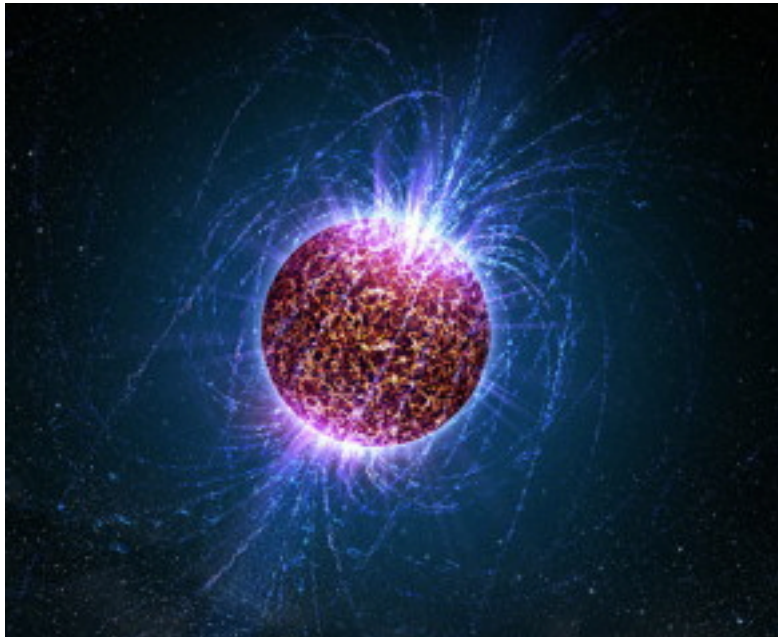


Constraints on the nuclear  
saturation parameters  
via neutron star observations

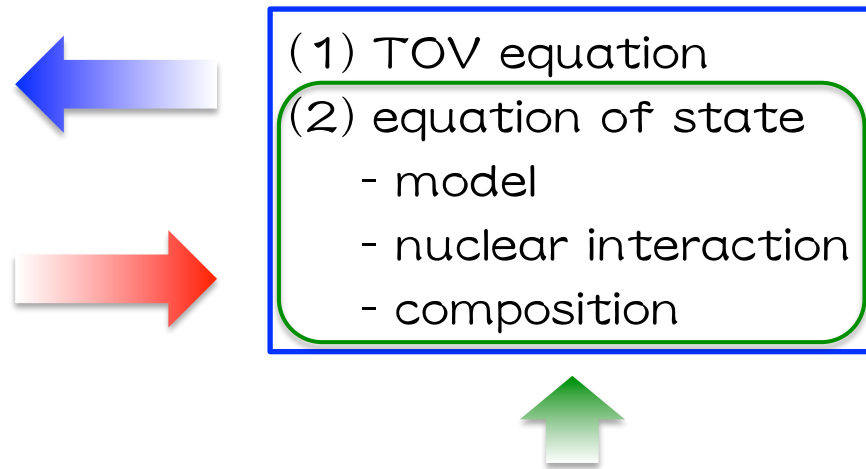
Hajime SOTANI (NAOJ)

K. Iida, K. Oyamatsu, K. Nakazato, & A. Ohnishi

# NS - EOS



- physics in NS crust
- low-mass NSs

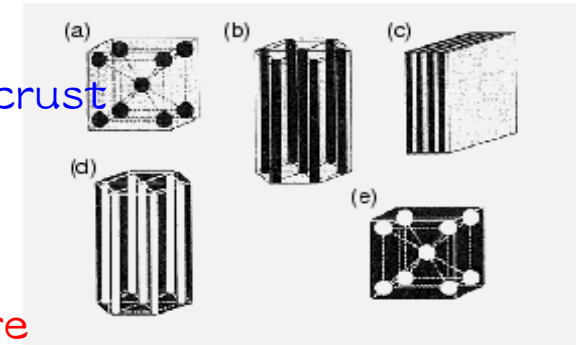
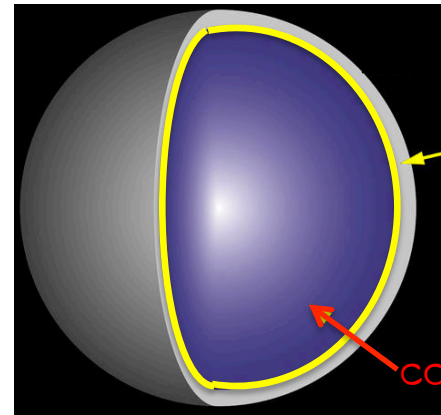


constraints from the terrestrial  
nuclear experiments  
∴  
properties around  
the saturation density

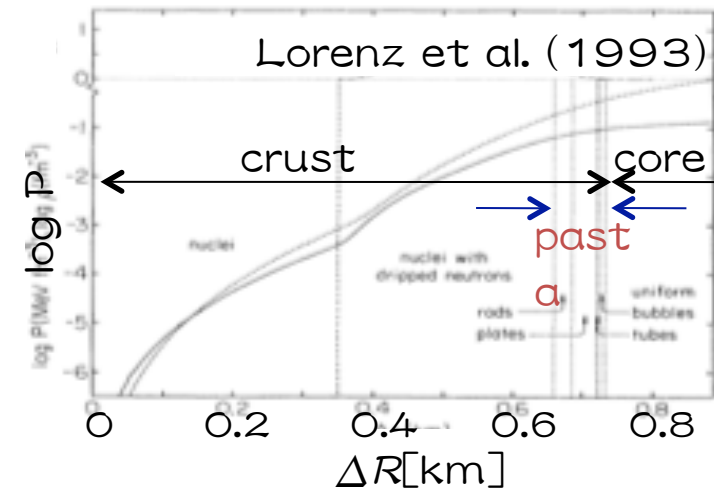
# Crust in NSs

# neutron stars

- Structure of NS
  - solid layer (crust)
  - nonuniform structure (pasta)
  - fluid core (uniform matter)
- Crust thickness  $\approx 1$  km
- Determination of EOS for high density (core) region could be quite difficult on Earth
- Constraint on EOS via observations of neutron stars
  - stellar mass and radius
  - stellar oscillations (& emitted GWs)



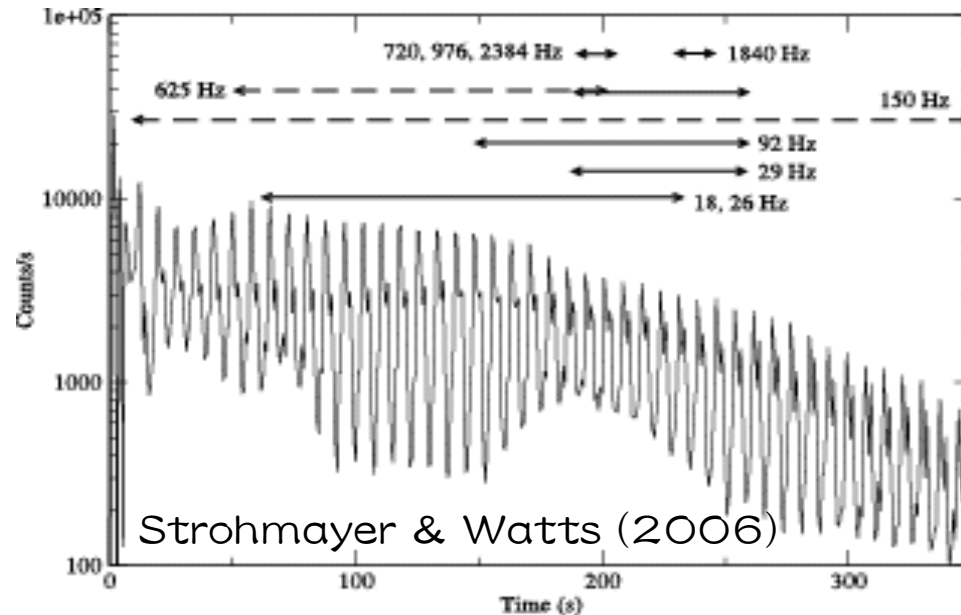
Oyamatsu (1993)



“(GW) asteroseismology”

# QPOs in SGRs

- Quasi-periodic oscillations (QPOs) in afterglow of giant flares from soft-gamma repeaters (SGRs)
  - SGR 0526-66 (5<sup>th</sup>/3/1979) : 43 Hz
  - SGR 1900+14 (27<sup>th</sup>/8/1998) : 28, 54, 84, 155 Hz
  - SGR 1806-20 (27<sup>th</sup>/12/2004) : 18, 26, 30, 92.5, 150, 626.5, 1837 Hz (Barat+ 1983, Israel+ 05, Strohmayer & Watts 05, Watts & Strohmayer 06)



- Crustal torsional oscillation ?
- Magnetic oscillations ?
- Asteroseismology
  - stellar properties ( $M$ ,  $R$ ,  $B$ , EOS ...)

# torsional oscillations

- axial parity oscillations
  - incompressible
  - no density perturbations

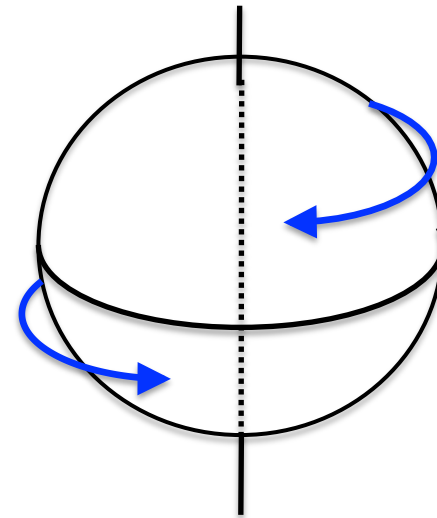
- in Newtonian case

(Hansen & Cioffi 1980)

$$\ell t_0 \sim \frac{\sqrt{\ell(\ell+1)\mu/\rho}}{2\pi R} \sim 16\sqrt{\ell(\ell+1)} \text{ Hz} \quad \ell t_n \sim \frac{\sqrt{\mu/\rho}}{2\Delta r} \sim 500 \times n \text{ Hz}$$

- $\mu$ : shear modulus
- frequencies  $\propto$  shear velocity  $v_s = \sqrt{\mu/\rho}$
- overtones depend on crust thickness
- one can consider torsional oscillations independently of core EOS
- effect of magnetic field
  - frequencies become larger

(Sotani+07, Gabler+12,13)

