上海交通大學
SHANGHAI JIAO TONG UNIVERSITY

Quark stars under strong magnetic fields

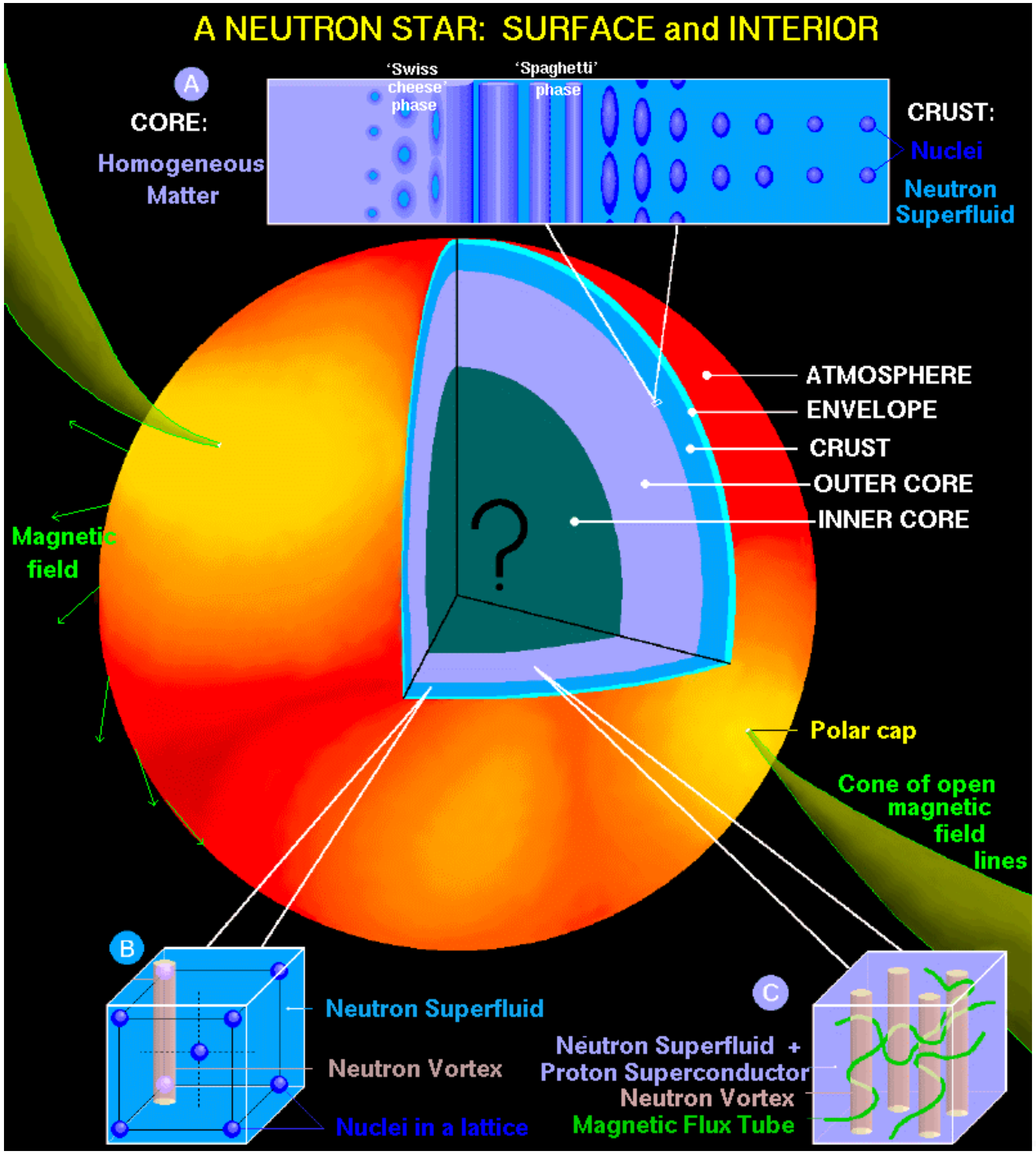
Peng-Cheng Chu(初鹏程)

(Department of Physics and Astronomy and Shanghai Key Laboratory for Particle Physics and Cosmology, Shanghai Jiao Tong University, Shanghai 200240, China
kyois@sjtu.edu.cn)

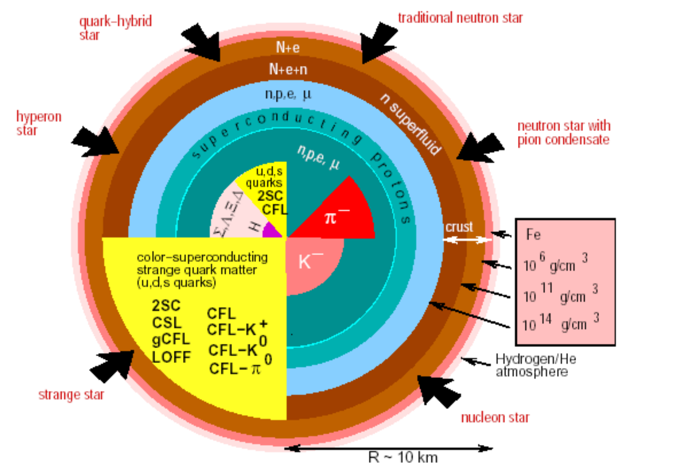
Collaborators:

Mei Huang 黄梅老师(IHEP) Lie-Wen Chen 陈列文老师 *(SJTU)*
Xin Wang 王欣老师 *(SJTU)*

Motivation to learn



Compact stars provide another way to explore the properties of strong interaction matter at high baryon density and low temperature.



Introductions

Compact stars: a unique astrophysical testing ground for strong interaction matter in nature.

Quark stars, hybrid stars & neutron stars.

Natur,467,1081(2010)
Sci,340,6131(2013)

Recent massive pulsars: PSR J1614−2230 $1.97 \pm 0.04 M_{\odot}$

PSR J0348+0432 $2.01 \pm 0.04 M_{\odot}$

Strong magnetic field, or finite temperature.

Phenomenological models are proposed to give the properties of quark matter.

u-d quark isospin symmetry

Asymptotic freedom & color confinement.

Since we can neither use PQCD to calculate the quark matter of the compact star with the low baryon number density, nor use lattice QCD because of the finite chemical potential

SQM under strong magnetic field

An important aspect of the physics of compact stars is that they could be endowed with strong magnetic field:

1. A certain class of neutron stars called magnetars can have even larger magnetic fields, reaching surface values as large as 10^{14} – 10^{15} G.

B. Paczynski, Acta Astron. **42**, 145 (1992);
C. Thompson and R. C. Duncan, Astrophys. J. **392**, L9 (1992); **473**, 322 (1996);
A. Melatos, Astrophys. J. Lett. **519**, L77 (1999).

2. The magnetic field will reach as large as in the core 10^{18} G as predicted.

Lai, D. and Shapiro, S., L., 1991, Astrophys. J. 383, 745

3. It is shown that the breaking of the O(3) rotational symmetry by the magnetic field results in a pressure anisotropy, which leads to the distinction between longitudinal- and transverse-to-the-field pressures.

SQM under strong magnetic field

We mention that the lagrangian density for noninteracting quarks (*u, d* and *s*) and leptons (*e* and μ) in an external magnetic field is

$$\mathcal{L} = \sum_{i=u,d,s,e,\mu} \bar{\psi}_i[\gamma^\mu(i\partial_\mu - q_i A_\mu) - m_i]\psi_i - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

Here we assume that the magnetic field is along *z* axis, and the vector potential is $A^\mu = (0, -By, 0, 0)$

so that we can get $\nabla \times \vec{A} = \vec{B} = (0, 0, B)$

SQM under strong magnetic field

After solving the Dirac equation, we can get the energy spectrum under magnetic field

$$E_{p,f} = \sqrt{p^2 + (2n + 1 - s)|q_f|B + m_f^2} = \sqrt{p^2 + 2k|q_f|B + m_f^2}$$

where n is the principal quantum number, s means spin, k=0,1,2... tells the Landau levels.

Then we will give the thermodynamic properties of magnetized SQM at zero temperature

SQM under strong magnetic field

We starts from the total thermodynamic potential of u-d-s quark matter at zero temperature

$$\Omega_i = -\sum_{\nu=0}^{\nu^i_{max}} \frac{g_i(|q_i|B)}{4\pi^2}\alpha_\nu \int_{-\infty}^{\infty} \mathrm{d}p_z[\mu_i^* - E_{p,i}]$$

$$= -\sum_{\nu=0}^{\nu^i_{max}} \frac{g_i(|q_i|B)}{2\pi^2}\alpha_\nu \left\{ \frac{1}{2}\mu_i^* \sqrt{\mu_i^{*2} - s_i(\nu,B)^2} - \frac{s_i(\nu,B)^2}{2} \ln\left[\frac{\mu_i^* + \sqrt{\mu_i^{*2} - s_i(\nu,B)^2}}{s_i(\nu,B)}\right]\right\}.$$

Where i is the sum of all flavors of quarks and leptons, Ω_i is the thermodynamic potential and μ_i^* is the fermi energy for quarks and leptons, and the upper Landau level is defined by

$$\nu^i_{max} \equiv \mathrm{int}\left[\frac{\mu_i^2 - m_i^2}{2|q_i|B}\right]$$

Confined density dependent quark model

CDDM $$m_q = m_{q0} + m_I = m_{q0} + \frac{D}{{n_B}^z}$$

$z = 1/3$

G.X.Peng,H.C.Chiang,J.J.Yang,L.Li. Phys.Rev.C 61 015201

$$\lim_{n_B \to \infty} m_I = 0 \qquad \lim_{n_B \to 0} m_I = \infty \qquad m_I = \frac{D}{{n_B}^z}$$

D *is the parameter determined by the stability arguments of SQM*

$z = 1/3$ *was derived based on the in-medium chiral condensates and linear confinement*

Confined isospin and density-dependent quark model

$$m_q = m_{q0} + m_I + m_{iso}$$
$$= m_{q0} + \frac{D}{{n_B}^{1/3}} - \tau_q \delta D_I n_B^{\alpha} e^{-\beta n_B}$$

The equivalent mass contains all the interaction of the quark matter.

P.C. Chu and L.W. Chen, Astrophys. J. **780**, 135 (2014)

$$\tau_q = \begin{cases} 1 & u\ quark \\ -1 & d\ quark \\ 0 & s\ quark \end{cases}$$

$$\delta = 3\frac{n_d - n_u}{n_d + n_u} = \frac{n_n - n_p}{n_n + n_p}$$

α and β should be positive so that we can get

$$\lim_{n_B \to 0} m_I = \infty \qquad \lim_{n_B \to \infty} m_I = 0$$

SQM under constant magnetic field

Then we can calculate the chemical potential μ_i for quarks and leptons.

$$\frac{d\mathcal{E}}{dn_i} = \mu_i = \mu_i^* + \sum_j \frac{\partial \Omega_j}{\partial m_j} \frac{\partial m_j}{\partial n_i}$$

and we get

$$\mu_u = \mu_u^* + D_I n_B^\alpha e^{-\beta n_B} [\frac{\partial \Omega_u}{\partial m_u} - \frac{\partial \Omega_d}{\partial m_d}] \frac{6 n_d}{(n_u + n_d)^2} + \mu_{den}$$
$$\mu_d = \mu_d^* + D_I n_B^\alpha e^{-\beta n_B} [\frac{\partial \Omega_d}{\partial m_d} - \frac{\partial \Omega_u}{\partial m_u}] \frac{6 n_u}{(n_u + n_d)^2} + \mu_{den}$$
$$\mu_s = \mu_s^* + \mu_{den}$$

SQM under constant magnetic field

We know that the $\mathcal{O}(3)$ rotational symmetry will be broken under strong magnetic field, and the pressure will split in two terms. Then we give the anisotropic pressure for SQM , the analytic form of the longitudinal and transverse pressure for SQM under CIDDM model, and we find there are additional parts coming from the density dependent equivalent mass.

