

Pulsar electrodynamics inferred from frequency-dependent circular polarization diversity

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ABSTRACT

Context. The nature of coherent radio emission remains challenging even after more than half a century of pulsar discovery. However, the general consensus asserts that single-pulse observations are essential for probing the magnetospheric dynamics, especially with the largest single-dish telescope presently available: Five-hundred-meter Aperture Spherical radio Telescope (FAST).

Aims. This paper is aimed at explaining the observed diversity of single pulse circular polarization and constraining the multiplicity and Lorentz factor of pulsar magnetospheric plasma, with the mode coupling model in the limiting polarization region.

Methods. Assuming that circular polarization only comes from wave mode coupling, we applied a Bayesian analysis to the FAST observed single pulse circular polarization spectra, including a numerical solution for the wave-mode coupling equations. We also analyzed the posterior probability distribution functions of the parameters.

Results. Although the model failed to quantitatively fit most circular polarization spectra, the circular polarization of different frequency evolution was successfully reproduced. For the three chosen pulsars, the Bayesian analysis constrained the multiplicity as approximately $10^0 \sim 10^2$, along with a Lorentz factor of approximately $10^{0.5} \sim 10^2$.

Conclusions. Pulsar circular polarization could be induced by wave mode coupling. The plasma flow responsible for coherent radio emission carries only a very small fraction of the pulsar spin-down energy loss.

Key words. plasmas – stars: neutron – pulsars: general

1. Introduction

Even after nearly six decades since the discovery of pulsars, the physics of cosmic coherent radio emission is not fully understood. This is likely due to the nature of pulsars, particularly with regard to pulsar surfaces (e.g., Xu et al. 2026). New hot topics, such as fast radio bursts (Sironi et al. 2021; Wang et al. 2022) and long-period radio transients (Qu & Zhang 2025; Zhong & Most 2026), could make this puzzling problem even more complex. Nevertheless, advanced facilities such as China's FAST help to resolve these big questions by providing high-quality pulsar data for probing pulsar magnetospheres. In these environments, the physical processes have not been fully clarified and the related parameters have not been determined with a high level of confidence. The circular polarization behaviors of single pulses detected by FAST are the focus of this paper to aid in our understanding of pulsar electrodynamics.

Linearly polarized radiation predominates in the pulsar magnetosphere, as coherent radio emission typically arises from one of two orthogonal linear polarization modes. Consequently, the circular polarization component is essential for understanding the underlying radiation mechanisms.

The observed circular components is, in fact closely, associated with pulsar magnetospheric properties and its generation mechanism is based on the modeling of emission mechanisms or wave propagation processes (e.g., Arons & Barnard 1986; Kazbegi et al. 1991; Lyubarskii & Petrova 1998; Melrose & Luo 2004; Benáček et al. 2026). Observational studies have already reported a great diversity among the circular polarization phenomenology of pulsar integrated profiles (Han et al. 1998; You & Han 2006), pulsar single pulses (Karastergiou et al. 2003), and fast radio bursts (Jiang et al. 2024). However, the comparisons between observations and theories are usually qualitative (or just on an order of magnitude), which stands in the way of an effective elimination of unhelpful models. A quantitative comparison between pulsar emission models and observational data would not only help us better understand the advantages and disadvantages of certain models, but it would also provide plausible ways of diagnosing pulsar magnetospheric plasmas, as some pioneer research has shown (Petrova 2003; Galishnikova et al. 2020). These studies have been carried out on the basis of integrated profiles and we hope to achieve good-quality diagnostics with single pulses.

In this paper, we use the mode coupling model in the limiting polarization region of pulsar magnetosphere to fit the longitude-resolved circular polarization spectra of single pulses observed by FAST. The role of the limiting polarization effect

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has been highlighted in studies of radio pulsar polarization patterns (Cheng & Ruderman 1979; Lyubarskii & Petrova 1998; Wang et al. 2010; Andrianov & Beskin 2010; Beskin & Philippov 2012). Certainly, the wave modes in strongly magnetized plasma are highly linear polarized (e.g., Arons & Barnard 1986). However, in the limiting polarization region of a magnetosphere, the inhomogeneity of the ambient plasma is strong enough to make the wave modes couple with each other during propagation. When the magnetic field orientation changes along the ray trajectory, the original linear polarization of one single wave mode fails to catch up with the magnetic field, which results in the emergence of another wave mode and finally induces a circular polarization component in the outgoing wave.

The paper is structured as follows. In Section 2, we describe the wave mode coupling model in the limiting-polarization region of the pulsar magnetosphere. We clarify the physical quantities and the assumptions of the model. Section 3 contains the information of the FAST data we use and how we process the data. In Section 4, we introduce our settings for the computational methods, including Bayesian analysis and numerical integration. Section 5 gives the results of Bayesian analysis. Section 6 presents a discussion of the results, including the implications for magnetospheric dynamics and particle accelerations. Section 7 concludes our findings.

2. The wave mode coupling model

The polarization transfer equations of waves propagating in astrophysical plasmas have been studied extensively (e.g., Sazonov 1969; Jones & O'Dell 1977). The wave-mode coupling in an inhomogeneous plasma, as an example, was first studied for the Earth ionosphere (e.g., Budden 1952) and later applied to investigations of astrophysical phenomena (e.g., Zheleznyakov 1996). Budden (1952) used the parameter ψ to describe how large the coupling is

$$\psi = \frac{i}{R_0^2 - 1} \frac{dR_0}{dh}, \quad (1)$$

where $R_0 = E_{O,y}/E_{O,x}$ describe the polarization of the ordinary (O) mode and h is a coordinate along the ray trajectory. A critical value of ψ is $k|n_O - n_E|/2$, where k is the wave number, while $n_{O,E}$ represents the refraction indices of the O mode and extraordinary (E) mode. When ψ increases, coupling becomes significant. In the case of pulsar magnetosphere, ψ is on the order of $1/r$, where r is the distance from neutron star centroid. So the criterion for mode coupling could be converted to

$$\frac{\omega}{c} |n_O - n_E| r \sim 1. \quad (2)$$

Equation (2) defines a limiting polarization radius, r_{pl} , which has been discussed in detail by Barnard (1986).

For the mode coupling equations, we used the formulae established by Lyubarskii & Petrova (1998) (i.e., Budden-Zheleznyakov approach in Beskin & Philippov 2012), expressed as

$$\begin{cases} \frac{da_x}{dz} = -iR[(b_x + q_y)^2 a_x + (b_x + q_y)(b_y - q_x)a_y], \\ \frac{da_y}{dz} = -iR[(b_x + q_y)(b_y - q_x)a_x + (b_y - q_x)^2 a_y], \end{cases} \quad (3)$$

where the parameter R contains the plasma parameters via

$$R = \frac{\omega_p^2}{2\omega c \gamma^3 (1 - \beta_z)^2} = \frac{4\pi k n_{GJ}(z) e^2 / m_e}{\omega c \gamma^3 (1 - (\mathbf{\Omega} \times \mathbf{r})_z / c - v_b b_z / c)^2}. \quad (4)$$

Table 1. Physical quantities in Eqs. (3)–(8).

Quantity	Meaning
$\mathbf{a}$	complex electric vector
$\mathbf{b}$	direction vector of magnetic field
$\mathbf{q}$	$\mathbf{b} \times (\mathbf{\Omega} \times \mathbf{r})/c$
$\mathbf{\Omega}$	angular velocity of pulsar rotation
ω	angular frequency of the propagating wave
γ	Lorentz factor of $e^\pm$
v_b	$e^\pm$ velocity component along magnetic field lines
n_{GJ}	$ \rho_{GJ} /e$, Goldreich-Julian plasma number density (Goldreich & Julian 1969)
κ	multiplicity
m_e	electron mass
c	vacuum speed of light
μ	magnetic dipole moment of the pulsar
α	inclination angle
ζ	angle between $\mathbf{\Omega}$ and $\mathbf{k}$
δ	angle between μ and $\mathbf{k}$
z_0	starting point of integration
z_1	ending point of integration

In Eqs. (3) and (4), the z axis is chosen to be along the ray trajectory. The meanings of physical quantities are presented in Table 1.

The derivation of Eq. (3) could be found in Lyubarskii & Petrova (1998) and Petrova & Lyubarskii (2000). This form of coupling equations are valid under the following assumptions:

1. Highly relativistic plasma: $\gamma^2 \gg 1$;
2. High frequency waves: $Rc/\omega \ll 1$ (similar discussions are made in Beskin et al. 1988);
3. The plasma is cold, with single Lorentz factor;
4. The plasma is strictly frozen on the magnetic field and the velocity can be written as $\mathbf{v} = v_b \mathbf{b} + \mathbf{\Omega} \times \mathbf{r}$;
5. Refraction is neglected in the limiting polarization region ($n_{O,E} \sim 1$).

In Lyubarskii & Petrova (1998), $\mathbf{b}$ and $\mathbf{q}$ could be derived in a simple form under two extra assumptions for the magnetic field and geometrical parameters:

1. $\delta \ll \zeta$;
2. The region of interest (limiting polarization region) is far enough from the neutron star surface, and rotation dominates the change of magnetic field along the ray trajectory.

