A Possibility of Quark Spin Polarized Phase in High Density Quark Matter

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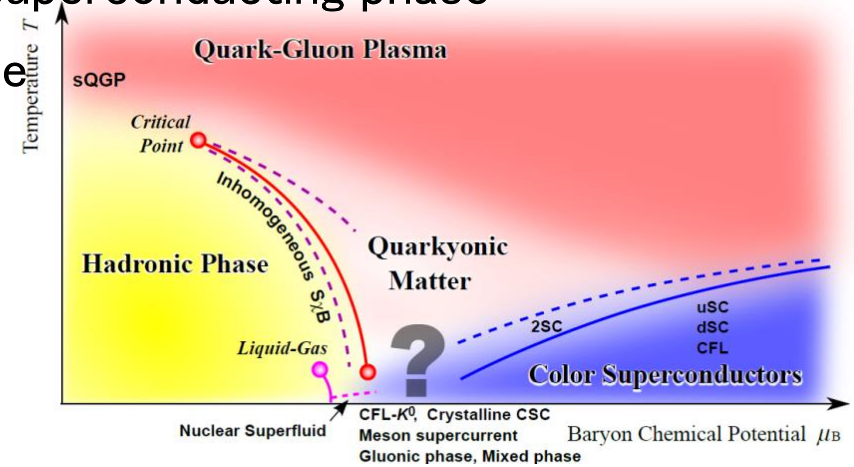
Introduction

- Quark and/or hadronic matter at finite density

  —low density･･･hadronic phase : chiral broken phase

  —high density･･･quark-gluon or color superconducting phase

  : chiral symmetric phase



@K.Fukushima and T. Hatsuda

- Possibility of quark spin polarization ?

  → Quark ferromagnetization ?

If possible, origin of spin polarization ?

from nuclear matter or from quark matter ?

Consider a possibility of spontaneous spin polarization in quark matter at high density

Introduction

- We show ・・・
  - ♪ tensor-type four-point interaction between quarks leads・・・

    ・・・NJL model with tensor interaction

    — even in chiral symmetric phase (quark mass is zero),

    spontaneous spin polarization occurs at high density

  cf. pseudovector interaction

  (cf, E.Nakano, T.Maruyama and T.Tatsumi, PRD 68 (2003) 105001)

  — spin polarization disappears in chiral symmetric phase

  due to quarks being massless (S.Maedan, PTP 118 (2007) 729)

  - ♪ spin polarized phase survives against two-flavor color superconductivity
  - ♪ spin polarized phase survives against color-flavor locking

Model – NJL model

- Nambu–Jona–Lasinio model with tensor–type interaction

$$\mathcal{L} = \mathcal{L}_{\rm kin} + \mathcal{L}_S + \mathcal{L}_V + \mathcal{L}_T$$
$$\mathcal{L}_{\rm kin} = i\bar{\psi}\gamma^\mu\partial_\mu\psi$$
$$\mathcal{L}_S = -G_S[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2]$$
$$\mathcal{L}_V = -G_V[(\bar{\psi}\gamma^\mu\vec{\tau}\psi)^2 + (\bar{\psi}\gamma_5\gamma^\mu\vec{\tau}\psi)^2]$$
$$\mathcal{L}_T = -G_T[(\bar{\psi}\gamma^\mu\gamma^\nu\vec{\tau}\psi)^2 + (\bar{\psi}i\gamma_5\gamma^\mu\gamma^\nu\psi)^2]$$

At high baryon density, chiral symmetry is restored

$\rightarrow \langle\overline{\psi}\psi\rangle = 0$: quarks are massless

and then, (S.Maedan, PTP 118 (2007) 729)

$\rightarrow \langle\overline{\psi}\gamma_5\gamma^{\mu=3}\vec{\tau}\psi\rangle = 0$: pseudovector condensate is zero

due to quark being massless

Model – tensor interaction

- Tensor interaction is retained : $L = L_{kin} + L_T$

$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - \frac{G}{4}(\bar{\psi}\gamma^\mu\gamma^\nu\vec{\tau}\psi)(\bar{\psi}\gamma_\mu\gamma_\nu\vec{\tau}\psi)$$

Here, $\gamma^1\gamma^2 = -i\Sigma_3 = -i\begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}$

Then, $\left\langle \overline{\psi}\gamma^1\gamma^2\vec{\tau}\psi \right\rangle \neq 0$ → quark spin polarization occurs

- Hereafter, μ =1,ν =2 are taken into account.

Effective potential – for spin polarization

- Generating functional Z

$$Z \propto \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left[i\int d^4x\left(\bar{\psi}i\gamma^\mu\partial_\mu\psi + \frac{G}{2}(\bar{\psi}\Sigma_3\tau_k\psi)(\bar{\psi}\Sigma_3\tau_k\psi)\right)\right]$$

- Inserting "auxiliary field" $F_k \left(= -G\langle\bar{\psi}\Sigma^3\tau_k\psi\rangle\right)$

$$1 = \int \mathcal{D}F_k \exp\left[-\frac{i}{2}\int d^4x\,(F_k + G(\bar{\psi}\Sigma_3\tau_k\psi))\,G^{-1}\,(F_k + G(\bar{\psi}\Sigma_3\tau_k\psi))\right]$$

Then, finally

$$Z \propto \int \mathcal{D}F_k \exp\left[i\int d^4x\left(-\frac{F_k^2}{2G} + \frac{1}{4i}\mathrm{tr}\ln\left(-p_0^2 + \epsilon_p^{(-)2}\right) + \frac{1}{4i}\mathrm{tr}\ln\left(-p_0^2 + \epsilon_p^{(+)2}\right)\right)\right]$$

$$\epsilon_p^{(\pm)} = \sqrt{\left((F_k\tau_k) \pm \sqrt{p_1^2 + p_2^2}\right)^2 + p_3^2}$$

: single-particle energy

Effective potential – for spin polarization

- Effective potential : $V[F]$

$$Z = \exp\left(i\Gamma[F]\right) \ , \qquad V[F] = -\frac{\Gamma[F]}{\int d^4x}$$