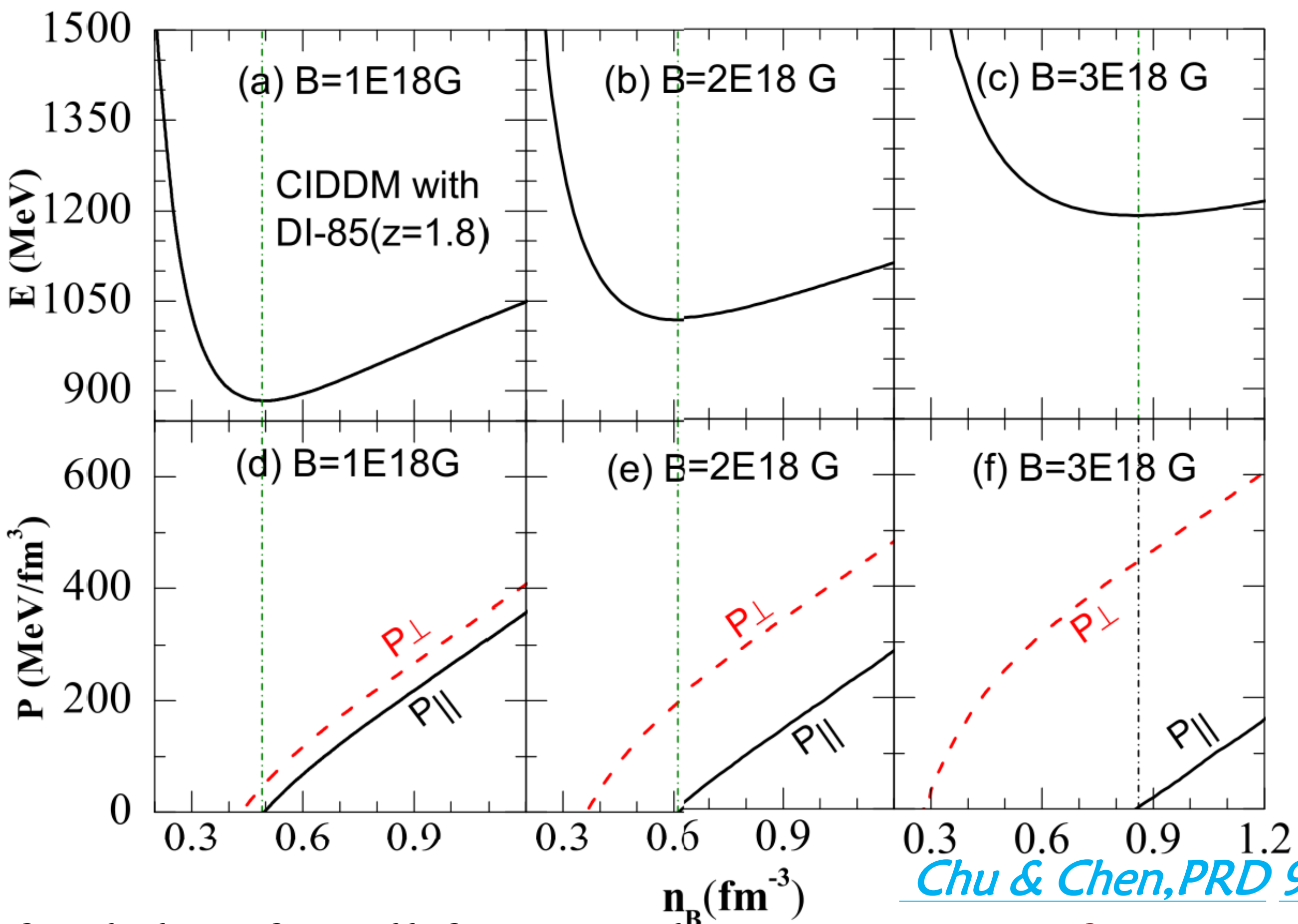
$$P_{\parallel} = \sum_i \mu_i n_i - \mathcal{E}_{tot},$$
$$P_{\perp} = \sum_i \mu_i n_i - \mathcal{E}_{tot} + B^2 - MB$$

Where $M_f = -\partial(\sum_i \Omega_i)/\partial B = \sum_i M_i$

is the system magnetization.

SQM under constant magnetic field

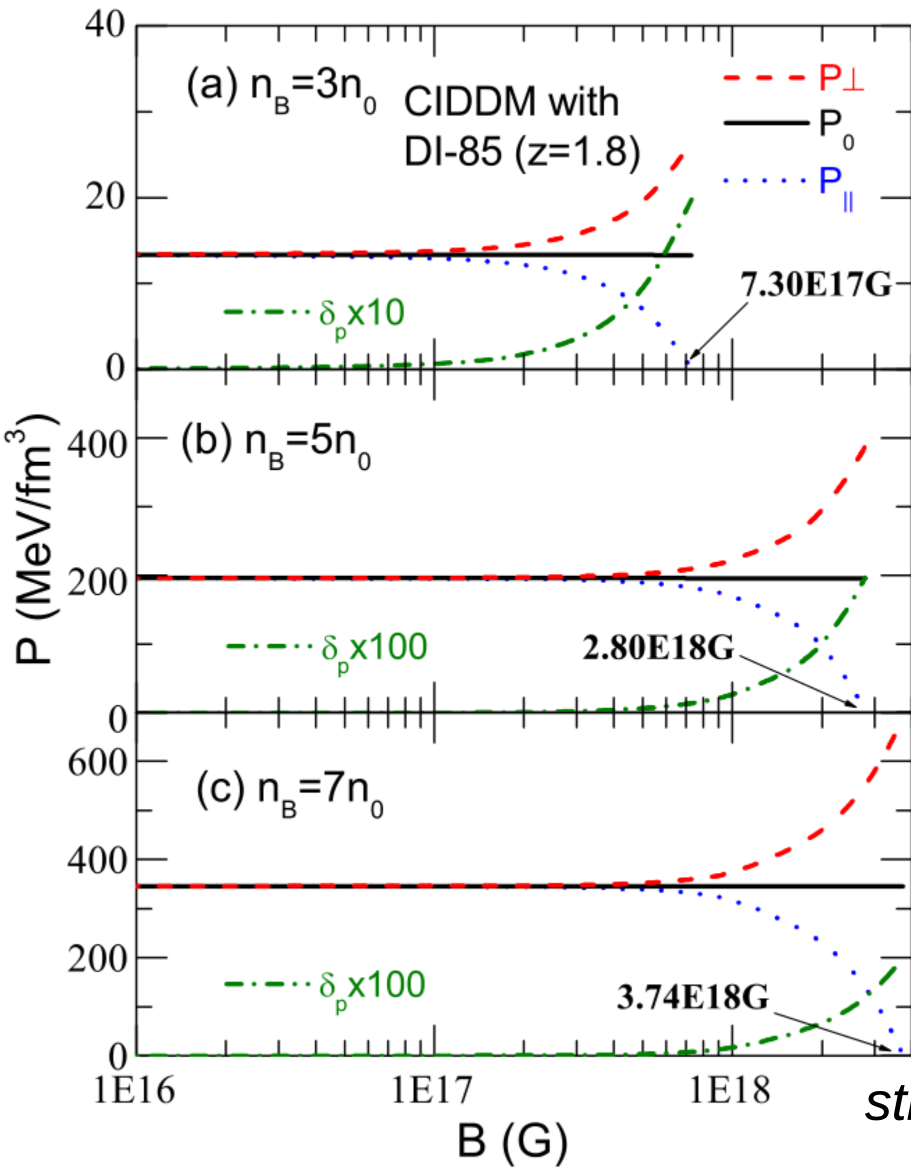
上海交通大学 SHANGHAI JIAO TONG UNIVERSITY



Chu & Chen, PRD 90, 063013(2014)

We can find that for all figures, the minimum of energy per baryon is exactly the zero longitudinal-pressure density, which is the requirement of thermodynamical self consistency.

SQM under constant magnetic field



pressure anisotropy

$$\delta_p = \frac{P_\perp - P_{||}}{(P_\perp + P_{||})/2}$$

The transverse pressure increases rapidly while the longitudinal pressure decreases rapidly with increment of B, leading to a rapid enhancement of δ_p

When the magnetic field strength B reaches a critical value of B_c, the longitudinal pressure will become negative and the system will become unstalbe. Therefore, B_c is the largest magnetic strength that a stable SQM in QSs can have.

Chu & Chen, PRD 90, 063013(2014)

SQM under density dependent magnetic field

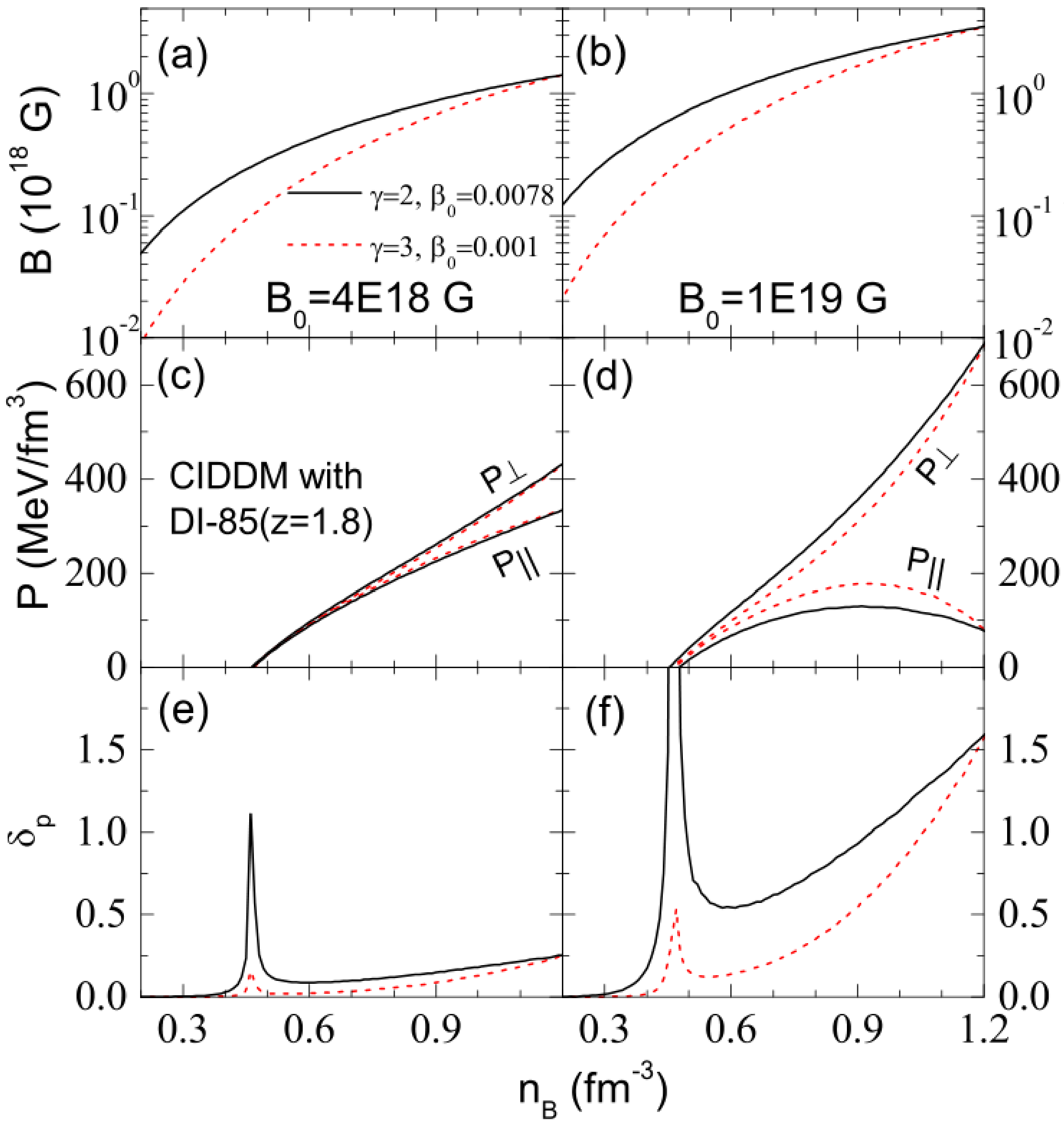
As people all accepted, large magnetic field $B=10^{15}$ G has been estimated at the surface of neutron stars. And the field may be as large as 10^{18}G in the core as predicted. So we use the density-dependent magnetic field as:

$$B = B_{surf} + B_0[1 - \exp\left(-\beta_0 (n_b/n_0)^\gamma\right)]$$

Debades Bandyopadhyay, Somenath Chakrabarty, and Subrata Pal, prl 1997,79,12
Bandyopadhyay, D., Pal, S. and Chakrabarty, S., 1998, J. Phys. G: Nucl. Part. Phys., 24, 1647

B_{surf} is the magnetic field on the surface of compact stars, and we set the value is 10^{15} G.

