

Microlensing pulsars

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ABSTRACT

We investigate the possibility that pulsars act as the lens in gravitational microlensing events towards the Galactic bulge or a spiral arm. Our estimation is based on the anticipated survey and observations of the Five hundred metre Aperture Spherical Telescope (FAST) and the Square Kilometer Array (SKA). Two different models of pulsar distribution are used. We find that the lensing rate is ≥ 1 event per decade, which is high enough to search for the real events. Therefore, microlensing observations that will focus on pulsars identified by FAST or SKA in the future are important. As an independent determination of pulsar mass, the future detection of microlensing pulsars should be significant in the history of studying pulsars, especially in constraining the state of matter (either hadronic or quark matter) at supra-nuclear densities. Observations of such events in the future using advanced optical facilities (e.g. the *James Webb Space Telescope* and the Thirty Meter Telescope) are highly recommended.

Key words: gravitational lensing: micro – stars: neutron – pulsars: general.

1 INTRODUCTION

Pulsars could be normal neutron stars or quark stars (Lattimer & Prakash 2004). It is very important to affirm or negate the existence of either neutron or quark stars in order to guide physicists in studying the nature of fundamental strong interaction. With regard to the possible ways of identifying quark stars (see, for example, Xu 2008, for a review), it should be very straightforward and clear to find low-mass quark stars, as neutron stars are essentially gravitation-bound while low-mass quark stars are mainly confined by strong interaction. If we can detect a pulsar-like star with mass $\lesssim 0.1 M_{\odot}$, then it is surely a quark star.

How do we measure the mass of stars? Up to now, only the masses of compact stars in binary systems have been determined, by either Keplerian or post-Keplerian parameters. The lowest mass detected of an eclipsing X-ray pulsar, SMC X-1, is $1.06_{-0.10}^{+0.11} M_{\odot}$, which is near the minimum mass expected for a neutron star produced in a supernova (van der Meer et al. 2007). However, most pulsars are isolated, and it is still a considerable challenge to measure their masses. It has been suggested that the mass of an isolated neutron star can be determined observationally from the redshift (as a function of the ratio of mass to radius, M/R) and pressure broadening (as a function of M/R^2) of an absorption spectrum. However, no atomic line has been detected with certainty in the thermal X-ray spectra (e.g. Xu 2002).

Interestingly, gravitational microlensing (e.g. Mao 2008), which makes use of the temporal brightening of a background star as

a result of an intervening object, can provide us with a powerful method to measure the masses of compact objects. The amplification A of a background star by the passage of a pulsar has the relation

$$A = \frac{u^2 + 2}{u\sqrt{u^2 + 4}}.$$

Here, $u = r'/R_E$ is a parameter, with the undeflected distance, r' , and the Einstein radius, R_E (Paczynski 1986), which is almost a function of pulsar mass M and r if the background star's distance $D \gg r$ (see equation 1). We can then directly obtain the mass distribution of the lens objects via the light curve (Glicenstein 2003). With the method of microlensing, not only can we determine the mass distribution of massive compact halo objects (MACHOs; Alcock et al. 1996) and massive objects in the Galactic Centre region (Wex, Gil & Sendyk 1996), but we can also measure the masses of isolated pulsars.

The possibilities of a microlensing neutron star have been discussed previously and several estimations of lensing rates have been presented in the literature (Horvath 1996; Schwarz & Seidel 2002). In this paper, we re-estimate the lensing rate by calculating the solid angle Einstein ring swept as a result of the pulsar's proper motion, using two different pulsar distribution models. In addition, our estimation also takes into account the anticipated observations of the Five hundred metre Aperture Spherical Telescope (FAST; Nan et al. 2006) and the Square Kilometer Array (SKA; Johnston et al. 2007). This is because these two new telescopes, FAST and SKA, will greatly enhance our ability to search and observe pulsars, and the number of pulsars with measured proper motion in the Galaxy is crucial to estimate the lensing rate. Lorimer et al. (2006) used the results from recent surveys with the Parkes Multibeam System to