- At finite density, introduce chemical potential : μ

$$L \rightarrow L + \mu\bar{\psi}\gamma^0\psi$$

Then, finally

$$V[F] = \frac{F^2}{2G} + 2N_c \int^F dF \int \frac{d^3p}{(2\pi)^3} \left[\frac{F - \sqrt{p_1^2 + p_2^2}}{\epsilon_p^{(-)}} \theta(\mu - \epsilon_p^{(-)}) + \frac{F + \sqrt{p_1^2 + p_2^2}}{\epsilon_p^{(+)}} \theta(\mu - \epsilon_p^{(+)}) \right]$$

where

$$F_k\tau \rightarrow F\tau_3 = F\tau \ , \qquad \tau = \begin{cases} 1 & \text{for up quark} \\ -1 & \text{for down quark} \end{cases}$$

Effective potential – for spin polarization

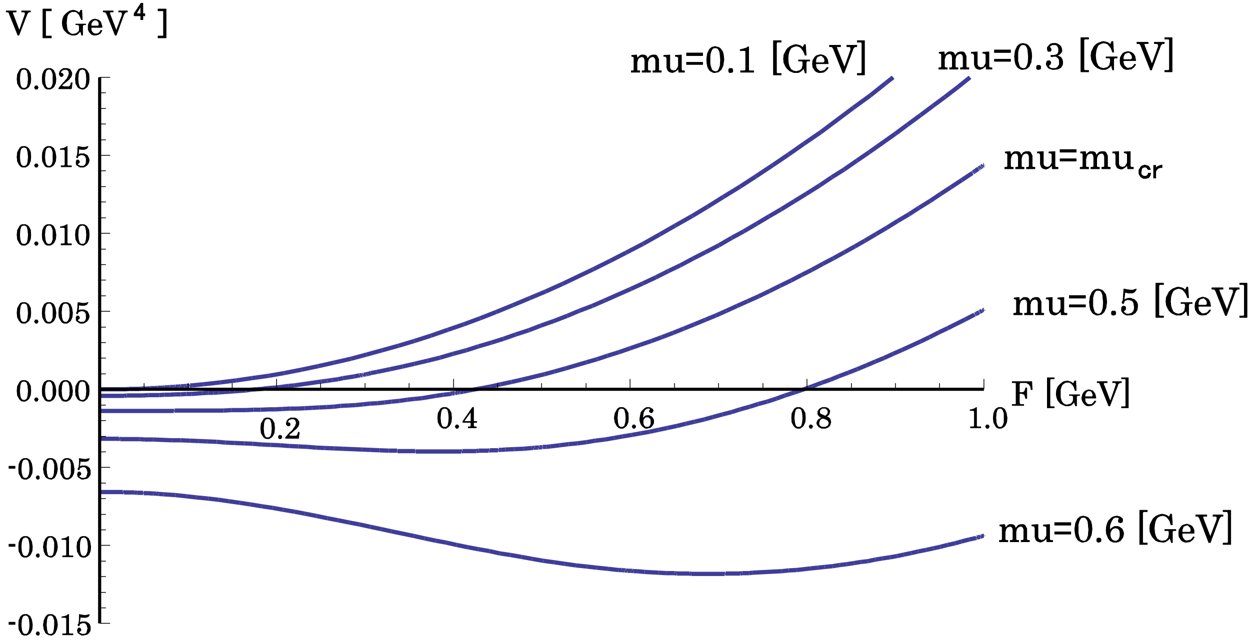
- Thermodynamic relation : pressure p

  : quark number density ρ_q

$$p = -V[F] \, , \qquad \rho_q = -\frac{\partial V[F]}{\partial \mu}$$

Numerical results – for spin polarization

- Effective potential : $V[F]$



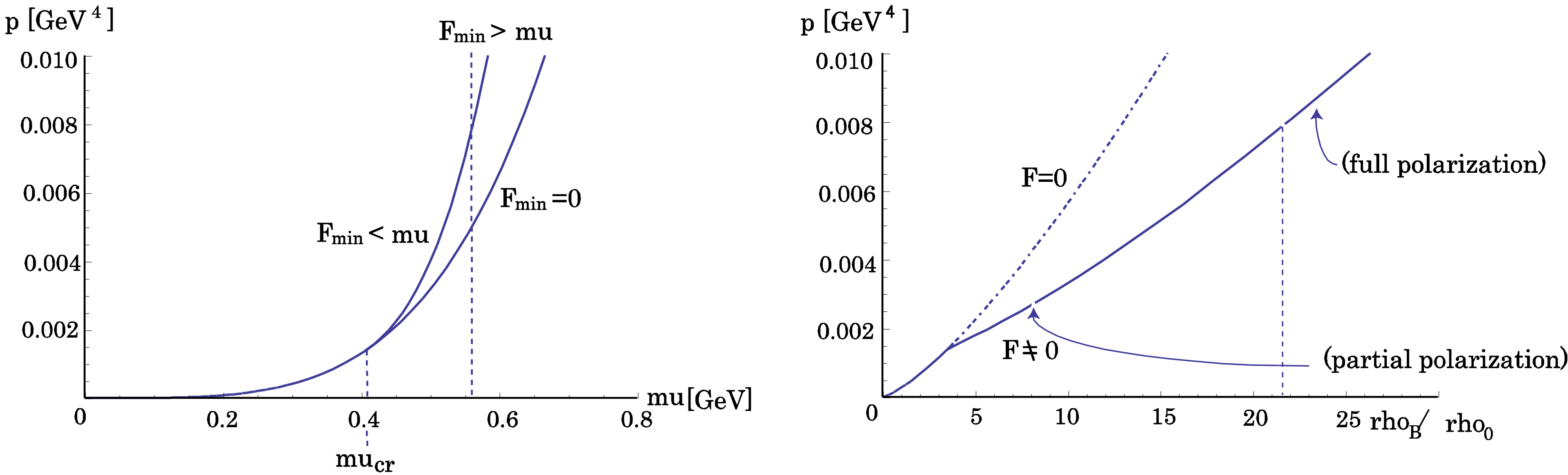
- Parameters used here

$$G = 20\,\mathrm{GeV}^{-2}$$

(with vaccume polarization, $G = 11.1\,\mathrm{GeV}^{-2}$ with cutoff $\Lambda = 0.631\,\mathrm{GeV}$)

Numerical results – for spin polarization

- Pressure vs chemical potential or baryon number density



- Critical density

G / GeV^{-2}	ρ_{cr}/ρ_0	μ_{cr} / GeV
15	5.34	0.468
20	3.47	0.406
25	2.48	0.363

$$\left(\rho_0 = 0.17\ \mathrm{fm}^{-3}\right)$$

Brief Summary

- We have shown ・・・
  - ♪ tensor−type four−point interaction between quarks leads・・・
    - — spontaneous quark spin polarization occurs at high density

Stability of spin polarized phase
--- two-flavor case

- High density (and low temperature) quark matter in two-flavor:

  → 2-flavor color superconducting (2SC) phase

Is the spin polarized phase survives
at high density against 2SC phase ?

