Braking PSR J1734–3333 with a possible fall-back disk *

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Abstract The very small braking index of PSR J1734–3333, $n = 0.9 \pm 0.2$, challenges the current theories of braking mechanisms in pulsars. We present a possible interpretation that this pulsar is surrounded by a fall-back disk and braked by it. A modified braking torque is proposed based on the competition between the magnetic energy density of the pulsar and the kinetic energy density of the fall-back disk. With this torque, a self-similar disk can fit all the observed parameters of PSR J1734–3333 with natural initial values of parameters. In this regime, the star will evolve to the region having anomalous X-ray pulsars and soft gamma repeaters in the $P - \dot{P}$ diagram in about 20 000 years and stay there for a very long time. The mass of the disk around PSR J1734–3333 in our model is about $10M_{\oplus}$, similar to the observed mass of the disk around AXP 4U 0142+61.

Key words: pulsars: individual (PSR J1734-3333) - stars: evolution - stars: neutron

1 INTRODUCTION

PSR J1734–3333 is a radio pulsar with a period of P = 1.17 s and a period derivative of $\dot{P} = 2.28 \times 10^{-12}$. In the $P - \dot{P}$ diagram, it is located between the normal radio pulsars and anomalous X-ray pulsars (AXPs) and soft gamma repeaters (SGRs). PSR J1734–3333 has been regularly observed since 1997 by using the 64 m telescope at Parkes and the 76 m telescope at Jodrell Bank. It has not glitched during these years, insuring accurate measurements of ν , $\dot{\nu}$ and $\ddot{\nu}$, where $\nu = 1/P$ is the pulsar rotation frequency, and $\dot{\nu}$ and $\ddot{\nu}$ are its first and second derivative, respectively. The braking index of a pulsar, $n \equiv \nu \ddot{\nu} / \dot{\nu}^2$, describes the dependence of the braking torque on rotation frequency. For PSR J1734–3333, $n = 0.9 \pm 0.2$, based on 13.5 years of observational data (Espinoza et al. 2011).

Because of observational difficulties, before PSR J1734–3333, reliable braking indices have only been published for seven pulsars (Lyne et al. 1993, 1996; Middleditch et al. 2006; Livingstone et al. 2007; Weltevrede et al. 2011). All of them have a value between 1 and 3 except PSR J0537–6910, which has glitched frequently (Middleditch et al. 2006). The value for braking index for PSR

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J1734–3333 is too small compared to that from spin-down models and the measured values of other pulsars. The case of pure magnetic braking with a constant dipolar magnetic field should have n = 3. Xu & Qiao (2001) considered the braking torques due to magnetodipole radiation and the unipolar generator, and predicted 1 < n < 3. Other factors may also affect the braking index, such as stellar wind, systematic variations in the moment of inertia or magnetic field, and non-dipolar braking (Manchester et al. 1985; Blandford & Romani 1988). Therefore, the small braking index of PSR J1734–3333 is a challenge to the current theory of braking mechanisms in pulsars.

From the definition of the braking index, one has $\dot{P} = kP^{2-n}$. When a pulsar evolves, its location on the $P - \dot{P}$ diagram will move according to n. The slope of the evolutionary path in the diagram is 2-n. If PSR J1734–3333 keeps the present value of braking index, it would dramatically evolve into the region that has AXPs and SGRs in $\sim 20\,000$ yr. Espinoza et al. (2011) suggested that PSR J1734–3333 may be a potential magnetar (Thompson & Duncan 1995, 1996), whose magnetic field has been buried under the surface due to large accretion shortly after the supernova explosion, and is relaxing out of the surface at present. The increase in the strength of the magnetic field may also result in the small value for braking index of 0.9. In this paper, we propose another interpretation for the small braking index: there may be a fall-back disk around PSR J1734–3333 which is braking the pulsar.

2 BRAKING PSR J1734-3333 BY A SELF-SIMILAR FALL-BACK DISK

2.1 The Fall-back Disk

It is reasonable to assume that all the mass could not be completely ejected during a supernova explosion and that a small amount of mass can fall back. Part of the fall-back material carries sufficient angular momentum and can rotate around the young neutron star, forming a fall-back disk. The mass of a fall-back disk is unclear, but could be larger than $10M_{\oplus}$ and should be smaller than $0.1M_{\odot}$ (Chevalier 1989; Lin et al. 1991; Wang et al. 2006). PSR J1734–3333 is a young pulsar with a characteristic age of $\tau_c \sim 8000$ yr. There may be a fall-back disk around it.

