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A polytropic model of quark stars

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ABSTRACT

A polytropic quark star model is suggested in order to establish a general framework in which theoretical quark star models could be tested by observations. The key difference between polytropic quark stars and the polytropic model studied previously for normal (i.e. non-quarkian) stars is related to two issues: (i) a constant term representing the contribution of vacuum energy may be added in the energy density and the pressure for a quark star, but not for a normal star; (ii) the quark star models with non-vanishing density at the stellar surface are not avoidable due to the strong interaction between quarks. The first one implies that the vacuum inside a quark star is different from that outside, while the second one is relevant to the effect of color confinement. The polytropic equations of state are stiffer than that derived in conventional realistic models (e.g. the bag model) for quark matter, and pulsar-like stars calculated with a polytropic equation of state could then have high maximum masses ($> 2M_\odot$). Quark stars can also be very low massive, and be still gravitationally stable even if the polytropic index, n, is greater than 3. All these would result in different mass-radius relations, which could be tested by observations. In addition, substantial strain energy would develop in a solid quark star during its accretion/spindown phase, and could be high enough to take a star-quake. The energy released during star-quakes could be as high as $\sim 10^{47}$ ergs if the tangential pressure is $\sim 10^{-6}$ higher than the radial one.

1. Introduction

It depends on the state of matter at supra-nuclear density to model pulsar's structure, which is unfortunately not certain due to the difficulties in physics although some efforts have been made for understanding the behavior of quantum chromo-dynamics (QCD) at high density. Of particular interest is whether the density in such compact stars could be high enough to result in unconfined quarks (quark matter). Stars composed of quarks (and possible gluons) as the dominant degrees of freedom are called quark stars, and there is possible observational evidence that pulsar-like stars could be quark stars (see reviews, e.g. [29,31,32]). But it is still a problem to model a realistic quark star for our lack of knowledge about the real state of quark matter.

The study of cold quark matter opens a unique window to connect three active fields: particle physics, condensed matter physics, and astrophysics. Many possible states (see, e.g. [2]) of cold quark matter are proposed in effective QCD models as well as in phenomenological models. An interesting suggestion is that quark matter could be in a solid state [12,17,20,30]. Solid relativistic stars are challenging astrophysicists since the stelar matter can not be well approximately by a perfect fluid and the conventional Tolman–Oppenheimer–Volkov (TOV) equation is thus not applicable. Nevertheless, in case of static and spherically symmetric gravity, the equilibrium equation could be similar to the TOV equation, by introducing a deviation between radial and tangential pressures (see, e.g. [34]). However, one has also to know the radial pressure, P, as a function of density ρ (and possible other parameters) in order to model a quark star in a solid state.

No realistic relation of $P(\rho)$ is available since no cold quark matter has been discovered experimentally and/or observationally with certainty, although many modeled relations between P and ρ are proposed in the literatures. Among the relations, a class of linear equations of state, $P = \kappa(\rho - \rho')$, is currently focused, with two free parameters κ and ρ' (see, e.g. [25,35]), in the framework of the bag model. Both relations derived in the bag model and in the density-dependent quark model [3] can be regarded as special cases of the linear relation of $P(\rho)$. Whatsoever, the linear equation could not be adequate if possible quark-clustering occurs in cold quark matter [30]. Such matter with clustered quarks could be in a fluid state at high temperature but in a solid state at sufficient low temperature. It should be worth noting that the interaction between quarks in a fireball with quarks and gluons is still very strong (i.e. the strongly coupled quark-gluon plasma [26]), according to recent achievements of relativistic heavy ion collision experiments. Such a strong coupling may naturally render quarks grouped in clusters. How can then one state a reliable P-ρ relation in order to establish a framework in which theoretical stellar models could be tested by observations if the quark-clustering effect is included?

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Astronomers faced a similar problem when trying to model non-quarkian normal stars (e.g. main sequent stars and white dwarfs). A polytropic model with equation of state, $P = K\rho^{\Gamma}$ ($\Gamma = 1 + 1/n$), had been extensively studied previously (see, e.g. [4]) for main sequent stars as well as white dwarfs under the Newtonian gravity. This model has also been extended under general relativity [24,28]. The polytropic models are valuable because they could help us to model stars composed of realistic matter, such as ideal gas, photon gas, and degenerate fermi gas. Would it be possible for us to do a parallel investigation for quark stars? As discussed previously, QCD, which is still developing in low-energy regime, should be involved to describe cold quark matter and the equation of state of quark stars is then uncertain up-to-now. Furthermore, it might be problematic to calculate the state of macroscopic quark stars in QCD because QCD is till a local theory [21]. Nonetheless, if quarks are clustered in quark stars where quarks are coupled strongly, the state of cold quark matter might be approximated phenomenologically by polytropic equations of state, since one may draw naively an analogy between the clusters in quark matter and the nuclei in normal matter.