Interplay between spin polarization and color superconductivity

- Lagrangian density with 2-flavor color superconductivity

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_T + \mathcal{L}_c$$

$$\mathcal{L}_c = \frac{G_c}{2} \sum_{A=2,5,7} \left((\bar{\psi} i\gamma_5 \tau_2 \lambda_A \psi^C)(\bar{\psi}^C i\gamma_5 \tau_2 \lambda_A \psi) + (\bar{\psi} \tau_2 \lambda_A \psi^C)(\bar{\psi}^C \tau_2 \lambda_A \psi) \right)$$

- Mean field approximation

$$\mathcal{L}^{MF} = \mathcal{L}_0 + \mathcal{L}_T^{MF} + \mathcal{L}_c^{MF} \ ,$$

$$\mathcal{L}_T^{MF} = -F(\bar{\psi}\Sigma_3\tau_3\psi) - \frac{F^2}{2G} \ , \quad \left(F = -G\langle\bar{\psi}\Sigma_3\tau_3\psi\rangle\right)$$

$$\mathcal{L}_c^{MF} = -\frac{1}{2} \sum_{A=2,5,7} (\Delta\bar{\psi}^C i\gamma_5\tau_2\lambda_A + h.c.) - \frac{3\Delta^2}{2G_c} \ ,$$

$$\Delta_A = \Delta_A^* = -G_c\langle\bar{\psi} i\gamma_5\tau_2\lambda_A\psi\rangle \ , \qquad \Delta = \Delta_2 = \Delta_5 = \Delta_7$$

For example, M.Kitazawa, T.Koide, T.Kunihiro and Y.Nemoto, PTP 108 (2002) 929

Interplay between spin polarization and color superconductivity

- Hamiltonian formalism

$$\mathcal{H}_{MF} - \mu\mathcal{N} = \mathcal{K}_0 + \mathcal{H}_T^{MF} + \mathcal{H}_c^{MF} \ ,$$
$$\mathcal{K}_0 = \bar{\psi}(-\gamma\cdot\nabla - \mu\gamma_0)\psi \ ,$$
$$\mathcal{H}_T^{MF} = -\mathcal{L}_T^{MF} \ , \qquad \mathcal{H}_c^{MF} = -\mathcal{L}_c^{MF}$$

- Hamiltonian for quark and antiquark

$$H_{MF} - \mu N = \sum_{\mathbf{p}\eta\tau\alpha} \left[(p-\mu)c^\dagger_{\mathbf{p}\eta\tau\alpha}c_{\mathbf{p}\eta\tau\alpha} - (p+\mu)\tilde{c}^\dagger_{\mathbf{p}\eta\tau\alpha}\tilde{c}_{\mathbf{p}\eta\tau\alpha}\right]$$
$$+ F\sum_{\mathbf{p}\eta\tau\alpha}\phi_\tau\left[\frac{\sqrt{p_1^2+p_2^2}}{p}\left(c^\dagger_{\mathbf{p}\eta\tau\alpha}c_{\mathbf{p}-\eta\tau\alpha} + \tilde{c}^\dagger_{\mathbf{p}\eta\tau\alpha}\tilde{c}_{\mathbf{p}-\eta\tau\alpha}\right) - \eta\frac{p_3}{p}\left(c^\dagger_{\mathbf{p}\eta\tau\alpha}\tilde{c}_{\mathbf{p}\eta\tau\alpha} + \tilde{c}^\dagger_{\mathbf{p}\eta\tau\alpha}c_{\mathbf{p}\eta\tau\alpha}\right)\right]$$
$$+ \frac{\Delta}{2}\sum_{\mathbf{p}\eta\alpha\alpha'\alpha''\tau\tau'}\left(c^\dagger_{\mathbf{p}\eta\tau\alpha}c^\dagger_{-\mathbf{p}\eta\tau'\alpha'} + \tilde{c}^\dagger_{\mathbf{p}\eta\tau\alpha}\tilde{c}^\dagger_{-\mathbf{p}\eta\tau'\alpha'} + c_{-\mathbf{p}\eta\tau'\alpha'}c_{\mathbf{p}\eta\tau\alpha} + \tilde{c}_{-\mathbf{p}\eta\tau'\alpha'}\tilde{c}_{\mathbf{p}\eta\tau\alpha}\right)\phi_\tau\epsilon_{\alpha\alpha'\alpha''}\epsilon_{\tau\tau'}$$
$$+ V\cdot\frac{F^2}{2G} + V\cdot\frac{3\Delta^2}{2G_c}$$

where $\eta = \pm 1 \cdots$ helicity, $\tau = \pm 1 \cdots$ isospin $(\phi_\pm = \pm 1)$, $\alpha \cdots$ color

Mean Field Approximation – for color-superconducting gap Δ and spin polarization F