Following Lyubarskii & Petrova (1998), we set the x axis along the initial $\mathbf{b} - (\mathbf{b} \cdot \mathbf{k}/k)\mathbf{k}$, in other words, we set the initial $b_y = 0$. The initial $\mathbf{b}$ is $(\delta/2, 0, 1 - \delta^2/8)$, the orientation of rotation axis is $\hat{\mathbf{\Omega}} = (\sin \zeta \sin \theta, \sin \zeta \cos \theta, \cos \zeta)$, and the orientation of magnetic dipole is $\hat{\mu} = (-\sin \delta, 0, \cos \delta)$. The angle θ satisfies $\hat{\mathbf{\Omega}} \cdot \hat{\mu} = \cos \alpha$. Then the rotation matrices are introduced as

$$\mathcal{M}_z(\theta) = \begin{pmatrix} \cos \theta & -\sin \theta & \\ \sin \theta & \cos \theta & \\ & & 1 \end{pmatrix} \quad \mathcal{M}_x(\zeta) = \begin{pmatrix} 1 & & \\ & \cos \zeta & -\sin \zeta \\ & \sin \zeta & \cos \zeta \end{pmatrix}. \quad (5)$$

Next, we have the evolution of $\mathbf{b}$ via

$$\begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix} = \mathcal{M}_z^{-1}(\theta) \mathcal{M}_x^{-1}(\zeta) \mathcal{M}_z(\Omega t) \mathcal{M}_x(\zeta) \mathcal{M}_z(\theta) \begin{pmatrix} \delta/2 \\ 0 \\ 1 - \delta^2/8 \end{pmatrix}, \quad (6)$$

where $t = \Omega(z - z_0)/c$. Expanding the expressions to the lowest order of Ωt , the components of $\mathbf{b}$ and $\mathbf{q}$ used in Eqs. (3) and (4) can then be written (for the case of $\alpha > \zeta$) as

$$\begin{cases} b_x(z) = \frac{\delta}{2} - \Omega \frac{z - z_0}{c} \sin \zeta \frac{\sqrt{\delta^2 - (\alpha - \zeta)^2}}{\delta} \text{sign}((\Omega \times \mu) \cdot \mathbf{k}), \\ b_y(z) = -\Omega \frac{z - z_0}{c} \sin \zeta \frac{\alpha - \zeta}{\delta}, \\ b_z(z) = \sqrt{1 - b_x^2 - b_y^2}, \end{cases} \quad (7)$$

$$\begin{cases} q_x(z) = \frac{\Omega z}{c} \sin \zeta \frac{\alpha - \zeta}{\delta}, \\ q_y(z) = -\frac{\Omega z}{c} \sin \zeta \frac{\sqrt{\delta^2 - (\alpha - \zeta)^2}}{\delta} \text{sign}((\Omega \times \mu) \cdot \mathbf{k}), \\ q_z(z) = 0. \end{cases} \quad (8)$$

Given the geometry parameters (α, ζ, δ) , plasma parameters (κ, γ) , rotation parameters $(\Omega, \dot{\Omega})$, and integration limits (z_0, z_1) , we can calculate $a_{x,y}$ at a frequency of $\nu = \omega/2\pi$. Thus, we can calculate Stokes parameters (I, Q, U, V) , which can then be compared with the observational data.

3. Data reduction

Among the parameters noted in the previous paragraph, $(\Omega, \dot{\Omega})$ can be determined once we have access to data from pulsars that are well timed. A number of other parameters could be determined by comparing the observed polarization with modeled polarization. Since mode coupling makes the polarization position angle (PA) deviate from the rotating vector model (RVM, Radhakrishnan & Cooke 1969), while producing circular polarization (e.g., Petrova 2003; Beskin & Philippov 2012), a full fitting should include both PA and V .

To reduce parameters, we divided the fitting procedure into two steps. First, we derived the geometry parameters (α, ζ, δ) from the RVM fitting of integrated pulse profiles. Second, we fixed the geometry parameters and derived the plasma parameters by fitting the circular polarization. In this work, we chose data samples from pulsars whose PA curves were well described by the RVM and we used their single pulses for study. We note that the propagation process occurs in one single pulse and circularly polarized signals with opposite handedness undergo mutual cancellation when translating them into averaged profiles.

Employing the catalog in Wang et al. (2023), we chose three pulsars that have FAST released data¹: B0301+19, J0631+1036, and B0656+14 (monogem). Their basic parameters are listed in Table A.1, and their locations on the P - $\dot{P}$ diagram are shown in Figure A.1 in Appendix A. The data processing (including folding, RFI mitigation, saturation mitigation, calibration, and timing) was performed with the software packages DSPSR (van Straten & Bailes 2011), PSRCHIVE (Hotan et al. 2004), and TEMPO2 (Hobbs et al. 2006). We follow the PSR/IEEE convention for the definition of Stokes parameters (van Straten et al. 2010). We used a Bayesian method to fit for the rotation measure (RM) of Faraday rotation (Luo et al. 2020) from the calibrated data. For the linear polarization intensity, L , total polarization intensity, P , and ellipticity angles, we corrected their biases using the formulae in Cao et al. (2025). The RVM fitting results

are also included in Table A.1 and the fitting details are given in Appendix A.

To improve the signal-to-noise ratio (S/N), we merged the 4096 (or 1024) frequency channels of the data into eight sub-bands. We calculated V/P and $\sigma_{V/P} = \sqrt{P^2 \sigma_V^2 + V^2 \sigma_P^2} / P^2$ for the fitting. The pulses used in our study were chosen to satisfy that the minimum $\sigma_{V/P}$ in the pulse profile is smaller than 0.2. The number of pulses chosen for three pulsars are given in Table A.1. For each pulse, we chose three pulse longitudes $\phi_{1,2,3}$, where $\sigma_{V/P}$ s are smallest, to get their V/P - ν spectra. Finally, we had $3 \times 8 = 24$ data points per pulse for the fitting procedure.

4. Computational settings

If we denote the mode coupling model as $\mathcal{M}$, parameters as Θ , and data as $\mathcal{D}$, then we can use the Bayes theorem to compute the posterior probability distribution function (PDF) via

$$P(\mathcal{M}, \Theta | \mathcal{D}) \propto P(\mathcal{M}, \Theta) P(\mathcal{D} | \mathcal{M}, \Theta), \quad (9)$$

where $P(\mathcal{M}, \Theta)$ is the prior PDF and $P(\mathcal{D} | \mathcal{M}, \Theta)$ is the likelihood function. The likelihood function could be written as

$$P(\mathcal{D} | \mathcal{M}, \Theta) \propto \exp \left(-\frac{1}{2} \sum_{\phi, \nu} \frac{[(V/P)_{\mathcal{D}}(\phi, \nu) - (V/P)_{\mathcal{M}, \Theta}(\phi, \nu)]^2}{\sigma_{V/P}^2(\phi, \nu)} \right). \quad (10)$$

Our $P(\mathcal{D} | \mathcal{M}, \Theta)$ contains the numerical solution of Eq. (3), where we used package *odes* (Malengier et al. 2018) for solving. We used the python package *emcee* (Foreman-Mackey et al. 2013) (version: 3.1.6) for Markov chain Monte Carlo (MCMC) computations.

We set five parameters in one fitting: κ , $\gamma/\kappa^{1/3}$, and $z_{0,(1,2,3)}$ for three chosen longitudes. Since the three chosen longitudes tend to be close to each other, we set κ and γ same for them (i.e., independently of longitudes). We did not use γ directly as parameter because, in practice, we find that κ and γ are sometimes highly correlated. The prior PDFs of the five parameters are chosen to be $\lg(\kappa) \sim \mathcal{U}(0, 7)$, $\lg(\gamma) \sim \mathcal{U}(0.5, 3)$, and $\lg(z_{0,(1,2,3)}/R_{\text{NS}}) \sim \mathcal{U}(1, 3)$, where $\lg$ is the base-10 logarithmic function and $\mathcal{U}(a, b)$ denotes a uniform distribution in the range (a, b) . In addition, R_{NS} is the neutron star radius and we set R_{NS} to 10 km. The details of our settings in solving the Bayesian analysis and Eq. (3) are given below.

4.1. The prior PDFs

The choice of the prior PDFs' ranges firstly satisfies $\gamma^2 \gg 1$ and $\kappa \geq 1$. We set wide ranges for γ and κ to explore the parameter space. Since the validity of Eq. (3) requires high frequency waves, we put an extra constraint on the prior PDFs via

$$R(\kappa, \gamma, z_0)c/\omega_{\min} < 0.1, \quad (11)$$

where $\omega_{\min} = 2\pi \cdot 1000$ MHz. We set 128 Markov chains for running the algorithm. To generate the required prior PDFs, we used *random* of *numpy* to generate random values within the ranges we set. If the generated random values satisfy Eq. (11), they are sampled.

4.2. From the pulse longitude ϕ to δ

With ϕ_0 obtained from the RVM fitting, the angle between wave vector and magnetic axis, δ , could be derived for each

¹ Their further information could be found on https://fast.bao.ac.cn/observation_log/search

longitude, ϕ ,

$$\cos \delta = \hat{k} \cdot \hat{\mu} = \sin \alpha \sin \zeta \cos(\phi - \phi_0) + \cos \alpha \cos \zeta. \quad (12)$$

4.3. Considering z_0 and z_1

In Petrova (2003), there were two additional assumptions employed when solving the mode coupling equation: The initial values were set at $z \sim 0$, or $z \ll z_p$. Furthermore, the final states refer to $z \rightarrow +\infty$. In practice, we have found that the solution of Eq. (3) is sensitive to z_0 ; therefore, we set z_0 as a parameter to be fitted for each pulse longitude, while z_1 is the altitude where the magnetic field is too weak to sustain the fourth assumption in Eq. (3) and could be estimated with the following equation (e.g., Petrova 2001):

$$\gamma\omega(1 - v_b(z_1)b_z(z_1)/c) = 0.1 \cdot \frac{eB(z_1)}{m_e c} = 0.1 \cdot \frac{eB_{\text{surf}}R_{\text{NS}}^3}{z_1^3 m_e c}, \quad (13)$$

where B_{surf} is the surface magnetic field strength, which could be derived from P and $\dot{P}$ by assuming that magnetic dipole radiation dominates the pulsar spin-down (e.g., Lorimer & Kramer 2012). Before running MCMC, we calculated z_1 using Eq. (13), of all $(\kappa, \gamma/\kappa^{1/3}, z_0, \phi)$ in the posterior PDFs. We chose the smallest z_1 (shown in Table A.1) as the ending point of all integrations during MCMC. Since z_1 actually depends on the pulse longitude, ϕ , we only give an approximate value in Table A.1.

