{\omega K}}{m_\omega} \right)^2 + \left( \frac{g_{\rho K}}{m_\rho} \right)^2 + \left( \frac{g_{\phi K}}{m_\phi} \right)^2 \right] (\mu f)^2 (1 - \cos \theta)\end{aligned}$$

Equations of motion for vector mean fields

$$m_\omega^2 \omega_0 = g_{\omega N}(\rho_n + \rho_p) + g_{\omega \Lambda} \rho_\Lambda + g_{\omega \Sigma^-} \rho_{\Sigma^-} - 2f^2 g_{\omega K} \mu_K (1 - \cos \theta)$$

$$m_\rho^2 R_0 = g_{\rho N}(\rho_p - \rho_n) + g_{\rho \Lambda} \rho_\Lambda - g_{\rho \Sigma^-} \rho_{\Sigma^-} - 2f^2 g_{\rho K} \mu_K (1 - \cos \theta)$$

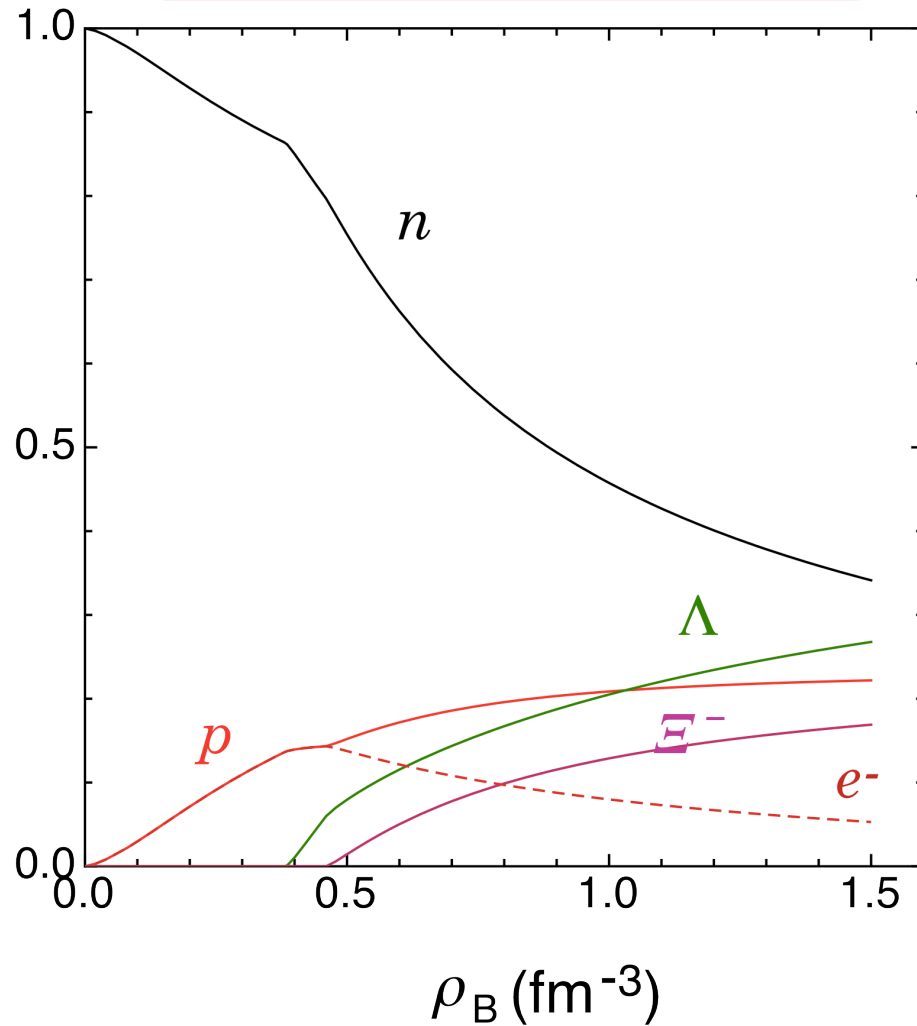
$$m_\phi^2 \phi_0 = g_{\phi \Lambda} \rho_\Lambda + g_{\phi \Sigma^-} \rho_{\Sigma^-} - 2f^2 g_{\phi K} \mu_K (1 - \cos \theta)$$

Vector interaction between  $K^-$  mesons and hyperons ( $\Sigma^-$  and  $\Xi^-$ ) works repulsively as far as  $\mu > 0$ , unfavorable for coexistence.