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derive a potentially detectable population of 30 000 normal pulsars. By using this number and assuming 30 000 potentially detectable millisecond pulsars in the Galaxy (Lyne et al. 1998), Smits et al. (2009a) estimated that an all-sky survey with only the 1-km core of the SKA located in the Southern hemisphere would detect about 14 000 normal pulsars and about 6000 millisecond pulsars. Under the same assumption, Smits et al. (2009b) noted that more than 7000 previously unknown pulsars could be discovered by FAST in the Galactic plane ($|b| < 10^\circ$) with an observation time of 1800 s per beam. Furthermore, by regular timing of these pulsars using FAST and SKA, accurate positions and proper motion can also be determined for them. Thus, we propose to search for possible microlensing events as a result of pulsars identified by FAST and SKA and to carry out microlensing observations coupled with radio observations using future optical projects (e.g. the *James Webb Space Telescope*¹ or the Thirty Meter Telescope Project²).

The lensing rate of our result indicates a high possibility of observing microlensing event as a result of pulsars in the future with the help of FAST and SKA. We expect an independent measurement of pulsar mass through these microlensing observations, especially to discover low-mass pulsars, in order to understand the real nature of pulsars.

2 LENSING RATE

We consider a pulsar at distance r with velocity v , microlensing a background star at distance D . The Einstein ring of this pulsar sweeps a solid angle S_N on the celestial sphere during a period time of t . The Einstein radius is

$$R_E = \sqrt{\frac{2R_S}{D}} r(D-r), \quad (1)$$

where $R_S = 2GM/c^2$ denotes the Schwarzschild radius. This solid angle S_N depends on the mass M , the distance r to the pulsar and also the distance D to the star:

$$S_N(M, v, r, D) = \frac{R_E vt}{r}. \quad (2)$$

We expect that a microlensing event occurs when a star in the background falls into this solid angle. We assume that our telescopes can identify a total of N pulsars and S deg² of the Milky Way is visible to it. Thus, the lensing rate per unit time can be estimated as

$$p = \frac{\sum_N S_N}{S} N_{\text{star}}, \quad (3)$$

where N_{star} denotes the total number of stars visible.

If we take pulsar distribution into account, the sum turns into a multiple integral on the space of position (r and D) and velocity (v). Then the lensing rate p can be rewritten as

$$p = \frac{\int_{\text{position}} \int_{\text{velocity}} S_N(\mathbf{v}, \mathbf{r}, \mathbf{D}) d\mathbf{v} d\mathbf{r} d\mathbf{D}}{S} N_{\text{star}}, \quad (4)$$

where $D \geq r$. We assume that the number density of stars in the Galactic bulge or a spiral arm is constant in the following estimation. Because the velocity distribution of pulsars is not a function of r and D , we can integrate the velocity distribution independently of position distribution. We can thus just apply a mean velocity in estimating the lensing rate. Hansen & Phinney (1997) suggested a

Maxwellian distribution for the kick velocities of pulsars:

$$f(v_{\text{kick}}) = \sqrt{\frac{2}{\pi}} \frac{v_{\text{kick}}^2}{\sigma^3} \exp\left(-\frac{v_{\text{kick}}^2}{2\sigma^2}\right). \quad (5)$$

Here, $\sigma = 190 \text{ km s}^{-1}$ and mean kick velocity $v_{\text{kick}} = 300 \text{ km s}^{-1}$ are chosen. Considering that only the part of the velocity vector that lies in the lens plane is effective, we should use the mean velocity projection on the lens plane (Schwarz & Seidel 2002)

$$v = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} v_{\text{kick}} \cos \alpha d\alpha, \quad (6)$$

where the projected velocity, v , is of the order of 200 km s^{-1} .