Some authors had applied the polytropic equations of state to model hybrid stars which have cores composed of unconfined quarks [14,36]. However, their polytropic equations of state are only used to describe normal phase (composed of normal baryons) and mixed phase (composed of denser baryon matter); for the quark phase inside the core, the equation of state is still the form of the bag model. In this paper, we will apply for the first time the polytropic equations of state for quark matter and calculate the structures of quark stars, with different polytropic indices, n. These stellar models could also be regarded as an extension to the quark star models with linear equations of state.

We are going to model quark stars in two separated ways. (i) The vacuum energy inside and outside quark matter is different. As proposed by Bambi [5], quark stars could represent an laboratory to investigate the cosmological constant problem if some compensate field exist to generate a constant-like term to compensate the difference of vacuum energy, and the mass-radius curves of quark stars will be different from the standard case. We generalize Bambi's idea to discuss generic polytropic equations of state. (ii) The vacuum inside and outside of a quark star is assumed the same, i.e. quark stars have no QCD vacuum energy. In both cases, a key difference between polytropic quark star and normal star models lies on the surface density $\rho_{\rm sur}$ ($\rho_{\rm sur} > 0$ for the former but $\rho_{\rm sur} = 0$ for the latter), since a quark star could be bound not only by gravity but also by additional strong interaction due to the strong confinement between quarks. Analogously, a quark star without QCD vacuum energy could be similar to an asteroid with a sharp surface where the density is also none-zero. The non-zero surface density is still natural in the case with the linear equation of state, where the binding effect is represented by the bag constant, B (and then $\rho_{\rm sur} = 4B$).

The stability of a polytropic star depends also on the surface density. It is well known that a normal star with zero surface density should be unstable if $n > 3$ in the Newtonian gravity, and that the case of $n = 3$ is still unstable in general relativity (see details in, e.g. [24]). However, in the models we will demonstrate, a quark star could still be stable even if $n > 3$.

The structures of related compact stars have been studied in general relativity by some authors. Durgapal and Bannerji [6] derived an analytical expression of mass-radius relation for isotropic stars in general relativity, and Herrera et al. [9,11] analyzed a set of solutions to the Einstein's equations for anisotropic matter. It is very interesting to model stars with anisotropic pressure for some physical reasons [22,23,27]. Harko and Mak [8,15] derived an analytical expression of mass-radius relation for anisotropic stars in general relativity, and discussed the constraints for the anisotropic parameter. They had also presented an exact analytical solution of the gravitational equations describing a static spherically symmetric anisotropic quark matter distribution [16]. These authors did not start with an equation of state, but studied the density and pressure in a more general framework based on the energy-momentum tensor. It is worth noting that the results obtained by above authors are still parameter- and assumption-dependent, even for the so-called exact solutions. We will alternatively study the problem, with an explicit form of equation of state. We also consider the case of anisotropic stars and compute the gravitational energy released during quakes of solid quark stars, with a parameter range given by Harko & Mak [8]. Numerical results show that, if the tangential pressure which is slightly larger than the radial one changes abruptly, the gravitational energy released could be high enough to power the supergiant flares observed from soft γ-ray repeaters.

This paper is arranged as follows. The details of polytropic model of quark stars of perfect and unperfect fluids, respectively, are presented in Sections 2 and 3. The numerical results are shown in Section 4. The paper is concluded in Section 4.

2. Stars of perfect fluid

2.1. Quark stars without QCD vacuum energy

If there is no difference between the vacuum inside and outside of a quark star, the equation of state for a quark star is the standard polytropic model, with a non-zero surface density, representing the strong confinement between quarks. In this point of view, quark stars could analogously be similar to asteroids: the electromagnetic force dominates over gravity in the later, while the strong interaction can not be negligible in the former. Consequently, both of those objects can have a sharp surface where the density goes down to zero in a negligible small scale.

Stars of perfect fluid in general relativity were discussed by Tooper [28], with an equation of state,

$$P = K\rho_g^{\Gamma}, \tag{1}$$

$$\rho = \rho_g c^2 + nP, \tag{2}$$

where ρ_g is the part of the mass density which satisfies a continuity equation and is therefore conserved throughout the motion, and $\Gamma = 1 + 1/n$. In the static case with spherically symmetry, with the space-time metric of the form,

$$ds^2 = e^{\nu}c^2dt^2 - r^2(d\vartheta^2 + sin^2\vartheta d\varphi^2) - e^{\lambda}dr^2, \tag{3}$$

the hydrostatic equilibrium condition is derived to be [19]

$$\frac{1 - 2GM(r)/c^2r}{P + \rho c^2}r^2\frac{dP}{dr} + \frac{GM(r)}{c^2} + \frac{4\pi G}{c^4}r^3P = 0, \tag{4}$$

and

$$M = \int_0^R \rho/c^2 \cdot 4\pi r^2 dr. \tag{5}$$

Similar to the Lane-Emden equation of normal stars, re-scale density ρ_g and radius r, as well as $M(r)$, by

$$\rho_g = \rho_{gc}\theta^n, r = \xi/A, \tag{6}$$

$$M(r) = \frac{4\pi\rho_{gc}}{A^3}\upsilon(\xi), \tag{7}$$

where

$$A^2 = \frac{4\pi G\rho_{gc}}{(n+1)\alpha c^2}, \quad \alpha = \frac{K\rho_{gc}^{\frac{1}{n}}}{c^2},$$