Mean field approximation

$$H_{\mathrm{MF}} - \mu N = \sum_{p\eta\tau\alpha} \left[(\varepsilon_p^{(\eta)} - \mu) a^\dagger_{p\eta\tau\alpha} a_{p\eta\tau\alpha} - (\varepsilon_p^{(\eta)} + \mu) \tilde{a}^\dagger_{p\eta\tau\alpha} \tilde{a}_{p\eta\tau\alpha} \right]$$

$$+ \frac{\Delta}{2} \sum_{p\eta\tau\tau'\alpha\alpha'\alpha''} f(\eta) \left[a^\dagger_{p\eta\tau\alpha'} a^\dagger_{-p\eta\tau'\alpha''} - \tilde{a}^\dagger_{p\eta\tau\alpha'} \tilde{a}^\dagger_{-p\eta\tau\alpha''} + h.c. \right] \epsilon_{\alpha\alpha'\alpha''} \epsilon_{\tau\tau'} \phi_\tau$$

$$+ V \cdot \frac{F^2}{2G} + V \cdot \frac{3\Delta^2}{2G_c}.$$

$$\varepsilon_p^{(\pm)} = \sqrt{p_3^2 + \left(F \pm \sqrt{p_1^2 + p_2^2} \right)^2} \quad , \quad f(\eta) = \frac{p + \eta e}{\varepsilon_p^{(\eta)}}, \quad \left(e = F \frac{\sqrt{p_1^2 + p_2^2}}{p} \right)$$

$$\Delta_A = \Delta_A^* = -G_c \langle \bar{\psi} i \gamma_5 \tau_2 \lambda_A \psi \rangle, \quad \Delta = \Delta_2 = \Delta_5 = \Delta_7$$

$$F \tau_k = -G \left\langle \overline{\psi} \Sigma^3 \tau_k \psi \right\rangle$$

$a_{p\eta\tau\alpha} \cdots$ positive energy states, $\tilde{a}_{p\eta\tau\alpha} \cdots$ negative energy states

Quark matter ･･･ positive energy particles are retained

Interplay between spin polarization and color superconductivity

- BCS state for positive energy particles

$$|\Psi\rangle = e^{S}|\Psi_0\rangle, \qquad |\Psi_0\rangle = \prod_{p\eta\tau\alpha\ (\varepsilon_p^{(\eta)}<\mu)} a^{+}_{p\eta\tau\alpha}|0\rangle,$$

$$S = \sum_{p\eta(\varepsilon_p^{(\eta)}>\mu)} \frac{K_{p\eta}}{2} \sum_{\alpha\alpha'\alpha''\tau\tau'} a^{+}_{p\eta\tau\alpha}\, a^{+}_{-p\eta\tau'\alpha'}\, \varepsilon_{\alpha\alpha'\alpha''}\, \varepsilon_{\tau\tau'}\phi_\tau + \sum_{p\eta(\varepsilon_p^{(\eta)}\le\mu)} \frac{\tilde{K}_{p\eta}}{2} \sum_{\alpha\alpha'\alpha''\tau\tau'} a_{p\eta\tau\alpha}\, a_{-p\eta\tau'\alpha'}\, \varepsilon_{\alpha\alpha'\alpha''}\, \varepsilon_{\tau\tau'}\phi_\tau$$

where $K_{p\eta} = K_{-p\eta}, \qquad \tilde{K}_{p\eta} = \tilde{K}_{-p\eta}$

and $\sin\theta_{p\eta} = \dfrac{\sqrt{3}K_{p\eta}}{\sqrt{1+3K_{p\eta}^2}}, \qquad \sin\tilde{\theta}_{p\eta} = \dfrac{\sqrt{3}\tilde{K}_{p\eta}}{\sqrt{1+3\tilde{K}_{p\eta}^2}}$

Interplay between spin polarization and color superconductivity

- Variational equations determine θ (K)

$$\frac{\partial}{\partial\theta_{p\eta}}\langle\Psi|H_{MF}-\mu N|\Psi\rangle=0\ , \qquad \frac{\partial}{\partial\tilde{\theta}_{p\eta}}\langle\Psi|H_{MF}-\mu N|\Psi\rangle=0$$

- Thermodynamic potential

$$\Phi(\Delta,F,\mu)=\frac{1}{V}\langle\Phi|H_{MF}-\mu N|\Phi\rangle$$

$$=2\cdot\frac{1}{V}\sum_{p\eta(\varepsilon_p^{(\eta)}\le\mu)}\left[2\,(\varepsilon_p^{(\eta)}-\mu)-\sqrt{(\varepsilon_p^{(\eta)}-\mu)^2+3\,\Delta^2\,f(\eta)^2}\right]$$

$$+2\cdot\frac{1}{V}\sum_{p\eta(\varepsilon_p^{(\eta)}>\mu)}\left[(\varepsilon_p^{(\eta)}-\mu)-\sqrt{(\varepsilon_p^{(\eta)}-\mu)^2+3\,\Delta^2\,f(\eta)^2}\right]+\frac{F^2}{2G}+\frac{3\Delta^2}{2G_c}$$

Interplay between spin polarization and color superconductivity

Gap equation

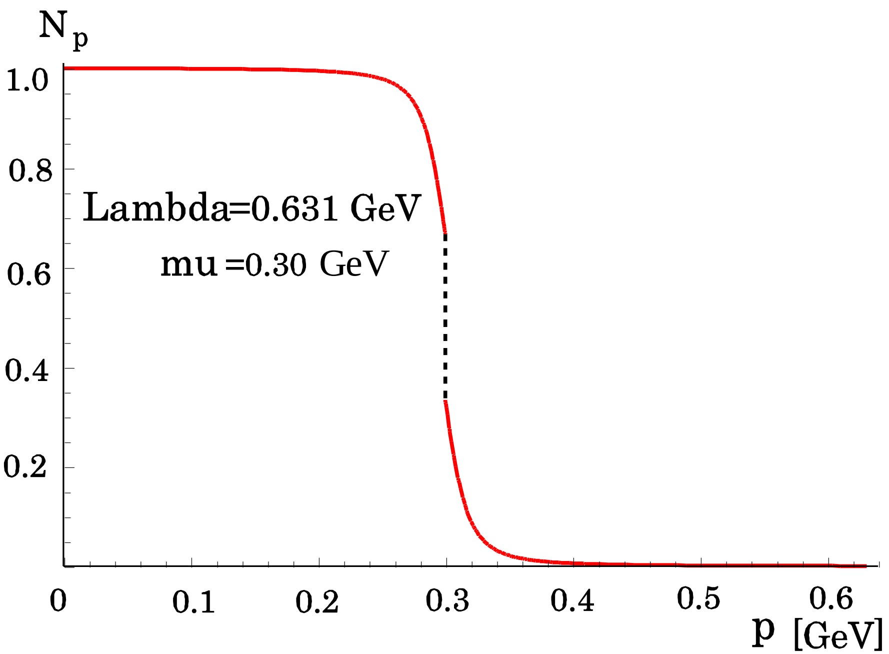
$$\frac{\partial}{\partial \Delta} \langle \Phi | H_{MF} - \mu N | \Phi \rangle = 0$$

