



廈門大學  
XIAMEN UNIVERSITY  
UNIVERSITAS AMOIENSIS

# Boiling of primordial quark nuggets in the early universe

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Cf. A. Li\*, T. Liu, P. Gubler and R. X. Xu, *Astropart. Phys.* 62, 115 (2015)

*"Quarks and Compact Stars"*

KIAA, Peking University, Beijing, China, Oct. 20-22, 2014

# Outline

- **Motivation**
- **Introduction**
  - The order of QCD phase transition
- **Boiling of primordial quark nuggets to hadrons**
  - Classical nucleation model; BHF for the hadron phase; CDDM for the quark phase.
  - The results
- **Summary and future plans**

# Motivation: Cosmic separation of phases

Witten, *PRD* 1984

- **(proposed)** Much of the baryon(A) number of the Universe condensed into quark nuggets (QNs) during the quark-hadron phase transition;
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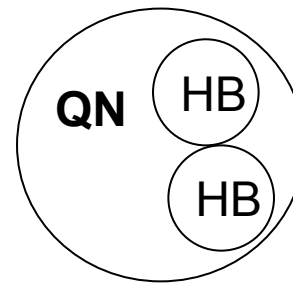
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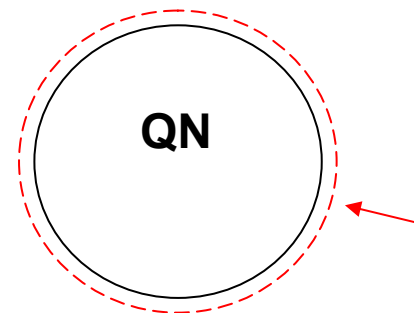
(nucleation of hadronic bubbles, namely **quark-hadron** phase transition);



Hot QCD environment ( $e^\pm, \nu, \gamma$ )

## 2. Surface evaporation

(depend on the dynamic of the neutrino-driven cooling)



Baryon accumulation

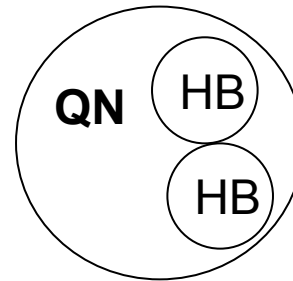
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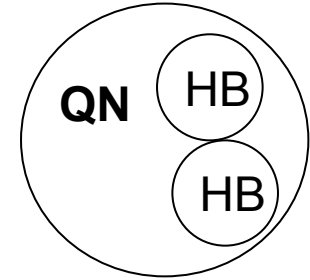
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## General arguments:

- Large  $A$ , easy to boil;
- Small  $A$ , easy to evaporate.

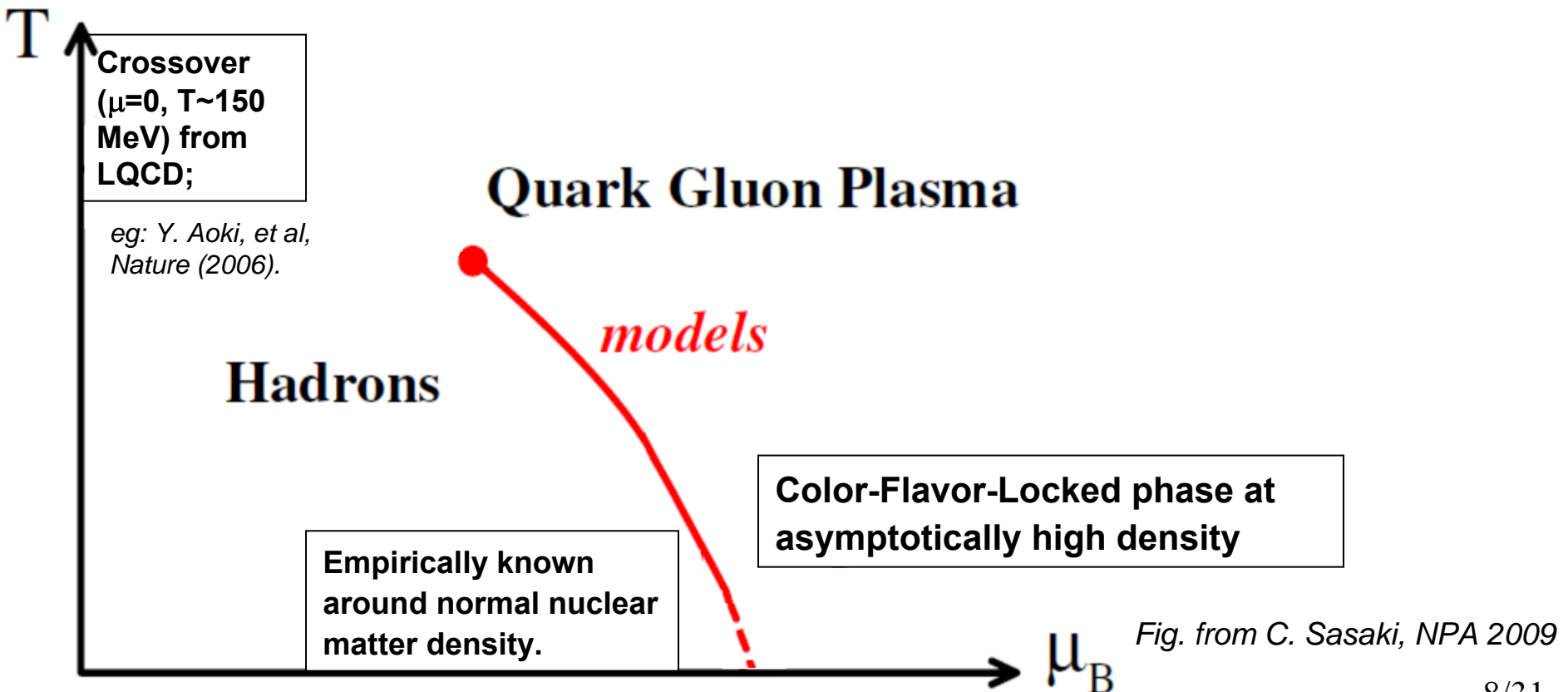
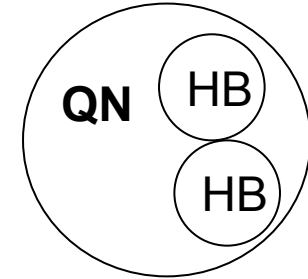
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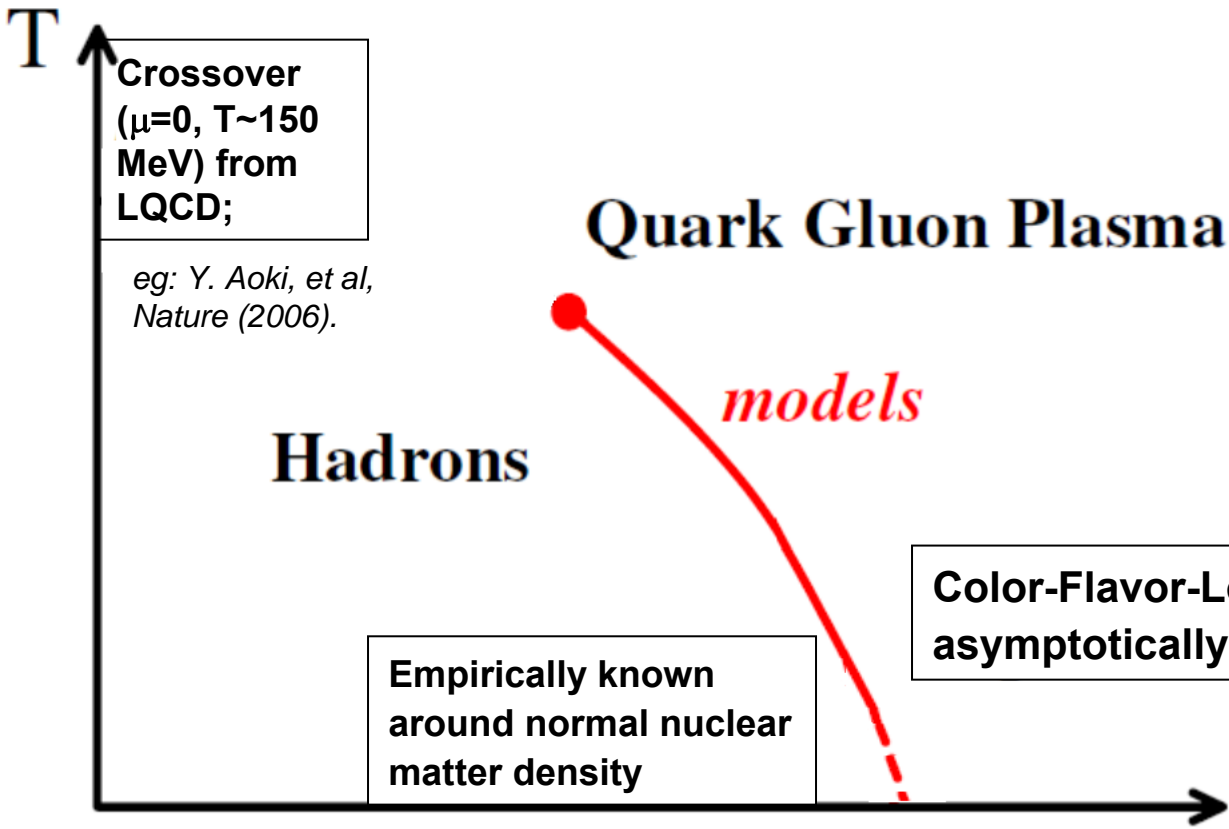
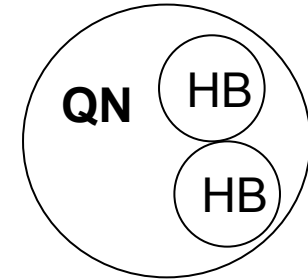
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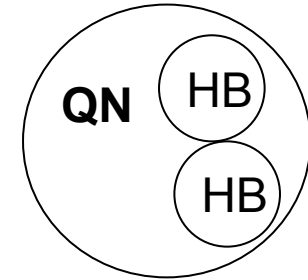
**Unknown:**