and ρ_{gc} is the rest mass density at the center, we can then obtain

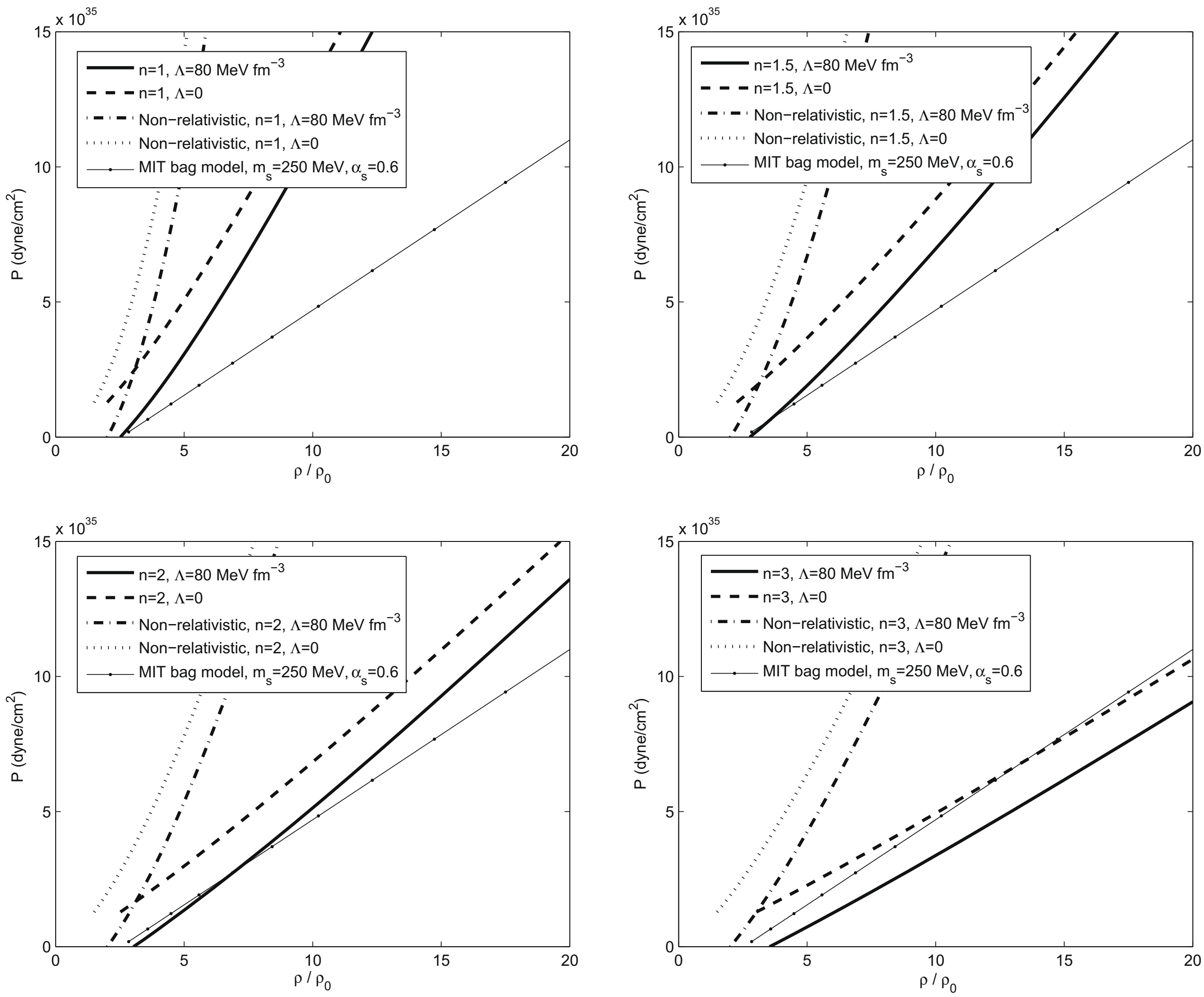

Fig. 1. To compare the equations of state discussed, including the polytropic states with $\Lambda = 80$ MeV (solid lines), $\Lambda = 0$ (dashed lines), the corresponding non-relativistic case with $\Lambda = 80$ MeV (dash-dotted lines), $\Lambda = 0$ (dotted lines), and that derived in the MIT bag model with the mass of strange quark $m_s = 250$ MeV and the strong coupling constant $\alpha_s = 0.6$ (thin lines with dots), for a given surface density $\rho_{\rm sur} = 1.5\rho_0$. Here and in the following figures, ρ_0 is the nuclear saturation density. It is evident that polytropic equations of state are stiff.

$$\frac{1-2(n+1)\alpha\upsilon/\xi}{1+(n+1)\alpha\theta}\xi^2\theta' + \upsilon + \alpha\xi^3\theta^{n+1} = 0, \tag{8}$$

$$\upsilon' = \xi^2\theta^n(1+n\alpha\theta), \tag{9}$$

with the initial conditions,

$$\theta(0) = 1, \upsilon(0) = 0. \tag{10}$$

The mass and radius of a star are evaluated at the point when the density reaches the surface density. In general, the surface of a star is defined by the position where the pressure is zero, or only the radial pressure is zero [9,10] in the anisotropic case where the radial pressure is different from the tangential one. In fact, the surface in our calculations may not be the real physical surface, since the pressure could not be zero there. However, this does not significantly affect the mass and radius we obtain, because a quark star has a sharp edge, and the pressure and density decrease to zero in a layer with thickness of few femto-meters.

