SEARCHING FOR SUB-MILLISECOND PULSARS: A THEORETICAL VIEW *

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Sub-millisecond pulsars should be triaxial (Jacobi ellipsoids), which may not spin down to super-millisecond periods via gravitation wave radiation during their lifetimes if they are extremely low mass bare strange quark stars. It is addressed that the spindown of sub-millisecond pulsars would be torqued dominantly by gravitational wave radiation (with braking index $n \simeq 5$). The radio luminosity of sub-millisecond pulsars could be high enough to be detected in advanced radio telescopes. Sub-millisecond pulsars, if detected, should be very likely quark stars with low masses and/or small equatorial ellipticities.

1. Introduction

Historically, the very idea of "*gigantic nucleus*" (neutron star) was first given by Landau more than 70 years ago, was soon involved in supernova study of Baade and Zwicky, and seems to be confirmed after the discovery of radio pulsars. Although this is a beautiful story, no convincing work, either theoretical from first principles or observational, has really proved this idea that pulsars are normal neutron stars, since nucleon (neutron or proton) was supposed to be point-like elementary particles in Landau's time but not now. An idea of quark stars, which are composed by free quarks rather than free nucleus, was suggested as more and more sub-nucleon phenomena were understood, especially the asymptotic freedom nature of strong interaction between quarks; and it is found that no problem exists in principle if pulsars are quark stars. Therefore, astrophysicist without bias should think equally neutron and quark stars to be two potential models for the nature of pulsar-like stars[1].

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One of the most important problems could then be differentiating neutron and quark stars in the new millennium astrophysics. Many likely ways to identifying a quark star are proposed[1], but the search of sub-millisecond pulsars would be an excellent key experiment. The reason for this is very intuitive and almost model-independent. Normal neutron stars are gravitationally confined, and thus can not spin with periods being less than a break period of $\sim 0.5 M_1^{1/2} R_6^{-3/2}$ ms (mass: $M_1 = M/M_\odot$, radius: $R_6 = R/10^6$ cm), but quark stars are chromatically confined, without limitation by the Kepler frequency. Additionally, a much high spin frequency may not damped significantly if a quark star has a very low mass. Therefore, sub-millisecond pulsars should be expected to be detected if pulsars are (low-mass) quark stars, with a rapid spin at birth, but could not exist if they are normal neutron stars.

We are viewing the implications of possibly future discovery of sub-millisecond pulsars in this paper, with (solid) quark star models to be focused for the nature of pulsar-like compact objects. The formation, evolution, and magnetospheric activity of sub-millisecond pulsars are also discussed. Because the pulse profiles of radio pulsars are highly modulated ($\sim 100\%$), with very small pulse-widths (a few $\sim 10^{\rm o}$), the timing precision could only be high enough to uncover sub-millisecond pulsars in radio band. We focus thus radio pulsars here.

2. Spindown of sub-millisecond pulsars

2.1. *Spin periods limited by gravitational wave emission*

Fluid pulsars. In Newtonian theory, a rapidly rotating fluid Maclaurin spheroid is secularly unstable to become a Jacobi ellipsoid, which is non-axisymmetric, if the ratio of the rotational kinetic energy to the absolute value of the gravitational potential energy $T/|W| > 0.1375$. For a Maclaurin spheroid with homogenous density ρ, the ratio $T/|W| = 0.1375$ results in an eccentricity $e = 0.81267$ (or $c/a = \sqrt{1-e^2} \simeq 0.6$) and thus in a critical frequency $\Omega_c \sim 5.6 \times 10^3$/s (i.e., spin period $P_c \sim 1.1$ ms) if $\rho = 4 \times 10^{14}$ g/cm^3. Sub-millisecond pulsars could then be Jacobi ellipsoids, to be tri-axial. In the general relativistic case[2,3], gravitational radiation reaction amplifies an oscillation mode, and it is then found that the critical value of $T/|W|$ for the onset of the instability could be much smaller than 0.1375 for neutron stars with mass of $\sim M_\odot$. This sort of non-axisymmetric stellar oscillations will inevitably result in gravitational wave radiation, and put limits on the spin periods.