4.4. Initial polarization state at z_0

The polarization state at z_0 refers to the pure linear polarization as a decoupled solution (Eq. 3.3 in Lyubarskii & Petrova 1998) of Eq. (3). We chose the initial polarization to be O mode and expressed as

$$\frac{a_x(z_0)}{a_y(z_0)} = \frac{b_x(z_0) + q_y(z_0)}{b_y(z_0) - q_x(z_0)}. \quad (14)$$

In practice, we fixed $a_x(z_0) = 1$.

4.5. emcee settings

We used the EnsembleSampler function of emcee for our computation. To achieve a quicker convergence, the update strategy of the coordinates of walkers are set to be a mixture of different moves: 60% of StretchMove (Foreman-Mackey et al. 2013), 20% of differential evolution Move (DEMove, Nelson et al. 2014), and 20% of snooker move using differential evolution (DESnoockerMove, ter Braak & Vrugt 2008). Details on these moves can be found in the emcee documentation². After some tests on the data, we finally set 10 000 steps for running and we omitted the first 6000 steps as burn-in steps, applicable to all pulses.

5. Results

Before applying the algorithm to real data, we performed a set of tests on simulated data. More details are given in Appendix B.

We applied the algorithm to pulses from the three chosen pulsars. For each pulse, we were able to obtain the maximum-likelihood parameters, Θ_m , according to the resulting posterior PDF of Bayesian analysis. The results could be divided into two

parts: the theoretical curve with a Θ_m versus data curve and the posterior PDF of parameters.

Let us first look at the pulses relatively well-described with the mode coupling model. An example (#751 of B0301+19) is shown in Figure 1, and more examples are shown in Figure C.2. The mode coupling model could reproduce the diversity of circular polarization spectra: the circular polarization could be left-handedness (#683 of B0301+19) or right-handedness (#751 of B0301+19) across the frequency band, be approaching 80% (#751 of B0301+19) or be close to zero (#35 of J0631+1036), and change its handedness when frequency changes (#1299 of B0301+19). The V/P versus ν curves could also be significantly different between neighbor phases (#1299 of B0301+19), under fixed κ and γ .

A badly fitted pulsation (#2164 of B0301+19) is also shown in Figure 1 (more in Figure C.2). In those V/P - ν curves, V/P varies vastly with frequency, but the mode coupling model only reproduces a part of them (e.g., the second plot of #2164 of B0301+19), or even totally fails to reproduce (#1235 of B0301+19). It is worth noting that the chosen phases in these two pulses exhibit PA jumps, which may indicate a more complex mode mixing than pure-mode coupling described by Eq. (3). We discuss this further in Section 6.

Three examples of posterior PDFs are presented in Figures 2 and C.1. The shapes of the posterior PDFs vary manifestly among different pulses: some are strongly multi-peaked (e.g., #683 of B0301+19), while some have much simpler shapes that are closer to single-peaked (e.g., #751 and #2164 of B0301+19). This is also reflected in the posterior PDFs of simulated data: in Figure B.1 of Appendix B, we see that z_3 is more tightly constrained than z_1 and z_2 . We used a clustering algorithm with hdbscan³ package to pick out three peaks in the posterior PDF of #683 of B0301+19, and to compare the modeling V/P - ν with the data points. The results are shown in Figure C.1 of Appendix C, indicating that different groups of parameters could lead to similar circular polarization spectra.

To evaluate the goodness of fitting, we calculated the reduced chi-square χ_r^2 of every pulse,

$$\chi_r^2 = \frac{1}{24 - 5} \sum_{\phi, \nu} \frac{[(V/P)_{\mathcal{D}}(\phi, \nu) - (V/P)_{\mathcal{M}, \Theta_m}(\phi, \nu)]^2}{\sigma_{V/P}^2(\phi, \nu)}. \quad (15)$$

An evident conclusion could be drawn from the χ_r^2 distribution (Figure 3): the mode coupling model is not accurate enough to quantitatively explain the circular polarization of most chosen pulses, with $\lg \chi_r^2$ significantly deviating from 0.

To explore whether the properties of the V/P - ν curve affect the goodness of parameter constraint, first we drew a correlations between the standard deviations, $\sigma_{P, \Theta}$, of the posterior PDFs of five parameters and the largest absolute V/P values ($|V/P|_{\text{max}}$). For each pulse, we also calculated the standard deviations of V/P values on each longitude and obtained the largest one $\sigma(V/P)_{\text{max}}$ out of three longitudes. The correlations between $\sigma_{P, \Theta}$ and $\sigma(V/P)_{\text{max}}$ are also plotted. The results for B0301+19 are shown in Figure 4. A larger circular polarization fraction or a higher varying circular polarization spectrum tend to constrain the parameters more tightly. We quantified the monotonic dependence between $|V/P|_{\text{max}}$ (and $\sigma(V/P)_{\text{max}}$) and $\sigma_{P, \Theta}$ using the Spearman rank correlation coefficient, ρ . To account for the nonuniform sampling in A, we assessed the statistical significance via a permutation test, in which the $\sigma_{P, \Theta}$ values are randomly shuffled while keeping $|V/P|_{\text{max}}$ (and $\sigma(V/P)_{\text{max}}$) fixed.

² <https://emcee.readthedocs.io/en/stable>

³ <https://hdbscan.readthedocs.io/en/latest/>

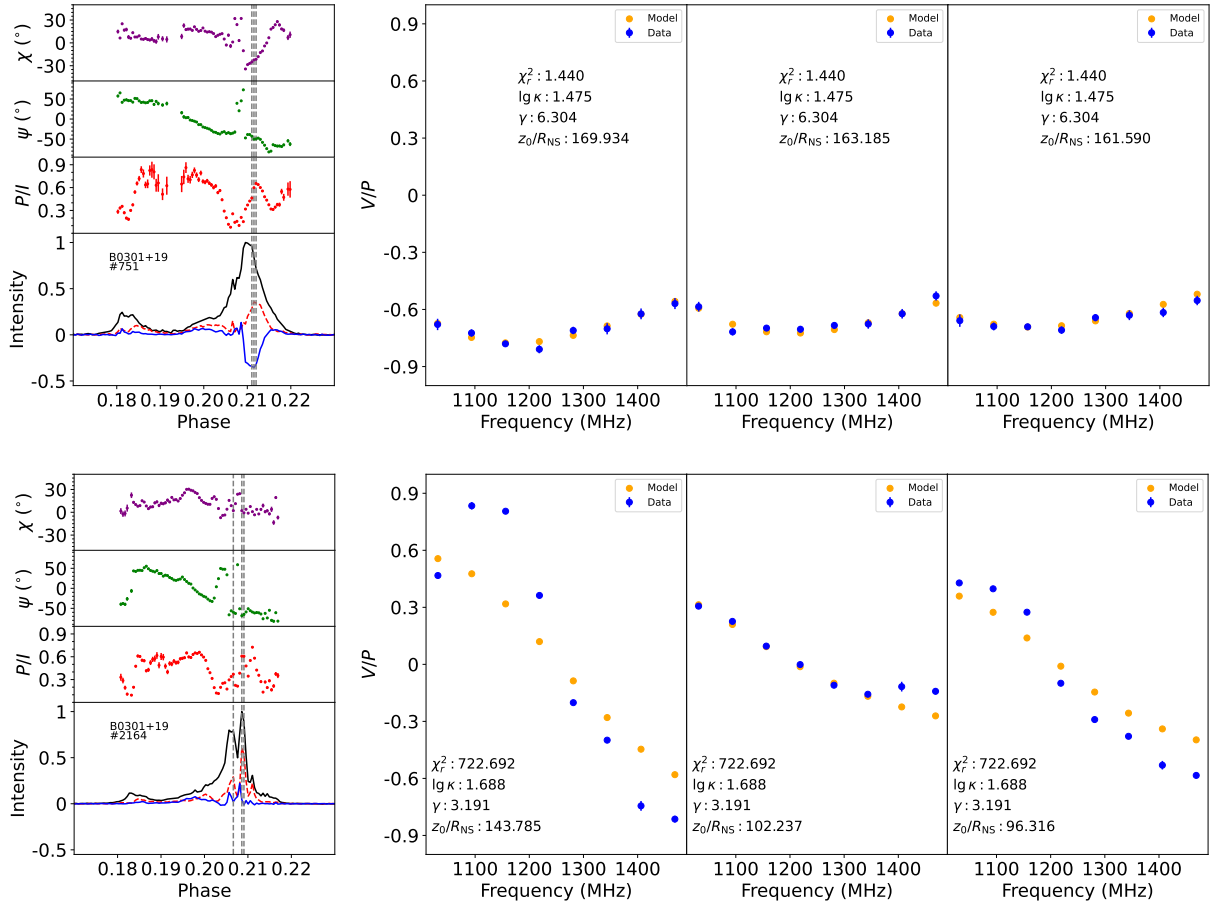


Fig. 1. Two pulses (6 V/P - ν curves) with fitting results. The pulsar name and the pulse number after # are marked in every figure. For every subfigure, the left panels are polarization profiles of single pulses, and the right plots are V/P - ν curves of three chosen pulse longitudes (blue dots with errorbars: observation data points; orange dots: modeling points with maximum-likelihood parameters). The maximum-likelihood parameters, and the χ_r^2 s for fitting the three phases are marked out in the right plots. In the Intensity panel: Black line shows the total intensity (I); red dashed line shows the linear polarization intensity ($L = \sqrt{Q^2 + U^2}$); blue line shows the circular polarization intensity (V). In addition, I , L , and V are normalized by the maximum total intensity of the respective profiles. The horizontal axis is the pulse phase (0 to 1 in a period). Red dots in the P/I panel with errorbars are the polarization degrees. Green dots in the ψ panel with errorbars are the polarization position angles (PA, $\psi = 0.5 \arctan(U/Q)$). Magenta dots in the χ panel with errorbars show the ellipticity angel (EA, $\chi = 0.5 \arcsin(V/\sqrt{Q^2 + U^2 + V^2})$).