2.2. Quark stars with QCD vacuum energy

The energy-momentum tensor has the form

$$\mathscr{T}_{\mu\nu} = \mathscr{T}^{\rm particles}_{\mu\nu} + \mathscr{T}^{\rm vacuum}_{\mu\nu}. \tag{11}$$

For a constant vacuum energy, the energy-momentum tensor can be written as

$$\mathscr{T}^{\rm vacuum}_{\mu\nu} = \Lambda g_{\mu\nu}, \tag{12}$$

and one finds

$$P^{\rm vacuum} = \mathscr{T}^{\rm vacuum}_{ii} = -\Lambda, \tag{13}$$

$$\rho^{\rm vacuum} = \mathscr{T}^{\rm vacuum}_{00} = \Lambda. \tag{14}$$

We can infer that no matter what forms the equations of state for the particles are, the contribution of vacuum is of the above form. Consequently, we write the equation of state as

$$P = K\rho_g^{1+\frac{1}{n}} - \Lambda, \tag{15}$$

$$\rho = \rho_g c^2 + nK\rho_g^{1+\frac{1}{n}} + \Lambda. \tag{16}$$

In this case, the density at surface (where pressure is zero) should also be non-zero. It is worth noting that these general equations of state of Eqs. (15) and (16) could be simplified into a few special ones: the form without QCD vacuum energy if $\Lambda = 0$, the linear (relativistic) form if $\rho_g = 0$, and the non-relativistic form if the second term in Eq. (16) is neglected (see the next sub-section for more discussions).