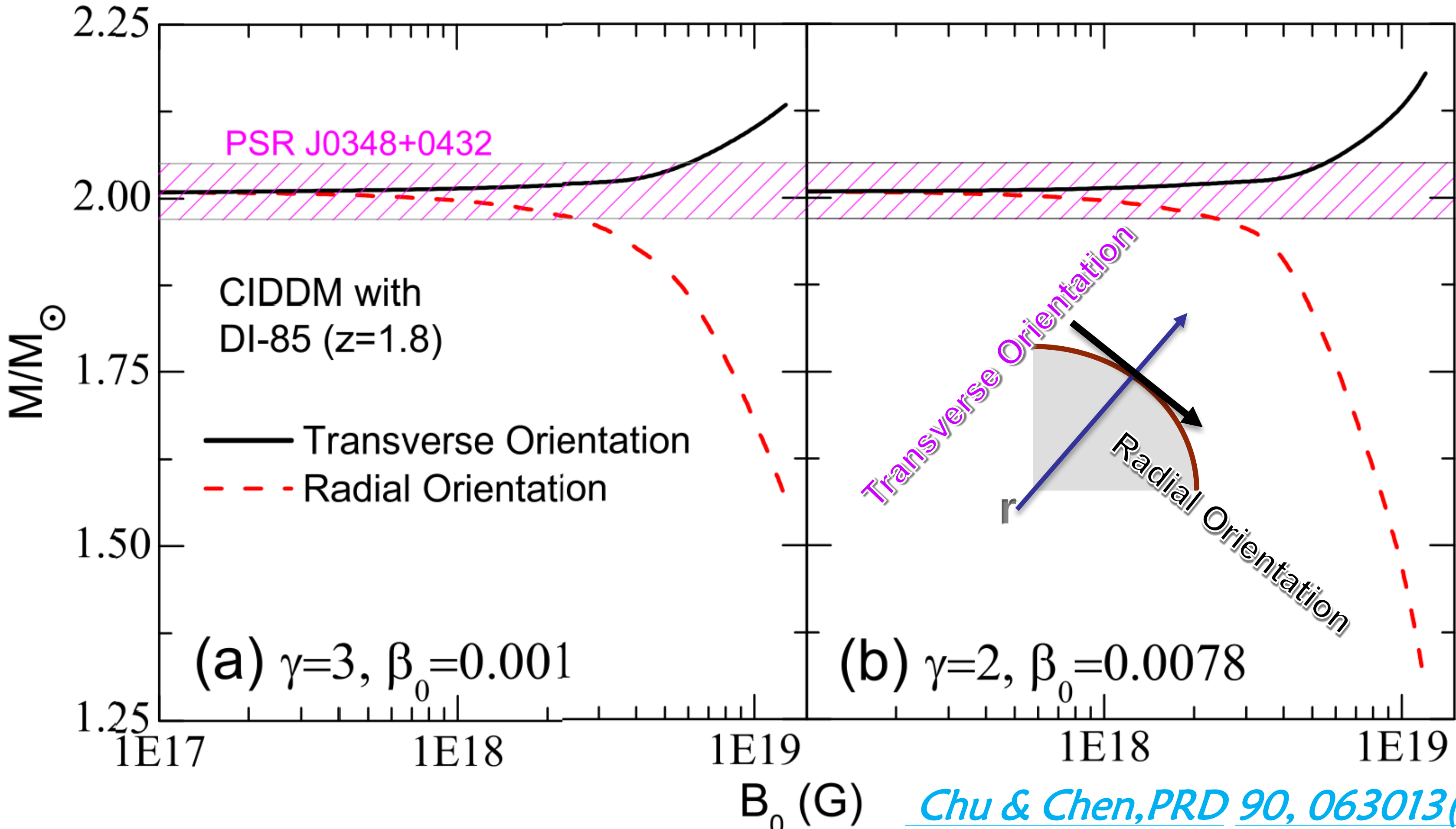
SQM under density dependent magnetic field



Baryon density dependence of the magnetic field strength B, longitudinal and transverse pressures as well as the pressure splitting factor δ_p for SQM in QSs using the slow B-profile (solid lines) and fast B-profile (dashed lines)

Chu & Chen, PRD 90, 063013(2014)

Quark stars under density dependent magnetic fields



Chu & Chen,PRD 90, 063013(2014)

Maximum mass of static QSs using the transverse and radial orientations of the magnetic fields as a function of B_0 with the fast B-profile (a) and the slow B-profile (b) within the CIDDM model with DI-85(z=1.8).

Summary

1. Within the confined-isospin-density-dependent-quark-mass model, we study the properties of strange quark matter (SQM) and quark stars (QSs) under strong magnetic fields.

2. Using a density-dependent magnetic field profile which is introduced to mimic the magnetic field strength distribution in a star, we study the properties of static spherical QSs by assuming two extreme cases for the magnetic field orientation in the stars

3. Our results indicate that including the magnetic fields with radial (transverse) orientation can significantly decrease (increase) the maximum mass of QSs

NJL model under strong magnetic fields

We study properties of 3-flavor system with external strong magnetic fields.

$$\mathcal{L} = \mathcal{L}_q + \mathcal{L}_e - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu^{ext} - \partial_\nu A_\mu^{ext}$$

$$A_\mu^{ext} = \delta_{\mu 2} x_1 B$$

The electron Lagrangian density is given by: $\mathcal{L}_e = \bar{e}[(i\partial_\mu - eA^\mu_{ext})\gamma^\mu]e$

The quark Lagrangian density is given by:

$$\mathcal{L}_q = \bar{\psi}_f [\gamma_\mu (i\partial^\mu - q_f A^\mu_{ext}) - \hat{m}_c] \psi_f + \mathcal{L}_4 + \mathcal{L}_6$$

NJL model under strong magnetic fields

where $$\mathcal{L}_4 = \mathcal{L}_S + \mathcal{L}_V + \mathcal{L}_{I,V}$$

$$\mathcal{L}_S = G_S \sum_{a=0}^{8} [(\bar{\psi}_f \lambda_a \psi_f)^2 + (\bar{\psi}_f i\gamma_5 \lambda_a \psi_f)^2]$$

$$\mathcal{L}_V = -G_V \sum_{a=0}^{8} [(\bar{\psi} \gamma^\mu \lambda^a \psi)^2 + (\bar{\psi} i\gamma^\mu \gamma_5 \lambda^a \psi)^2]$$

$$\mathcal{L}_{IV} = -G_{IV} [(\bar{\psi} \gamma^\mu \vec{\tau} \psi)^2 + (\bar{\psi} \gamma_5 \gamma^\mu \vec{\tau} \psi)^2]$$

$$\mathcal{L}_{det} = -K\{\det{}_f[\bar{\psi}_f(1+\gamma_5)\psi_f] + \det{}_f[\bar{\psi}_f(1-\gamma_5)\psi_f]\}$$

NJL model under strong magnetic fields

The quark mass is determined by the gap equation

$$M_f = m_f - 4G_S\sigma_f + 2K\sigma_j\sigma_k$$

and the chiral condensate is given as

$$\sigma_f = \langle\bar{\psi}_f\psi_f\rangle = -i\int\frac{d^4p}{(2\pi)^4}\mathrm{tr}\frac{1}{(\not{p} - M_f + i\epsilon)}$$

Then we can get the pressure for quark part as

$$\begin{aligned} p_q &= -2G_S(\sigma_u^2 + \sigma_d^2 + \sigma_s^2) + 4K\sigma_u\sigma_d\sigma_s \\ &+ 2G_V(n_u^2 + n_d^2 + n_s^2) + G_{IV}(n_u - n_d)^2 \\ &+ (\Omega_{ln}^u + \Omega_{ln}^d + \Omega_{ln}^s) \end{aligned}$$

NJL model under strong magnetic fields

Under strong magnetic field, the corresponding Debye mass of the Longitudinal gluon field has the screening mass:

$$m_g^2(eB) = \sum_f |q_f| \frac{g_{eff}^2}{4\pi^2} |eB|,$$

V. A. Miransky and I. A. Shovkovy, Phys. Rev. D **66**, 045006 (2002).

Once considering the nonzero temperature and density:

$$m_g^2(T, \mu, eB) = g^2(aT^2 + b\mu^2 + ceB)$$

Then the corresponding pressure contribution from gluons is

$$p_g(T, \mu; eB) = a_0\mu^2 eB + b_0\mu^4 + c_0 T^2 eB + d_0\mu^2 T^2 + e_0 T^4$$

At zero temperature, we can get

$$p_g(T = 0, \mu; eB) = a_0(\mu^2 eB + \mu^4)$$

NJL model under strong magnetic fields

It is shown that the breaking of the O(3) rotational symmetry by the magnetic field results in a pressure anisotropy, which leads to the distinction between longitudinal- and transverse-to-the-field pressures.

$$p_{||} = p - \frac{1}{2}B^2,$$

E.J. Ferrer, V. de la Incera, J.P. Keith, I. Portillo and P.L. Springsteen, Phys. Rev. C 82, 065802 (2010);

$$p_{\perp} = p + \frac{1}{2}B^2 - MB,$$

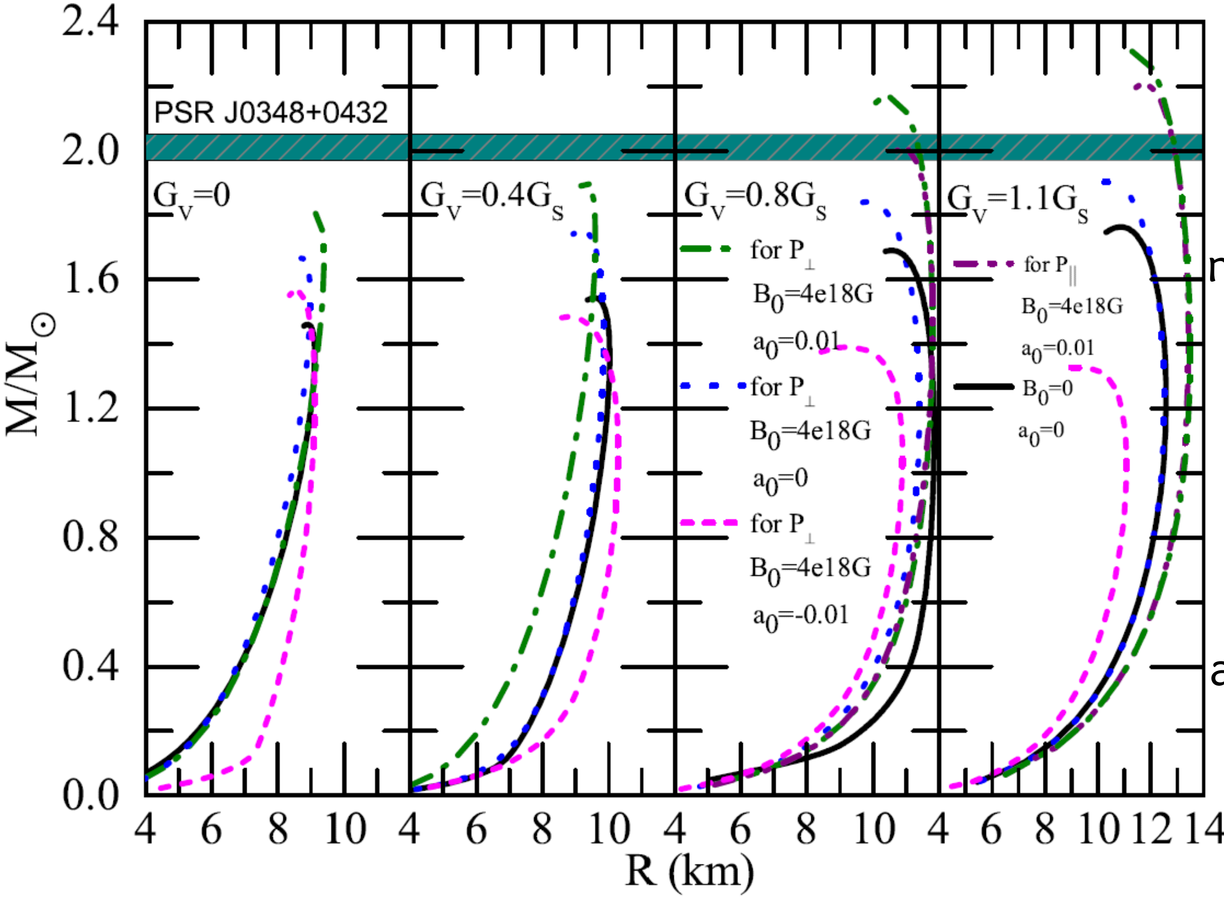
Peng-Cheng Chu, Lie-Wen Chen and Xin Wang, Physical Review D 90, 063013, (2014).

where: $p = p_q + p_l + p_g - p_0$

$$p_0 = -\Omega_0 = -\Omega(T = 0, \mu = 0, B = 0)$$

$$M = -\partial\Omega/\partial B = \sum_{i=u,d,s,l} M_i$$

NJL model under strong magnetic fields



Since the pressure anisotropy from density- dependent magnetic field is not big, we can calculate the properties of QSs under magnetic field by using isotropic TOV equation approximately, and we find the mass difference of magnetars by using transverse and longitudinal pressure is also very small.

arXiv:1409.6154v1

Mass-radius relation for magnetars within different coupling constants and magnetic fields by using transverse pressure and longitudinal pressure respectively.