- The **order** of the QCD phase transition at  $\mu \neq 0$ ?
- The existence of a QCD critical point? Its location?
- The coincidence of the transition of chiral & deconfinement?
- ...

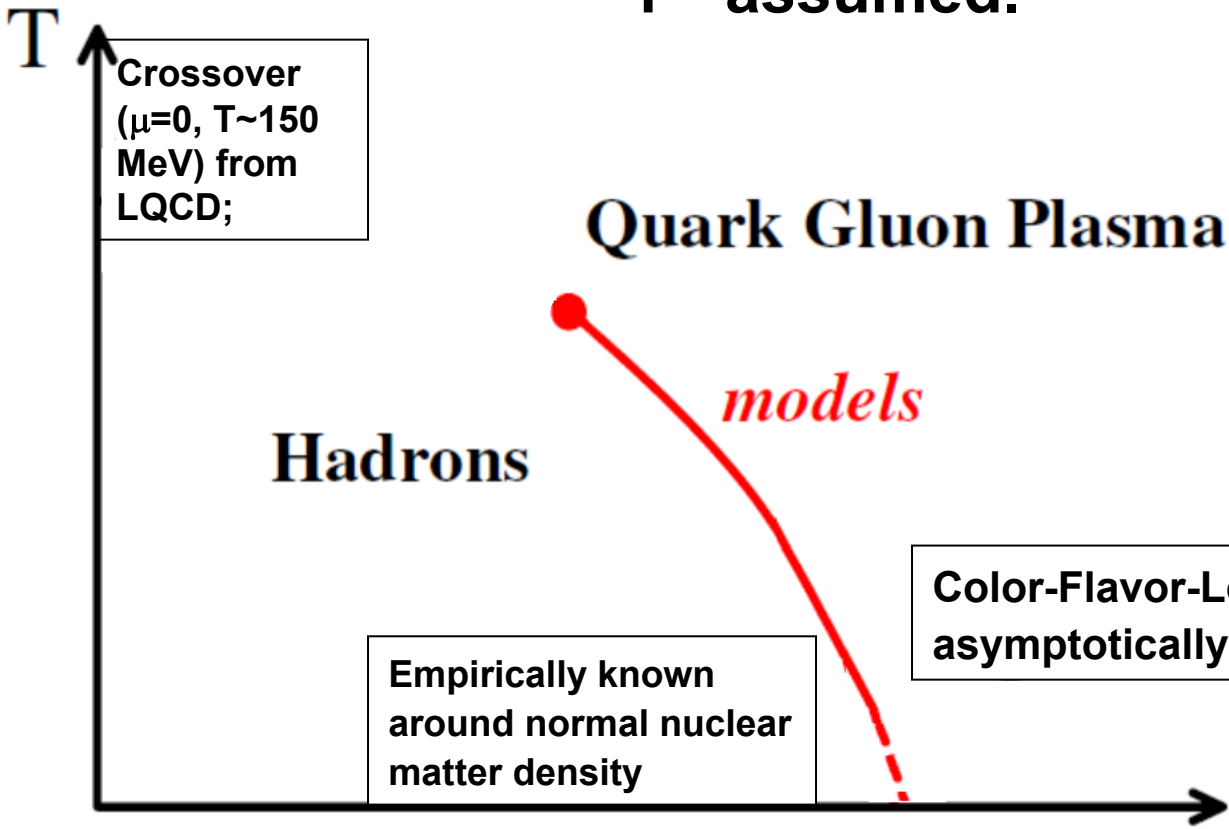
Fig. from C. Sasaki, NPA 2009

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1<sup>st</sup> assumed.



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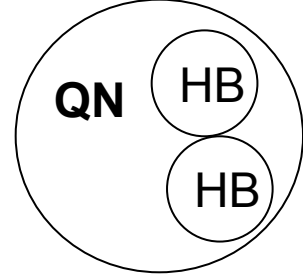
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# **Boiling of primordial quark nuggets**

**in the early universe ( $\mu \neq 0$ )**

# The boiling of quark nuggets



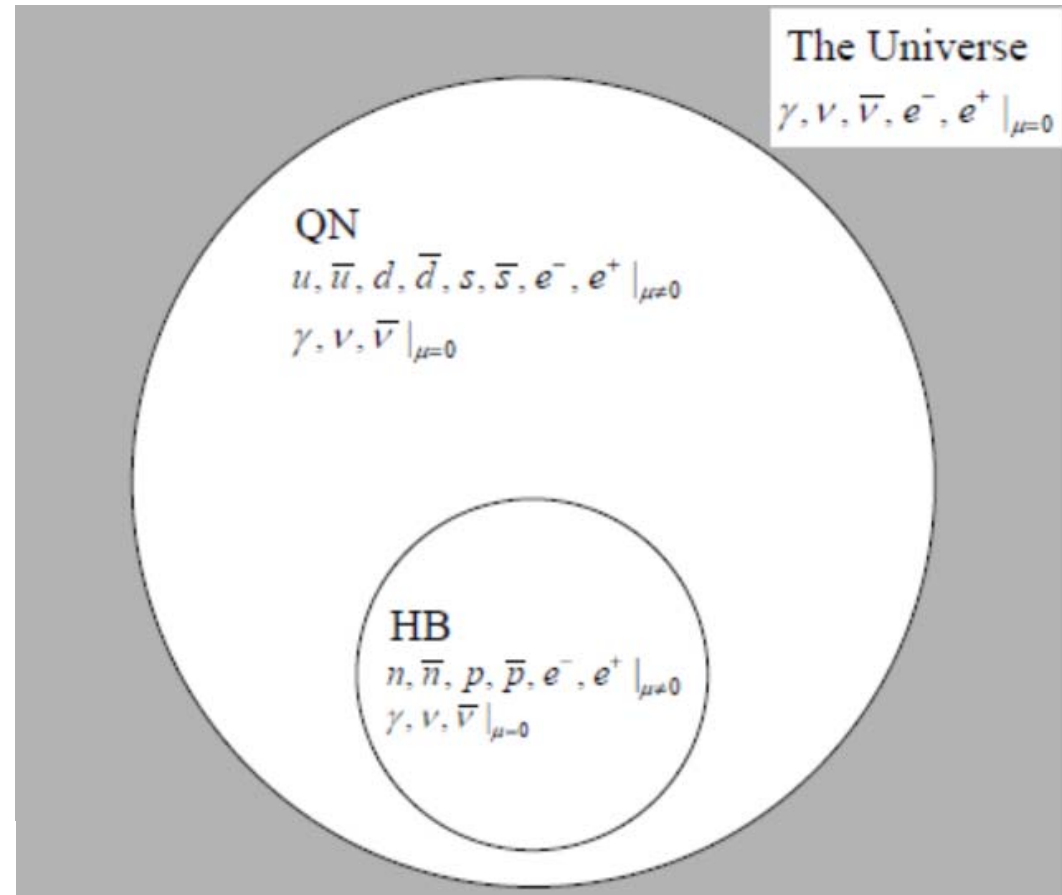
- **Earlier studies ignored** interactions between quarks and used phenomenological models for the hadron phase.