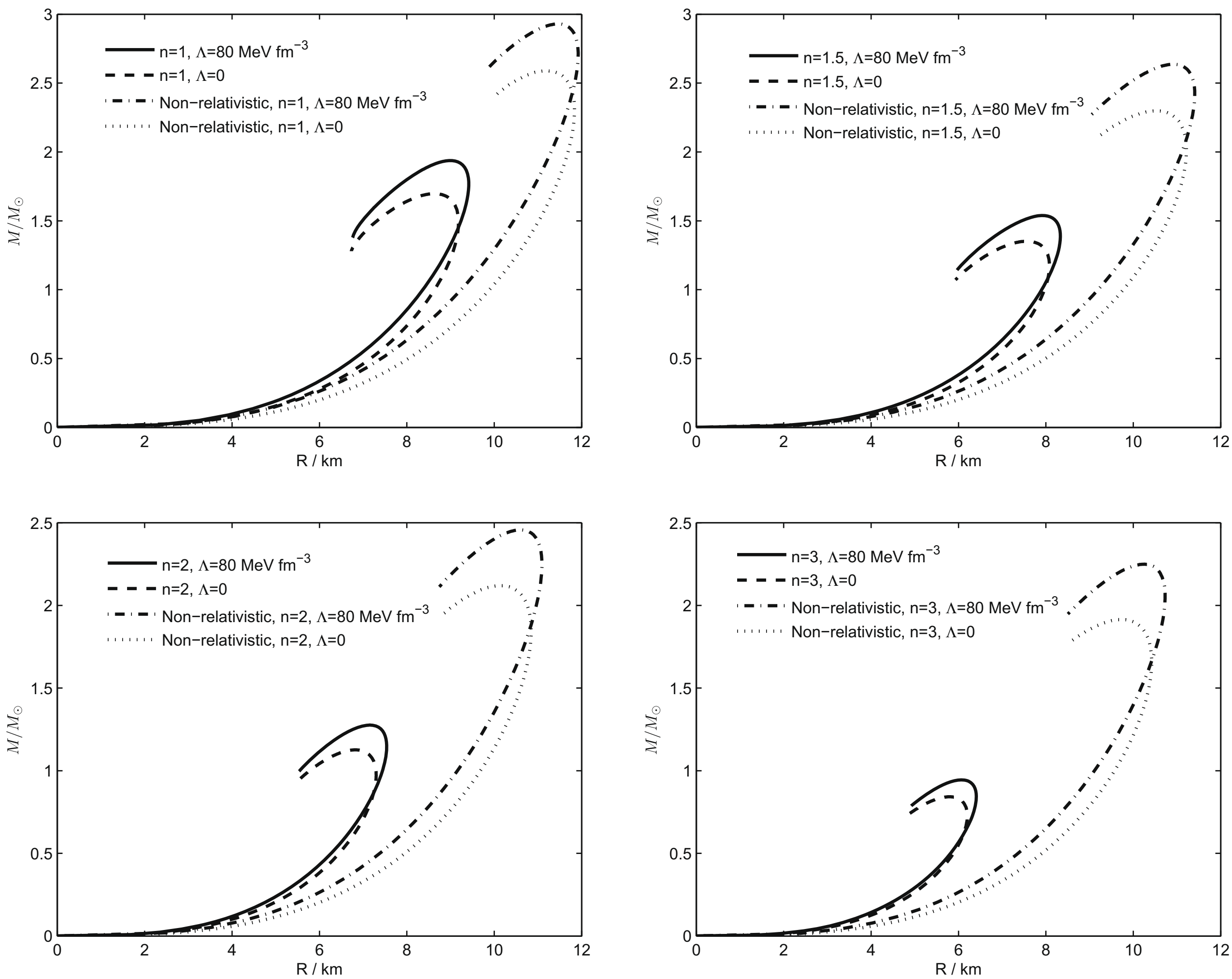

Fig. 2. Mass-radius relations for different polytropic indices, n, with $\rho_{\mathrm{sur}} = 1.5\rho_0$. Solid lines are $\Lambda = 80$ MeV fm^{-3}, dashed lines are for $\Lambda = 0$, dash-dotted lines are for non-relativistic case with $\Lambda = 80$ MeV fm^{-3}, and dotted lines are for non-relativistic case with $\Lambda = 0$.

The hydrostatic equilibrium conditions are also determined by Eqs. (4) and (5). But in this case, we cannot derive differential equations for density and radius, and can only numerically calculate the structure of a quark star from the center to the surface to obtain the mass and radius.

2.3. Comparison with MIT bag model

In the MIT bag model, quark matter are composed of massless up and down quarks, massive strange quarks, and electrons. Quarks are combined together by an extra pressure, denoted by the bag constant B, which is the vacuum energy density similar to the Λ-parameter in the polytropic model. For the comparison, we apply the formulae given by Alcock [1] to calculate the equation of state, with strange quark mass $m_s = 250$ MeV and the strong coupling constant $\alpha_s = 0.6$ for indications.

Note that the K-parameter in Section 2.2 could be calculated in the way of, from Eq. (15),

$$K = \Lambda \cdot \rho_{\mathrm{sur}}^{-\left(1+\frac{1}{n}\right)}, \tag{17}$$

where ρ_{sur} is the surface density, since at the surface the pressure is zero. This density should be determined by the behavior of the elementary strong interaction. In the calculation, we assume that Λ has the same value as the bag constant B of MIT bag model, and choose the surface density, $\rho_{\mathrm{sur}} = 1.5\rho_0$, where ρ_0 is the nuclear matter density. The K-parameter could be *larger* if the quark self-confinement effect is included, and the value determined by Eq. (17) is the minimal and the maximum masses presented in Figs. 2 and 3 could also be higher. For the sake of simplicity, we suppose a value of K from Eq. (17), and assume the K-parameter is the same for both cases with and without vacuum energy under a same polytropic index n. If quark-cluster inside a quark star are very massive, the kinetic energy density could be negligible compared to the rest mass energy density, i.e. the equation of state could be non-relativistic, and the total energy density would include only the rest mass energy density[1] in Eq. (16).

The equations of state discussed above are shown in Fig. 1. An obvious conclusion is that the equations of state of polytropic form could be stiffer than that of the MIT bag model, especially in the non-relativistic case. A stiffer equation of state would lead to a larger maximum mass of quark stars.

2.4. Gravitational energy in general relativity

The gravitational energy in general relativity was calculated by Tooper [28]. In general relativity, the integrating of space volume should be different from that in the Newtonian gravity due to

[1] An example similar to this non-relativistic equation of state is of the matter in white dwarfs where the energy density is dominated by the rest mass of nuclei, while the electron gas contributes to the pressure.