Thank you!

Symmetry energy of quark matter

Energy per baryon can be expanded in isospin asymmetry

$$E(n_B, \delta, n_s) = E_0(n_B, n_s) + E_{\rm sym}(n_B, n_s)\delta^2 + \mathcal{O}(\delta^4)$$

The total energy density of the SQM is expressed as

$$\begin{aligned}\epsilon_{uds} = & \frac{g}{2\pi^2}\int_0^{(1-\delta/3)^{\frac{1}{3}}\nu} \sqrt{k^2 + {m_u}^2}k^2 dk \\ & + \frac{g}{2\pi^2}\int_0^{(1+\delta/3)^{\frac{1}{3}}\nu} \sqrt{k^2 + {m_d}^2}k^2 dk \\ & + \frac{g}{2\pi^2}\int_0^{\nu_s} \sqrt{k^2 + {m_s}^2}k^2 dk.\end{aligned}$$

Symmetry energy of quark matter

The quark symmetry energy is expressed as

$$E_{\mathrm{sym}}(n_B, n_s) = \frac{1}{2!}\frac{\partial^2 E(n_B, \delta, n_s)}{\partial \delta^2}\bigg|_{\delta=0} = \left[\frac{\nu^2 + 18 m D_I {n_B}^{\alpha} e^{-\beta n_B}}{18\sqrt{\nu^2 + m^2}} + A + B\right]\frac{3n_B - n_s}{3n_B},$$

For CIDDM

$$A = \frac{9m^2}{2\nu^2\sqrt{\nu^2 + m^2}}(D_I {n_B}^{\alpha} e^{-\beta n_B})^2,$$

$$B = \frac{9}{4\nu^3}\left[\nu\sqrt{\nu^2 + m^2} - 3m^2 ln\left(\frac{\nu\sqrt{\nu^2 + m^2}}{m}\right)\right] \times (D_I {n_B}^{\alpha} e^{-\beta n_B})^2,$$

$$m = m_{u0}\ (\mathrm{or}\ m_{d0}) + \frac{D}{{n_B}^{1/3}}$$

Symmetry energy of quark matter

For **CDDM** model, the **quark symmetry energy** is reduced to

$$E_{\mathrm{sym}}(n_B, n_s) = \frac{1}{18}\frac{\nu^2}{\sqrt{\nu^2+m^2}}\frac{3n_B - n_s}{3n_B}$$

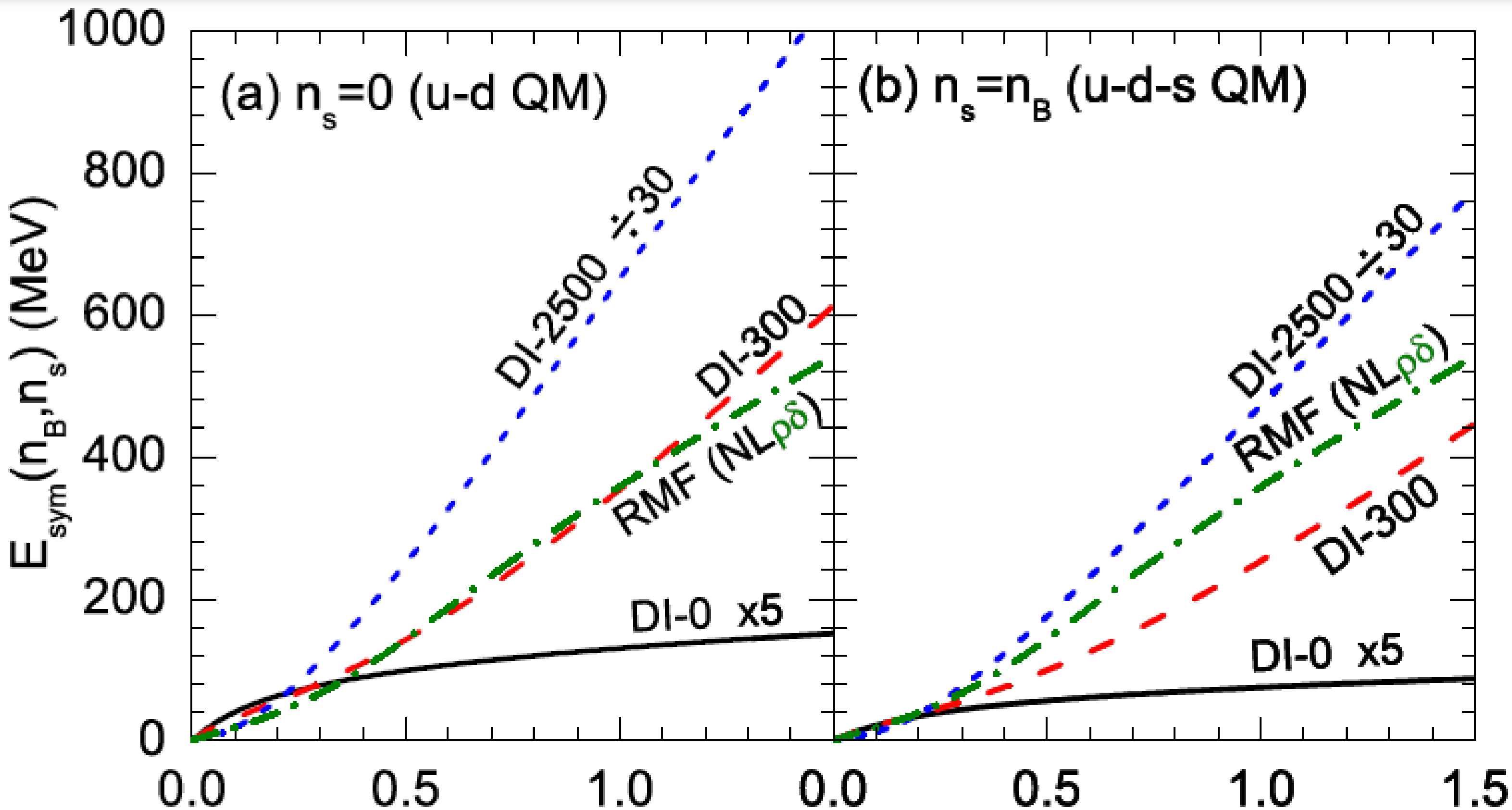
For two-flavor u-d quark matter, the quark symmetry energy is :

$$E_{\mathrm{sym}}(n_B) = \frac{1}{18}\frac{\nu^2}{\sqrt{\nu^2+m^2}}$$

$n_s = 0$

Symmetry energy of quark matter

The symmetry energy of different sets of parameters of CIDDM and relativistic mean field model with interaction NLρδ



Chu&Chen arXiv:1212.1388v1

The quark symmetry energy is much bigger than that of nuclear matter

β-equilibrium condition & the stability of SQM

The weak beta-equilibrium condition can be expressed as

$$\mu_u + \mu_e = \mu_d = \mu_s$$

For SQM

The electric charge neutrality condition can be written as

$$\frac{2}{3}n_u = \frac{1}{3}n_d + \frac{1}{3}n_s + n_e$$

μ is too heavy to appear

The chemical potential can be obtained as

$$\mu_i = \frac{d\epsilon}{dn_i}$$

β-equilibrium condition & the stability of SQM

The chemical potential in CIDDM can be obtained as

$$\mu_u = \sqrt{\nu^2 + m_u^2} + \frac{1}{3}\sum_{j=u,d,s} n_j f\left(\frac{\nu_j}{m_j}\right) \times \left[-\frac{D}{3n_B^{4/3}} - \tau_j D_I \delta(\alpha n_B^{\alpha-1} - \beta n_B^{\alpha})e^{-\beta n_B}\right] + D_I n_B^{\alpha} e^{-\beta n_B}\left[n_u f\left(\frac{\nu_u}{m_u}\right) - n_d f\left(\frac{\nu_d}{m_d}\right)\right] \times \frac{6n_d}{(n_u+n_d)^2}.$$

$$\mu_d = \sqrt{\nu^2 + m_d^2} + \frac{1}{3}\sum_{j=u,d,s} n_j f\left(\frac{\nu_j}{m_j}\right) \left[-\frac{D}{3n_B^{4/3}} - \tau_j D_I \delta(\alpha n_B^{\alpha-1} - \beta n_B^{\alpha})e^{-\beta n_B}\right] + D_I n_B^{\alpha} e^{-\beta n_B}\left[n_d f\left(\frac{\nu_d}{m_d}\right) - n_u f\left(\frac{\nu_u}{m_u}\right)\right] \times \frac{6n_u}{(n_u+n_d)^2},$$

$$\mu_s = \sqrt{{\nu_s}^2 + {m_s}^2} + \frac{1}{3}\sum_{j=u,d,s} n_j f\left(\frac{\nu_j}{m_j}\right) \left[-\frac{D}{3n_B^{4/3}} - \tau_j D_I \delta(\alpha n_B^{\alpha-1} - \beta n_B^{\alpha})e^{-\beta n_B}\right]$$

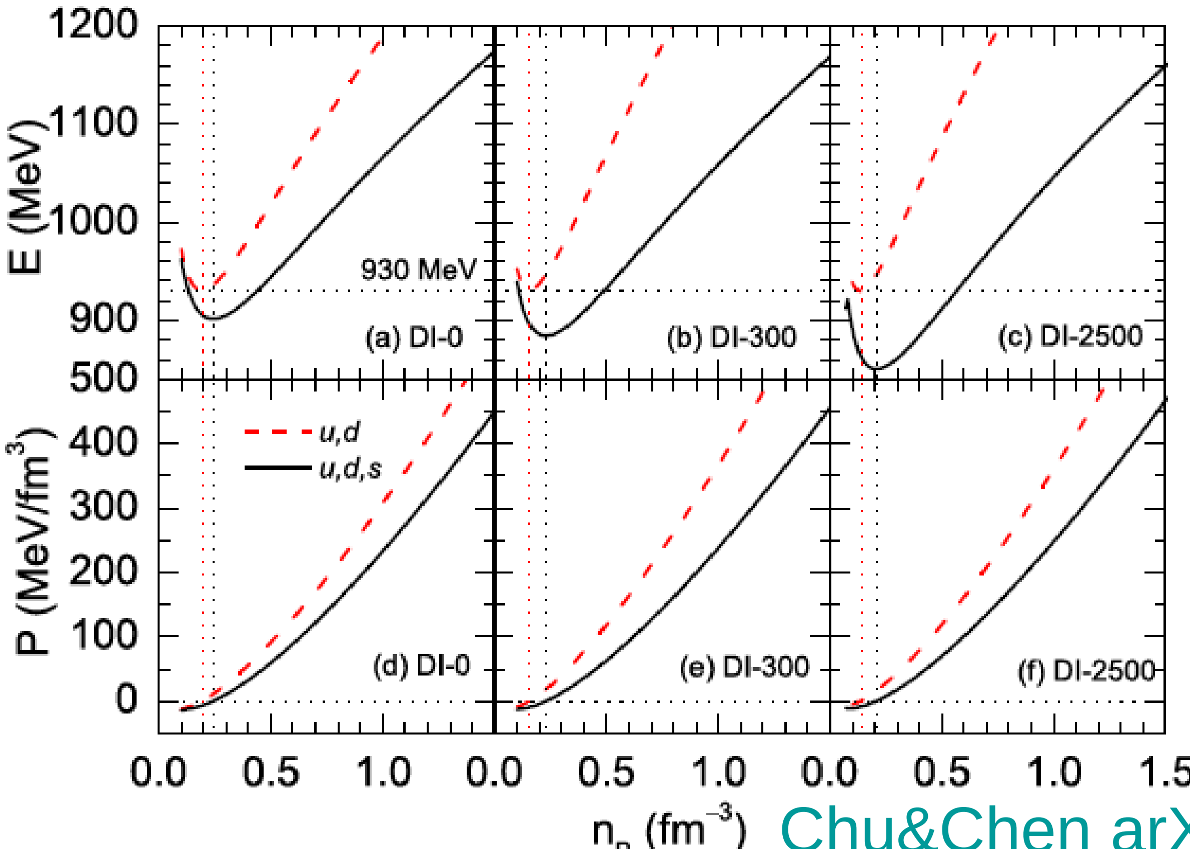
$$\mu_e = \sqrt{3\pi^2{\nu_e}^2 + {m_e}^2}$$

上海交通大学 SHANGHAI JIAO TONG UNIVERSITY

Energy per baryon and pressure VS. baryon density

The absolute stable conditions :

The minimum of energy per baryon of the u-d quark matter is larger than 930 MeV while SQM is less than 930 MeV.