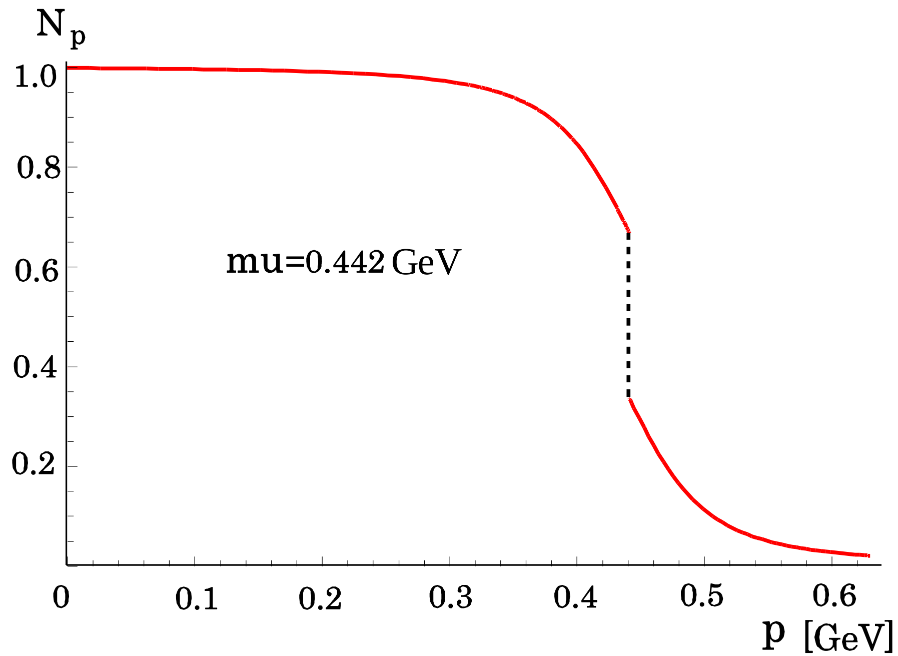
Namely,

$$\Delta \left[2 \cdot \frac{1}{V} \sum_{p\eta=\pm}^{\Lambda} \frac{f(\eta)^2}{\sqrt{(\varepsilon_p^{(\eta)} - \mu)^2 + 3\Delta^2 f(\eta)^2}} - \frac{1}{G_c} \right] = 0$$

Numerical results
– for interplay between spin polarization and 2SC

- Occupation number for 2SC phase





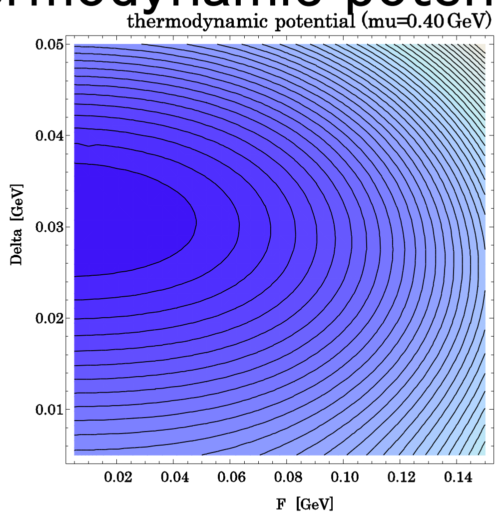
Usual cutoff Λ =0.631 GeV is valid for this calculation

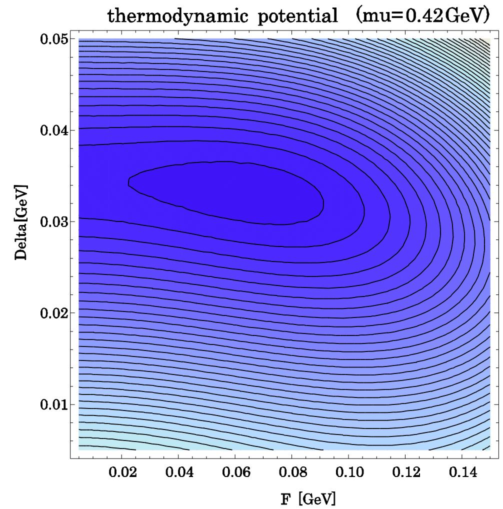
Λ / GeV	G / GeV^{-2}	G_c / GeV^{-2}
0.631	20.0	6.6

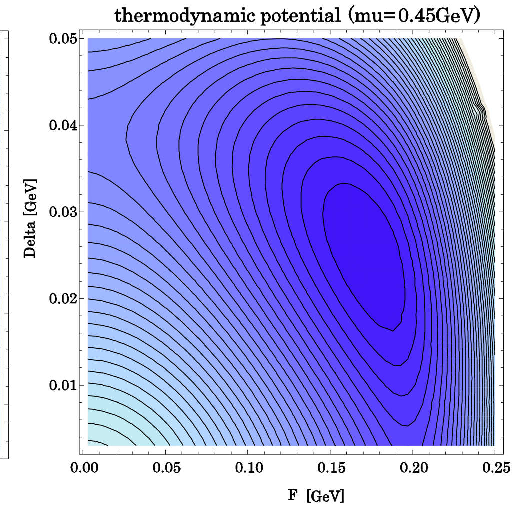
Numerical results

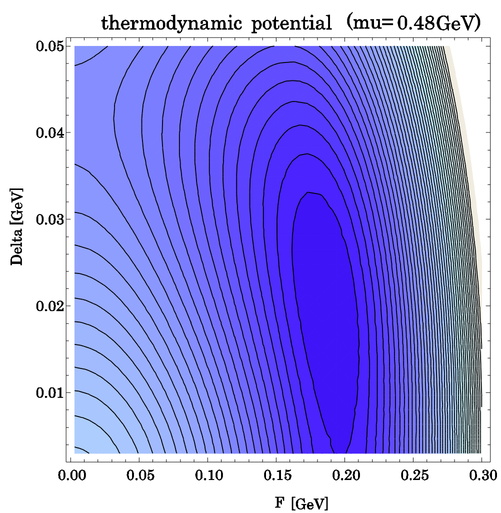
− for interplay between spin polarization and 2SC