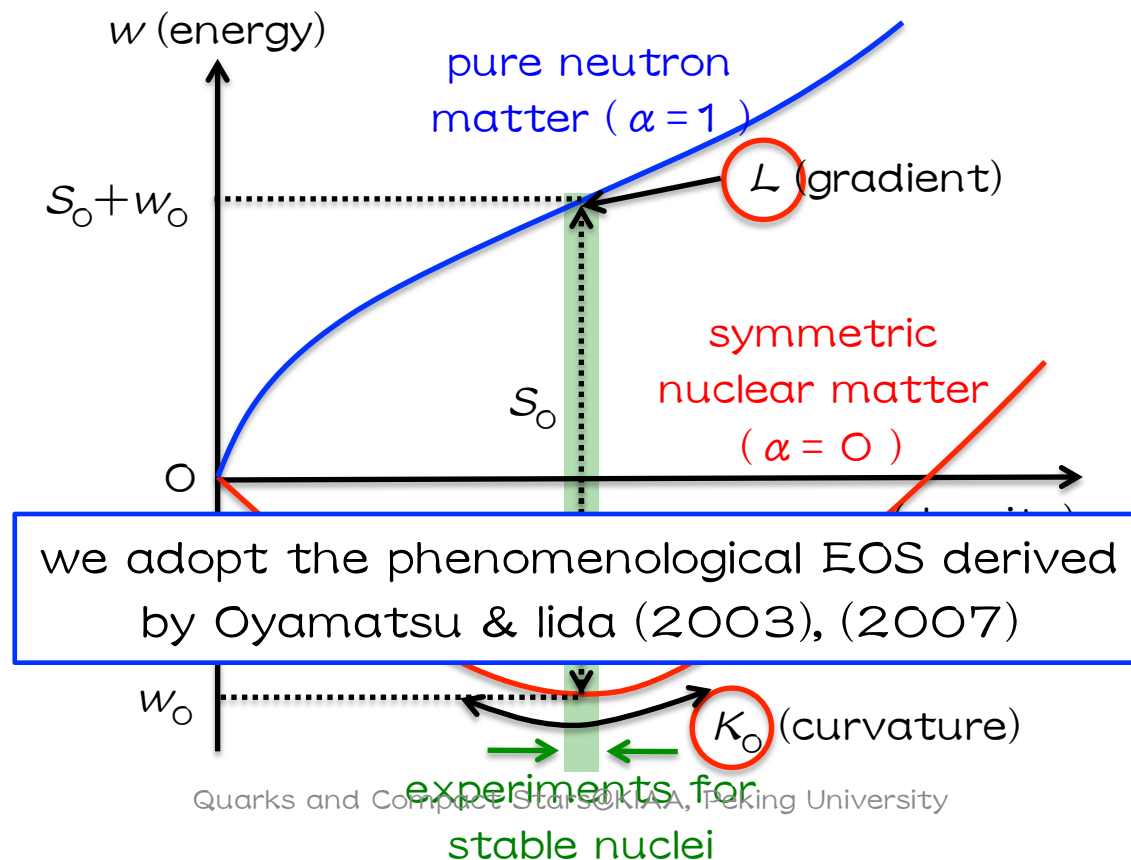


# EOS near the saturation point

- Bulk energy per nucleon near the saturation point of symmetric nuclear matter at zero temperature;

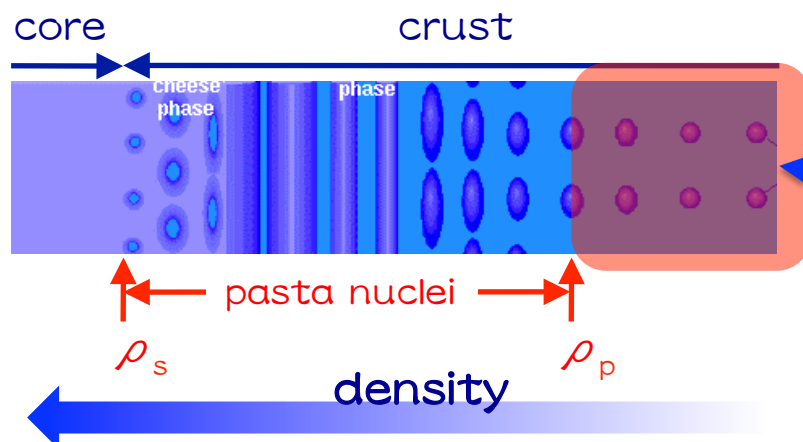
$$w = w_0 + \frac{K_0}{18n_0^2} (n - n_0)^2 + \left[ S_0 + \frac{L}{3n_0} (n - n_0) \right] \alpha^2$$

incompressibility
symmetry parameter



# What we do

- EOS for core region is still uncertain.
- To prepare the crust region, we integrate from  $r=R$ .
  - $M, R$  : parameters for stellar properties
  - $L, K_0$  : parameters for crust EOS (Oyamatsu & Iida (2003), (2007))
    - For  $L \gtrsim 100\text{MeV}$ , pasta structure almost disappears
- In crust region, torsional oscillations are calculated.
  - considering the shear only in spherical nuclei.
  - frequency of fundamental oscillation  $\propto v_s$  ( $v_s^2 \sim \mu/H$ )
  - calculated frequencies could be lower limit



for bcc lattice

$$\mu = 0.1194 \frac{n_i (Ze)^2}{a}$$

$n_i$  : number density of quark droplet

$Z$  : charge of quark droplet

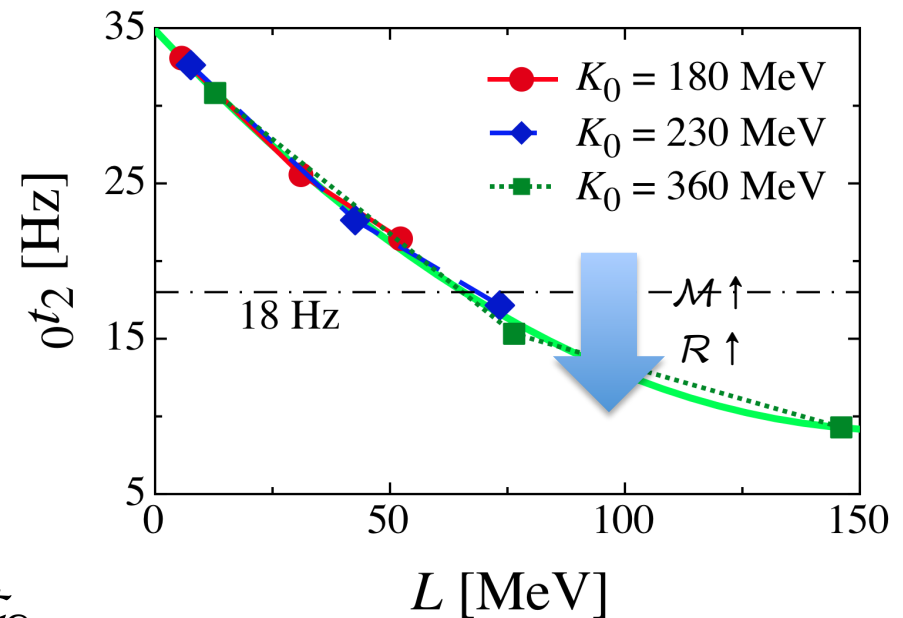
$a$  : Wigner-Seitz radius



# ${}_0t_2$ without superfluidity

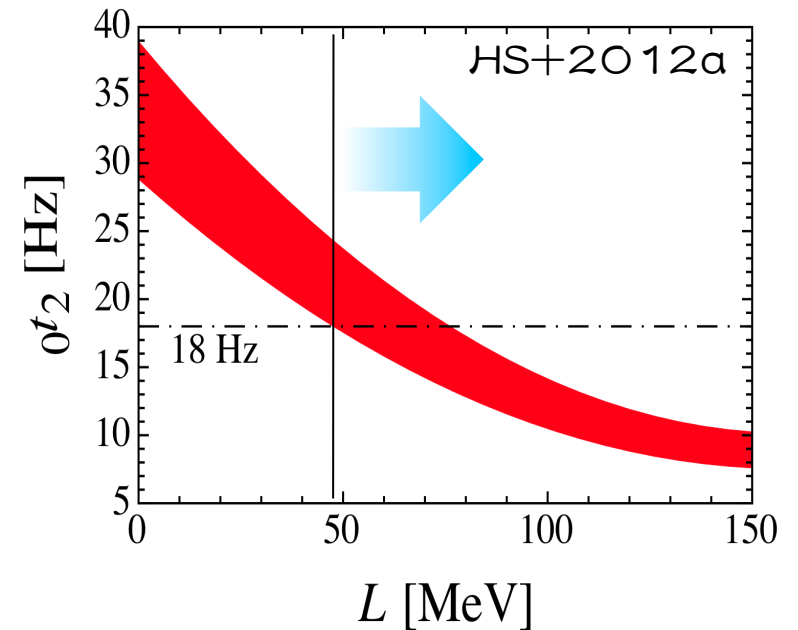
HS+2012a

- For  $M=1.4M_\odot$  &  $R=12\text{km}$ , calculated frequencies  ${}_0t_2$
- ${}_0t_2$  is almost independent of the value of  $K_0$
- For  $R=10\sim 14\text{ km}$  and  $M/M_\odot=1.4\sim 1.8$ , similar dependence on  $K_0$
- One can write fitting line
- Focus on  $L$  dependence of  ${}_0t_2$
- ${}_0t_2$  becomes smaller with larger  $R$  and  $M$ .



# Constraint on $L$

- For  $R=10\text{km}\sim 14\text{km}$  &  $M/M_{\odot}=1.4\sim 1.8$ ,  ${}_{\circ}t_2$  are calculated
- Assuming that the observed QPOs would come from torsional oscillations
- ${}_{\circ}t_2$  is the smallest frequency among a lot of torsional oscillations
  - ${}_{\circ}t_2$  should be equal to or smaller than the smallest observed QPOs frequency



- Consequently,  $L \gtrsim 50 \text{ MeV}$ .
  - For  $L \gtrsim 50 \text{ MeV}$ , pasta region could be very narrow
  - Modification due to the pasta effect should be small
  - This is first constraint in the symmetry parameter with astronomical observations

# Effect of superfluidity

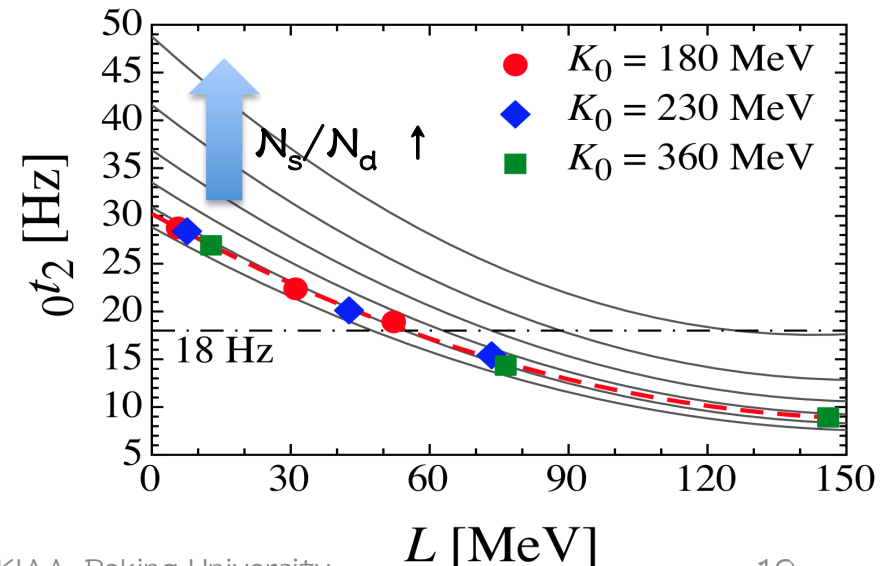
HS+2012b

- For  $\rho \gtrsim 4 \times 10^{11} \text{ g cm}^{-3}$ , neutron could drip from nuclei
- Some of dripped neutron play a role as superfluid
- Effective enthalpy affecting on the shear oscillations could be reduced