β equilibrium condition

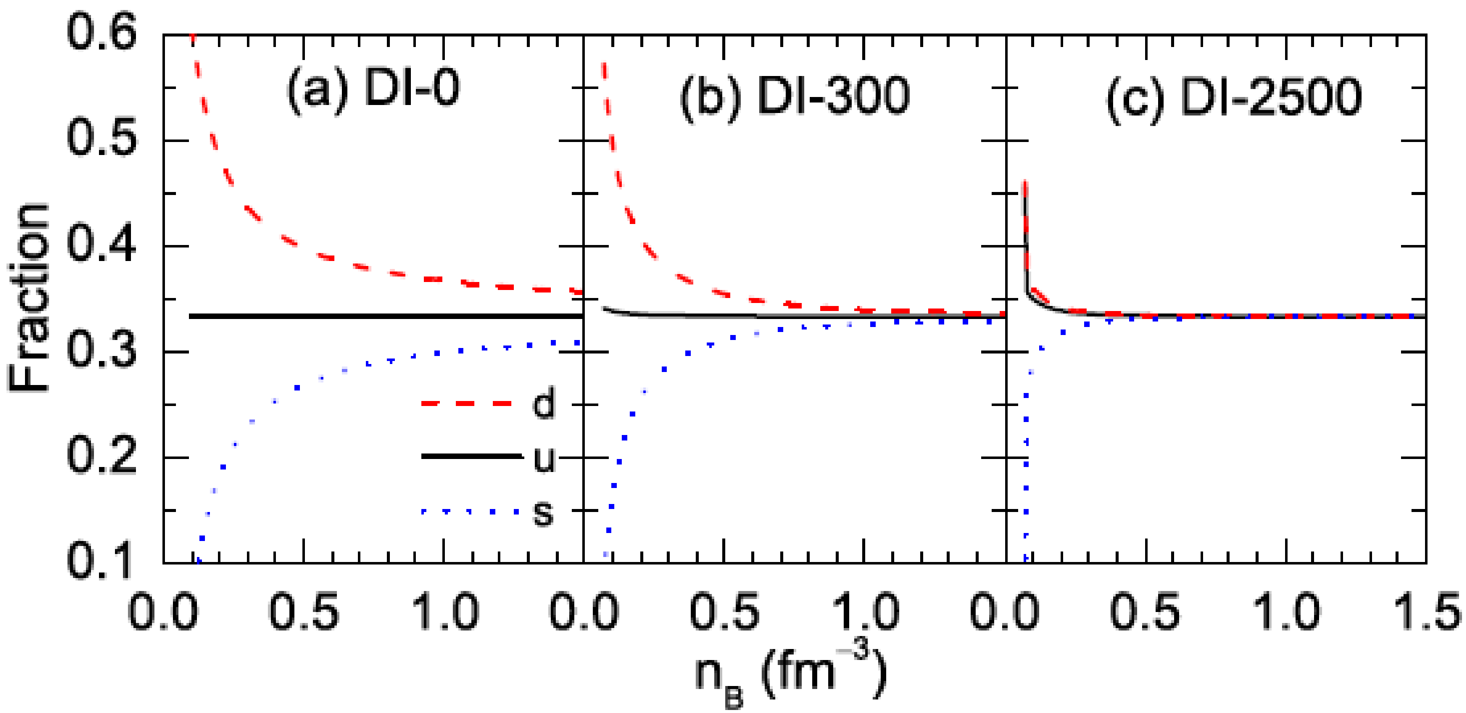
Chu&Chen arXiv:1212.1388v1

Baryon density at minimum energy per baryon is exactly zero-pressure density, which is consistent with the requirement of thermo dynamical self-consistency.

上海交通大學 SHANGHAI JIAO TONG UNIVERSITY

Quark fraction VS. baryon density in SQM for CIDDM

The difference among u , d and s quark fractions becomes smaller when the quark matter symmetry energy is increased.
The bigger the symmetry is, the smaller the isospin asymmetry is.

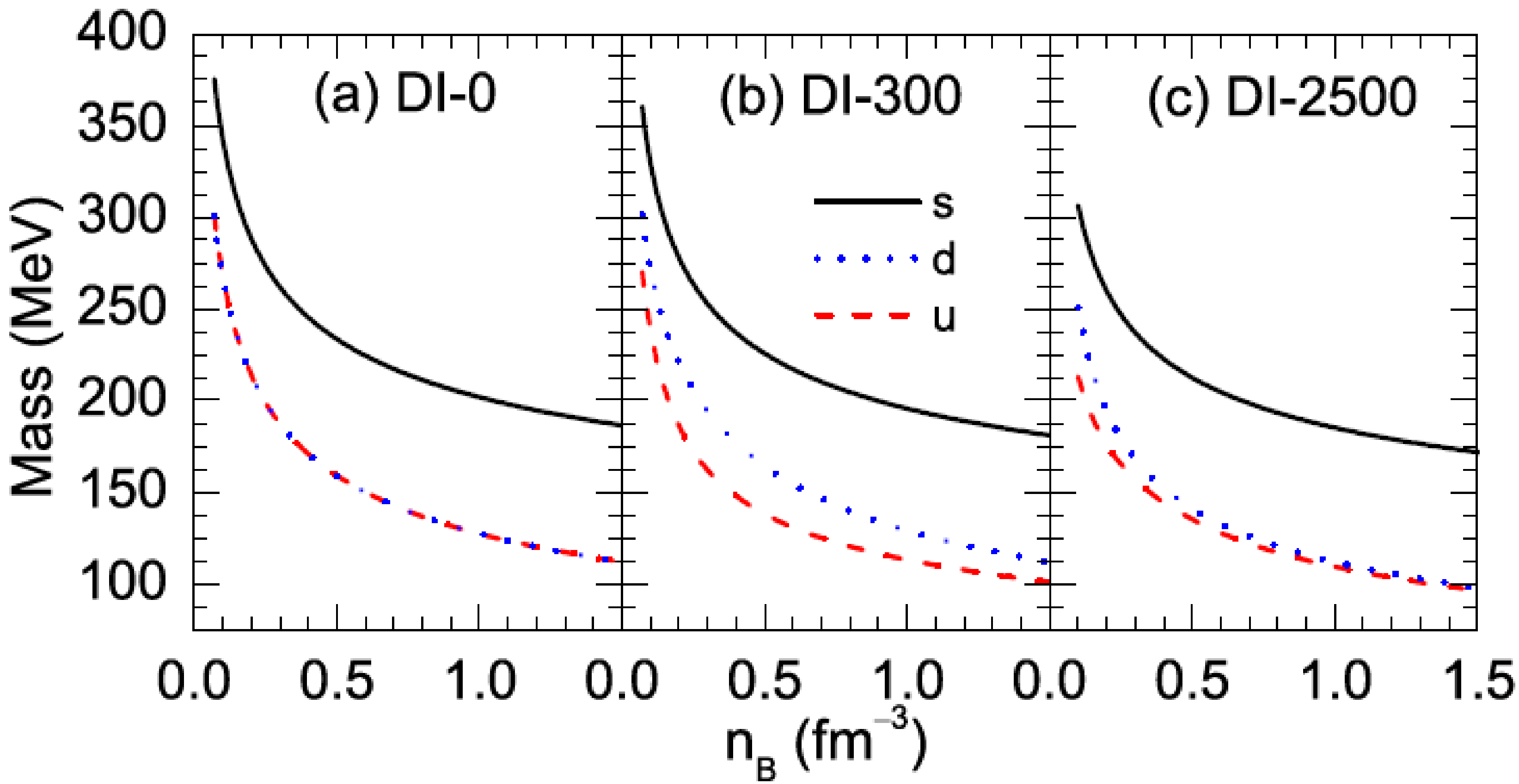


Just like the Color-Flavor-Locked phase

Chu&Chen arXiv:1212.1388v1

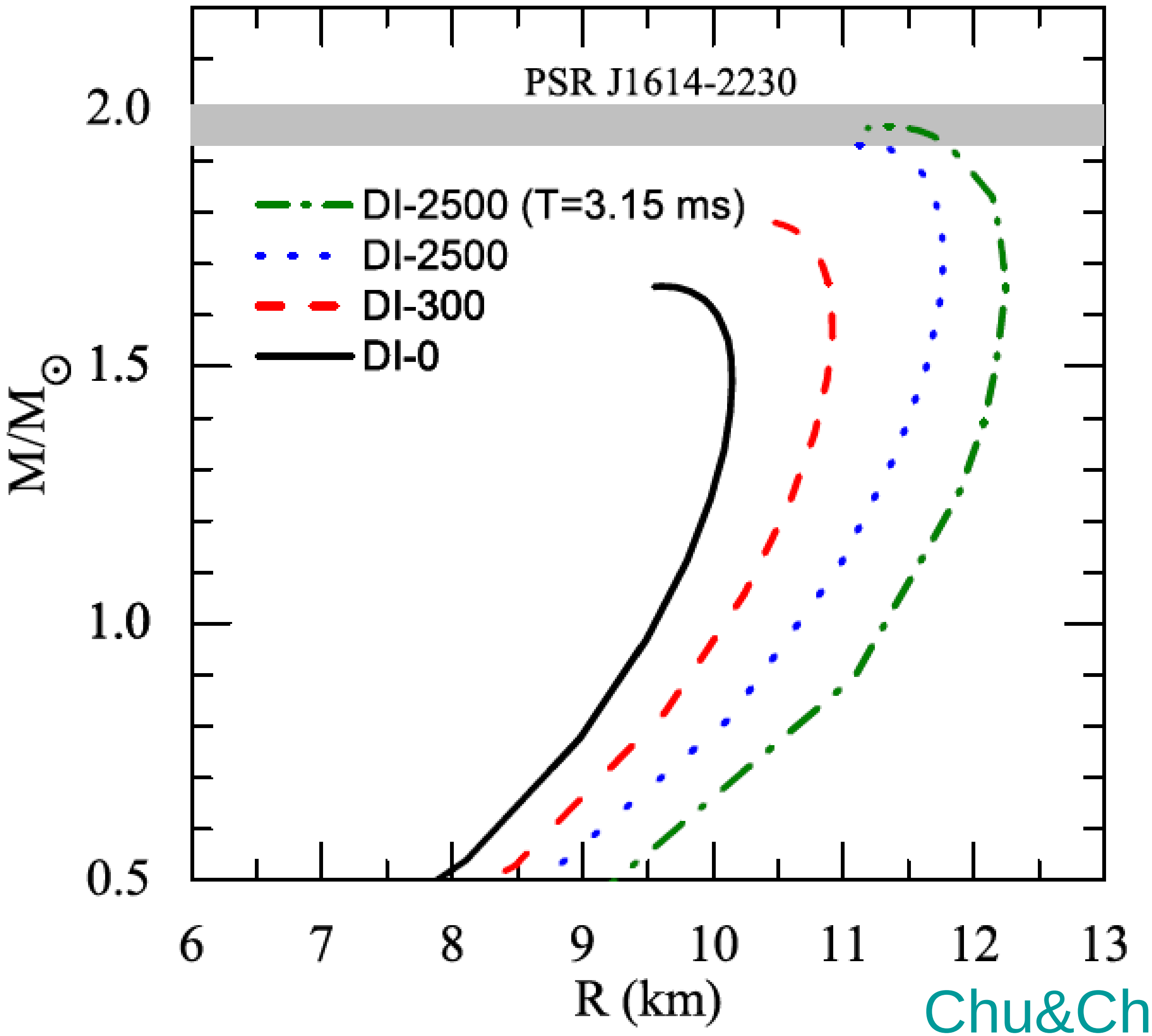
Equivalent quark mass VS. baryon density in SQM

The equivalent mass decreases when baryon density increases, it is the feature of confinement in this model. We can see there is a isospin dependent split between u and d quarks.