*Alcock & Olinto, PRD 1989*

*Madsen & Olesen, PRD 1991*

*Lugones & Horvath, PRD 2004*

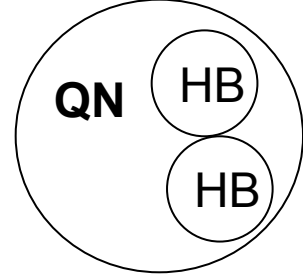
- **Boiling possible or not still uncertain.**



Zero chemical potential ( $\mu = 0$ ) represents that the relevant particles are thermally produced.

# Classical nucleation theory

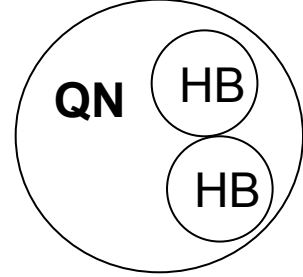
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# Classical nucleation theory

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- Whether boiling is important or not depends on the formation rate of **critical** bubbles;
- Formulas:

the work done to form a bubble of radius  $r$ :  $W = \frac{4\pi}{3} r^3 (P_q - P_h) + 4\pi\sigma r^2$

The work of a critical-size bubble can be obtained by maximizing  $W$  w.r.t.  $r$ :

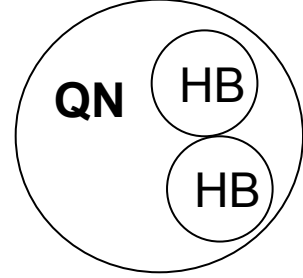
$$r_c = \frac{2\sigma}{P_h - P_q}$$

## Formation rate of critical bubbles:

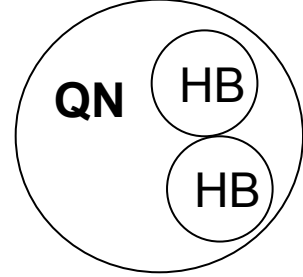
$$p \approx \epsilon^4 \exp(-W_c / T) \quad \begin{array}{l} \epsilon \text{ is a "characteristic energy" in the kinetics;} \\ \epsilon \sim T \end{array}$$

# To determine the critical $A_{\text{boil}}$ ;

*Madsen & Olesen, PRD1991*



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- **Formation rate** of critical bubbles:  $p \approx T^4 \exp(-W_c/T)$
- Boiling is efficient provided that the total surface area of bubbles  $\alpha_b$  exceeds the nugget surface area  $\alpha_n$ .

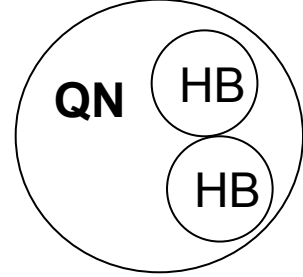
$$\alpha_b \approx V n_b 4\pi r_c^2 \quad \begin{array}{l} V \sim A \text{ MeV}^{-3} \\ n_b \approx 0.1 M_P T^2 \exp(-W_c/T) \end{array} \quad \text{Constant baryon density assumed in the nuggets.}$$

where  $0.1 M_P / T^2$

the duration of the epoch (of order the expansion time)

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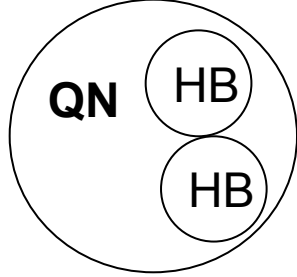
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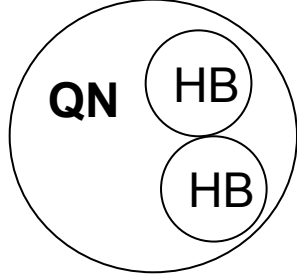
$$A_{\text{boil}} \approx 7.90 \times 10^{-61} \frac{\Delta P^6}{T^6 \sigma^6} \exp \left[ 16\pi \frac{\sigma^3}{T \Delta P^2} \right]$$



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- **Pressure difference** between the phases calculated from the equation:

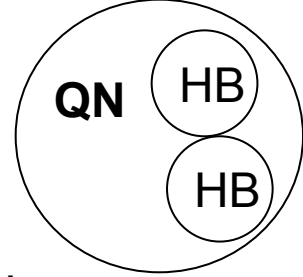
$$\mu_q(T, P_q) = \mu_h(T, P_h)$$

- **Surface tension**  $\sigma$  of QN can be calculated consistently as the free energy per unit surface area, from all fermion species ( $i = u; d; s; e$ ).

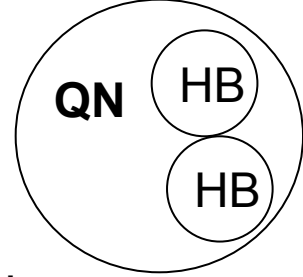
*L. D. Landau and E. M. Lifschitz, <Statistical Physics>*

$$\sigma_i = \frac{3T}{8\pi} \times \int_0^\infty \left( 1 - \frac{2}{\pi} \arctan \frac{k}{m_i} \right) \ln \left[ 1 + \exp \left( -\frac{e_i(k) - \mu_i}{T} \right) \right] k dk.$$

# Pressure difference between the phases



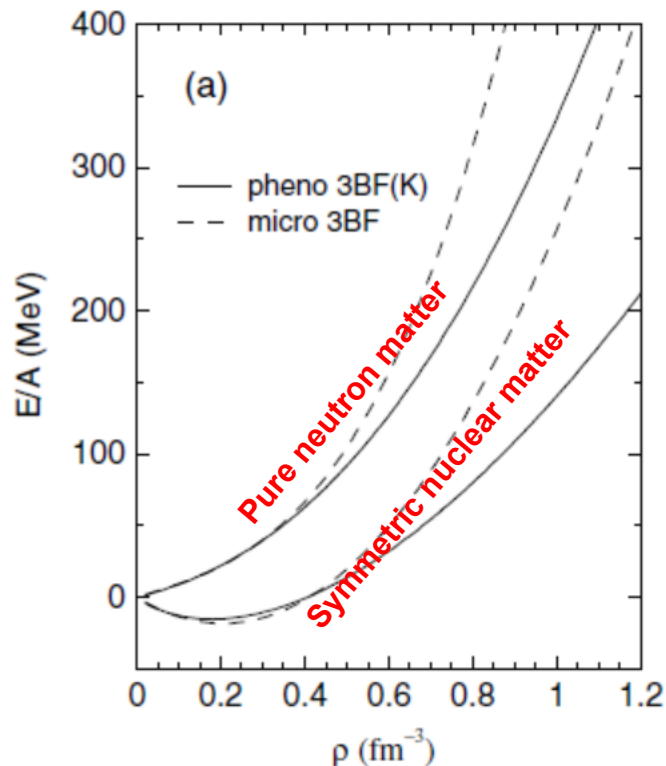
- Based on our previous calculations of hybrid neutron stars (Peng, AL, Lombardo, *PRC* 2008):
  - Using the **BHF** nuclear many-body approach for hadron phase;
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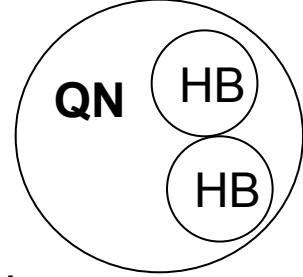
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## NNN 3 body force (3BF) included in non-rel. BHF



- 3BF substantially improve saturation;
- Micro 3BF stiffer than pheno 3BF at high densities: **Hadron EoS dependence.**

A. Li, G. F. Burgio, U. Lombardo,  
and W. Zuo, *PRC* 2006



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## Confined-density-dependent-mass (**CDDM**) model

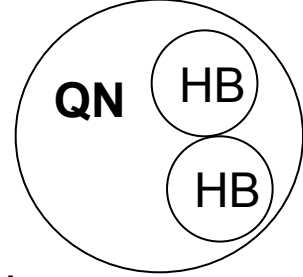
$$m_q = m_{q0} + \frac{\langle H_1 \rangle_{n_b} - \langle H_1 \rangle_0}{\sum_q [\langle \bar{q}q \rangle_{n_b} - \langle \bar{q}q \rangle_0]}$$

$$\equiv m_{q0} + m_1,$$

**G.X. Peng** et al, *PRC* 1999

X. J. Wen, et al., *PRC* 2005

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$$m_I(n_b, T) = \frac{D}{n_b^z} \left[ 1 - \frac{8T}{\lambda T_c} \exp\left(-\lambda \frac{T_c}{T}\right) \right]$$

$$\equiv m_{q0} + m_I,$$

- Stability window for  $D^{1/2}$ : **(158- 270)**MeV;
- Lower** limit from nuclear physics,  
at  $P = 0$ , non-strange nuclear matter should be stable against decay to  $(ud)$  quark matter.

