

Pulsars and Gravitational Waves

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The relationship between pulsar-like compact stars and gravitational waves is briefly reviewed. Due to regular spins, pulsars could be useful tools for us to detect \sim nano-Hz low-frequency gravitational waves by pulsar-timing array technique; besides, they would also be \sim kilo-Hz high-frequency gravitational wave radiators because of their compactness. The wave strain of an isolate pulsar depends on the equation state of cold matter at supra-nuclear densities. Therefore, a real detection of gravitational wave should be very meaningful in gravity physics, micro-theory of elementary strong interaction, and astronomy.

Keywords: Pulsars; Gravitational waves; Neutron stars

1. Introduction

Since the human mind first awakened from slumber, it has never ceased to feel the profound nature of space-time, especially the time-consciousness, in both philosophy and physics. However, a physical concrete of space-time is clarified only after Einstein's first insight: the space-time is a four dimensional continuum and the rule of the motion is the pure geometrical constrain that free particles follow the geodesics of the space-time, while the response of the space-time continuum to the matter is determined by the Einstein's equation in which a linear function of space-time curvature is in proportion to the energy-momentum of the matter.

It is worth noting that the nature of space-time is a debating topic starting earlier than the general relativity and would not be terminated only by Einstein's pure geometrical arguments. Guided by different perceptions of space-time philosophies, many gravity theories are proposed,¹ with different interpretations of equivalence principle. Even in the general relativity, the equivalent principle is still a matter of debate. Therefore, experimental

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tests of gravity theories, including in strong field and with fast motion, are critical to differentiate or falsify the gravity theories. Such experimental environments are only available in astrophysics, especially related to compact stars known as white dwarfs, pulsars/neutron stars, and black holes.

Topics of gravitational waves relevant to the pulsar astronomy are focused on in this review. The binary pulsar tests for gravity theories are given in §2. Pulsars as tools of detecting and as sources of gravitational waves are presented in §3 and §4, respectively. Future prospects are discussed in §5.

2. Pulsars, binary pulsars, and tests of gravity theories

Soon after the discovery, pulsars are identified as a class of fast rotating rather than pulsating compact objects. It was used to believe that pulsars are neutron stars composed by hadronic matter, because of the very limited knowledge in sub-nucleon research at that time, but this view might not be true² since the lowest compact states could be of quark matters with strangeness rather than that of neutron liquid. Observationally, there are two main categories of pulsars: the millisecond pulsars (MSPs) and the normal pulsars. Normal pulsars have rotational period from a few tens of milliseconds to a few seconds, while the MSPs' periods range from about 1 millisecond to a few tens milliseconds.³ Long term timing monitoring shows that the MSPs are more stable rotators compared to the normal pulsars, which might due to both observational reasons and pulsar intrinsic physics. For MSPs, the difference between model-expected time of arrival (TOA) of radio pulses and observed TOA is usually less than 10 percent of their periods, and the most stable MSPs can achieve ~ 100 ns level on the time scale of a few years.⁴

It turns out that most of the MSPs are in binary system. One particular interesting system is the recently discovered binary pulsar system, J0737-3039AB, where both of the two stars are radio pulsars.⁵ The J0737 is also a highly relativistic celestial system. Binary pulsars with possible pulsar companions are listed in Table 1, obtained from ATNF pulsar catalog.⁶

Armed with such a kind of stable celestial clocks (i.e., pulsars) relativistically orbiting their companions, one can then test gravity theories in the case of strong gravitational fields, as illustrated in the classical system of PSR B1913+16.⁷ In the J0737 system, two pulsars orbit each other with a period of 2.5 hours and a very low orbital eccentricity. Up to known, this J0737 system becomes the most relativistic binary system, the details of which can be found in the review by Kramer and Wex (2009).⁸