Chu&Chen arXiv:1212.1388v1

Mass − radius relation for quark star in CIDDM



The symmetry energy increases, when DI increases.

Chu&Chen arXiv:1212.1388v1

Maximum rotational frequency for empirical formula

Chu&Chen arXiv:1212.1388v1

	DI-0	DI-300	DI-2500
$M/M_\odot(static)$	1.65	1.78	1.93
$R(km)(static)$	9.60	10.40	11.12
$Central\ density(fm^{-3})$	1.31	1.11	1.06
f_{max} (Hz)	1680	1547	1458
$M/M_\odot$ $(at\ f_{max})$	1.78	2.12	2.43
$R(km)$ $(equator\ at\ f_{max})$	9.93	11.6	14.2

$$\Omega_{max} = 7730(M_\odot^{stat}/M_\odot)^{1/2}(R_{M\odot}^{stat}/10\mathrm{km})^{-3/2}\ \mathrm{rad}\cdot s^{-1}$$

Gourgoulhon, E., Haensel, P., Livine, R., Paluch, E., Bonazzola, S., & Marck, J. A. 1999, A&A, 349 , 851

The maximum rotational frequency decreases with DI while the mass and radius increase with DI .

This is the maximum mass DI-2500 can support.

Effect of the quark mass scaling parameter

As mentioned before, the quark mass scaling parameter z is phenomenological in the CDDM model , and in principle it should be determined by non-perturbative QCD calculations.

$$CIDDM \quad z \neq 1/3$$

$$\begin{aligned} m_q &= m_{q0} + m_I + m_{iso} \\ &= m_{q0} + \frac{D}{{n_B}^z} - \tau_q \delta D_I n_B^{\alpha} e^{-\beta n_B} \end{aligned}$$

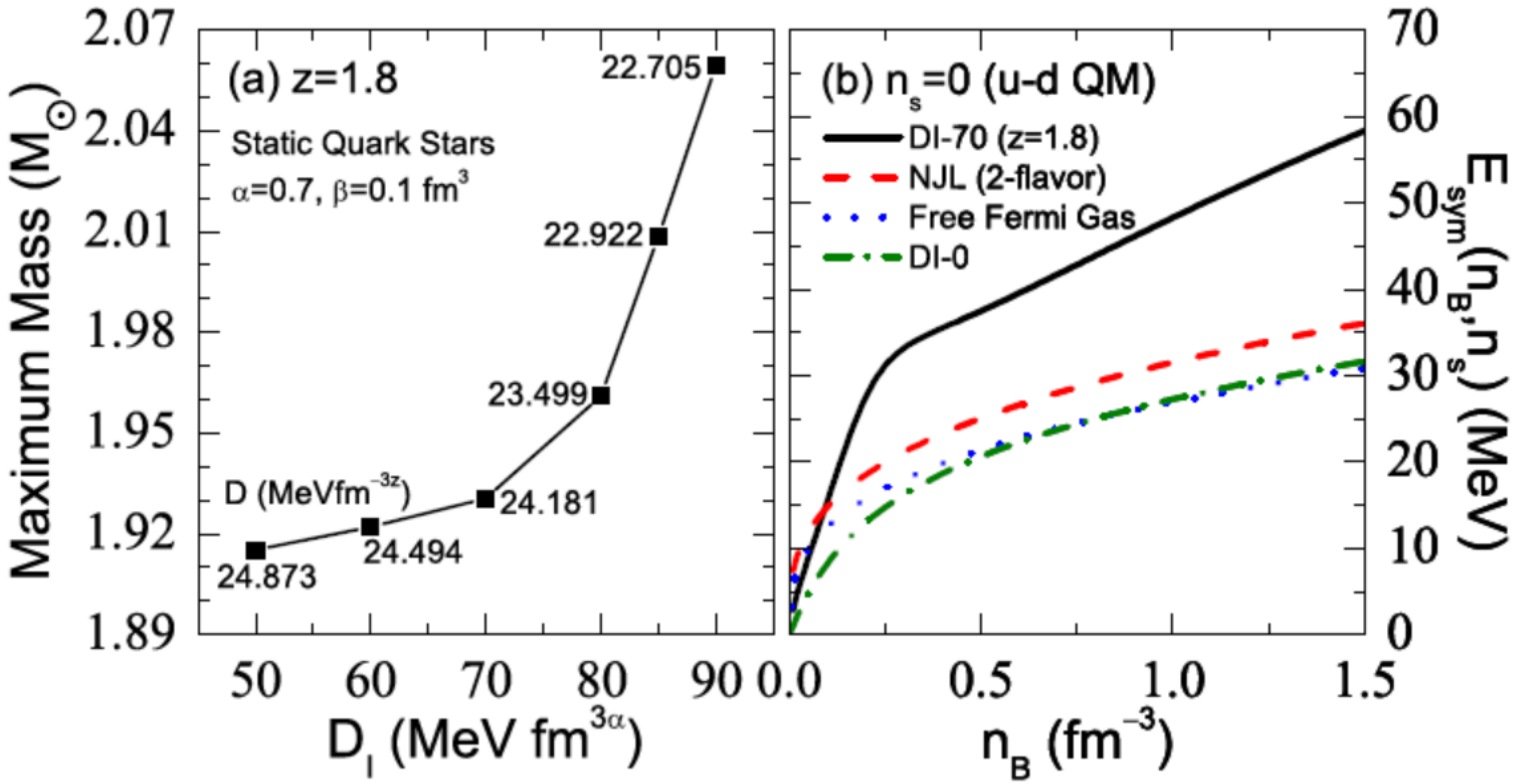
Effect of the quark mass scaling parameter

For CDDM model (DI=0), it cannot describe the PSRJ1614-2230 as a quark star, even though the z parameter can be varied freely.

Li, A., Xu, R. X., & Lu, J. F. 2010, Mon. Not. R. Astron. Soc. 402, 2715

We look for the minimum value of DI(the smallest quark matter symmetry energy) that is necessary to support a QS with nearly 2 solar mass in the CIDDM model

Effect of the quark mass scaling parameter



Furthermore, for fixed values of the parameters *D* and *DI* , varying the scaling parameter *z* can significantly change the maximum mass of QS's and we find that *z* = 1.8 generally gives rise to the largest QS maximum mass.

Extended NJL model with vector channel

The model Lagrangian density we consider is (for quarks):

$$\mathcal{L}_{NJL} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + G_s \sum_{a=0}^{8}[(\bar{\psi}\lambda^a\psi)^2 + (\psi i\bar{\gamma_5}\lambda^a\psi)^2] - K[\det\bar{\psi}(1+\gamma_5)\psi + \det\bar{\psi}(1-\gamma_5)\psi]$$

$$-G_{I,v}(\bar{\psi}\gamma^\mu\gamma_{3f}\psi)^2 \begin{cases} - g_v(\bar{\psi}\gamma^\mu\psi)^2 \\ - G_v \sum_{a=0}^{8}[(\bar{\psi}\gamma^\mu\lambda^a\psi)^2 + (\bar{\psi}i\gamma^\mu\gamma_5\lambda^a\psi)^2] \end{cases}$$

Mean-field approximation

$$\begin{aligned} \mathcal{L}_{MFA} = {} & \bar{\psi}_f[\gamma_\mu i\partial^\mu - \hat{M} - 4G_v\gamma_0\hat{\rho} - 2g_v\gamma_0(\rho_u + \rho_d + \rho_s) - 2G_{I,v}\gamma_0\tau_{3f}\rho_f]\psi_f \\ & - 2G(\phi_u^2 + \phi_d^2 + \phi_s^2) + 4K\phi_u\phi_d\phi_s + 2G_v(\rho_u^2 + \rho_d^2 + \rho_s^2) \\ & + g_v(\rho_u + \rho_d + \rho_s)^2 + G_{I,v}(\rho_u - \rho_d)^2 \end{aligned}$$

Extended NJL model with vector channel

For quark pressure：

$$p_f = -F_f = \theta_u + \theta_d + \theta_s - 2G(\phi_u^2 + \phi_d^2 + \phi_s^2) + 4K\phi_u\phi_d\phi_s + 2G_v(\rho_u^2 + \rho_d^2 + \rho_s^2) + g_v(\rho_u + \rho_d + \rho_s)^2 + G_{I,v}(\rho_u - \rho_d)^2$$

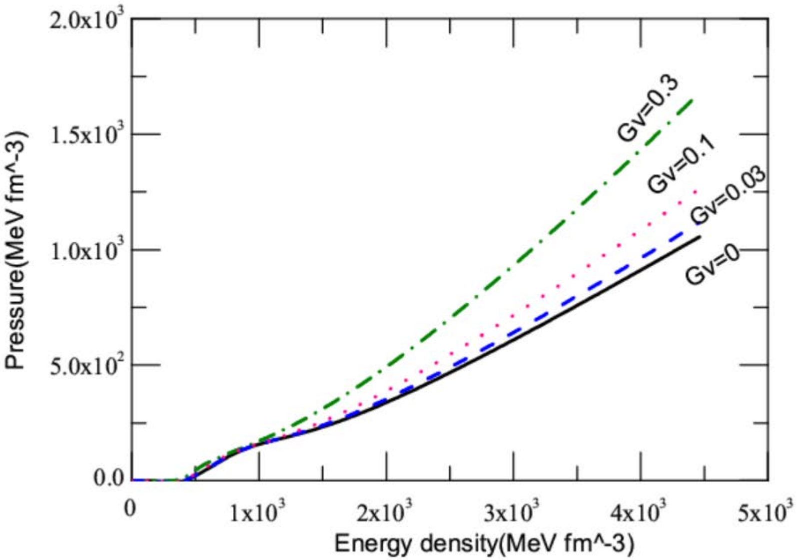
Where：

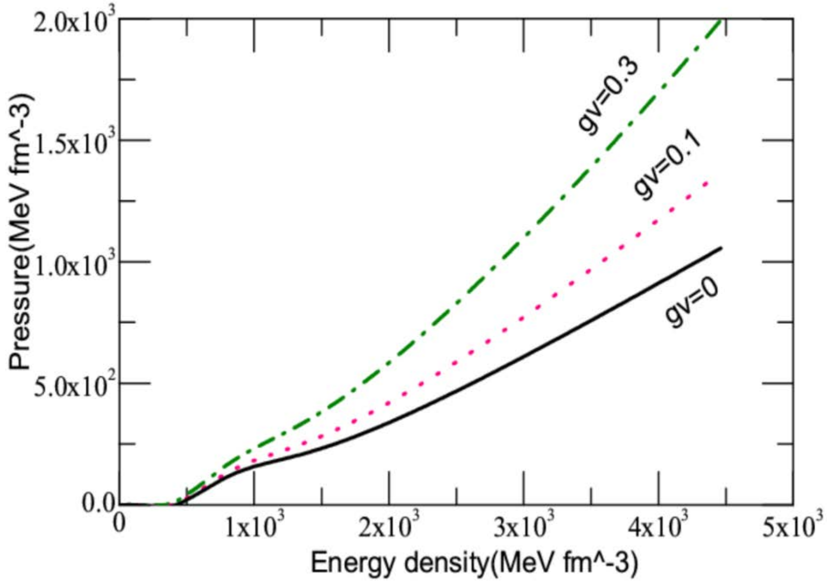
$$\theta_i = -\frac{i}{2}\mathrm{tr}\int \frac{d^4p}{(2\pi)^4} \ln\left(\vec{p}^2 + \hat{M}_i^2 - (p_0 + \tilde{\mu}_i)^2\right)$$