**Upper** limit from vacuum quark condensation.

- LQCD favored area (161 MeV~195 MeV).

Aoki, et al. hep-lat/13108555

**G.X. Peng** et al, *PRC* 1999

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# Parameter

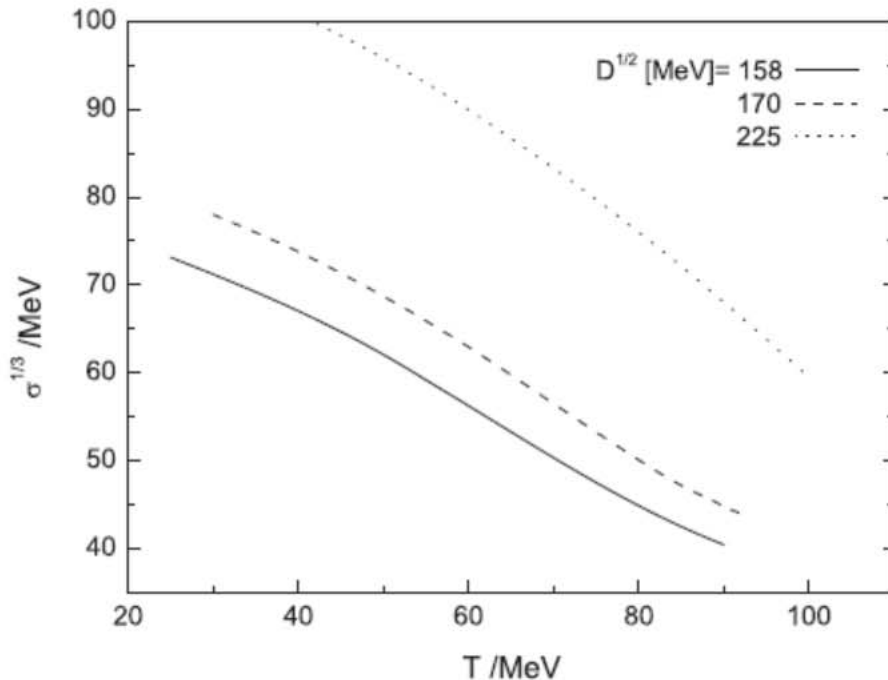
- Two hadron EoSs: Stiff (with Micro 3BF) and Soft (with Pheno 3BF);
- $D^{1/2} = 158\text{MeV}$ : Absolutely stable strange quark matter;  
    **170MeV**: LQCD favored area (161 MeV~195 MeV);  
    225MeV: Near the upper limit of  $D$  parameter.



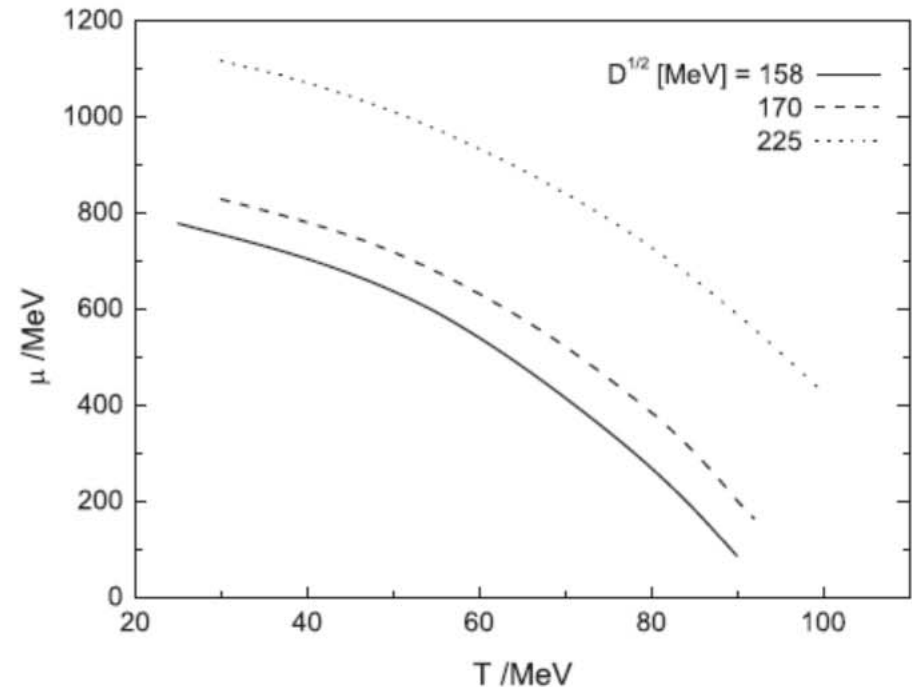
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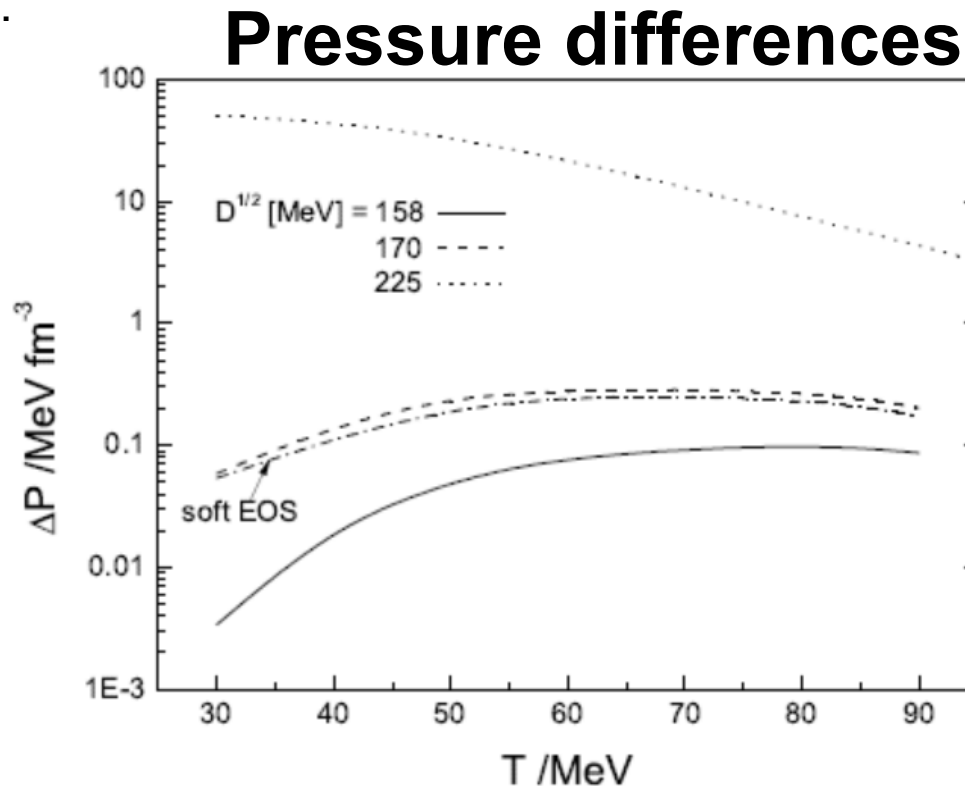