Fall-back disks are very difficult to detect due to their extremely weak emission. About half of the AXPs and SGRs have observed infrared/optical counterparts (Wang & Chakrabarty 2002; Israel et al. 2004, 2005; Mereghetti 2011a). Wang et al. (2006) observed the mid-infrared emission from a debris disk around AXP 4U 0142+61, the brightest known AXP. The disk mass is of the order of $\sim 10M_{\oplus}$. Kaplan et al. (2009) detected the counterpart to AXP 1E 2259+586 at 4.5 μ m, which can be explained by a model with a passive X-ray-heated dust disk that was developed for 4U 0142+61. From a theoretical point of view, the existence of disks around AXPs and SGRs can help to explain their distribution on the $P - \dot{P}$ diagram, e.g. the origin of SGR 0418+5729 with a dipolar magnetic field less than 7.5×10^{12} G (van der Horst et al. 2010; Rea et al. 2010; Alpar et al. 2011). It is possible that PRS J1734–3333 will evolve to be an AXP or SGR, and now has a disk around it that has not yet been detected.

When a fall-back disk forms around a pulsar, the accretion rate will evolve over time. Cannizzo et al. (1990) found that the accretion rate \dot{M} declines self-similarly according to $\dot{M} \propto t^{-\alpha}$, where $\alpha > 1$, when the disk is under the influence of viscous processes. We parameterize the accretion rate in the same way as Chatterjee et al. (2000), i.e.

$$M = M_0, \qquad 0 < t < T, \dot{M} = \dot{M}_0 (t/T)^{-\alpha}, \qquad t \ge T,$$
(1)

where \dot{M}_0 is a constant accretion rate. Chatterjee et al. (2000) supposed T is the dynamical time in the inner parts of the disk, which is about 0.001 s. Menou et al. (2001) pointed out that a more appropriate choice for T is instead the viscous timescale, which is much larger than the dynamical time. We note that the choice for T has no significant effect on the spin evolution of a pulsar if one supposes the same value for $\dot{M}_0 T^{\alpha}$. In this paper, we take T = 1000 s to be of the order of the viscous timescale. With these assumptions, the total or initial disk mass is

$$M_{\rm d,0} = \int_0^\infty \dot{M} dt = \frac{\alpha}{\alpha - 1} \dot{M}_0 T, \tag{2}$$

and the residual disk mass at time $t, t \ge T$, is

$$M_{\rm d}(t) = \int_t^\infty \dot{M} dt = \frac{\dot{M}_0 T^\alpha}{\alpha - 1} t^{1 - \alpha}.$$
(3)

In this model, super-Eddington accretion could occur in the early accretion phase. The initial values of parameters \dot{M}_0 and T are coupled, and they are insensitive to the results at a late stage. Cannizzo et al. (1990) found $\alpha = 19/16$ and 1.25 for a disk in which the opacity is dominated by electron scattering and for a Kramers' opacity, respectively. We choose $\alpha = 7/6$, which is very near the previously stated values and could simplify the calculations, as shown by Chatterjee et al. (2000). Our calculations show that small changes in the value of α do not significantly affect the results.

2.2 The Braking Torque

The fall-back disk will interact with the pulsar's magnetosphere, and influence the spin evolution of the pulsar, i.e.

$$I\dot{\Omega} = N,\tag{4}$$

where I and $\hat{\Omega}$ are the moment of inertia and angular velocity derivative of the pulsar, and N is the torque acting on the pulsar by the disk. To study the spin evolution of a pulsar in detail, we introduce two important radii: one is the corotation radius $R_{\rm co}$ where the corotation velocity equals the Keplerian velocity, and the other is the magnetospheric radius $R_{\rm m}$ inside which the accretion disk cannot exist. The corotation radius

$$R_{\rm co} = (GM_*/4\pi^2)^{1/3} P^{2/3} \approx 1.5 \times 10^8 M_1^{1/3} P^{2/3} \,\rm cm, \tag{5}$$