We used a p-value to describe the probability of achieving a more negative ρ in the permutation test. The p-value p is computed as $(k + 1)/(N_{\text{perm}} + 1)$, where k is the number of permuted samples yielding a correlation coefficient more extreme than the observed one, and N_{perm} is the number of times of permutation. The ρ and p are marked in all panels of Figure 4. The observed correlation is found to be significantly more negative than expected under the null hypothesis of no correlation.

The distributions of Θ_{ms} s, separated for three pulsars, are shown in Figure 3. The fitting results are different for them. The distribution of κ for J0631+1036 is peaked at the lower limit 10^0 , while the distribution of γ for B0656+14 is even more concentrated around the lower limit value $10^{0.5}$. Regarding what is shown in Figure 3, the χ_r^2 distribution of B0656+14 generally deviates from $\chi_r^2 = 10^0$, the model fails to quantitatively describe the chosen pulses of B0656+14.

6. Discussions

6.1. Diversity of single pulse circular polarization

The highly variable single pulse morphology hints at the variations among the magnetospheric parameters between pulse and

pulse, as well as the variety of parameters, leading to the diversity of single pulse circular polarization. The V/P change of both absolute value and handedness could be attributed to the change of the amplitude ratio and phase lag between a_x and a_y . The model fails to quantitatively explain all pulses; this is understandable since the model is greatly simplified.

The goodness of the parameter constraint depends on the V/P - ν curve properties. To understand this phenomenon from the model side, we calculated the V/P - ν curves with a $20 \times 20 \times 20$ grids of values for $\lg \kappa \in (0, 7)$, $\lg \gamma \in (0.5, 3)$, and $\lg(z_0/R_{NS}) \in (0, 3)$. Then, for each pair of $(\lg \kappa, \lg \gamma)$, we found the maximum $|V/P|$ and the maximum $\sigma(V/P)$, and plotted them: the result is shown in the right two plots of Figure 4. We can conclude that larger $|V/P|_{\text{max}}$ and larger $\sigma(V/P)_{\text{max}}$ have less of a parameter degeneracy than the smaller ones. They require the amplitude ratio and phase lag between a_x and a_y to simultaneously be in an appropriate range.

6.2. The effect of orthogonal modes mixture on V/P - ν curves

As already pointed out in Section 5, the two badly fitted cases in Figures 1 and C.2 are associated with 90° PA jumps. The PA jumps indicate the incoherent (or

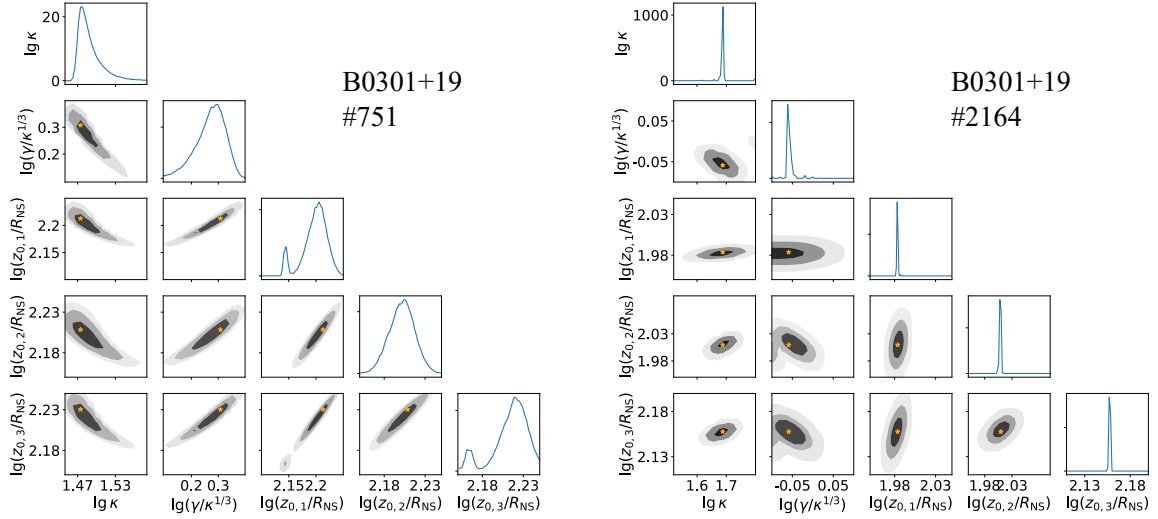


Fig. 2. Posterior PDFs of two pulsars in Figure 1. The pulsar names and pulse numbers are marked in each plot. The blue curves are marginal PDFs of parameters and the grey contours are 2D PDFs of each two parameters. The contour profiles mark the credibility regions of probabilities 50%, 80%, and 94%. The orange stars represent the maximum-likelihood parameters.

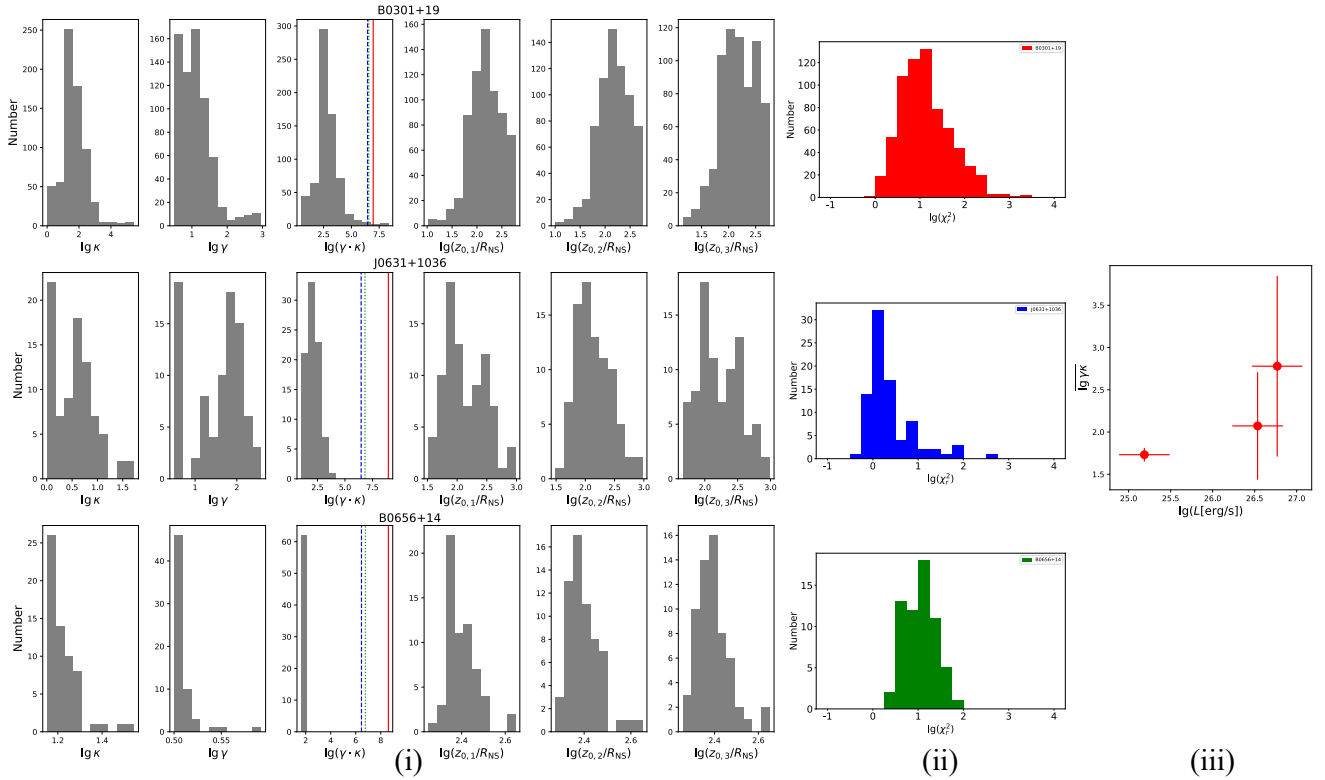


Fig. 3. (i) Distributions of maximum-likelihood parameters of MCMC results for three pulsars. The third panels of three plots are distributions of multiplicity times Lorentz factor (logarithmic). In each $\lg(\gamma \cdot \kappa)$ distributions: the red vertical line represents the maximum potential drop in polar cap region times the unit charge, e , divided by electron rest energy $m_e c^2$; the blue vertical dashed line is the inner gap potential drop $\times e/(m_e c^2)$ given in Ruderman & Sutherland (1975); the green vertical dotted line is the charge starvation zone potential drop $\times e/(m_e c^2)$ given in Arons & Scharlemann (1979). For details, see Section 6.4. (ii) Common logarithmic values of reduced chi-square distributions for the fittings of three pulsars, in the form of histograms. For the horizontal axis, The $(-1, 4)$ range is divided into 20 bins. (iii) The correlation between averaged $\kappa\gamma$ and radio luminosity L for three pulsars. For details please refer to Section 6.4.