In the following simulations, we use two different models of pulsar number density from Hartman et al. (1997), which resemble the models from Narayan (1987) and Johnston (1994); see also Schwarz & Seidel (2002). The first model of pulsar distribution is

$$n_{p1}(R) = \frac{1}{2\pi R_W^2} \exp\left(-\frac{R}{R_W}\right), \quad (7)$$

where R is the radial distance of the pulsar to the Galactic Centre in the Galactic plane, and $R_W = 5 \text{ kpc}$. The second model is

$$n_{p2}(R) = \frac{c_{p2}}{2\pi R_W^2} \exp\left[-\frac{(R - R_{\text{max}})^2}{2R_W^2}\right], \quad (8)$$

where $R_W = 1.8 \text{ kpc}$ and $R_{\text{max}} = 3.5 \text{ kpc}$. The normalization constant is $c_{p2} = 0.204$ for the given choice of R_{max} . For the z -dependence, we apply

$$n_z(z) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2} \frac{z^2}{\sigma^2}\right), \quad (9)$$

with $\sigma = 0.45 \text{ kpc}$ (Lyne et al. 1998; Schwarz & Seidel 2002).

We transform the pulsar distribution into the spherical coordinates (r, θ, ϕ) with the Sun at the origin using

$$z = r \sin \theta, \quad (10)$$

and

$$R^2 = r^2 \cos^2 \theta + R_{\text{SC}}^2 - 2r R_{\text{SC}} \cos \theta \cos \phi, \quad (11)$$

where R is the radial distance of the pulsar to the Galactic Centre in the Galactic plane, z is the height and $R_{\text{SC}} = 8.5 \text{ kpc}$. For pulsar distribution n_{p1} , we have

$$\begin{aligned} p &= \frac{NN_{\text{star}}}{S} \int \int \int n_{p1}(R) n_z(z) r^2 \sin \theta dr d\theta d\phi \\ &\times \frac{1}{r^2} \frac{vt}{4\pi r} \int_0^r \sqrt{\frac{2R_S}{D}} r(D-r) dD \\ &= \frac{NN_{\text{star}} vt}{2\pi S R_W^2 \sqrt{2\pi}\sigma} \int \int \int \\ &\times \exp\left(-\frac{\sqrt{r^2 \cos^2 \theta + R_{\text{SC}}^2 - 2r R_{\text{SC}} \cos \theta \cos \phi}}{R_W}\right) \\ &\times \exp\left(-\frac{1}{2} \frac{r^2 \sin^2 \theta}{\sigma^2}\right) \sin \theta dr d\theta d\phi \\ &\times \frac{\sqrt{2R_S r}}{4\pi} \int_0^1 \sqrt{\frac{y-1}{y}} dy, \end{aligned} \quad (12)$$

¹<http://www.jwst.nasa.gov/>

²<http://www.tmt.org/>

where $y = D/r$. If we define $x = r/R_{\text{SC}}$, then the expression can be reduced to

$$\begin{aligned}
 p = & \frac{NN_{\text{star}}\nu t R_{\text{SC}}\sqrt{2R_{\text{S}}R_{\text{SC}}}}{8\pi^2 S R_{\text{W}}^2\sqrt{2\pi}\sigma} \\
 & \times \int \int \int \exp\left(-\frac{\sqrt{x^2\cos^2\theta + 1 - 2x\cos\theta\cos\phi}}{R_{\text{W}}/R_{\text{SC}}}\right) \\
 & \times \exp\left(-\frac{1}{2}\frac{x^2\sin^2\theta R_{\text{SC}}^2}{\sigma^2}\right) \sin\theta dx d\theta d\phi \int_0^1 \\
 & \times \sqrt{\frac{y-1}{y}} dy. \tag{13}
 \end{aligned}$$

We consider that FAST is expected to detect about 7770 pulsars in about $70 \times 10 \text{ deg}^2$ of the Milky Way in less than a year of observing time (Nan et al. 2006), while SKA is expected to detect about 15000 pulsars in $290 \times 10 \text{ deg}^2$ of the Milky Way. If we choose $M = 1 M_{\odot}$, $r \in [0, 5 \text{ kpc}]$, $t = 1 \text{ yr}$, $N = 15000$, $N_{\text{star}} = 10^{11}$, $\nu = 200 \text{ km s}^{-1}$ and $S = 2000 \text{ deg}^2$, then we have $p \approx 12$ events per year from equation (13).