where G is the gravitational constant, M_* is the mass of the star, and M_1 is the mass of the star in units of solar mass. The magnetospheric radius (Ghosh & Lamb 1979)

$$R_{\rm m} \approx 0.52 r_{\rm A} = 8.7 \times 10^7 \mu_{30}^{4/7} M_1^{-1/7} \dot{M}_{18}^{-2/7} \,\rm cm, \tag{6}$$

where $r_A = \mu^{4/7} (2GM_*)^{-1/7} \dot{M}^{-2/7}$ is the Alfvén radius inside which the flow is dominated by the magnetic field, μ_{30} is the magnetic moment of the star in units of 10^{30} G cm³ (for a dipole magnetic field, $\mu = BR^3$) and \dot{M}_{18} is the accretion rate in units of 10^{18} g s⁻¹.

The sizes of $R_{\rm co}$ and $R_{\rm m}$ will change with the evolution of the pulsar and the disk, thereby the accretion process would undergo three successive phases: the accretion phase, the propeller phase and the tracking phase. At the early stage, when $R_{\rm m} < R_{\rm co}$, the accretion flow can fall onto the star's surface, and the gravitational energy is released. In the accretion phase, \dot{M} is usually larger than the Eddington limit ($\dot{M}_{\rm E} \approx 10^{18}$ g s⁻¹). In a spherically symmetric case with Eddington accretion, the star would spin down very quickly, braked by the strong coupling between the magnetic field and the ionized surroundings (Liu et al. 2012). In the case of a disk with Eddington accretion, the braking torque is still unclear. Fortunately, this stage is very short (in this paper, about several months to several years) and the spin period changes very little. Thus, we can simply ignore the effect of this stage and only suppose that the disk evolves self-similarly after the short initial accretion phase. The magnetospheric radius $R_{\rm m}$ increases faster than the corotation radius $R_{\rm co}$, and will exceed $R_{\rm co}$ after the accretion phase. When $R_{\rm m} > R_{\rm co}$, most of the accretion flow cannot reach the star's surface, and the material would be ejected out by the propeller effect (Illarionov & Sunyaev 1975). The

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propeller effect can brake the star to a relatively long spin period (typically several seconds), until $R_{\rm m}$ approximates $R_{\rm co}$ and the propeller torque fades away. $R_{\rm m}$ could never be equal to $R_{\rm co}$, but changes with $R_{\rm co}$. This is the tracking phase. During the whole evolution process, the propeller phase is the most important stage that brakes the pulsar.

The propeller effect between a rotating magnetic neutron star and the accreted material was first proposed by Illarionov & Sunyaev (1975) to explain why the number of galactic X-ray sources is much less than the expected number of neutron stars and black holes in binary systems. They suggested that most neutron stars in binary systems are in the propeller phase. The accreted material is thrown away from the neutron star by the rotating magnetosphere, thus the stars do not have a high enough X-ray luminosity to be detected. Manchester et al. (1995) found evidence for a spin-down of PSR B1259–63, driven by propeller torque in a binary system with a highly eccentric orbit. Nevertheless, the value of propeller torque is still unclear.

The braking torque may be directly calculated by integrating the azimuthal magnetic stress in the coupling region. However, this approach has to make some very strong assumptions because there are too many uncertainties (see Ertan & Erkut 2008; Çalişkan et al. 2013). The torque may also be indirectly estimated by calculating the rate of angular momentum loss removed by the ejected material. For example, if the disk material rotates with Keplerian angular frequency $\Omega_{\rm K}$ before it drops to $R_{\rm m}$, corotates with the star when it drops to $R_{\rm m}$ and is thrown away from the neutron star, it will take angular momentum away from the star, which acts as a braking torque (Menou et al. 1999; Chatterjee et al. 2000)

$$N = \dot{J} = 2\dot{M}R_{\rm m}^2\Omega_{\rm K}(R_{\rm m})\left[1 - \frac{\Omega}{\Omega_{\rm K}(R_{\rm m})}\right],\tag{7}$$

where J is the rate of angular momentum exchange and Ω is the angular frequency of the star. In this formula, the torque is zero when Ω equals $\Omega_{\rm K}(R_{\rm m})$, and would increase with the difference between Ω and $\Omega_{\rm K}(R_{\rm m})$ increasing. This is reasonable and feasible. However, according to this formula, the kinetic energy density of the corotating material at $R_{\rm m}$ would become much larger than the magnetic energy density there when $\Omega \gg \Omega_{\rm K}(R_{\rm m})$, which seems to contradict our general understanding of this situation. Considering both reasonable and perhaps unreasonable factors, we modify the propeller torque with an additional parameter χ , i.e.