partially coherent) superposition of orthogonal polarization modes (OPMs) (e.g., Cheng & Ruderman 1979; Karastergiou et al. 2005; Oswald et al. 2023). In our computation, we assumed that before propagating into the limiting polarization region, the wave is a pure O-mode. This assumption

may break down at PA-jumping longitudes for two plausible reasons: (1) the amplitude ratio of OPMs is expected to be frequency-dependent (Petrova 2001) and (2) the partially coherent superposition of OPMs itself also induces circular polarization components (Dyks 2017; Oswald et al. 2023;

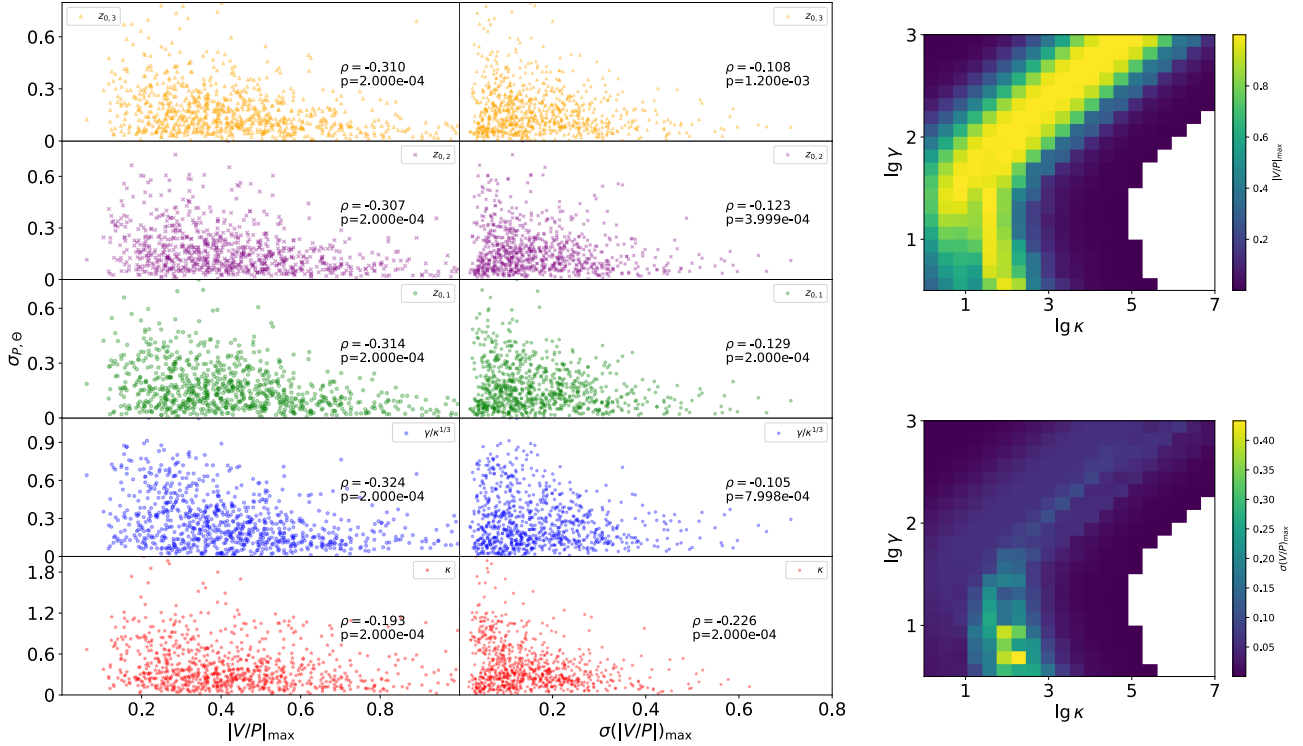


Fig. 4. Left two plots: Correlations between standard deviations of posterior PDFs of five modeling parameters and the maximum circular polarization fractions in pulses (left), or the maximum standard deviations of circular polarization fraction spectra in pulses (right). The Spearman rank correlation coefficients, ρ , and the p-value of permutation tests are marked in all panels. For details, see the main text. Right two plots: Maximum $|V/P|$ and the maximum $\sigma(V/P)$ calculated from the mode coupling model, for pairs of $(\lg \kappa, \lg \gamma)$. The calculation details are given in the text. The blank regions in the plots are the parameter spaces, where $Rc/\omega \ll 1$ is violated.

Cao et al. 2025); in addition, the frequency dependence of V/P could be more complex than what could be fitted by the simple mode coupling model we adopted in this work. This qualitatively explains the large χ_r^2 of the fitting of two pulses in Figure 1.

6.3. The physical meaning of z_0

Here, z_0 is the point where mode coupling becomes important. From this aspect, z_0 seems to be the limiting polarization radius defined by Eq. (2). We note that Eq. (2) is only an order-of-magnitude estimate and does not allow for an exact determination of z_0 . A more detailed modeling of pulsar magnetosphere may be needed for further discussions of z_0 .

6.4. Implications of derived plasma parameters on pulsar theories

The multiplicity κ and Lorentz factor γ values of the three pulsars are different. In particular, κ is around 10^0 to 10^2 and γ is around $10^{0.5}$ to 10^2 . Their κ values are generally in accordance with former simulations and theories of the pair cascade in the pulsar magnetospheres Daugherty & Harding (1982), Gurevich & Istomin (1985), well below the maximum multiplicity estimated by former modelings, usually around 10^5 (Beskin et al. 1993; Timokhin & Harding 2015, 2019). In addition, Petrova (2003) reported $\kappa \sim 1-100$ for PSR B0329+54, which is close to our results. While Galishnikova et al. (2020) gives $\kappa \sim 10^3$ for PSR J1906+0746. A plausible explanation for the relatively small multiplicity of some pulsars is that the radiation we measured escapes through low-density “holes” in

the highly nonuniform plasma distributions created by pair cascades, since the nonuniformity is generally suggested in simulations (e.g., Timokhin & Arons 2013; Philippov et al. 2020; Benáček et al. 2024).

The question of why our Bayesian analysis does not give larger κ and γ values might also be related to the energy supplement to the pair cascade. Overall, $\kappa n_{\text{GJ}} \gamma m_e c^2$ could be regarded as the energy density of the secondary particles and should be restricted by the acceleration processes above the pulsar polar cap. Assuming that the maximum acceleration potential drop is Φ and that the initial particle number density is n_{GJ} , we would expect to have $\kappa n_{\text{GJ}} \gamma m_e c^2 < e \Phi n_{\text{GJ}}$, i.e., $\kappa \gamma < e \Phi / m_e c^2$. We used three values of Φ for testing the inequality. The first one is the potential difference between the magnetic axis and the polar cap boundary (e.g., Beskin 2018):

$$\Phi_{\text{PC}} = \frac{1}{2} (\Omega R_{\text{NS}} / c)^2 R_{\text{NS}} B_{\text{surf}}. \quad (16)$$

The second one is the inner gap potential drop given by Ruderman & Sutherland (1975):

$$\Phi_{\text{RS}} \approx 1.6 \times 10^{12} \left(\frac{B_{\text{surf}}}{10^{12} \text{ G}} \right)^{-1/7} \left(\frac{2\pi/\Omega}{1 \text{ s}} \right)^{-1/7} \text{ V}. \quad (17)$$

The third one is the charge starvation zone potential drop given by Arons & Scharlemann (1979):

$$\Phi_{\text{RS}} \approx 5 \times 10^{12} \left(\frac{2\pi/\Omega}{0.1} \right)^{-3/8} \text{ V}. \quad (18)$$

The $\lg(e\Phi/m_e c^2)$ values are plotted in Figure 3. We also compared the power carried by plasma particles, P_e , with the rotating

energy loss rate, $\dot{E}$, of pulsars, assuming for the purposes of this study that $\dot{E}$ is equal to dipole energy loss $B_{\text{surf}}^2 \Omega^4 R_{\text{NS}}^6 / 6c^3$ via

$$\frac{P_e}{\dot{E}} = \frac{\kappa n_{\text{GI}}(z = R_{\text{NS}}) \gamma m_e c^2 \pi R_{\text{PC}}^2 c}{-I_* \Omega \dot{\Omega}} = \frac{3 \kappa \gamma m_e c^4}{e B_{\text{surf}} \Omega^2 R_{\text{NS}}^3} \sim \frac{\kappa \gamma}{\Phi_{\text{PCE}} / m_e c^2}, \quad (19)$$

where $R_{\text{PC}} = R_{\text{NS}} \sqrt{\Omega R_{\text{NS}} / c}$ is the polar cap radius and I_* is the neutron star moment of inertia. Most derived $\kappa \gamma$ values are below those $\lg(e\Phi / m_e c^2)$ values; therefore, the $P_e / \dot{E}$ ratio is small. This indicates that acceleration in the inner magnetosphere ($z \lesssim 1000 R_{\text{NS}}$) is far less efficient than the acceleration taking place in the outer magnetosphere or beyond the light cylinder.

Since radio emission usually originates from inner magnetosphere, we explored the correlation between $\kappa \gamma$ and radio luminosity. For each pulsar we chose, the radio luminosity could be estimated as

$$L \approx (S_{1400} / W_{50}) \Delta \nu D^2 2\pi (1 - \cos \pi W_{50}) \approx S_{1400} \Delta \nu D^2 \pi^3 W_{50}, \quad (20)$$

where S_{1400} is the mean flux density at 1400 MHz and $\Delta \nu = 500$ MHz is observation bandwidth, while D is the distance of pulsar from us and W_{50} is the full width at half maximum (FWHM) of the pulse profile in the form of fraction (e.g., $0.05 = 5\%$ of the period). We obtained S_{1400} and D from the PSRCAT website⁴ (Manchester et al. 2005), and set $W_{50} = 0.05$. The correlation between $\kappa \gamma$ and L is plotted in Figure 3. The vertical errorbar was calculated from the inferred $\kappa \gamma$ distribution in Figure 3, while the horizontal errorbar is set as $\lg 2$. There seems to be a positive correlation between $\kappa \gamma$ and L . This tendency is reasonable because radio emission should originate from the particles we measured, $\kappa \gamma$; namely, the inner magnetospheric plasma particles.