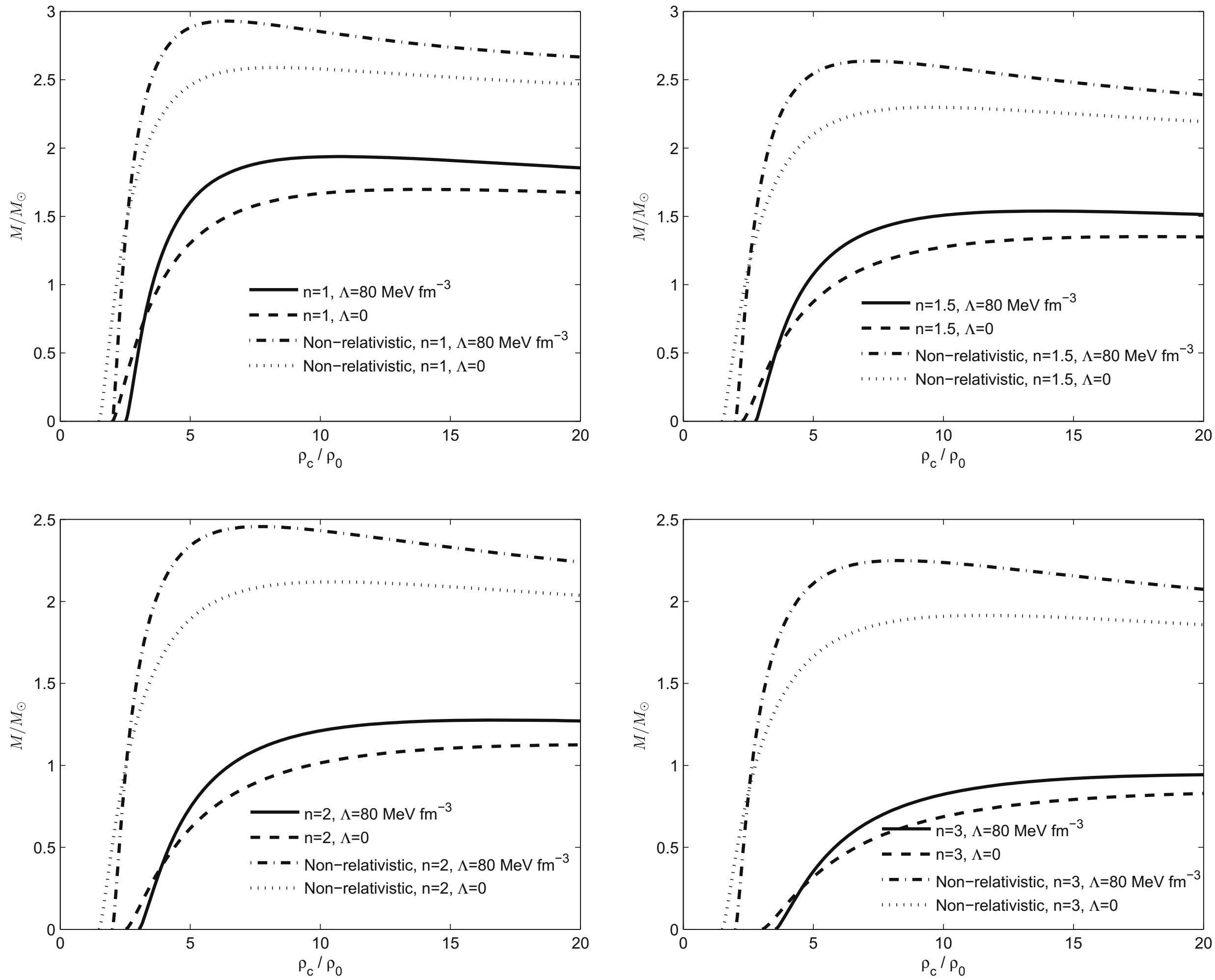

Fig. 3. Mass-central density relations for different polytropic indices, n, with $\rho_{\rm sur} = 1.5\rho_0$. Solid lines are $\Lambda = 80$ MeV fm^{-3}, dashed lines are for $\Lambda = 0$, dash-dotted lines are for non-relativistic case with $\Lambda = 80$ MeV fm^{-3}, and dotted lines are for non-relativistic case with $\Lambda = 0$. Stars with central densities greater than that of stars with maximum masses are gravitationally unstable.

the space-time curvature around a massive star. The proper energy, E_0, is obtained by integrating the energy density over elements of proper spatial volume,

$$E_0 = E_{0g} + E_{0k}, \tag{18}$$

where the rest energy, E_{0g}, of the system and its microscopic kinetic energy, E_{0k}, are

$$E_{0g} = M_{0g}c^2 = 4\pi\int_0^R \rho_g c^2 e^{\lambda/2} r^2 \mathrm{d}r, \tag{19}$$

$$E_{0k} = 4\pi\int_0^R nPe^{\lambda/2} r^2 \mathrm{d}r, \tag{20}$$

where λ can be calculated by

$$e^{-\lambda} = 1 - \frac{2GM}{c^2 r}. \tag{21}$$

The total energy of a star with mass $M = M(R)$ is $E = Mc^2$, and its gravitational energy, Ω, is the difference between the total energy and the proper energy,

$$\Omega = Mc^2 - E_0. \tag{22}$$

3. Stars with an anisotropic pressure

Fluid within inhomogeneous pressure is imperfect, and we will consider only the case of spherical symmetry, that the tangential and radial pressure are not equal. In this case the hydrostatic equilibrium condition reads (e.g. [34])

$$\frac{1 - 2GM(r)/c^2 r}{P + \rho c^2}\left(r^2\frac{dP}{dr} - 2\varepsilon rp\right) + \frac{GM(r)}{c^2} + \frac{4\pi G}{c^4} r^3 P = 0, \tag{23}$$

where ε is defined by $P_\perp = (1+\varepsilon)P$, and P is the radial pressure and $P_\perp$ is the tangential one.

Combine the hydrostatic equilibrium condition and equation of state, one can calculate the structures of quark stars with and without QCD vacuum energy.

4. Numerical results

Based on the formulae presented in Section 2.1, the mass-radius relations for various index, n, can be calculated. We are applying the Runge–Kutta method of order 4 to solve the differential equations, until the density reaches the surface density. For the case in Section 2.2, we numerically calculate from the center to the surface and obtain then the mass and radius.