To test the gravity theories, one must compare the predicted TOA of

Table 1. Parameters for possible pulsar-neutron star systems

Name	P	P_b	a	e
J0737-3039A	0.022699	0.1023	1.4150	8.778e-02
J0737-3039B	2.773461	0.1023	1.5161	8.778e-02
J1518+4904	0.040935	8.6340	20.0440	2.495e-01
B1534+12	0.037904	0.4207	3.7295	2.737e-01
J1756-2251	0.028462	0.3196	2.7564	1.806e-01
J1811-1736	0.104182	18.7792	34.7827	8.280e-01
B1820-11	0.279829	357.7620	200.6720	7.946e-01
J1829+2456	0.041010	1.1760	7.2380	1.391e-01
J1906+0746	0.144072	0.1660	1.4202	8.530e-02
B1913+16	0.059030	0.3230	2.3418	6.171e-01
B2127+11C	0.030529	0.3353	2.5185	6.814e-01

Note: P is the pulsar period in unit of second, P_b is the orbit period in unit of days, a is the projected semi major axis in unit of light seconds, and e is the orbital eccentricity.

a theoretical model with the observation. In this way, one needs thus the binary motion dynamical models, in which we put the gravity theory in. One can then calculate the theoretical pulse TOA at the solar barycenter. Note that, from a pulsar to the barycenter, various processes set in, including the photon propagation effects due to the gravitational field of both the binary system and solar system, the dispersion of pulsar radio signal due to interstellar medium and solar wind, and so on. One needs also the solar system ephemeris to convert the pulsar TOA at the radio telescope to the barycenter. The modeled TOA will then be compared with observed ones to see if the gravity theories is able to account for the observation.

In reality, thanks to the phenomenological framework of post-Keplerian (PK) parameters, with which gravity theories can be approximated, we can independently measure these PK parameters by fitting the TOA data. A gravity test is then via checking the self-consistency of PK parameters for a particular gravity theory. There are 7 PK parameters which are possibly measurable in the near future: advance of periastron $\dot{\omega}$, gravitational redshift parameter γ , Shapiro delay parameters r and s , orbit period derivative \dot{P}_{orb} , spin-orbital coupling induced precession $\Omega_{\text{so,p}}$ and relativistic orbit deformation δ_{θ} . These PK parameters, except δ_{θ} , have been measured in the 0737 system.

All the 7 parameters measured are functions of two unknown parameters: pulsar masses, m_a and m_b . A double neutron star system is then overdetermined if one detects three or more PK parameters. It is worth noting that the double pulsar system of J0737 offers an extra Keplerian constrain, the mass ratio between two star, $R(m_a, m_b) = m_a/m_b = x_b/x_a$,

with x the projected semimajor axes. Two recent reviews^{8,9} are valuable in the topic of testing gravity with binary pulsars.

3. Detecting gravitational waves with pulsars

Directly detecting gravitational wave (GW) is the Holy Grail of present experimental researches, not only in gravity physics but also in astronomy. With the efforts since 1960s,¹⁰ recent equipments (e.g., LIGO¹¹) may finally allow us to directly detect GWs although there is no confirmed detection now yet. In this section, we will review the ability of detecting gravitational waves using *pulsar timing array* (PTA). Potential roles of testing gravity with PTA are also presented here.

GW is actually a perturbation of space-time, fully characterized by a wave-like metric perturbation. Detecting GW is thus identical to measure the wave-like metric perturbation^a which can be performed by comparing geodesics of two test objects approaching to and departing from each other. Such experiments fall into four categories: 1. Tracing the motion of two free-falling test objects (e.g. LIGO, LISA, GEO, TAMA, and so on), 2. Detecting the deformation of finite extend solid body (e.g. Bar detector, Sphere detector, and so on), 3. Measuring the Doppler shift of electromagnetic signals from distance free-falling objects (e.g. Doppler tracking of satellite, pulsar timing array, laser ranging, LISA), 4. Checking the perturbation of a cosmological system (e.g. cosmic background B mode detection, weak lensing survey). Among all these possible ways, PTA is one of the promising techniques to directly detect gravitational waves, being unique to detect GW at nano-Hertz band.¹²