Our algorithm could potentially be applied to more pulsars. However, since there might be other various mechanisms contributing to the pulsar circular polarization, more detailed modeling should be carried out. Finally, we note that B0656+14 has been observed to have extended high-energy emission (pulsar wind nebula and pulsar halo, Birzan et al. 2016; Abeysekara et al. 2017). Modeling the radiation spectra of pulsar wind nebulae can also constrain the multiplicity (de Jager 2007; Spencer & Mitchell 2025). Therefore, the multiplicity of B0656+14 may be also derived from high-energy observations and be compared with the result of this study in the future.

7. Conclusions

We performed a quantitative analysis of the circular polarization of single pulses observed by FAST, based on wave-mode coupling equations in the magnetospheres of pulsars, put forward by Lyubarskii & Petrova (1998) in the limiting polarization of a pulsar magnetosphere. Furthermore, we explored the details of the model properties. We found that the model could reproduce the diversity of single pulse circular polarization and could give some constraint to the plasma parameters, multiplicity, κ , and Lorentz factor, γ . The goodness of fit and the goodness of constraint are all related to the exact shapes of circular polarization spectra curves. Our derived plasma parameters indicate that the inner magnetospheric flow power responsible for coherent radio emission is only a very small fraction of the pulsar spin-down

energy loss and the power seems to be positively correlated with spin-down rate.

Over the past fifty years, various models have been developed for explaining pulsar radio emission properties. However, quantitative studies of pulsar radio emission signals are still insufficient. Now that FAST, MeerKAT, and other facilities are producing high-quality radio pulsar data, it is now imperative to test previously established theories with high-quality data to gain a more comprehensive understanding of pulsar magnetospheres.

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References

- Abeysekara, A. U., Albert, A., Alfaro, R., et al. 2017, *Science*, **358**, 911
- Andrianov, A. S., & Beskin, V. S. 2010, *Astron. Lett.*, **36**, 248
- Arons, J., & Barnard, J. J. 1986, *ApJ*, **302**, 120
- Arons, J., & Scharlemann, E. T. 1979, *ApJ*, **231**, 854
- Barnard, J. J. 1986, *ApJ*, **303**, 280
- Benáček, J., Timokhin, A., Muñoz, P. A., et al. 2024, *A&A*, **691**, A137
- Benáček, J., Jessner, A., Pohl, M., et al. 2026, *A&A*, **707**, A316
- Beskin, V. S. 2018, *Phys. Usp.*, **61**, 353
- Beskin, V. S., & Philippov, A. A. 2012, *MNRAS*, **425**, 814
- Beskin, V. S., Gurevich, A. V., & Istomin, I. N. 1988, *Ap&SS*, **146**, 205
- Beskin, V. S., Gurevich, A. V., & Istomin, Y. N. 1993, *Physics of the Pulsar Magnetosphere* (Cambridge: Cambridge University Press)
- Birzan, L., Pavlov, G. G., & Kargaltsev, O. 2016, *ApJ*, **817**, 129
- Blaskiewicz, M., Cordes, J. M., & Wasserman, I. 1991, *ApJ*, **370**, 643
- Budden, K. G. 1952, *Proc. Roy. Soc. London Ser. A*, **215**, 215
- Cao, S., Jiang, J., Dyks, J., et al. 2024, *ApJ*, **973**, 56
- Cao, S., Jiang, J., Dyks, J., et al. 2025, *ApJ*, **983**, 43
- Cheng, A. F., & Ruderman, M. A. 1979, *ApJ*, **229**, 348
- Daugherty, J. K., & Harding, A. K. 1982, *ApJ*, **252**, 337
- de Jager, O. C. 2007, *ApJ*, **658**, 1177
- Dyks, J. 2017, *MNRAS*, **472**, 4598
- Everett, J. E., & Weisberg, J. M. 2001, *ApJ*, **553**, 341
- Foreman-Mackey, D., Hogg, D. W., Lang, D., & Goodman, J. 2013, *PASP*, **125**, 306
- Galishnikova, A. K., Philippov, A. A., & Beskin, V. S. 2020, *MNRAS*, **497**, 2831
- Goldreich, P., & Julian, W. H. 1969, *ApJ*, **157**, 869
- Gurevich, A. V., & Istomin, I. N. 1985, *Zhurnal Eksperimentalnoi i Teoreticheskoi Fiziki*, **89**, 3
- Han, J. L., Manchester, R. N., Xu, R. X., & Qiao, G. J. 1998, *MNRAS*, **300**, 373
- Hobbs, G. B., Edwards, R. T., & Manchester, R. N. 2006, *MNRAS*, **369**, 655
- Hotan, A. W., van Straten, W., & Manchester, R. N. 2004, *PASA*, **21**, 302
- Jiang, J. C., Xu, J. W., Niu, J. R., et al. 2024, *Natl. Sci. Rev.*, **12**, nwae293
- Johnston, S., Mitra, D., Keith, M. J., Oswald, L. S., & Karastergiou, A. 2024, *MNRAS*, **530**, 4839
- Jones, T. W., & O’Dell, S. L. 1977, *ApJ*, **214**, 522
- Karastergiou, A., Johnston, S., & Kramer, M. 2003, *A&A*, **404**, 325
- Karastergiou, A., Johnston, S., & Manchester, R. N. 2005, *MNRAS*, **359**, 481
- Kazbegi, A. Z., Machabeli, G. Z., & Melikidze, G. I. 1991, *MNRAS*, **253**, 377
- Lorimer, D. R., & Kramer, M. 2012, *Handbook of Pulsar Astronomy* (Cambridge: Cambridge University Press)
- Luo, R., Wang, B. J., Men, Y. P., et al. 2020, *Nature*, **586**, 693
- Lyubarskii, Y. E., & Petrova, S. A. 1998, *Ap&SS*, **262**, 379

⁴ <https://www.atnf.csiro.au/research/pulsar/psrcat/>

- Malengier, B., Kišon, P., Tocknell, J., et al. 2018, *J. Open Source Softw.*, **3**, 165
- Manchester, R. N., Hobbs, G. B., Teoh, A., & Hobbs, M. 2005, *AJ*, **129**, 1993
- Melrose, D. B., & Luo, Q. 2004, *MNRAS*, **352**, 915
- Mitra, D., Melikidze, G. I., & Basu, R. 2023, *MNRAS*, **521**, L34
- Nelson, B., Ford, E. B., & Payne, M. J. 2014, *ApJS*, **210**, 11
- Oswald, L. S., Karastergiou, A., & Johnston, S. 2023, *MNRAS*, **525**, 840
- Petrova, S. A. 2001, *A&A*, **378**, 883
- Petrova, S. A. 2003, *A&A*, **408**, 1057
- Petrova, S. A., & Lyubarskii, Y. E. 2000, *A&A*, **355**, 1168
- Philippov, A., Timokhin, A., & Spitkovsky, A. 2020, *Phys. Rev. Lett.*, **124**, 245101
- Qu, Y., & Zhang, B. 2025, *ApJ*, **981**, 34
- Radhakrishnan, V., & Cooke, D. J. 1969, *Astrophys. Lett.*, **3**, 225
- Ruderman, M. A., & Sutherland, P. G. 1975, *ApJ*, **196**, 51
- Sazonov, V. N. 1969, *Soviet Astron.*, **13**, 396
- Sironi, L., Plotnikov, I., Nättilä, J., & Beloborodov, A. M. 2021, *Phys. Rev. Lett.*, **127**, 035101
- Spencer, S. T., & Mitchell, A. M. W. 2025, *A&A*, **694**, A324
- Sun, S. N., Wang, N., Yan, W. M., & Wang, S. Q. 2025, *ApJ*, **983**, 179
- ter Braak, C. J. F., & Vrugt, J. A. 2008, *Stat. Comput.*, **18**, 435
- Timokhin, A. N., & Arons, J. 2013, *MNRAS*, **429**, 20
- Timokhin, A. N., & Harding, A. K. 2015, *ApJ*, **810**, 144
- Timokhin, A. N., & Harding, A. K. 2019, *ApJ*, **871**, 12
- van Straten, W., & Bailes, M. 2011, *PASA*, **28**, 1
- van Straten, W., Manchester, R. N., Johnston, S., & Reynolds, J. E. 2010, *PASA*, **27**, 104
- Wang, C., Lai, D., & Han, J. 2010, *MNRAS*, **403**, 569
- Wang, W.-Y., Yang, Y.-P., Niu, C.-H., Xu, R., & Zhang, B. 2022, *ApJ*, **927**, 105
- Wang, P. F., Han, J. L., Xu, J., et al. 2023, *RAA*, **23**, 104002
- Xu, Z.-H., Wang, W.-Y., Cao, S.-S., et al. 2026, *RAA*, **26**, 035014
- You, X.-P., & Han, J.-L. 2006, *Chin. J. Astron. Astrophys.*, **6**, 237
- Yuan, M., Zhu, W., Kramer, M., et al. 2023, *ApJ*, **949**, 115
- Zheleznyakov, V. V. 1996, *Radiation in Astrophysical Plasmas* (Kluwer Academic Publishers)
- Zhong, Y., & Most, E. R. 2026, *ApJ*, **999**, L2

Appendix A: Information of 3 chosen pulsars, and RVM fitting

Three selected pulsars are B0301+19, J0631+1036, and B0656+14 (monogem). Their basic parameters are listed in Table A.1 and their locations on the $P - \dot{P}$ diagram are shown in Figure A.1.