A kind of oscillation mode, socalled r-mode, is focused on in the literatures[4,5,6]. The r-mode oscillation is also called as the Rossby waves that are observed in the Earth's ocean and atmosphere, the restoring force of which is the Coriolis force. This instability may increase forever if no dissipation occurs. Therefore, whether the instability can appear and how much the oscillation amplitude is depend on the interior structure of pulsars, which is a tremendously complicated issue in supranuclear physics.

The Kepler frequency of low-mass bare strange stars could be approximately a constant,

$$\Omega_0 = \sqrt{\frac{GM}{R^3}} = 1.1 \times 10^4 \ \ \mathrm{s}^{-1}, \tag{1}$$

where the average density is taken to be $\sim 4 \times 10^{14}$ g/cm^3. Correspondingly, the spin period $P_0 = 2\pi/\Omega_0 \simeq 0.6$ ms $< P_c$. It is found by Xu[7] that the gravitational wave emissivity of quark stars is mass-dependent. The r-mode instability could not occur in fluid bare strange stars with radii being smaller than ~ 5 km (or mass of a few $0.1 M_\odot$) unless these stars rotates faster than the break frequency (in fact, a more effective gravitational wave emission mode occurs if $P <\sim P_0$, see Eq.(5)). These conclusions do not change significantly in the reasonable parameter-space of bag constant, strong coupling constant, and strange quark mass.

Some recent observations in X-ray astronomy could hint the existence of low-mass bare strange stars[8]. The radiation radii (of, e.g., 1E 1207.4-5209 and RX J1856.5-3754) are only a few kilometers (and even less than 1 km). No gravitational wave emission could be detected from such fluid stars even they spin only with a period of ~ 1 ms.

Solid pulsars. A protoquark stars should be in a fluid state when their temperatures are order of 10 MeV, but would be solidified as they cool to very low temperatures[9,?]. Assuming the initial ellipticity of a solid quark star keeps the same as that of the star just in its fluid phase, strain energy has to develop when a solid quark star spins down. Quake-induced glitches of the quark star occur when the strain energy reaches a critical value[10], and we thus suggest that the stellar ellipticity would be approximately determined by the conventional Maclaurin spheroids (for $P \gg 1$ ms)

$$\varepsilon(P) \simeq \frac{5\Omega^2}{8\pi G\rho} \simeq 3 \times 10^{-3} P_{10\mathrm{ms}}^{-2}, \tag{2}$$

where the spin period $P = 2\pi/\Omega = P_{10\mathrm{ms}} \times 10$ ms, provided that the density $\rho \simeq 4 \times 10^{14}$ g/cm^3 is a constant.

A pulsar must be non-axisymmetric in order to radiate gravitationally. A triaxial pulsar, with deformation ellipticity ϵ_e in its equatorial plane, or a wobbling pulsar, either freely or forcedly, may thus radiate gravitational waves, the frequency of which is 2Ω for the former but is $\Omega + \Omega_{\rm prec}$ (the precession frequency $\Omega_{\rm prec}$ is orders of magnitude smaller than Ω) for the later. This wave results in a perturbed metric, which is order of h_0 being given by[11],

$$h_0 = \frac{128\pi^3 G\rho_0}{15c^4} \cdot \frac{R^5}{dP^2}(\epsilon_e \text{ or } \epsilon\theta) \approx 2.8 \times 10^{-20} R_6^5 d_{\rm kpc}^{-1} P_{\rm 10ms}^{-2}(\epsilon_e \text{ or } \epsilon\theta), \quad (3)$$

where approximations $I \simeq 0.4MR^2$ and $M \simeq 4\pi R^3\rho_0/3$ are applied for solid quark stars in the right equation, the pulsar's distance to earth is $d = d_{\rm kpc} \times 1$ kpc, θ is the wobble angle.

For normal neutron stars, ϵ_e and $\epsilon\theta$ are supported by crustal shear stress and magnetic pressure. However for solid quark stars, this mechanisms may not work due to a relatively negligible magnetic and Coulomb forces. Nevertheless, glitches of solid quark stars could also produce bumps, with a maximum ellipticity[12],

$$\epsilon_{\rm max} \sim 10^{-3}(\frac{\sigma_{\rm max}}{10^{-2}})R_6^{-6}(1 + 0.084R_6^2)^{-1}, \quad (4)$$

where $\sigma_{\rm max}$ is the stellar break strain. This ellipticity is larger for low-mass quark stars due to weaker gravity. This maximum ellipticity could be much smaller than the ellipticity of Maclaurin spheroids with $P <\sim 1$ ms (sub-millisecond pulsars). Therefore, for the sake of simplicity, we assume that the real ellipticity of a solid quark star could be $\varepsilon(P)$, due to stress releases through star-quake induced glitches[10], in the discussion below.