Certainly, the total number of stars visible is less than 10^{11} if the effect of extinction is included. Based on the photometric maps of the Galactic bulge released by the Optical Gravitational Lensing Experiment (OGLE) project, which contain the photometry of about 30 million stars from 49 fields covering 11 deg^2 in different regions of the Galactic bulge (Udalski et al. 2002), we estimate that in the region of $S = 2000 \text{ deg}^2$, 10^9 stars would be visible. Then, the lensing rate should be $p \geq 1$ events per decade according to equation (13). The lensing rate density as a function of θ and ϕ is shown in Fig. 1.

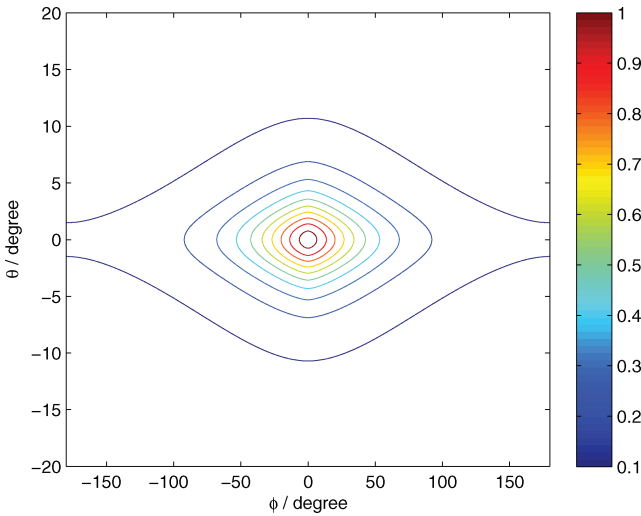


Figure 1. The lensing rate density as a function of θ and ϕ in the model of pulsar distribution described by equation (7), where θ is the azimuth angle and ϕ is the polar angle. The colour bar denotes the lensing rate in units of the number of events per decade per deg^2 .

For pulsar distribution n_{p2} in the model of pulsar distribution described by equation (8), we have accordingly

$$\begin{aligned}
 p = & \frac{NN_{\text{star}}}{S} \int \int \int n_{p2}(R)n_z(z)r^2 \sin\theta dr d\theta d\phi \\
 & \times \frac{1}{r^2} \frac{\nu t}{4\pi r} \int_0^r \sqrt{\frac{2R_{\text{S}}}{D}} r(D-r) dD \\
 = & \frac{NN_{\text{star}}c_{p2}\nu t}{2\pi S R_{\text{W}}^2\sqrt{2\pi}\sigma} \int \int \int \\
 & \times \exp\left[-\frac{\left(\sqrt{r^2\cos^2\theta + R_{\text{SC}}^2 - 2rR_{\text{SC}}\cos\theta\cos\phi} - R_{\text{max}}\right)^2}{2R_{\text{W}}^2}\right] \\
 & \times \exp\left(-\frac{1}{2}\frac{r^2\sin^2\theta}{\sigma^2}\right) \sin\theta dr d\theta d\phi \frac{\sqrt{2R_{\text{S}}r}}{4\pi} \int_0^1 \\
 & \times \sqrt{\frac{y-1}{y}} dy. \tag{14}
 \end{aligned}$$

From the same process as for n_{p1} of equation (7), then we have $p \geq 2$ events per decade if we choose $M = 1 M_{\odot}$, $r \in [0, 5 \text{ kpc}]$, $t = 1 \text{ yr}$, $N = 15000$, $N_{\text{star}} = 10^{11}$, $\nu = 200 \text{ km s}^{-1}$ and $S = 2000 \text{ deg}^2$, and take the effect of extinction into account. The corresponding lensing rate density as a function of θ and ϕ is shown in Fig. 2.