## Baryon chemical potential



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Softer Hadron EoS

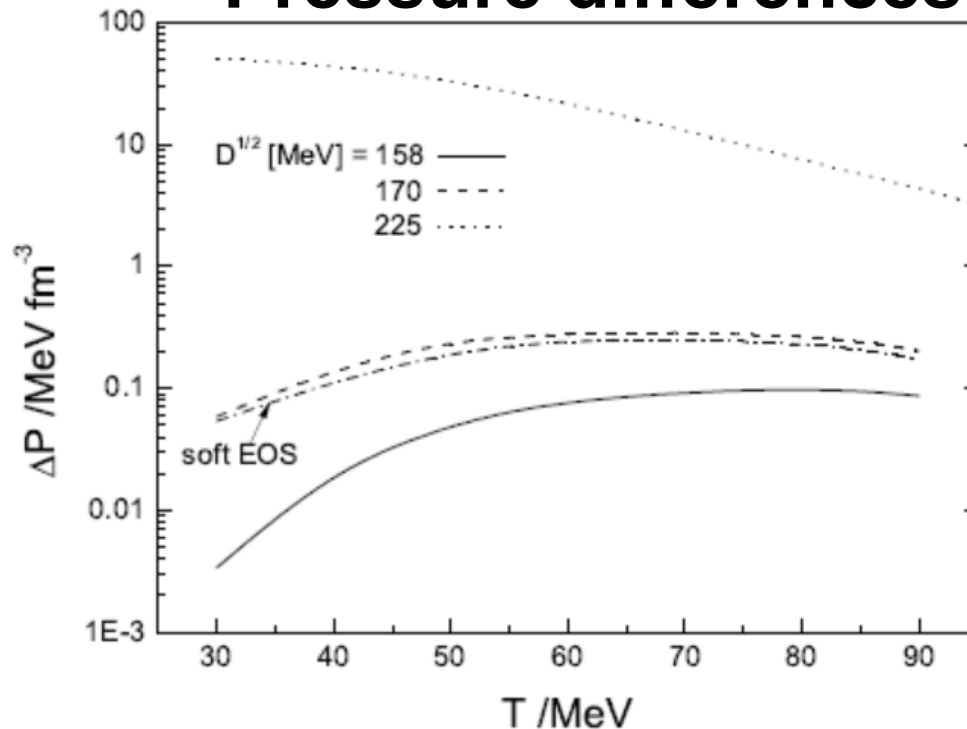


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- More sensitive to  $D$  parameter, than to the hadron EoS.

## Pressure differences



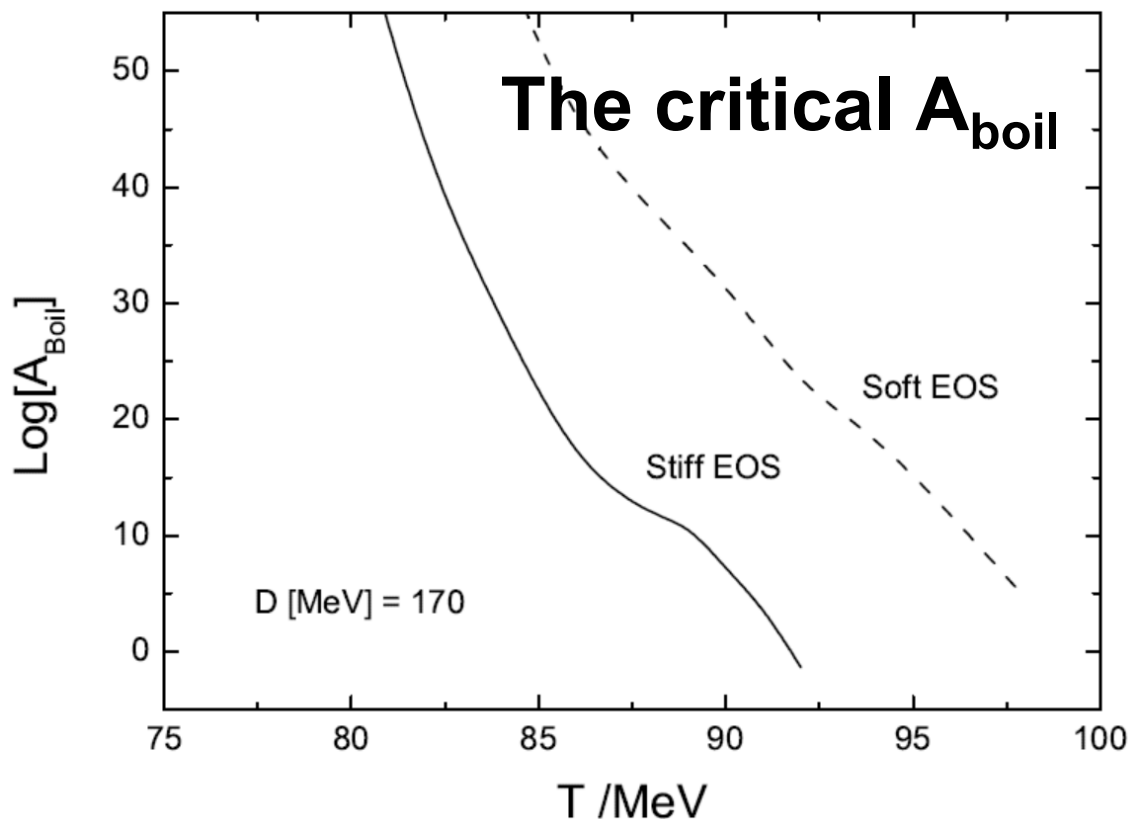
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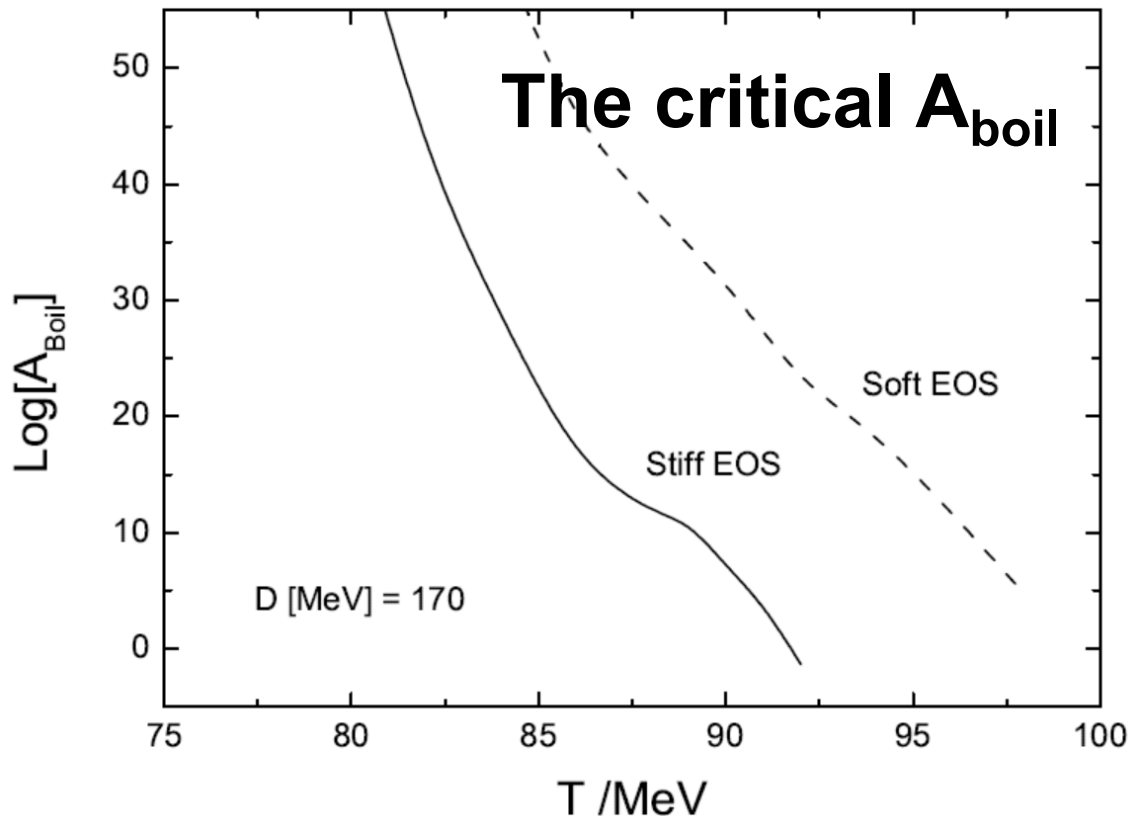
↓  
**Larger work for  
a critical-size bubble**