$$N = 2\dot{M}R_{\rm m}^2\Omega_{\rm K}(R_{\rm m})\left\{1 - \left[\frac{\Omega}{\Omega_{\rm K}(R_{\rm m})}\right]^{\chi}\right\},\tag{8}$$

where the power exponent χ is smaller than one, i.e. $0 < \chi < 1$. This torque is more moderate than those with $\chi \ge 1$ (Chatterjee et al. 2000; Ertan & Erkut 2008). We expect torque with $0 < \chi < 1$ fits the observations better than that with $\chi \ge 1$.

2.3 The Results

The period evolution of a pulsar can be calculated using Equations (4) and (6), given the accretion history from Equation (1) and the torque formula (8). The magnetic dipole radiation torque is insignificant compared with the propeller torque, especially when it spins slowly. We thus ignore the magnetic dipole radiation in the following calculations.

Figure 1 shows the evolution paths in the $P - \dot{P}$ diagram of a neutron star surrounded by a fall-back disk, with initial values of parameters $P_0 = 50$ ms, $M_1 = 1.4$, T = 1000 s and $\alpha = 7/6$. We tried $\chi = 1/3$ and $\chi = 1/2$, whose paths can go across the location of PSR J1734–3333, and the braking indexes coincide with the observed value $n = 0.9 \pm 0.2$ (Espinoza et al. 2011). For comparison, the case of $\chi = 1$ (Chatterjee et al. 2000) and the case of $\chi = 2$ (Ertan & Erkut 2008) are also calculated and shown in Figure 1, in which the braking indexes cannot be equal to the



Fig. 1 Evolution paths of a neutron star surrounded by a fall-back disk with different propeller torques, given initial values of parameters $P_0 = 50$ ms, T = 1000 s and $\alpha = 7/6$. To cross the location of PSR J1734–3333 in the diagram, different values of χ need different values of disk mass and magnetic field strength. The fitting parameters and main results are given in Table 1. The two short lines near the star indicate the observed upper and lower limits of the braking index of PSR J1734–3333. The information about radio pulsars is taken from the ATNF pulsar catalogue (Manchester et al. 2005, *http://www.atnf.csiro.au/research/pulsar/psrcat/*), data of AXPs and SGRs from the McGill SGR/AXP Online Catalog (*http://www.physics.mcgill.ca/~pulsar/magnetar/main.html*) and X-ray Dim Isolated Neutron Stars from Mereghetti (2011b).

observed value of PSR J1734–3333. As Figure 1 shows, the propeller phase can be divided into three stages according to the slope of the evolution path in the $P - \dot{P}$ diagram. In the first stage, the path goes down quickly until the slope increases from very negative to zero. In the second stage the path goes up, where the slope increases from zero to a value larger than 1, i.e. n < 1, and decreases to zero at the end. In the last stage the path goes down again, and the slope keeps decreasing, until the neutron star evolves to the tracking phase (Chatterjee et al. 2000). The fitting parameters and main results are given in Table 1. All the parameters and results smoothly vary with χ .

It is difficult to judge whether $\chi = 1/2$ or $\chi = 1/3$ is the most likely solution, though it seems that $\chi = 1/3$ fits the braking index better, because the real disk and the braking torque are unlikely to be as simple as described in the models. Nevertheless, the torques with $0 < \chi < 1$ fit the observations much better than those with $\chi \ge 1$. This agrees with our expectation when we propose modifying the torque by adding χ .



Fig. 2 Period evolution of a neutron star with $P_0 = 50$ ms surrounded by a disk with different parameters given in Table 1.