The lensing rates estimated above are for the microlensing events in the Galactic Centre. Considering that the observations of FAST are limited to the spatial arms and that pulsars mainly distribute on the disc of the Milky Way, we then transform the pulsar distribution into cylindrical coordinates with the Sun at the origin using

$$R^2 = r^2 + R_{\text{SC}}^2 - 2rR_{\text{SC}}\cos\phi. \tag{15}$$

Using the cylindrical coordinates, we have

$$\begin{aligned}
 p = & \frac{NN_{\text{star}}}{S} \int \int n(r, \phi)n_z(r)r dr d\phi \frac{1}{r^2} \frac{\nu t}{r} \\
 & \times \int_0^r \sqrt{\frac{2R_{\text{S}}}{D}} r(D-r) dD. \tag{16}
 \end{aligned}$$

For pulsar distributions of both n_{p1} and n_{p2} , we perform the calculation in the way we have previously, and we choose

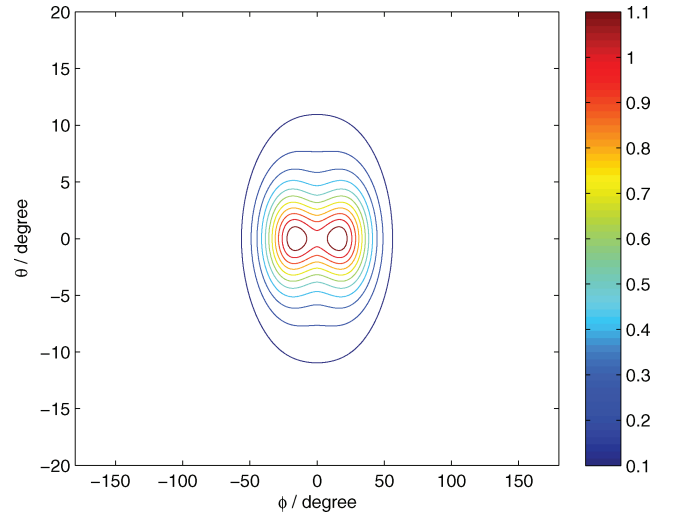


Figure 2. Same as in Fig. 1, but in the model of pulsar distribution described by equation (8).

$M = 1 M_{\odot}$, $r \in [0, 5 \text{ kpc}]$, $t = 1 \text{ yr}$, $N = 10\,000$, $v = 200 \text{ km s}^{-1}$ and $S = 2000 \text{ deg}^2$. Considering the effect of extinction, we choose $N_{\text{star}} = 10^9$. Finally, we can also obtain a lensing probability of $p \geq 1$ events per decade for FAST.

3 PROPOSAL FOR DISCOVERING MICROLENSING PULSARS

We have estimated the probability of observing microlensing pulsars using FAST and SKA. Besides the encouraging lensing rate according to our results, a feasible searching strategy is also essential for discovering a microlensing pulsar. We therefore propose to carry out microlensing pulsar observations coupled with FAST and SKA observations in the future, with which we are hopeful of determining the mass of isolated pulsars.

Microlensing pulsar observations should be based on the anticipated data of FAST and SKA. We propose to systematically list the information of new pulsars discovered by FAST and SKA in the future, including the position, proper motion and distance, and high proper motion pulsars should be especially focused on. The position of these pulsars in the next few decades could also be predicted. Then we could compare the predicted position of these pulsars with the position of stars in the Galactic bulge and spiral arms, and the background star candidates of microlensing events, whose position would be quite close to the predicted position of pulsars, could be picked up. With these candidates, we should formulate a plan to monitor certain background stars for predicted microlensing events in a certain period of time in the future.