As we have shown, MSPs are very stable celestial clock in the Galaxy. GWs perturb the background space-time of the Galaxy, such that pulsar pulse signals get red or blue Doppler shift along the path from pulsar to earth. It turns out that such GW-induced frequency shift only involves the metric perturbation at the pulsar and that at the earth. The GW-induced frequency shift can be obtained to be¹³

$$\frac{\Delta\omega(t)}{\omega} = \frac{\hat{\mathbf{n}}^i \hat{\mathbf{n}}^j}{2(1 + \mathbf{n}_{\mathbf{g}} \cdot \hat{\mathbf{n}})} [h_{ij}(t, 0) - h_{ij}(t - D/c, \mathbf{D})], \quad (1)$$

^aIt should be born in mind that detecting of GW is not detecting any types of metric perturbation. GW detection focuses on detecting the oscillatory part of the metric perturbation with strain h decrease as r^{-1} , such that gravitational wave could carry energy and momentum to the infinity.

where ω is the pulsar angular frequency of spin, $\hat{\mathbf{n}}$ is the pulsar direction, $\mathbf{D} = D\hat{\mathbf{n}}$ with D the distance to the pulsar, \mathbf{n}_g is the GW propagation direction, and h is the perturbation of metric. The GW-induced timing residuals in pulsar TOA is therefore $R = \int \Delta\omega/\omega dt$.

Due to the intrinsic noises and possible non-modeled accelerations of pulsars, it is unlikely that one can use $R(t)$ of a single pulsar to detect GWs. Nevertheless, magic happens if we correlate the residuals of R_i and R_j of two pulsars. From Eq. (1), one can have a correlation of $\langle R_i R_j \rangle = C(\theta)\sigma^2$ for two different pulsars in general relativity, with σ the RMS (root-mean-square) of a single pulsar's residual. Note that the correlation $C(\theta)$ is a determined function only involving the angular, θ , between two pulsars,¹⁴ and this correlation $C(\theta)$ certainly plays a vital role in detecting GW using a array of pulsar timing data (PTA) since the shape of $C(\theta)$ is uniquely determined by a gravity theory and there is no other physical processes to make the pulsar signal correlated for two pulsars widely separated with a distance of several thousand light years away from each other^b. In the general relativity theory of gravity, fortunately, the correlation $C(\theta)$ has a very simple form of^{13,14}

$$C(\theta) = \frac{3x \log x}{2} - \frac{x}{4} + \frac{1}{2}(1 + \delta(x)), \quad (2)$$

where $x = (1 - \cos\theta)/2$.

We may make sense of the $C(\theta)$ -curves from simple symmetric reasons. If a monochromatic general relativistic GW is propagating along 'z-axis' direction (there will be 180° symmetry and 90° anti-symmetry in x-y plane), then correlation $C(\theta)$ between two pulsars with 0° or 180° angular separation are positive, while $C(\theta)$ for 90° will be negative. This make a U-shaped $C(\theta)$.¹⁴ Note that $C(180^\circ) \neq C(0)$ which will be explained later. One can then measure such multi-pulsar correlation to detect GWs. Jenet et al. (2005)¹² had investigated the statistical properties of such detection processes. Their results show that regular timing observations of 40 pulsars each with a timing accuracy of 100 ns will be able to make a direct detection of the predicted stochastic background from coalescing black holes within 5 years. We compare the detection abilities for GW detectors in Fig. 1, for GW background due to coalescing supermassive binary black holes (BBH).

From symmetry arguments above, it is clear that the shape of $C(\theta)$ de-

^bThe imperfectness of terrestrial clock and un-modeled solar system dynamics may introduce also correlation between measured R_i of pulsars, however the angular dependence of such correlations is very different from that $C(\theta)$ presented in Eq.(2).