It is worth noting that the non-zero surface density of quark stars play an important role in the computation. This density should be determined by the behavior of the elementary strong interaction, and is then an uncertain parameter. In the calculation as following, we choose the surface density, $\rho_{\rm sur} = 1.5\rho_0$, where ρ_0 is the nuclear matter density.

In calculating the gravitational energy, we numerically integrate from the center to the surface for both cases with and without QCD vacuum energy.

4.1. Mass-radius relations for stars of perfect fluid

The mass-radius curves for both cases with and without QCD vacuum energy are shown in Fig. 2. It is evident from the calculation that the maximum mass of quark star decreases as the index, n, increases. This is understandable. A small n means a large Γ, and the pressure is relatively lower for higher values of n. Lower pressure should certainly support a lower mass of star.

It could have observational implications that the maximum mass of quark stars with polytropic equations of state are larger than that derived in conventional model. Recently the results of 19 years of Arecibo timing for two pulsars in the globular cluster NGC 5904 (M5) had been reported by Freire et al. [7]. They confirmed that for one of the binary pulsars (M5B) the mass is $2.08 \pm 0.19M_\odot$ in 1σ, and concluded that this mass for the pulsar would exclude most "soft" equations of state for dense neutron matter. However, a quark star in the polytropic model could be more massive than in previously derived realistic models (e.g. the MIT bag model) because of a stiffer equation of state, and the maximum mass could be larger than $2M_\odot$. It is worth emphasizing that the maximum mass of quark star depends on the value of K-parameter. Although in this paper we derive its minimum value under some assumptions, its real value is still uncertain. On this point of view, even the pulsars of masses larger than $1.4M_\odot$ have been observed, the case of $n = 3$ could not be ruled out because the value of K could be larger than the value we use.

On the gravitational stability. A polytropic star, with a state equation of $P \propto \rho^\Gamma$, supports itself against gravity by pressure, PR^2 (note: the stellar gravity $\propto M/R^2 \propto \rho R$). Certainly, a high pressure (and thus large Γ or small n) is necessary for a gravitationally stable star, otherwise a star could be unstable due to strong gravity. Actually, in the Newtonian gravity, a polytropic normal star (with $\rho_{\rm sur} = 0$) is gravitationally unstable if $n > 3$, but the star should be still unstable if $n = 3$ when the GR effect is included [24].

A polytropic quark star with non-zero surface density or with QCD vacuum energy, however, can still be gravitationally stable even if $n \geqslant 3$. A quark star with much low mass could be self-bound dominantly, and the gravity is negligible (thus not being gravitationally unstable). As the stellar mass increases, the gravitational effect becomes more and more significant, and finally the star could be gravitationally unstable when the mass increases beyond the maximum mass. In order to see the central density-dependence of stability, the calculated mass-central density curves are shown in Fig. 3.

Sound speed. From the $P - \rho$ relation in Section 2.1, we can derive the ratio of sound speed to speed of light, which cannot be greater than 1,

$$\left(\frac{v_s}{c}\right)^2 = \frac{dP}{d\rho} = \frac{n+1}{n}\,\frac{P}{\rho + P} \leqslant 1. \tag{24}$$

Similarly, also from the relation given in Section 2.2 we can come to

$$\left(\frac{v_s}{c}\right)^2 = \frac{dP}{d\rho} = \frac{n+1}{n}\,\frac{P+\Lambda}{\rho + P} \leqslant 1. \tag{25}$$

Both of the two inequations lead to

$$(1 - n^2)K\rho_g^{\frac{1}{n}} \leqslant nc^2. \tag{26}$$

The equation above holds if $n \geqslant 1$, that means that the causality keeps for $n \geqslant 1$.

4.2. Gravitational energy released during a star-quake

Based on various manifestations of pulsar-like stars, a solid state of cold quark matter was conjectured [30]. A solid stellar object would inevitably result in star-quakes when strain energy develops to a critical value, and a huge of gravitational and elastic energies would then be released. One way to accumulate both shear and bulk forces in a solid quark star is during an accretion process: strain develops remarkably in massive stars, for which the gravitational effect is not negligible. A solid star could additionally support the accreted matter against gravity by these forces, unless the forces become so strong that a star-quake occurs. This is the so-called *AIQ* (Accretion-Induced star-Quake) mechanism proposed previously [33,34], which might be responsible to the bursts (even the supergiant flares) and glitches observed in soft γ-ray repeaters/anomalous X-ray pulsars (see a recent review by [18]). In addition, the glitches of radio pulsars could also be the results of star-quakes [37].

How to calculate the energy released during an AIQ? Theoretically, anisotropic fluid stars could be introduced for, e.g. the presence of type 3A superfluid [13], different kind of phase transitions [27], and pion condensation [23]. For the general relativistic configurations, when the interactions between particles could be treated relativistically, the fluid could also be anisotropic [22]. Previous theoretical results for anisotropic fluid, in a simple case with spherical symmetry, could still be adaptable to estimate the AIQ-released energy of a solid star.