↓  
**Lower probability  
of bubbles nucleation**

↓  
**Larger  $A_{\text{boil}}$**

# Results

- Two hadron EoSs: Stiff (with Micro TBF) and Soft (with Pheno TBF);
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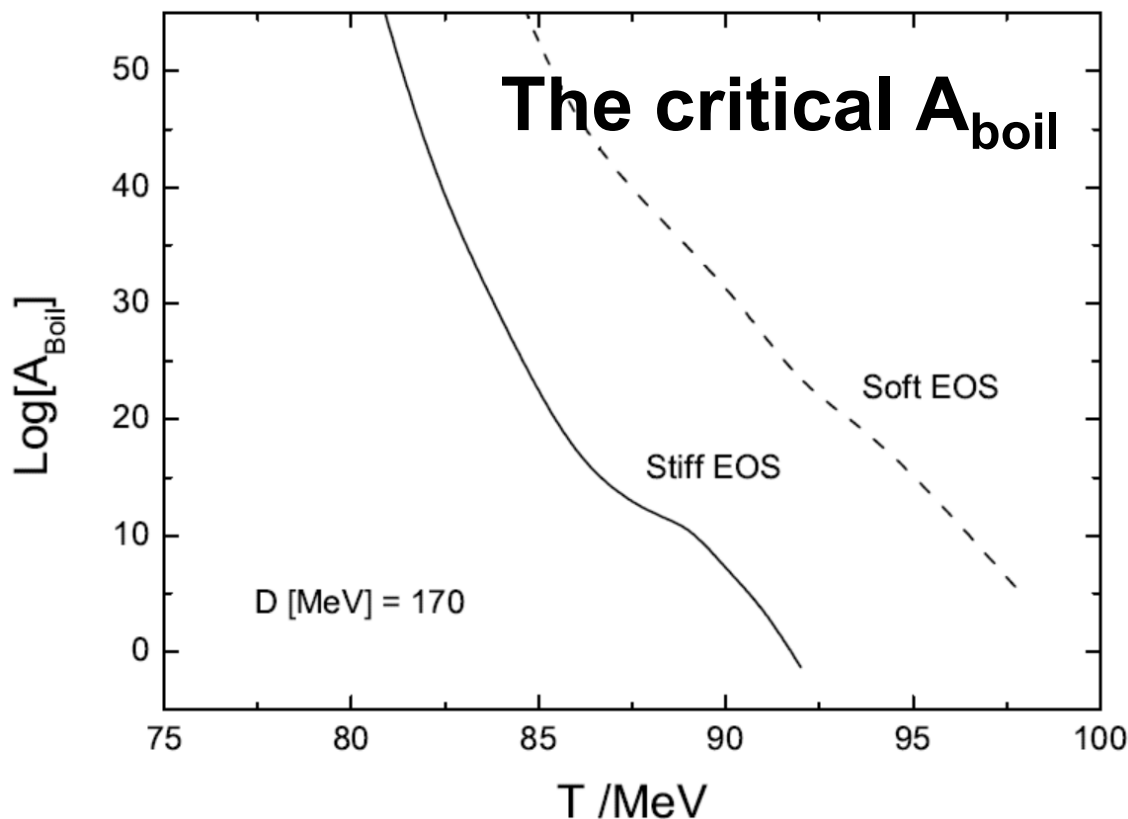


- **Surprisingly!**  
Other  $D$  values result in an extremely large  $A_{\text{boil}}$ , and boiling might not happen;
- Boiling might be important only **~170MeV** (values from Lattice).

# Results

- **Further comments:**

QNs might **unlikely** a candidate for DM, since only boiling can destroy them very sufficiently (Companied by yet unknown consequences for the spectrum of the emitted photons).



- **Surprisingly!** Other  $D$  values result in an extremely large  $A_{\text{boil}}$ , and boiling might not happen;
- Boiling might be important only  **$\sim 170\text{MeV}$**  (values from Lattice).

# Summary and and future plans

- Boiling revisited using updated micro. many-body theory;
- Boiling very possibly important for destroying (large) primordial QN;
- + Evaporation;
- Hyperon effect;
- Color superfluity effect (for  $T < T_{\Delta}$ );
- Other important aspects from the audience.

**Thank you very much!**



校園鳥瞰



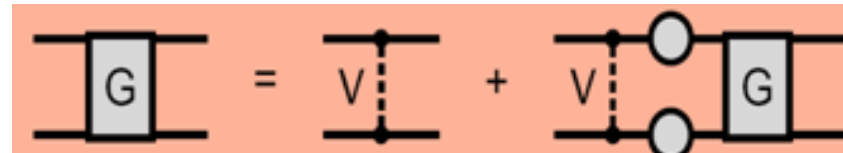
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# Brueckner-Hartree-Fock (BHF) model 58 -present

(P. Ring's lectures)

## Input quantities

- Interaction  $V$
- Baryon density  $\rho = \rho_n + \rho_p$
- Asymmetry parameter  $\beta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$



$$G(\rho, \beta; \omega) = v + v \sum_{k_a k_b} \frac{|k_a k_b\rangle Q(k_a, k_b) \langle k_a k_b|}{\omega - e(k_a) - e(k_b) + i\eta} G(\rho, \beta; \omega)$$

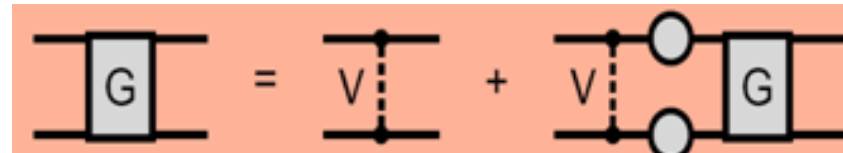
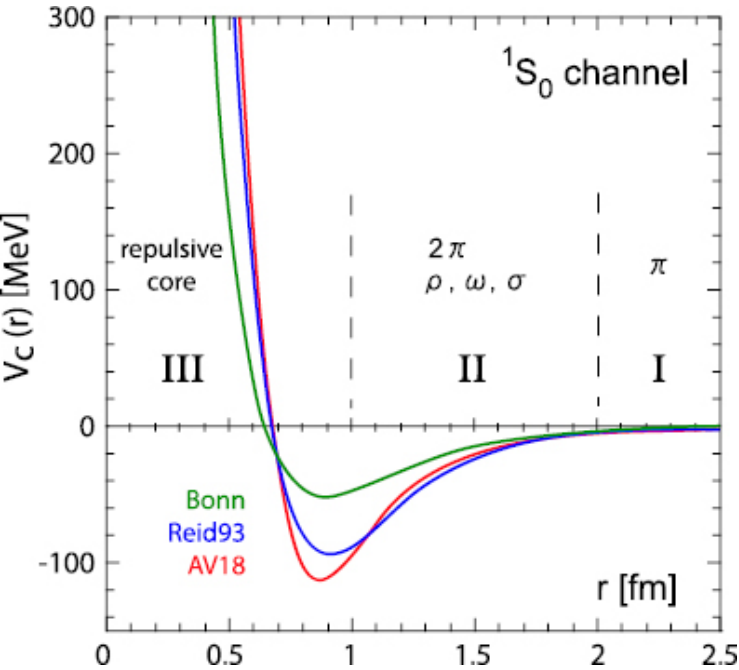
- $G$ : In-medium effective Interaction matrix
- $Q$ : Pauli operator  $Q(k_a, k_b) = [1 - n(k_a)][1 - n(k_b)]$
- $\omega$ : Starting energy
- $e$ : s.p. energy

$$e(k) = \frac{\hbar^2 k^2}{2m} + \sum_{k'} n(k') \text{Re} \langle k k' | G[e(k) + e(k')] | k k' \rangle_A$$

- A theory based on independent nucleon pair, for handling the repulsive core of nuclear force.

# Brueckner-Hartree-Fock (BHF) model 70 -present

(P. Ring's lectures)