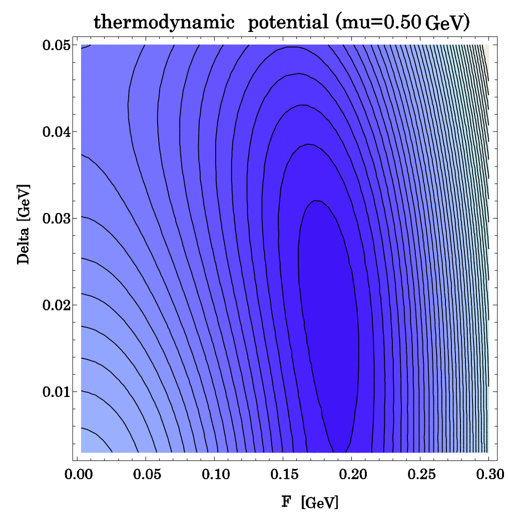
- Thermodynamic potential











Summary 1

- We have shown ・・・
  - ♪ tensor–type four–point interaction between quarks leads・・・
    - — spontaneous spin polarization occurs at high density even in chiral symmetric phase (quark mass is zero),
  - ♪ spin polarized (SP) phase survives against two–flavor color superconducting (2SC) phase at high density
  - ♪ the order of phase transition from 2SC to SP may be the second order

Stability of spin polarized phase
--- three-flavor case

- High density (and low temperature) quark matter in three-flavor:

  → color-flavor locked (CFL) phase may exist

Is the spin polarized phase survives
at high density against CFL phase ?

Interplay between spin polarization and color superconductivity

- Lagrangian density with 3-flavor color superconductivity

$$L = \bar{\psi} i\gamma^{\mu}\partial_{\mu}\psi - \frac{G}{4}\left(\bar{\psi}\gamma^{\mu}\gamma^{\nu}\lambda_k^f\psi\right)\left(\bar{\psi}\gamma_{\mu}\gamma_{\nu}\lambda_k^f\psi\right) + \frac{G_c}{2}\left(\bar{\psi} i\gamma_5\lambda_a^c\lambda_k^f\psi^C\right)\left(\bar{\psi}^C i\gamma_5\lambda_a^c\lambda_k^f\psi\right)$$

- Mean field approximation

$$L = \bar{\psi} i\gamma^{\mu}\partial_{\mu}\psi + L_T^{MF} + L_c^{MF}$$

$$L_T^{MF} = -\sum_{k=3,8} F_k\left(\bar{\psi}\Sigma_3\lambda_k^f\psi\right) - \frac{1}{2G}\sum_{k=3,8} F_k^2$$

$$\Sigma_3 = -i\gamma^1\gamma^2 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}, \quad F_3 = -G\left\langle \bar{\psi}\Sigma_3\lambda_3^f\psi \right\rangle, \quad F_8 = -G\left\langle \bar{\psi}\Sigma_3\lambda_8^f\psi \right\rangle$$

$$L_c^{MF} = -\frac{1}{2}\sum_{(a,k)=\{2,5,7\}}\left(\left(\Delta_{ak}^*\left(\bar{\psi}^C i\gamma_5\lambda_a^c\lambda_k^f\psi\right) + h.c.\right) + \frac{1}{2G_c}\left|\Delta_{ak}\right|^2\right)$$

$$\Delta_{ak} = -G_c\left\langle \bar{\psi}^C i\gamma_5\lambda_a^c\lambda_k^f\psi \right\rangle$$

Interplay between spin polarization and color superconductivity

- Hamiltonian formalism

$$\mathcal{H}_{MF} - \mu\mathcal{N} = \mathcal{K}_0 + \mathcal{H}_T^{MF} + \mathcal{H}_c^{MF} ,$$
$$\mathcal{K}_0 = \bar{\psi}(-\gamma\cdot\nabla - \mu\gamma_0)\psi ,$$
$$\mathcal{H}_T^{MF} = -\mathcal{L}_T^{MF} , \qquad \mathcal{H}_c^{MF} = -\mathcal{L}_c^{MF}$$

- Hamiltonian for quark and antiquark

$$H = H_0 - \mu N + V_{CFL} + V_{SP} + V\cdot\frac{1}{2G}\left(F_3^2 + F_8^2\right) + V\cdot\frac{3\Delta^2}{2G_c}$$

$$H_0 - \mu N = \sum_{p\eta\tau\alpha}\left[\left(|p| - \mu\right)c^+_{p\eta\tau\alpha}c_{p\eta\tau\alpha} - \left(|p| + \mu\right)\tilde{c}^+_{p\eta\tau\alpha}\tilde{c}_{p\eta\tau\alpha}\right]$$

$$V_{CFL} = \frac{\Delta}{2}\sum_{p\eta}\sum_{\alpha\alpha'\alpha''}\sum_{\tau\tau'}\left(c^+_{p\eta\tau\alpha}c^+_{-p\eta\tau'\alpha'} + c_{-p\eta\tau'\alpha'}c_{p\eta\tau\alpha} + \tilde{c}^+_{p\eta\tau\alpha}\tilde{c}^+_{-p\eta\tau'\alpha'} + \tilde{c}_{-p\eta\tau'\alpha'}\tilde{c}_{p\eta\tau\alpha}\right)\varepsilon_{\alpha\alpha'\alpha''}\varepsilon_{\tau\tau'}\phi_p$$

$$V_{SP} = \sum_{p\eta\tau\alpha}F_\tau\left[\frac{\sqrt{p_1^2 + p_2^2}}{|p|}\left(c^+_{p\eta\tau\alpha}c_{p-\eta\tau\alpha} + \tilde{c}^+_{p\eta\tau\alpha}\tilde{c}_{p-\eta\tau\alpha}\right) - \eta\frac{p_3}{|p|}\left(c^+_{p\eta\tau\alpha}\tilde{c}_{p\eta\tau\alpha} + \tilde{c}^+_{p\eta\tau\alpha}c_{p\eta\tau\alpha}\right)\right]$$

where

$\eta = \pm 1 \cdots$ helicity , $\tau = u, d, s \cdots$ flavor , $\alpha \cdots$ color $\left(\phi_p = -\phi_{\bar{p}} = 1\right)$