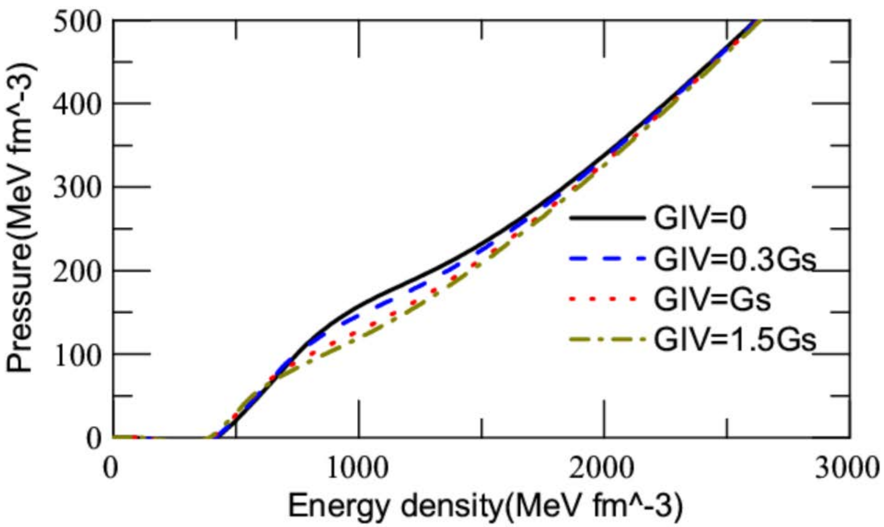
$$= -i\int \frac{d^4p}{(2\pi)^4} \sum_{i=u,d,s} \mathrm{tr}\ln\left\{\frac{1}{T}[\not{p} - \hat{M}_i + \gamma_0\tilde{\mu}_i]\right\}$$

Extended NJL model with vector channel

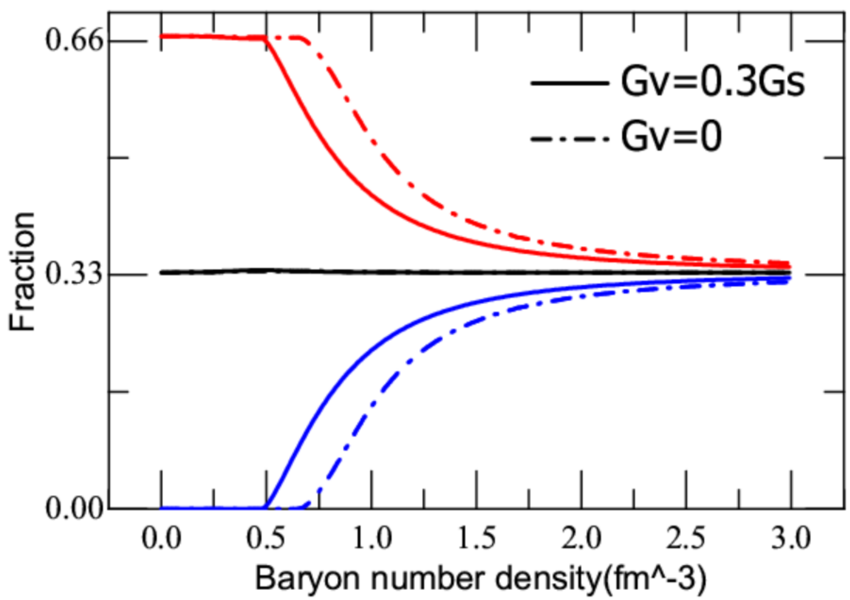
Under beta-equilibrium condition(three different channels):

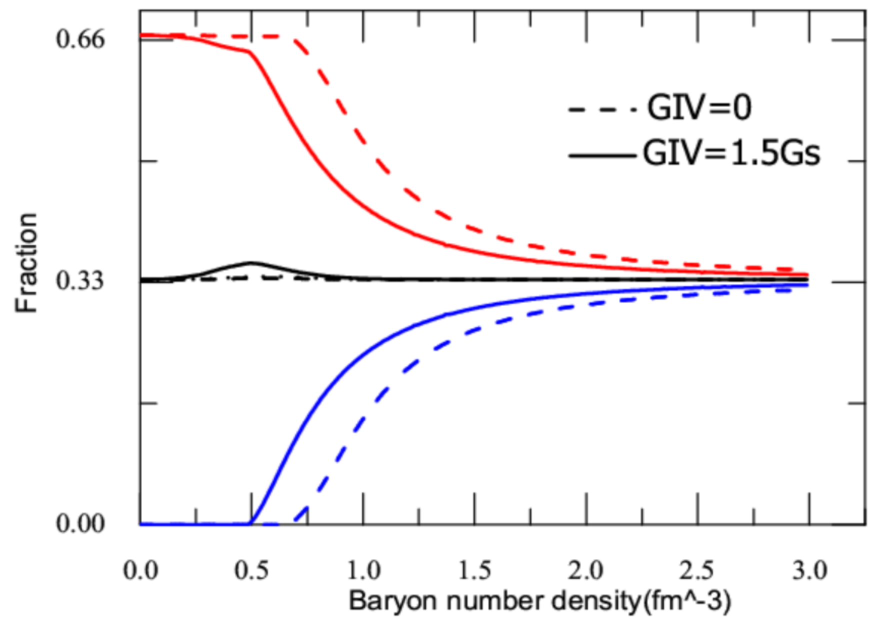




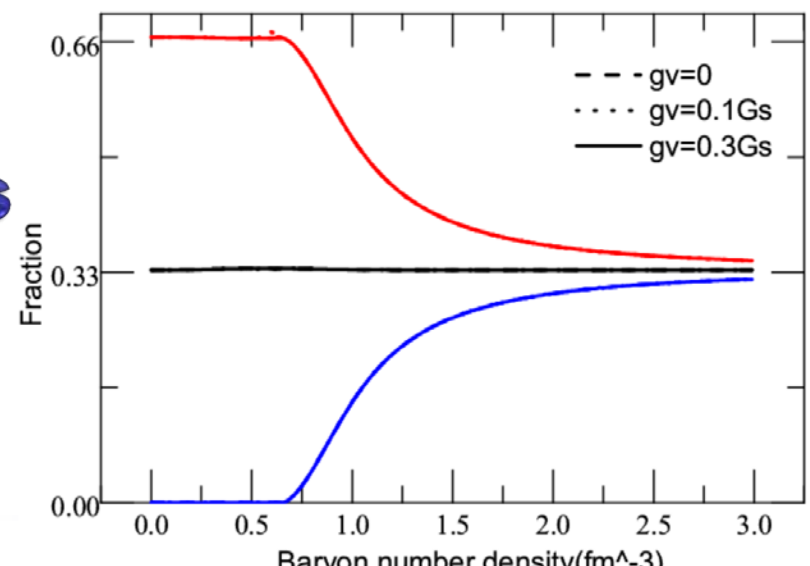


Extended NJL model with vector channel

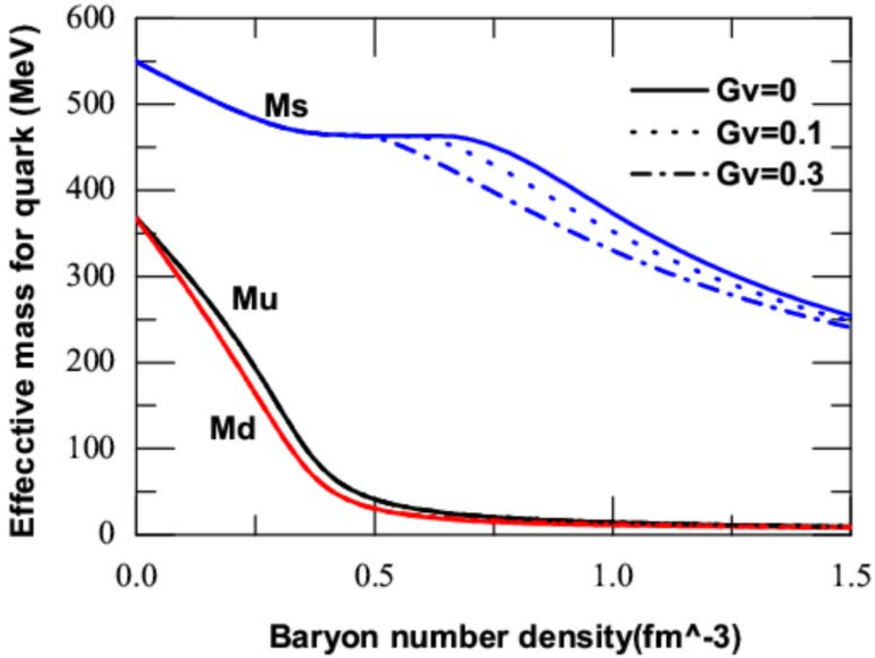


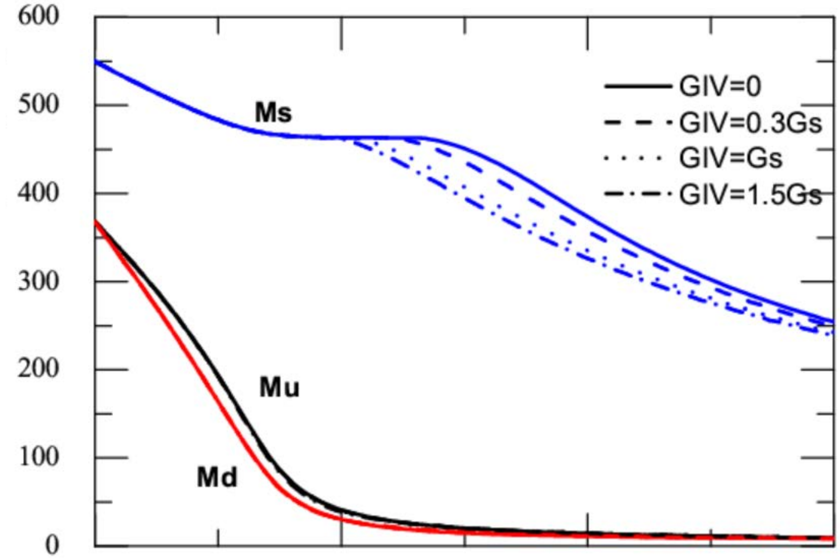


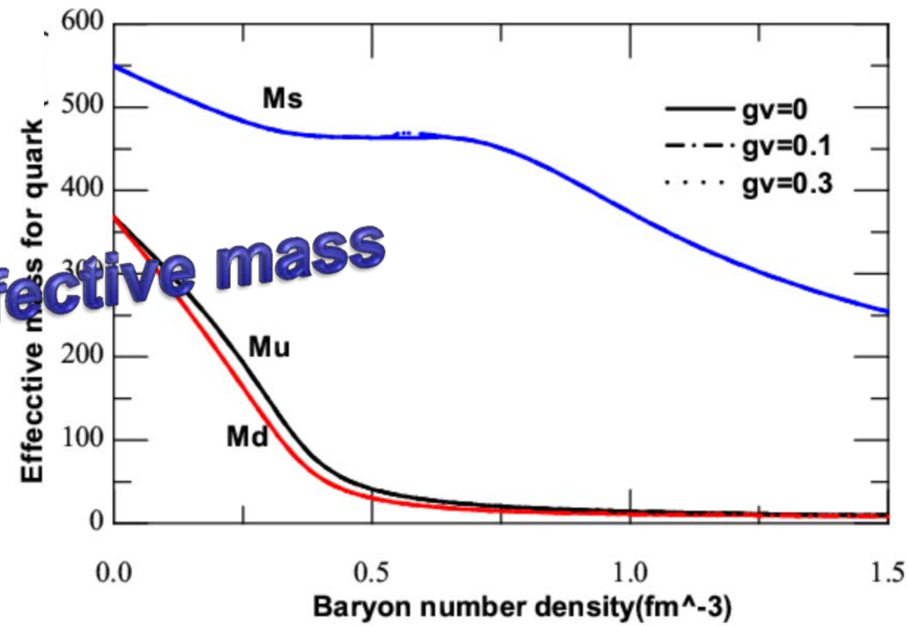
Fraction with different channels
gv is not important