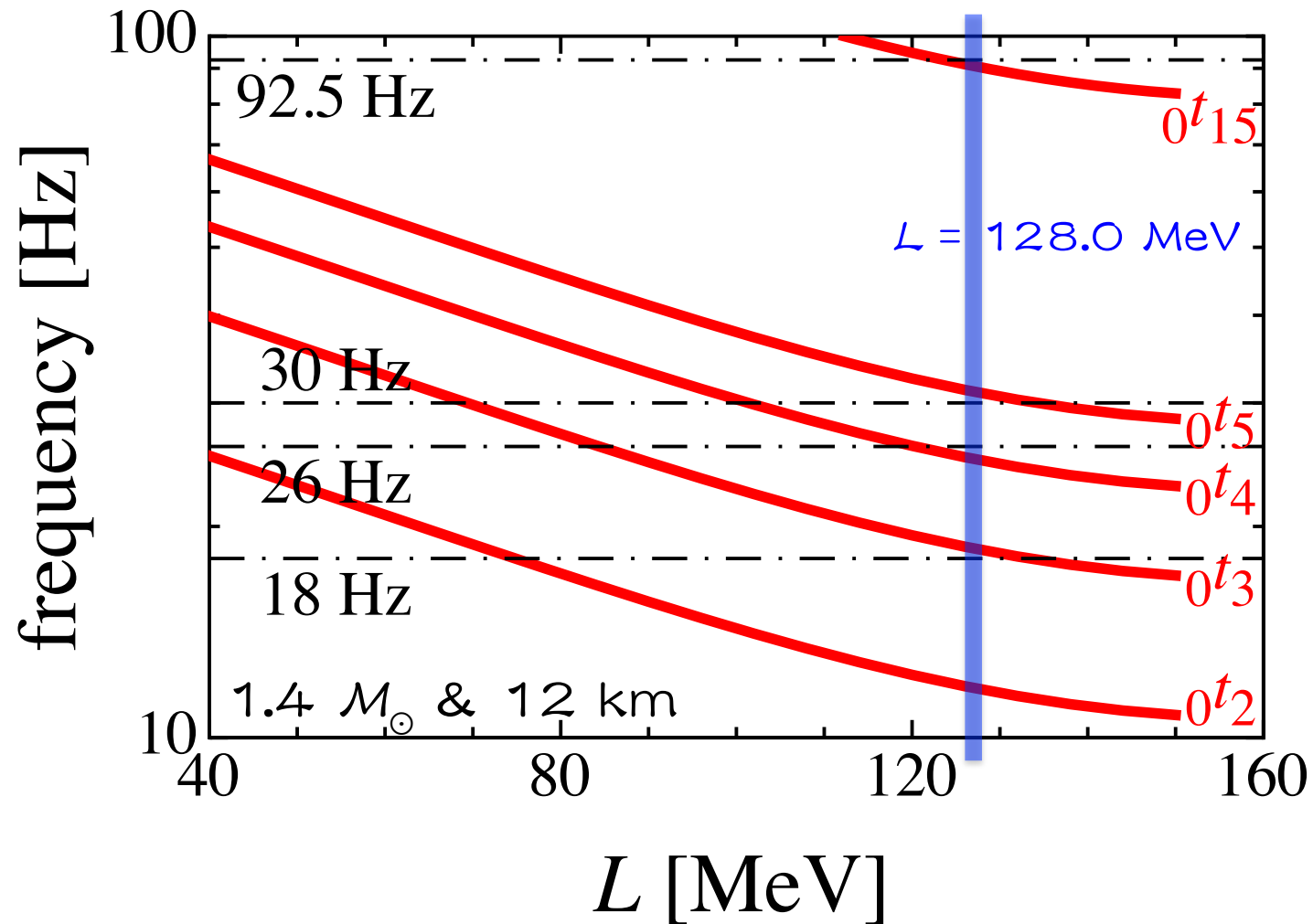
- shear speed ( $v_s^2 \sim \mu/H$ ) increases due to the effect of superfluidity

$$y'' + \left[ \left( \frac{4}{r} + \Phi' - \Lambda' \right) + \frac{\mu'}{\mu} \right] y' + \left[ \frac{\epsilon + p}{\mu} \omega^2 e^{-2\Phi} - \frac{(\ell+2)(\ell-1)}{r^2} \right] e^{2\Lambda} y = 0.$$

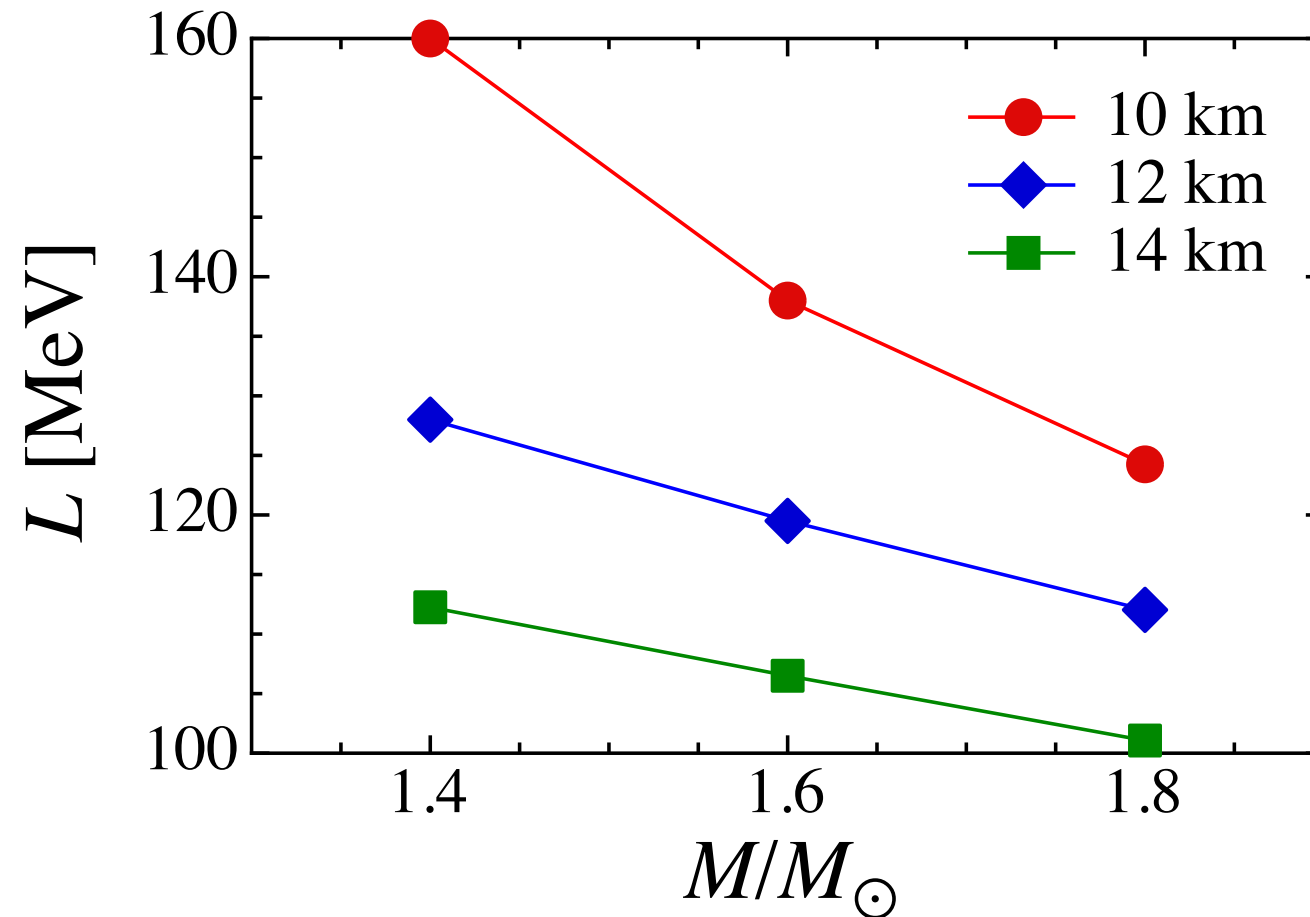
- $t_l$  could also increase due to the effect of superfluidity
- While, the fraction of superfluid neutron in dripped neutron is still unknown...
  - Chamel (2012): superfluid neutron are not so much ( $\sim 10\text{-}30\%$ ?)
- $t_l$  with using a parameter of  $N_s/N_d$  for  $R=14\text{km}$  &  $M=1.8M_\odot$