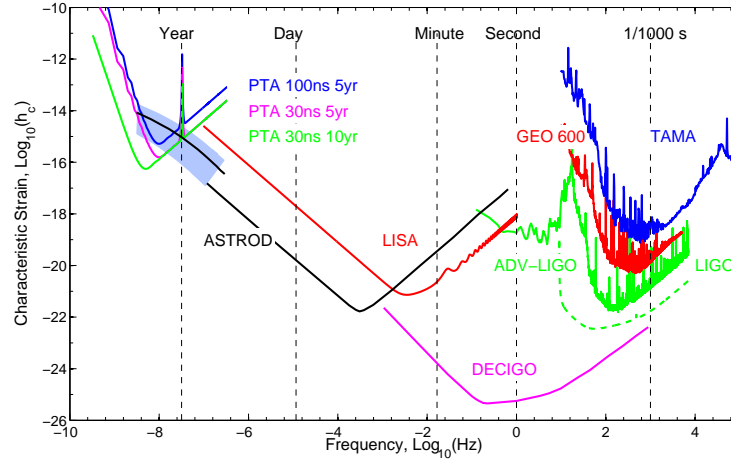


Fig. 1. A comparative for abilities of detecting GWs for various GW detectors, where the x-axis and y-axis are the GW frequency and the characteristic strain of GW respectively. Most of the labeling are self-explanatory. The PTA curves are at in the left-top corner, where the length of data and pulsar average timing noise level are also labeled.

depends on the polarization of GW (see Fig. 2). Einsteins theory of gravity predicts waves of the distortion of space-time with two degrees of polarization; alternative theories predict more polarizations, up to a maximum of six.¹⁵ Lee et al. (2008)¹³ analyzed such polarization effects and conclude that for biweekly observations made for five years with rms timing accuracy of 100 ns, detecting non-Einsteinian modes will require: 60 pulsars in the case of the longitudinal mode; 60 for the two spin-1 ‘shear’ modes; and 40 for the spin-0 ‘breathing’ mode. Further more, they showed that one can test gravity theories by checking GW polarization, i.e., to discriminate non-Einsteinian modes from Einsteinian modes, we need 40 pulsars for the breathing mode, 100 for the longitudinal mode, and 500 for the shear mode. These requirement is beyond present observation technology, but could be easily achieved using SKA or FAST telescope.^{16,17}

Another interesting topic on detecting GWs using PTA is about the dispersion relation of GW,¹⁸ since the function of $C(\theta)$ and the detection statistics depends also the mass of graviton. It is found¹⁸ that $C(180^\circ)$ increases to match the value of $C(0)$ as the graviton mass increases (see Fig. 3 for details). In the case of massless GW background, we know that the GW has 180° degree symmetry due to the polarization property, but

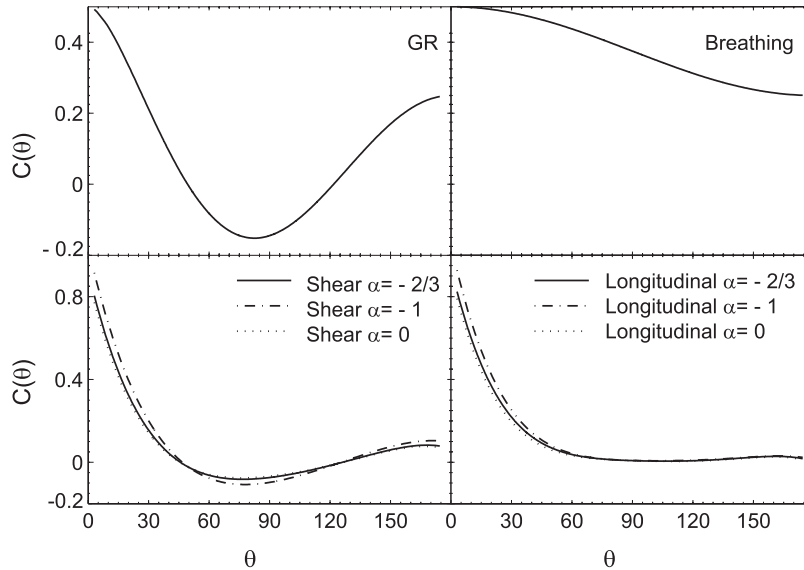


Fig. 2. The $C(\theta)$ curves for different kinds of GW polarization, with α denoting the power index of the GW background.