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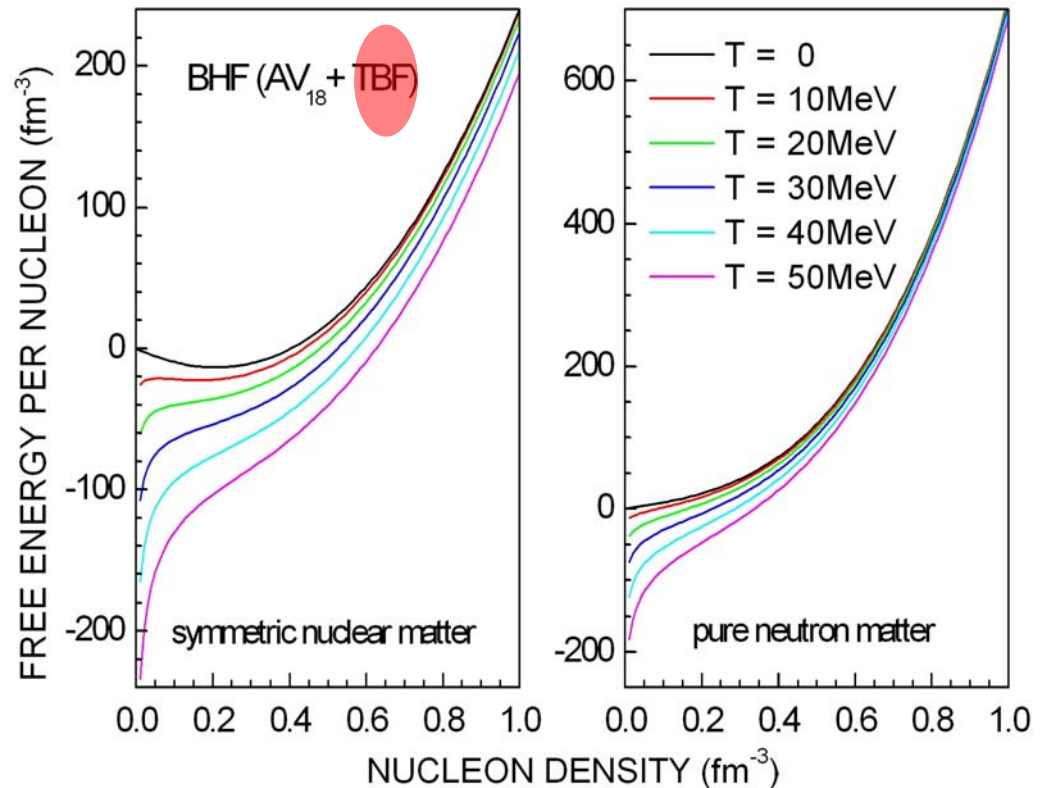
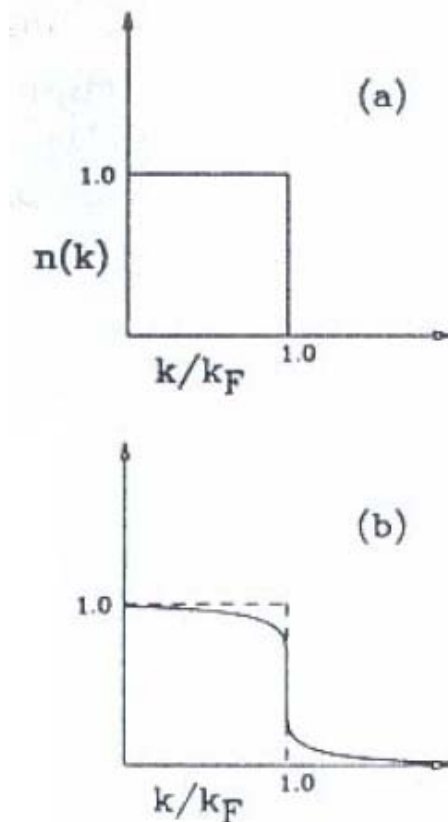
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- A theory based on independent nucleon pair, for handling the repulsive core of nuclear force.

# Finite-temperature BHF model with TBF

$$n(k) = \begin{cases} 1 & k \leq k_F \\ 0 & k > k_F \end{cases} \quad \longrightarrow \quad f_\tau(k, T, \rho, \beta) \equiv [1 + \exp(\frac{e_\tau(k) - \mu_\tau}{T})]^{-1}$$



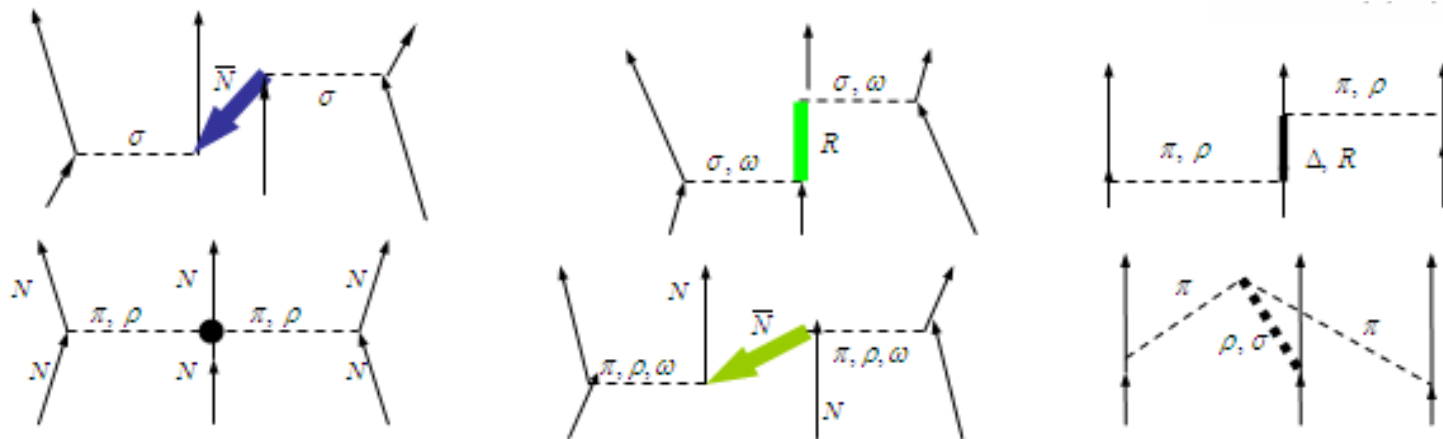
# TBF (Three body force)

(Schulze's talk)

- Microscopic TBF;**

Exchange of  $\pi, \rho, \sigma, \omega$  via  $\Delta$  (1232),  $R$ (1440),  $NN$  with parameters compatible with 2BF (Paris, V18,...)

P. Grangé et al., *PRC* 1989



- Phenomenological TBF.**

Directly add the TBF operators  $V_{ijk} = V_{ijk}^{2\pi} + V_{ijk}^R$ ,

Only  $2\pi$ -TBF + phenomenological repulsion

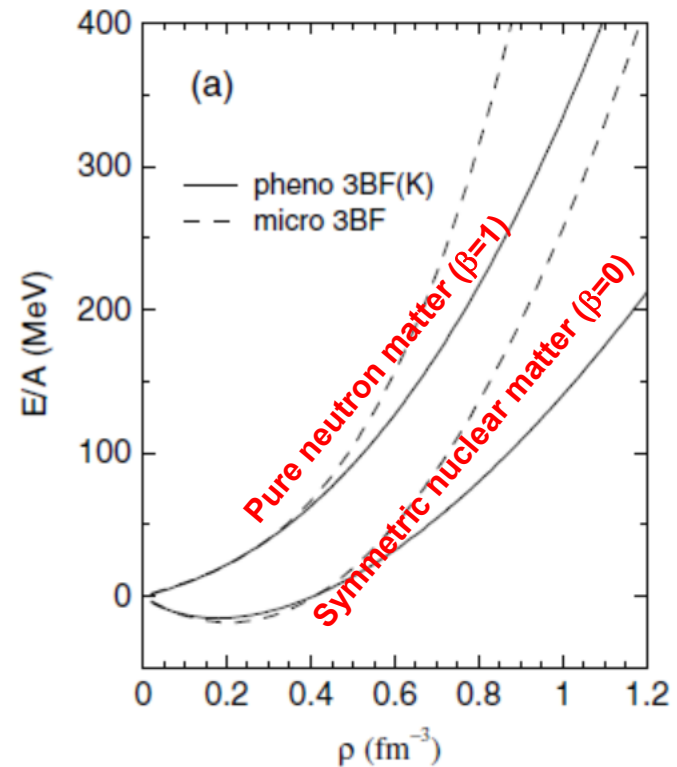
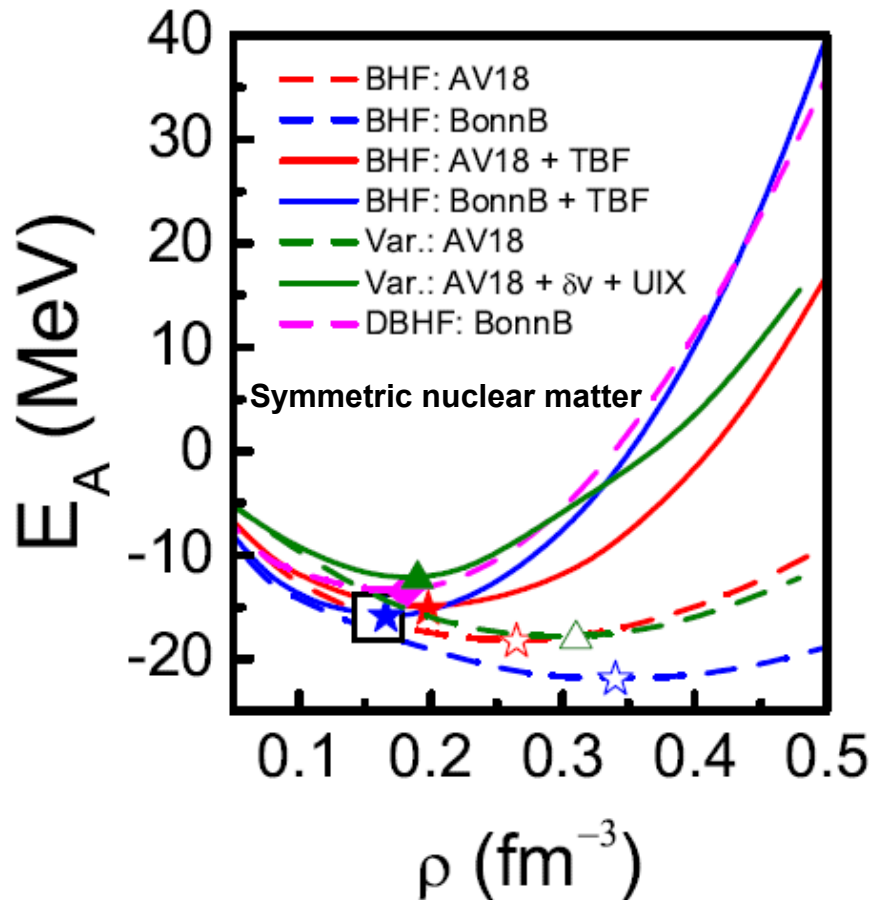
Parameters fitted from saturation point.