Table A.1. Parameters of chosen pulsars

Name	Period (s)	$\dot{P}$ (s/s)	N_{channel}	Obs. date (UTC)	z_1/R_{NS}	$\alpha(^{\circ})$	$\zeta(^{\circ})$	ϕ_0	$N_{\text{p,chosen}}$
B0301+19	1.39	1.30×10^{-15}	4096	20210912	~ 560	52.92	50.23	0.199	679
J0631+1036	0.288	1.05×10^{-13}	1024	20230110	~ 1040	177.3	176.9	0.750	85
B0656+14	0.385	5.49×10^{-14}	4096	20230430	~ 1090	177.6	176.7	0.769	62

Notes. N_{channel} refers to the number of frequency channels. Obs. date (UTC) indicates the observation data in UTC. α is the magnetic inclination angle, ζ is the view angle, and ϕ_0 is the phase (0 to 1) of the fiducial plane where the spin axis, magnetic axis and line-of-sight are coplanar. Finally, $N_{\text{p,chosen}}$ is the number of pulses chosen for the Bayesian analysis.

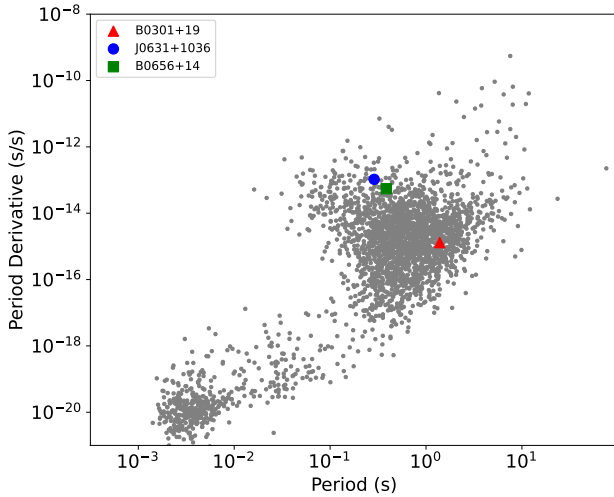


Fig. A.1. $P - \dot{P}$ diagram. The grey dots are from the PSRCAT (Manchester et al. 2005) website (<https://www.atnf.csiro.au/research/pulsar/psrcat/>). The red triangle, blue dot, and green square mark the three pulsars chosen for our study.

The RVM fitting algorithm is same as that in Cao et al. (2024) and Cao et al. (2025): for each pair of (α, ζ) , we calculate the minimum reduced chi-square (χ_r^2) by varying ϕ_0 and ψ_0 . For RVM fitting, We chose PA values from pulse longitudes where linear polarization L is significant enough: $L/\sigma_L > 10$. This basic algorithm works relatively well for J0631+1036 and B0656+14. Their χ_r^2 distributions along with integrated profiles are shown in Figure A.2.

When applying the algorithm to B0301+19, we find that the minimum χ_r^2 is always larger than 50, even when we chose only highly linearly polarized pulse bins to produce a highly linearly polarized profile and use its PA (the method used in Mitra et al. 2023 and Johnston et al. 2024). The radiative geometry of B0301+19 has been measured in former papers (Everett & Weisberg 2001; Yuan et al. 2023; Sun et al. 2025), and their results are not all the same. Although not significant in our chosen observation, B0301+19 profile has an interpulse, which is also pointed out in Yuan et al. (2023), Sun et al. (2025). The interpulse longitudes does not satisfy $L/\sigma_L > 10$. But to

better constrain the geometry, we pick two PA points within the interpulse range, and add them into the highly linearly polarized ($L/I > 0.8$ and $L/\sigma_L > 10$) PA curve. We do not use the algorithm of calculating minimum χ_r^2 for (α, ζ) , but just directly use least-squares method to fit the PA curve. The α , ζ , and ϕ_0 are given in Table A.1, and the pulse profile with RVM is also shown in Figure A.2. The fitting result is close to that in Blaskiewicz et al. (1991), Yuan et al. (2023).

Appendix B: Bayesian analysis of simulated Data

To test how the model works and how the parameters will be constrained, we generated some simulated V/P versus ν curves from the model. We set $\kappa = 450$, $\gamma = 4.2$, $z_{0,1} = 120R_{\text{NS}}$, $z_{0,2} = 115R_{\text{NS}}$, and $z_{0,3} = 412R_{\text{NS}}$. We then added Gaussian noise to the simulated V/P values, with standard deviation σ , and do Bayesian analysis on them. We set 20 simulated $V/P - \nu$ curves using the parameters of B0301+19, for $\sigma = 0.01$ and $\sigma = 0.03$ separately.

The χ_r^2 s of the fitting of simulated data lie within the range of (0.4, 1.2). Examples of the fitted curves v.s. simulated curves, and of posterior PDFs are shown in Figure B.1. The fitting is generally good, with the fitted curves close to simulated curves. The posterior PDFs are sometimes multi-peaked, which reflects the degeneracy of parameters of the model.

The distributions of the maximum-likelihood parameters of 20 simulations are shown in Figure B.2. The mode coupling model seems to constrain the parameters differently. In the case of our simulation, the κ and γ are not as well constrained as $z_{0,(1,2,3)}$.

Appendix C: Comparison of parameters of multi-peaks in posterior PDFs, and more examples of fitted pulses

The analysis of three peaks in the posterior PDF of #683 of B0301+19 is shown in Figure C.1. Four pulses with fitted V/P curves are shown in Figure C.2.

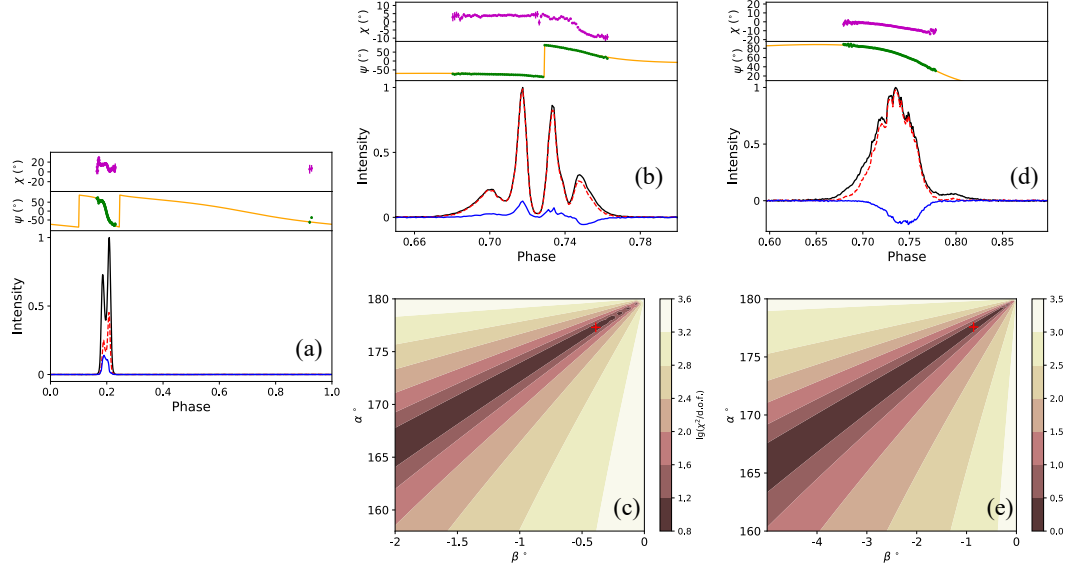


Fig. A.2. Integrated pulse profiles and RVM fitting results of the three pulsars. (a)(b)(d) are profiles of B0301+19, J0631+1036, and B0656+14, and (c)(e) are χ^2 distributions for (α, ζ) pairs of J0631+1036 and B0656+14. The meanings of lines and dots in profiles are same as those in Figure 1, except for the orange-colored RVM curves of fitted parameters.