Extended NJL model with vector channel

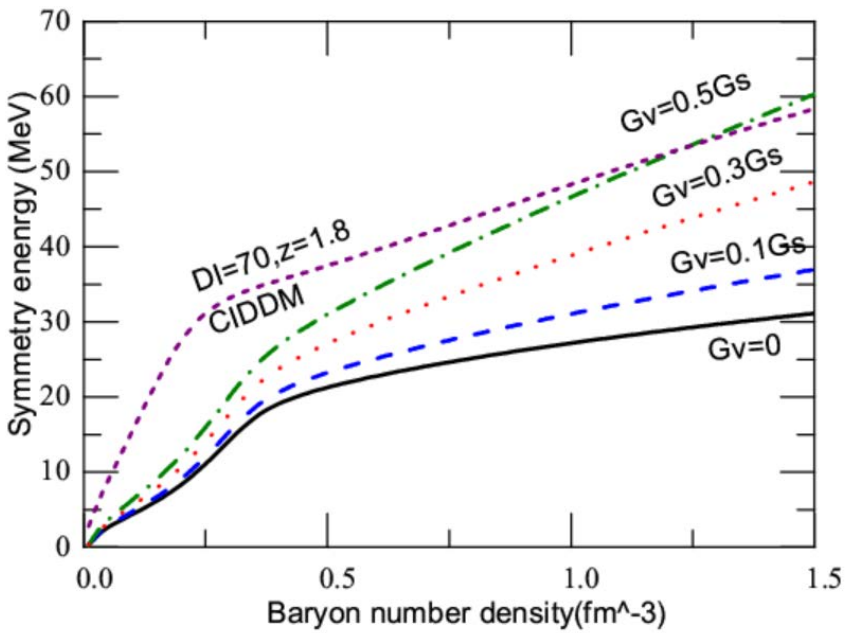


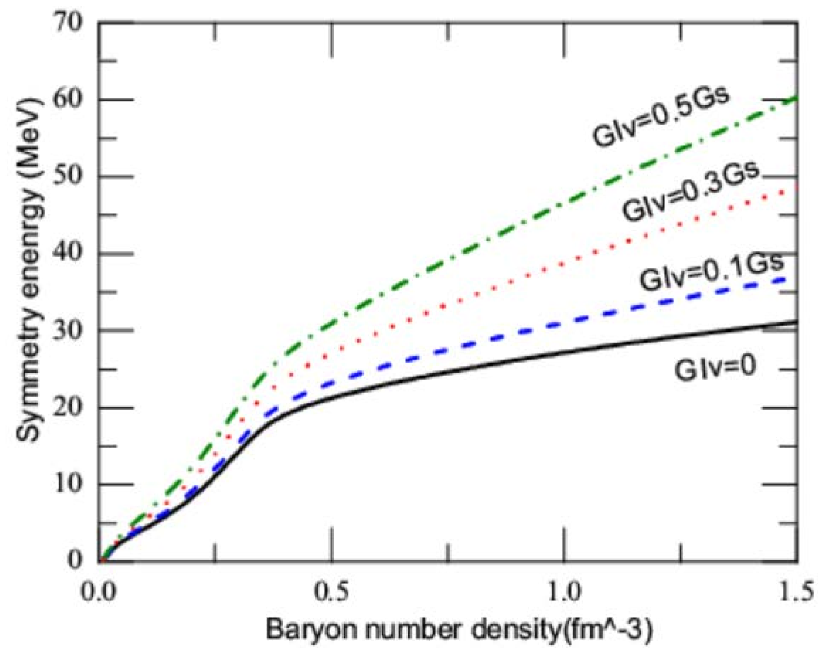




gv does not change the effective mass

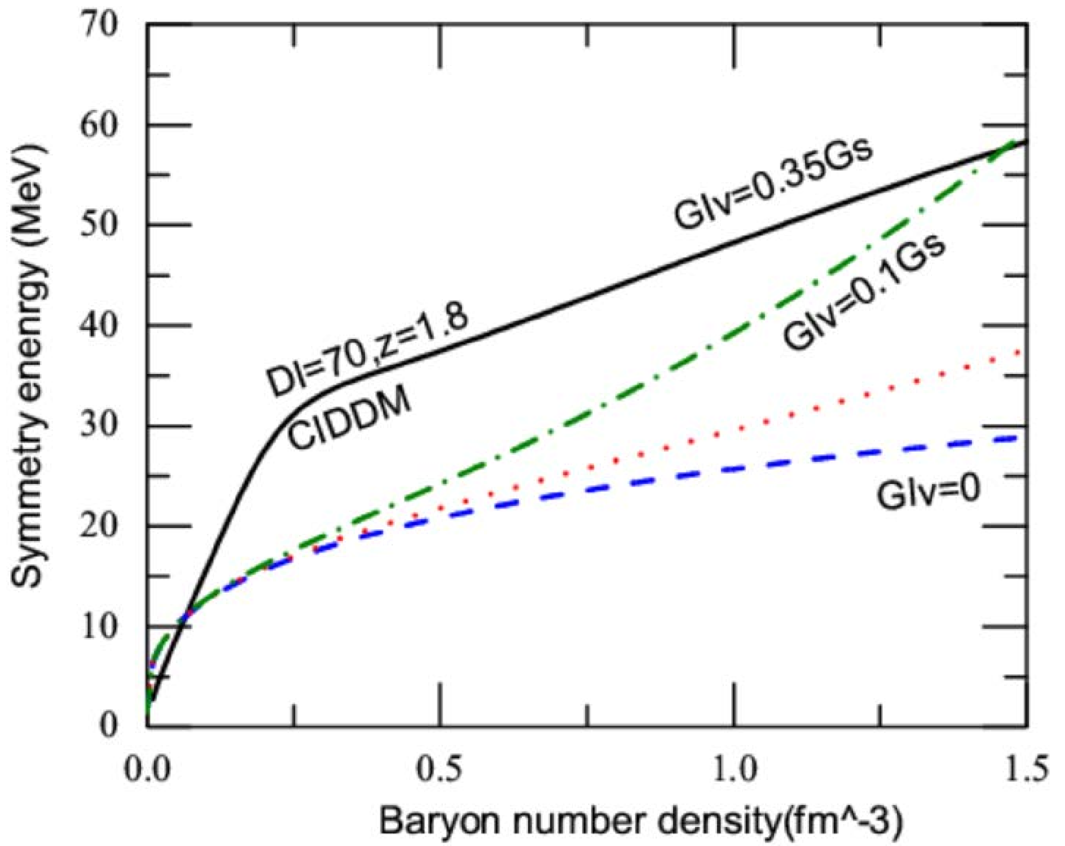
Extended NJL model with vector channel





Symmetry energy with different channels

Extended NJL model with vector channel



Symmetry energy with different channels SU(2)

Summary

1.We extend the confined-density-dependent-mass model in which the quark confinement is modeled by the density-dependent quark masses to include isospin dependence of the quark mass.

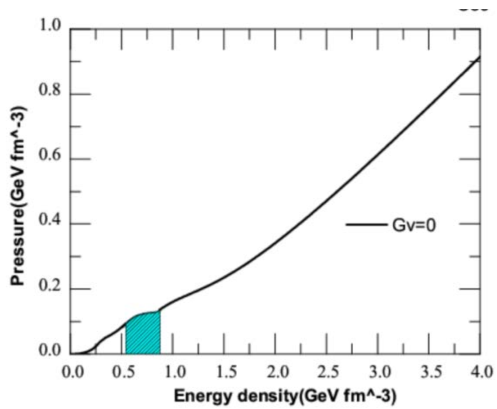
2.Within the confined isospin and density-dependent-mass model, we study the quark matter symmetry energy, the stability of strange quark matter, and the properties of quark stars.

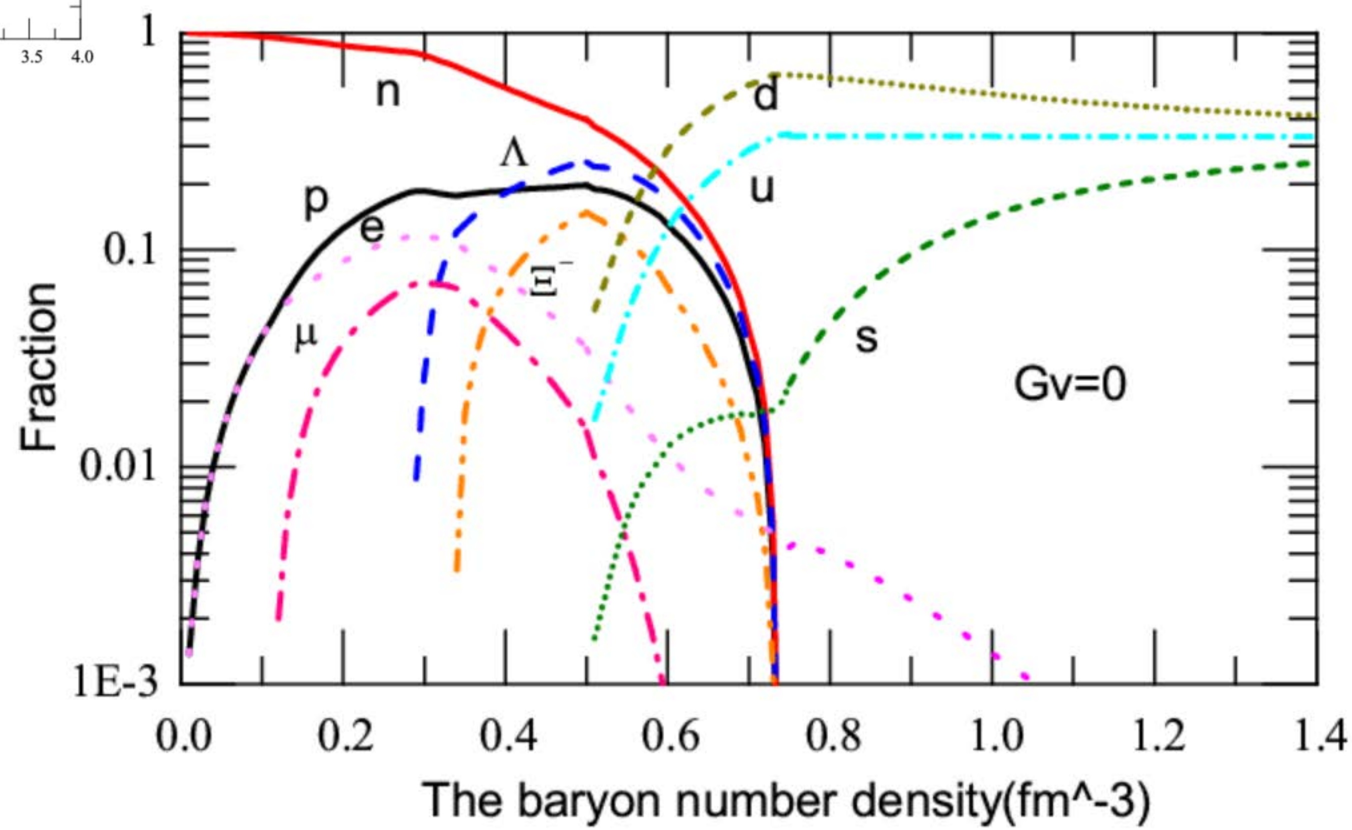
*3.*In particular, we find that PSR J1614-2230 can be well described by a quark star within the confined-isospin-density-dependent-mass model with the parameter set DI-2500, indicating that the quark matter symmetry energy might be much stronger than the nuclear matter symmetry energy .

4. If the mass scaling parameter *z* can be varied freely, the quark matter symmetry energy could be smaller than the nuclear matter symmetry energy but its strength should be at least about twice that of a free quark gas or normal quark matter with inconventional Nambu-Jona-Lasinio model in order to describe the PSR J1614-2230 as a quark star

Thank you!

Extended NJL model with vector channel





Extended NJL model with vector channel