LIGO is sensitive to hight frequency waves, which recently puts upper limits on h_0 for 28 known pulsars through the second LIGO science run[13]. The upper limits are order of 10^{-24}, which means approximately an limit of $\epsilon R_6^5 < 10^{-4}$. Only three normal pulsars are targeted; others are millisecond pulsars. The upper limits of masses and radii for millisecond pulsars are constrained[7] by the second LIGO science run. Specially, the radius of the fastest rotating pulsar, PSR B1937+21, could be smaller than ~ 2 km if its wobble angle θ is between $1^{\rm o}$ and $10^{\rm o}$.

Energy of gravitational wave is not as instinctive as the perturbed metric, h_0. Nonetheless, for triaxial sub-millisecond pulsars, the luminosity of gravitational waves radiation from a solid pulsar could be obtained[14], and the total rotation energy loss via gravitational and magnetospheric (photons and particles) emission is

$$-I\Omega\dot{\Omega} = \frac{32G\Omega^6 I^2 \epsilon_e^2}{5c^5} + \frac{2}{3c^3}\mu_m^2 M^2 \Omega^4, \tag{5}$$

where μ_m is the magnetic momentum per unit mass. It is worth mentioned that the work relevant to rapid rotating Jacobi ellipsoids was done by many authors[15,16,17,18,19]. The ratio of the term due to gravitational wave and that due to magnetospheric activity in Eq.(5) is

$$f_r = \frac{192GR_{\rm eff}^4 \epsilon_e^2}{125c^2\mu_m^2}\Omega^2 \simeq 4.5\times10^{11} R_{\rm effkm}^4 \epsilon_e^2 \mu_{m-6}^{-2} P_{\rm ms}^{-2}, \tag{6}$$

where an effective radius $R_{\rm eff} = R_{\rm effkm} \times (1\ {\rm km})$ is defined through $I = 2MR_{\rm eff}^2/5$, $\mu_m = \mu_{m-6} \times (10^{-6}\ {\rm G\ cm^3\ g^{-1}})$, $P = 2\pi/\Omega = P_{\rm ms} \times (1\ {\rm ms})$. It is evident from Eq.(6) that the braking torqued by gravitational wave dominates the spindown of a sub-millisecond pulsar unless $\epsilon_e \ll 1$ and/or $R_{\rm eff} \ll 1$ km. The braking index for gravitational wave emission is $n \equiv \Omega\ddot{\Omega}/\dot{\Omega}^2 = 5$ from Eq.(5), and this feature of $n > 3$ could be evidence for gravitational wave in return. Therefore, gravitational wave radiation alone could result in an increase of the rotation period of sub-millisecond pulsars,

$$\dot{P} = \frac{K}{P^3}, \quad \text{with } K \equiv \frac{512\pi^4 GI\epsilon_e^2}{5c^5} \simeq \frac{4096\pi^5 G}{75c^5}\rho\epsilon_e^2 R_{\rm eff}^5, \tag{7}$$

where the pulsar mass is approximated by $M \simeq 4\pi R_{\rm eff}^3\rho/3$.