why $C(0^\circ) \neq C(180^\circ)$? It turns out that GW propagation breaks up this 180° symmetry by the geometric factor in Eq. (1), which reads $1 + \hat{\mathbf{e}}_z \cdot \hat{\mathbf{n}}$ for the massless case. For the case of a massive GW background, the geometric factor reads $1 + \frac{c}{\omega_g} \mathbf{n}_g \cdot \hat{\mathbf{n}}$, where the graviton mass reduces the asymmetry. For the limiting case, where the GW frequency is just at the cut-off frequency, the dispersion relation tells us that such a GW is not propagative, then the 180° symmetry is restored. Therefore, we would expect that the correlation function are of 180° symmetry for very massive gravitons.

Lee et al. (2009) further find that it is possible to measure graviton mass using PTA and one will get 90% probability to differentiate between the results for massless graviton and that for graviton heavier than 3×10^{-22} eV, if biweekly observation of 60 pulsars are performed for 5 years with pulsar RMS timing accuracy of 100 ns in the future.

As we have shown that PTA can be constructed to measure the alternative polarization modes of GW and the GW dispersion relation. These measurements provide tests for gravity theory in the weak field/high velocity region, which are different from that of the solar system tests (i.e., the

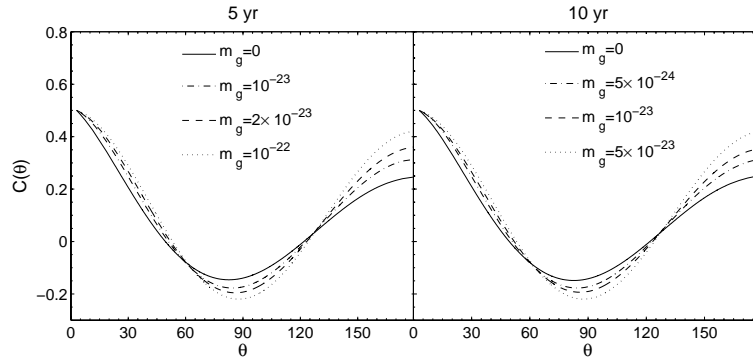


Fig. 3. The curves of $C(\theta)$ for different graviton masses.

weak field/slow velocity case) and the binary pulsar tests (i.e., the strong field/slow velocity cases), because it is not completed to describing GW using post-Newtonian formalism and the scalar and tensor sectors of gravity theories are different.¹⁹

4. Gravitational wave radiation from pulsars

Pulsars are not only as tools to detect GWs, but also strong GW sources because of their compactness and the rapid mass changes. Indirect evidence for GW from binary pulsars has been discussed in §2, whereas a direct detection of GW from pulsars with ground-based facilities should be meaningful. It is recognized that the GW amplitudes of isolate pulsars depend on the equation of state (EoS) of cold matter at supra-nuclear density, which is strongly related to the understanding of QCD (quantum chromo-dynamics) at low energy scale, still another challenge for physicists today. A mixture of quantum (QCD) and gravity (relativity) makes this project more funny.

We are still not sure about the nature of pulsar-like compact stars though discovered since 1967. It is conventionally believed that these compact stars are normal neutron stars composed of hadronic matter, but one can not rule out the possibility that they are actually quark stars of quark matter² (see, e.g., a review²⁰). Quark stars with strangeness are popularly discussed in literatures, which are called as strange (quark) stars. The EoS of realistic quark matter in compact stars, depending non-perturbative QCD, was supposed to be of Fermi gas or liquid, but could be of classical solid in order to understand different manifestations of pulsar-like stars.^{21,22}