# identification of SGR 1806-20

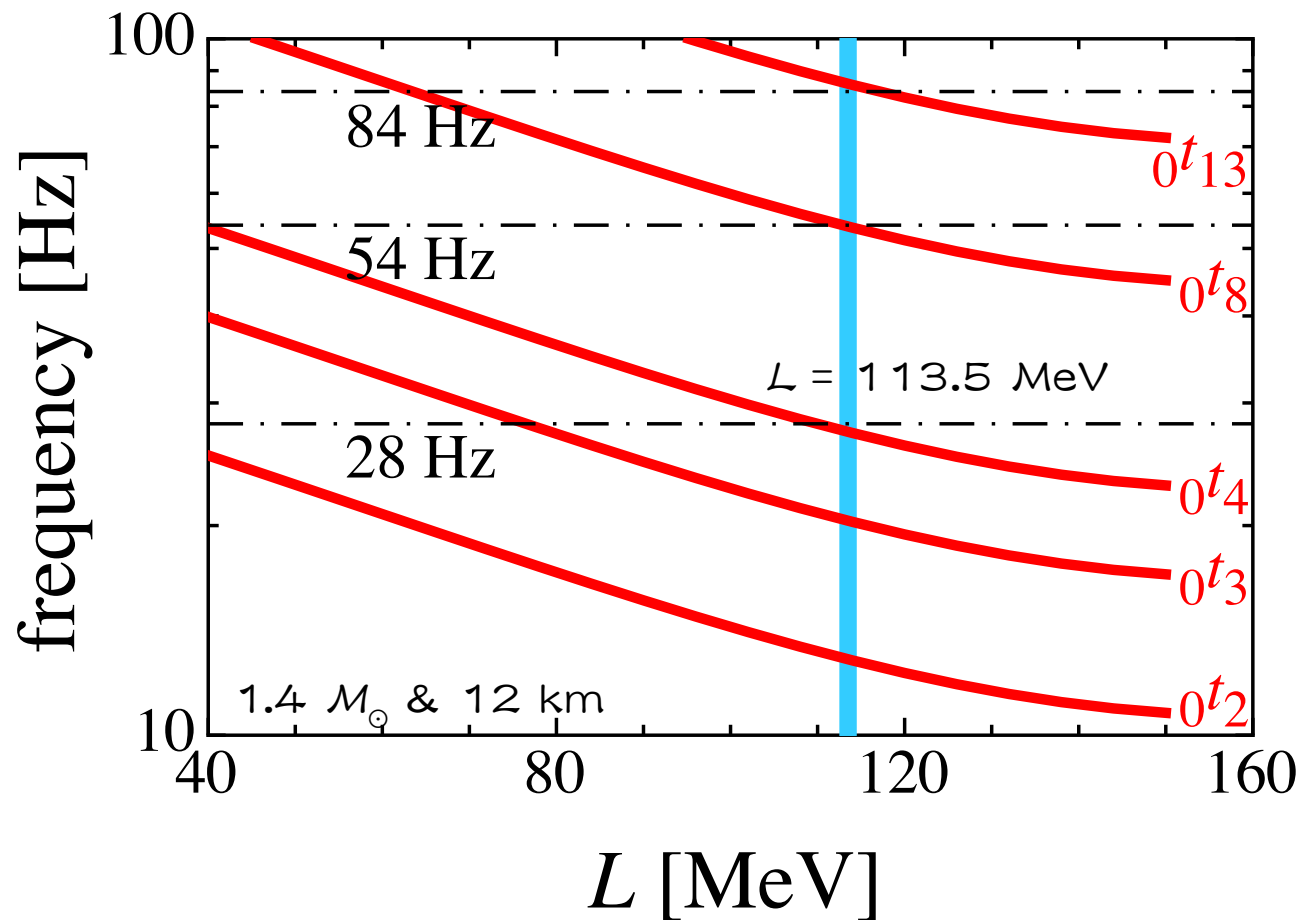


# constraint on $L$ via SGR 1806-20

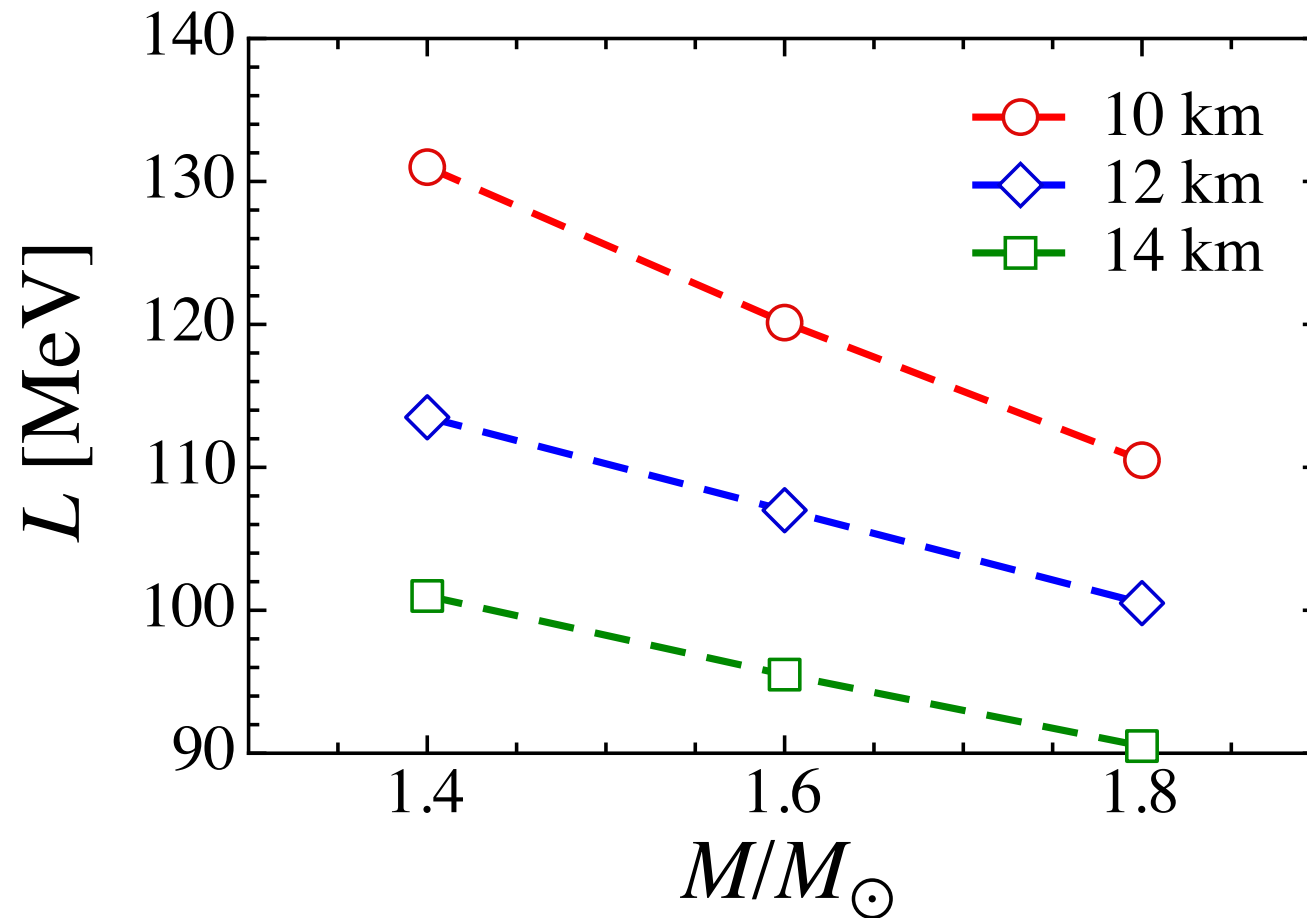


→  $101.1 \text{ MeV} \leq L \leq 160.0 \text{ MeV}$

# identification of SGR 1900+14

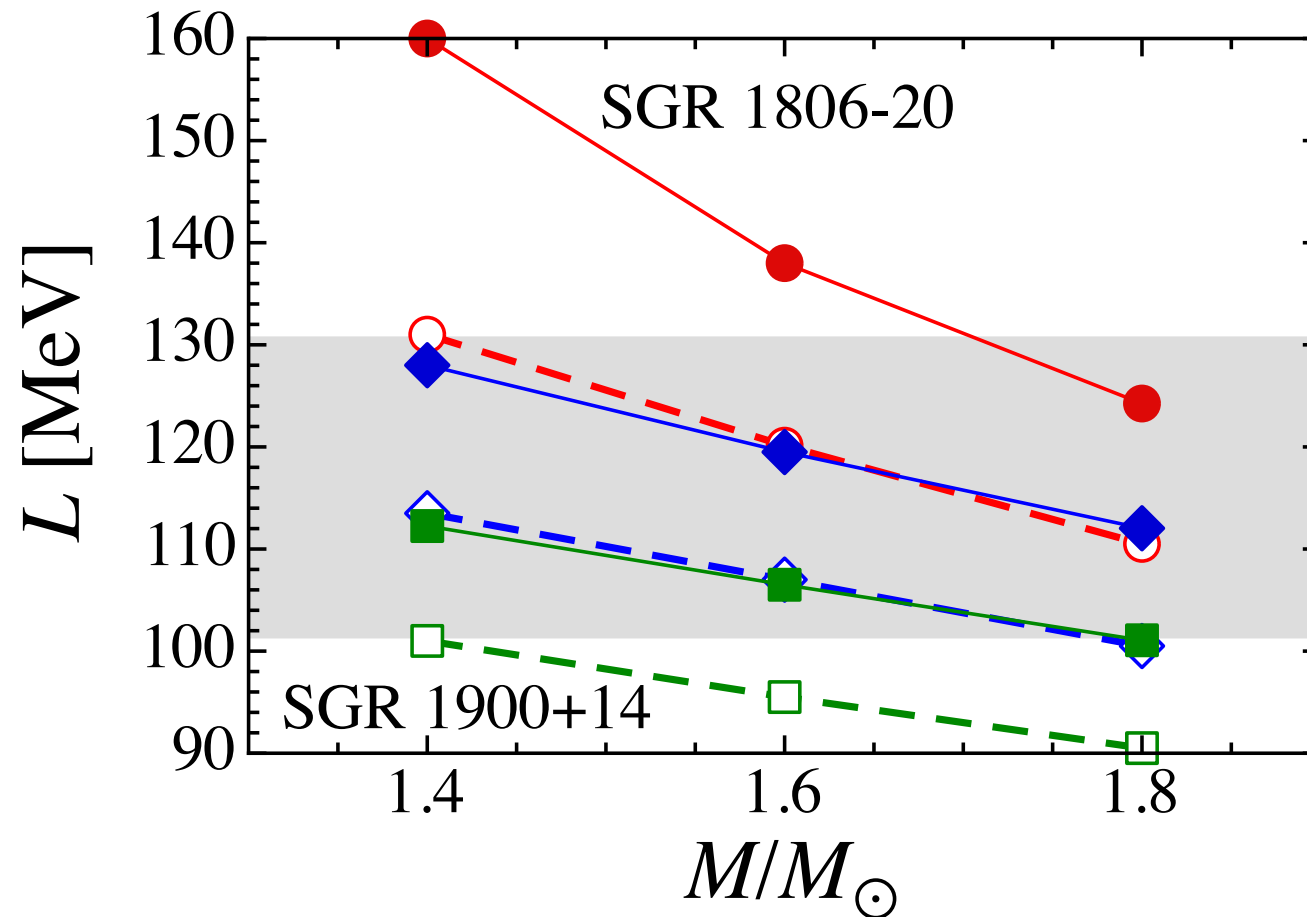


# constraint on $L$ via SGR 1900+14



➔  $90.5 \text{ MeV} \leq L \leq 131.0 \text{ MeV}$

# allowed region for $L$

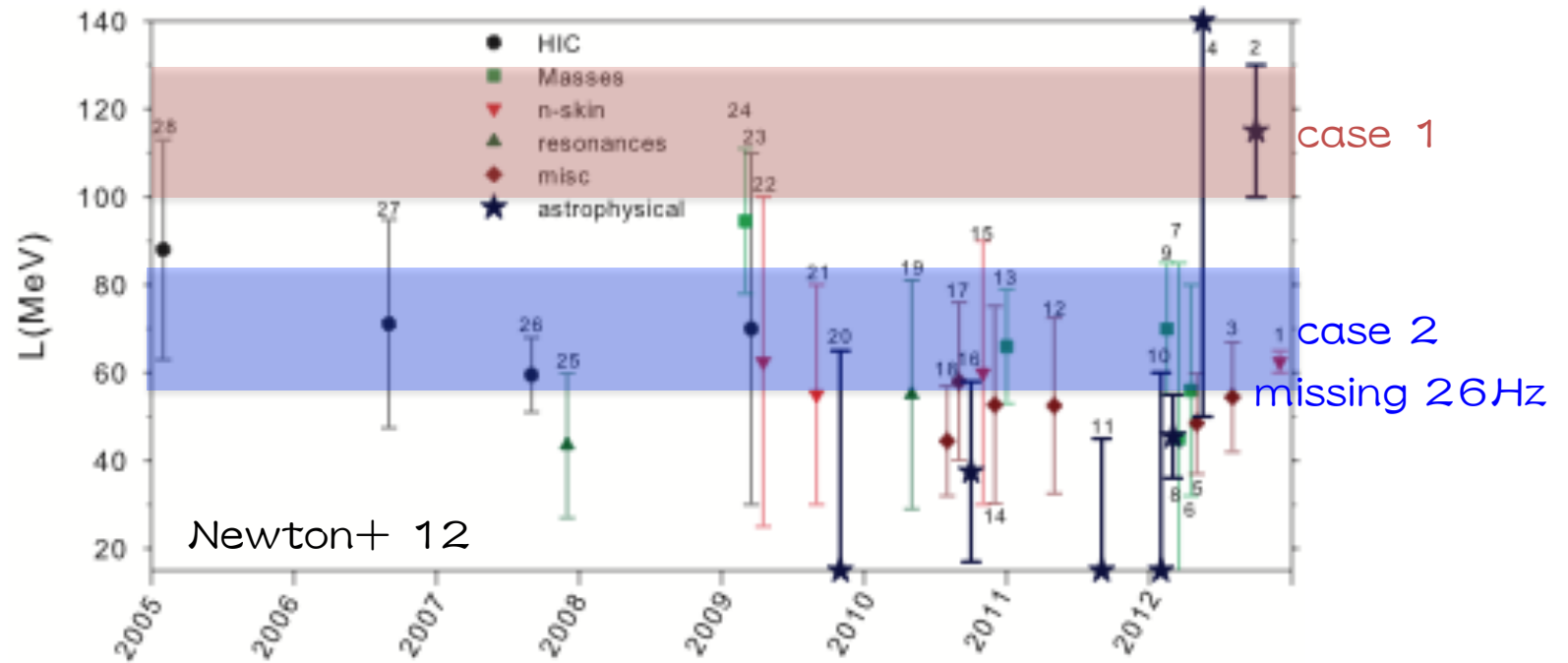


→  $101.1 \text{ MeV} \leq L \leq 131.0 \text{ MeV}$

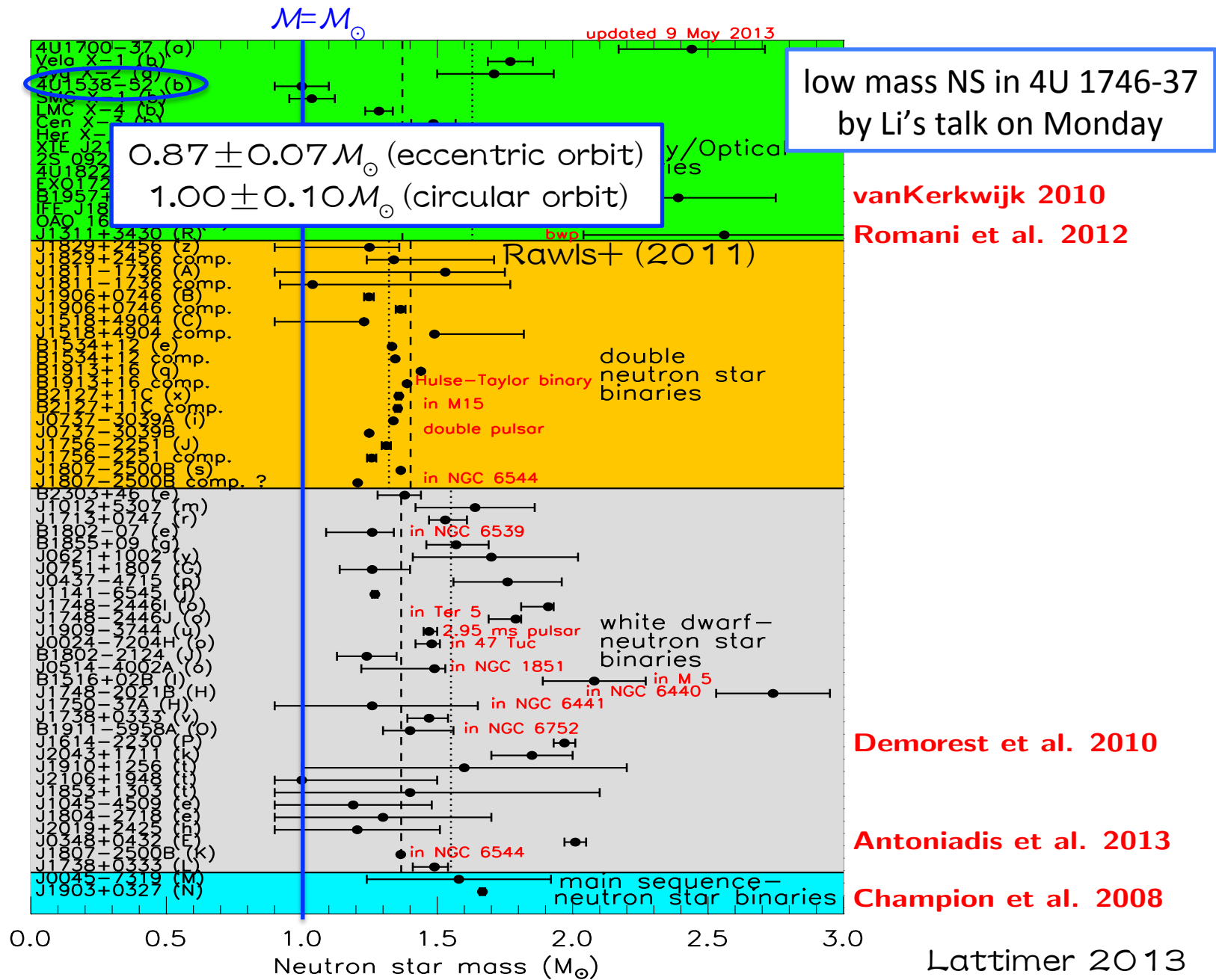


# other constraints on $L$

- other constraints suggests  $L \sim 60 \pm 20$  MeV ?
  - our results may be larger than the previous experimental constraints.

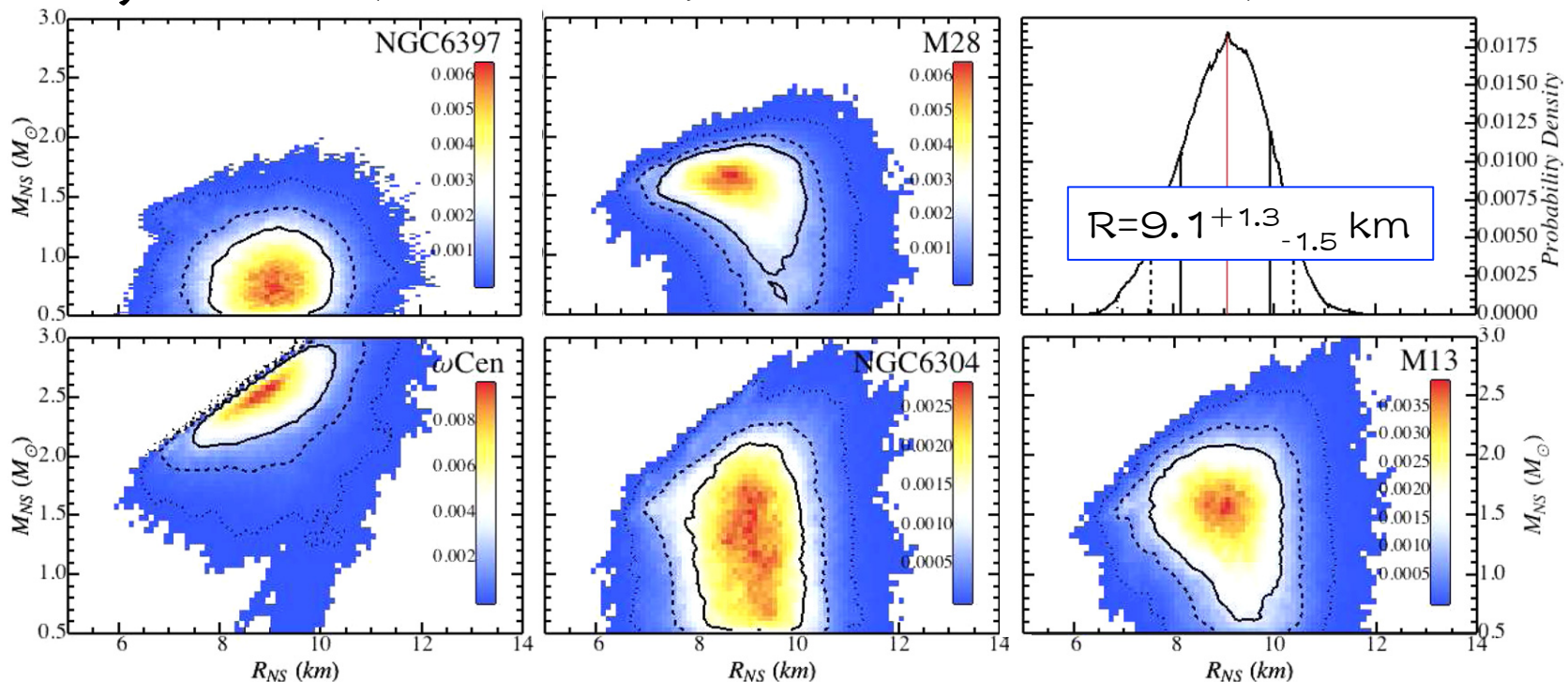


# Low-mass NSs

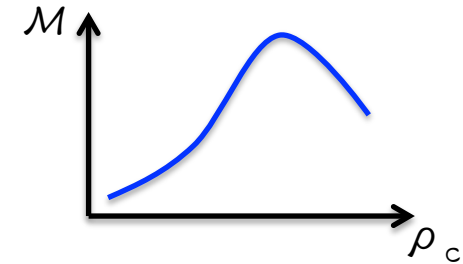


# observations of NSs

- candidates of low-mass NSs have been also discovered in binary system (Lattimer & Prakash 2011)
- radiation radius of X-ray source (Rutledge+ 2002)  
e.g.)  $R_\infty = 14.3 \pm 2.1 \text{ km}$  : CXOU 1326 19.7-4729 10.8 in omega Cen
- $M$  &  $R$  from thermal spectra from quiescent low-mass X-ray binaries (Guillot+ 2013; Lattimer & Steiner 2013)



# low-mass NS models



- low-mass NSs
  - low-central density
  - EOS for low-density region plays an important role
  - may be able to discuss the stellar models without the EOS for high density region → this is an advantage to consider low-mass NSs!
- EOS of nuclear matter for  $\rho \lesssim \rho_0$  (normal nuclear density) would be determined with reasonable accuracy by terrestrial nuclear experiments.
  - saturation parameters may be constrained via such terrestrial experiments.
- For  $\rho \lesssim 2 \rho_0$ , one may almost neglect an uncertainty of three nucleon interaction (Gandolfi+ 2012) and contribution from hyperon (or quark etc...).