Based on the analytical study of anisotropic matter, Harko and Mak [8,15] discussed the constraints for the anisotropic parameter defined in Section 3, ε, and presented an exact analytical solution for the gravitational equations of a static spherically symmetric anisotropic quark matter star [16]. It is found that the ε-value could be as high as 10^{-2}. Applying the formulae in Sections 2.4 and 3, we can obtain the gravitational energy released during a star-quakes in both case with and without QCD vacuum energy, for example, with $n = 1$. The gravitational energy released for

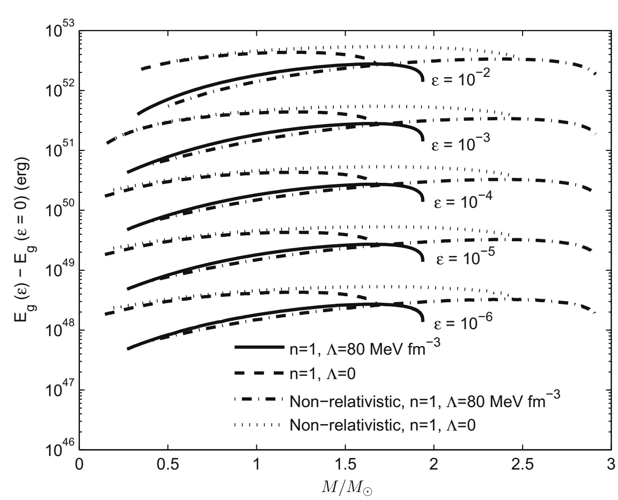

Fig. 4. The gravitational energy difference between stars with and without anisotropic pressures, which may be released during sequential star-quakes. Solid lines are $\Lambda = 80$ MeV fm^{-3}, dashed lines are for $\Lambda = 0$, dash-dotted lines are for non-relativistic case with $\Lambda = 80$ MeV fm^{-3}, and dotted lines are for non-relativistic case with $\Lambda = 0$.

quark stars of linear equation of state with two different vacuum energy has been calculated in Xu et al. [34]. We approximate that the rest energy (corresponds to the total baryon mass) E_{0g} does not change during a star-quake, $E_{0g} = M_0c^2$, since the released energy is much smaller than E_{0g}.

The gravitational energy difference between stars with $\varepsilon \neq 0$ and with $\varepsilon = 0$ are shown in Fig. 4. Three supergiant flares from soft γ-ray repeaters have been observed, with released photon energy being order of $\sim 10^{47}$ ergs. Our numerical results imply that for all the parameters we chosen, the released energy could be as high as the observed.

5. Conclusions and discussions

Because of the difficulty to obtain a realistic state equation of cold quark matter at a few nuclear densities, we have tried to apply polytropic equations of state to model quark stars in this paper. The polytropic equations of state to quark stars are studied in two separated cases: the vacuum inside and outside quark matter is the same or not. In addition, the quark-clustering could lead to the non-relativistic equation of state. The differences between the those cases could be significant in the mass-radius relations, and may be tested by observations. It could additionally provide a way to probe the properties of QCD vacuum.

The polytropic equations of state are stiffer than that derived in previous realistic models, so they could lead to more massive quark stars with masses $> 2M_\odot$. Consequently, even when some massive pulsars have been observed, it still can not rule out the possibility that pulsar-like stars are quark stars. Though a normal star with zero surface density can only be gravitationally stable if the polytropic index $n < 3$, a quark star with non-zero surface density could still be stable even if $n \geqslant 3$. A solid quark star may break if its strain energy develops to a critical value, and we calculate the gravitational energy released during quakes and find that the energy could be as high as $\sim 10^{47}$ ergs if the anisotropic parameter, ε, could be order of 10^{-6}. Such a huge of energy would be liberated during an AIQ (accretion-induced star-quake) process, to be probably responsible to the bursts and glitches observed in soft γ-ray repeaters/anomalous X-ray pulsars. The general relativity effect has been included to simulate the polytropic quark stars.

The nature of pulsars is unfortunately still a matter of controversy, even more than 40 years after the discovery of pulsar. Although quark stars cannot be ruled out, both theoretically and observationally, and pulsars are potential idea laboratories to study the elementary strong interaction, we are lacking a *general* framework in which theoretical stellar models could be tested by observations. Polytropic quark star model is the one we try to establish. Future advanced observations may help to constrain the uncertain parameters, e.g. the polytropic index n, the coefficient K, the surface density $\rho_{\rm sur}$, and even the vacuum energy Λ.

One of the daunting challenges nowadays is to understand the fundamental strong interaction between quarks, especially the QCD in the low-energy limit, since the coupling is asymptotically free in the limit of high-energy. The state of cold matter at a few nuclear densities is still an unsolved problem in the low-energy QCD. In effective QCD models, BCS-type quark pairing was proposed to form at a Fermi surface of cold quark matter, and the shear moduli of the rigid crystalline color super-conducting quark matter could be 20–1000 times larger than those of neutron star crusts [17]. However, quark-clusters are phenomenologically suggested to form in cold quark matter [30]. The state of such cold quark matter might be approximated by polytropic equations of state since one may draw naively an analogy between the clusters in quark matter and the nuclei in normal matter. Certainly, it would be very interesting to observationally distinguish between those two kinds of solid quark matter.

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