Mean Field Approximation – for color-superconducting gap Δ without spin polarization F (=0)

Mean field approximation for quasi-particle operators

$$H_{CFL}=H_0-\mu N+V_{CFL}+V\cdot\frac{3\Delta^2}{2G_c}$$

$$=\frac{1}{2}\sum_{|p|>\mu}\left[9\bar{\varepsilon}_p-\sqrt{\bar{\varepsilon}_p^2+4\Delta^2}-8\sqrt{\bar{\varepsilon}_p^2+\Delta^2}\right]+\sum_{|p|>\mu}\left[\sqrt{\bar{\varepsilon}_p^2+4\Delta^2}\,d_{p;1}^{+}d_{p;1}+\sum_{a=2}^{9}\sqrt{\bar{\varepsilon}_p^2+\Delta^2}\,d_{p;a}^{+}d_{p;a}\right]$$

$$+\frac{1}{2}\sum_{|p|<\mu}\left[9\bar{\varepsilon}_p-\sqrt{\bar{\varepsilon}_p^2+4\Delta^2}-8\sqrt{\bar{\varepsilon}_p^2+\Delta^2}\right]+\sum_{|p|<\mu}\left[\sqrt{\bar{\varepsilon}_p^2+4\Delta^2}\,d_{p;1}^{+}d_{p;1}+\sum_{a=2}^{9}\sqrt{\bar{\varepsilon}_p^2+\Delta^2}\,d_{p;a}^{+}d_{p;a}\right]+V\cdot\frac{3\Delta^2}{2G_c}$$

$$\bar{\varepsilon}_p=p-\mu,\quad \Delta_{\alpha''\tau''}=G_c\sum_{p\eta\alpha\alpha'\tau\tau'}\left\langle c_{-p\eta\alpha'\tau'}c_{p\eta\alpha\tau}\right\rangle\varepsilon_{\alpha\alpha'\alpha''}\varepsilon_{\tau\tau'\tau_{\alpha''}}\phi_p;\quad \Delta=\Delta_{1u}=\Delta_{2d}=\Delta_{3s}$$

Thermodynamic potential for F=0

$$\Phi_0=\frac{1}{V}\left\langle H_{CFL}\right\rangle$$

$$=\frac{1}{2V}\sum_{|p|>\mu}\left[9\bar{\varepsilon}_p-\sqrt{\bar{\varepsilon}_p^2+4\Delta^2}-8\sqrt{\bar{\varepsilon}_p^2+\Delta^2}\right]+\frac{1}{2V}\sum_{|p|<\mu}\left[9\bar{\varepsilon}_p-\sqrt{\bar{\varepsilon}_p^2+4\Delta^2}-8\sqrt{\bar{\varepsilon}_p^2+\Delta^2}\right]+\frac{3\Delta^2}{2G_c}$$

Mean Field Approximation – for spin polarized gap F without CFL condensate Δ (=0)

Thermodynamic potential for Δ=0

$$\Phi_F = 3 \cdot \frac{1}{V} \sum_{p,\eta=\pm,\tau=u,d,s} \left(\varepsilon_{p\tau}^{(\eta)} - \mu\right)\theta\left(\mu - \varepsilon_{p\tau}^{(\eta)}\right) + \frac{1}{2G}\left(F_3^2 + F_8^2\right)$$

$$\varepsilon_{p\tau}^{(\eta)} = \sqrt{p_3^2 + \left(F_\tau + \eta\sqrt{p_1^2 + p_2^2}\right)^2} \quad , \quad F_\tau = \left(F_3 + \frac{1}{\sqrt{3}}F_8\right)\delta_{\tau u} + \left(-F_3 + \frac{1}{\sqrt{3}}F_8\right)\delta_{\tau d} - \frac{2}{\sqrt{3}}F_8\delta_{\tau s}$$

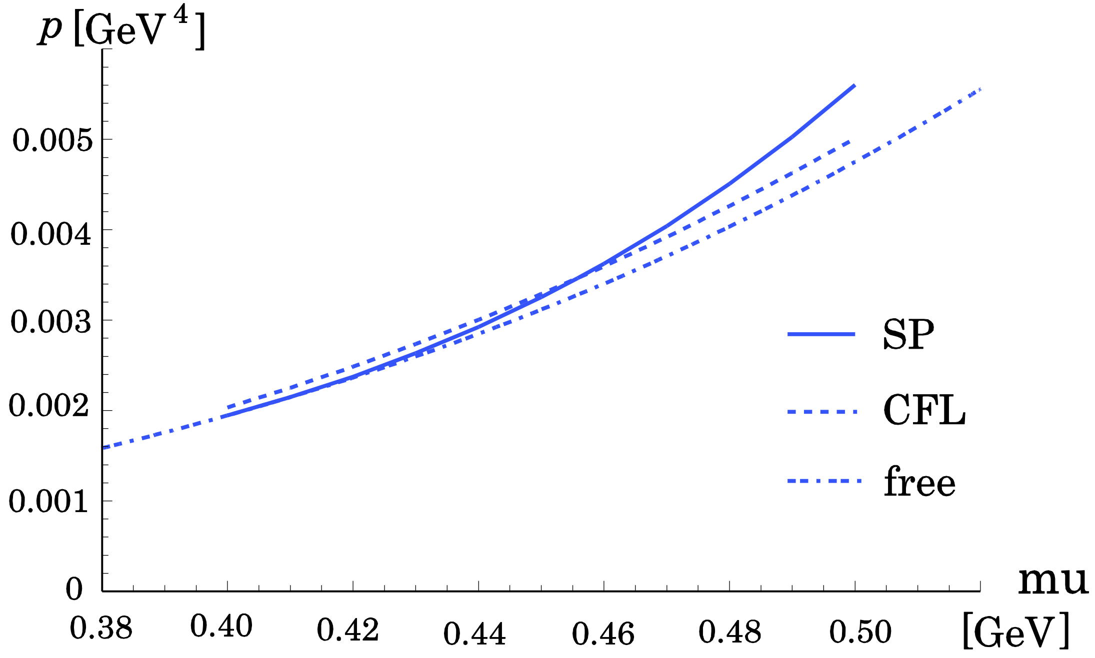
Gap equations for Φ_0 (CFL), Φ_F (SP)

$$\frac{\partial \Phi_0}{\partial \Delta} = 0 \,, \text{ or } \frac{\partial \Phi_F}{\partial F_3} = \frac{\partial \Phi_F}{\partial F_8} = 0$$