Fig. 3 Evolution paths of a neutron star with different magnetic field strengths and disk masses, which are braked by a disk torque with $\chi = 1/3$. The same parameters are $P_0 = 50$ ms, $M_1 = 1.4$, T = 1000 s and $\alpha = 7/6$. Left panel: $M_{d,0} = 525M_{\oplus}$; Right panel: $\mu_{30} = 5.8$.

Figure 2 shows the period evolution with the different parameters given in Table 1. The period of a neutron star surrounded by a fall-back disk could evolve to several seconds in several tens of thousands of years and could stay in this period range for a very long time.

Figure 3 shows the evolution paths of a pulsar with different magnetic field strengths and initial disk masses, where $\chi = 1/3$. The paths are similar and change smoothly with the parameters. In the second stage, the slopes of the paths vary little and are all similar to the observations of PSR J1734–3333, which means that the braking indexes of pulsars in the propeller phase with self-similar fall-back disks are all about 1.

χ	μ_{30}	$M_{\mathrm{d},0}$ (M_{\oplus})	P (s)	$\dot{P} \\ (10^{-12})$	n	t (kyr)	$M_{\rm d}(t)$
1/3 1/2 1 2	5.8 4 1.7 0.8	525 292 117 93	1.17 1.17 1.17 1.17	2.28 2.28 2.28 2.28 2.28	0.97 1.10 1.52 2.25	34.2 30.2 22.9 13.4	14 8 3.3 2.9

Table 1 Fitting Parameters and Main Results

Notes: μ_{30} and $M_{d,0}$ are the only two free fitting parameters. P, \dot{P}, n, t and $M_d(t)$ are the fitted results when the pulsar passes the location of PSR J1734–3333.

3 CONCLUSIONS AND DISCUSSION

In this paper we proposed a modified formula for the propeller torque considering the energy densities of the magnetosphere and the accreted material, which was motivated by previous works (Menou et al. 1999; Chatterjee et al. 2000; Alpar 2001; Ertan & Erkut 2008). With such a torque, a self-similar fall-back disk can make the braking index of a pulsar very small, which fits the observed P, \dot{P} and nof PSR J1734–3333 very well. The results are stable and insensitive to initial values of parameters.

Shortly after we presented our work on arXiv (arXiv:1211.4185), Çalişkan et al. (2013) presented another fall-back disk solution for the small braking index of PSR J1734–3333. However, the torque used in their paper was doubtable. Ertan & Erkut (2008) obtained the torque based on at least two assumptions: the disk material extends to the corotation radius R_{co} and the azimuthal pitch of the magnetic field in the region between R_{co} and r_A is constant, which are somewhat unreasonable. As a result, Çalişkan et al. (2013) needs a small disk whose mass is much smaller than the mass of the disk around AXP 4U 0142+61 (Wang et al. 2006) to fit the observed parameters of PSR J1734– 3333, and the fitting is very sensitive to the initial conditions (see fig. 4 of Çalişkan et al. (2013) and Fig. 3 of this paper for comparison).

As shown in Table 1, the initial disk masses are smaller than $0.1M_{\odot}$ which agrees with Chevalier (1989) and Lin et al. (1991). When PSR J1734–3333 evolves into the AXP and SGR region about 20 000 years later, the disk mass would be of the order of $10M_{\oplus}$, similar to the observed disk mass around AXP 4U 0142+61 (Wang et al. 2006). The propeller torque of a fall-back disk would modify the period derivative, which makes the apparent dipole magnetic field strength much stronger than the real field strength. Our model implies that the magnetic field strength of PSR J1734–3333 is only about 10^{12} G, much smaller than the value expected from pure magnetic braking.

As Figures 1 and 3 show, the evolution paths sharply go down in the last stage, when the pulsar evolves into the region of AXPs and SGRs in the $P - \dot{P}$ diagram. At that time, the inner disk is very close to the corotation radius, which makes it be possible that a large part of the material forming the inner disk passes through the corotation radius and falls onto the surface of the neutron star. Thus the star may act as an AXP then, just like Chatterjee et al. (2000) and Alpar (2001) have suggested. If this pulsar really becomes an accretion powered AXP in the future, the braking torque will be more complex than the torque used in our model, thus it will be more difficult to predict the evolutionary path of a pulsar in the region of AXPs and SGRs. We leave this issue to future work since this paper is mainly devoted to explaining the small braking index.

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