Besides QCD, that pulsars are quark stars should also be meaningful in

GW physics.²³ (i). *GW being EoS-dependent.* Rotation (r) mode instability, which would result in GW radiation, may occur in fluid quark stars if the bulk and shear viscosities of quark matter is not sufficiently high, but no r -mode instability occurs in solid quark stars. Even in case of solid quark stars, the GW amplitude is relevant to the quadrupole deformations²⁷ (e.g., mountain building on stellar surface) sustained by elastic or magnetic forces on stellar surface. A quark or neutron star with quadrupole deformation would be a GW radiator if it has precession either free or torqued, and the precession amplitude (or the angle between spin axis and spindle of inertia ellipsoid) is determined by EoS²⁴ and determines GW strain. (ii). *GW being mass-dependent.* A very difference between quark and normal neutron stars is that the latter is gravity-bound while the former is confined additionally by self strong interaction, that results in the fact that quark stars could be very low massive²⁵ (even to be of $\sim 10^{-3}M_{\odot}$) but neutron star cannot. Low mass quark stars, either in liquid or solid states, are surely very weak GW emitters. This mass-depend nature makes it more complex to constrain EoS of target compact stars by negative results of LIGO GW detections. The points of above (i) and (ii) are certainly very useful for us to observationally distinguish quark stars from normal neutron stars in the future.

Pulsars spin usually at frequencies $> 10^0$ Hz, and we thus are interested in LIGO to detect their GWs, from Fig. 1. There are two kinds of GWs from pulsar-like compact stars: continuous GWs due to spin and bursting GWs due to stellar catastrophic events (e.g., star quake²⁶ or binary coalescence). It is worth noting that all the upper limits estimated from LIGO science runs depend on simulated waveform types of GWs (i.e., astrophysical GW radiative mechanisms). For continuous GWs, the waveforms could be better understood, and their searches are significantly more sensitive, especially when informed by observational photon astronomy and theoretical astrophysics.²⁸ The waveform of bursting GW is a matter of debate,²⁹ and such kind of GW searches is also focused by LIGO, especially on the super-flares of soft gamma-ray repeaters.³⁰

5. Summary and Future prospects

Pulsars could be useful tools to detect GWs by PTA technique, they would also be strong GW radiators; a real detection of gravitational wave should be very meaningful in gravity physics, micro-theory of elementary strong interaction, and astronomy. A successful detection of GW by PTA may provide a test of GW polarization and measure the graviton mass. Thought indirect evidence for GWs from pulsar timing in binaries has been obtained,

a direct GW detection of pulsar-like stars is also expected as persistent or transient sources. The strain of GW from an isolated compact star depends on the equation of state of cold matter at supra-nuclear densities.

Pulsar timing array projects are promising for detect GWs. We can achieve $h_c = 10^{-15}$ region for several year continuous pulsar timing monitoring. If bi-weekly observations are made for five years with RMS timing accuracy of 100 ns, then 40 pulsars are required for general relativistic modes, 60 for the longitudinal mode; 60 for the two spin-1 shear modes; and 40 for the spin 0 breathing mode. Additionally, we may measure the graviton mass through PTA techniques. With a 5-year observation of 100 or 300 pulsars, we can detect the graviton mass being higher than 2.5×10^{-22} and 10^{-22} eV, respectively. Ultimately, a 10-year observation of 300 pulsars allows us to probe the graviton mass at a level of 3×10^{-23} eV.

For the task of measuring the GW polarization and the graviton mass, there is one critical requirement: a large sample of stable pulsars. Thus the on-going and coming projects like the Parkes PTA,³¹ the European PTA,³² the Large European Array for Pulsars,³³ the FAST^{16,17} and the SKA would offer unique opportunities to detect the GW background and to probe into the nature of GWs, both physical (the GW polarization and the graviton mass) and astronomy (the GW sources).

Other important requirements for a successful PTA include high stability of pulsar intrinsic noises and a low measurement noise. We need pulsar survey with better sky coverage as well as good observing system, especially with better band width³⁴ to get better signal to noise ratio and to subtract the interstellar medium effects. Better radio frequency interference filtering technology will also be very helpful such that we can use the full band data and reduce the terrestrial contamination. Better timing techniques (such as timing in full Stokes parameters³⁵) could also be preferred.

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