The solution of P from Eq.(7) is $P^4 = P_i^4 + 4Kt$ (P_i denotes the initial spin period), which is shown in Fig. 1 for different parameters of ϵ_e and $R_{\rm eff}$ ($\rho = 4 \times 10^{14}$ g/cm^3 is assumed). We see that the period increase rate is proportional to $(\epsilon_e^{2/5} R_{\rm eff})^5$, a much small value of ϵ_e and/or a low mass could make it possible that sub-millisecond pulsars exist in the Universe. Since $K \propto \rho R_{\rm eff}^5 \sim MR_{\rm eff}^2$, P would increase much faster for normal neutron stars than that for quark stars because of the sharp difference between the mass-radius relations of these two kinds of stars, if the equatorial ellipticities, ϵ_e, are almost the same. The origin of ϵ_e could be due to the secular instability of Maclaurin spheroid (to become Jacobi ellipsoid finally) before solidification, or the glitch-induced bumpy surface after solidification. Detailed calculation on ϵ_e is necessary and essential for us to know the spindown features of sub-millisecond pulsars as well as to estimate the gravitational wave strength emitted from such stars. It is worth noting that quark nuggets (stars) born during cosmic QCD phase separation may exist as super-millisecond pulsars due to spindown in Hubble time ($\sim 10^{10}$ years) although they might still keep sub-millisecond spin if their values of K is small enough.

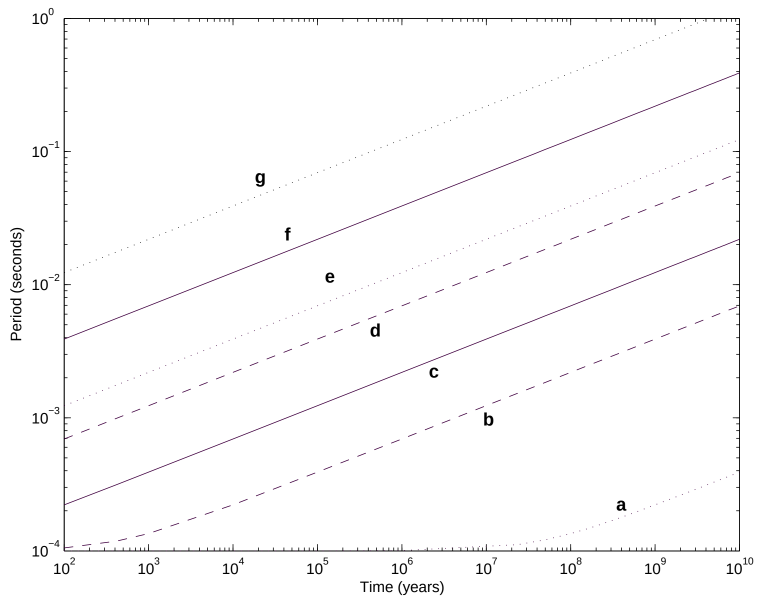

Figure 1. Gravitational-wave-radiation-induced period evolution of sub-millisecond pulsars with an initial period $P_i = 0.1$ ms for different parameters of equatorial ellipticity, ϵ_e, and effective radius, $R_{\rm eff}$, but a fixed density $\rho = 4 \times 10^{14}$ g/cm^3. The lines are labelled from "**a**" to "**g**". "a": $\epsilon_e = 10^{-3}$, $R_{\rm eff} = 0.01$ km; "b": $\epsilon_e = 10^{-3}$, $R_{\rm eff} = 0.1$ km; "c": $\epsilon_e = 0.01$, $R_{\rm eff} = 0.1$ km; "d": $\epsilon_e = 0.1$, $R_{\rm eff} = 0.1$ km; "e": $\epsilon_e = 10^{-3}$, $R_{\rm eff} = 1$ km; "f": $\epsilon_e = 0.01$, $R_{\rm eff} = 1$ km; "g": $\epsilon_e = 0.1$, $R_{\rm eff} = 1$ km.

2.2. *Propeller-torqued spindown*

Besides spindown mechanisms due to gravitational wave radiation and to magnetospheric activity, another one could also work when a sub-millisecond pulsar is in a propeller phase. Propeller torque acts on a pulsar through magnetohydrodynamical (MHD) coupling at magnetospheric boundary with Alfvén radius, and accreted matter inward has to been ejected at the boundary. The matter going outward gains kinematic energy during this process, and the pulsar losses then its rotation energy.

A real difficulty to estimate quantitatively propeller-torqued spindown is to know the accretion rate, $\dot{M}$. The accreted matter could be either the debris captured during the birth of pulsars or the inter-stellar medium. Such accretion details, including the MHD coupling, have not been known with certainty yet.