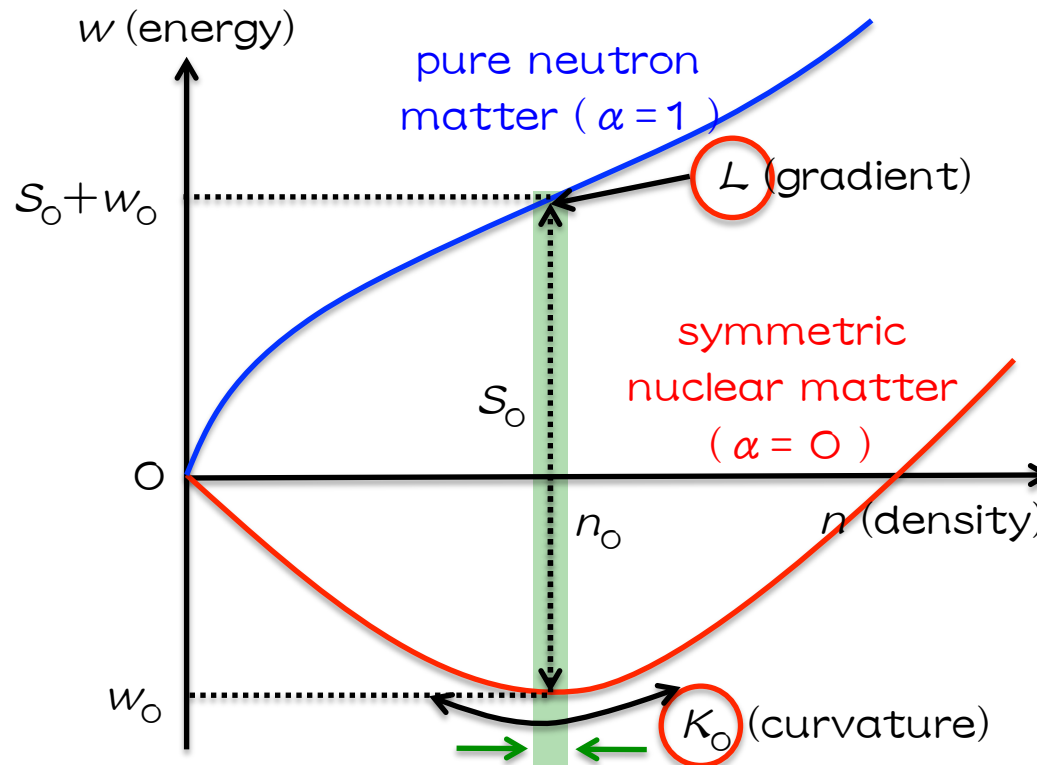
➡ we focus on the NS models for  $\rho \lesssim 2 \rho_0$

# EOS near the saturation point

- Bulk energy per nucleon near the saturation point of symmetric nuclear matter at zero temperature;

$$w = w_0 + \frac{K_0}{18n_0^2} (n - n_0)^2 + \left[ S_0 + \frac{L}{3n_0} (n - n_0) \right] \alpha^2$$

incompressibility
symmetry parameter



# unified EOS modes

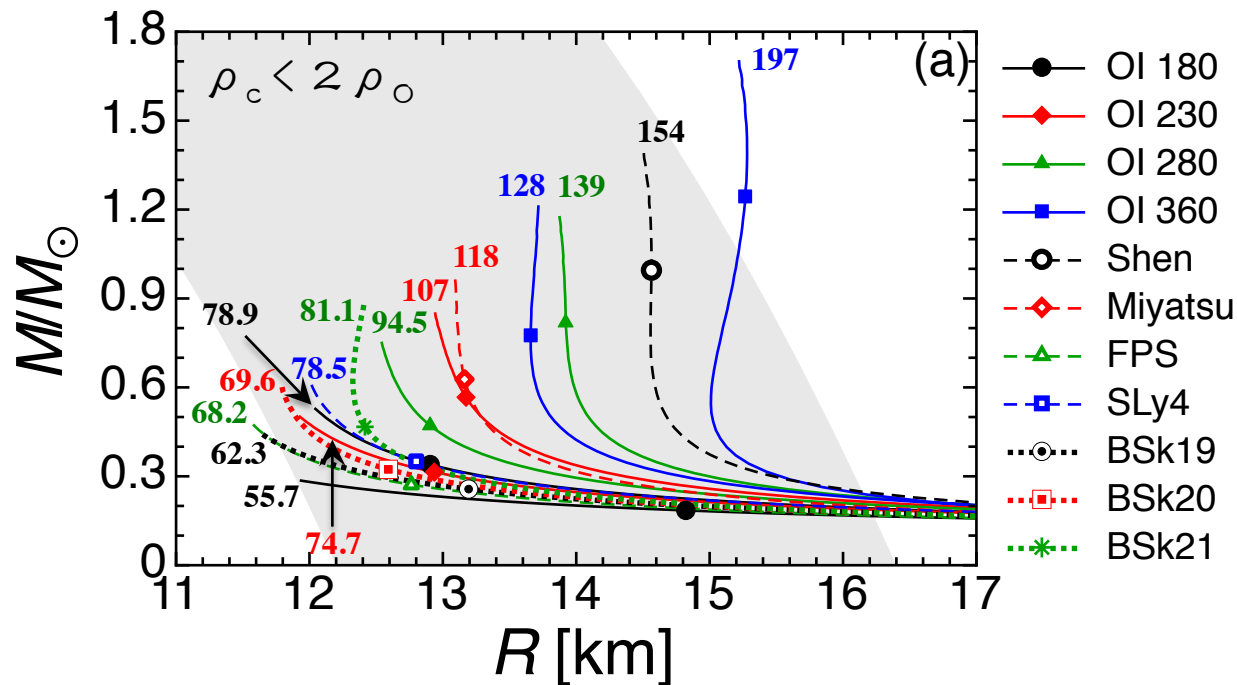
- unified-EOS models
  - based on the EOSs of nuclear matter with specific values of  $K_0$  &  $L$
  - consistent with empirical data of masses and radii of stable nuclei
  - describing both the crustal and core regions of NS
  
- we especially focus on
  - phenomenological EOS with various  $K_0$  &  $L$  (Oyamatsu & Iida 2003; 2007)
  - EOSs based on relativistic mean field models
    - Shen EOS (Shen+ 1998)
    - Miyatsu EOS (Miyatsu+ 2013)
  - Skyrme-type effective interaction
    - FPS (Pethick+ 1995),
    - SLy4 (Douchin & Haensel 2001)
    - BSk19, BSk20, BSk21 (Potekhin+ 2013)

EOSs based on the different theoretical models

# MR relations

- NS models are constructed with various sets of  $K_0$  &  $L$
- We can find the specific combination of  $K_0$  &  $L$  describing the low-mass NSs,

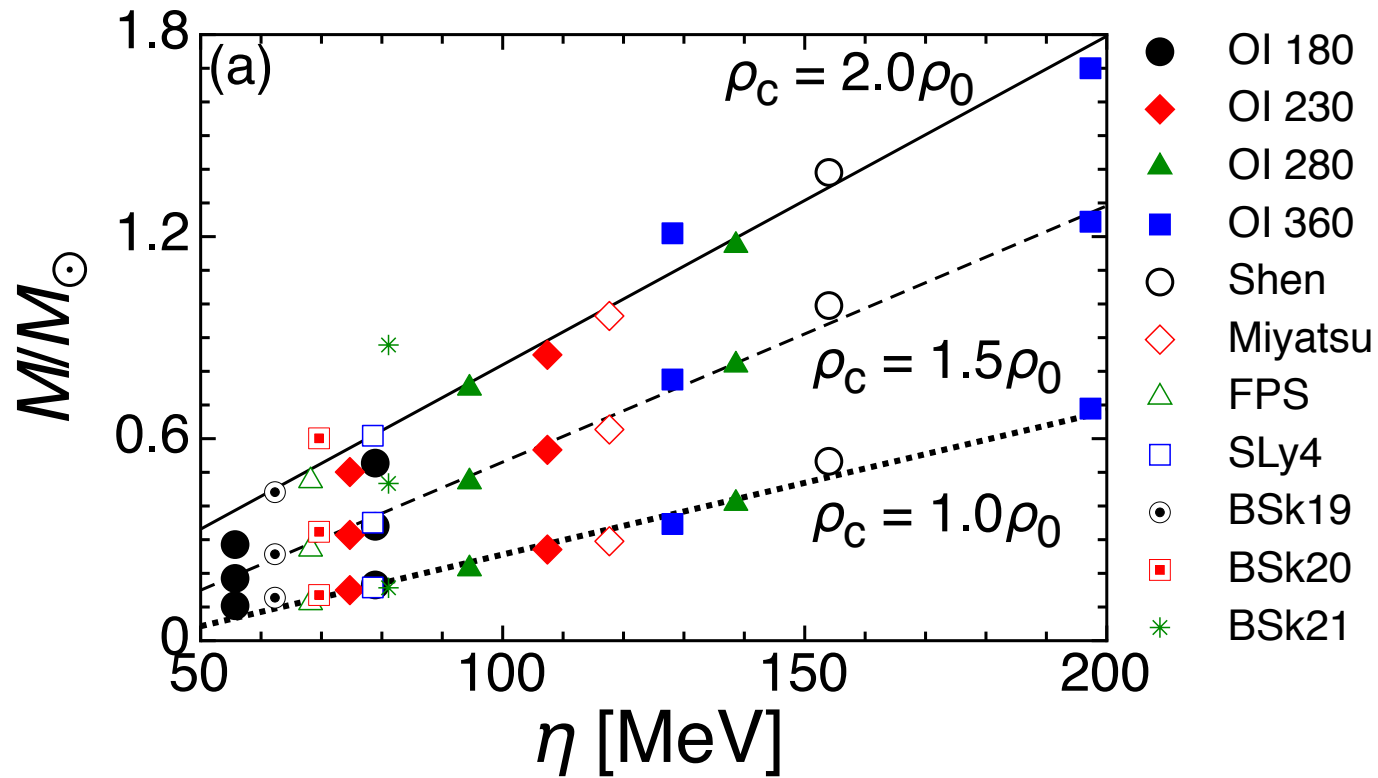
$$\eta = (K_0 L^2)^{1/3}$$



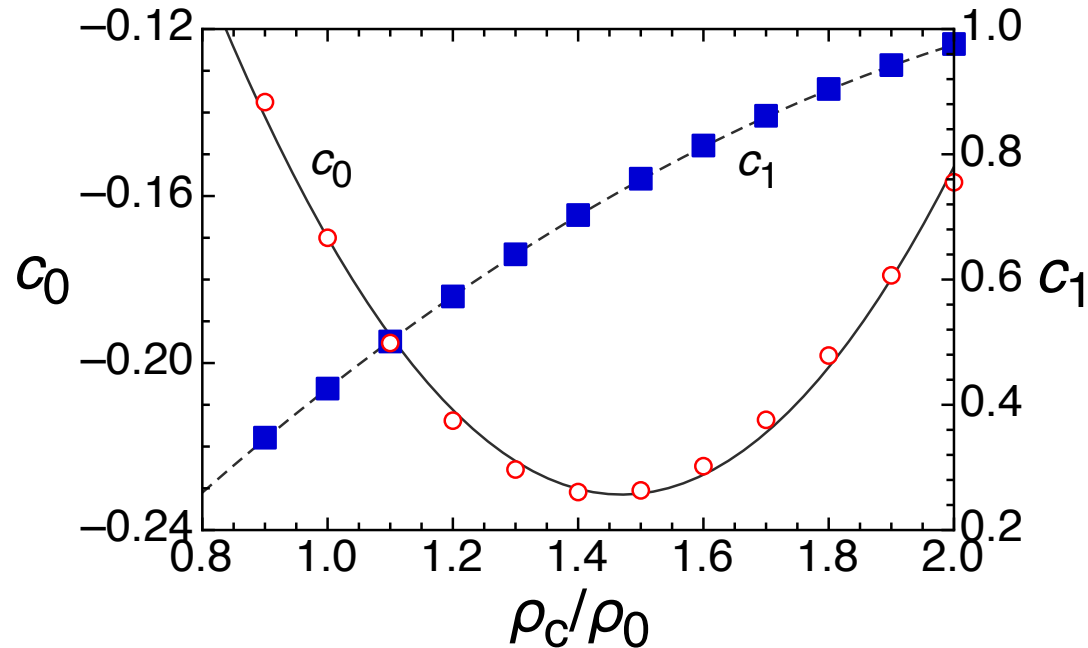
EOS	$K_0$ (MeV)	$L$ (MeV)	$\eta$ (MeV)
OI-EOS	180	31.0	55.7
	180	52.2	78.9
	230	42.6	74.7
	230	73.4	107.4
	280	54.9	94.5
	280	97.5	138.6
Shen	360	76.4	128.1
	360	146.1	197.3
Shen	281	114	154.0
Miyatsu	274	77.1	117.7
FPS	261	34.9	68.2
	SLy4	230	45.9
BSk19	237	31.9	62.3
BSk20	241	37.4	69.6
BSk21	246	46.6	81.1