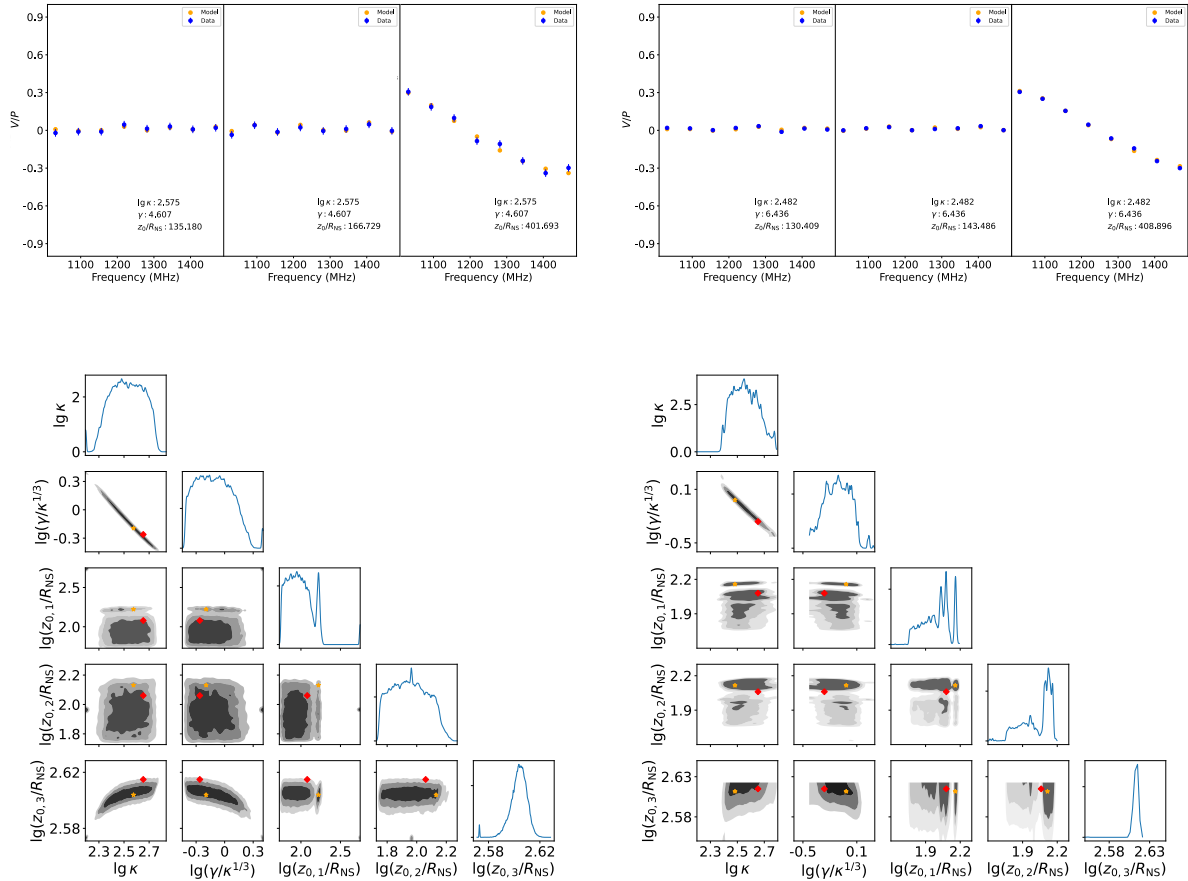


Fig. B.1. Bayesian analysis results of a simulated sample with $\sigma = 0.03$ (left) and $\sigma = 0.01$ (right). The upper plots: simulated V/P s (blue dots with errorbars) v.s. modeled V/P s (orange dots) with maximum-likelihood parameters. The maximum-likelihood parameters are marked in the plots. The lower plot: posterior PDFs of five parameters. The meanings of plots are same as those in Figure 2. The red diamonds represent the parameters set for simulation, and the orange stars represent the maximum-likelihood parameters.

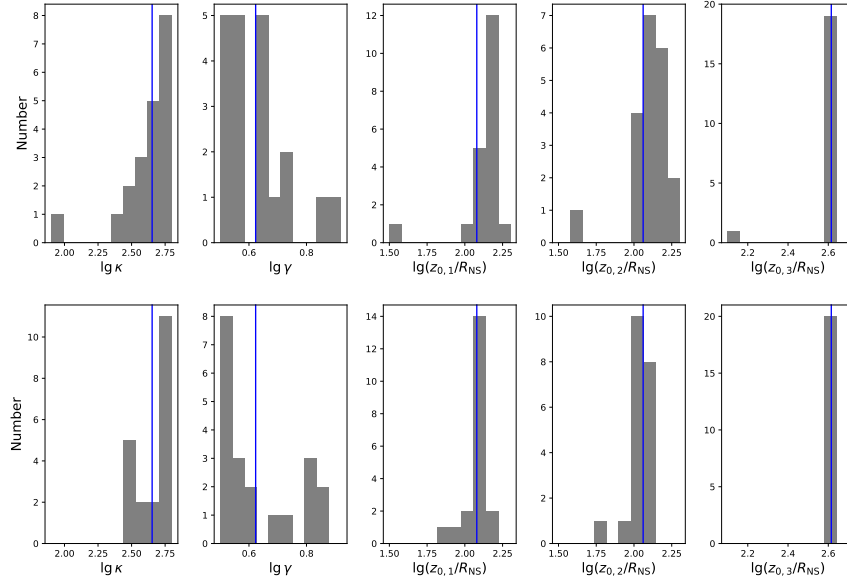


Fig. B.2. The distributions of maximum-likelihood parameters of 20 simulations, for $\sigma = 0.03$ (upper) and $\sigma = 0.01$ (lower). The blue vertical lines mark the reference parameters set for simulations.

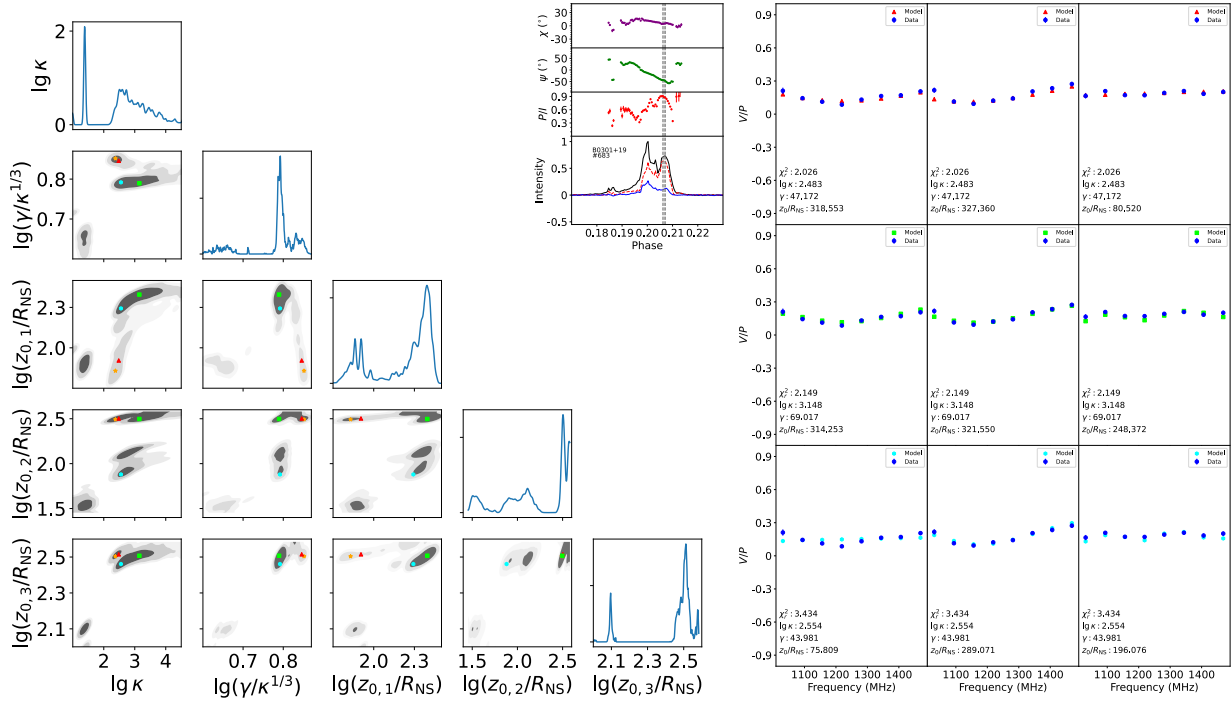


Fig. C.1. Left: Posterior PDFs of #683 of B0301+19. The meanings of plots are same as those in Figure 2, except that the red triangle, the green squares, and the cyan regular hexagon are markers of three 5D peaks chosen by using clustering package *hdbscan*. The comparison between real data (with errorbars) and modeled $V/P - \nu$ curves (with corresponding parameters marked with the red triangle, the green squares, and the cyan regular hexagon). The pulse profile is also plotted here, and the meanings of lines and dots are same as those in Figure 1.

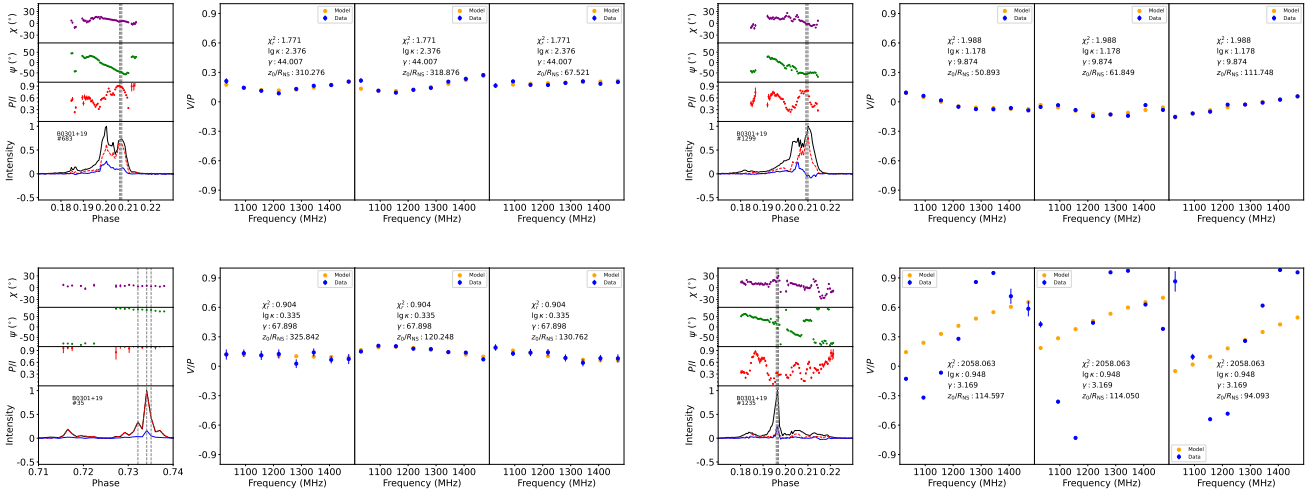


Fig. C.2. Four pulses (12 V/P - ν curves) with fitting results. The meanings of plots are same as those in Figure 1.