3. Magnetospheric activity of sub-millisecond pulsars

The potential drop in the open-field-line region is essential for the magnetospheric activity of sub-millisecond pulsars. In case of approximately constant $\mu_{\rm m}$, the potential drop between the center and the edge of a polar cap can be expressed as[8],

$$\phi = \frac{64\pi^3}{3c^2}\bar{B}\mu_m R_{\rm eff}^3 P^{-2} \simeq 2.2\times 10^{13}({\rm volts})\ \mu_{m-6}R_{\rm effkm}^3 P_{\rm ms}^{-2}, \qquad (8)$$

where the bag constant $\bar{B} = 60$ MeV/fm$^3 \simeq 10^{14}$ g/cm^3 (i.e., $\rho/4$). It is well known that pair production mechanism is a key ingredient for pulsar radio emission. A pulsar is called to be "death" if the pair production condition can not be satisfied. Although a real deathline depends upon the dynamics of detail pair and photon production, the deathline can also be conventionally taken as a line of constant potential drop ϕ. Assuming a critical drop $\phi_c = 10^{12}$ volts, a sub-millisecond pulsar with $P = 0.1$ ms could still be active even its radius is only 0.08 km, in case of $\mu_{m-6} = 1$.

The potential drop in the open field line region would be much higher than that presented in Eq.(8) if the effect of inclination angle is included[20]. Note that this conclusion favors the magnetospheric activity of sub-millisecond pulsars.

Part of the power of the magnetospheric activity is in the electromagnetic emission of radio band. If the radio power accounts for $\eta \approx 10^{-10\sim-5}$ times of the magnetospheric activity[21], the radio luminosity is then, from Eq.(5),

$$L_{\rm radio} = \eta\frac{512\pi^6}{27c^3}\mu_m^2\rho^2 R_{\rm eff}^6 P^{-4} \simeq 1.1\times 10^{32}\eta({\rm erg/s})\ \mu_{m-6}^2 R_{\rm effkm}^6 P_{\rm ms}^{-4}, \quad (9)$$

where $\rho = 4\times 10^{14}$ g/cm^3. This power is in the same order of that of normal radio pulsars observed[21] even the radius of sub-millisecond pulsars is less than 1 km. Although sub-millisecond pulsars could be radio loud, one needs a very short sampling time, and has to deal with then a huge amount of data in order to find a sub-millisecond pulsar. Due to its large receiving area and wide scanning sky, the future radio telescope, *FAST* (five hundred meter aperture spherical telescope), to be built in Yunnan, China, might uncover sub-millisecond radio pulsars.

4. Conclusions and discussions

We show that sub-millisecond pulsars should be in Jacobi ellipsoidal figures of equilibrium. It is addressed that the spindown of sub-millisecond pulsars would be torqued dominantly by gravitational wave radiation, and that such pulsars may not spin down to super-millisecond periods via gravitation wave radiation during their lifetimes if they are extremely low mass bare strange quark stars. It is possible, based on the calculation of Fig. 1, that isolated super-millisecond pulsars could be quark nuggets (stars) born during cosmic QCD phase separation (via spindown in Hubble timescale).

The radio luminosity of sub-millisecond pulsars could be high enough to be recorded in advanced radio telescopes (e.g., the future *FAST* in China). Sub-millisecond pulsars would not be likely to be normal neutron stars. It could then be a clear way of identifying quark stars as the real nature of pulsars to search and detect sub-millisecond radio pulsars.

Where to find sub-millisecond radio pulsars? This is a question related to how sub-millisecond pulsars origin. Actually, a similar question, which was listed as one of Lorimer-Kramer's 13 open questions in pulsar astronomy[21], is still not answered: How are isolate millisecond pulsars produced? More further issues are related: Should sub-millisecond pulsars be in globular clusters? Can millisecond and sub-millisecond pulsars form during cosmic QCD separation? How to estimate an initial period of quark star in this way? Could AIC (accretion-induced collapse) of white dwarfs produce pulsars with periods < 10 ms? A recent multi-dimensional simulations of AIC was done[22] in the normal neutron stars regime, but a quark-star version of AIC simulation is interesting and necessary. Another interesting idea is: could low mass quark stars form during the fission of a progenitor quark star? (Quark matter produced in this way might be chromatically charges?). All these ideas are certainly interesting, and could *not* be ruled out simply and quickly by first principles.

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