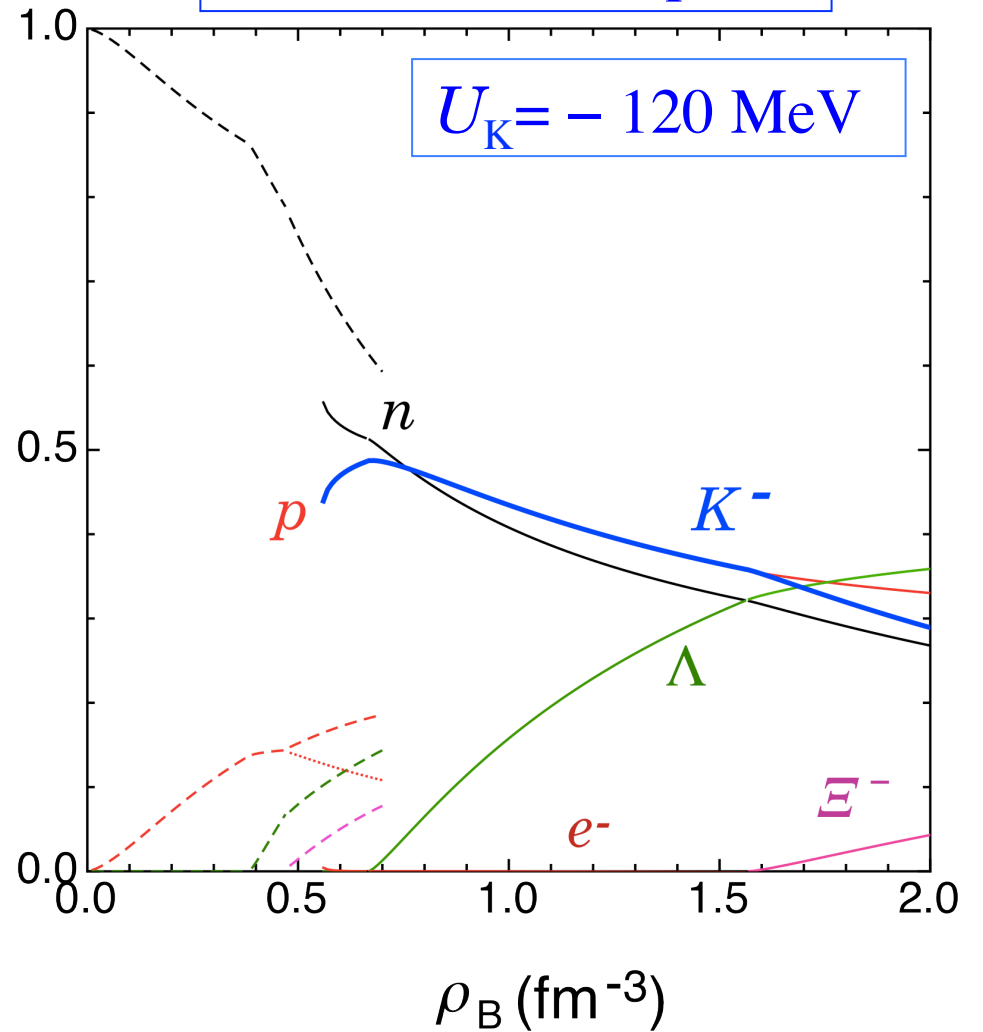
Development of kaon condensates suppresses vector attraction

## 3-2. Particle fractions

Normal hyperonic matter



Kaon-condensed phase



## Problems: Too soft EOS resulting from coexistence of kaon condensates and hyperons

$K^-$ -baryon attractive interactions, especially, **the S-wave scalar attractions** lead to additional softening of the EOS as compared with the case of **hyperonic matter**.

The soft EOS cannot give massive compact stars ( $\sim 2 M_{\odot}$ ).

**Strong repulsion between baryons and suppression of attractive  $\bar{K}$  - B interaction** at high densities are needed.