# mass formula



$$\frac{M}{M_\odot} = c_0 + c_1 \left( \frac{\eta}{100 \text{ MeV}} \right)$$



$$\frac{M}{M_\odot} = 0.371 - 0.820u_c + 0.279u_c^2 - (0.593 - 1.254u_c + 0.235u_c^2) \left( \frac{\eta}{100 \text{ MeV}} \right)$$

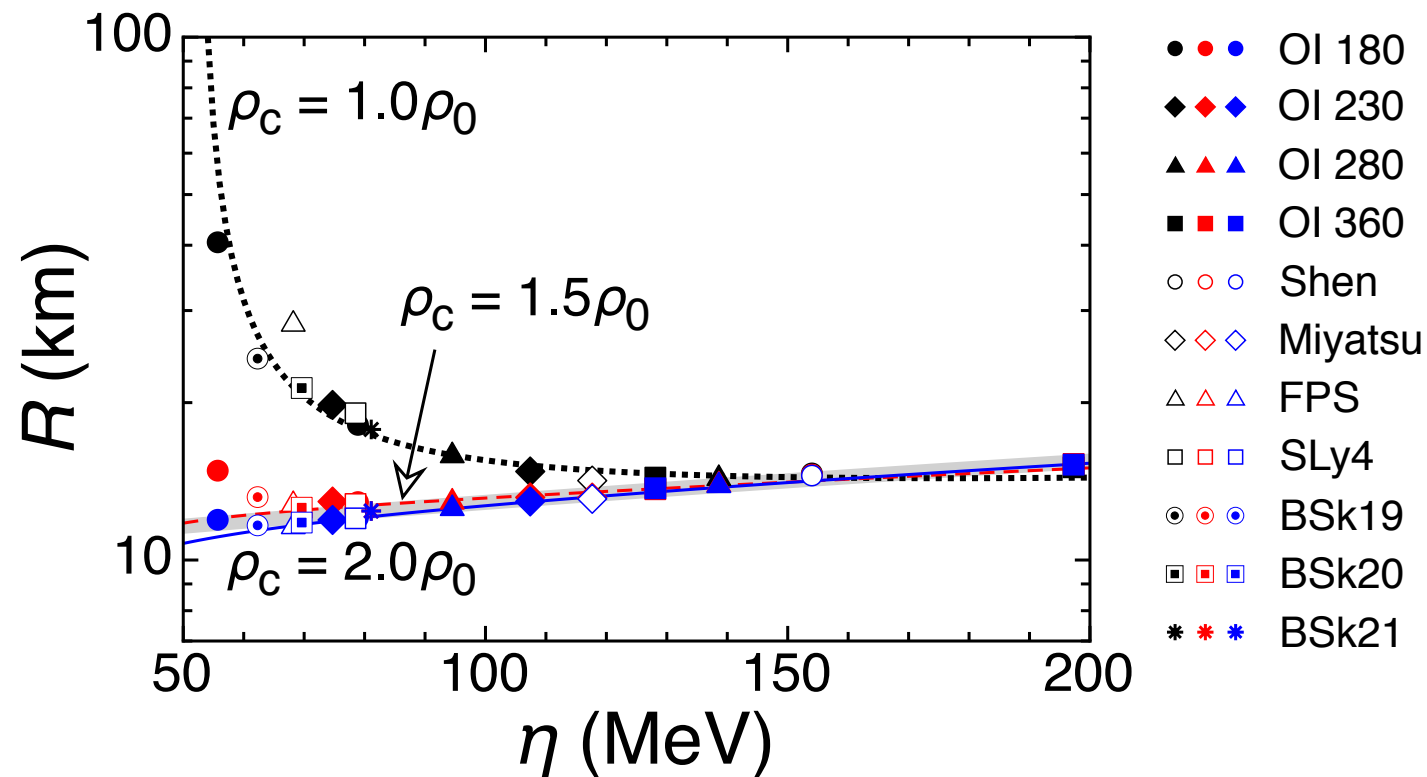
$$z = 0.00859 - 0.0619u_c + 0.0255u_c^2 - (0.0429 - 0.108u_c + 0.0120u_c^2) \left( \frac{\eta}{100 \text{ MeV}} \right)$$

$$z = 1/\sqrt{1 - 2GM/Rc^2} - 1$$

- via the simultaneous observations of  $M$  &  $z$  (or  $R$  or  $R_\infty$ ), one could extract the values of  $\eta$  &  $\rho_c$  !!

# radii of low-mass NSs

- with using the formulas of mass and gravitational redshift, one can also predict the radius of NS.



# how to determine $R$

- Unlike  $M$ ,  $R$  is generally much more difficult to determine
- Thermal emission from NS surface must be one of the good chances to obtain the information associated with  $R$ .
  - thermonuclear X-ray bursts at NS surfaces
  - photospheric radius expansion
  - quiescent low-mass X-ray binaries

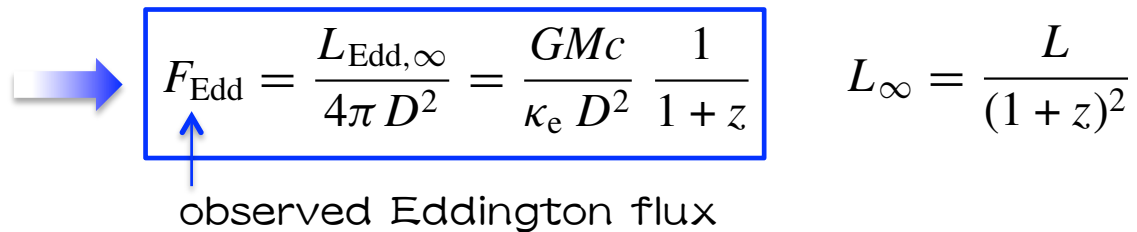
# how to determine ( $M$ , $R$ ) 1

- Assuming that Eddington limit reaches at the stellar surface...
- Eddington luminosity

$$L_{\text{Edd}} = \frac{4\pi GMc}{\kappa_e} (1+z) = 4\pi R^2 \sigma_{\text{SB}} T_{\text{Edd}}^4 \quad 1+z = (1 - 2GM/Rc^2)^{-1/2}$$

$\kappa_e = 0.2(1+X) \text{ cm}^2 \text{ g}^{-1}$  electron Thomson scattering opacity

$X$  : hydrogen mass fraction

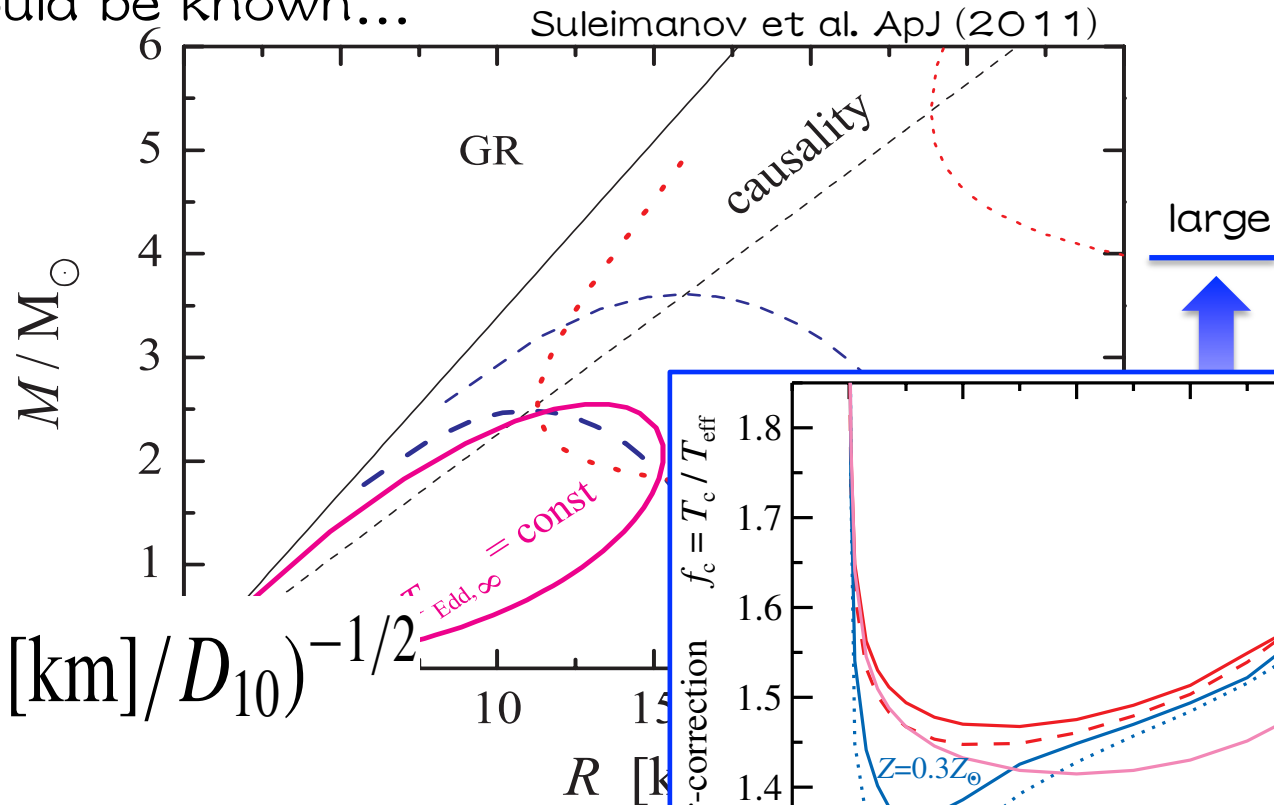

$$F_{\text{Edd}} = \frac{L_{\text{Edd},\infty}}{4\pi D^2} = \frac{GMc}{\kappa_e D^2} \frac{1}{1+z} \quad L_{\infty} = \frac{L}{(1+z)^2}$$

observed Eddington flux

- $X$  depends on an atmosphere model
  - pure hydrogen:  $X = 1$
  - pure helium:  $X = 0$
  - solar  $H/He + Z = 0.3Z_{\odot}$  :  $X = 0.74$ , where  $Z_{\odot} = 0.0134$

# how to determine $(M, R)$ 1

- if  $D$  would be known...