Numerical results

− for interplay between spin polarization and CFL

- Pressure p vs chemical potential μ



- Parameters used here

Λ / GeV	G / GeV^{-2}	G_c / GeV^{-2}
0.631	20.0	6.6

Order of phase transition

– second order perturbation on CFL phase with respect to SP term

- Hamiltonian under consideration

$$H = H_{CFL} + H_{SP} \,, \qquad H_{SP} = \sum_{p\eta\alpha\tau} F_\tau \frac{\sqrt{p_1^2 + p_2^2}}{|p|} c^+_{p\eta\tau\alpha} c_{p-\eta\tau\alpha}$$

Here, H_{SP} is regarded as perturbation term

- First order perturbation = 0
- Second order perturbation

$$E_{corr} = \sum_i \frac{\langle \Phi | H_1 | i \rangle \langle i | H_1 | \Phi \rangle}{E_0 - E_i}$$

E_0; ground state energy , $|i\rangle$; intermediate (excited) state , E_i ; excited state energy

Order of phase transition

– second order perturbation on CFL phase with respect to SP term

- Thermodynamic potential

$$\Phi = \Phi_0 + \frac{1}{V} E_{corr} + \frac{1}{2G}\left(F_3^2 + F_8^2\right)$$

$$= \Phi_0 + \left(c_3 + \frac{1}{2G}\right) F_3^2 + \left(c_8 + \frac{1}{2G}\right) F_8^2$$

mu / GeV	c_3 + 1/(2G)	c_8 + 1/(2G)
0.40	0.015734	0.0076297
0.42	0.014873	0.0060047
0.44	0.014015	0.0043828
0.4558	0.0133487	0.0031934
0.46	0.013174	0.0027882
0.48	0.012367	0.0012519

coefficients of F_3 and F_8 are always positive

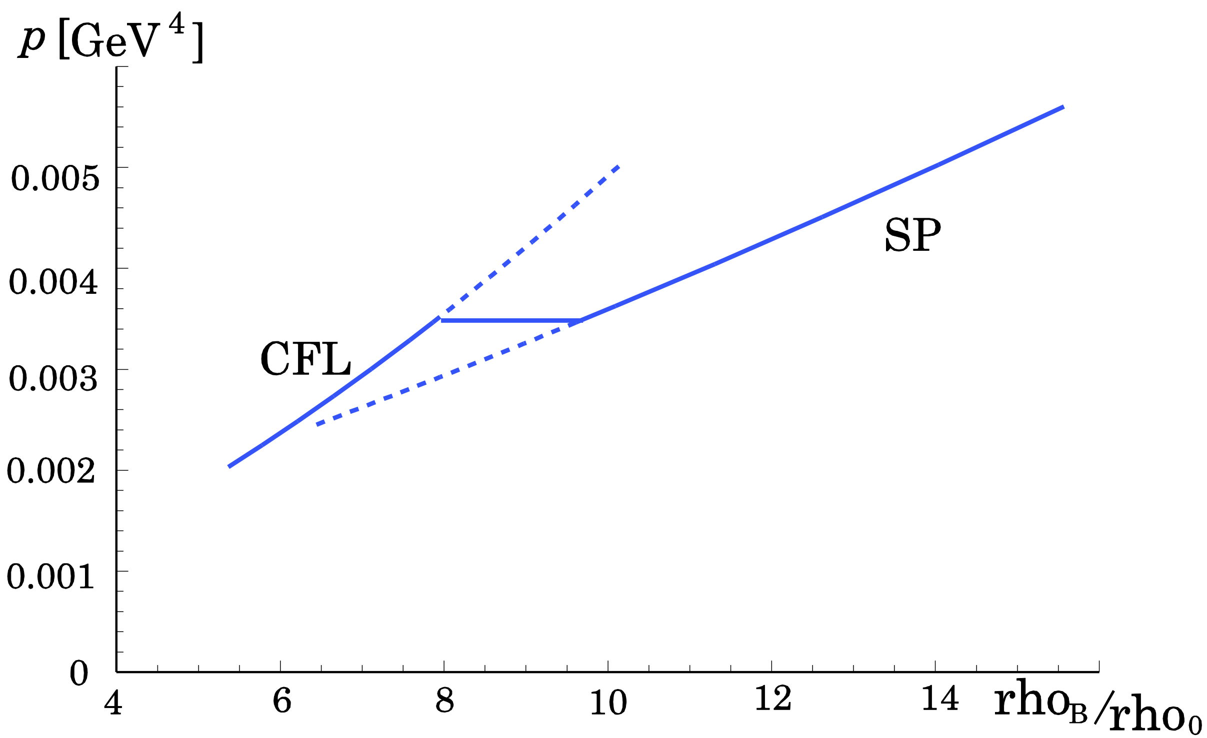
$\rightarrow \Delta \neq 0,\ \ F_3 = F_8 = 0$ is local minimum

Order of phase transition

— second order perturbation on CFL phase with respect to SP term

- Thus, $\Delta \neq 0$ and $F_3 = F_8 = 0$ ・・・・ stable

  Then, the phase transition may be the first order



baryon number density divided by normal nuclear density

Summary 2

- We have shown ・・・
  - ♪ tensor-type four-point interaction between quarks leads・・・
    - — spontaneous spin polarization occurs at high density even in chiral symmetric phase (quark mass is zero),
  - ♪ spin polarized (SP) phase survives against color-flavor locking (CFL) phase at high density
  - ♪ the order of phase transition from CFL to SP may be the first order