Microlensing pulsar observations could be carried out on ongoing lensing projects, and future advanced facilities are also expected to benefit the observations. The magnification of a background star could be about 0.3, on the condition that a pulsar is just arriving at the edge of the Einstein ring (Mao 2008), and the time scale of the microlensing event should be about 15 days. The observation can then be carried out on current telescopes in the infrared band where the interstellar extinction is much weaker (Horvath 1996; Udalski et al. 2002). Besides ongoing gravitational lensing projects, we could also look forward to future projects, especially the Thirty Meter Telescope (TMT) project. The TMT project, which is scheduled for the next decade, will greatly promote our study of gravitational lensing on cosmology, galaxy formation and the distribution of lensing mass. The TMT's great capabilities of resolving smaller, fainter source populations will allow a much higher sky density of background sources to be used (Carlberg 2004). This will directly enhance the possibility of discovering microlensing pulsar events. Therefore, the TMT together with FAST and SKA will provide us with a good opportunity to observe microlensing pulsar events and to determine the mass of isolated pulsars.

We note that the proposed microlensing pulsar observations are costless, that is to say, we do not need to monitor a target all the time. Our estimation has shown the possibility of microlensing a pulsar. The key to the observation is predicting the microlensing events, based on the large and sensitive data base of FAST and SKA, and then monitoring microlensing candidates of background stars in a predicted short period of time. Even though the prediction cannot be precise and the lensing rate is relatively small, the costless observation is still meaningful and should be carried out.

4 DISCUSSION AND CONCLUSIONS

We have estimated the lensing rates of pulsars towards the Galactic bulge and spiral arms. Our estimation shows that, for FAST and SKA, the lensing rates ($\propto N$) are ≥ 1 events per decade at least, much higher than the estimations made previously in the literature. The number of pulsars with measured proper motion, N , is crucial to estimate the rate of lensing events. In Table 1, we summarize the number of pulsars expected to be detected by FAST and SKA, based on previous survey simulation results. In addition, the lensing rate should increase significantly if the population of rotating radio transients (RRATs; Keane 2010) is included, as the total number of RRATs is at least several times that of galactic active radio pulsars (McLaughlin 2006). RRATs, which can be precisely located by timing and dispersion measurements, should then be one of the key targets of FAST and SKA.

We also note that the lensing rates we have shown are based on photometric microlensing events. If we consider astrometric microlensing, which makes use of the shift of the centroid of the combined images of the light source, the cross-section would increase by a factor of $\sim 10^2$ (Horvath 1996; Schwarz & Seidel 2002). As our lensing rate is proportional to the cross-section, the astrometric microlensing probability would be $\sim 10^2$ times higher than that presented previously in Section 2. It could then be realistic to search for astrometric microlensing pulsars and to detect the events in the future using facilities with very high position precision.

The lensing rates indicate that we can hope to measure the mass of an isolated pulsar in the future using the method of microlensing. We propose to perform catalogue comparison and microlensing prediction for pulsars identified by SKA and FAST in the future, and to carry out microlensing observations coupled with radio observations in order to detect microlensing pulsar events and to measure the masses of isolated pulsars with advanced optical facilities.

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Table 1. Summary of the previous survey simulation results of FAST and SKA. Numbers in parentheses represent known pulsars. l , b and Dec. are the longitude, latitude and declination in the galactic coordinate system, respectively.

	Detectable pulsars All sky	FAST ^a		SKA ^b	
		20° < l < 90° $ b \leq 10^\circ$	20° < l < 90° $ b \leq 10^\circ$	0° < l < 85° and 155° < l < 360° $ b \leq 5^\circ$	Dec. < 50°
Normal pulsar	$\sim 30\,000^c$	~ 5700 (352)	~ 7000 (418)	$\sim 11\,000$	$\sim 14\,000$
Millisecond pulsar	$\sim 30\,000^d$	~ 550 (14)	~ 770 (20)	~ 4000	~ 6000

^aResults from Smits et al. (2009b).

^bResults from Smits et al. (2009a).

^cResults from Lorimer et al. (2006).

^dResults from Lyne et al. (1998).

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