## 4. Discussion and summary

### Repulsive effects on EOS

(1) Kaon-Baryon sector -- suppression of  $K^-$  - B attractions ---

(i)  $K^-$  potential depth  $U_K \sim -87 \text{ MeV}$   $\longrightarrow$   $\rho_B^C \sim 12 \rho_0$   
(  $\langle N | \bar{s}s | N \rangle \sim 0$  ,  $\Sigma_{KN} \sim 280 \text{ MeV}$  )

$U_K = -120 \text{ MeV}$   $\longrightarrow$   $\rho_B^C \sim 4.5 \rho_0$

(ii) S-wave  $K^-$  - B vector int.

Consequence from chiral symmetry

Vector interaction between  $K^-$  mesons and hyperons ( $\Sigma^-$  and  $\Xi^-$ ) works repulsively as far as  $\mu > 0$ , leading to suppression of kaon condensates

Development of kaon condensates suppresses vector attraction

But, such suppression of  $K^-$  - baryon attractions is not enough for making the EOS stiffer.

**Strong repulsion between baryons** at high densities are needed.

(Recent Lattice QCD)  $\bar{s}s$  content in the nucleon is small.

[R. D. Young, A. W. Thomas, Nucl. Phys. A844(2010) 266c.]


$$\Sigma_{KN} = \frac{1}{2}(m_u + m_s)\langle N|\bar{u}u + \bar{s}s|N\rangle$$

$$\approx \frac{1}{2}\left(1 + \frac{m_s}{m_u}\right)\left(\frac{\Sigma_{\pi N}}{1 + m_d/m_u} + \frac{m_u}{m_s}\sigma_s\right) \quad \sigma_s = m_s\langle N|\bar{s}s|N\rangle \sim 0$$

$$\Sigma_{\pi N} = m_l\langle N|\bar{u}u + \bar{d}d|N\rangle \quad m_l = \frac{1}{2}(m_u + m_d)$$

In case  $\Sigma_{\pi N} = 64 \pm 7$  MeV [M.M.Pavan et al., PiN Newslett. 16, 110(2002).]

→ Scalar int.:  $\Sigma_{KN} \sim 280$  MeV for  $m_s/m_l = 25$ ,  $m_d/m_u = 2$   
 $(U_K \sim -80\text{MeV})$

• In flight (K<sup>-</sup>, N) KEK, BNL   
 deep K-nucleus potential,  $U_K \sim -200$  MeV [ T. Kishimoto et al.,  
 (analysis of missing mass spectra) Prog. Theor. Phys. 118 (2007), 181. ]

# Making EOS stiffer at high density

## (2) Baryon-baryon sector

(i) Phenomenological universal YNN, YYN, YYY repulsions

[ S. Nishizaki, Y. Yamamoto and T. Takatsuka,

Prog. Theor.Phys. 108 (2002) 703. ] )

(cf : RMF extended to BMM, MMM type diagrams)

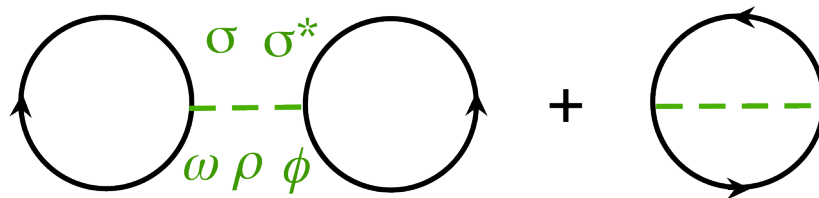
[K. Tsubakihara and A. Ohnishi, arXiv:1211.7208.]

(ii) relativistic Hartree-Fock

Introduction of tensor coupling of vector mesons

Cf. for hyperonic matter,

[T. Miyatsu, T. Katayama, K. Saito, Phys. Lett.B709 242(2012).]



## Connection to quark matter

### (3) Relation between kaon condensation in hadronic matter and that in quark matter

Hadron phase and quark phase are connected with cross-over region

 Massive stars ( $\sim 2 M_{\odot}$ )

[ K. Masuda, T. Hatsuda, T. Takatsuka, *Astrophys. J. Lett.* 764, 12 (2013).]

Coexistence of kaon-condensates  
and hyperons for hadronic phase  Strange quark matter  
(kaon-condensates in quark matter)