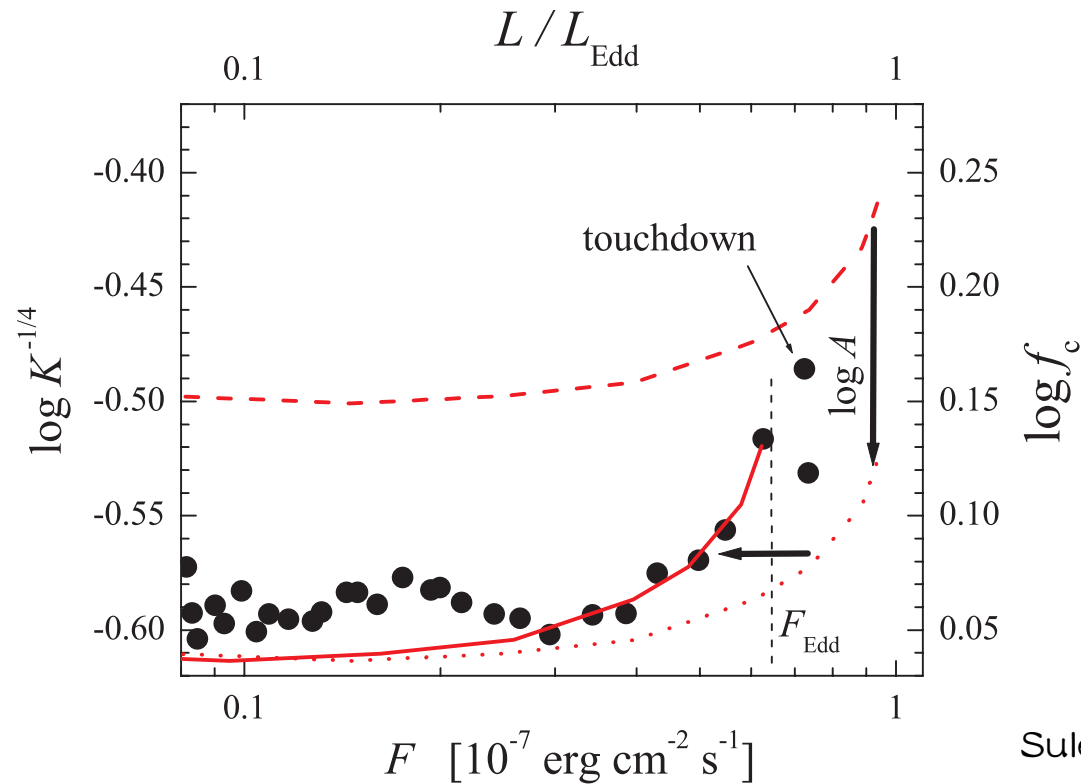


$$A = (R_\infty[\text{km}] / D_{10})^{-1/2}$$

- BUT, the determination of the effective luminosity reaches Eddington limit is quite uncertain...

# Suleimanov idea

- in order to minimize the theoretical uncertainties, the whole cooling track is adopted to determine the values of  $F_{\text{Edd}}$  &  $A$



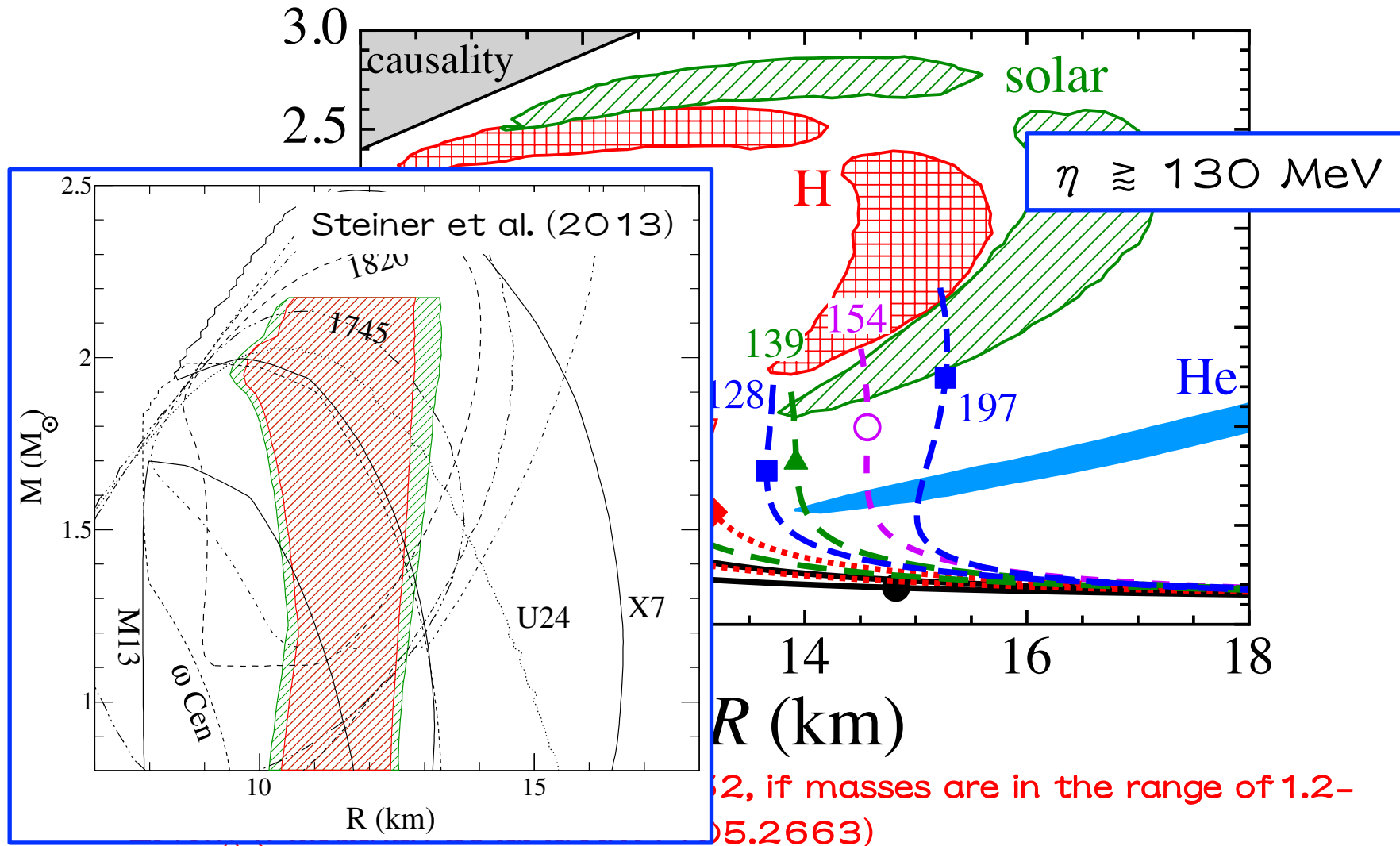
Suleimanov et al.  
ApJ (2011)

# X-ray burster 4U 1724-307

- in the globular cluster Terzan 2
  - solar H/He + subsolar metal abundance  $Z = 0.3Z_{\odot}$  (Ortolani et al. 97)
- Distance
  - $D = (5.3 - 7.7) \pm 0.6$  kpc (Kuchinski et al. 95, Ortolani et al. 97)
- data observed by Rossi X-ray Timing Explorer (RXTE)

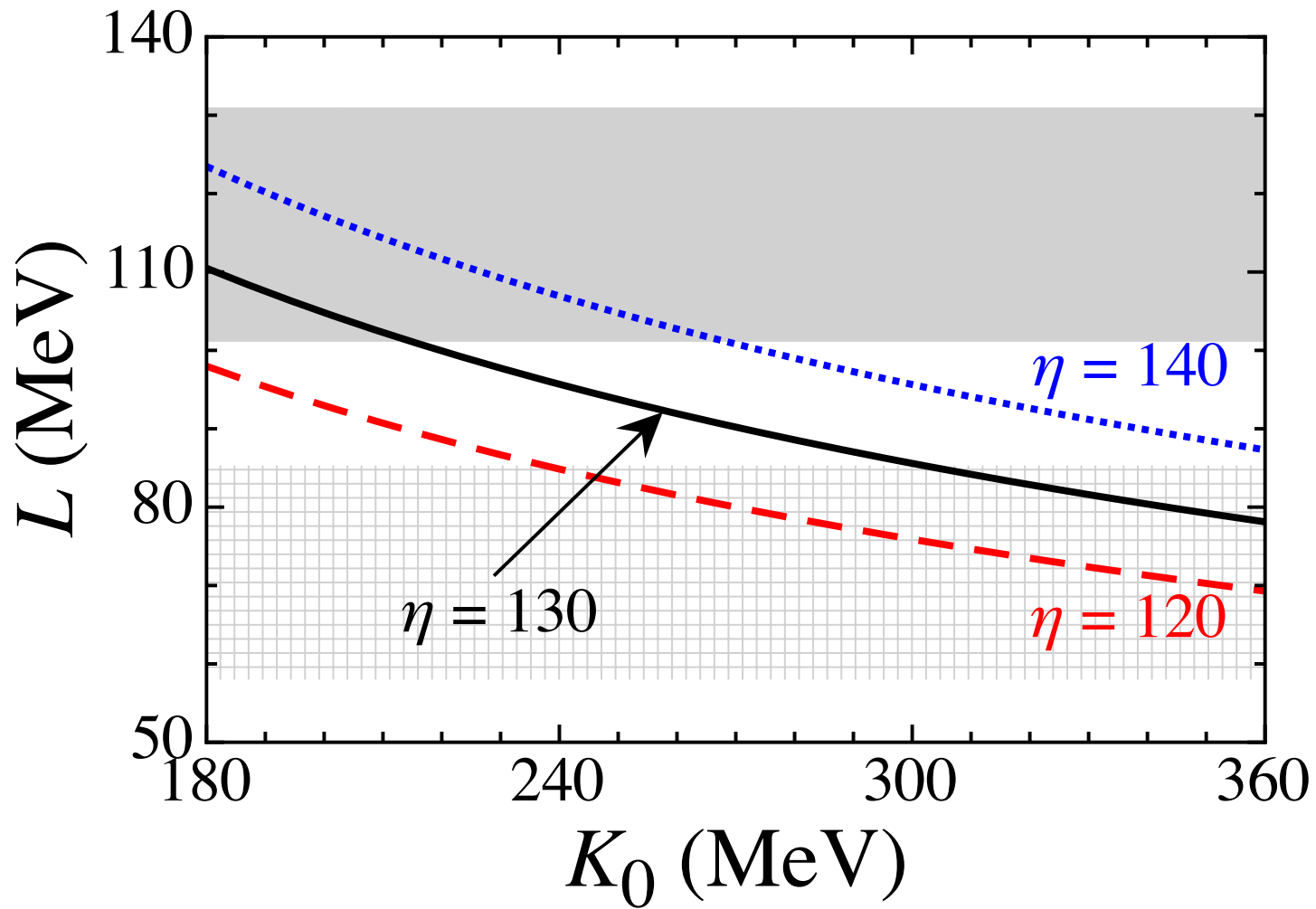


# allowed region in MR relation



2, if masses are in the range of 1.2-1.52663)

# constraint on $(L, K_0)$



# summary

- neutron stars are good candidates to examine the physics under the extreme state.
  - QPOs in SGRs may be good examples to adopt the asteroseismology
- comparing the torsional oscillations to the observational evidences, we can get the constraint on  $L$  as  $L \gtrsim 50 \text{ MeV}$ .
- superfluid effect enhances the frequencies of torsional oscillations.
  - $100 \lesssim L \lesssim 130 \text{ MeV}$ , if all QPOs come from torsional oscillations
  - $58 \lesssim L \lesssim 85 \text{ MeV}$ , if QPOs except for 26 Hz QPO come from torsional oscillation
- we find a good parameter to describe a low-mass NS
  - using the mass-radius constraint obtained by Suleimanov et al., we show a possibility to make a constraint on the nuclear saturation parameters
  - consistent with the constraints obtained from the QPO frequencies observed from the giant flares in soft-gamma repeaters