## TBF: Nucleon structure effect + Relativistic effect

# TBF included in BHF

(Lombardo's talk)

- TBF substantially improve saturation;
- Micro TBF more stiffer than pheno TBF at high densities.



- «Recipe» for neutron star structure calculation:

Brueckner calculation:  $\epsilon(\rho, x_e, x_p, x_\Lambda, x_\Sigma, \dots); x_i = \frac{\rho_i}{\rho}$   
 ↓  
 Chemical potentials:  $\mu_i = \frac{\partial \epsilon}{\partial \rho_i}$   
 ↓  
 Beta-equilibrium:  $\mu_i = b_i \mu_n - q_i \mu_e$   
 Charge neutrality:  $\sum_i x_i q_i = 0$   
 ↓  
 Composition:  $x_i(\rho)$   
 Equation of state:  $p(\rho) = \rho^2 \frac{d(\epsilon/\rho)}{d\rho}(\rho, x_i(\rho))$   
 ↓  
 TOV equations:  $\frac{dp}{dr} = -\frac{Gm}{r^2} \frac{(\epsilon + p)(1 + 4\pi r^3 p/m)}{1 - 2Gm/r}$   
 $\frac{dm}{dr} = 4\pi r^2 \epsilon$   
 ↓  
 Structure of the star:  $\rho(r), M(R)$  etc.

- M(R) relation is **unique** to the underlying EoS;
- Very **massive** NS observed recently.

# Confined-density-dependent-mass (CDDM) model

$$H_{\text{QCD}} = H_k + \sum_q m_{q0} \bar{q}q + H_I \quad \leftarrow \begin{array}{c} \text{The variation of} \\ \text{the quark mass with} \\ \text{density mimics the} \\ \text{strong interaction} \\ \text{between quarks.} \end{array} \rightarrow H_{\text{eqv}} = H_k + \sum_q m_q \bar{q}q$$

$$m_q = m_{q0} + \frac{\langle H_I \rangle_{n_b} - \langle H_I \rangle_0}{\sum_q [\langle \bar{q}q \rangle_{n_b} - \langle \bar{q}q \rangle_0]}$$

$$\equiv m_{q0} + m_I$$

Quark confinement

$$\lim_{n_b \rightarrow 0} m_I = \infty$$

Asymptotic freedom

$$\lim_{n_b \rightarrow \infty} m_I = 0$$

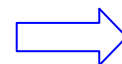
$$m_I = \frac{D}{n_b^{\frac{2}{3}}}$$

G.X. Peng et al, *PRC* (1999)

$$\langle H_I \rangle_{n_b} - \langle H_I \rangle_0 = \frac{1}{2V} \iint_V v(r) (3n_b dr_1) (3n_b dr_2) \quad (1)$$

$$\sum_q [\langle \bar{q}q \rangle_{n_b} - \langle \bar{q}q \rangle_0] = \sum_q [1 - \langle \bar{q}q \rangle_0 / \rho'_q] n_b \equiv A n_b \quad (2)$$

a linear confinement potential  $v(r) = \alpha r \quad (3)$



$$m_I = \frac{18\pi}{A} n_b \frac{R^4}{4} \propto \frac{1}{n_b^{1/3}}$$

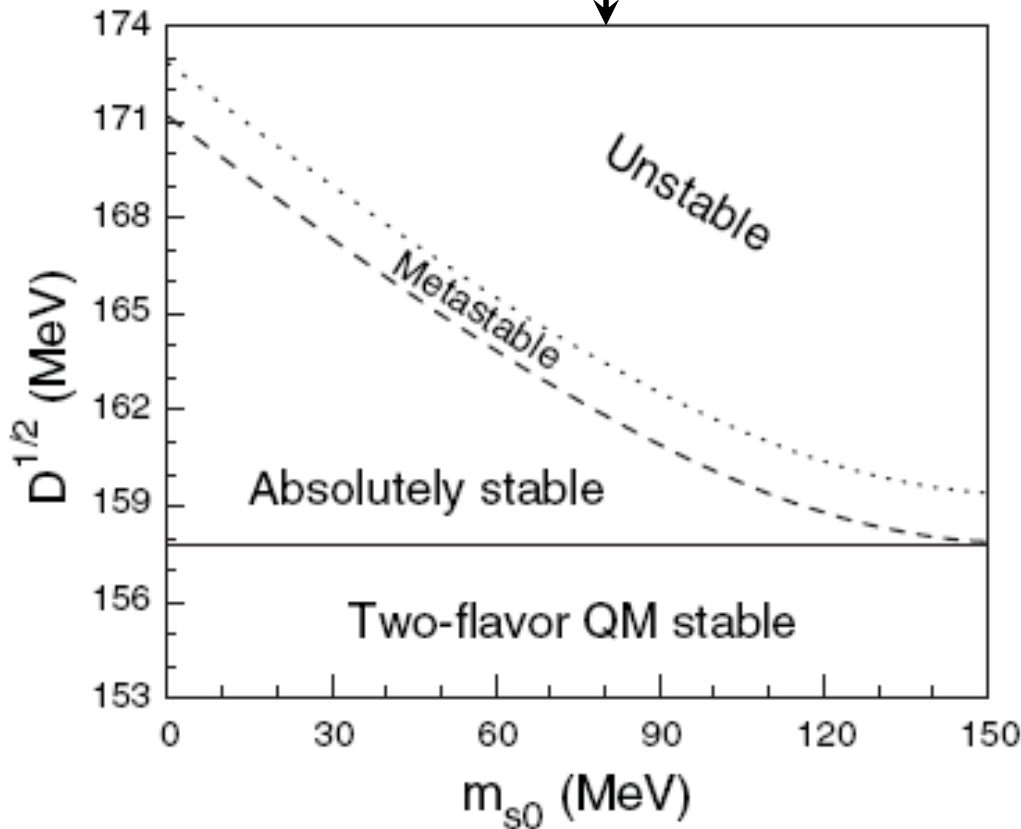
Quark model with chiral mass scaling

High-order perturbative interactions included, see C. J. Xia, G. X. Peng, et al. *PRC* 2014



# Confined-density-dependent-mass (CDDM) model

- Stability window for  $D^{1/2}$ : **(158- 270)MeV**;
- **Lower** limit from nuclear physics, **Upper** limit from vacuum quark condensation;



Peng, AL, Lombardo, *PRC* 2008

$$D = \frac{3(2/\pi)^{1/3} \sigma_0 m_\pi^2 f_\pi^2}{-\sigma_N \sum_q \langle \bar{q}q \rangle_0}$$

- Finite-temperature extension

X. J. Wen, et al., *PRC* 2005

$$m_I(n_b, T) = \frac{D}{n_b^z} \left[ 1 - \frac{8T}{\lambda T_c} \exp\left(-\lambda \frac{T_c}{T}\right) \right]$$

# Cosmic separation of phases

Witten, *PRD* 1984

**H**: High-temperature **quark** phase;

**L**: Low-temperature **hadron** phase;

